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An imaging $K$-band survey — I: The catalogue, star and galaxy counts

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ABSTRACT
We present results from a large area (552◦) imaging $K$-band survey of faint objects. The survey is a high galactic latitude blank-field sample to a $5\sigma$ limit of $K \simeq 17.3$. The methods for constructing the infrared survey are described, including flatfielding, astrometry, mosaicing and photometry. Also described are optical CCD observations which cover the survey to provide optical-infrared colours of almost all the objects in the sample. Star-galaxy discrimination is performed and the results used to derive the infrared star and galaxy counts. $K$-band “no-evolution” galaxy-count models are constructed and compared with the observed data. In the infrared, there is no counterpart for the large excess of faint galaxies over the no-evolution model seen in optical counts. In contrast, the $K$ counts require little or no evolution in the luminosity or space density of galaxies, although it is shown that the count predictions can be remarkably insensitive to evolution under certain reasonable assumptions. Finally, model predictions for $K$-selected redshift surveys are derived.

1 INTRODUCTION
The development of 2-dimensional near-infrared detectors has finally made it possible to survey substantial areas of the sky at these wavelengths to cosmologically interesting depths. This is the first in a series of papers describing a project designed to map several hundred square arcminutes of sky to a depth of $K \simeq 17$. This paper in particular is concerned with the details of the construction and calibration of this catalogue, and the associated optical CCD imaging for all the objects.

Such a survey has a multitude of uses. Firstly, there are serendipitous searches for populations of objects which might be quite bright in the infrared but absent, or very faint, in the deepest optical surveys. Such populations could include, for example, protogalactic objects and low-luminosity stars and brown dwarfs.

It has been proposed that protogalaxies (hereafter PG’s) should have flat spectra similar to giant Hii regions (Koo 1986). The density of such objects on the sky should be very high, at least $(Koo 1986)$. The density of such objects on the sky should be very high, at least $(Koo 1986)$. The methods for constructing the infrared survey are described, including flatfielding, astrometry, mosaicing and photometry. Also described are optical CCD observations which cover the survey to provide optical-infrared colours of almost all the objects in the sample. Star-galaxy discrimination is performed and the results used to derive the infrared star and galaxy counts. $K$-band “no-evolution” galaxy-count models are constructed and compared with the observed data. In the infrared, there is no counterpart for the large excess of faint galaxies over the no-evolution model seen in optical counts. In contrast, the $K$ counts require little or no evolution in the luminosity or space density of galaxies, although it is shown that the count predictions can be remarkably insensitive to evolution under certain reasonable assumptions. Finally, model predictions for $K$-selected redshift surveys are derived.

Secondly the survey provides a catalogue of galaxies selected by near-infrared flux, which allows a new approach to the study of galaxy evolution. It is now well established that the faint galaxy number-magnitude counts in the optical show a blue excess population over that predicted without evolution (e.g. Peterson et al. 1979, Kron 1980, Hall & Mackay 1984, Tyson 1988). Deep redshift surveys by Broadhurst et al. (1988), Colless et al. (1990) and Lilly et al. (1991) are beginning to show that the excess is not due to high redshift objects, as would be expected in simple models of luminosity evolution in the stellar populations, but appears to be due to lower-luminosity objects with a higher space density.

At redder wavelengths the light becomes dominated by stars with progressively longer lifetimes. The near-infrared light from a galaxy is dominated by the “old-stellar population”. This consists of old, evolved, stars on the giant branch with lifetimes of gigayears. Thus the relative contribution
from on-going star-formation to the total light will be much less. In the \(I\)-band (~1\(\mu\)m) the excess in the counts over no-evolution is already much less than in the \(B\)-band (e.g. Tyson, 1988). This leads us to expect that the \(K\)-band light will be relatively insensitive to star-formation. Thus the \(K\) light should provide an excellent tracer of direct evolution in galaxy density.

The paper is laid out as follows: Section 2 discusses the construction of the infrared survey. The choosing of field centres from Schmidt plates and the method adopted for observing are discussed. The data reduction procedures peculiar to infrared array detectors are detailed and the methods of mosaicking and performing photometric and astrometric calibrations are described.

Section 3 describes the CCD observations that were performed to obtain accurate colours for all the infrared selected objects. Photometry, astrometry and matching with the infrared data are detailed.

Section 4 derives the infrared star and galaxy counts for the survey and corrects them for observed error rates, from the spectroscopy, in star-galaxy classification.

Section 5 derives galaxy count predictions from literature spectral evolution models and compared in \(K\) and \(b\_j\) with the survey data and other published data. The effect of clustering on the galaxy counts are considered. More general parametric models of density and luminosity evolution in the galaxy population are also developed and compared with the data.

Finally the results and conclusions are summarized in Section 6.

2 THE INFRARED SURVEY

The design of an infrared survey is a trade off between area and depth due to the limitations of observing time. It is also heavily constrained by the desire to obtain optical information on the same set of objects.

On a 4m optical telescope, a spectrum good enough to yield a redshift from absorption features in a night’s observing (30 000s) can be obtained down to \(B\sim 23.5\) or even \(B\sim 24\) if the object has very strong emission lines. In simple passively evolving galaxy models (e.g. Rocca-Volmerange and Guideroni 1988) the galaxies can be as red as \(B - K\sim 5 - 6\) at redshifts of order 0.5–1. This implies a limit of \(K\sim 18\) for complete samples. At this magnitude a galaxy with the characteristic luminosity \(L^\ast\) is seen at \(z\sim 0.3\), so the sample is deep enough to be cosmologically interesting. Because of the desire to study a sample selected in the infrared rather than make infrared observations of an optically limited sample, it is necessary to include the reddest objects. This sets the approximate limiting magnitude for the present survey.

In order to address the question of galaxy evolution a fairly large sample of at least \(\sim 50 - 100\) galaxies would be required. More galaxies are observed by covering a larger area rather than by integrating longer — the number of detected objects in a background limited observation rises only \(\propto t^2\) for a uniform population in a simple Euclidean universe, which provides an upper bound to the observed number-magnitude optical counts. Thus the estimated surface density at \(K = 18\) of \(\sim 1\) galaxy/arcmin\(^2\) (based on optical surface densities and assumed colours) implies several hundred galaxy would have to be surveyed to provide such a large sample. Due to the limited size of the detectors it was decided to survey as large an area of sky as possible in one infrared band only, the 2.2\(\mu\)m \(K\)-band, and supplement this with optical CCD imaging and spectroscopy.

2.1 Observations

The infrared observations for this project were all made over the period 1987–1988 using the Infrared Camera IRCAM at the 3.8 m United Kingdom Infrared Telescope on Mauna Kea, Hawaii. IRCAM uses a 62 \(\times\) 58 indium antimonide (InSb) array for imaging observations in the 1–5\(\mu\)m band (McLean et al. 1986). A pixel size of 1.2\arcsec/pixel was chosen to cover the largest area possible with reasonable sampling and without field vignetting.

For IRCAM at \(K\) in the rms noise per pixel in 1 second was equivalent to \(K = 17.8\) (Casali et al. 1987) which predicts a 5\(\sigma\) detection at the required depth of \(K = 18.0\) in 5 minutes. This Fig. allows for both shot and read noise making approximately equal contributions because of the exposure being broken down into 10 second segments between readouts. Any longer would saturate the detector from the sky signal, causing non-linearity.

With such a small detector it is necessary to cover the sky in a mosaic pattern, ideally with overlaps to cross-calibrate photometry and astrometry between different nights. A simple rectilinear grid with overlaps was ruled out as there would be insufficient bright stars in the overlaps — stellar distribution models (Bahcall & Soneira 1980) predict only 0.3–1 stars/arcmin\(^2\) in the fields brighter than \(B = 20\).

Thus a sparse rather than a full mosaic strategy was adopted. The images were taken in a 2 \(\times\) 2 pattern around preselected bright stars so that each frame would have the star in one of its four corners — about 10 pixels from the edge. Each quarter was observed on different nights so that the reference star provided a photometric and astrometric cross-calibration. The disadvantage of this approach is that the sky is filled in less efficiently with gaps in regions devoid of bright stars. This introduces a slight bias towards bright stars owing to the effects of star clusters and multiple-star systems. Galaxy positions are not correlated with those of bright stars so they are unbiased by this procedure.

Reference stars were selected from COSMOS scans of U.K. Schmidt plates in equatorial fields well separated around the sky in RA. These were chosen either from Kapteyn’s Selected Areas (Kapteyn 1906) or from areas which were to be surveyed with the LDSS-1 multislit spectrograph by Colless et al. (1990), obtained by private communication. Since the UKIRT mosaicing software limits the maximum offset from a mosaic centre to \(\pm 500\) the areas used were further subdivided into 10\(^2\) \(\times\) 10\(^2\) zones, either by gridding the Selected Areas or using the LDSS-1 sub-field centres.

