

# Josephson effect in an atomic Fulde-Ferrell-Larkin-Ovchinnikov superfluid

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We study theoretically two spatially separate quasi-one-dimensional atomic Fermi gases in a double-well trap. By tuning independently their spin polarizations, a Fulde-Ferrell-Larkin-Ovchinnikov (FFLO) superfluid or a Bardeen-Cooper-Schrieffer (BCS) superfluid may be formed in each well. We seek the possibility of creating a spatially tunable atomic Josephson junction between two superfluids, which is supposed to be realizable via building a weak link at given positions of the double-well barrier. We show that within mean-field theory the maximum Josephson current is proportional to the order parameter in two wells. Thus, the spatial inhomogeneity of the FFLO order parameter in one well may be directly revealed through the current measurement with the position-tunable link. We anticipate that this type of Josephson measurement can provide useful evidence for the existence of exotic FFLO superfluids. Possible experimental realizations of the Josephson measurements in atomic Fermi gases are discussed.

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## I. INTRODUCTION

Strongly attractive Fermi gases with imbalanced spin components are ubiquitous systems in diverse fields of physics [1]. They are the building-blocks of atomic nuclei, the matter in neutron stars, and even the quark-gluon plasma that comprised the early Universe. Imbalanced Fermi gases also appear in solid-state superconductors subjected to either an internal exchange field or external magnetic field. A recent example that attracted intense attentions is the trapped atomic gases of neutral fermions with unequal or polarized spin populations [2,3]. Owing to the flexibility in the control of the constituents and interaction strengths, atomic Fermi gases provide the most promising place for observing many exotic forms of matter.

The ground state of polarized Fermi gases remains elusive [4]. The mismatched Fermi surfaces in a polarized environment cannot guarantee the standard Bardeen-Cooper-Schrieffer (BCS) mechanism, which requires a pairing of two fermions on the same Fermi surface with opposite spins. Various exotic forms of pairing have been suggested [5–9], such as the Fulde-Ferrell-Larkin-Ovchinnikov (FFLO) state with spatially varying order parameters [5], deformed Fermi surface [6], interior gap [7] or Sarma superfluidity [8], and phase separation [9].

Among these, the FFLO state is of particular interest since the Cooper pairs may condense into a state with a finite center-of-mass momentum. The search for the FFLO state has lasted for more than four decades in many branches of physics. In the condensed-matter community, experimental evidences of its existence have been reported in the heavy fermion superconductor CeCoIn<sub>5</sub>. The observations include the specific heat, magnetization, and penetration depth measurements [10]. Recent theoretical studies suggest that such a phase is more favorable in the low-dimensional systems [11–32]. Particularly, it dominates in the quasi-one-dimensional (1D) polarized gases [12,13]. Following these suggestions, most recently, strong evidence for the FFLO superfluid has been found in ultracold atoms at Rice University [33] by trapping a two-component mixture of ultracold <sup>6</sup>Li atoms in an array of 1D tubes. At temperatures  $T \sim 0.1T_F$ , where  $T_F$  is the Fermi

temperature, the measured density profiles exhibit a partially polarized core surrounded by wings composed of either a completely paired BCS superfluid or a fully polarized normal gas, in excellent agreement with the theoretical predictions given by Orso [12] and by the present authors [13].

However, the Rice experiment was done with an intermediate interaction strength, where the 1D binding energy is much larger than the Fermi energy. Therefore, more accurate measurements are required at weaker interactions (and hence lower temperatures), together with a new definitive identification scheme for the FFLO order parameter. These should be based on phase-sensitive measurements that can directly reveal the spatial variations of the phase of the order parameter. One possibility is the Josephson effect [34].

In this work we propose an *atomic Josephson effect* to detect the existence of the exotic FFLO superfluids. We consider two spatially separate quasi-1D atomic Fermi gases with tunable spin polarizations in a tight double-well potential, where the lateral motion of fermions is frozen, while axial motion is weakly confined. A weak link at a specific position  $x_0$  may be created by superimposing a narrow dipole dimple potential to allow tunneling. Figure 1 presents a schematic view of the configuration, together with the potential and particle density profiles. Its possible realization will be addressed later.

The underlying physics of our proposal is easily understood using Ginsburg-Landau (GL) theory [35]. Assuming that the order parameters or condensate wave functions in the left and right wells are described by  $\Psi_{\text{BCS}}(x)$  and  $\Psi_{\text{FFLO}}(x)$ , respectively, the Josephson current by GL theory is

$$I_J = \text{Im} \left[ J \int_{x_0 - \Delta x/2}^{x_0 + \Delta x/2} dx \Psi_{\text{BCS}}^*(x) \Psi_{\text{FFLO}}(x) \right], \quad (1)$$

where  $J$  is the characteristic tunneling amplitude and is determined by the small overlap of the condensate wave functions along the lateral direction.  $\Delta x$  is the width of the narrow tunneling link at position  $x_0$ . The BCS order parameter in Eq. (1) is essentially spatially independent, while the FFLO one is oscillatory in real space with period  $2\pi\hbar/q_{\text{FFLO}}$ , where  $q_{\text{FFLO}}$  is the center-of-mass momentum.

