

Observer-based Robust Finite-time Cooperative Consensus Control for Multi-agent Networks

Suiyang Khoo, Lihua Xie
School of Electrical and Electronic Engineering
Nanyang Technological University
Singapore.
khooyang@yahoo.com, ELHXIE@ntu.edu.sg

Zhihong Man, Shengkui Zhao
School of Electrical and Computer Systems Engineering
Monash University
Selangor, Malaysia.
man.zhihong@eng.monash.edu.my, zhaoshengkui9@yahoo.com

Abstract—This paper studies the finite-time consensus tracking control for multi-agent networks. The time-varying control input and the velocity of the leader is unknown to any follower. Only the position of the leader is known to its neighbors. We first propose a new finite-time multiple-surface sliding mode observer to estimate the leader's velocity. It is seen that the estimation error of the observer can converge to zero in a finite time. Then, we prove that finite-time consensus tracking of multi-agent networks can be achieved on a new terminal sliding mode surface. Simulation results are presented to validate the analysis.

Index Terms—Cooperative consensus control; Sliding mode control; Finite-time observer; Finite-time convergence

I. INTRODUCTION

Cooperative consensus control of multi-agent networks poses significant theoretical and practical challenges. First, the research objective is to develop a system of subsystems rather than a single system. Second, the communication bandwidth and connectivity of the team are often limited, and the information exchange among agents may be unreliable. Third, arbitration between team goals and individual goals needs to be negotiated. Fourth, the computational resources of each individual agent will always be limited [1]. In recent years, there has been an increasing research interest in the consensus control design of multi-agent networks [2-10].

In reality, the velocity and the time-varying control input of the active leader in a multi-agent network may not be able to be measured. Furthermore, the network topology might be directed, and the time required for the network to reach consensus is finite. The objective of this paper is to address the following issues: (i) How to design finite-time observer for multi-agent networks such that the velocity of the leader can be estimated in a finite time. (ii) Under what conditions, a non-smooth control algorithm can be developed to guarantee the directed leader-follower multi-agent network to reach consensus in a finite time. (iii) How to design this finite-time control algorithm systematically. Our interest in these three issues is motivated by the preliminary research work in [11] on finite-time consensus design for first order systems, the work in [12] on finite-time consensus for second order systems with undirected communication topology, and the work in [13], [14] on integral observer design for leader-follower multi-agent systems.

In this paper, we show that for leader-follower multi-agent network dominated by second order systems, it is possible to design a finite-time multiple-surface sliding mode observer [15], [16] to estimate the leader's velocity and this estimated information can be used to design a new terminal sliding mode (TSM) surface such that finite-time consensus can be achieved on this TSM surface [17-19]. This conclusion is proved based on the Lyapunov theory for finite-time stability. Our proposed control scheme is robust to system uncertainties and input disturbances. Since not all followers in the network with directed communication topology have directed communication with the leader, our results assume that, the agents in the network only need to communicate with their neighbors and not the entire community.

The remainder of this paper is organized as follows. Section 2 reviews some concept of graph theory and Lyapunov theory for finite-time stability. The new multiple-surface sliding mode observer is proposed in Section 3. In Section 4, the new TSM surface for multi-agent system is proposed and the design of the observer-based robust finite-time consensus control (ORFCC) algorithm to guarantee finite-time consensus tracking of multi-agent network is discussed in detail. Section 5 gives numerical examples to illustrate our results. Concluding remarks are given in Section 6.

II. BACKGROUND AND PRELIMINARIES

In this section, we introduce some basic concepts in algebraic graph theory for multi-agent networks and review some terminologies related to the notion of finite-time stability and the corresponding Lyapunov stability theory.

