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2-D IIR Filter Bank Design by An LMI Approach

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***Abstract*—This paper is concerned with the problem of the two-channel 2-D IIR filter bank design. Using a systematic optimization approach, which is solved in terms of an LMI, the 2-D IIR synthesis filters are designed such that the systems are alias free and then optimized to achieve approximate perfect reconstruction.**

***Index Terms*—2-D IIR filter banks, linear matrix inequality (LMI), state-space representation.**

I. Introduction

One-dimensional (1-D) filter banks have extensive applications include coding and compression of images and video data [4], [9]. Activities in the area of Multidimensional (M-D) filter banks started in 1980s [5]. 2-D FIR filter banks have been extensively studied in the literature because these filter banks are always stable and have an exact linear phase response [2], [12]. In comparison with 2-D FIR filter banks, 2-D IIR filter banks usually need a lower filter order to meet the same magnitude specifications, although these filter banks is more complicated, and there are also design methods for these filter banks [1], [8]. In [1], parameterizations in frequency domain terms as well as in terms of state space descriptions of filters are provided for designing 2-D IIR filter banks. In particular, many studies have been done on 2-D FIR and IIR perfect reconstruction (PR) filter banks using a two-channel filter bank design [1], [2], [3], [6], [7], [8]. For example, a technique to design FIR filter banks using transformations of variables is presented in [7], and [8] extends the transformation of variables technique to IIR filters. Very simple design methods are given both for FIR and IIR cases in [6].The two-channel 2-D FIR and IIR perfect reconstruction problem using Gröbner bases is given in [2], [3]. Recently, an LMI approach for the design of 2-D FIR filter banks is considered in [11].

In this paper, we study the two-channel 2-D IIR filter banks for the diamond filter bank and the quadrant filter bank. Using a systematic optimization approach, which is solved in terms of an LMI, two synthesis filters are designed such that the systems are alias free and then optimized to achieve approximate perfect reconstruction. The advantage of this method is that it can guarantee the stability and causality for the designed synthesis filters.

First of all, we describe the 2-D two-channel IIR filter banks problem and preliminaries in the section 2. The method for designing synthesis filters in terms of an LMI and a corresponding algorithm are presented in section 3. In section 4, an example is provided to demonstrate the applicability of the proposed methods. Finally, we give the conclusions in section 5.

Notation. Throughout this paper, for Hermitian matrices X and Y, the notation $X \geq Y$ (respectively, $X > Y$) means that the matrix $X - Y$ is positive semi-definite (respectively, positive definite). I is the identity matrix with appropriate dimension. The superscript "T" represents the transpose and the complex conjugate transpose. For the sake of easing the notation of partitioned symmetric matrices, we use an asterisk $(*)$ to represent each of its symmetric blocks. Matrices, if not explicitly stated, are assumed to have compatible dimensions.

II. Problem Formulation and Preliminaries

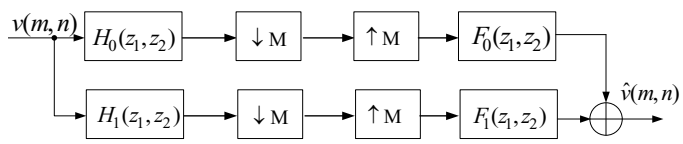

Fig. 1. Two-channel 2-D filter bank.

Consider the 2-D two-channel filter bank in Fig. 1, where the decimation matrix M is a 2×2 integer matrix with $|\det M| = 2$. In the figure, $v(m,n)$ is the input signal, $H_0(z_1,z_2)$ and $H_1(z_1,z_2)$ are analysis filters, $F_0(z_1,z_2)$ and $F_1(z_1,z_2)$ are synthesis filters used to reconstruct the signal $v(m,n)$, and $\hat{v}(m,n)$ is the reconstructed signal. In the absence of the channel noises, perfect reconstruction is said to be achieved if for some integers $d_1 \geq 0$, $d_2 \geq 0$, $\hat{v}(m,n) = v(m-d_1, n-d_2)$. For the diamond filter bank, M is of the quincunx type $M = \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}$, and the alias transfer function $A(z_1,z_2)$ and the distortion function $T(z_1,z_2)$ are given by

