Three-dimensional trapping of Mie metallic particles by the use of obstructed laser beams

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In this article, we propose a method for three-dimensional optical trapping of metallic Mie particles using an obstructed laser beam. It is found from the ray-optics model that Mie gold, silver, and copper particles can be trapped against gravity in the focal region of a high numerical-aperture objective illuminated by a centrally obstructed Gaussian (TEM$_{00}$-mode) beam. The axial trapping force of the three types of metallic particles is maximized in the near-infrared wavelength region. The maximum axial trapping efficiency increases with the size of the center obstruction and the aperture angle of an objective. Axial trapping force on Mie metallic particles is enhanced by a factor of two if an obstructed doughnut (TEM$_{01}^{*}$-mode) beam is employed. The experimental condition for achieving three-dimensional trapping is also discussed. © 2002 American Institute of Physics. [DOI: 10.1063/1.1428801]

I. INTRODUCTION

The idea of laser trapping is to utilize a laser beam focused by a high numerical-aperture microscope objective to induce a confining force on a dielectric or metallic particle in the focal region. This technique, sometimes termed laser tweezers, has led to a strong impact on physical and biological research. For a dielectric particle, three-dimensional trapping can be achieved in the focal region regardless of particle size. In particular the axial trapping efficiency of Mie dielectric particles (i.e., the size of particles is comparable to the wavelength of a trapping beam) is enhanced when the illumination beam is centrally obstructed or has a doughnut (i.e., a TEM$_{01}^{*}$-mode) intensity distribution.

However, the trapping situation becomes complicated when a metallic particle is used. It has been demonstrated both experimentally and theoretically that three-dimensional trapping can be achieved for Rayleigh metallic particles (i.e., the size of particles is much smaller than the wavelength of a trapping beam). If the size of metallic particles is comparable to or larger than the trapping wavelength, i.e., for Mie metallic particles, trapping can be achieved only in two dimensions when the focus of the trapping beam is located near the bottom of a metallic particle.

The difference of trapping performance between Mie dielectric and metallic particles is caused by the fact that gradient and scattering forces are dominant, respectively, in the former and latter cases. According to the ray-optics model valid for Mie particles, gradient and scattering forces contribute to pulling and pushing forces, respectively, along the propagating (axial) direction of the trapping beam. Because of the dominance of scattering force, the net axial force on a Mie metallic particle always behaves in a pushing nature, leading to two-dimensional trapping of metallic particles. However, the projection of scattering force in the axial direction decreases with the angle $\theta$ of a ray of convergence. Consequently, one can use a central obstruction to remove the stronger components of pushing force resulting from smaller angles of rays of convergence. As a result, it is possible to employ an obstruction of appropriate size to enhance the contribution of the pulling force from gradient force, so that a net pulling force occurs for three-dimensional trapping.

The aim of this article is to present a theoretical investigation, in terms of the ray-optics model, into three-dimensional trapping of a Mie metallic particle under obstructed-beam illumination. In Sec. II, the radiation pressure efficiency under obstructed Gaussian beam illumination is calculated for Mie gold, silver, and copper particles. The effect of the trapping wavelength of a trapping beam and the numerical aperture of a trapping objective on the maximum axial trapping efficiency is also considered. In Sec. III, the effect of an obstructed doughnut beam on three-dimensional trapping of three types of Mie metallic particles is numerically explored. A condition for experimental implementation of three-dimensional trapping of Mie metallic particles is estimated in Sec. IV. Finally, a conclusion is put forward in the last section.

