Analysis of Bubble Flow
in Metallurgical Operations Using Multivariate
Statistical Techniques

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Declaration

The candidate hereby declares that the work in this thesis, presented for the award of the degree of Doctor of Philosophy submitted in the Mathematics Discipline, Faculty of Engineering and Industrial Sciences, Swinburne University of Technology: is that of the candidate alone and has not been submitted previously, in whole or in part, in respect of any other academic award and has not been published in any form by any other person except where due reference is given, and has been carried out during the period from Feb 2007 to May 2010 under the supervision of Professor Geoffrey Brooks, Dr. William Yang (CSIRO) and Dr. Alireza Bab-Hadiashar.

Certification

This is to certify that the above statement made by the candidate is correct to the best of our knowledge.
Professor Geoffrey Brooks
Dr. William Yang.
Abstract

The behaviour of bubbles within metallurgical vessels is important to crucial aspects of their operations, such as mass transfer, heat transfer and splash generation. Physical models have been used to investigate different aspects of bubbling and provide data for verification of mathematical models. In industry, spout eye area, which is formed while the gas escapes from the liquid surface during bottom gas stirring process, has been used to monitor the process. Vibration signals on the wall of vessels have also been measured in industry to monitor the gas flow. Sound signals of the bubbling have been correlated with different behaviour of the gas bubbles inside the bath, such as bubble formation, distortion, coalescence, volumetric oscillation and detachment. It is clear that these three types of signal, i.e. image from the disturbed top surface, sound of the bubbling and vibration on the wall of the vessel are all generated from the same physical process, and indicate some aspect of the bubbling phenomena. This study focuses on the investigation of the combined effect of all these three types of signals, which were collected simultaneously in well controlled cold model experiments, based on multivariate statistical analysis techniques. The aim of this study is to investigate the possibility of monitoring bubble flow with a combined signal, which depends on the variables that can be reliably measured, and if possible, how to simplify this combined signal from the large data base which carries all the information of the process.

Cold modelling experiments were performed to establish techniques to analyse all these three types of signals simultaneously and quickly. A cylindrical cold model with a diameter of 420 mm and height of 500 mm, based on both dimensional and dynamic similarity criteria, was used to collect different types of signals.
simultaneously over a wide range of flowing conditions. The depth of the water bath which simulates steel was kept at 210 mm, and motor oil, which simulated slag, the height varied from 5 mm to 20 mm. Pressured gas were injected from the bottom of the vessel through a nozzle with a diameter of 3 mm, and the volume flow rate varied from 2.0 l/min to 20.0 l/min. Images of the disturbed top surface and sound of the bubbling signals were collected by a digital video camera installed above the vessel and vibration signals were collected by an accelerometer installed on the wall of the vessel. The size of the spout eye area was calculated by a threshold technique developed in this study, which takes approximately 0.1 second to analyse each frame of the image files in average, and the sound and vibration signals were pre-treated in both time domain and frequency domain.

Principal Component Analysis (PCA) technique was applied in this study to investigate the data base collected from the cold model experiments. The results from PCA demonstrated that the three types of signals are highly correlated and can be combined into one latent variable, which explains most (about 86%) of the total variation of the cold model experiments, and this latent variable can indicate the stirring process inside the bath effectively, because there exists a clear linear relationship ($R^2=0.96$) between this latent variable (dominant principal component) and stirring power which was calculated from the same cold model data. Since there are established relationships between the overall mass transfer coefficients and inclusion removal rate with stirring power, it should be possible to predict the metallurgical operations inside the bath using this latent variable.

The possibility of indicating the stirring process by just one or two channels of signals was further investigated and the PCA results showed that the signals from just one channel can only provide limited indication of the system, however, the
combined signal from sound and vibration can capture most of the variation of the process (about 88% of the total variation except the image signals), and there is also a clear linear relationship ($R^2=0.95$) between stirring power and the latent variable which combined the signals from sound and vibration. This finding suggests new type of sensor can be developed, which can be applied to monitor the gas stirring process and provide a feedback signal for the control system, based on combination of vibration and sound signals alone. This finding is particularly important for the pyrometallurgical operations where it is difficult to install a digital camera above the vessel.

The relationship between stirring power and Froude number, which is generally applied as a dynamic criteria for the gas bubbling phenomena, was investigated and the results showed that there is a clear linear relationship between the combined signal and Froude number specifically defined. A new variable “$BF$ (bubbling factor)” was defined in this study based on the combined signal of sound intensity and vibration magnitude. PCA results based on the cold model data showed that $BF$ can capture most variation of the process (91.3%), and a strong linear relationship was found between bubbling factor and stirring power ($R^2=0.92$), which demonstrate that $BF$ can monitor the bubble flow effectively and can be applied to predict the metallurgical operations inside the bath. Additionally, statistical analysis based on the cold model data showed that the sampling period can be reduced to 2.0 seconds to collect sufficient information about the stirring process inside the bath, which means that the bubbling factor can give a feed back signal every two seconds. These findings should facilitate the development of online sensors that monitor the stirring process quickly and effectively, based on sound and vibration signals from the process.
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It is said that although we have a long way to go in our life, however there are only a few critical steps which may affect you for a period of time or, may affect the rest of your life. I think the time I was introduced to Professor Geoff Brooks is one of such critical moment in my life. I was supposed to finish a quick degree and continue with my academic career in the field of water jetting technology. However, I suddenly realized that its application was very limited and so was my career. Geoff appeared in my life at the right time, and I fortunately became his first Ph. D student at Swinburne. I can never thank him enough for his wonderful guidance and extensive help during the long way toward this degree, particularly, the encouragement I received from him every time we have a talk.