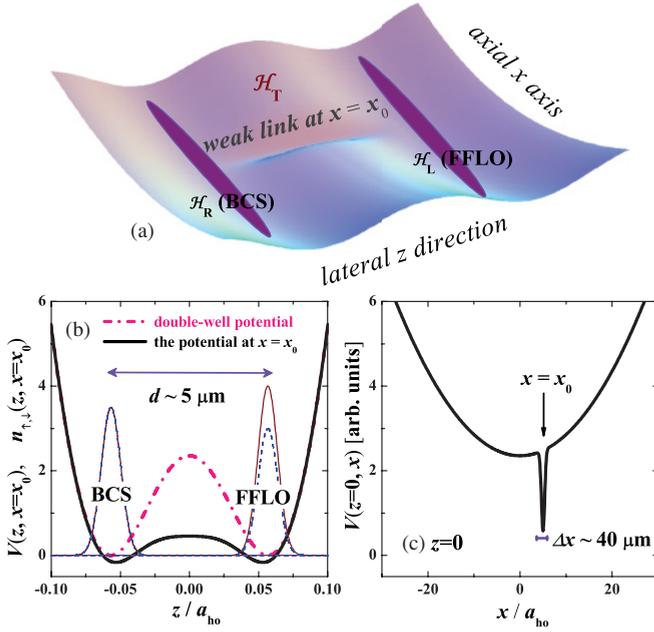


FIG. 1. (Color online) Schematic view of a proposed atomic Josephson junction in quasi-1D atomic Fermi gases with a number of total atoms  $N \sim 10^3$ . (a) The double-well potential landscape. Two needle-like (BCS and FFLO) superfluids are located in the left and right wells, respectively. Reducing the double-well barrier at a specific position  $x = x_0$  by superimposing a narrow dipole potential, the Cooper pairs in superfluids can tunnel back and forth between wells, leading to a Josephson current. (b) Lateral distributions of the potential (red thick dot-dashed line) and spin-up (red thin solid line) and spin-down (blue dashed line) particle density profiles away from the weak link at  $x_0$ . The interwell distance in experiments would be about  $5 \mu\text{m}$ . The potential with superposition of a dimple potential is shown by the black thick solid line. Then the particle densities and the order parameters can have appreciable overlaps within the weak link. The length scale  $a_{\text{ho}} = \sqrt{\hbar/m\omega}$  is around  $40 \mu\text{m}$  for  ${}^6\text{Li}$  atoms, where  $\omega$  is the trapping frequency in the axial direction. (c) Axial profile of the “harmonic” plus “dimple” potential at  $z = 0$ .

Thus, provided that  $\Delta x \ll 2\pi\hbar/q_{\text{FFLO}}$ , the measurement of the maximum Josephson current results directly  $\Psi_{\text{FFLO}}(x = x_0)$ . By displacing axially the harmonic traps and consequently changing the position of weak link  $x_0$ , a series of measurements therefore reveal the whole spatial inhomogeneity of the FFLO order parameter. As shown in Fig. 2, the simple GL picture is verified by very complicated microscopic calculations, which will be outlined in detail in the following.

Further manipulation of temperature, number of total atoms, or interaction strengths via a Feshbach resonance may lead to an *atomic superfluid-normal junction*. In this case, the difference in chemical potentials between wells, resulting from the different total number of atoms, plays the role of the voltage. Therefore, analogous to the differential conductance measurements in superconductors [35], the derivative of single-particle tunneling currents with respect to the number difference provides a direct measurement of the density of states (DOS) of superfluids. We show that the characteristic two-energy-gap structure in the DOS of the FFLO state can be clearly determined, giving an independent means of identifying its existence.