II. RADIATION PRESSURE EFFICIENCY FOR AN OBSTRUCTED GAUSSIAN BEAM

Throughout this article we assume that a trapping beam propagates downwards and that this direction is termed the positive axial direction. The polarization direction of the trapping beam is along the $x$ direction as shown in Fig. 1. There are two main methods for calculating trapping force: one is based on electromagnetic wave theory and the other is based on ray optics. The latter describes the force on a particle according to the momentum exchange between light and matter caused by absorption, reflection, refraction and
scattering. The magnitude and direction of the force are determined using the law of momentum conservation. It has been experimentally demonstrated that this method is applicable for Mie particles.\(^8\)

Having considered the effect of skin depth of Mie metallic particles, we can express the scattering force \(F_s\) and the gradient force \(F_g\) caused by a ray as\(^{21}\)

\[
F_s = \frac{nP}{c} \left[ 1 + R \cos(2\theta) \right],
\]

\[
F_g = \frac{nPR}{c} \sin(2\theta),
\]

respectively. Here \(F_s\) and \(F_g\) are the force components parallel and perpendicular to the direction of an incident ray, respectively. \(\theta\) is the angle of incidence of a ray. \(P\) is the power of the trapping beam, and \(n\) is the refractive index of the medium in which a particle is suspended. \(R\), the reflectance on the surface of a metallic particle,\(^{22}\) is different for beams polarized perpendicular or parallel to the plane of incidence.

The total trapping force on a particle can be evaluated by integrating the contribution of all rays within the cone of a trapping objective illuminated by a laser beam of an intensity profile given by \(I(r)\), where \(r\) is the radial coordinate over the aperture of the trapping objective. If a circular obstruction is coaxially placed at the aperture of the trapping objective, the integration of the gradient and scattering forces is performed only between the maximum angle of convergence \(\phi_{\text{max}}\), determined by the numerical aperture of the trapping objective, and the minimum angle of convergence \(\phi_{\text{min}}\) defined by the radius of the obstruction. For a Gaussian beam, the intensity profile is given by

\[
I(r) = I_0 \exp\left(-\frac{2r^2}{r_0^2}\right),
\]

where \(I_0\) is the intensity at the center of the beam and \(r_0\) is the beam waist normalized by the radius of the objective aperture. If \(r_0\) is large, the beam distribution over the aperture becomes uniform.

Finally, the strength of the trapping force, and its components can be expressed by a dimensionless radiation pressure efficiency \(Q\) given by\(^9,21\)

\[
Q = \frac{Fc}{nP},
\]

Consider a water-immersion objective of numerical aperture (NA = 1.2) and spherical metallic Mie particles suspended in water of refractive index 1.33. Three types of metallic particles (gold, silver, and copper) were chosen. Their refractive indices at wavelength 800 nm are 0.08+1i4.56, 0.27+1i5.79, and 0.26+1i5.26, respectively.\(^{23}\)

The distribution of the total radiation pressure efficiency for a gold particle at different trapping positions in the \(x-z\) plane is detailed in Fig. 1. \(x\) and \(z\) are the transverse and axial trapping positions normalized by the radius of a particle and originating from the center of the particle, respectively. The right-hand side of the force mapping has been omitted due to symmetry with respect to the \(z\) axis. The direction of arrows points to the direction of trapping force. As expected, the trapping force caused by an unobstructed Gaussian beam \(\text{[Fig. 1(a)]}\) always acts to push the particle in the direction of the beam propagation.\(^{21}\) When the beam is centrally obstructed \(\text{[Fig. 1(b)]}\) (here \(e\) is defined as the ratio of the radius of the central obstruction to the radius of the objective aperture), a pulling or lifting force occurs near the bottom of the particle, which means that axial trapping is possible in this region.

Figure 2 depicts the axial (\(x = 0\)) and transverse (\(z = 1\)) radiation pressure efficiencies for a gold particle as a function of the trapping position for an obstructed beam. It is
seen that the axial radiation pressure efficiency is minimized when the trapping beam is placed near the bottom of the particle (i.e., near \( z = 1 \)). There are high and low axial positions \( z_h \) and \( z_l \) at which the axial trapping force is zero. If no other force was exerted on the particle, the particle could be axially trapped at these two positions while the net transverse trapping force is also zero, which leads to three-dimensional trapping. However, in practice, a particle is trapped between the two zero-force positions in order to overcome the effect of other force (see Sec. IV for a detailed discussion).