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Nomenclature

\( A \) - spout eye area (m\(^2\))
\( A^* \) - relative spout eye area
\( A_p^* \) - predicted value of non-dimensional ladle eye area
\( A_c^* \) - calculated value of non-dimensional ladle eye area
\( A_e \) - area of exposed plume eye (m\(^2\))
\( A_{es} \) - exposed spout area measured at slag air interface (m\(^2\))
\( A_s \) - spout area (surface area between liquid metal and air) (m\(^2\))
\( A(t) \) - total transformed area (cm\(^2\)) at the time of observation \( t \)
\( A_0 \) - orifice cross section (m\(^2\))

\( \bar{A}^* \) - non-dimensional eye area

\( A_p \) - area of bubble plume at interphase level (m\(^2\))

\( a \) - numerical constant

“\( a \)” (in Matlab program) - relative spout eye area

\( b \) - numerical constant

\( C_d \) - drag coefficient

\( D_0 \) - diameter of orifice (m)

\( \bar{d}_{ Bj} \) - mean bubble diameter (m)

\( Fr \) - Froude number

\( f_n \) - frequency of oscillation (Hz)

\( f(\theta) \) - volume factor

\( F_d \) - drag force (N)

\( F_s \) - surface tension force (N)

\( F_i \) - inertial force (N)
\( F_p \) - pressure force (N)
\( F_b \) - buoyancy force (N)
\( g \) - gravitational constant (m/s\(^2\))
\( \Delta G_s \) - energy required to form the drop/vapour interface (J)
\( \Delta G_p \) - energy gained because of the new phase (J)
\( \Delta G_{Total} \) - total energy for the free bubble
\( \Delta G_v \) - energy gained per unit volume of the new phase (J/m\(^3\))
\( h \) - height of upper layer or top layer (slag/oil) (m)
\( H \) - height of bulk fluid (liquid steel/water) (m)
\( h_d \) - height of the dome (m)
\( k \) - Boltzmann constant (J/K)
\( k \) - empirical coefficient
\( n \) - number of samples
\( N_c \) - dimensionless capacitance number
\( N_0 \) - initial number of heterogeneous nuclei present per cm\(^2\) of area
\( P \) - pressure inside the drop (Pa)
\( P_0 \) - absolute liquid pressure (Pa)
\( P \) - pressure of the chamber (Pa)
\( P_\infty \) - pressure of the liquid flow at sufficiently distance (Pa)
\( p_\nu (T_c) \) - saturated vapour pressure (Pa)
\( p \) - pressure inside the bubble (Pa)
\( P^* \) - pressure of the liquid (Pa)
\( Q \) - gas flow rate at nozzle exit (l/min)
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\( r \) - discrepancy of the predicted value of the non-dimensional ladle eye area
\( r \) - radius of the bubble (m)
$r^*$ - critical drop radius ($m$)

$r_0$ - bubble radius ($m$)

$R^2$ - square of multiple correlation coefficient

- coefficient of multiple determination

$R_d$ - departure radius,

$R_s$ - radius of the bubble foot ($m$)

$R_0$ - mean radius of the bubble ($m$)

$r_n$ - amplitude of oscillation associated with the $n^{th}$ order ($m$)

“$s$” (in Matlab program) - sound intensity

“$sf$” (in Matlab program) - sound spectrum

$S_n(\theta, \varphi)$ - surface harmonic of order of $n$ describing the variation of the bubble

$T$ - absolute temperature (K)

$U_p$ - plume velocity ($m/s$)

$u_x$ - velocity of the liquid at sufficiently distance ($m/s$)

$u_{Bj}$ - mean bubble rising velocity ($m/s$)

$u_{cl}$ - axial mean velocity of water ($m/s$)

$u_r$ - relative bubble rising velocity ($m/s$)

$v$ - volume of the bubble ($m^3$)

$V_c$ - volume of the chamber ($m^3$)

“$v$” (in Matlab program) - vibration spectrum

“$vf$” (in Matlab program) - vibration magnitude

$z$ - axial height ($m$)
Greek letters

\( \alpha_0, \alpha_1, \alpha_2, \alpha_n \) - empirical constant

\( \alpha = \frac{P}{P_0} \) - pressure ratio

\( a(t) \) - corresponding fractional transformed areas

\( \sigma \) - cavitation number

\( \rho \) - density of liquid steel (density of the bulk liquid) (kg/m\(^3\))

\( \Delta \rho = \rho_L - \rho_G \) - density difference between liquid and gas (kg/m\(^3\))

\( \rho_L \) - density of liquid (kg/m\(^3\))

\( \rho_G \) - density of gas (kg/m\(^3\))

\( \Delta \rho \) - density difference between liquid and bubble (kg/m\(^3\))

\( \rho_v \) - vapour density (kg/m\(^3\))

\( \tau \) - time (s)

\( \gamma \) - unit interfacial tension (J/m\(^2\))

\( \gamma \) - surface tension (N/m)

\( \gamma \) - ratio of specific heats of gas

\( \theta \) - contact angle

\( \eta \) - viscosity of the liquid (m\(^2\)/s)
CHAPTER 1

Project Introduction

The behavior of bubbles within metallurgical vessels is important to crucial aspects of their operations, such as, mass transfer, heat transfer, splash generation, reaction of phases with atmosphere and refractory degradation. Cold models of pyrometallurgical operations have been developed to study these aspects of the process and to provide data for verification of mathematical models\(^1, 2\). For example, the spout eye area where the bubble collapses and escapes to the air, which can be picked up by the image from the disturbed top surface, has been investigated to improve the fundamental understandings of the stirring process\(^2, 3\). Acoustic data from an industrial cell has been related to the phenomena measured in a cold model and used as a signal for process control\(^4\). Vibration signals from the outside wall of the metallurgical vessel were able to indicate the stirring energy inside the vessel and have been applied in industry to help control the stirring process\(^5-7\).

It is clear that all these three types of signals, \(i.e\). image of the disturbed top surface, sound of the bubbling and vibration on the wall of the vessel were indirect effects of the bubbling phenomena which are generated by the injection of pressurized gas at the bottom of the vessel; therefore, they should be related to each other. There are currently limited techniques available for simultaneously analyzing the visual and acoustic data from cold models\(^8\). The development of techniques for rapid evaluation of all the effects of bubble flow would not only provide useful data for analyzing cold modeling experiments but also potentially lead to the development of techniques for controlling industrial systems.

Techniques for rapid analysis of images using multivariate statistical methods have been developed in the last decade and applied to the metallurgical industry.
These techniques are based on viewing an image as a data matrix and analyzing this data for underlying relationships ("latent variables"). In principle, it should be possible to simultaneously analyze other related data, such as acoustic data and vibration data, in the same way. Though there are significant challenges related to sampling, time dependency and data scaling that need to be addressed. In this study, these challenges will be addressed through development of multivariate statistical analysis techniques for a simple bubbling geometry (bottom bubbling of a cylindrical vessel) and testing where possible the techniques developed on the actual industrial applications.

The aims and objectives of this study are as follows:

1. Development of effective techniques to provide rapid image analysis of bubble flow in cold models of metallurgical processes.