A. Concepts in graph theory and multi-agent systems

Consider a multi-agent system consisting of one leader and n followers. To solve the coordination problems and to model the information exchange between agents, graph theory is introduced here. Let $\mathcal{G} = \{\mathcal{V}, \mathcal{E}\}$ be a directed graph, where $\mathcal{V} = \{1, 2, \dots, N\}$ is the set of nodes, node i represents the i th agent, \mathcal{E} is the set of edges, and an edge in \mathcal{G} is denoted by an ordered pair (i, j) . $(i, j) \in \mathcal{E}$ if and only if the i th agent can send information to the j th agent directly, but not necessarily vice versa. In contrast to a directed graph, the pairs of nodes in an undirected graph are unordered, where the edge

(i, j) denotes that agent i and j can obtain information from each other. Therefore, an undirected graph can be viewed as a special case of a directed graph. A directed tree is a directed graph, where every node has exactly one parent except for the root, and the root has a directed path to every other node. A directed spanning tree of \mathcal{G} is a directed tree that contains all nodes of \mathcal{G} [24].

$A = (a_{ij}) \in \mathfrak{R}^{n \times n}$ is called the weighted adjacency matrix of \mathcal{G} with nonnegative elements where $a_{ii} = 0$ and $a_{ij} \geq 0$ with $a_{ij} > 0$ if there is an edge between the i th agent and the j th agent. Let $D = \text{diag}\{d_1, \dots, d_n\} \in \mathfrak{R}^{n \times n}$ be a diagonal matrix, where $d_i = \sum_{j=1}^n a_{ij}$ for $i = 1, \dots, n$. Then the Laplacian of the weighted graph can be defined as

$$L = D - A. \quad (1)$$

The connection weight between the i th agent and the leader is denoted by b_i with $b_i > 0$ if there is an edge between the i th agent and the leader. The following theorems present the existing results on Laplacian matrix and graph theory.

Theorem 1: [25] The Laplacian matrix L of a directed graph $\mathcal{G} = \{\mathcal{V}, \mathcal{E}\}$ has at least one zero eigenvalue and all of the nonzero eigenvalues are in the open right half plane. Further, L has exactly one zero eigenvalue if and only if \mathcal{G} has a directed spanning tree.

Theorem 2: [1] The directed graph $\mathcal{G} = \{\mathcal{V}, \mathcal{E}\}$ has a directed spanning tree if and only if $\{\mathcal{V}, \mathcal{E}\}$ has at least one node with a directed path to all other nodes.

In this paper, we assume that the leader is active, in the sense that its state keeps changing throughout the entire process [14]. In general the behavior of the leader is independent of the followers. The dynamics of the leader is described as follows:

$$\begin{aligned} \dot{x}_0 &= v_0, & x_0 &\in \mathfrak{R}^m, \\ \dot{v}_0 &= u_0, & v_0 &\in \mathfrak{R}^m, \end{aligned} \quad (2)$$

where x_0 is the position and v_0 is the velocity of the leader. The dynamics of the i th follower agent is described by

$$\begin{aligned} \dot{x}_i &= v_i, & x_i &\in \mathfrak{R}^m, \\ \dot{v}_i &= u_i + \delta_i, & v_i &\in \mathfrak{R}^m, \end{aligned} \quad i = 1, \dots, n, \quad (3)$$

where δ_i represents the disturbance and $u_i (i = 1, \dots, n)$, the control inputs. For further analysis, we assume that $\|\delta_i\| \leq D < \infty$.

B. Lyapunov theory for finite-time stability

Here, we recall some Lyapunov theorem for finite-time stability of nonlinear systems, which was discussed previously in [26], [27]. The classical Lyapunov stability theory is only applicable to a differential equation whose solution from any initial condition is unique [28]. A well-known sufficient condition for the existence of a unique solution of a nonlinear differential equation $\dot{x} = f(x)$ is that the function $f(x)$ is locally Lipschitz continuous. The solution of such nonlinear differential equation can have at most asymptotic convergence rate.

Since finite-time stability guarantees that every system state reaches the system origin in a finite time, finite-time stability has a much stronger requirement than asymptotic stability. The following theorem presents the sufficient conditions for finite-time stability.

Theorem 3: [29] Consider the non-Lipschitz continuous nonlinear system $\dot{x} = f(x)$ with $f(0) = 0$. Suppose there are C^1 function $V(x)$ defined on a neighbourhood of the origin, and real numbers $c > 0$ and $0 < \alpha < 1$, such that

- (1) $V(x)$ is positive definite,
- (2) $\dot{V}(x) + cV^\alpha \leq 0$.