$$\begin{aligned} A(z_1,z_2) &= \frac{1}{2}H_0(-z_1,-z_2)F_0(z_1,z_2) \\ &\quad + \frac{1}{2}H_1(-z_1,-z_2)F_1(z_1,z_2), \qquad (1) \\ T(z_1,z_2) &= \frac{1}{2}H_0(z_1,z_2)F_0(z_1,z_2) \\ &\quad + \frac{1}{2}H_1(z_1,z_2)F_1(z_1,z_2). \qquad (2) \end{aligned}$$

and for the quadrant filter bank, with the choice of $M = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}$, the alias transfer function $A(z_1,z_2)$ and the dis-

tortion function $T(z_1, z_2)$ are given by

$$\begin{aligned} A(z_1,z_2) &= \frac{1}{2}H_0(-z_1,z_2)F_0(z_1,z_2) \\ &\quad +\frac{1}{2}H_1(-z_1,z_2)F_1(z_1,z_2), \quad (3) \\ T(z_1,z_2) &= \frac{1}{2}H_0(z_1,z_2)F_0(z_1,z_2) \\ &\quad +\frac{1}{2}H_1(z_1,z_2)F_1(z_1,z_2). \quad (4) \end{aligned}$$

The system is alias free if the alias transfer function $A(z_1, z_2) = 0$. The system has perfect reconstruction if it is alias free and the distortion function $T(z_1, z_2)$ is a delay [5]. In this paper, the 2-D IIR filter bank design problem is formulated as follows. Given two 2-D IIR filters $H_0(z_1, z_2)$ and $H_1(z_1, z_2)$, design two 2-D IIR filters $F_0(z_1, z_2)$ and $F_1(z_1, z_2)$, such that

$$H_0(z_1,z_2)F_0(z_1,z_2) + H_1(z_1,z_2)F_1(z_1,z_2) = z_1^{-d_1}z_2^{-d_2}, \quad (5)$$

$$H_0(-z_1,-z_2)F_0(z_1,z_2)+H_1(-z_1,-z_2)F_1(z_1,z_2) = 0, \quad (6)$$

for the diamond filter bank, and

$$H_0(z_1,z_2)F_0(z_1,z_2) + H_1(z_1,z_2)F_1(z_1,z_2) = z_1^{-d_1}z_2^{-d_2}, \quad (7)$$

$$H_0(-z_1,z_2)F_0(z_1,z_2) + H_1(-z_1,z_2)F_1(z_1,z_2) = 0. \quad (8)$$

for the quadrant filter bank, and for some integers $d_1 > 0$, $d_2 > 0$. Let

$$\begin{aligned} J &= \left\| z_1^{-d_1}z_2^{-d_2} - R(z_1,z_2) \right\|_\infty \\ &= \sup_{\omega_1,\omega_2\in[-\pi,\pi]} \left| e^{-jd_1\omega_1}e^{-jd_2\omega_2} - R(e^{j\omega_1}, e^{j\omega_2}) \right| \end{aligned}$$

where

$$R(z_1,z_2) = H_0(z_1,z_2)F_0(z_1,z_2) + H_1(z_1,z_2)F_1(z_1,z_2).$$

Note that (6) and (8) hold if

$$\begin{bmatrix} F_0(z_1,z_2) \\ F_1(z_1,z_2) \end{bmatrix} = F(z_1,z_2) \begin{bmatrix} H_1(-z_1,-z_2) \\ -H_0(-z_1,-z_2) \end{bmatrix}. \quad (9)$$

and

$$\begin{bmatrix} F_0(z_1,z_2) \\ F_1(z_1,z_2) \end{bmatrix} = F(z_1,z_2) \begin{bmatrix} H_1(-z_1,z_2) \\ -H_0(-z_1,z_2) \end{bmatrix}. \quad (10)$$

for some $F(z_1, z_2)$. An optimization formulation for handling above conditions is thus

$$\begin{aligned} &\min_{F_0(z_1,z_2),F_1(z_1,z_2)} J \\ &= \min_{F(z_1,z_2)} \left\| z_1^{-d_1}z_2^{-d_2} - H(z_1,z_2)F(z_1,z_2) \right\|_\infty, \quad (11) \end{aligned}$$

where

$$\begin{aligned} H(z_1,z_2) &= H_0(z_1,z_2)H_1(-z_1,-z_2) \\ &\quad -H_1(z_1,z_2)H_0(-z_1,-z_2), \quad (12) \end{aligned}$$

for the diamond filter bank, and

$$H(z_1,z_2) = H_0(z_1,z_2)H_1(-z_1,z_2) - H_1(z_1,z_2)H_0(-z_1,z_2), \quad (13)$$