The candidate reference stars were selected from the plate to a faint limit of \(R = 17–18\), and a bright limit 1–2 magnitudes brighter so that a similar surface density was obtained in each field. The actual stars used were chosen to be well separated so as to minimise the overlap between adjacent IRCAM fields centred on them. Typically there were 10–20 of these per 10\(^2\) \(\times\) 10\(^2\) zone.
The survey was carried out in three observing runs in October 1987, 1988 and March 1988. The limited data obtained in October 1987 were not used in the final survey due to problems with electronic artifacts on images and with bad weather. In the October 1988 run (RA 22:02h) a total of 662 frames, of suitable quality for the final survey, were obtained covering $\sim 366^2$. About 50% of these data were obtained in photometric conditions and the rest were taken in slightly cirrus. In the March 1988 run (RA 09:17h) a total of 205 survey frames were obtained covering $\sim 228^2$. In the October run the frames consisted mainly of $2 \times 2.5$ minute integrations while in the March run single 2.5 minute exposures were taken, thus giving less deep. The individual field centers of the $10^7 \times 10^2$ zones are listed in Table 1 along with their galactic latitude and the centres of all the individual frames, together with exposure times and reference star field IDs, are listed in Table 2.

### 2.2 Data Reduction

As with CCD data, bias and dark frames have to be subtracted from the IRCAM frames. These frames contain considerably more structure than they do in the optical. The dark current ($\approx 100 \text{e}^- \text{sec}^{-1}$) is slightly non-linear with time, so a dark frame of the same exposure as the data is subtracted, rather than scaling a different exposure dark. There is also a small non-linearity in the photon detection rate, the correction for which is well determined and applied automatically at the telescope.

In the infrared the sky is much brighter and the fractional $\sqrt{N}/N$ shot noise consequently much smaller. This means the flatfields have to have higher fractional accuracy to reach this limit. Also the colour of the sky changes due to variable OH emission lines which significantly alters the flatfield on timescales of 10–15 minutes. Because of this, the optimum strategy was to construct a new flatfield every 4–6 frames by median filtering the data frames to remove objects, excluding regions around known reference stars. Using this procedure the final stacked frames had a rms equal to that predicted from shot noise (from sky level as measured off the frame) plus readout noise which implies a flat-field accuracy of $\leq 5 \times 10^{-4}$.

Once the flatfielding was complete a fixed set of bad pixels was removed by interpolation. IRCAM also suffers from intermittent “hot” pixels which vary in sensitivity and location from frame to frame. These were removed in the image detection stage of the reduction (see Section 2.3).

### 2.3 Image detection

Image detection was done, following Tyson 1988, by setting a threshold above the sky background and joining up pixels higher than this into discrete objects. Those objects with area greater than a certain number of pixels are selected. Experimentation showed a threshold of twice the global frame rms and an area cut of 3 pixels to be optimal in selecting objects but rejecting bad pixels.

Because of the large pixel scale it was not found necessary to pre-smooth the images before object detection. However some of the IRCAM images were slightly vignetted resulting in a background gradient across the chip which was removed by applying a low-pass $17 \times 17$ pixel median filter. This also removed some background patterns due to the electronics (see Section 3.4).

### 2.4 Photometry

It was decided to adopt a fixed aperture magnitude scheme for the photometry, this having the advantages that it is simple to define and easy for other observers to check. Deriving total magnitudes using a growth curve and extrapolating to infinite aperture is problematic owing to sky-noise and contamination from other objects. Provided the aperture is more than a few arcseconds the correction will be small for point sources and the magnitude is well-defined and seeing independent for galaxies.

The other common scheme, using isophotal magnitudes, systematically underestimates the total flux of faint objects as the area above the threshold isophote gets smaller with magnitude. Another systematic effect which partially compensates for this is that an isophote can only include positive and never negative sky noise. However isophotal magnitudes are difficult to check or interpret as, unlike fixed aperture magnitudes, the position of the isophote varies depending on the noise.

Two sets of standard stars were used to calibrate the photometry. The UKIRT standard star list provided a list of commonly used bright ($K \approx 6–7$) standards. In addition, fainter stars from Leggett & Hawkins (1988) and from Hawkins (private communication, 1988) were used which provided stars in the North and South Galactic Poles and Hyades cluster areas. These have magnitudes in the range $K \approx 10–14$ and using them meant that the linearity of the detector could be checked to faint magnitudes.

In order to determine the aperture size to be used for the photometry extinction curves of zeropoint against airmass were plotted for a series of increasing apertures (4", 5", 6", 8", 10" and 12"). As the apertures increase the rms scatter about the best fit line tends to a constant due to the diminishing effect of small variations in the seeing.

The infrared and optical data in the October fields had a range of seeing values of 1"–2" respectively. Adopting a 4" diameter aperture for standards and objects and taking a Gaussian PSF from the images this gives typical zeropoint errors of 0.03 magnitudes from the range of seeing involved. In the March data the seeing was a lot worse — as bad as 3–4" in both the infrared and optical data and so an 8" aperture was used for the photometry.

The data taken in non-photometric conditions had to be calibrated against the photometric data using the reference stars. In some cases these were not bright enough to do this accurately as the original Schmidt plate selected stars were too blue to be bright in $K$.

To calibrate these data it was decided to interpolate the zeropoints from neighbouring frames with brighter reference stars. The zeropoint was determined for each frame where the reference star was brighter than 10σ using the median magnitude of the star from photometric nights. Next, the zeropoints were smoothed by taking the median value of magnitude of the star from photometric nights. The typical scatter of the zeropoints about these median values was $\sim 0.1$ magnitudes as expected, thus the final zeropoint is good to $0.1/\sqrt{5} = 0.04$ magnitudes. Finally, the zeropoints of frames with reference stars fainter
than 10σ were interpolated from the smoothed median values.

2.5 Mosaicing

Once the zeropoints were determined, up to 8 sub-images had to be mosaiced together — 2 × 2.5 minute exposures of each 2 × 2 mosaic. Using the reference star centroids, the sub-images were registered to the nearest pixel. Higher accuracy was found not to be necessary as the seeing was typically rather greater than 1 pixel. The resultant flux in each mosaic pixel was computed as the sum of the counts in each sub-image weighted by the image rms and scaled according to the difference in zero-points.

Image detection and photometry for the mosaiced images was similar to that for the individual sub-images. One complication is that some portions of the mosaic with different overlap factors have different amounts of noise and this had to be allowed for in the image detection, though this was not so important for the photometry. This was done by dividing each image by a "mask" image, whose pixel values reflected the local noise average and then using the result for the image detection. The photometry was performed on the original image. The mask was constructed by replacing each pixel with the square root of the median value in a local box of the squared flux — because the background is subtracted and the mean is zero this forms a robust estimator of the rms. It was found by experimentation that a 9 × 9 box worked best and was least sensitive to objects while correctly reflecting the local noise.

As well as the overlaps internal to the 4-mosaics there was a small amount of overlap between some adjacent 4-mosaics which resulted in the same object appearing twice in the final catalogue. To remove these duplicates (3% of the objects), objects within 3″ of a given object on a different 4-mosaic were deleted from the catalogue. It should be noted that this means that pairs would not be detected across 4-mosaic boundaries.

2.6 Astrometry

The objects on each IRCAM image were matched up with objects detected on COSMOS scans of UKSTU plates to derive the coordinate transformation between the two systems. As there were few objects on each IRCAM image only a 4 parameter orthogonal transformation was used (rotation, scale, x and y offset). This is non-linear but for small angles and scale changes is approximately linear, so a least squares solution was fitted to the residuals and iterated.

Once the rotation and scale had been determined for each individual frame with sufficient objects, global median values were determined for these parameters and the fits repeated to determine the best fit offsets. The rms position residuals which resulted from this process were typically 0.3–0.7″ as expected from the typical measurement accuracy of the UKSTU plates and the 1 pixel registration.

3 CCD IMAGING OBSERVATIONS

Optical CCD images were made of almost all the fields surveyed in the infrared. This allows the study of the colours of the infrared-selected sample and the relation to optically-selected samples. Also optical imaging allows the possibility of more reliable star-galaxy separation based on the relative sizes of star and galaxy profiles (see Section 3.5).

3.1 Observations

All the CCD observations were performed on the 2.5 m Isaac Newton Telescope (INT) on La Palma over the period in April 1988, September 1988 and April 1989 using the RCA 512 × 320 chip at a 0.74″ pixel scale.

Because of the high density of IRCAM fields in the October data it was decided to cover the 9 existing 10′ × 10′ fields using an overlapping 2 × 4 grid pattern. In contrast the March fields had a much lower density and the optimum strategy was to place the CCD so as to maximize the number of IRCAM fields. The overlaps were still quite large and could be used for cross-calibration. Additionally a number of positions with short exposures and large overlaps were taken in photometric conditions to facilitate calibration.

