Topological superfluid in one-dimensional spin-orbit-coupled atomic Fermi gases

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We investigate theoretically the prospect of realizing a topological superfluid in one-dimensional spin-orbit-coupled atomic Fermi gases under a Zeeman field in harmonic traps. In the absence of spin-orbit coupling, it is well known that the system is either a Bardeen-Cooper-Schrieffer superfluid or an inhomogeneous Fulde-Ferrell-Larkin-Ovchinnikov (FFLO) superfluid. Here we show that with spin-orbit coupling it could be driven into a topological superfluid, which supports zero-energy Majorana modes. However, in the weakly interacting regime the topological superfluid prefers to stay at the trap edge, in contrast to a FFLO superfluid, which occurs near the trap center. As a result, it is unlikely to experimentally observe an inhomogeneous FFLO superfluid with topological order without specifically tailoring the geometry or other parameters of the Fermi cloud.

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I. INTRODUCTION

Topological superfluids are new states of matter that have attracted intense attention in recent years [1,2]. They have a full pairing gap in the bulk and exotic gapless excitations at the edge, the so-called Majorana fermions, which obey non-Abelian statistics [3,4]. These excitations are immune to decoherence caused by local perturbations. By properly braiding excitation quasiparticles, topological quantum information might be processed. As a result, topological superfluids could provide an ideal platform for topological quantum computation [5,6]. Because of this potential application, the realization of topological superfluids in a well-controlled environment is highly desirable.

Theoretically, there are a number of proposals on realizing a topological superfluid in two-dimensional (2D) settings, including the use of 2D p-wave pairing [7,8], proximity coupling to a conventional s-wave superconductors for the surface state of three-dimensional (3D) topological insulators [9–11], and 2D atomic Fermi gases with strong Rashba spin-orbit coupling [12,13]. It is also possible to create a topological superfluid in one-dimensional (1D) solid-state systems by suitably engineering spin-orbit coupling of electrons, such as InAs wires and banded carbon nanotubes [14–17]. The purpose of this work is to examine the possibility of observing topological superfluids in 1D ultracold atomic Fermi gases [18], which may be regarded as highly controllable quantum simulators of the corresponding 1D solid-state systems. We note that 1D atomic Fermi gases can now be routinely created in cold-atom laboratories [19]. The spin-orbit coupling for neutral atoms may also be generated by using the so-called non-Abelian synthetic gauge fields technique [20,21].

Even in the absence of spin-orbit coupling the 1D ultracold atomic Fermi gas is of great interest. It hosts a Bardeen-Cooper-Schrieffer (BCS) superfluid and an exotic inhomogeneous Fulde-Ferrell-Larkin-Ovchinnikov (FFLO) superfluid [19,22–28], respectively, in the case of balanced and imbalanced spin populations. Here we show that by adding spin-orbit coupling both superfluids can turn into a topological superfluid. We discuss in detail the resulting zero-energy Majorana edge modes and possible experimental signature. We also explore the possibility of creating an inhomogeneous topological superfluid with a spatially oscillating FFLO order parameter. Unfortunately, the spin-orbit coupling seems to suppress the FFLO order parameter. As a result, in the weakly interacting regime we always find the same topological superfluid with a uniform order parameter whether the initial state is a BCS or FFLO superfluid. Our study is based on the self-consistent solution of fully microscopic Bogoliubov–de Gennes (BdG) equations [25,28]. It enables ab initio simulations under realistic experimental conditions.

This paper is organized as follows. In Sec. II, we present the model Hamiltonian and the BdG equations. In Sec. III, we discuss the phase diagram at a sufficiently large spin-orbit coupling and the phase transition from BCS superfluid to topological superfluid. Finally, in Sec. V we provide conclusions and some final remarks.