2. Establishing multivariate statistical techniques for simultaneous analysis of image, acoustic and vibration data generated from bubble flow in cold models.

3. Investigating the effect of combining all the three types of signals and the possibilities to indicate the stirring process by this combined signal.

4. Addressing the online analysis of the bubble flow and investigate the possibilities of indicating the stirring process with limited signals, *i.e.* by removal of one channel of the signals, or just by one channel of the signals.

5. Evaluating the use of these techniques in industrial systems.

The study was started by surveying the literature on bubbling fundamentals, digital signal analysis, and multivariate analysis. This literature review is presented in Chapter 2, 3 and 4 of this thesis. Research issues are addressed in Chapter 5. A clear cylindrical vessel which is based on ladle geometry was
constructed and used to establish the techniques for analyzing the image, acoustic and vibration signal of the bubbling phenomena simultaneously. The experimental methodology and techniques of analysis developed are presented in Chapter 6. The signal analysis and results are presented in Chapter 7. Discussion regarding to monitoring the bubble flow in metallurgical operations is presented in Chapter 8. Industrial trials of the image analysis techniques developed in this study are described in Chapter 9. Chapter 10 summaries the conclusions of this study and give some suggestions regarding further research and industrial development.

The following papers have resulted from this study:

- Modeling of ladle eye phenomena, Chemeca, Sept 28th-Oct 1st, 2008, Newcastle City Hall, Newcastle, Australia
- Rapid image analysis of ladle eye area using a threshold technique, accepted by Ironmaking and Steelmaking: Processes, Products and Applications on Nov 3, 2009
- Development of Online Sensors for Bubble Stirred Vessels, TMS 2010 Annual meeting, Feb 14th-18th, 2010, Washington State Convention Center, Seattle, WA, USA
- Online Monitoring of the Ladle Stirring, AISTech, May 3rd-6th, 2010, David L. Lawrence Convention Center, Pittsburgh, PA, USA
- Online analysis of stirring processes in ladle metallurgy, accepted by Metallurgical and Materials Transactions B on May 26, 2010
CHAPTER 2

Bubbling Fundamentals

2.1 Introduction

A bubble is a globule of one substance in another, usually gas in a liquid, and the word “bubble” can be equally applied to gas surrounded by a bilayer of surfactant in a second gas phase, and to gas surrounded entirely by a liquid, with or without surfactant, though this distinction might be lost at very high bubble volume fractions\(^{[13]}\). Bubble stirring is common in metallurgical processes, due to its operational convenience and potential for bath mixing derived from the intrinsic buoyancy energy of bubbles\(^{[14]}\). In general, the objectives of bubble stirring are to achieve homogenization of temperature and composition, accelerate chemical reactions and promote the removal of inclusions\(^{[15]}\).

Bubble phenomena have been extensively studied and there have been several reviews on the formation, growth, and detachment of bubbles, mostly because bubble phenomena are common and important in various industrial applications, ranging from chemical engineering\(^{[16-18]}\) to mineral processing\(^{[14, 19]}\) and manufacturing\(^{[20]}\). In this chapter, a general review is provided regarding bubble formation, bubble plume profile, spout eye size and acoustic signals given out by bubbling flow inside cylindrical vessels which are filled with two layers of liquid and stirred by gas injected at the bottom. Current bubble flow control systems in ladle metallurgy stations are also briefly reviewed.

2.2 Formation of Bubbles

The formation of bubbles has been a topic of many investigations and it was categorized by various approaches. Zhang\(^{[21]}\) divided the formation of bubbles
into three cases, viz. formation within a liquid, at a liquid-solid interface and at the tip of an immersed lance. Kulkarni and Joshi\textsuperscript{[22]} comprehensively reviewed the bubble formation at a single submerged orifice, particularly highlighting the formation of bubble in Newtonian as well as non-Newtonian stagnant and flowing liquids. Brimacombe and his coworkers\textsuperscript{[14]} regarded the bubble as dispersed gas phase in liquid phase, and comprehensively reviewed bubble formation in metallurgical systems. However, a more general description of the bubble formation was given by Lubetkin\textsuperscript{[13]}, which will be briefly summarized below.

According to Lubetkin\textsuperscript{[13]}, bubble formation can be broadly separated into two categories, spontaneous bubble formation and non-spontaneous bubble formation\textsuperscript{[13]}. In the first case, the free energy of the system is reduced by the appearance of bubbles. While in the latter case, the free energy of the system is increased with the formation of the bubbles.

The spontaneous bubble formation has been further categorized into the following six quite separate and essentially independent sources. It is noted that in the last two cases, the new (bubble) phase already exists, true nucleation is thus avoided\textsuperscript{[13]}.

- Homogeneous nucleation
- Heterogeneous nucleation
- Cavitation
- Electrolysis
- Harvey nuclei
- Pre-existing and colloidally stable free bubbles

If the free energy of the system is increased by the appearance of the bubbles, it is said that the bubbles are formed non-spontaneously in the thermodynamic sense\textsuperscript{[13]}. Lubetkin further categorized non-spontaneous bubble formation into
sparging, entrainment, and attrition\textsuperscript{13}.

—Sparging was defined as the insertion of gas bubbles directly into a liquid by pumping gas into the bulk liquid\textsuperscript{13}, which is termed as “stirring” in metallurgical operations. Accumulated knowledge of stirring vessel is well documented in a review paper by Brimacombe and his coworkers\textsuperscript{14}. Several studies have been made for the formation of bubbles in liquid and gas interactions\textsuperscript{23-25}. The important factors affecting bubble formation have been comprehensively reviewed by Kulkarni and Joshi\textsuperscript{22}, they include:

- Liquid properties, including viscosity of the liquid, surface tension of the liquid, and liquid density
- Gas density
- Orifice configuration, including orifice construction and type of the orifice, orifice diameter, orifice chamber volume, orifice submergence, orifice contact angle, orifice orientation and material of construction of the orifice

Mittoni\textsuperscript{26} also reviewed the physical factors affecting bubble formation covering most of the factors mentioned above, particularly highlighting the chemical reactivity between gas and liquid, which can have a larger impact on the gas-liquid interaction under certain circumstances, such as injection of ammonia and nitrogen mixture into water. Zhang\textsuperscript{21} presented a comprehensive review in his thesis regarding to bubble formation models and various models for evaluating bubble size at different conditions, and concluded that any factor influencing bubble size will also influence bubble sound.