The maximum axial trapping efficiency \( Q_{\text{max}} \) (defined to be the minimum axial radiation pressure efficiency in Fig. 2) and the corresponding axial position \( z_{\text{max}} \) for different values of the central obstruction \( \varepsilon \) are shown in Table I for gold, silver, and copper particles. The negative and positive values of \( Q_{\text{max}} \) mean the lifting and pushing situations, respectively. It is seen from Table I that the absolute value of \( Q_{\text{max}} \) increases with \( \varepsilon \) while the axial trapping region (defined as the distance between the high and low zero-force positions) becomes large. It should be pointed out that the position corresponding to \( Q_{\text{max}} \) does not change appreciably as \( \varepsilon \) increases.

The refractive index for a metallic particle is wavelength dependent, as shown in Fig. 3. Consequently, the magnitude of \( Q_{\text{max}} \) may be minimized within a wavelength region. This feature is described in Fig. 4 for three types of metallic particles, showing that \( Q_{\text{max}} \) reaches a minimum approximately within the wavelength region from 800 to 900 nm if the size of the obstruction is larger than a certain value. Figure 4 also exhibits that it may be possible to achieve three-dimensional trapping for gold and silver particles in the visible wavelength region but is difficult to achieve this type of trapping for copper particles.

Figure 5 shows the wavelength dependence of the maximum axial trapping efficiency \( Q_{\text{max}} \) for an oil-immersion objective. Although the numerical aperture of the trapping objective is illuminated by an obstructed Gaussian beam (NA = 1.2), axial trapping objective for trapping is advantageous because it provides a large angle of convergence and reduces spherical aberration if particles are suspended in water.

### III. AXIAL TRAPPING EFFICIENCY FOR AN OBSTRUCTED DOUGHNUT BEAM

A doughnut beam, i.e., a TEM\(_{10}^0\) -mode beam, exhibits a phase singularity at the center of the beam,\(^{11,12}\) leading to zero intensity at the singularity. It has been shown that if a trapping objective is illuminated by a doughnut beam, axial trapping force on a dielectric particle is enhanced\(^8\) because a doughnut intensity distribution enhances the contribution of rays with a large angle of convergence. Such a contribution should become more pronounced for metallic particles because of the dominance of scattering force.\(^{20}\)

A doughnut beam has an intensity profile given by

### TABLE I. Trapping parameters for Mie gold, silver, and copper particles using a water-immersion objective (NA = 1.2) illuminated by a uniform laser beam.

<table>
<thead>
<tr>
<th>Trapping parameters</th>
<th>Gold</th>
<th>Silver</th>
<th>Copper</th>
</tr>
</thead>
<tbody>
<tr>
<td>( Q_{\text{max}} )</td>
<td>( e = 0.9 )</td>
<td>( e = 0.8 )</td>
<td>( e = 0.7 )</td>
</tr>
<tr>
<td>( z_{\text{max}} )</td>
<td>-0.63</td>
<td>-0.45</td>
<td>-0.28</td>
</tr>
<tr>
<td>( z_h )</td>
<td>0.96</td>
<td>0.98</td>
<td>1.02</td>
</tr>
<tr>
<td>( z_l )</td>
<td>1.10</td>
<td>1.16</td>
<td>1.18</td>
</tr>
<tr>
<td>( A_{\text{area}} )</td>
<td>0.62</td>
<td>0.72</td>
<td>0.83</td>
</tr>
<tr>
<td>( A_{\text{area}} )</td>
<td>19%</td>
<td>36%</td>
<td>51%</td>
</tr>
<tr>
<td>( Q_{\text{max}} \times A )</td>
<td>0.12</td>
<td>0.16</td>
<td>0.14</td>
</tr>
</tbody>
</table>
where $I_0$ is the normalization factor determined by the incident intensity and the condition of energy conservation, $r$ is the radial coordinate on the aperture of the trapping objective, and $r_0$ is the spot size of the doughnut beam. Both are normalized by the radius of the objective aperture. In practice, a doughnut beam can be generated by a computer-generated hologram or by a liquid crystal phase modulator.