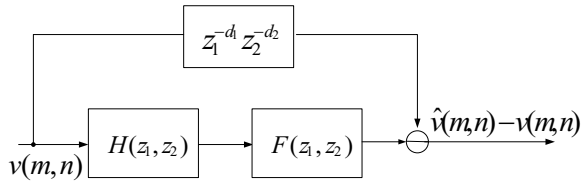

Fig. 2. Error system.

for the quadrant filter bank (see Fig.2). In this case, we call $F_0(z_1, z_2), F_1(z_1, z_2)$ by H_∞ filters.

Then, the 2-D IIR filter bank design problem statement is as follows: Given stable and causal 2-D IIR analysis filters $H_0(z_1, z_2)$ and $H_1(z_1, z_2)$ and given two tolerable 2-D shift d_1 and d_2, design stable and causal 2-D IIR synthesis filters $F_0(z_1, z_2)$ and $F_1(z_1, z_2)$ to minimize J.

Remark 1: Note that the above optimization problem leads to aliasing-free filter bank. Thus, the design of a filter bank reduces to designing synthesis banks such that the distortion error is minimized.

III. Main Results

In this section, we will present a method for designing stable and causal 2-D IIR synthesis filters.

A. Equivalent state-space computations

For a 2-D system, its Roesser state-space representation is defined by the following equations

$$\begin{aligned} \begin{bmatrix} x^h(i+1,j) \\ x^v(i,j+1) \end{bmatrix} &= A \begin{bmatrix} x^h(i,j) \\ x^v(i,j) \end{bmatrix} + Bw(i,j), \quad (14) \\ z(i,j) &= C \begin{bmatrix} x^h(i,j) \\ x^v(i,j) \end{bmatrix} + Dw(i,j), \quad (15) \end{aligned}$$

where $x^h(i,j) \in R^{n_h}$, $x^v(i,j) \in R^{n_v}$ are the horizontal state vector and the vertical state vector, respectively; $z(i,j) \in R^l$ is the measured output vector; $w(i,j) \in R^q$ is the input vector.

To simplify the presentation, we use the packed notation. Thus $\begin{bmatrix} A & B \\ C & D \end{bmatrix}$ stands for the transfer function

$$G(z_1,z_2) = C\left[I\left(z_1,z_2\right) - A\right]^{-1}B + D, \quad (16)$$

where $I(z_1, z_2) = diag\{z_1 I_{n_h}, z_2 I_{n_v}\}$.

Now, denote the state-space representation for $z_1^{-d_1}z_2^{-d_2}$, $H(z_1, z_2)$ and $F(z_1, z_2)$ as

$$\begin{aligned} z_1^{-d_1}z_2^{-d_2} &:= \begin{bmatrix} A_D & B_D \\ C_D & 0 \end{bmatrix}, \quad (17) \\ H(z_1,z_2) &:= \begin{bmatrix} A_H & B_H \\ C_H & D_H \end{bmatrix}, \quad (18) \\ F(z_1,z_2) &:= \begin{bmatrix} A_F & B_F \\ C_F & D_F \end{bmatrix}, \quad (19) \end{aligned}$$

respectively, where $A_D \in R^{n_D\times n_D}, B_D \in R^{n_D\times 1}, C_D \in R^{1\times n_D}, n_D = n_{D_h}+n_{D_v}$, and $A_H \in R^{n_H\times n_H}, B_H \in R^{n_H\times 1}, C_H \in R^{1\times n_H}, D_H \in R, n_H = n_{H_h}+n_{H_v}$, and

$A_F \in R^{n_F \times n_F}, B_F \in R^{n_F \times 1}, C_F \in R^{1 \times n_F}, D_F \in R, n_F = n_{F_h} + n_{F_v}$.