The mechanics of bubble formation at a submerged orifice depend strongly on the flow properties of the gas phase\textsuperscript{14}. The system capacity number $N_c$ was suggested as a quantitative criteria for constant flow and constant pressure gas
injection circumstances\cite{27}.

\[
N_c = \frac{\Delta \rho g V_s}{\rho_G A_0 c^2} \tag{2-1}
\]

Where, \( \Delta \rho = \rho_L - \rho_G \) density difference between liquid and gas (kg/m\(^3\))

\( V_s \) -sub-nozzle chamber volume (m\(^3\))

\( g \) -gravitational constant (m/s\(^2\))

\( A_0 \) -orifice cross section (m\(^2\))

\( c \) -sonic velocity (m/s)

The gas injection system is considered to operate at constant flow condition when \( N_c < 1 \) and at constant pressure condition if \( N_c > 9 \)\cite{14, 27}.

The mechanism of bubble formation under constant flow conditions depends on the rate of gas injection\cite{14}. At low gas flow rates which normally require capillary injection to minimize nozzle flooding, the bubble volume is determined by a balance of buoyancy and surface tension forces, therefore, the volume of the bubble can be obtained by the following equation\cite{28}.

\[
V = \frac{\pi d_0 \sigma \cos \theta}{\Delta \rho g} \tag{2-2}
\]

Where, \( d_0 \) -orifice diameter (m), \( \sigma \) -surface tension (N/m\(^2\)), \( \theta \) -contact angle

For relatively high gas flow rates which are close to real tuyere operations, the surface tension can be neglected in comparison with the inertia of the bubbles\cite{29}, the volume of the bubble at detachment \( V_d \) can be predicted by the following equation\cite{14}:

\[
V_d = 1.378Q^{\frac{5}{6}}g^{-\frac{3}{5}} \tag{2-3}
\]

Where, \( Q \) is the gas flow rate (m\(^3\)/s).
The bubble volume under constant pressure conditions can be determined by means of a force balance similar to the case of constant flow systems, provided the gas flow rate is related to the steady chamber pressure, $P_c$, through the following orifice equation\cite{14}:

$$Q = K_0 \sqrt{P_c - \rho_L g (h - s) - \frac{2\sigma}{a}}$$ \hspace{1cm} (2-4)

Where, $h$ -depth of liquid (m) \hspace{0.5cm} $a$ -bubble radius (m)

$s$ -displacement of bubble center from orifice plate (m)

The orifice constant $K_0$ has been determined experimentally for a steady state flow of gas through the orifice in the absence of the liquid phase\cite{14}.

Due to the unsteady state thermal effects and non-wetting properties of liquid metals, the scaling up of aqueous model results to metallurgical systems is difficult\cite{14}. However, the correlations for bubble volume in aqueous systems are generally valid for liquid metals provided that the outer diameter of the nozzle is substituted for the inner diameter to account for the non-wettability of liquid metal\cite{14}. The performance of the system is determined by a combined effect of gas flow rate and system capacitance, as summarized by Brimacombe in Table 2.1\cite{14}, in which the bubble Reynolds number is defined as:

$$Re = \frac{du}{\nu}$$ \hspace{1cm} (2-5)

Where, $d$ -bubble diameter (m) \hspace{1cm} $u$ -bubble velocity (m/s)

$\nu$ -liquid kinematic viscosity (m$^2$/s)
Table 2.1 Effects of gas flow rate \((Q)\) and gas chamber volume \((V_s)\) on bubble diameter \((d)\) in aqueous solutions\[^{14, 30}\]

<table>
<thead>
<tr>
<th>(V_s)</th>
<th>(Q)</th>
<th>Low (Q)</th>
<th>Transitional (Q)</th>
<th>Intermediate (Q)</th>
<th>High (Q)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Small (V_s)</td>
<td>Equation (2-6)</td>
<td>Equation (2-7)</td>
<td>Equation (2-8)</td>
<td>Equation (2-9)</td>
<td></td>
</tr>
<tr>
<td>Intermediate (V_s)</td>
<td>Equation (2-10)</td>
<td>Equation (2-11)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Large (V_s)</td>
<td>Equation (2-12)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The equations are shown below:

\[
d = \left( \frac{6d_o \sigma}{\rho_L g} \right)^{\frac{1}{3}} \text{ for } N_C < 1, f < 100 \quad (2-6)
\]

\[
d = \sqrt[3]{\frac{3d_o \sigma}{\rho_L g} + \frac{9d_o^2 \sigma^2}{\rho_L^2 g^2} + K_c \sqrt{\frac{Q^2 d_o}{g}}} \text{ for } N_C < 1 \quad (2-7)
\]

\[
d = 0.54 \left( Qd_o^{0.5} \right)^{0.289} \text{ for } 0.7 < Qr_0^{0.5} < 100 \quad (2-8)
\]

\[
d = 0.7 \text{Re}^{-0.05} \text{ for } \text{Re} > 10^4 \quad (2-9)
\]

\[
d = \left( \frac{6d_o \sigma}{\rho_L g} \right)^{\frac{1}{3}} N_C^{-\frac{1}{2}} \text{ for } < 1N_C < 9, We < 2.4N_C^{-1} \quad (2-10)
\]

\[
d = \sqrt[3]{\frac{3d_o \sigma N_C}{\rho_L g} + \frac{9d_o^2 \sigma^2 N_C^2}{\rho_L^2 g^2} + K_c \sqrt{\frac{Q^2 d_o}{g}}} \quad (2-11)
\]

\[
d = 3.8 \left( \frac{d_o \sigma}{\rho_L g} \right)^{\frac{1}{3}} \text{ for } N_C > 9, We < 16 \quad (2-12)
\]

Mazumdar and Guthrie\[^{31}\] thoroughly reviewed the physical and mathematical models relating to gas stirred ladle systems. The influence of operating variables...
(viz., gas flow rate, vessel geometry, location of gas injection nozzle, etc.) on bath hydrodynamics and associated transport process under practical ladle refining conditions are now known with reasonable certainty[31]. However, according to their review of various models for bubble flow inside a cylindrical vessel, the expectation that a single analytical relationship should hold for all gas-injection configurations is unrealistic[26].