Then, the origin is locally finite-time stable, and the settling time, depending on the initial state $x(0)$, satisfies

$$T(x(0)) \leq \frac{V(x(0))^{1-\alpha}}{c(1-\alpha)}, \quad (4)$$

for all $x(0)$ in some open neighborhood of the origin.

III. HIGH GAIN MULTIPLE-SURFACE SLIDING MODE OBSERVER

Let us consider the following second order system

$$\begin{aligned} \dot{x} &= v, \\ \dot{v} &= u. \end{aligned} \quad (5)$$

Based on the concepts of multiple-surface sliding mode controller [15], [16] and observer design [20 – 23], we first define the 2 sliding mode variables as

$$s_1 = x - \hat{x}, \quad (6)$$

$$s_2 = v - \hat{v}, \quad (7)$$

where \hat{x} and \hat{v} represent the state estimation of x and v , respectively. Then the observer can be designed as:

$$\begin{aligned} \dot{\hat{x}} &= \hat{v} + \hat{u}_1 \\ \dot{\hat{v}} &= \hat{u}_2 \end{aligned} \quad (8)$$

with

$$\begin{aligned} \hat{u}_1 &= k_1 \text{sign}(s_1) + c_1 s_1^\alpha, \\ \hat{u}_2 &= k_2 \text{sign}(\hat{u}_1) + c_2 \hat{u}_1^\alpha, \\ k_1 &\geq |s_2|, k_2 \geq |u|, \\ c_1 &> 0, c_2 > 0, \\ 0 &< \alpha < 1. \end{aligned} \quad (9)$$

Theorem 4: Considering system (5). If the observer is designed as in (6–9), then for any initial conditions $(x(0), v(0))$, the states \hat{x} and \hat{v} converge toward the states x and v , respectively in a finite time.

Proof: (sketch) Consider the following Lyapunov function

$$\hat{V}_1 = 1/2s_1^2. \quad (10)$$

Differentiating \hat{V}_1 with respect to time, we have

$$\begin{aligned} \dot{\hat{V}}_1 &= s_1 \dot{s}_1 \\ &\leq -2c_1 \hat{V}_1^{\frac{\alpha+1}{2}}. \end{aligned} \quad (11)$$

Since $0 < \frac{\alpha+1}{2} < 1$, by Theorem 3, s_1 converges to zero in a finite time. On this first sliding mode surface,

$$\hat{u}_1 = s_2. \quad (12)$$

Take the Lyapunov function

$$\hat{V}_2 = 1/2s_2^2 \quad (13)$$

and consider the time derivative of \hat{V}_2 :

$$\begin{aligned} \dot{\hat{V}}_2 &= s_2 \dot{s}_2 \\ &\leq -2c_2 \hat{V}_2^{\frac{\alpha+1}{2}}. \end{aligned} \quad (14)$$

By Theorem 3, s_2 converges to zero in a finite time. On the second sliding mode surface,

$$\hat{v} = v, \quad (15)$$

$$\hat{u}_2 = u. \quad (16)$$

■

Remark 1: Not only the velocity, but also the time-varying control input of the leader and the follower can be reconstructed using the proposed observer.

IV. OBSERVER-BASED ROBUST FINITE-TIME CONSENSUS TRACKING ALGORITHM

It is seen in Section 2 that the network considered here consists of $n+1$ agents where an agent indexed by 0 acts as the leader and the other agents indexed by $1, \dots, n$, are referred to as the followers. The topology relationships among the leader and followers is described by a directed graph $\mathcal{G} = \{\mathcal{V}, \mathcal{E}\}$ with $\mathcal{V} = \{0, 1, \dots, n\}$ and the adjacent matrix

$$A = \begin{bmatrix} 0 & 0 & \dots & 0 \\ a_{10} & a_{11} & \dots & a_{1n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n0} & a_{n1} & \dots & a_{nn} \end{bmatrix} \in \mathfrak{R}^{(n+1) \times (n+1)}. \quad (17)$$