Complete coverage of all the IRCAM data was obtained in R, in addition complete B coverage was obtained in the March fields (V and I were also observed for a limited subset of the March fields). A few of the October fields were also covered in B. Note only the field 851STARS,2 did not have any optical coverage, if this field is excluded the infrared survey area is 551.9′. Table 3 summaries the magnitude limits reached and the equivalent noise levels.

The images were reduced following standard CCD procedures. They were bias and dark frame subtracted and then divided by a flatfield. A master flatfield was constructed for each night by median filtering the normalized data frames in each band and the photon shot noise limit was reached. The RCA chip had one bad column which was interpolated over.

Standard stars were selected from the list of Landolt (1983) to match the colours typical of faint galaxies. Fitting a linear extinction law gave a rms residual of 0.01–0.05 magnitudes for photometric nights, and the values of the absolute zeropoint (extrapolated to zero airmass) and extinction were consistent between nights and runs. A substantial colour term between $B_{\text{LAN}}$ and $B_{\text{CCD}}$ was found, but not in $V$, $R$ or $I$. The relation defined by:

$$B_{\text{LAN}} = B_{\text{CCD}} - \beta (B - R)_{\text{LAN}}$$

$$R_{\text{LAN}} = R_{\text{CCD}} - \beta (R - I)_{\text{LAN}}$$

where $\beta$ was consistent between nights and observing runs and has a mean value of −0.082. Tabulated magnitudes of the catalogue objects are given as $B_{\text{CCD}}$ as not all objects were paired in $B$ and $R$.

3.2 Image Detection and Photometry

The image detection proceeded exactly as described in Section 2.3 for the infrared images. Because of the smaller pixels (0.74″) the object profiles were better sampled, so fewer real objects were lost and fewer spurious features were picked up as verified by visual checking of the images. Some of the frames suffered from background variations induced by the presence of dust on the camera window. These were effectively removed by subtraction of a 20″ × 20″ median filtered
Aperture photometry was performed on the objects using a 4" diameter aperture for the October data and 8" for the March data and object catalogues were constructed for each image. These were then paired with Schmidt plate data to obtain the rotation and scale transformations as described in Section 2.6 for the infrared data.

### 3.3 Cross-calibration of CCD data — a new non-iterative technique

As only 30% of the nights were photometric it was necessary to cross-calibrate large parts of the photometry via objects in common between overlapping frames. It was found that the most robust method, of determining the magnitude zeropoint offset was to take the median magnitude difference of the brightest 10 objects, matched with a 3" tolerance. Using a median made this insensitive to the presence of ≤1–2 matching errors per frame. Cross checking photometric data it was found that the typical shifts were of the order ±0.05 mags between frames.

For the October fields, which had a simple 2 × 4 grid arrangement of frames, the non-photometric images were matched up with adjacent frames according to the arrangement in the grid. The zeropoint errors of each matching were found to be of order 0.05 magnitudes. Not every frame was immediately adjacent to a photometric frame. The maximum number of frames matched across to calibrate a particular image was 4 in a few extreme cases — this would give a systematic error of 0.1 mags in the zeropoint at most.

The geometry of the March fields was considerably more complex so a more general method of cross-calibrating data was devised. This involves allowing the zeropoints of all the non-photometric frames to be free parameters and then finding a least-squares solution for them which minimizes the magnitude overlap residuals via a matrix inversion.

Consider \( n \) frames of which \((1 \ldots m)\) are uncalibrated and \((m+1 \ldots n)\) are calibrated and let \( \Delta_{ij} \) be the magnitude difference between frames \( i \) and \( j \).

\[
\Delta_{ij} = (\text{Mag}_i - \text{Mag}_j) \text{pairs} \quad \text{(Note: } \Delta_{ij} = -\Delta_{ji})
\]

Let \( a_i \) be the floating zeropoint of frame \( i \), with \( a_i = 0 \) if \( i > m \).

If we define the overlap function:

\[
\theta_{ij} = \begin{cases} 
1, & \text{if frames } i \text{ and } j \text{ overlap} \\
0, & \text{if no overlap} \\
0, & \text{if } i = j
\end{cases}
\]

Then the sum of squares to be minimized is:

\[
S = \sum_{i=1}^{n} \sum_{j=1}^{n} w_{ij} \theta_{ij} (\Delta_{ij} + a_i - a_j)^2
\]

where \( w_{ij} \) are the weights used for each match. In this case they were set equal to unity although they could, for example, be set according to the errors on each overlap.

Differentiating with respect to \( a_i \) gives the matrix equation:

\[
\sum_{j=1}^{m} A_{ij} a_j = b_i
\]

where

\[
A_{ij} = w_{ij} \theta_{ij} - \delta_{ij} \sum_{k=1}^{n} w_{jk} \theta_{jk} \\
b_i = \sum_{j=1}^{n} w_{ij} \theta_{ij} \Delta_{ij}
\]

This gives a single-step solution for the \( a_i \). It is not required that all the CCD frames be contiguous. If there are disconnected groups of frames then the matrix becomes block-diagonal and the different solutions become independent of each other. If there are uncalibrated frames which do not join up in any way onto calibrated frames then the matrix becomes singular so it cannot be inverted. These frames have to be removed from the list.

The overall rms residual (\( \epsilon \)) of the solution is given by:

\[
\epsilon = \frac{\sum_{i=1}^{n} \sum_{j=1}^{n} w_{ij} \theta_{ij} (\Delta_{ij} + a_j - a_i)^2}{\sum_{i=1}^{n} \sum_{j=1}^{n} w_{ij} \theta_{ij}}
\]

A simple example is useful in order to clarify this technique. Consider the arrangement of CCD images shown in Fig. 1, where the * marks the calibrated images. The matrix equation in this case is:

\[
\begin{pmatrix} -2 & 1 & 0 & 0 \\
1 & -2 & 0 & 0 \\
0 & 0 & -1 & 1 \\
0 & 0 & 1 & -2 \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{pmatrix} = \begin{pmatrix} \Delta_{12} + \Delta_{16} \\ \Delta_{21} + \Delta_{26} \\ \Delta_{34} \\ \Delta_{43} + \Delta_{45} \end{pmatrix}
\]

which results in the solutions:

\[
\begin{align*}
a_1 & = \frac{2}{3} \Delta_{61} + \frac{1}{3} (\Delta_{21} + \Delta_{62}) \\
a_2 & = \frac{2}{3} \Delta_{62} + \frac{1}{3} (\Delta_{12} + \Delta_{61}) \\
a_3 & = \Delta_{43} + \Delta_{54} \\
a_4 & = \Delta_{54}
\end{align*}
\]

These results make intuitive sense. In the left-hand case the offsets are linear combinations of the offsets along two different paths from the calibrated image. In the right-hand case there is only one path and the offsets are just the sum of the individual offsets along that path. The matrix has decomposed into block-diagonal form as the two groups of frames are disconnected.

The rms magnitude residual on each image from different overlaps can be computed as:

\[
\text{rms}_i = \frac{\sum_{j=1}^{n} w_{ij} \theta_{ij} (\Delta_{ij} + a_j - a_i)^2}{\sum_{j=1}^{n} w_{ij} \theta_{ij}}
\]
The typical rms residual was found to be 0.02–0.04 magnitudes, depending on band. Histogram plots made for each band confirmed that there was no problem with anomalously large values.

In conclusion it appears that the zeropoint errors on the non-photometric data have been determined to < 0.05 mags by two similar methods in the October and March fields.

3.4 Multicolour Catalogues

Multicolour catalogues were made by successively pairing the infrared catalogue with the optical catalogues for the different bands, always based on the infrared position. As the nearest object to the infrared position was taken this automatically removed objects multiply covered in the CCD data. The final paired infrared-optical magnitudes are shown in Table 4. For the October data the optical coverage was complete in $R$ (except for 581 STARS) but only partial in $B$. Out of the 576 infrared objects, 167 of them (the “$K$-only” objects) were detected solely in the $K$-band and not in $R$ or $B$; for some reason there were infrared detections which were not present at $R < 23$. Of course if these were genuine galaxies they would have to be very red ($R - K \geq 6$) and so they would be very interesting as candidate protogalaxies (see Introduction).

The images were examined visually to try to ascertain whether these were due to any obvious flaw in the reduction procedures. Most of the $K$-only objects (∼ 95%) were found not to be real, but comprised a variety of image artifacts:

1) Hot pixels manifest themselves as spikes consisting of usually 1, but occasionally a group of 2–3, bright pixels. Despite the 3-pixel area cut in the object detection many of them still make it into the catalogue. Some frames seemed to have large-scale correlated noise streaks caused by problems with the readout electronics. These can be broken up into a large number of spurious objects by the image detection algorithm. At the detection stage this was partially compensated for by the subtraction of the median-filtered sky background. The worst images were not used.

2) Due to the faint isophote scheme used in the object detection a number of close objects would be separately detected at $K$ but not in the optical. This is because for bright $K$ objects would be many magnitudes above the sky in the optical, so merging into a single $R$ detection leaving one surplus $K$ detection.