II. MODEL HAMILTONIAN AND BDG EQUATIONS

We consider a trapped two-component 1D atomic Fermi gas under a non-Abelian gauge field (spin-orbit coupling) and Zeeman field, described by the model Hamiltonian,

\[ \mathcal{H} = \int dx \psi^\dagger(x) \left[ \mathcal{H}_0^S(x) - h \sigma_z + \lambda k \sigma_y \right] \psi(x) + g_{1D} \int dx \psi^\dagger(x) \psi^\dagger(x) \sigma_z \psi_1(x) \psi_1(x), \]  

(1)

where \( \psi(x) \equiv [\psi^\dagger_1(x), \psi_1(x)] \) denotes collectively the creation field operators for spin-up and spin-down atoms. In the single-particle Hamiltonian (i.e., the first line of the above equation), \( \mathcal{H}_0^S(x) \equiv -\hbar^2/2m \partial^2 / \partial x^2 + m \omega^2 x^2 / 2 - \mu \) describes the single-particle motion in a harmonic trapping potential \( m \omega^2 x^2 / 2 \), and in reference to the chemical potential \( \mu \), the strength of the Zeeman field is denoted by \( h, \lambda k \sigma_y \equiv -i\lambda (\partial / \partial x) \sigma_y \) is the spin-orbit coupling term with coupling strength \( \lambda \), and \( \sigma_z \) and \( \sigma_y \) are the 2 \times 2 Pauli matrices. The second line of the equation is the interaction Hamiltonian,
with the (attractive) interaction strength given by the s-wave scattering length: \( g_{1D} = -\frac{2\hbar^2}{(ma_{1D})} \).

The model Hamiltonian equation (1) can be realized straightforwardly with cold fermionic atoms. It is a direct generalization of the standard model Hamiltonian for a 1D spin-imbalanced Fermi gas, through the inclusion of a non-Abelian synthetic gauge field \( \lambda k \sigma_j \). Experimentally, a bundle of 1D spin-imbalanced atomic Fermi gases can now be manipulated using 2D optical lattices [19]. The generalization of the synthetic gauge field \( \lambda k \sigma_j \) has already been demonstrated in a 3D Bose gas of \(^{87}\text{Rb}\) atoms [20]. In addition, its realization in fermionic atoms has been proposed [21]. Therefore, all the techniques required to simulate Eq. (1) are within current experimental reach.

To understand the 1D superfluidity in the presence of spin-orbit coupling, we calculate elementary excitations within the mean-field BdG approach [25,28]. The wave function of low-energy fermionic quasiparticles \( \Psi_\eta(x) \) with energy \( E_\eta \) is solved by

\[
\mathcal{H}_\text{BdG} \Psi_\eta(x) = E_\eta \Psi_\eta(x),
\]

where \( \Psi_\eta(x) \equiv [u_\eta \gamma \eta(x), u_{1\eta}(x), v_{1\eta}(x), v_\eta(x)]^T \) in the Nambu spinor representation and the BdG Hamiltonian \( \mathcal{H}_\text{BdG} \) reads, accordingly,

\[
\mathcal{H}_\text{BdG} = \begin{bmatrix}
\mathcal{H}_0^\eta(x) - h & -\lambda \partial / \partial x & 0 & -\Delta(x)

\lambda \partial / \partial x & \mathcal{H}_0^\eta(x) + h & 0 & -\Delta(x)

0 & 0 & \Delta(x) & 0

-\Delta^*(x) & -\lambda \partial / \partial x & -\mathcal{H}_0^\eta(x) + h & -\lambda \partial / \partial x
\end{bmatrix}.
\]