Then, the state-space representation for $z_1^{-d_1} z_2^{-d_2} - H(z_1, z_2)F(z_1, z_2)$ in (11) can be written as

$$z_1^{-d_1} z_2^{-d_2} - H(z_1, z_2)F(z_1, z_2) := \begin{bmatrix} \bar{A} & \bar{B} \\ \bar{C} & \bar{D} \end{bmatrix}, \quad (20)$$

where

$$\bar{\Omega} = \begin{bmatrix} I_{n_h} & 0 & 0 & 0 \\ 0 & 0 & I_{n_{F_h}} & 0 \\ 0 & I_{n_v} & 0 & 0 \\ 0 & 0 & 0 & I_{n_{F_v}} \end{bmatrix}, \quad (21)$$

$$\bar{A} = \bar{\Omega}\tilde{A}\bar{\Omega}^T, \bar{B} = \bar{\Omega}\tilde{B},\ \bar{C} = \tilde{C}\bar{\Omega}^T,\ \bar{D} = -D_F D, \quad (22)$$

$$\tilde{A} = \begin{bmatrix} A & 0 \\ B_F C & A_F \end{bmatrix}, \tilde{B} = \begin{bmatrix} B \\ B_F D \end{bmatrix}, \quad (23)$$

$$\tilde{C} = \begin{bmatrix} C - D_F C & -C_F \end{bmatrix}. \quad (24)$$

and

$$\Omega = \begin{bmatrix} I_{n_{H_h}} & 0 & 0 & 0 \\ 0 & 0 & I_{n_{D_h}} & 0 \\ 0 & I_{n_{H_v}} & 0 & 0 \\ 0 & 0 & 0 & I_{n_{D_v}} \end{bmatrix} \quad (25)$$

$$A = \Omega\hat{A}\Omega^T, B = \Omega\hat{B},\ C = \hat{C}\Omega^T, \quad (26)$$

$$D = D_H,\ L = \hat{L}\Omega^T, \quad (27)$$

$$\hat{A} = \begin{bmatrix} A_H & 0 \\ 0 & A_D \end{bmatrix}, \hat{B} = \begin{bmatrix} B_H \\ B_D \end{bmatrix}, \quad (28)$$

$$\hat{C} = \begin{bmatrix} C_H & 0 \end{bmatrix},\ \hat{L} = \begin{bmatrix} 0 & C_D \end{bmatrix}. \quad (29)$$

where $A \in R^{n \times n}, B \in R^{n \times 1}, C \in R^{1 \times n}, L \in R^{1 \times n}, n_h = n_{D_h} + n_{H_h}, n_v = n_{D_v} + n_{H_v}, n = n_h + n_v$.

Denote the 2-D system $(\bar{\Sigma}) := \begin{bmatrix} \bar{A} & \bar{B} \\ \bar{C} & \bar{D} \end{bmatrix}$, and its transfer function is $\bar{G}(z_1, z_2) = \bar{C}\left[I(z_1, z_2) - \bar{A}\right]^{-1}\bar{B} + \bar{D}$. Obliviously, H_∞ norm of $z_1^{-d_1} z_2^{-d_2} - H(z_1, z_2)F(z_1, z_2)$ simply is that of the 2-D system $(\bar{\Sigma})$. In other words, solving the optimization problem in (11) requires one to find A_F, B_F, C_F, D_F to minimize H_∞ norm of the 2-D system $(\bar{\Sigma})$.

B. 2-D IIR synthesis filter design

We start this subsection by recalling a lemma which will be essential in the proof of the main results.

Lemma 1: [10] Given a scalar $\gamma > 0$, the 2-D system $(\bar{\Sigma})$ is stable and $\left\|\bar{G}(z_1, z_2)\right\|_\infty < \gamma$ if there exist a block-diagonal matrix $P = diag\{P_h, P_v\} > 0$ with $P_h \in R^{(n_h+n_{F_h}) \times (n_h+n_{F_h})}$ and $P_v \in R^{(n_v+n_{F_v}) \times (n_v+n_{F_v})}$, and matrix $\Phi = diag\{\Phi_h, \Phi_v\}$ with $\Phi_h \in R^{(n_h+n_{F_h}) \times (n_h+n_{F_h})}$ and $\Phi_v \in R^{(n_v+n_{F_v}) \times (n_v+n_{F_v})}$, satisfying

$$\begin{bmatrix} P - \Phi - \Phi^T & 0 & \Phi^T\bar{A} & \Phi^T\bar{B} \\ * & -I & \bar{C} & \bar{D} \\ * & * & -P & 0 \\ * & * & * & -\gamma^2 I \end{bmatrix} < 0, \quad (30)$$

In the following, we can get the design of $F(z_1, z_2)$ using an LMI approach.