3) The other major cause of artifacts is multiple reflections within the camera which allows bright stars to produce secondary ghost images. This can be checked by re-observation with the image on a different portion of the detector. If it was a ghost it will not have moved to the correct place.

From the most promising “$K$-only” objects seven were re-imaged in service observing time to check on the reality of the detections, — none of these objects were re-confirmed so it appears most likely that none of the $K$-only detections correspond to genuine objects.

Given this knowledge, for the March data it was decided to cut deeper into the signal/noise in object detection and avoid the extra spurious objects by tightening the optical pairing constraint. The final catalogue was generated with detection thresholds > 2.5σ (equivalent to $K = 17.3$ in a 4′′ aperture) and 2′′ pairing tolerance and contained 663 objects of which 281 were detected in at least one of the optical bands. If the catalogue is restricted to 5σ then there are 312 objects of which 195 were paired in at least one optical band. 74 of these $K$-only objects were checked visually (the other 43 being due to the lack of optical coverage in one field) and only 4 emerged as candidate objects, the other being ghosts, hot pixels, etc.

It is possible to get an upper limit to the contamination of the paired objects, if it is assumed that all the $K$-only objects are spurious. Given a $R$-band optical density of ∼ 8 objects/arc min down to the flux limit, and the pairing box size, then the expected number of spurious $K$ pairs is $\approx 0.04 \times$ the number of spurious $K$-only objects. Thus in the March field catalogue (256 $(K, R)$ pairs) there are ∼ 15 spurious pairs. This can be checked by increasing the pair box to 3″, as the positional accuracy is good to 0.5″ then the increase in $(K, R)$ pairs should be predominately due to the random matching up of genuine $R$ objects with spurious $K$-only objects. Thus ∼ 19 extra pairs are expected whist 17 are found, which confirms the estimate of the contamination rate.

3.5 Star-Galaxy Classification

Automated classification of objects into stars and galaxies was performed using a technique based on image size. The area, in pixels, above the threshold isophote was determined for each catalogue object. At bright magnitudes galaxies cover a much larger area than stars, so a histogram of objects against area is strongly bimodal. At fainter magnitudes the galaxy peak moves down in area and merges with the stellar peak as the galaxies become unresolved. For each object, with magnitude $M$ and on frame $j$ a parameter $y$ was calculated:

$$y \equiv \log_{10} \text{[Object area]} - \log_{10} [A_\star (M, j)]$$

where $A_\star$ is the averaged area of an object on the stellar locus at that magnitude on the same image. Since the point-spread-function varies from frame to frame this locus is calculated separately for each one.

The star-galaxy classification was performed using the paired $R$-band CCD data rather than directly on the $K$ data as each CCD image had a much larger numbers of objects per frame (several hundred as opposed to 1–2) which made the determination of the stellar locus less noisy. The CCD data also had a smaller pixel scale and generally better seeing.

The procedure adopted was to read in all the optical objects (not just those paired with infrared objects) and bin each image in magnitude and $\log_{10}$ [image area]. Next the binned data were smoothed slightly along the magnitude axis by applying a top hat filter of width 1 magnitude. Then the stellar locus in each magnitude bin and image was determined from the objects with least area. It was taken as the area value of the second smallest object if the total number of objects in the magnitude bin was > 10 or else the lowest object.

Fig. 2 shows plots of $y$ vs $R$ magnitude. Note that for $R > 20$ only a third of the points are plotted — this is done so the structure in the plot can be more easily discerned among the rapidly increasing number of objects. It
can be seen that there is a well defined stellar locus down to at least $R \simeq 20$. Fainter than this the galaxies become unresolved (for 2-$''$ seeing) and the quantisation of area becomes apparent in the plots.

A conservative (in the sense of including stars rather than missing galaxies) value of $y=0.3$ was chosen to separate stars from galaxies. Additionally in the final selection all objects which were saturated on the $R$-band CCD image were classified as stars because galaxies do not have a high enough central surface brightness to saturate for these observations.

Though this procedure is crude and rather semi-empirical it does serve as a useful measure to ensure that not too many stars are observed in subsequent spectroscopy at bright magnitudes ($R < 20$). It is not intended to be, and does not need to be an accurate measure, as the cut is conservative and a representative fraction of objects classified as “stars” were included in the spectroscopy in any case as a check.

### 3.6 Summary Final Catalogue

A total area of $552''$ (after correcting for overlaps) was surveyed with 826 IRCAM images. Using a median filtering technique the images were flattened to $\lesssim 1$ part in $10^4$ and all the frames were essentially limited by sky shot and readout noise. The limiting depth was $K = 17.5$ (3-$\sigma$ in a $4''$ aperture) over 64% of the area and $K = 17.1$ over the rest. The 1-$\sigma$ surface brightness limit of the survey was thus $19.7-20.1$ mags/$''$.

634 objects (including 179 reference objects) were detected and matched up with optical images. These are listed in Table 4 together with the optical magnitudes and star-galaxy classification derived in section 3. Using overlaps between photometric and non-photometric data the magnitude zeropoints on all the frames were determined to $\lesssim 0.1$ mag and photometric catalogues were constructed with a positional accuracy of $\simeq 0.5''$.

### 4 K-BAND STAR AND GALAXY COUNTS

Galaxy counts are a classical tool of cosmology. The determination of the 2-$\mu m$ counts is important because, in principle, they are much less sensitive to the evolution of stellar populations than the optical galaxy counts. This allows them to be used as a direct test of cosmological models as well as to probe galaxy evolution. There has been little work done in this area until recently, because the recent arrival of infrared array detectors only now allows a large area to be surveyed quickly.

There are several important points addressed directly by the $K$-band counts:

1) **Geometry.** The insensitivity to evolution means the counts are an excellent probe of the geometry of the Universe. It may be possible to obtain new constraints on $\Omega_0$, and also test non-standard world models such as those including the Cosmological Constant.

2) **Normalization.** One problematical issue in galaxy count studies is the true local space density of galaxies. The range of estimates from differing surveys typically vary by a factor of two (Efstathiou *et al.*, 1988).

For number-magnitude count predictions this normalization is usually allowed to float — instead the models are normalized to the data at some intermediate magnitude. This of course affects the size of the excess over no-evolution and, hence, the amount of evolution required. The problem is that if the models are normalized at faint enough $B$ magnitudes to avoid problems from local fluctuations in galaxy density, then one is sufficiently deep that spectral evolution is already important. As there should be less spectral evolution in $K$ it would be possible to obtain a more reliable normalization, on larger scales, and transfer this to the $B$-band to determine just how much evolution is actually needed.

2) **Extinction.** Light at 2-$\mu m$ is relatively insensitive to dust extinction — $A_K \simeq 0.1A_V$. And unlike longer far-infrared wavelengths, such as those surveyed by the IRAS satellite, there is little thermal emission from warm dust $\lesssim 100$ K as seen in many starburst galaxies (Rowan-Robinson & Crawford 1991).

3) **Luminosity Evolution.** The $K$ data should allow the quantification of just how much luminosity evolution in the $B$ light of galaxies, is allowed. This should manifest itself in a greater excess over no-evolution in the $B$ number-magnitude counts than in $K$, and, equivalently, in the evolution of the $B-K$ colours. Also by selecting galaxies in a band insensitive to star-formation it should be possible to find out the true fraction of objects with strong emission lines such as [OIII], which make up such a large proportion of the Broadhurst *et al.* (1988) and Colless *et al.* (1990) samples.

4) **Density evolution.** The $K$ data also provide a probe of the amount of evolution in the space density of galaxies. This will cause an excess in the number-magnitude counts over no-evolution which, unlike luminosity evolution, would be the same in $B$ and $K$.

### 4.1 Observed K-band counts and corrections.

The raw counts of galaxies and stars are listed in Table 5. In total there are 287 galaxies and 167 stars in the all the fields. These are counts for objects paired in the $K$ and $R$ bands; as is discussed in Section 3.4 there is no evidence for any of the unpaired $K$ detections being real. The counts are given in bins of 1 magnitude, the completeness is discussed below. Poisson errors are assumed for these — the real error will, of course, be somewhat larger due to the effect of clustering but this is not large compared to the Poisson error. This is discussed further in section 5.2).

The objects used as astrometric and photometric references in the construction of the infrared survey (column “R” in Table 4) present a significant difficulty in the count analysis. Clearly, as a result of using references in this way, the survey is biased to contain too many bright objects. However, it is not correct simply to exclude these objects from the analysis: in the limit that the mosaic survey became a completely filled area, all the reference objects would clearly be retained as genuine survey members. In general, for a survey with a filling factor $f$, it is obvious that the the reference objects should be given a weight which is on the order of $f$ ($\sim 20$ per cent in our case). We now show how to make a precise estimate of the weight to apply in practice.
For this analysis, we need to know three things: (i) the surface density of candidate reference objects, from which we can estimate the number of objects of this sort expected in an unbiased survey, \( \bar{N} \); (ii) the number of such objects actually used as references, \( N_R \); (iii) the number of such objects which were serendipitously included in the survey, in addition to the preselected references, \( N_S \). The correct weight to give to the reference objects is then clearly just the fraction of the reference objects needed to supply the difference between the expected number of reference-type objects and the number included serendipitously:

\[
w = \frac{\bar{N} - N_S}{N_R}.
\]

The relevant numbers are as follows: the surface density of candidate references gives \( \bar{N} = 218 \). \( N_R \) was 207 and \( N_S \) was 144, yielding an estimate of \( w = 0.36 \). The correction must be applied using this value of \( w \) for all \( K \) bins, although the actual value of the correction to each \( K \) bin obviously depends on the number of reference stars and galaxies in the bin, which is listed in Table 5.