Here \( \Delta(x) = -(g_{1D}/2) \sum_\eta [u_{\eta \gamma \eta}(x) f(E_\eta) + u_{1\eta}(x) f(-E_\eta)] \) is the order parameter, and \( f(x) \equiv 1/[e^{x/(k_B T)} + 1] \) is the Fermi distribution function at temperature \( T \). The order parameter is to be solved self-consistently together with the number equation for the chemical potential, \( \partial \mathcal{H} / \partial n \equiv \int dr [n_\uparrow(r) + n_\downarrow(r)] = \mathcal{N} \), where \( \mathcal{N} \) is the total number of atoms and the density of spin-\( \sigma \) atoms is given by \( n_\sigma(x) = (1/2) \sum_\eta [u_{\sigma \gamma \eta}(x) f(E_\eta) + |v_{\eta \gamma \eta}(x)|^2 f(-E_\eta)] \). We note that the use of Nambu spinor representation leads to an inherent redundancy built into the BdG Hamiltonian [2]. \( \mathcal{H}_\text{BdG} \) is invariant under the particle-hole transformation: \( u_\sigma(x) \rightarrow v^*_\sigma(x) \) and \( E_\eta \rightarrow -E_\eta \). Therefore, every eigenstate with energy \( E \) has a partner at \( -E \). These two states describe the same physical degrees of freedom, as the Bogoliubov quasiparticle operators associated with them satisfy \( \Gamma_E = \Gamma^\dagger_{-E} \). This redundancy has been removed by multiplying a factor of 1/2 in the expressions for the order parameter and atomic density.

The BdG equation (2) can be solved by expanding \( u_\sigma(x) \) and \( v_\eta(x) \) in the basis of 1D harmonic oscillators. On such a basis, Eq. (3) is converted to a secular matrix. On such a basis, Eq. (3) is converted to a secular matrix. A matrix diagonalization then gives the desired quasiparticle energy spectrum and wave functions. Numerically, we have to truncate the summation over the energy levels \( \eta \). For this purpose, we adopt a hybrid strategy developed earlier by us for an imbalanced Fermi gas without spin-orbit coupling [25,28]. We introduce a high-energy cutoff \( E_c \), above which a local-density approximation (LDA) is used for the high-lying energies and wave functions. This leads to an effective coupling constant in the gap equation, \( \Delta(x) = -g_{1D}(x)/2 \sum_\eta [u_{\eta \gamma \eta}(x) f(E_\eta) + u_{1\eta}(x) f(-E_\eta)] \), where \( \mathcal{N} \) is now restricted to \( |E_\eta| < E_c \). We refer to Ref. [25] for further details of \( g_{1D}(x) \) and the LDA atomic density.

In harmonic traps, it is useful to characterize the interaction strength by using a dimensionless interaction parameter [25], \( \gamma \equiv -mg_{1D}/(\hbar^2 n_0) = 2/(n_0 a_{1D}) \), where \( n_0 \) is the zero-temperature center density of an ideal two-component Fermi gas with equal spin populations \( N/2 \). In the Thomas-Fermi approximation (or LDA), \( n_0 = 2 N^{1/3} / (\pi a_{ho}) \) and \( \sqrt{\gamma} \equiv \sqrt{\gamma} / (m \hbar \omega_0) \) is the characteristic oscillator length of the trap. Therefore, the dimensionless interaction parameter is given by

\[
\gamma = \frac{1}{\pi N^{1/3}} \left( \frac{a_{ho}}{a_{1D}} \right),
\]

We note that, for a 1D atomic Fermi gas created using 2D optical lattices, the typical dimensionless interaction strength is about \( \gamma = 3.5 \) [19,25]. Throughout this paper, we shall take a slightly smaller value of \( \gamma = \pi/2 \) \( \approx 1.6 \) in order to validate the mean-field treatment. It is also convenient to use the Thomas-Fermi energy \( E_F = (N/2) \hbar \omega_0 \) and Thomas-Fermi radius \( x_F = N^{-1/3} a_{ho} \) as the units for energy and length, respectively. For the spin-orbit coupling, we use a dimensionless parameter \( \lambda k_F / E_F \), where \( k_F = \sqrt{2m E_F} \) is the Thomas-Fermi wave vector. We have performed numerical calculations for a Fermi gas of \( N = 100 \) fermions in traps at both zero temperature and finite temperature. In the following, we present only the zero-temperature results, as the inclusion of a finite but small temperature (i.e., \( T = 0.1 T_F \)) essentially does not affect the results. The Fermi energy is \( E_F = (N/2) \hbar \omega_0 \) and has used \( 3N = 300 \) 1D harmonic oscillators as the expansion functions. These parameters are already sufficiently large to ensure the accuracy of calculations.