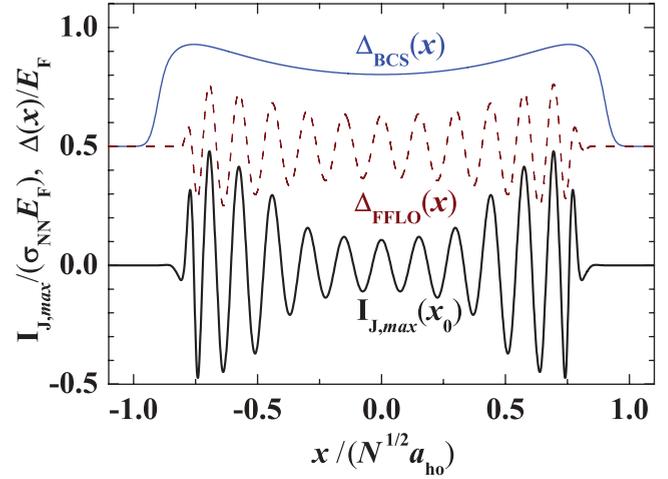


FIG. 2. (Color online) Underlying physics of the Josephson effect, as a probe of the exotic FFLO superfluids. With a BCS and a FFLO superfluid placed on the left and right sides of the junction, respectively, the calculated maximum Josephson current is roughly proportional to the FFLO order parameter. We have chosen the number of fermions on each well  $N_L = N_R = 128$ , compared to the realistic number of  $N \sim 10^3$ . The interaction strengths are  $\gamma_L = \gamma_R = 1.6$ . The order parameter is normalized by the Fermi energy  $E_F = N_L\hbar\omega/2$ , and is shifted upward by an amount of  $0.50E_F$  for clarity. The spin polarization in the FFLO superfluid is 0.25.  $\sigma_{NN}$  is the conductance of a corresponding normal junction. See, for example, the text in Sec. III A.

Our results are obtained by solving mean-field Bogoliubov-de Gennes (BdG) equations for each well, while treating the tunneling through the weak link within linear response theory. Our calculations are performed specifically for atomic Fermi gases. However, as the FFLO physics is a fundamental issue that is of importance to many research fields it can have potential implications beyond ultracold atoms.

## II. THEORETICAL MODEL

We assume that the dipole dimple potential is a small perturbation and the resulting weak link at  $x_0$  does not disturb considerably the distributions of the order parameter and particle density profile in each well. The atomic Josephson junction in Fig. 1(a) then is well described by a tunneling Hamiltonian. By integrating out the lateral degree of freedoms in a tight-binding approximation, it takes three terms

$$\mathcal{H} = \mathcal{H}_L + \mathcal{H}_T + \mathcal{H}_R, \quad (2)$$

$$\mathcal{H}_T = V_0 \sum_{\sigma} [\psi_{L\sigma}^{\dagger}(x_0)\psi_{R\sigma}(x_0) + \text{H.c.}], \quad (3)$$

where  $\sigma = \uparrow, \downarrow$  is the spin index. The terms  $\mathcal{H}_L$  and  $\mathcal{H}_R$  are, respectively, the Hamiltonians for fermions on the left and right sides of the junction and can be expressed in terms of the operator  $\psi_{L\sigma}$  and  $\psi_{R\sigma}(x)$ . They contain all the many-body interactions. Assuming that the width  $\Delta x$  is the smallest length scale, in  $\mathcal{H}_T$  we approximate all operators  $\psi(x) \approx \psi(x_0)$  and introduce a transfer parameter  $V_0 = J\Delta x$ . This is valid as far as  $\Delta x \ll 2\pi\hbar/q_{\text{FFLO}}$ . We shall take a small constant transfer parameter, which corresponds to the small overlap of two order

parameters. The overlap could depend weakly on the position of the weak link. However, the assumption of a fixed transfer parameter is sufficient to capture the qualitative feature of the Josephson effect.

In each well the ground state of an attractive gas of  $N = N_\uparrow + N_\downarrow$  fermions with polarization  $P = (N_\uparrow - N_\downarrow)/N$  is conveniently determined by using the BdG formalism [35] that describes the quasiparticle wave functions  $u_\eta(x)$  and  $v_\eta(x)$  with a contact interaction  $g$  (the well index is suppressed for clarity),

$$\begin{bmatrix} \mathcal{H}_\uparrow^0 - \mu_\uparrow & \Delta(x) \\ \Delta^*(x) & -\mathcal{H}_\downarrow^0 + \mu_\downarrow \end{bmatrix} \begin{bmatrix} u_\eta(x) \\ v_\eta(x) \end{bmatrix} = E_\eta \begin{bmatrix} u_\eta(x) \\ v_\eta(x) \end{bmatrix}, \quad (4)$$