The magnitude of these upward corrections is highest at intermediate \( K \) magnitudes as expected from the selection criteria used for the stars. It peaks at \( +2.2\sqrt{N} \) (where \( \sqrt{N} \) is the random Poisson error) in the brightest bin containing any galaxies (\( 14 \leq K < 15 \)) - this is where the most reference objects lie compared to the rapidly diminishing number of field galaxies. In all other bins the systematic reference object correction is \( \lesssim \) the random error so the effect on the number-magnitude relation is small.

Star-galaxy separation is vital to the counts as stars outnumber galaxies for \( K < 16 \). The reference stars provide a useful independent check on the \( R \) CCD star-galaxy classification as they were independently classified by the COSMOS image analysis software based on the Schmidt plate data. There were 37 “galaxies” out of 116 reference objects in the October fields, of which 3/21 were later identified spectroscopically as stars. In the March fields the galaxy fraction was 11/63; none of these “galaxies” were examined spectroscopically. It appears that the misclassification rate of the CCD star-galaxy separation is consistent between the reference objects and the rest (see below), and that the COSMOS classification is much less reliable than the CCD classification.

The CCD classification failure rate can be estimated from early spectroscopic results, to be fully presented in Paper II. We can define parameters \( \alpha \) as the fraction of “stars” which turn out to be really galaxies and \( \beta \) as the fraction of “galaxies” which turn out to be really stars. For the entire identified spectroscopic sample (\( R \lesssim 21 \)) \( \alpha = 4 \pm 2\% \) and \( \beta = 20 \pm 6\% \), averaging over all \( K \) magnitudes, the errors being based on Poisson statistics. \( \beta \) is much larger than \( \alpha \) because a deliberately conservative star-galaxy cut was chosen for completeness reasons, i.e. to include stars rather than lose galaxies. The effect of this error on the normalization of the galaxy counts is about half that of the Poisson errors in each magnitude bin. However what is most important is the differential change in star-galaxy misclassification with \( K \) magnitude — the slope of the galaxy relation is much more important than the normalization. This misclassification can be corrected for by using the relations:

\[
N'_g = N_g(1 - \beta) + \alpha N_s
\]

\[
N'_s = N_s(1 - \alpha) + \beta N_g
\]

where \( N_s \) and \( N_g \) are the original star and galaxy numbers in each magnitude bin and \( N'_g \) and \( N'_s \) are the corrected values (note \( N'_s + N'_g = N_s + N_g \) of course). The errors \( \Delta \alpha, \Delta \beta, \sqrt{N_s} \) and \( \sqrt{N_g} \) can all be folded in to give errors on \( N'_g \) and \( N'_s \) which are now slightly in excess of their Poisson values. (In the figures that follow the corrected error bars appear smaller. This is not in fact the case as this is a log plot: the actual absolute errors increase).

\( \alpha \) and \( \beta \) should be strong functions of \( R \), the band used to do the separation. To correct the counts we need estimates of average values in the different \( K \) ranges. It can be seen from Fig. 3, which shows the classification image-area parameter \( y \) against \( K \) magnitude, that the star and galaxy loci are well separated down to \( K \simeq 15 \) and that the misclassification should apply mostly in the \( K > 15 \) regime. In the spectroscopic sample 12 out of 15 of the misclassified objects had \( K > 15 \). For the final corrected counts all objects with \( K < 15 \) (94) were inspected visually to check for errors and only 4 were found. The \( K > 15 \) bins were corrected using the values \( \alpha = 7.7 \pm 7.7\% \) (1 object) and \( \beta = 23.3 \pm 8.8\% \) derived from the spectroscopy in this magnitude range. These values are consistent with the \( K < 15 \) values, and those for the entire sample, indicating that any systematics in \( \alpha \) and \( \beta \) are of lesser importance than the random error, which is incorporated into the final error.

Another issue to be considered with regard to the reference objects is the effect of galaxy clustering — since 27\% of our reference objects are unintentionally galaxies clustering will introduce an excess of nearby objects. This can be easily checked by comparing the raw counts in fields with and without galaxies as references — this is shown in Fig. 4. It can be seen there is no significant clustering effect.

Table 5 shows the final counts of stars and galaxies after applying these corrections, together with the propagated errors. These counts are plotted in Fig. 5 which compares stars and galaxies.

There are also, in principle two more corrections to be made:

1) The true counts are multiplied by a selection function which represents the probability of detection as a function of magnitude. Because the objects are \( 2\sigma \) detections in at least 3 pixels this produces a set of peak flux selected objects.

2) Once the objects are detected then the magnitude is measured in a larger aperture introducing a magnitude scatter that introduces a Malmquist-like correction (Eddington 1940) that biases the observed distribution away from the true one.

Because our detections have high significance we expect the size of these corrections to be small for \( K < 17 \). Because they are small we can estimate their amplitude by using the noise and detection parameters for the different parts of the survey and making a rough estimate for the number-magnitude relation fainter than \( K = 17 \).

At \( K = 17 \) the log number-magnitude slope is close to \( \simeq 0.45 \); if we allow a generous range of slope from 0.3 to 0.6 we get a range of integrated correction for the final
16 < K < 17 bin from +2% to −1%. Since these corrections are negligible compared to the random error we do not apply them. For K > 17 the incompleteness rapidly becomes very large as the survey limits are approached and we do not plot the counts at these magnitudes.

The final corrected star counts are also shown in Fig. 6 split according to the different galactic latitudes in the survey. Although the reference stars have been allowed for it should be born in mind there could still be a residual bias in these counts from stellar binaries and clusters. Nevertheless, the star counts can be compared with the model of Bahcall & Soneira (1980), these were modified to the infrared by I.N. Reid (private communication, 1990). Two models are shown both consisting of various stellar components:

1) “Normal model” — an old disk stellar with age 2–10 Gyr, scale height of 350 pc, luminosity function from Reid (1987) and weight at zero scale height (w) = 0.80. Plus an intermediate age disk (0.3–2 Gyr, 250 pc, w = 0.17), plus an old extended thick disk (2–3 Gyr, 1000 pc, w = 0.015) and plus a halo component with local density 0.15% of the disk and disk-shaped luminosity function.

2) “Extended flat LF” — the same as (1) except that the luminosity function is assumed to be flat beyond $M_V \geq +13$ (Dahn et al., 1986) which gives the maximum possible contribution of low-mass late-type red M-stars.

These are also shown in Fig. 6 and it can be seen that in most cases the agreement is good though the normalization appears to be slightly too high in some cases. This is worse in the $09^h$ field which is also the field at lowest galactic latitude. Though clearly these first direct K star counts have insufficient objects to distinguish these two models it is encouraging that they agree so well with predictions made by an extrapolation of optical stellar data.

5 CONSTRAINTS ON GEOMETRY AND GALAXY EVOLUTION FROM THE K COUNTS

Fig. 7 shows the K-band counts determined from this survey together with those of Cowie et al. (1990) and Cowie (1991). Cowie et al’s strategy is complementary to ours as their program was to go very deep (∼50 hours integration) in tiny fields (∼4°). Together, our surveys define the galaxy counts very well over 8 magnitudes. Cowie et al. perform star-galaxy separation based on colours, but their stellar contribution is not significant in this magnitude range: the star counts reach a plateau at $K = 16$ while the galaxies continue to increase. Also shown is the number-magnitude determination of Jenkins & Reid (1991) based on a confusion limited survey of 113° to $K = 19$ using the single channel photometer UKT9 on UKIRT with a 19.6′′ diameter aperture. Jenkins & Reid analysed the flux density distribution and fitted a parametric form to $n(m)$ (after allowing for predicted star counts), ±1σ errors being determined from Monte-Carlo simulations.

Their final results, with errors, are shown in Fig. 7 as dotted lines. Note there is a very satisfactory agreement between the 3 independent determinations in the slope. The survey area of Jenkins & Reid is 113°, this is distributed over 11 widely separated patches of sky in an attempt to get a fair sample. The work presented here is based on 552° and is almost equally distributed over 6 patches spaced around the celestial equator at high galactic latitude. For the present purposes the normalization based on the larger area will be adopted.