### III. PHASE DIAGRAM AT A GIVEN SPIN-ORBIT COUPLING

The most salient feature of a spin-orbit-coupled Fermi gas is the appearance of topological superfluidity and a zero-energy Majorana fermion mode under an appropriate Zeeman field. The quasiparticle operators of Majorana fermions are real and satisfy \( \gamma = \gamma^\dagger \), which means that a quasiparticle is its own antiparticle [3,4]. Mathematically, we can always write
a complex ordinary fermion operator $c$ in terms of two real Majorana fermions $\gamma_1$ and $\gamma_2$, such as $c = \gamma_1 - i\gamma_2$. An ordinary fermion may therefore be viewed as a bound state of two Majorana fermions, which in general cannot be deconfined. However, the deconfinement does happen in a topological superfluid, leading to two Majorana fermions localized at the two edges of a topological superfluid. This can be clearly seen with the help of the particle-hole redundancy of the BdG equation [2,13]. Let us imagine that we have a zero-energy solution $E = 0$. Because of the particle-hole redundancy $\Gamma_{\text{E}} = \Gamma_{\text{E}}^\dagger$, we will immediately have $\Gamma_{\text{E}} = \Gamma_{\text{E}}^\dagger$, exactly the defining feature of a Majorana fermion. We note that zero-energy Majorana fermions should always come in pairs since the original model Hamiltonian describes ordinary fermions only and each Majorana fermion is just half of an ordinary fermion. It is straightforward to check from the BdG Hamiltonian that the wave functions of two paired Majorana fermions should satisfy $u_{\sigma}(x) = v_{\sigma}(x)$ and $u_{\sigma}(x) = -v_{\sigma}^*(x)$, respectively. The former follows the particle-hole symmetry, while the latter is required to express an ordinary fermion by two Majorana fermions [29].

The emergence of topological order in a 1D spin-orbit-coupled Fermi gas is easily understood from its analogy with a 1D chiral $p$-wave superfluid. In the weakly interacting regime, the latter has a winding spin texture (and therefore a topological defect) in momentum space and is a prototype of topological superfluids [7]. Though the underlying pairing in atomic Fermi defect) in momentum space and is a prototype of topological superfluids. This can be clearly seen with the help of the particle-hole redundancy of the BdG equation [2,13]. Let us imagine that we have a zero-energy solution $E = 0$. Because of the particle-hole redundancy $\Gamma_{\text{E}} = \Gamma_{\text{E}}^\dagger$, we will immediately have $\Gamma_{\text{E}} = \Gamma_{\text{E}}^\dagger$, exactly the defining feature of a Majorana fermion. We note that zero-energy Majorana fermions should always come in pairs since the original model Hamiltonian describes ordinary fermions only and each Majorana fermion is just half of an ordinary fermion. It is straightforward to check from the BdG Hamiltonian that the wave functions of two paired Majorana fermions should satisfy $u_{\sigma}(x) = v_{\sigma}(x)$ and $u_{\sigma}(x) = -v_{\sigma}^*(x)$, respectively. The former follows the particle-hole symmetry, while the latter is required to express an ordinary fermion by two Majorana fermions [29].