Theorem 1: Suppose that $H(z_1, z_2)$ is given as above. Then for a given scalar $\gamma > 0$, there exists a n_Fth-order $F(z_1, z_2)$ such that

$$\left\|z_1^{-d_1} z_2^{-d_2} - H(z_1, z_2)F(z_1, z_2)\right\|_\infty < \gamma$$

if there exist matrices $Q_{11} = diag\{Q_{11h}, Q_{11v}\} > 0$, $Q_{22} = diag\{Q_{22h}, Q_{22v}\} > 0$, $Q_{12} = diag\{Q_{12h}, Q_{12v}\}$, and matrices $X_1 = diag\{X_{1h}, X_{1v}\}$, $X_2 = diag\{X_{2h}, X_{2v}\}$, $Y = diag\{Y_h, Y_v\}$, N, M, Z, K, such that the following LMI holds

$$\begin{bmatrix} \Pi_1 & 0 & L_1 & L_2 \\ * & -I & S & -MD \\ * & * & -\Pi_2 & 0 \\ * & * & * & -\gamma^2 I \end{bmatrix} < 0, \quad (31)$$

where

$$\Pi_1 = \begin{bmatrix} Q_{11} - X_1 - X_1^T & Q_{12} - \hat{Y} - X_2^T \\ * & Q_{22} - Y - Y^T \end{bmatrix},$$

$$L_1 = \begin{bmatrix} X_1 A + \hat{Z}C & \hat{K} \\ X_2 A + ZC & K \end{bmatrix}, \Pi_2 = \begin{bmatrix} -Q_{11} & -Q_{12} \\ * & -Q_{22} \end{bmatrix},$$

$$L_2 = \begin{bmatrix} X_1 B + \hat{Z}D \\ X_2 B + ZD \end{bmatrix}, S = \begin{bmatrix} C - MC & -N \end{bmatrix},$$

$$\hat{Y} = \begin{bmatrix} Y \\ 0_{(n-n_F) \times n_F} \end{bmatrix},\ \hat{K} = \begin{bmatrix} K \\ 0_{(n-n_F) \times n_F} \end{bmatrix},$$

$$\hat{Z} = \begin{bmatrix} Z \\ 0_{(n-n_F) \times n_F} \end{bmatrix}.$$

Then $F(z_1, z_2)$ can be designed by

$$A_F = Y^{-1}K, B_F = Y^{-1}Z,\ C_F = N, D_F = M. \quad (32)$$

Proof: By Lemma 1, $\left\|z_1^{-d_1} z_2^{-d_2} - H(z_1, z_2)F(z_1, z_2)\right\|_\infty < \gamma$ if there exist matrices $P = diag\{P_h, P_v\} > 0$ with $P_h \in R^{(n_h+n_{F_h}) \times (n_h+n_{F_h})}$ and $P_v \in R^{(n_v+n_{F_v}) \times (n_v+n_{F_v})}$, and matrix $\Phi = diag\{\Phi_h, \Phi_v\}$ with $\Phi_h \in R^{(n_h+n_{F_h}) \times (n_h+n_{F_h})}$ and $\Phi_v \in R^{(n_v+n_{F_v}) \times (n_v+n_{F_v})}$, such that

$$\begin{bmatrix} P - \Phi - \Phi^T & 0 & \Phi^T\bar{A} & \Phi^T\bar{B} \\ * & -I & \bar{C} & \bar{D} \\ * & * & -P & 0 \\ * & * & * & -\gamma^2 I \end{bmatrix} < 0. \quad (33)$$