Denote $\bar{\mathcal{G}} = \{\bar{\mathcal{V}}, \bar{\mathcal{E}}\}$ as the subgraph of \mathcal{G} , which is formed by the n followers, where

$$\bar{A} = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{bmatrix} \in \mathfrak{R}^{(n) \times (n)}. \quad (18)$$

In this paper, for simplicity, we assume that

$$a_{ij} = \begin{cases} 1 & \text{if agents } i \text{ and } j \text{ are connected,} \\ 0 & \text{otherwise.} \end{cases} \quad (19)$$

Meanwhile, the connection weight between agent i and the leader is denoted by \bar{B} where

$$\bar{B} = \text{diag}\{b_1, b_2, \dots, b_n\} \quad (20)$$

such that

$$b_i = \begin{cases} 1 & \text{if agent } i \text{ is connected to the leader,} \\ 0 & \text{otherwise.} \end{cases} \quad (21)$$

For further analysis, we have the following assumption.

Assumption 1: The position of the leader, x_0 is available to its neighbors only.

Using Theorem 1 and Theorem 4, we are able to prove the following theorem that state under what condition, consensus can be reached in a finite-time.

Theorem 5: If the directed graph \mathcal{G} has a directed spanning tree and multiple sliding mode variables (6) and (7) reached the multiple sliding mode surfaces, then there exist a terminal sliding variable vector and a nonsingular TSM control law for the leader-follower system (2) and (3) such that on the TSM surface, consensus can be reached in a finite-time.

Proof: Let $\mathcal{E}_1 \triangleq [e_1^1, \dots, e_n^1]^T$ and $\mathcal{E}_2 \triangleq [e_1^2, \dots, e_n^2]^T$, and by defining

$$e_i^1 \triangleq \sum_{j=1}^n a_{ij}(x_i - x_j) + b_i(x_i - x_0), \quad (22)$$

$$e_i^2 \triangleq \sum_{j=1}^n a_{ij}(v_i - v_j) + b_i(v_i - \hat{v}_0), \quad (23)$$

the error dynamics of the interconnection graph can be expressed as:

$$\begin{aligned} \dot{\mathcal{E}}_1 &= \mathcal{E}_2 \\ \dot{\mathcal{E}}_2 &= (\bar{L} + \bar{B})U + (\bar{L} + \bar{B})\delta + \bar{B}1u_0. \end{aligned} \quad (24)$$

Then, by designing the terminal sliding variable as

$$s_i = e_i + (e_i^2)^\alpha, \quad \text{for } i = 1, \dots, n, \quad (25)$$

the terminal sliding variable vector can be written as:

$$S = \mathcal{E}_1 + \mathcal{E}_2^\alpha. \quad (26)$$

One can simply choose the control input as

$$U = [u_1^0, \dots, u_n^0]^T + [u_1^1, \dots, u_n^1]^T, \quad (27)$$

with

$$u_i^0 = - \left(\sum_{j=1, j \neq i}^n a_{ij} + b_i \right)^{-1} \frac{(e_i^2)^{2-\alpha}}{\alpha}, \quad (28)$$

$$\begin{aligned} u_i^1 &= - \left(\sum_{j=1, j \neq i}^n a_{ij} + b_i \right)^{-1} \left[\sum_{j=1, j \neq i}^n a_{ij} (-u_j) \right. \\ &\quad \left. + (2nD + b_i \bar{u}_0 + \kappa_1) \text{sign}(s_i) \right], \end{aligned} \quad (29)$$

$$\kappa_1 > 0, \quad (30)$$

and this results in

$$\begin{aligned} U &= - [\bar{D} + \bar{B}]^{-1} \left[\frac{(\mathcal{E}_2)^{2-\alpha}}{\alpha} - \bar{A}U \right. \\ &\quad \left. + (\text{diag}(2nD + \kappa_1) + \bar{B}1\bar{u}_0) \text{sign}(S) \right]. \end{aligned} \quad (31)$$