2.3 Bubble Plume Profiles for bottom stirring vessel

Bottom gas injection via porous nozzles has been widely used in metal refining systems[32-34]. The bubble plume profiles (the dispersion patterns of bubbles generated from a round porous nozzle in a water bath) can be classified into three types with respect to volumetric gas flow rate, as schematically illustrated in Fig 2.1[33, 34]. In the low gas flow rate regime, small discrete bubbles are generated and they rise in the bath without coalescence and disintegration (Fig 2.1 (a)). In the medium flow rate regime, small bubbles are generated but some of them cluster and coalesce in a large bubble while rising in the bath (Fig 2.1 (b)). In the high gas flow rate regime, bubbles forming at the nozzle exit are merged into one large packet, which envelopes the entire nozzle surface but starts breaking up as soon as it lifts off from the nozzle, mainly due to very intense turbulent liquid motion induced by preceding bubbles (Fig 2.1 (c))[34].

![Fig 2.1 Schematic illustration of bubble plume profile in three gas flow rate regimes][34]
Iguchi and his coworkers\textsuperscript{[34]} investigated the bubble plume profile from round porous nozzles using a still and a high-speed video camera. The characteristics of rising bubbles were represented by gas holdup $\alpha$, bubble frequency $f_B$, and mean bubble rising velocity $\bar{u}_B$ etc., which were measured with a two needle electro-resistivity probe. It was found that the bubble plume profiles (dispersion pattern) were dependent on the pore diameter of the porous nozzle, and were hardly influenced by the bath surface pressure. However, the characteristic variables were dependent on the volumetric gas flow rate at the porous nozzle exit\textsuperscript{[34]}. They further defined “gas holdup” as the volume percentage of gas in a gas-liquid mixture, and denoted the gas holdup on the centerline of the bubbling jet as $\alpha_{cl}$\textsuperscript{[34]}. The bubble flow profile in a cylindrical vessel was characterized as momentum region, transition region, buoyancy region and surface region, which are shown schematically in Fig 2.2.\textsuperscript{[35-37]}

![Diagram of bubble flow field in a cylindrical vessel](image)

Fig.2.2 Classification of the bubble flow field in a cylindrical vessel\textsuperscript{[36]}

Anagbo and his coworkers\textsuperscript{[38]} investigated bubble plume characteristics by means of Laser Doppler Velocimeter (LDV), and demonstrated that the velocity of the liquid circulation patterns is a weak function dependent on the dispersion regime.
The schematic plume profile at a moderate gas flow rate (0.5 Nm$^3$/min) into an aqueous model is illustrated in Fig 2.3$^{[31]}$.  

Fig 2.3 Bubble plume profile at moderate gas flow rate$^{[31]}$

The gas-liquid two phase region has been subdivided into four physically distinct regions, viz., primary region (1), free bubble region (2), plume region (3) and spout region (4) respectively, of these, the plume region is the largest, which is characterized by dispersed spherical cap bubbles in an air-water mixture$^{[31]}$. It is generally accepted that in the immediate vicinity of the nozzle, the input gas kinetic energy and the mode of gas injection are important variables; while in the fully developed region, these variables will have practically no influence on the overall development of the gas-liquid two phase region, such as bubble size, bubble spatial distribution and rise velocities$^{[31]}$. Bubbles in the fully developed region will tend to establish a dynamic range of sizes in the spherical cap regime, the equilibrium size of which are determined by the thermo-physical properties
of the system and not by the inlet operating variables, such as gas injection device and orifice diameter\cite{31}.

### 2.4 Spout Eye Size

“Spout” is defined as the region at surface in which the gas escapes into the environment during gas stirring of a metal bath\cite{39}, and —spout eyes” are the open parts of the meniscus, uncovered by slag\cite{39}. The spout eye is usually termed —ladle eye” in industry applications\cite{12, 40}. Controlling the ladle eye area is crucial to ensure the product quality in industrial practice, because it can be the site of undesirable reactions between the metal and air. For example, oxygen dissolved into molten steel from an exposed eye can increase the level of inclusions and work against sulfur removal. However, the spout eye area is also the place for alloy addition, which prevents losses through entrainment in the slag or entrapment in a slag crust\cite{15}. Spout eye size has been a topic of several investigations\cite{2, 3, 10-12, 39-47}, and there is an ongoing debate about the appropriate correlations for predicting spout eye size\cite{2, 43, 47}.

![Fig 2.4 Schematic diagram of cold model experiment setup\cite{39}](image)

Yonezawa and Schwerdtfeger\cite{39} investigated the spout region with the objective
of establishing relationships to predict the dimensions of the spout and the spout eye as a function of process conditions. Data were obtained from both cold models and industrial measurements on a 350 ton ladle. Mercury and silicon oil were used to simulate metal and slag. A video camera was employed to measure the spout and spout eye, as per the experimental set up shown schematically in Fig 2.4.\textsuperscript{[39]} Electro-resistivity probes were used to measure the height of the spout for the cold model and dissolved tube length techniques were applied for plant measurements of the slag. The results indicated that spout eye formation and spout eye size are highly dynamic, as a consequence of the discontinuous gas discharge at the nozzle and of the subsequent disintegration of the bubbles. Non-dimensional correlations were established for the time averaged spout eye area size\textsuperscript{[39, 42]}. Subagyo and his coworkers\textsuperscript{[42]} further claimed to have improved the correlation by assuming the geometry of the plume profile, which is schematically shown in Fig 2.5.\textsuperscript{[42]}

\begin{figure}[h]
  \centering
  \includegraphics[width=0.5\textwidth]{fig2.5.png}
  \caption{Schematic diagram of gas stirred ladle metallurgy\textsuperscript{[42]}}
\end{figure}

It has been widely accepted that the spout eye area fluctuates with time because of the stochastic nature of the bubbling process and the accumulated average stabilized after approximately 100 readings\textsuperscript{[2]}. Yonezawa and Schwerdtfeger\textsuperscript{[39]} developed the following semi-empirical representation of average open spout eye

15
area through dimensional analysis technique and correlation with cold model experiment results and industrial data. Their correlation is based on the Froude number, which is the ratio of inertial force and the gravitational force, and is given as \(^{[39]}\):

\[
\frac{A_{es}}{hH} = f\left(\frac{Q^2}{gh^5}\right) \tag{2-13}
\]

Where, \(A_{es}\) - exposed spout area measured at the slag air interface (\(m^2\))

\(h\) - height of the upper layer or top layer (slag/oil) (m)

\(H\) - height of the bulk fluid (liquid steel/water) (m)

\(Q\) - gas flow rate at nozzle exit (\(m^3/s\))

\(g\) - gravitational constant (\(m/s^2\))