Pre- and post-multiplying (33) by $diag\{\bar{\Omega}^T, I, \bar{\Omega}^T, I\}$ and its transpose, the matrix inequality (33) is equivalent to

$$\begin{bmatrix} Q - \Psi - \Psi^T & 0 & \Psi^T\tilde{A} & \Psi^T\tilde{B} \\ * & -I & \tilde{C} & \bar{D} \\ * & * & -Q & 0 \\ * & * & * & -\gamma^2 I \end{bmatrix} < 0, \quad (34)$$

where

$$\Psi = \bar{\Omega}^T\Phi\bar{\Omega},\ Q = \bar{\Omega}^T P\bar{\Omega}.$$

Let $\Psi = \begin{bmatrix} X_1 & \hat{Y} \\ X_2 & Y \end{bmatrix}$, which is consistent with $\tilde{A}$. Similarly, Q has the blocked form $\begin{bmatrix} Q_{11} & Q_{12} \\ Q_{12}^T & Q_{22} \end{bmatrix}$ corresponding to the structure of Ψ. Substituting the above structure of Ψ and Q

into (34), and letting $YA_F := K$, $YB_F := Z$, $C_F := N$, $D_F := M$, then we can obtain (31). This completes the proof.

Remark 2: The optimization problem (11) can be solved via the following LMI optimization problem

$$\min_{\substack{Q_{11},Q_{12},Q_{22},\\ X_1,X_2,M,\\ N,Y,Z,K}} \gamma : \{31\} \tag{35}$$

If the solution of (35) is obtain, then the filter $F(z_1, z_2)$ can be designed in (32).

Now, from the above theory, we give the following design procedure.

Procedure

Step 1: Compute $H(z_1, z_2)$ from (2) or (13).

Step 2: Compute A_D, B_D, C_D, A_H, B_H, C_H, D_H and A, B, C, D in (17), (18) and (26)-(29).

Step 3: Solve the optimization problem (35), to deduce the matrices A_F, B_F, C_F, D_F from (32).

Step 4: Compute $F(z_1, z_2)$ from (16).

Step 5: Compute $F_0(z_1, z_2)$ and $F_1(z_1, z_2)$ from (9) or (10).

Remark 3: It is well known that the stability is a significant requirement in IIR filter design. Note that from Theorem 1, the LMI (31) always guarantee the stability of $\bar{A}$, whose eigen values are that of A and A_F. Then, the designed filter $F(z_1, z_2)$ is always stable.

The following example verify the feasibility and effectiveness of this method.

IV. Example

Consider a 2-D two-channel IIR diamond filter bank with the following low pass filter $H_0(z_1, z_2)$ and high pass filter $H_1(z_1, z_2)$ [6]:

$$\begin{aligned} H_0(z_1, z_2) &= \frac{1}{2}\left[z_1^{-7} + \beta_1(z_1, z_2)\beta_2(z_1, z_2)\right], \\ H_1(z_1, z_2) &= z_1^{-11} - \beta_1(z_1, z_2)\beta_2(z_1, z_2)H_0(z_1, z_2), \end{aligned}$$

where

$$\begin{aligned} \beta_1(z_1, z_2) &= \frac{0.025 - 0.094z_1^{-1}z_2^{-1} + 0.473z_1^{-2}z_2^{-2} + z_1^{-3}z_2^{-3}}{1 + 0.473z_1^{-1}z_2^{-1} - 0.094z_1^{-2}z_2^{-2} + 0.025z_1^{-3}z_2^{-3}}, \\ \beta_2(z_1, z_2) &= \frac{0.025z_2^{-3} - 0.094z_1^{-1}z_2^{-2} + 0.473z_1^{-2}z_2^{-1} + z_1^{-3}}{z_2^{-3} + 0.473z_1^{-1}z_2^{-2} - 0.094z_1^{-2}z_2^{-1} + 0.025z_1^{-3}} \end{aligned}$$

The magnitude responses of $H_0(z_1, z_2)$ and $H_1(z_1, z_2)$ are shown in Fig. 3 and Fig. 4. Let $d_1 = 17$, $d_2 = 1$, using the above procedure, firstly, we can compute $H(z_1, z_2) = -z_1^{-17}$. Then, by Theorem 1 or the design procedure summarized at the end of Section III, we can obtain $F(z_1, z_2) = -z_2^{-1}$, and finally, the synthesis filters $F_0(z_1, z_2)$ and $F_1(z_1, z_2)$ given below

$$\begin{aligned} F_0(z_1, z_2) &= -z_2^{-1}H_1(-z_1, -z_2), \\ F_1(z_1, z_2) &= z_2^{-1}H_0(-z_1, -z_2). \end{aligned}$$

The magnitude responses of $F_0(z_1, z_2)$ and $F_1(z_1, z_2)$ are shown in Fig. 5 and Fig. 6.