where  $\mathcal{H}_{\uparrow,\downarrow}^0 = -\hbar^2 \nabla^2 / 2m + m\omega^2 x^2 / 2 + g_{1d} n_{\uparrow,\downarrow}(x)$  is the single particle Hamiltonian under the axial harmonic trap and Hartree potential. The chemical potentials are shifted as  $\mu_{\uparrow,\downarrow} = \mu \pm \delta\mu$  to account for the unequal population  $N_{\uparrow,\downarrow}$ . The order parameter  $\Delta(x)$  and  $\mu_{\uparrow,\downarrow}$  are calculated by self-consistency equations for the gap,  $\Delta(x) = g_{1d} \sum_\eta u_\eta(x) v_\eta^*(x) f(E_\eta)$ , and for the densities  $n_\uparrow(x) = \sum_\eta |u_\eta(x)|^2 f(E_\eta)$  and  $n_\downarrow(x) = \sum_\eta |v_\eta(x)|^2 f(-E_\eta)$ , with  $f(x) = 1/(\exp[x/k_B T] + 1)$  being the Fermi function. These must be constrained so that  $\int dx n_{\uparrow,\downarrow}(x) = N_{\uparrow,\downarrow}$ . We note that the unequal chemical potentials break the time-reversal symmetry. Thus, the sum over energy levels is done for all the eigenstates with both positive and negative energies  $E_\eta$ . Generally, the interaction strength is parameterized by a dimensionless coupling constant  $\gamma = -mg_{1d}/(\hbar^2 n_0)$ , where  $n_0$  is the center density of an ideal gas. In the weak or intermediate coupling regimes (i.e.,  $\gamma \lesssim 10^0$ ), the mean-field BdG theory appears to be very accurate. More rigorous treatment should include the pair fluctuations beyond mean field [36–40]. Figure 3 shows the mean-field results of density profiles of a gas at  $\gamma = 1.6$  and  $P = 0.05, 0.25$ , compared

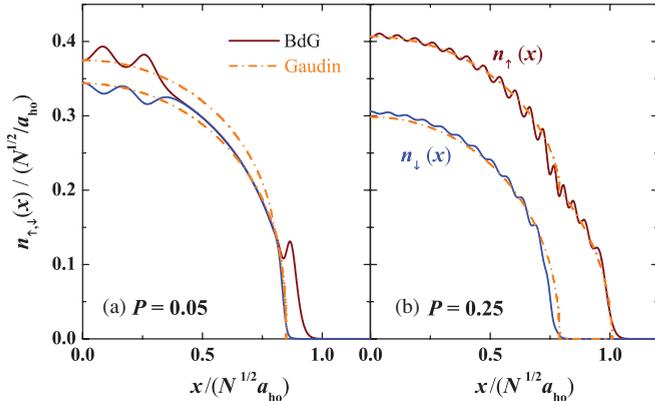


FIG. 3. (Color online) Zero temperature mean-field density profiles (solid lines) of a polarized gas at  $N = 128$  and  $\gamma = 1.6$  compared with the results from Gaudin solutions (dot-dashed lines) [13,16]. The oscillation in the density profiles, the so-called Friedel oscillation, is due to finite size effect. It becomes negligibly small for a large enough number of atoms. In contrast, the oscillation in the order parameter (scaled by the Fermi energy), the unambiguous signature of the FFLO phase, is robust with respect to the change of the number of total atoms. We refer to Sec. IV A in Ref. [20] and Sec. III A in Ref. [21] for a more detailed discussion of the Friedel oscillation.

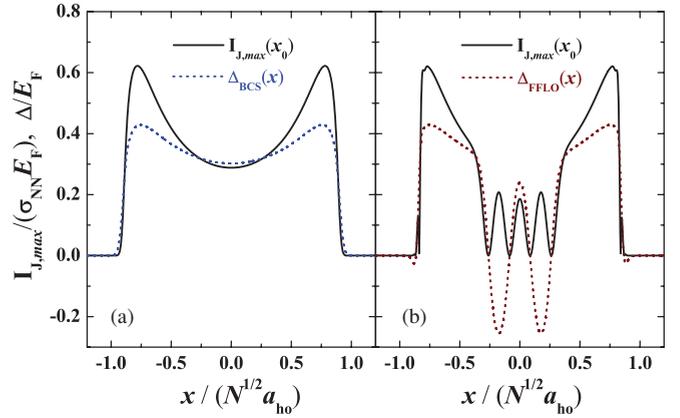


FIG. 4. (Color online) Maximum Josephson current of a junction with identical superfluids of either (a) BCS or (b) FFLO type, as a function of the position of the weak link. The spin polarization in (b) is 0.05. The order parameters (dashed lines) are plotted to emphasize their similarity to the current (i.e.,  $I_{J,max} \propto |\Delta(x_0)|$ ). The other parameters are the same as in Fig. 2.

with that obtained from exact Gaudin solutions and local density approximation [13,16]. The agreement is reasonable. Both theories predict two-shell structures with a partially polarized superfluid at the trap center and either a fully paired (small  $P$ ) superfluid or a fully polarized (large  $P$ ) normal state at the edge. The mean-field order parameters at these polarizations are given in Fig. 4(b) (for  $P = 0.05$ ) and Fig. 2 (for  $P = 0.25$ ), respectively. Their spatial variation identifies clearly that the partially polarized phase at center is indeed a FFLO superfluid.