According to Yonezawa and Schwerdtfeger\(^{[39]}\), when the Froude number is between 0.01 and 10,000, and nozzle diameter is 0.5 mm, the following semi-empirical equation can be used\(^{[39]}\):

\[
\log\left(\frac{A_{es}}{hH}\right) = \alpha_0 + \alpha_1 \log\left(\frac{Q^2}{gh^5}\right) + \alpha_2 \log\left(\frac{Q^2}{gh^5}\right)^2 + \alpha_3 \log\left(\frac{Q^2}{gh^5}\right)^3 \tag{2-14}
\]

Where, \(\alpha_0, \alpha_1, \alpha_2, \alpha_3\) - empirical constants

Subagyo and his coworkers\(^{[42]}\) claimed to have improved the above equation by assuming the shape of the plume to be conical, which is shown in Fig 2.5, and forming the following semi-empirical correlation\(^{[42]}\):

\[
\frac{A_{es}}{(H + h)^2} = f\left(\frac{Q^2}{gh^5}\right) \tag{2-15}
\]

\[
\frac{A_{es}}{(H + h)^2} = (0.02 \pm 0.002)\left(\frac{Q^2}{gh^5}\right)^{0.375 \pm 0.0136} \tag{2-16}
\]
Mazumdar and Evans\textsuperscript{[3]} formed a new equation based on equating the kinetic energy of the rising plume to the potential energy associated with the creation of a spout at the bath surface, which is shown schematically in Fig 2.6\textsuperscript{[3]}.

Through some simplified geometrical considerations, they derived the following expression for the dimensionless exposed eye area in a thin slag covered ladle\textsuperscript{[3]}:

\[
\frac{A_e}{H^2} = K \left( 1 - 2 \cdot \frac{h}{H} \cdot \frac{1}{Fr} \right)^2
\]  
(2-17)

Where, \(A_e\) - the area of the exposed plume eye (m\(^2\))

\(K\) - empirical coefficient, \(Fr\) - Froude number

\(h\) - height of the upper layer or top layer (slag/oil) (m)

\(H\) - height of the bulk fluid (liquid steel/water) (m)

Using the control volume technique and forming momentum balance equations of the plume in the vertical direction, as shown in Fig 2.7 and Fig 2.8, Krishnapisharody and Irons proposed the following equation\textsuperscript{[2]}:

\[
\frac{A_e}{A_p} = a + b \left( \frac{\rho \cdot U_p^2}{\Delta \rho \cdot gh} \right)^{1/2}
\]
(2-18)

Where, \(A_p\) - area of the plume (m\(^2\)) \(a, b\) - numerical constants

\(U_p\) - plume velocity (m/s) \(g\) - gravitational constant (m/s\(^2\))
ρ - density of the liquid steel (density of the bulk liquid) (kg/m³)
Δρ - density difference between the liquid steel and slag (kg/m³)
h - height of the upper layer or top layer (slag/oil) (m)

Fig 2.7 Schematic diagram of the eye formation

Fig 2.8 Control volume for momentum balance

A critical review regarding to the existing semi-empirical correlations between non-dimensional spout eye area and operating parameters was performed by Xu et al. [48], as part of this study, and is presented as Appendix 1. The comparison of different mathematical models with the same industrial data and cold model experiment revealed that the Krishnapisharody and Irons’ model provides the best prediction for the industrial results, while, Subagyo et al. model predicts the result from the cold model experiments more successfully than the other studies. All of the models developed so far depend on parameters which can not be reliably measured in industry and thus are limited in their ability to accurately predict industrial phenomena. However, it is generally accepted that the spout
eye area size increases with gas flow rate and decreases with increasing upper phase thickness\cite{2, 12, 44}.

A Multivariate Image Analysis (MIA) technique was proposed by Brooks and Subagyo for ladle eye detection\cite{15, 40, 49}, and the original objectives was to develop a semi-empirical prediction for ladle eye size as function of gas flow rate and geometry of the vessel\cite{15, 42}. However, this approach stimulated them to design a new system for online monitoring of the ladle eyes\cite{15}.

An image (RGB format) can be digitalized into three congruent matrices in which, each pixel is specified by three values —one each for the red, green and blue components of the pixel’s color\cite{9}. Consequently an image can be regarded as a matrix with dimensions $n \times m \times 3$ mathematically, which is illustrated in Fig 2.9\cite{10}.

![Schematic diagram of image structure](image)

Fig 2.9 Schematic diagram of image structure\cite{10}

A multivariate statistical method, Principal Component Analysis (PCA), which will be further reviewed in Chapter 4, was applied for image monitoring and analysis by MacGregor and his coworkers\cite{9}. An algorithm was proposed by Brooks and Subagyo\cite{10, 15} to calculate the spout eye area size for high temperature systems, and computational time less than 30 seconds has been achieved in 2003\cite{10, 15, 40}. A MIA approach based on PCA was further investigated by Graham et al.\cite{12}, which they claimed to be able to provide rapid online estimates of the ladle eye area.
2.5 Bubbling Acoustics

Acoustic signals originate from the variation of the medium pressure, which will be further reviewed in Chapter 3. The sound of bubbling is strongly related with the bubble oscillation, and the emission of acoustic signals by bubbles on formation and deformation is well known\textsuperscript{[50-52]}. The research for bubbles oscillation can be traced back to 1917. Lord Rayleigh derived the following equation for bubble dynamics to describe the motion of the radius of the bubble $R(t)$ as a function of time $t$\textsuperscript{[53-56]}.

$$R R^2 + \frac{3}{2} \dot{R} = \frac{1}{\rho} \left( P_g - P_0 - P(t) - 4 \nu \frac{\ddot{R}}{R} - 2 \gamma \frac{\dot{R}}{R} \right) \quad (2-19)$$

Where,

- $\nu$ - viscosity of the liquid ($m^2/s$)
- $\rho$ - density of the liquid ($kg/m^3$)
- $P$ - pressure of the bubble ($N/m^2$)
- $\gamma$ - surface tension ($N/m^2$)

Strasberg\textsuperscript{[57]} argued that gas bubbles entrained in liquid generate sound under a variety of circumstances, the sound radiated by a bubble was the result of the simple pulsation in volume and the oscillatory motion of the bubble wall, which was represented by a sum of surface harmonics. He developed the following equation to represent the instantaneous radius $r$ at any point on the bubble surface\textsuperscript{[57]}.