FIG. 1. (Color online) Phase diagram at a given spin-orbit coupling $\lambda k_F/E_F = 1$, determined from the behavior of the lowest eigenenergy of Bogoliubov quasiparticle spectrum $\min|\langle E_i|\rangle$. As the Zeeman field increases, the system evolves from a conventional BCS superfluid to a topological superfluid and finally to a normal state. The insets on the top and bottom show the quasiparticle spectrum at $h/E_F = 0.3$ and 0.5, respectively.

of the lowest eigenenergy of the quasiparticle energy spectrum. As shown in the top inset, at a small Zeeman field the energy spectrum is gapped. However, by increasing the Zeeman field above a critical value of $h \sim 0.35E_F$, the lowest eigenenergy becomes exponentially small. Four quasiparticle modes with nearly zero energy appear, as seen clearly from the bottom inset. By further increasing the Zeeman field ($h > 0.65E_F$), the system will be driven into a normal state with a negligible superfluid order parameter.

The appearance of the topological superfluid can also be monitored by the calculation of $h - h_c(x)$, where $h_c(x) = \sqrt{\mu(x)^2 + \Delta(x)^2}$ is the local critical Zeeman field for a local uniform cell at position $x$ with the local chemical potential $\mu(x) \equiv \mu - \omega^2 x^2/2$ and order parameter $\Delta(x)$. The local uniform cell would be in the topological superfluid state if $h > h_c(x)$. In Fig. 2, we present $h - h_c(x)$ and $\Delta(x)$ at different phases. In accord with Fig. 1, at a small field $h = 0.3E_F$ [Fig. 2(a)], $h < h_c(x)$ for any position $x$, and the whole Fermi cloud is in the conventional superfluid. At the field $h = 0.5E_F$ [Fig. 2(b)], we find $h > h_c(x)$ at the two wings of the harmonic trap, and therefore there are two blocks of topological superfluid, as highlighted by the hatching. At an even large Zeeman field [Fig. 2(c)], the area of $h > h_c(x)$ extends over the whole system. However, the superfluid order parameter becomes so small that the system can no longer be viewed a superfluid. We note that, at large attractive interactions where the order parameter is not destroyed by a large Zeeman field, it is possible to have a single topological superfluid throughout the whole Fermi cloud.

B. Majorana fermions

In each of the topological superfluid phases, we should find two Majorana fermion modes, well localized at the two edges. At the Zeeman field $h = 0.5E_F$, we therefore could have four Majorana fermions, as indicated by the energy spectrum in the
FIG. 2. (Color online) Spatial dependence of the critical Zeeman field $h - h_c(x)$ (solid lines) and the superfluid order parameter $\Delta(x)$ (dot-dashed lines) at $\lambda k_F/E_F = 1$ and at three Zeeman fields, $h/E_F = 0.3, 0.5,$ and $0.8$. The hatching highlights the areas in which the atoms are in the topological superfluid state.

bottom inset in Fig.1. The wave functions of these Majorana fermions are shown in Figs. 3 and 4 for states localized at $x \simeq \pm 0.5 x_F$ and $\pm 1.1 x_F$, respectively. It is interesting that the wave functions of two paired Majorana fermions, for example, those located at $x \simeq -0.5 x_F$ and $x \simeq +0.5 x_F$ (Fig. 3), tend to interfere with each other [13,32]. This quasiparticle interference or tunneling leads to the splitting of degenerate zero-energy Majorana modes to a finite but exponentially small energy: $E_{ZES} \simeq \pm 3.3 \times 10^{-5} E_F$. The tunneling between the paired Majorana fermions at the outer wing of the trap, $x \simeq \pm 1.1 x_F$, is more difficult (see Fig. 4), so the energy splitting is much smaller, i.e., $E_{ZES} \simeq \pm 7.2 \times 10^{-10} E_F$. It is readily seen that the paired wave functions satisfy either $u_x(x) = v_x^*(x)$ or $u_o(x) = -v_o^*(x)$, as anticipated by the required symmetry of Majorana wave functions.