### III. JOSEPHSON EFFECT AND SINGLE-PARTICLE TUNNELING

The main observable of interest, the rate of transferred atoms from, for example, the right well to left well, is defined by  $I(t) = \langle d\hat{N}_L(t)/dt \rangle$ . In analogy to superconductors where the flow of electrons out of the superconductor establishes an electrical current, we call  $I$  the current. By rewriting the transfer Hamiltonian  $\mathcal{H}_T = \sum_\sigma (A_\sigma + A_\sigma^\dagger)$  where  $A_\sigma = \psi_{L\sigma}^\dagger(x_0)\psi_{R\sigma}(x_0)$ , the equation of motion leads to  $d\hat{N}_L(t)/dt = i[\mathcal{H}_T(t), \hat{N}_L(t)] = i \sum_\sigma [A_\sigma(t) - A_\sigma^\dagger(t)]$ . Bearing in mind that the link at  $x_0$  is weak so that the transfer of atoms can be treated as a perturbation, we use the linear response theory [41] in which the current  $I(t) = -i \int_{-\infty}^t dt' \langle [d\hat{N}_L(t)/dt, \mathcal{H}_T(t')] \rangle_0 = \sum_{\sigma\sigma'} \int_{-\infty}^t dt' \langle [A_\sigma(t) - A_\sigma^\dagger(t), A_{\sigma'}(t') + A_{\sigma'}^\dagger(t')] \rangle_0$ . Here the subscript 0 in the average refers to the unperturbed systems  $\mathcal{H}_L$  and  $\mathcal{H}_R$ . Two contributions can be easily identified: the normal single-particle current and the Josephson current of Cooper pairs. The explicit expression of the latter is given by [41]  $I_J(t) = \exp[i(\varphi_R - \varphi_L) + i2(\mu_R - \mu_L)t/\hbar] \sum_\sigma \int_{-\infty}^t dt_1 \exp[-i(\mu_{R,-\sigma} - \mu_{L,-\sigma})(t - t_1)/\hbar] \langle [\tilde{A}_\sigma(t), \tilde{A}_{-\sigma}(t_1)] \rangle_0 + c.c.$ , where we introduce an interaction representation with respect to  $\mathcal{H}_L$  and  $\mathcal{H}_R$ , and represent it by a tilde in the operators. The global phases of order parameters  $\varphi_L$  and  $\varphi_R$  are made explicit in  $I_J(t)$ . Their difference, together with the factor  $2(\mu_R - \mu_L)t/\hbar$ , drives the

direct- and/or alternating-current Josephson currents even at the zero chemical potential difference between wells.

### A. Josephson tunneling

We first concentrate on the Josephson tunneling with the same number of particles in each well, for which the single-particle tunneling is blocked. With the help of the Wick theorem, in the statistical average of  $I_J(t)$  we split the four fermionic field operators. The integration of the average over time  $t_1$  can then be expressed in terms of the retarded correlation functions [41],

$$\chi_{\downarrow\uparrow}(\Omega) = \sum_{ij} (u_i v_i^*)_L (u_j^* v_j)_R \frac{[f(E_i) - f(E_j)]}{+\Omega + E_i - E_j}, \quad (5)$$

$$\chi_{\uparrow\downarrow}(\Omega) = \sum_{ij} (u_i v_i^*)_L (u_j^* v_j)_R \frac{[f(E_i) - f(E_j)]}{-\Omega + E_i - E_j}, \quad (6)$$

where the indices  $i$  and  $j$  refer to, respectively, the energy levels in the left and right wells, and the subscripts  $L$  and  $R$  are the indices for wells. We have abbreviated  $u = u(x_0)$  and  $v = v(x_0)$ . Consequently, the Josephson current can be calculated as [41]

$$I_J(t) = I_J^{\max} \sin[(\varphi_R - \varphi_L) + 2(\mu_R - \mu_L)t/\hbar], \quad (7)$$

where the maximum current

$$I_J^{\max} = \frac{2V_0^2}{\hbar} [\chi_{\downarrow\uparrow}(\mu_{R\downarrow} - \mu_{L\downarrow}) + \chi_{\uparrow\downarrow}(\mu_{R\uparrow} - \mu_{L\uparrow})]. \quad (8)$$

The Josephson current thus oscillates in phase with a peak value  $I_J^{\max}$ . It is clear from the expressions (5) and (6) that correlation functions become roughly a product of two order parameters if we approximate the gap equation  $\Delta(x) \propto \sum_{\eta} u_{\eta}(x) v_{\eta}^*(x)$ . This particular structure emphasizes the microscopic origin of the GL theory of the Josephson effect.