Using the same method and condition described in Sec. II, one can obtain the mapping of the total radiation pressure efficiency in the $x$-$z$ plane. It is shown in Fig. 6(a) that without using an obstruction, there is no pulling force near the bottom of a gold particle. As expected, a pulling force occurs in that region if the doughnut beam is centrally obstructed [Fig. 6(b)]. The dependence of the maximum axial trapping efficiency $Q_{\text{max}}$ on the trapping wavelength is shown in Fig. 7 for gold, silver, and copper particles. Although the dependence in Fig. 7 is similar to that in Fig. 4, the magnitude of $Q_{\text{max}}$ in Fig. 7 is increased by a factor of two compared with that in Fig. 4. The same factor of the enhancement can be obtained if an oil-immersion objective illuminated by an obstructed doughnut beam is used for trapping (Fig. 8).

IV. DISCUSSION

It is clear from Table I that the absolute value of the maximum axial trapping efficiency $Q_{\text{max}}$ increases monotonically with the size of the obstruction. According to Eq. (3) the maximum axial trapping force is directly proportional...
to the illumination power over the aperture of the trapping objective. As a result, the illumination power is decreased with the size of the obstruction. For example, the illumination power for a uniform beam, i.e., for a beam with \( r_0 \to \infty \), is directly proportional to \((1 - \varepsilon^2)\) for a laser of given power. Thus, when \( \varepsilon \) approaches 1, i.e., for a thin ring beam, one may not necessarily get the maximum axial trapping force; an optimized value of \( \varepsilon \) is needed, as displayed in Table I.

It is noted from Table I that the optimized value of \( \varepsilon \) is 0.8 under uniform beam illumination, giving the maximum axial trapping force for the three types of the Mie metallic particles. From this maximum axial trapping force, an estimation of the trapping power from a laser can be made by taking into account buoyancy force and gravity. Finally, the power required to lift a particle against gravity is found to be

\[
P \geq 9.3 \times 10^9 \frac{\rho r_p^3}{Q_{\text{max}}(1 - \varepsilon^2)},
\]

where \( Q_{\text{max}} \) is the maximum efficiency given in Table I.
where \( \rho \) and \( r_p \) are the density and radius of a particle, respectively.

Consider that the density of gold, silver, and copper particles is 19300, 10490, and 8960 kg/m\(^3\), respectively. For \( r_p = 1.5 \) \( \mu \)m, the trapping power over the objective aperture is greater than 3.9 mW for gold particles, 2.2 mW for silver particles, and 1.7 mW for copper particles at a wavelength of 800 nm, which should be easily achieved with a commercially available laser.

In practice, an absorptive particle such as a metallic particle experiences a radiometric force caused by the heating effect under laser beam illumination.\(^\text{25}\) It has been estimated\(^\text{21}\) that the radiometric force on Mie metallic particles may be an order of magnitude larger than the axial trapping force. However, the heating effect can be reduced if a doughnut beam is employed because of its zero intensity on the axis. In the meantime, using a doughnut beam also increases the axial trapping force according to Figs. 7 and 8.

V. CONCLUSION

Based on the ray-optics model, which is a good approximation for Mie particles, we have demonstrated that axial trapping (pulling) force can exist near the bottom of Mie gold, silver, and copper particles when a near-infrared trap-
ping beam is centrally obstructed. Therefore, it is possible to achieve three-dimensional trapping of those particles of diameter 3 μm provided that the laser illumination power is greater than 3.5 mW. It is also shown that the use of an obstructed doughnut beam leads to an increase of the maximum axial trapping force by a factor of two, and may reduce radiometric force caused by the heating effect.

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