C. Density distribution and local density of states

We now consider the possible experimental signature for observing a topological superfluid and the associated Majorana fermions. The useful experimental tools include in situ absorption imaging and spatially resolved radio-frequency (rf) spectroscopy [33], which give, respectively, the density distribution and the local density of states of the Fermi cloud [34].
FIG. 5. (Color online) The spin-up and spin-down density distributions, \(n_\uparrow(x)\) (dashed lines) and \(n_\downarrow(x)\) (solid lines), and their difference \(\Delta n(x) = n_\uparrow(x) - n_\downarrow(x)\) (dot-dashed lines) are shown at (a) the conventional superfluid phase, (b) topological superfluid state, and (c) normal state. The density distributions are in units of the Thomas-Fermi density \(n_0 = 2N_1^{1/2}/(\pi a_0)\). The spin-orbit coupling is \(\lambda k_F/E_F = 1\).

In Fig. 5, we plot the spin-up \(n_\uparrow(x)\) and spin-down \(n_\downarrow(x)\) density distributions and their difference \(\Delta n(x) = n_\uparrow(x) - n_\downarrow(x)\) at different phases. While the shape of the spin-up density distribution \(n_\uparrow(x)\) is nearly unchanged across different phases, in the topological superfluid phase [see Fig. 5(b) at \(h = 0.5E_F\)] the spin-down density distribution \(n_\downarrow(x)\) shows an interesting bimodal structure. It decreases rapidly when the atoms enter the topological area from the center. Accordingly, a broad dip appears in the density difference around the trap center. The bimodal distribution in \(n_\downarrow(x)\) may be regarded as a useful and convenient feature to identify the topological superfluid. However, it is not a characteristic feature for identifying the Majorana modes, as the contribution of the Majorana modes to the density distribution is negligibly small, i.e., relatively at the order of \(N^{-1/2}\).

A practical way to probe the Majorana fermions is to measure the local density of states using the spatially resolved rf spectroscopy [33,34], with which we anticipate that the contributions of Majorana fermions will be well isolated in both the energy domain and real space. The local density of states for spin-up and spin-down atoms is defined by

\[
\rho_\sigma(x,E) = \frac{1}{2} \sum_\eta \left[ |u_{\sigma\eta}|^2 \delta(E - E_\eta) + |v_{\sigma\eta}|^2 \delta(E + E_\eta) \right].
\]

In Fig. 6, we report the local density of states in the topological superfluid state. Near the zero energy, the contributions from Majorana fermions are clearly visible and are well separated from other quasiparticle contributions by an energy gap \(\Delta \sim 0.1E_F\). It is interesting to note that the Majorana modes at \(x \simeq \pm 1.1x_F\) and \(\pm 0.5x_F\) contribute to \(\rho_\uparrow(x,E)\) and \(\rho_\downarrow(x,E)\), respectively. This can be understood from the wave function of Majorana modes, as shown in Figs. 3 and 4. The wave functions at \(\pm 0.5x_F\) are dominated by the spin-down

FIG. 6. (Color online) Linear contour plot (in arbitrary units) for the local density of states (a) of spin-up atoms \(\rho_\uparrow(x,E)\) and (b) of spin-down atoms \(\rho_\downarrow(x,E)\). The contributions from Majorana fermions are highlighted by circles. Here the Fermi cloud is in the topological superfluid state with parameters \(h = 0.5E_F\) and \(\lambda k_F/E_F = 1\). In the calculation, the \(\delta\) function in \(\rho_\sigma(x,E)\) has been simulated by a Lorentz distribution with a small energy broadening \(\Gamma = 0.01E_F\).

FIG. 7. (Color online) Phase diagram at a given Zeeman field \(h = 0.4E_F\), determined from the behavior of the lowest eigenenergy of Bogoliubov quasiparticle spectrum \(\min(|E_\eta|)\). As the spin-orbit coupling increases, the system evolves from a FFLO superfluid to a topological superfluid. The inset shows the critical Zeeman field \(h - h_c(x)\) and the order parameter \(\Delta(x)\) at \(\lambda k_F/E_F = 0.3\), where the Fermi gas is in the FFLO superfluid state.
spin-orbit coupling? is the fate of such a FFLO superfluid when we switch on the oscillating order parameter. It is therefore natural to ask, what field can be an inhomogeneous FFLO superfluid with an ground state of an imbalanced 1D Fermi gas under a Zeeman field?