A simple expression for  $I_J^{\max}$  can be derived when we assume identical uniform BCS superfluids in both wells,  $I_J^{\max} = (\pi/2)\sigma_{NN}\Delta$ , where  $\sigma_{NN} = (V_0^2/\pi)[2m/(\hbar^3 E_F)]$  may be viewed as the conductance of a normal junction, with  $E_F$  being the Fermi energy. Therefore the value of the gap can be determined by measuring  $I_J^{\max}$ . In the presence of traps or inhomogeneous superfluids, one has to resort to the numerical calculations. Figure 4 presents the maximum Josephson currents for identical BCS or FFLO superfluids in both wells. Roughly we find  $I_{J,\max} \propto |\Delta(x_0)|$ . A more promising scheme is provided in Fig. 2, where a BCS superfluid is set in the left well as a reference system, while the order parameter of a FFLO superfluid in the right well is to be determined. As a result of the flat distribution of the BCS order parameter, we anticipate  $I_J^{\max} \propto \Delta_{\text{FFLO}}(x_0)$ , which is confirmed numerically. Therefore, the spatial inhomogeneity of FFLO phases can be precisely detected by the Josephson effect at varying positions of the weak link. This constitutes the main result of the present paper.

### B. Single-particle tunneling

We evaluate next the single-particle tunneling current. Again, by the use of correlation functions, one ends up with a

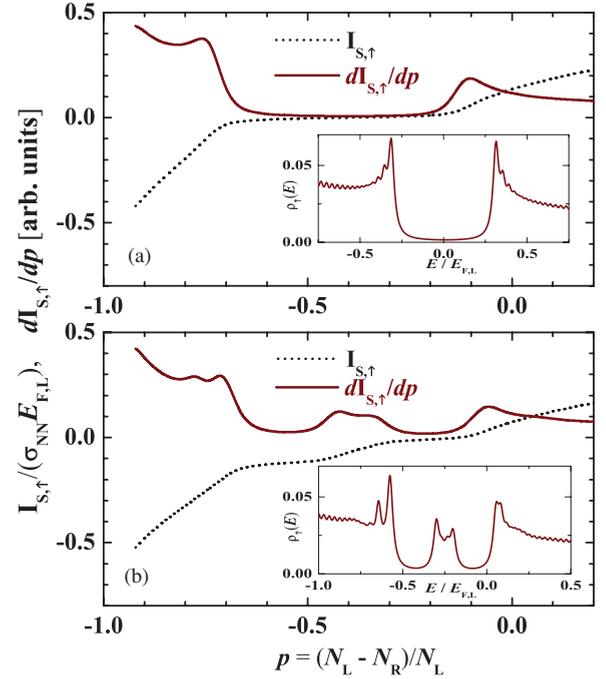


FIG. 5. (Color online) Spin-up single-particle current and its derivative as a function of the imbalance between wells. An unpolarized normal gas with varying number of particles  $N_R$  is set in the right well. We put in the left well a (a) BCS superfluid or (b) a FFLO superfluid at polarization 0.05, at  $N_L = 128$  and  $\gamma_L = 1.6$ .

familiar expression [41]  $I_S = I_{S,\uparrow}(\mu_{R\uparrow} - \mu_{L\uparrow}) + I_{S,\downarrow}(\mu_{R\downarrow} - \mu_{L\downarrow})$ , where

$$I_{S,\sigma} = \frac{2\pi V_0^2}{\hbar} \int_{-\infty}^{+\infty} d\epsilon \rho_{L\sigma}(\epsilon + \Omega) \rho_{R\sigma}(\epsilon) [f(\epsilon) - f(\epsilon + \Omega)],$$

and  $\rho_{\uparrow}(\epsilon) = \sum_{\eta} |u_{\eta}(x_0)|^2 \delta(\epsilon - E_{\eta})$  and  $\rho_{\downarrow}(\epsilon) = \sum_{\eta} |v_{\eta}(x_0)|^2 \delta(\epsilon + E_{\eta})$  are the spin-up and spin-down DOS at the position of the weak link. We are interested in the superfluid-normal junction, where the DOS of the well in a normal state is essentially a constant. Thus, the derivative of currents with respect to the chemical potential difference or number difference between wells provides a direct measurement of the DOS in the superfluid well. This is a phenomenon reminiscent of the scanning tunneling microscopy; here the role of the tip is played by the normal gas in one well. Figure 5 predicts such measurements for BCS and FFLO superfluids at the trap center  $x_0 = 0$ . All the features in the spin-up DOS of superfluids (insets in the figure) are faithfully recovered in the differential conductance  $dI_{S,\uparrow}/dp$ , which is calculated by changing the relative number difference between wells  $p = (N_L - N_R)/N_L$ . In particular, the midgap state or two-energy-gap structure in the FFLO DOS, a salient feature due to the spatially modulated order parameter, is clearly visible in  $dI_{S,\uparrow}/dp$ . This presents an independent check of the existence of FFLO phases.