V. Conclusions

In this paper we have considered the problem of the two-channel 2-D IIR filter bank design. The synthesis filters have been given in terms of solutions to an LMI. This method can guarantee the stability and causality for the designed synthesis filters.

VI. Acknowledgements

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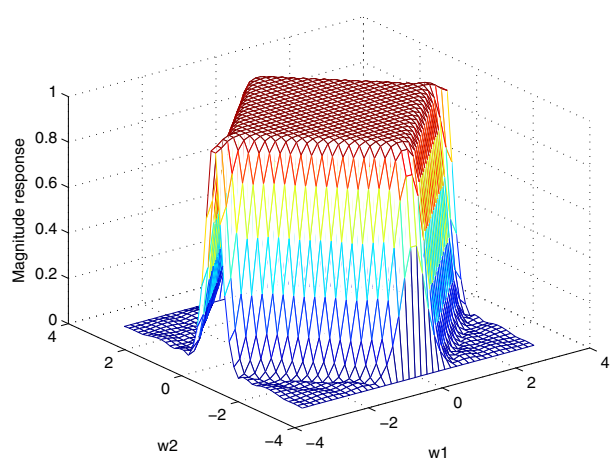

Fig. 3. Magnitude response of $H_0(z_1, z_2)$.

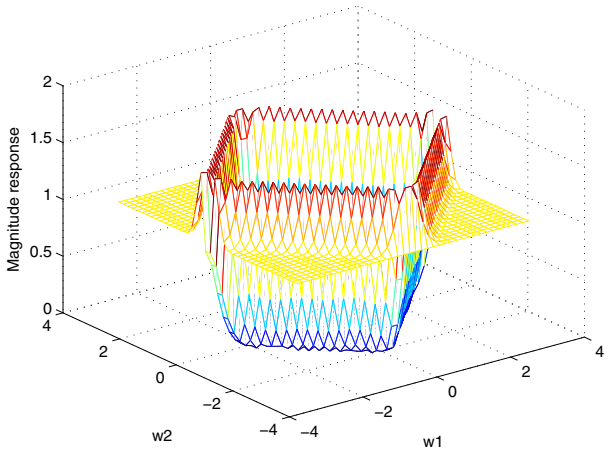

Fig. 4. Magnitude response of $H_1(z_1, z_2)$.

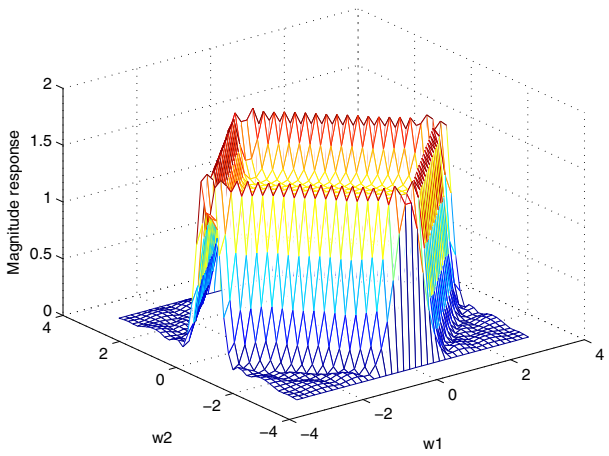

Fig. 5. Magnitude response of $F_0(z_1, z_2)$.

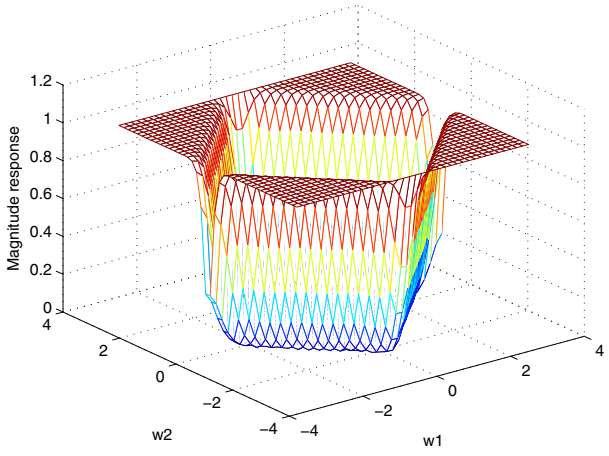

Fig. 6. Magnitude response of $F_1(z_1, z_2)$.