In the absence of spin-orbit coupling, it is known that the lowest eigenenergy becomes exponentially small, suggesting a topological superfluid. However, in this case, the order parameter no longer oscillates in real space, as shown in Figs. 8(b) and 8(c). The noncoexistence of topological order and FFLO superfluidity is because a topological superfluid prefers to stay at the trap edge, while a FFLO superfluid occurs near the trap center. As a result, it is difficult to create an inhomogeneous topological superfluid with spatially oscillating order parameter in a 1D spin-orbit-coupled Fermi gas if we do not tailor specifically the geometry or other parameters of the Fermi cloud.

IV. PHASE DIAGRAM AT A GIVEN ZEEMAN FIELD

We now consider the possibility of observing a topological superfluid with a spatially oscillating order parameter [24,25]. In the absence of spin-orbit coupling, it is known that the ground state of an imbalanced 1D Fermi gas under a Zeeman field can be an inhomogeneous FFLO superfluid with an oscillating order parameter. It is therefore natural to ask, what is the fate of such a FFLO superfluid when we switch on the spin-orbit coupling?

In Fig. 7, we present the phase diagram at a given Zeeman field \( h = 0.4E_F \), determined again by tracing the behavior of the lowest eigenenergy of the quasiparticle spectrum as a function of the spin-orbit coupling. The density distributions and order parameter are reported in Fig. 8 for three values of spin-orbit coupling. At small spin-orbit coupling, we find a stable FFLO order parameter, which is modified slightly by the spin-orbit coupling. However, in the area where \( \lambda k_F / E_F = 0.3 \) is nonzero, the criterion for a topological superfluid \( h > h_*(x) \) is always unsatisfied, as seen from the inset for the case of \( \lambda k_F / E_F = 0.3 \). This excludes the coexistence of a FFLO superfluid and topological order. As a result, the energy spectrum is gapped and \( \min(|E_n|) > 0 \). By increasing the spin-orbit coupling above \( \lambda k_F / E_F \approx 0.6 \), we observe that the lowest eigenenergy becomes exponentially small, suggesting a topological superfluid.

V. CONCLUSIONS

In conclusion, we have investigated theoretically the properties of a 1D imbalanced Fermi gas under a non-Abelian synthetic gauge field. We have predicted that by suitably tuning the strength of spin-orbit coupling and Zeeman field, it is possible to create a topological superfluid, which hosts Majorana zero-energy fermions at its edge. The order parameter in the topological superfluid is always of the conventional Bardeen-Cooper-Schrieffer type, as the spin-orbit coupling tends to destroy inhomogeneous Fulde-Ferrell-Larkin-Ovchinnikov pairing. Experimentally, the topological superfluid may be identified from the bimodal distribution of the spin-down atomic density by using \textit{in situ} absorption imaging. The associated Majorana fermions may be detected by applying the spatially resolved radio-frequency spectroscopy, which would show a well-isolated signal at zero energy.

We would like to emphasize that the ultracold atomic Fermi gas with a non-Abelian synthetic gauge field is an ideal platform for creating topological superfluid and manipulating Majorana fermions because of its unprecedented controllability and flexibility. This system can now be readily realized in ultracold-atom laboratories.

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[29] The ordinary fermion operator at $E = 0$ is given by $c = \Gamma_0 + \bar{\Gamma}_0$. By defining Majorana operators $\gamma_1 = \Gamma_0$ and $\gamma_2 = i\bar{\Gamma}_0$, we express $c = \gamma_1 - i\gamma_2$, as anticipated. For $\bar{\Gamma}_0$, we must have $\bar{\Gamma}_0 = -\bar{\Gamma}_0$. The associated wave functions satisfy $u_\sigma(x) = -v_\sigma^*(x)$.