We consider the possibility of vortices forming spontaneously in the condensate through the process of evaporative cooling, without external intervention.

The presence of vortex states can be detected quantitatively by transforming the spatial lattice into a lattice which uses the angular momentum eigenstates as a basis set. Figure 1 gives the angular momentum distribution for a particular two-dimensional trajectory. In this run, a vortex with $j = -1$ appears at about one quarter of the way through the simulation, and persists until the end.

As is usual in quantum mechanics, only the ensemble averages of the simulations have an operational meaning. Thus, while individual stochastic realizations have a definite coherent state, the ensemble averages have no absolute phase information. Similarly, the existence of varying angular momentum occupation numbers between individual trajectories indicates that this can vary in the quantum distribution of angular momenta over the whole ensemble. We investigate the resulting higher-order coherence properties.

In summary, we have demonstrated a new technique for real-time quantum dynamical simulations of Bose-condensation of large numbers of interacting atoms. Sampling errors and lattice size restrictions mean there are necessary limitations on these first simulations. The results, as well as showing evidence for highly non-classical behavior in a first principles simulation of BEC formation, illustrate the possibility of spontaneous vortex formation in small evaporatively cooled condensates.


\[
\langle x | \Psi_{\{\ell\}} \rangle = \left( \frac{-i \hbar \nabla^2}{2m} + V_0 (2k_x x) \right) \Psi_{\{\ell\}} + \frac{4 \pi \hbar a}{m} | \Psi_{\{\ell\}} \rangle | \Psi_{\{\ell\}} \rangle
\] 

By solving Eqs. (1), (2) and (3), we show that, for example, or rather “polaronic” band gaps are created for probe light due to the spatial periodicity of the ground state of the lattice BEC. However being different from the ordinary periodic dielectric, the lattice BEC can experience elementary excitations which lead to a perturbative distortion of the periodicity of the lattice BEC. Such non-periodic perturbations are similar to lattice defects in solid state physics. Here they will produce defect states inside the photonic band gaps. By analyzing the elementary excitations of the lattice BEC, we study the frequency spectrum of photonic band gaps and defect states induced by such excitations in the lattice BEC. These photonic defect states may technically have the potential to be employed to acquire knowledge about the quantum degeneracy in an optical lattice.

Here the field $\Phi$ represents an atomic species of mass $m$ in a potential $V_N(x)$, in one internal state, while $\Psi$ represents a dimer species of mass $M = 2m$, in a single vibrational and rotational state, in a potential $V_\Psi(x)$. The coupling constant $\gamma$ represents a formation rate for the dimer, in the $S$-wave scattering limit, while $\kappa$ represents the effective self-interaction of the atomic field. In the absence of any trap, the potentials are uniform, and $\hbar \rho = \hbar (V_\Psi - 2V_\Phi)$ is the formation energy of the dimer species.

One way to understand the behavior of this quantum many-body system is to look for energy eigenstates of the original Hamiltonian, in the limit of a large momentum cutoff $k_m$. These must simultaneously be the eigenstates of $N \rightarrow f \Phi(x) \Phi(x) + 2\Psi(x)$, conserving the generalized particle number $N$. Solving this, we can show rigorously that in the limit of free space propagation, an $N$-boson ground state exists—by finding exact upper and lower bounds on the Hamiltonian energy. Since these coincide in three dimensions, the (idealized) quantum ground state energy is exactly:

$$E_0 = -N m \chi^2 k_m / (6 \pi^2).$$

Here we have assumed that $k_m \gg (\chi m/2 \hbar)^2$, and used a variational ansatz of the form given previously. The ansatz gives us the true ground state energy in the limit $k_m \to \infty$ (for any finite $\kappa$), however, with finite $k_m$, it is not necessarily the lowest possible energy.

In order to show this, we consider a coherent or mean-field ansatz, with broken symmetry, of the form $|\phi> = \exp\{i \int dx \Psi(x)|\phi> + i \chi(x)|\phi>^\dagger(x)|\phi>\}$. Provided $\theta(x)$, $\phi(x)$ are chosen to minimize the classical Hamiltonian, they can give a lower energy than previously—although still finite. This calculation makes use of the known result that the classical parametric Hamiltonian is always bounded below, and the bound is given by the soliton energy for exact phase matching $\rho = 0$. This soliton energy is estimated by means of a Gaussian variational ansatz. The physics is considerably simplified in the region where the term in $\chi$ is dominant which we note should not involve too large a contribution from the repulsive term, and tends to destabilize soliton formation. In this region (i.e., assuming $\kappa \sim \rho = 0$), we obtain a coupled atom-molecular Bose condensate energy of:

$$E_0^2 = -CN^2 (\hbar^2/m)(\chi m/\hbar)^3/2,$$

where $C \approx 1.2 \times 10^{-7}$. The relevant length scale is nearly identical for the two coupled condensates, and is given by $l_1 = 1.7 \times 10^2 (m \chi/\hbar)^{1/2}/N$.

To obtain a stable coupled atom-molecular condensate, we require $E_0^2 < E_0^2$, which occurs at a critical boson number $N > N_c = \sqrt{N_m (8 \pi^2 \chi / m \hbar)}$. To give some numerical results we consider $m = 10^{-25} \text{ kg}$, and use a $\chi$-value estimate of about $\chi = 10^{-10} m \chi/\hbar$, which is well within the range of current BEC experiments.

At low particle density, the formation of individual dressed molecules is favored, as atoms couple to molecules in a particle-like way. At large couplings $\chi$, and at large density (but not too large so that $S$-wave scattering is dominant) the coherent coupling of two entire condensates occurs—just as in nonlinear optics.

In this domain, provided other recombination processes are negligible, there are strong, coherent and nonlinear wave-like interactions between the atomic and the molecular Bose condensates. For these parameters, it even appears possible to form stable, three-dimensional, Bose-Einstein solitons (soliton). We give a numerical simulation in Fig. 1.


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Fig. 1. 3D solitary waves for the coherently coupled atomic (left) and molecular (right) condensates, with a Gaussian initial condition.

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Weak force detection using a Bose-Einstein condensate

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We present a potential scheme to detect weak forces by employing a BEC confined in a double-well potential. Initially the condensate is prepared in a coherent superposition of having all the atoms in one well or the other. We allow the condensate to experience a phase shift for a certain time $\tau$ under the influence of the force. The phase shift can be detected by using the technique analogous to the Ramsey interference. The interference fringes can be read out by performing a homodyne measurement on the condensate localized in one well. We have analyzed the limitations on the accuracy of the scheme and show that a high-precision measurement can be achieved if the condensate contains a large number of coherently condensed atoms.