Consider the Lyapunov function $V_1 = \frac{1}{2}S^T S$. Using (24), (26),

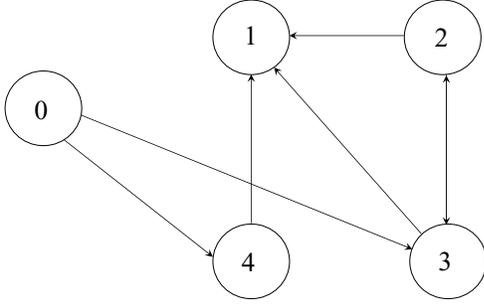


Fig. 1. Information-exchange topology among leader and followers

and (31), a simple computation gives

$$\begin{aligned}
\dot{V} &= S^T \left\{ \mathcal{E}_2 + \alpha \text{diag}(\mathcal{E}_2^{\alpha-1})[(\bar{L} + \bar{B}) \right. \\
&\quad \left. \times (U + \delta) - \bar{B}\mathbf{1}u_0] \right\} \\
&= S^T \left\{ \mathcal{E}_2 + \alpha \text{diag}(\mathcal{E}_2^{\alpha-1}) \left[-\frac{(\mathcal{E}_2)^{2-\alpha}}{\alpha} - \left(\text{diag}(2n\mathcal{D} \right. \right. \right. \\
&\quad \left. \left. \left. + \kappa_1) + \bar{B}\mathbf{1}\bar{u}_0) \right) \text{sign}(S) + (\bar{L} + \bar{B})\delta - \bar{B}\mathbf{1}u_0 \right] \right\} \\
&\leq -\alpha \sum_{k=1}^n |s_k| (e_k^2)^{\alpha-1} (2n\mathcal{D}) - \alpha \sum_{k=1}^n |s_k| (e_k^2)^{\alpha-1} (\kappa_1) \\
&\quad -\alpha \sum_{k=1}^n |s_k| (e_k^2)^{\alpha-1} (b_k \bar{u}_0) + \alpha \sum_{k=1}^n |s_k| (e_k^2)^{\alpha-1} (n\mathcal{D}) \\
&\quad + \alpha \sum_{k=1}^n |s_k| (e_k^2)^{\alpha-1} (n\mathcal{D}) + \alpha \sum_{k=1}^n |s_k| (e_k^2)^{\alpha-1} (b_k \bar{u}_0) \\
&= -\alpha \sum_{k=1}^n |s_k| (e_k^2)^{\alpha-1} (\kappa_1). \tag{32}
\end{aligned}$$

which implies that the TSM surface $S = 0$ can be reached in a finite time.

We claim that on this new TSM surface, consensus tracking of multi-agent system can be reached in finite time. To prove this claim, consider the Lyapunov function $V_2 = 1/2\mathcal{E}_1^T \mathcal{E}_1$. On the TSM surface,

$$\mathcal{E}_2 = -\mathcal{E}_1^{\frac{1}{\alpha}}, \tag{33}$$

it follows that

$$\begin{aligned}
\dot{V}_{\mathcal{E}_1} &= -\mathcal{E}_1^T \mathcal{E}_1^{\frac{1}{\alpha}} \\
&\leq -2^{\frac{1+\alpha}{2\alpha}} \left(V_{\mathcal{E}_1} \right)^{\frac{1+\alpha}{2\alpha}}. \tag{34}
\end{aligned}$$

By Theorem 3,

$$\lim_{t \rightarrow T} |\mathcal{E}_1| = 0, \quad \lim_{t \rightarrow T} |\mathcal{E}_2| = 0. \tag{35}$$

After $\mathcal{E}_1 = 0$, with $e_i^1 \triangleq \sum_{j=1}^n a_{ij}(x_i - x_j) + b_i(x_i - x_0)$ in

mind, it is easy to see that

$$(\bar{L} + \bar{B}) \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix} = \bar{B}\mathbf{1}x_0. \tag{36}$$

Since $\bar{L}\mathbf{1} = 0$, we have

$$\begin{aligned}
(\bar{L} + \bar{B}) \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix} &= (\bar{L}\mathbf{1} + \bar{B}\mathbf{1})x_0 \\
&= (\bar{L} + \bar{B})\mathbf{1}x_0. \tag{37}
\end{aligned}$$