The optical galaxy counts are shown in Fig. 8. This is a new compilation of the most recently available data and matches well previously published compilations. The faint points are taken from the deep CCD counts of Tyson (1988), Lilly et al. (1991) and Metcalfe et al. (1991). At intermediate magnitudes are the counts of Jones et al. (1991) based on deep AAT plates. The brightest points are taken from the two largest surveys in existence, both based on southern UK Schmidt plate data, the 946° Edinburgh-Durham Southern Galaxy Catalogue (Collins & Nichol, 1991) and the 4300° APM Galaxy Survey (Maddox et al. 1990). Note these two surveys are not independent: the EDSGC sky coverage is a smaller subset of the APM survey, analysed independently.

The errors bars are determined from the observed field-rms scatter where given. All the counts have been transformed to the Schmidt $b_J$-band using the transformations given in the references. The plate data are for isophotal magnitudes (see references for surface brightness limits) while the CCD data represent total magnitudes. The agreement between the normalization of different determinations is now of the order of $\leq 50\%$, much better than in previous compilations of older data.

5.1 Non-evolving prediction

The first step is to compare the counts to the prediction of a model which assumes an unchanging galaxy population. For this calculation two ingredients are required: the K-correction and the local galaxy luminosity function.

The K-correction corrects luminosity distances for the redshifting of a galaxy’s spectrum through the fixed observed passband. It is given by:

$$K(z) = -2.5 \log_{10} \left[ \frac{1}{(1+z)} \int_{-\infty}^{\infty} T_\lambda f_\lambda \frac{d\lambda}{(1+z)} \right]$$

where $T_\lambda$ is the bandpass transmission as a function of wavelength and $f_\lambda$ is the galaxy’s spectrum. This spectrum was taken from the final epoch of the evolving galaxy models generated by Rocca-Volmerange and Guideroni (1988). At late times these approximately match the optical and near-infrared colours of local galaxies and provide a similar optical K-correction to that of Broadhurst et al (1988). In the K-band the K-corrections of the different types are very similar and the no-evolution model is thus insensitive to the exact mixture.

The type-dependent luminosity function was taken from King & Ellis (1985) with the relative mix of morphological types calculated from that observed in the local population ($b_J < 16.75$) by Shanks et al. (1984). This is summarized in Table 6. This luminosity function is very similar to that used by Broadhurst et al. and those of Efstathiou et al. (1988) and Loveday et al. (1992) though they found less type dependence. The K-band luminosity function was constructed by applying model $B-K$ colours to the $B$-band luminosity...
function — this is very similar to the \( K \) luminosity derived from the Durham-AAT redshift survey (R.M Sharples, private communication, 1990).

The normalization, \( \phi^* \), is usually derived from local redshift surveys along with the rest of the luminosity function, and Efstathiou et al. (1988) do this for five different surveys. They gave a mean \( \phi^* \) of \((1.56 \pm 0.34) \times 10^{-2} \ h^3 \text{Mpc}^{-3}\) although the dispersion among estimates from the differ- ing surveys amounts to a factor of two. Additionally from a large APM-selected redshift survey Loveday et al. (1992) find \( \phi^* = (1.4 \pm 0.17) \times 10^{-2} \ h^3 \text{Mpc}^{-3}\). An alternative approach is to choose a value of \( \phi^* \) which normalizes the prediction to the bright end of the counts. Note that the \( b_j \) and \( K \) normalisations are, in principle, not independent — the same total number of galaxies integrated down to zero flux must be seen in \( b_j \) and \( K \), this requires the same value of \( \phi^* \) to be used.

Taking \( \phi^* = 1.5 \times 10^{-2} \ h^3 \text{Mpc}^{-3}\) the \( K \) and \( B \)-band no-evolution models are plotted with the counts in Fig. 7 and Fig. 8. The \( b_j \) counts show the well-known result of a huge galaxy excess compared to the non-evolving prediction. If the curves are normalized as shown then this excess is a factor of 3 at \( b_j = 22 \) and 10 at \( b_j = 28 \). It can be seen at once that the observed \( K \) counts are a much better match to the non-evolving prediction than are the counts in \( b_j \). This follows the trend seen in optical counts in other bands (e.g. Tyson 1988) where the galaxy excess is greatest in \( b_j \) and progressively less in \( R \) and \( I \) filters.

It can be seen that changing to a non-standard world model — e.g. introducing a cosmological constant or using a "Tired light" model of the redshift would not simultaneously rectify both the \( K \) and \( B \)-band counts. While increasing the volume element would increase the number of faint \( B \) galaxies, this would have the same effect in \( K \) and thus overpredict the counts in this band.

We shall not consider the blue counts in any detail in the remainder of this paper. It is clear that any explanation for the excess of blue galaxies must involve some star formation to give the galaxies additional ultraviolet output at moderate to high redshifts. Only a small number of high-mass stars are required to accomplish this. However such an event will have very little effect on the properties of galaxies in the infrared. The \( K \)-band counts are therefore much better suited to setting constraints on cosmological geometry or cosmological evolution of the mass function. We consider these topics below.

### 5.2 Galaxy Clustering and count normalization

So far, we have not considered the extent to which our counts (and those of others) could be in error owing to galaxy clustering; different areas of sky will have counts which differ from the global average, and we need to estimate how large these fluctuations can be. This is relatively straightforward in principle, given some knowledge of the power spectrum for galaxy clustering. The process of making a magnitude-limited survey over some area of the sky corresponds to a convolution of the galaxy density field with some window function. Hence, the fractional variance \( \langle \sigma^2 \rangle \) in the resulting number of galaxies is given by an integral over the power spectrum times the azimuthally averaged squared transform of the window function determined the volume sampled:

\[
\sigma^2 = \int \Delta^2(k) \frac{dk}{k} \langle |W_k|^2 \rangle.
\]

We have used a dimensionless notation for the power spectrum: \( \Delta^2(k) \) is the contribution to the fractional density variance per \( \ln k \) (see Peacock 1991). The window function is a product of radial and angular selection functions:

\[
W(r) = \phi(r) f(\theta), \quad \text{so that} \quad k\text{-space window is}
\]

\[
W_k = \int \frac{\int r^2 \phi(r) f(\theta) e^{ikr} \, dr \, d\theta}{\int f(\theta) \, d\theta \int r^2 \phi(r) \, dr}.
\]

We assume here an \( \Omega = 1 \) model and take \( r \) to be the comoving radius, so that the comoving spatial geometry is Euclidean. To evaluate the necessary integrals for our exact angular selection and realistic radial selection is a tedious exercise. Fortunately, insight into what is happening can be combined with acceptable accuracy in a simple analytical model.

For the radial selection, \( \phi(r) \propto r^{-1/2} \exp(-[r/r^*]^2) \) is often adopted as a reasonable approximation to the integral Schechter function. For the final \( k \)-space result, it is adequate to model the angular selection as convolution with an angular Gaussian: \( f(\theta) \propto \exp(-\theta^2/2R^2) \), where \( R^2 \approx A/12 \) approximates the effect of a square of area \( A \) (this relation between \( R \) and survey area is almost independent of the exact survey geometry in simple cases). We consider the appropriate value of \( R \) for our data below. If we further assume that the angle \( R \) is small, then an excellent approximation (\( \leq 1 \) per cent error) to the azimuthally-averaged value of \( |W_k|^2 \) is

\[
\langle |W_k|^2 \rangle = \left[ 1 + (kr^*)^2/2 \right]^{-2} \left[ 1 + (kr^*)^2/3 \right]^{-1/3}.
\]

Even for an exact evaluation of the window for a square survey region, this formula is within about 10 per cent of the correct answer. The above expression shows that the power is reduced by one power of \( k \) for wavelengths less than the depth of the survey, and by four additional powers of \( k \) if the wavelength is also smaller than the typical transverse size of the survey cone. For realistic power spectra, the dominant fluctuations will be those where the second type of filtering starts to become important.