## IV. POSSIBLE EXPERIMENTAL REALIZATIONS OF THE ATOMIC JOSEPHSON JUNCTION

We are now in a position to discuss the experimental realization of the atomic Josephson effect. The major experimental

challenge is the reach of a quasi-1D fermionic superfluid at the lowest experimentally accessible temperature, which is about  $0.05T_F$ . To have reasonable superfluid transition temperatures, a feasible way of tuning the interatomic interactions is required, such as Feshbach resonances. On the other hand, an optical lattice may be used to effectively reduce the dimensionality of atomic Fermi gases.

Thus, we start from a fermionic  ${}^6\text{Li}$  gas with a number of total atoms  $N \sim 10^3$  in an optical dipole trap, which was realized recently by the Hulet group at Rice University [3]. The optical trap is highly elongated, with an aspect ratio of radial to axial trapping frequencies  $\omega_{\perp}/\omega \sim 50$  [3]. Then, we consider the superposition of a deep periodic potential along the radial (lateral) direction. A double-well configuration will be formed. By suitably choosing the depth and periodicity of the optical lattice, the aspect ratio may increase to several hundreds or up to a thousand. This technique has already been applied successfully to create a bosonic Josephson junction for  ${}^{87}\text{Rb}$  gases with a similar number of total atoms, but in a much less anisotropic optical trap [42]. Next, a narrow weak link between wells may be built up by adding a tight dipole dimple microtrap [43] at a specific position. The position of the link could be easy to displace. We note that, prepared in this way, the typical interwell distance will be about several micrometers. For such a short distance, it is difficult to manipulate independently the spin polarization in each well by a radio-frequency sweep [2,3]. In this respect, the Josephson measurement between two FFLO superfluids, as shown in Fig. 4(b), seems to be more feasible, compared to the proposal outlined in Figs. 1 and 2.

We turn to estimate some realistic experimental parameters. To observe the Josephson oscillations, it is necessary to fulfill two conditions: (i) the number of fermions involved in the oscillations, to be measured by phase-contrast imaging [2,3] should be large enough to be easily detected, but small enough to ensure the validity of the linear response theory; and (ii) the width of the weak link  $\Delta x$  should be much smaller than the period of FFLO order parameter  $2\pi\hbar/q_{\text{FFLO}}$ .

Typically, the total number of atoms in one well would be around  $N \sim 10^3$ , and the frequency of axial trap  $\omega \sim 2\pi \times 1$  Hz, which is much smaller than the radial frequency  $\omega_{\perp} \sim 2\pi \times 10^3$  Hz so that the quasi-1D condition  $N\omega \leq \omega_{\perp}$  nearly holds [44]. The tunneling barrier is on the order of  $\hbar\omega_{\perp}$ . Further, we select the total spin polarization  $P = 0.05$ . For  ${}^6\text{Li}$  atoms with these parameters we find  $2\pi\hbar/q_{\text{FFLO}} \sim 2\pi/(k_{\uparrow} - k_{\downarrow}) \sim 2\pi/(N^{1/2}P)a_{\text{ho}} \sim 160 \mu\text{m}$ , where  $a_{\text{ho}} = \sqrt{\hbar/m\omega} \sim 40 \mu\text{m}$  is the harmonic oscillator length along the axial direction. This FFLO period is much larger than the width of dimple potential  $\Delta x$  that is about several ten micrometer, and therefore the condition  $\Delta x \ll 2\pi\hbar/q_{\text{FFLO}}$  can be well satisfied. Roughly, at the trap center there are about 100 fermions in each FFLO period. Choosing  $\Delta x \sim 40 \mu\text{m}$ , we expect about 25 particles, on average, on one side of the tunneling window in each well.

The maximum Josephson current  $I_{J, \text{max}}$  depends critically on the weak link at position  $x_0$ . To estimate it, we quote the parameters from previous Josephson measurements for bosonic  ${}^{87}\text{Rb}$  atoms,  $I_J \sim 5 \times 10^4 \text{ s}^{-1}$  in Ref. [42] and  $I_J \sim 3 \times 10^6 \text{ s}^{-1}$  in Ref. [45]. Due to the use of a narrow dimple potential, the maximum Josephson current in our configuration should be much reduced. It is not unreasonable

to estimate  $I_{J, \text{max}} \sim 10^3 \text{ s}^{-1}$ , whose value is two or three orders of magnitude smaller than that of bosonic Josephson junctions [42,45]. On the other hand, assuming a time scale of oscillation  $\sim 0.05 \text{ s}$  [42], the number of transferred fermions is about 50, an order of magnitude smaller than the total number of atoms. This number may be within the present experimental detection limits (i.e., the Josephson oscillation in a Bose-Einstein condensate with a similar number of atoms and time scale has already been observed [42]).