This implies that

$$\begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix} = \mathbf{1}x_0, \tag{38}$$

and thus, consensus is reached in a finite time. \blacksquare

V. NUMERICAL EXAMPLES

This section presents some simulation results to illustrate the performance of the proposed ORFCC algorithms. Here, we consider one leader indexed by 0 and four followers indexed by 1, 2, 3, and 4, respectively. Suppose that the leader dynamics is

$$\begin{aligned}
\dot{x}_0 &= v_0 \\
\dot{v}_0 &= u_0, \tag{39}
\end{aligned}$$

and the dynamics of i th follower is described as follows:

$$\begin{aligned}
\dot{x}_i &= v_i \\
\dot{v}_i &= u_i + 0.01 \sin(x_i), \quad i = 1, 2, 3, 4. \tag{40}
\end{aligned}$$

Let the initial condition of the four followers indexed by 1, 2, 3, and 4 be $x_1(0) = 1$, $x_2(0) = 1.2$, $x_3(0) = 2$, and $x_4(0) = -1.2$, respectively. Suppose that the information-exchange topology among the agents is given by Fig.1, where the information of leader is available only to followers 3 and 4. Note that follower 4 has no directed path to all other followers, but there exist a directed path from the leader to all followers. The adjacent matrix of the graph can be written as

$$A = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix}. \tag{41}$$

The Laplacian of the follower system can be written as:

$$\bar{L} = \begin{bmatrix} 3 & -1 & -1 & -1 \\ 0 & 1 & -1 & 0 \\ -1 & -1 & 2 & 0 \\ -1 & 0 & 0 & 1 \end{bmatrix}, \tag{42}$$

and the diagonal matrices for the interconnection relationship between the leader and the followers is

$$B = \text{diag}(0 \ 0 \ 1 \ 1). \tag{43}$$

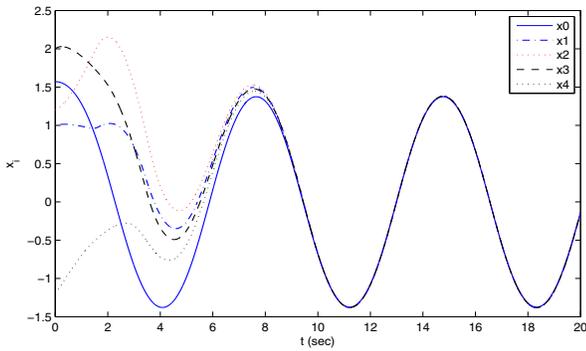


Fig. 2. Position tracking of four followers with input disturbances

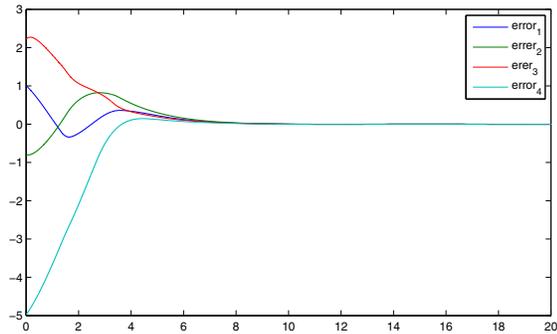


Fig. 3. Tracking error of four followers with input disturbances

The simulation results are obtained with the time-varying control input to the leader be designed as:

$$u_0 = -\frac{\sin(x_0)}{1 + \exp^{-t}}, \quad (44)$$

where $x_0 = \frac{\pi}{2}$ and $v_0 = 0$. Fig. 2 and fig. 3 show the results of the proposed ORFCC algorithm in (26–29). It is seen that the followers can track the leader in a finite-time under the noisy condition.

VI. CONCLUSIONS

This paper has presented an observer-based robust finite-time consensus control scheme for leader-follower multi-agent systems. A new high gain multiple-surface sliding mode observer is proposed. Based on the observer's estimated results, a new terminal sliding mode surface is proposed for the system to ensure finite-time consensus under the condition that only the neighbors have access to the leader and these followers might not have a directed path to all other followers. The velocity and the control input of the leader is assumed unknown to any followers. Simulation results have validated the analysis. The proposed ORFCC algorithm can be easily applied to practical control of multi-robot systems.

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