We have applied the above expression to a simple model for the clustering power spectrum, which corresponds to a power-law correlation function \( \xi(r) = (r/r_0)^{-\gamma} \):

\[
\Delta^2(k) = 0.903(kr_0)^{-\gamma}.
\]

We take a slope of \( \gamma = 1.8 \) and a normalization of \( r_0 = 5 \ h^{-1} \text{Mpc} \). Empirically it appears that the observed power spectrum is convex on large scales (Peacock 1991), so using a power law will be conservative in the sense of overestimating the effect of clustering on the galaxy counts. Provided \( R \) is small, the integral can be performed to yield

\[
\sigma^2 = 2.36 \left( \frac{r_0}{R^2} \right) \left( \frac{r_0}{R^2} \right)^{0.8}.
\]

We can now put some numbers into this equation: the bright blue counts are based on the APM survey of 4300 square degrees, which corresponds to \( R = 0.33 \) radians. Our present survey lies in 6 zones, for each of which the area is on average \( 99 \text{M} \), corresponding to \( R = 8.4 \times 10^{-4} \) radians.
However, each zone consists of a sparsely-filled mosaic with a filling factor of about 0.25; it turns out that the transform of the angular selection function is dominated by this larger area (see equation 4.5 of Kaiser & Peacock 1991); we therefore adopt a value of $R$ twice the above figure. Furthermore, since we have 6 widely-spaced zones, the overall rms form of the angular selection function is dominated by this a filling factor of about 0.25; it turns out that the trans-
ure. Lastly, the deep $K$-band counts are based on 4 frames each of area 1.4 arcmin$^2$, corresponding to $R = 1.0 \times 10^{-4}$. Inserting these figures gives our final results:

$$
\sigma \simeq (r^*/13h^{-1}\text{Mpc})^{-0.9} \quad \text{(APM)}
$$

$$
\sigma \simeq (r^*/51h^{-1}\text{Mpc})^{-0.9} \quad \text{(This paper)}
$$

$$
\sigma \simeq (r^*/22h^{-1}\text{Mpc})^{-0.9} \quad \text{(Cowie)}
$$

A more practical measure of depth than $r^*$ is the median redshift of a survey. For this selection function, the median comoving distance is 0.968$r^*$, and so the critical depths above correspond to median redshifts of 0.0042, 0.017, and 0.08 respectively. More interesting are perhaps the redshifts where the rms uncertainty reaches about 20 per cent; this represents the bright limit for useful conclusions given our limited survey areas. For APM this is a median redshift of 0.026; for our survey it is 0.11. According to our no-evolution models, such median redshifts correspond to magnitude limits of $b_J = 15$ and $K = 13.7$.

These calculations have interesting implications for the normalization, $\phi^*$, and for conclusions concerning evolution in general. We need to ignore the brighter parts of the counts in setting the normalization: clearly the adopted is a compromise between normalizing at low redshift so as to avoid evolutionary effects but wanting to be certain that the effects of clustering are completely negligible. If we use the magnitudes calculated above where the clustering uncertainty is small, the implied value of the normalization is (consistent between both wavebands) is

$$
\phi^* = (1.5 \pm 0.2) \times 10^{-2} h^3 \text{Mpc}^{-3}.
$$

This agrees with the empirical value of $\phi^*$ adopted in Section 5.1 and provides additional support for the values found by Efstathiou et al. (1988) and Loveday et al. (1992) in optical redshift surveys. It is encouraging that $\phi^*$ is the same in both $b_J$ and $K$, this indicates that our assumption of negligible evolution to the depth where clustering becomes unimportant is valid. The normalization is at a higher redshift in $K$ (0.11 vs 0.026 in $b_J$) but because the slope of the no-evolution model is so closer in $K$ to the data than it is in $b_J$ the effects of evolution will be much less. The normalization would also match up with that of future larger area infrared surveys unless the $K$-count slope changed markedly at brighter magnitudes.

Shanks (1990) and Maddox et al. (1990) noted that the slope of the $b_J$ counts between $b_J = 15$ and $b_J = 20$ was sufficiently steep that either there was very strong evolution or a large local hole in the galaxy distribution. Loveday et al. (1992) directly estimated the radial density variation from $b_J = 15$ to $b_J = 17$ from their redshift survey and find no evidence for a hole in this range. It appears from the above argument that a hole is inconsistent with the observed large-scale power in galaxy clustering and that this is additionally supported by the infrared data. We can turn the argument around and ask what values of $\gamma$ and $r_0$ are required to cause a 100% effect at $b_J = 15$? It turns out that either $\gamma \sim 0.3$ or $r_0 \sim 40 \text{h}^{-1} \text{Mpc}$ (separately) is required — both of these would increase the power by $\Delta^2(k) \sim 0.3$ on scales of $\sim 80 \text{h}^{-1} \text{Mpc}$ (the depth at $b_J = 15$) compared to the $\Delta^2(k) \lesssim 0.01$ observed (Peacock 1991).

Thus it appears that it is difficult to reconcile the steep slope of the optical counts with observed galaxy clustering and we must consider the possibility of strong evolution. Obviously the infrared data will provide new constraints on evolutionary models, so it is this we must consider next.

### 5.3 Luminosity evolution

We first consider the limits set by the $K$-counts to models of luminosity evolution in which galaxies were brighter in the past. Lilly & Longair (1984) suggested from a study of radio galaxies that the brightening amounted to 1 magnitude by $z = 1$. It is convenient to parameterise the luminosity evolution by a simple functional form. Many schemes have been used, for example Broadhurst et al. (1992) who choose a function which is exponential with time. However the exact form of the relation is unimportant, as the counts are mainly sensitive to the first order linear term with redshift. We choose to investigate this directly using:

$$
L^* \propto 1 + bz
$$

For $z \ll 1$ this is exactly equivalent to the functional form of Broadhurst et al., the higher order terms in $z^2$ etc. being small. However we are applying this in the infrared not the optical and do not wish to attribute physical significance to $b$. Since we wish to investigate what minimum departure from the no-evolution model is allowed this seems to us a sensible course.

In $K$, how big a value of $b$ is required to match passive evolution of the galaxy population, as the stars get younger with increasing redshift? The 1 magnitude brightening of Lilly & Longair can be regarded as an upper limit — the effect of increased star-formation in $K$ is in the opposite sense to the passive evolution, i.e. to decrease the mean luminosity with redshift rather than increase it as it does in $B$. This may seem counter-intuitive but does make sense — the $K$ light is approximately proportional to the number of stars in the galaxy because the light is dominated by long-lived stellar populations. Passive evolution makes these stars younger and brighter in the past, but the effect of star-formation is to reduce the number of stars in the past and hence make the galaxy dimmer. This was checked quantitatively by running some simple Bruzual (1983) models — an upper bound of $\approx 1$ magnitude by $z = 1$ was found for a passively evolving model in an $\Omega_0 = 0.1$ cosmology. Increasing $\Omega_0$ or the star-formation rate reduces this and makes $b$ smaller. This upper bound corresponds to $b = 1.5$, and is plotted in Fig. 9 along with $b = 0.6$ (0.5 magnitudes to $z = 1$) and $b = 6.3$ (1.5 magnitudes) to compare the effect of other amounts of nett positive luminosity evolution on the number-magnitude counts. Note that Fig. 9 has a slope of 0.6 (the Euclidean slope) subtracted from the data and the models so as to reduce the number of orders of magnitude plotted and to allow small discrepancies to be more easily discerned.
Galaxies to \( z \simeq 1 \) are seen approximately to \( K \simeq 18 \) in these models, so attention should be focused on this region, rather than the deeper counts of Cowie. It can be seen that only very mild amounts of luminosity evolution are consistent with the data: \( b \lesssim 0.6 \) (0.5 magnitudes) for \( 0 \leq \Omega \leq 1 \). Larger values of \( b \) can be tolerated only if normalization is regarded as a free parameter; however, the normalization \((\phi^* = 1.5 \times 10^{-2} \, h^3 \, \text{Mpc}^{-3})\) is already at the bottom end of the range of values allowed by the optical data. Thus it appears that the result of Lilly & Longair is not applicable to the field galaxy population.

### 5.4 Merger evolution

The infrared counts provide, in principle, an approximate measure of the mass function of galaxies. As the old stars which dominate the \( K \) light evolve only very slowly, \( K \) light should trace the galactic stellar mass and be insensitive to small amounts of star formation. The \( K \) light is also insensitive to dust extinction (\( A_K \simeq 0.1 A_V \)) so all the light is being seen, even if the interstellar medium is heavily disrupted by a galaxy merger (e.g. Thronson et al. 1990). The \( K \) counts thus in principle set limits on changes with epoch to the mass function of galaxies.

The evolution in the optical luminosity function is best described empirically by pure number-density evolution, i.e. evolution in the value of \( \phi^* \). (Lilly, Cowie & Gardner 1991). This is because the number-magnitude counts increase over no-evolution while the mean redshift remains unchanged. Functional forms of number-density evolution have been used to fit the optical data, e.g. Koo (1990) who using \( \phi^* \propto (1 + z)^m \) found \( m = 1.5-2 \). Such models are inconsistent since what they assume is merging plus a no-evolution K-correction. However, galaxies with free gas could not fail to form stars during a merger, which would change the luminosity, especially in \( b_j \). Since the faint galaxies are indeed bluer, i.e. the excess is less in redder bands, a proper model of number-density evolution should seek to account for this.

At first sight, the \( K \) counts strongly constrain such models: a pure change in the number density of galaxies quickly overpredicts the \( K \) counts. This is a general effect — if a particular model of pure density evolution, which fits the \( b_j \) data, is simply translated to \( K \) then it produces too many faint galaxies in that band.