$$r(t, \theta, \phi) = R_0 + \sum_n r_n S_n(\theta, \phi) \exp(2\pi f_n t) \quad (2-20)$$

Where, $R_0$ - mean radius of the bubble ($m$)

$S_n(\theta, \phi)$ - surface harmonic of order of $n$ describing the variation of the
bubble surface with angle $\theta$ and $\varphi$

$r_n$ - amplitude of oscillation associated with the $n^{th}$ order ($m$)

$f_n$ - oscillation frequency (Hz)

The appearance of the bubble wall oscillating in modes corresponding to individual orders of $n$ is shown in Fig 2.10\textsuperscript{[57]}.

\begin{figure}[h]
\centering
\includegraphics[width=0.8\textwidth]{shape_of_bubble.png}
\caption{Shape of an oscillating bubble for various orders $n$ (The bubble wall is shown in section, with mean position as a solid circle and the two extreme positions of oscillating wall as dotted lines)\textsuperscript{[57]}}
\end{figure}

According to Strasberg\textsuperscript{[57]}, a gas bubble formed at a nozzle emits a short pulse of sound. The pulse starts just as the bubble separates from the nozzle, and has the character of a damped sinusoidal oscillation with the natural frequency of volume pulsation. For typical conditions, the frequency will be in the range of 500 Hz to several thousand Hz. The pressure amplitude will decay at a rate of one or two decibels per millisecond. When bubbles divide or coalesce, a short pulse of sound is generated as in the case of bubble formation; however, the peak sound pressure is much smaller\textsuperscript{[57]}. He also claimed that significant sound pressures are associated only with volume pulsations of the bubble, whereas, oscillations in the shape of the bubble do not result in appreciable sound by assuming that the size of the bubble is sufficiently small so that the sound pressure and the external pressure are uniform around the bubble wall, yet sufficiently large that surface tension and the conduction of heat out of the bubble are negligible. The
predictions were verified by the experiment of sound generated by air bubbles formed on a nozzle, using oscillographs and high speed photography.\textsuperscript{[57]}

The emission of an acoustic signal by bubbles on formation and deformation has been a topic for a number of investigations, and a comprehensive review of bubble acoustics is given by Leighton\textsuperscript{[58]}. The fundamental relationship between bubble acoustic frequency and radius was found by Minnaert\textsuperscript{[51]} by equating the potential energy of the compressed gas at one node of the oscillation cycle, with the kinetic energy of the fluid set in motion around the bubble at the antinode, giving the following equation:\textsuperscript{[51]}

\[ f = \frac{1}{2\pi r_0} \sqrt{\frac{3\gamma P_0}{\rho}} \quad (2-21) \]

Where,

\( f \)– acoustic frequency of bubbling (Hz) \quad \( r_0 \)– bubble radius (m)

\( \gamma \)– ratio of specific heats for the gas

\( P_0 \)– absolute liquid pressure (Pa), \( \rho \)– density of the liquid (kg/m\(^3\))

It should be noticed that surface tension is not involved in the above equation, according to Longuet-Higgins \textit{et al.}\textsuperscript{[50]}, it is because the surface tension is a second-order effect, it is usually neglected except for microscopic bubbles\textsuperscript{[59]}.

Leighton and Walton\textsuperscript{[52]} suggested that the sound spectrum produced by bubbles in the environment could be used to calculate the size of the bubbles. As many industrial processes in chemical engineering involve aeration to enhance reactions and the rate of mass transfer, knowledge of the bubble size is very important to control the process, as it decides the overall surface area which controls the rate of mass transfer.

\[ \]
It was found that under most conditions in industrial aerators, for the large bubbles (2-4 mm radius) which are typically sparged rapidly from a nozzle, the acoustic frequency was not a constant and varied during the pulse, implying that the assumption of linear oscillation inherent in Minnaert equation (Equation 2-14) is not valid in practical cases. Significant errors are introduced if a simplistic approach, like applying the Fourier transformation to the whole pulse, is used to determine the bubble size\[^{[60]}\].

Acoustic bubble sizing in a turbulent jet was trialed by Pandit et al.\[^{[61]}\], they correlated the bubble size distribution in a two-phase bubble flow with the sound spectrum measured by a hydrophone and spectrum analyzer. Two types of bubble flow have been considered, \textit{i.e.} horizontal two-phase flow in a pipeline and a two-phase turbulent axisymmetric jet. It was found that the frequency of the sound increases as the bubble size diminishes\[^{[61]}\]. The advantages of this technique, as compared to alternative methods, such as photographic measurements, include the suitability of the technique for use in existing opaque pipe work without requiring major modifications and minimal disturbance of the flow. The technique can also be used to predict the gas-liquid mass transfer coefficient in pipelines\[^{[61]}\].

Manasseh \textit{et al.}\[^{[8, 62-64]}\] extensively investigated the precise mechanism of sound emission by bubbles on formation and detachment. They confirmed the idea that bubbles oscillate volumetrically and emit sound at a frequency characteristic of its radius. According to their experiments on continuously sparged bubbles and numerical simulations, the initial drop in the acoustic signal pressure is found to be due to the contraction of the tip of the bubble during the neck-breaking process, however, the peak in acoustic signal pressure is found to be caused by a jet of liquid that penetrates the bubble after neck breaking\[^{[53]}\]. They related the precisely-timed photographs to the acoustic signal produced on bubble formation to actual physical processes, and found that the initial fall in pressure is
associated with the neck-breaking process and the rapid retraction of the tip of the bubble once it has detached\cite{8}.

![Graph showing acoustic signal over 10 ms with labels a, b, c, d corresponding to the photograph in Fig 2.12\cite{8}]

**Fig 2.11** Acoustic signal over 10 ms, labels a, b, c, d correspond to the photograph in Fig 2.12\cite{8}

![Photograph of bubble detachment stages with labels a, b, c, d]

**Fig 2.12** Photograph of the later stages of bubble detachment, showing the jet tip forming a droplet, corresponding acoustic signal shown in Fig 2.11\cite{8}

Fig 2.11 shows the first 2 ms of the acoustic signal produced by bubble formation at a rate of $13.57 \pm 0.05 \ Hz^{[8]}$, it has been band-pass filtered between 200 Hz and 3
kHz. The dash lines in Fig 2.11 show the time of the series of four photographs in Fig 2.12.