It is worth noting that the proposed atomic Josephson experiment requires a series of measurements (shots) with varying position of the weak link. Thus, the experiment relies on a good repeatability from shot to shot. For example, the nodes and antinodes of the oscillating FFLO order parameter should always occur at the same point in the cloud. However, the node of the FFLO wave functions may be sensitive to some experimental imperfections such as fluctuations in number imbalance and in temperature. This would wash out the FFLO signal of the measurement.

We check in Fig. 6 how the FFLO signal is affected by the shot to shot fluctuations in atom numbers at zero temperature. As a concrete example, we consider a BCS superfluid with a fixed number of atoms in the left well and a FFLO superfluid in the right well. The atom number in the FFLO superfluid is allowed to vary within a certain range. The curve in Fig. 6 shows the maximum Josephson current after averaging over 16 configurations. We find that our scheme is robust if the number fluctuation is less than 10%, within the experimental resolution of measuring the atom number. In particular, we find that the FFLO oscillation at the trap center persists up to the 30% fluctuation in atom numbers.

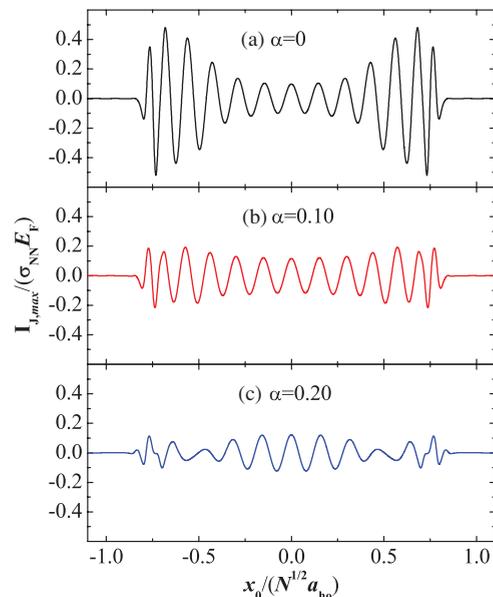


FIG. 6. (Color online) Averaged maximum Josephson current at several shot to shot number fluctuations,  $\alpha = 0, 0.10$ , and  $0.20$ . The BCS superfluid in the left well has a fixed atom number  $N_L = N = 128$ , while the FFLO superfluid in the right well is subjected to a number fluctuation, with which the atom number  $N_R$  is allowed to vary in the range  $[N(1 - \alpha), N(1 + \alpha)]$ . The two wells have the same interaction strength set by  $\gamma_L = 1.6$ . The spin polarization in the FFLO superfluid is fixed to  $p = 0.25$ .

We finally discuss two issues concerning the temperature and interactions. (i) In contrast to the three-dimensional (3D) case, the 1D FFLO state is notably stable in response to a nonzero temperature. In magnitude the critical temperature of the FFLO state is at the same order (i.e., a half or one third) of its unpolarized counterpart  $T_{\text{BCS}} \sim 4.54e^{-\pi^2/2\gamma} T_F$ , where  $T_F$  is the Fermi temperature. Given an intermediate interaction  $\gamma = 1.6$ , we estimate a critical temperature  $T_{\text{FFLO}} \sim 0.10T_F$  at the trap center, which is well above the lowest temperature reported so far. We anticipate a lower transition temperature inside the weak link because of reduced density. However, it can be much enhanced by increasing  $\gamma$ . (ii) In practice,  $g_{1d}$  is parameterized by a 3D scattering length  $a_{3d}$ ,  $g_{1d} = 2\hbar^2\omega_{\perp}a_{3d}/(1 - Aa_{3d}/a_{\perp})$ , where  $a_{\perp} = \sqrt{\hbar/m\omega_{\perp}}$  and  $A \simeq 1.0326$ . The denominator indicates a confinement-induced Feshbach resonance that occurs when  $a_{3d} \sim a_{\perp}$  [46,47], as observed experimentally [48]. For  ${}^6\text{Li}$ , using a Feshbach resonance the 3D scattering length  $a_{3d}$  and hence the dimensionless coupling constant  $\gamma$  can be changed at will.

## V. CONCLUSION

In summary, we have proposed a scheme to realize the atomic Josephson junction to detect the possible existence of the exotic FFLO superfluids in quasi-1D atomic Fermi gases. Though there are several difficulties for realizing such a junction configuration, considering the rapid developments in cold-atom experiments, we anticipate that they will be overcome in the near future. Our proposal opens the possibility for creating ultracold Fermi gases for practical applications, such as precision measurement and interferometry.

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