If any change in galaxy number density is due to mergers, then it is possible to construct a more realistic model by requiring that the total amount of light in the \( K \) luminosity function be conserved. The key assumption here is that the combined \( K \) light from two systems before a merger is approximately the same as the \( K \) light afterwards; \( K \) light is thought of as tracing stellar mass. Clearly this will be false at some level, but it is a more natural starting point than pure number-density evolution where one effectively assumes that the light doubles. For a Schechter function

\[
L_{\text{TOT}} = \int_{-\infty}^{\infty} L \phi(L) dL \propto \phi^* L^*
\]

so any formalism which conserves \( \phi^* L^* \) as a function of \( z \) will do. To investigate evolution in the number density the following parameterisation was adopted:

\[
\phi^* \propto 1 + Q z
\]

\[
L^* \propto \frac{1}{1 + Q z}
\]

Note that this actually implements luminosity evolution as the galaxies break up into smaller units at high \( z \). This is similar to the approach of Guideroni & Rocca-Volmerange (1991) except that they modelled the optical \( b_j \) counts rather than \( K \) — to get from the mass function to the \( b_j \) light requires complex models of spectral evolution and an assumption for the star-formation rate which may depend in a complicated way on the merger history. By modelling \( K \) we reduce the importance of such effects and just consider the mass function (in stars).

Fig. 10 shows the results of such models with \( Q = 2 \), 4 and 7 and \( \phi^* = 1.5 \times 10^{-2} \, h^3 \, \text{Mpc}^{-3} \), the normalization used before. Note that the effect is to bend up the faint end in such a way that the slope is a better match to the data. The normalization is a little low, but this could be rectified as we do have some freedom to normalize up. Note also that the number-magnitude counts are insensitive to large values of \( Q \). This is because the effect of such merging is to stretch the model along a line of slope 0.4 in the log \( n - m \) plane — i.e. the surface brightness of the population on the sky is conserved. Because this is close to the observed count slope, a wide range of \( Q \) can be consistent with the count data.

Of course such models are extremely crude: all that is specified is that galaxies of all masses double their mass in similar times. It does not specify how many mergers actually happen in that time — solutions where galaxies always merged with units of the same mass or digested many much smaller subunits can not be distinguished. But it does demonstrate an important point, namely that from the count slope alone it is not easy to tell whether galaxies at \( z = 1 \) exist in many subunits.

The existence of radio-galaxies at high-z provides a lower limit on the amount of merging if the \( K \) luminosity function evolves in a self-similar way. Radio-galaxies have a comoving space density at \( z \gtrsim 1 \) of \( 10^{-9} \, h^3 \, \text{Mpc}^{-3} \) (Dunlop & Peacock 1990). Their mean absolute magnitude is \( -24.0 \), \( \simeq 0.5 \) magnitudes brighter than the \( M^* \) for normal galaxies with a scatter of 0.5 magnitudes (Lilly & Longair 1984). If we assume half the radio-galaxies are brighter than the mean this places an observational lower limit on the number density of galaxies with \( M < -24.0 \).

If there is too much self-similar evolution there will not be enough galaxies of any kind brighter than \( M = -24 \) to account for the observed radio-galaxies. This is a strong constraint due to the rapid exponential fall-off in the Schechter luminosity function at \( M < M^* \). Putting in the figures this constrains the self-similar evolution in \( \phi^* \) and \( L^* \) to be less than a factor of \( \sim 6 \) at \( z \sim 1 \). For our linear form this corresponds to \( Q \lesssim 5 \), for the exponential form of Broadhurst et al. (1992) this is \( Q \lesssim 4 \) since this increases faster at higher \( z \). This implies that if the preferred amount of evolution in Broadhurst et al. \((Q = 4)\) is allowed all luminous galaxies at high-z would have to be radio-galaxies, and radio-galaxies would just be defined by a mass cutoff at early times. Given the peak in radio AGN activity at \( z \sim 2 \) (Dunlop & Peacock 1990) it is certainly not implausible to
suggest that all massive galaxies would be active if we are seeing them soon after their assembly.

A direct way to measure the evolution in the $K$ luminosity function is to measure the redshift distribution. Fig. 11 shows the predicted redshift distributions for a $16 < K < 17$ selected galaxy sample for increasing values of $Q$. Such merging models predict that the mean redshift of $K$ selected samples will be less than the no-evolution prediction because galaxies become less luminous in the past. Such a comparison, with actual survey data, will be made in Paper II.

Such a merging model can naturally explain the $b_j$ counts and redshift distributions if there is enough enhancement to the $b_j$ light to compensate for the decrease in galaxy mass, thus keeping the mean redshift at the no-evolution value. This would have to be via some merger-driven starburst mechanism, which would also explain the galaxies in the $b_j$ excess being spectroscopically evolved (e.g. Broadhurst, Ellis & Glazebrook (1992)).

6 CONCLUSIONS

We have presented a $K$-band survey complete to $K \simeq 17.3$ over $552 \square Mpc^2$, and have constructed galaxy counts from a sample of 481 galaxies and stars.

It is possible to explain the infrared number-magnitude galaxy counts in a model based upon local data on optical galaxies. Unlike the blue counts, the $K$ counts are well matched by a non-evolving galaxy model. If the models are normalized at bright magnitudes using our data, the deep $K$ counts of Cowie are only a factor of $\sim 2$ at most (for $\Omega = 1$) in excess of the non-evolving model, in contrast with the factor of $\sim 10$ excess seen in $b_j$. This follows the trend previously seen as number-magnitude counts were extended to redder optical bands.

The bright counts exclude pure $K$ luminosity evolution in excess of 0.5 magnitudes to $z = 1$. However this does not rule out models with active star-formation in the past — these predict less luminosity evolution than simple passive stellar evolution. Models of density evolution, and models with non-standard volume elements, which explain the $b_j$ number counts and redshift distributions by increasing the number of galaxies in the past, generally overpredict the faint $K$ counts. It is possible to match the slope by invoking a merging type evolutionary model, but only if the powerful assumption is made that merging conserves $K$ light (i.e. $\approx$ stellar mass). It is argued that this is a more logical starting point than empirical number-density evolution. In this case the $K$ counts are insensitive to the amount of merging. This can be tested by an infrared-selected redshift distribution, such a test will be carried out in Paper II.

The infrared counts favour a local space density of galaxies $\phi^* = 1.5 \pm 0.2 \times 10^{-2} h^3$ Mpc$^{-3}$. This value is in good agreement with the value determined by Efstathiou et al. (1988). This normalization is much better defined than in the optical as the counts are very close to the no-evolution prediction and so the value is insensitive to evolution, provided the density and luminosity scale as $\phi^* \propto 1/L^*$.  

7 ACKNOWLEDGEMENTS

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REFERENCES


FIGURES

Fig. 1 Example geometry of CCD frames for the least-square cross-calibration technique.

Fig. 2 The star-galaxy image classification parameter $y$ (see text) plotted against CCD $R$ magnitude. Note for $R > 20$ only a third of the points are plotted.

Fig. 3 The star-galaxy image classification parameter $y$ (see text) plotted against IRCAM $K$ magnitude.

Fig. 4 Raw galaxy counts in fields with and without galaxies as references (renormalised according to area). The reference star and reference galaxy points are shown slightly offset from the raw points for clarity.

Fig. 5 Final star and galaxy counts for the survey.

Fig. 6 The star counts plotted for different galactic co-ordinates and compared with models of I.N. Reid (see text).

Fig. 7 The $K$ galaxy counts compared with those of Cowie et al. (1990, 1991) and Jenkins & Reid (1991). Note the latter are shown by dotted lines marking the $\pm 2\sigma$ limits from their statistical analysis of sky fluctuations. No-evolution predictions are shown.

Fig. 8 The $b_j$ galaxy counts from a number of authors together with no-evolution predictions.

Fig. 9 Parametric models of luminosity evolution compared with the $K$ galaxy counts.

Fig. 10 Parametric models of merger evolution compared with the $K$ galaxy counts.

Fig. 11 The predicted number-redshift distributions for a $16 < K < 17$ selected sample for no-evolution and increasing amounts of merging. Note the shift of the mean redshift to lower values.

TABLES

Table 1 Centres of the $10' \times 10'$ survey zones.

Table 2 The centres of all individual IRCAM frames together with exposure times in seconds and an ID corresponding to the field — in the sense that there is one unique ID per reference star/galaxy if they were all detected. Because many IRCAM frames have the same reference object they have the same field ID (see section 2.1 for details).

Table 3 Magnitude limits reached, sky brightness (magnitudes/$''$) observed and equivalent noise levels for the optical CCD observations.

Table 4 The final catalogue, in RA order, of all detected $K$ sources together with optical magnitudes were available. Column “G” is whether or not the object is a galaxy and column “R” is whether or not the object was selected in as a reference (see text for details). The ID number is unique for each object, the field ID numbers (FID) are the same as in Table 2.

Table 5 The raw counts of galaxies ($N_g$), stars ($N_s$), reference galaxies ($N_{gr}$), reference stars ($N_{sr}$) and final corrected counts galaxies ($N'_g$) and stars ($N'_s$) with errors.

Table 6 Luminosity function used in the no-evolution models.
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K. Glazebrook, J.A. Peacock, C.A. Collins and L. Miller
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