An alternative explanation for sound generation is a nonlinear process that transfers energy from shape distortions. According to nonlinear theory, the sound frequency is twice the basic frequency of the distortion mode and the sound amplitude is proportional to the square of the distortion amplitude. Hence, for the bubbles which range in radius 0.1 mm to 10 mm, the radian frequency lie between 2×10³ Hz and 2×10⁵ Hz, corresponding to frequencies ω/2π lying between 0.3 and 30 kHz.

Fedotkin et al. investigated the acoustic oscillations of the bubbling processes and particularly studied the resonance characteristics of the vessel and the liquid contained in it. They asserted that multiple reflections of the acoustic signals from the walls produce interference distortions. Therefore, the sound which was produced by the pressure variation at the instant of growth and break off of the bubble may be affected by the resonance properties of the vessel, and the presence of the bubbles in the liquid also may alter the acoustic properties of the liquid and the resonance characteristics of the vessel. However, if the inner walls of the vessel are coated with a Porolon (a textile product for acoustic insulation) layer of thickness of one centimeter, the frequency response has a linear behavior. In the frequency range 6-20 kHz, the resonating action of the vessel is attenuated by 20-30 dB; and in such a vessel, the acoustical characteristics of bubbling are practically free of the influence of the resonance properties of the vessel.

In summary, the sound of bubble flow may come from the following sources:

- **Formation of the bubbles**
  
  A short pulse of sound will be emitted just as the bubble separates from the nozzle, and has the character of a damped sinusoidal oscillation at the
natural frequency of volume pulsation. The frequency is in the range of 500 Hz to several thousand Hz and the pressure amplitude will decay very quickly\[67\].

- **Free bubble volumetric oscillation**
  It is characterized by the Minnaert frequency\[59\]. The sound signal can be analyzed to predict the radius of the bubbles.

- **Shape distortion**
  It produces a monopole radiation of sound at second order, its frequency is twice the basic frequency of the distortion mode, and the sound amplitude is proportional to the square of the distortion amplitude\[65\].

- **Detachment**
  An initial drop in pressure is associated with the neck-breaking process and the rapid retraction of the tip of the bubble once it has detached\[53\].

- **Bubble coalescence**
  As the gas flow rate increase, the size and number of the secondary bubbles increase, and the sound amplitude also increase. The emission of high-amplitude sound coincides with the coalescence of a primary bubble with a smaller secondary one\[64\].

### 2.6 Vibrations Caused by Bubbling

The bubble stirring process not only lead to the variation of the spout eye area at the disturbed top surface but also lead to the acoustic signals, which has been reviewed in section 2.4 and 2.5 respectively. Additionally, the bubbling process causes vibrations on the wall of the vessel.

It was found that the force of gas bubbles rising through a liquid in a container
and breaking the surface causes the container to vibrate at one or more characteristic frequencies, depending on the inertial mass, and the physical properties of the liquid, the gas and the container\textsuperscript{[6]}. Mucciardi and his coworkers\textsuperscript{[68, 69]} investigated the use of accelerometers in monitoring flow conditions due to gas stirring in a bulk of liquid. In their experiments, an accelerometer was attached to the wall of the liquid container to monitor liquid-gas interactions for the following three different processes:

- Mixing of steel in teeming ladles
- Powder injection techniques of volatile reagents such as magnesium and calcium
- Decarburization of hot metal in steelmaking converters

They concluded that the relative magnitude of the vibrations can provide a valuable clue about the liquid-gas interactions that occur in a vessel\textsuperscript{[69]}, and demonstrated that the mixing power input, or the degree of agitation, imparted to melt is proportional to a representative value of the accelerometer signal raised to the power of 1.6\textsuperscript{[68]}. However, the vibration signals may be distorted by many extraneous vibrations in an industrial environment, such as arc reheating, crane movement, motor noise, insertion of alloying materials or sampling tubes\textsuperscript{[6]}. TruStir\textsuperscript{TM} \textsuperscript{[70]} was patented by Kemeny, F.L et al. (Nupro Corporation) for measuring the ladle full of steel as it is being stirred, isolating relevant frequency ranges and integrating to provide real time base amplitude of this integral, and relating this amplitude to the require stirring energy. A vibration controlled stirring system was also developed by Stelco LMF (Ladle Metallurgy Facility)\textsuperscript{[7]}, which applied three accelerometers attached to a ladle magnetically during the stirring process to detect ladle vibrations in three directions, they reported that the system resulted in reduced argon gas consumption during reheating, and increased reheating arc stability at the same time\textsuperscript{[7]}.
2.7 Current Bubble Flow Control Systems

Current control systems for bubbling in Ladle Metallurgy Stations (LMS) are still dominated by manual procedures which emphasize process stability rather than process optimization\cite{71}. For example, in a typical Ladle Metallurgy Station (LMS), the argon stirring system is usually controlled by measuring gas line pressure, back pressure, and gas flow rate. The system is reliable if there are no pressure fluctuations and if all the gas flows into the liquid steel as intended. However, this is usually not the case\cite{6}. The porous refractory element (plugs) may produce highly variable resistance to gas flow, depending on its physical conditions, which are listed as follows\cite{6}:

- The porous refractory element may get partially plugged with refractory particles, frozen metal or slag
- The pores or channels through which the gas flows may enlarge due to erosion during use and thereby offer less resistance to flow with time
- The connections between pipes, hook-ups, and refractory components may develop leaks during use
- The gas is under pressure, and it may find another porous path through the ladle refractory, allowing a portion of it to escape without entering the liquid steel

Consequently the system may demonstrate a certain level of stirring, as indicated by the volumetric gas flow rate, but this may be different from the actual degree of stirring in the ladle\cite{6}. In normal operations, visual check is used to determine that the desired level of stirring is being achieved. The ladle refining station operator varies the stirring level until he observes a desired level of stirring. This control relies on a subjective evaluation of the degree of turbulence at the surface of the steel\cite{6}.
A comprehensive review on the automation and mechanization in the steelmaking process was given by Fujii et al.\textsuperscript{[72]}. A new sampling method using the sublance during the blow, which carries the information of the molten steel, brought about remarkable improvements in terms of operation results, cost and work load in Nippon steel\textsuperscript{[72]}, however, it is difficult to express the interrelated data by mathematical equations, and artificial intelligence programs such as expert systems were incorporated and these advances made it possible for unskilled operators to operate the BOF process. SIEMENS\textsuperscript{[73]} automation system takes the approach of individual gas flow control system to provide an optimum of working safety and operational convenience, however, it still depends on the readings of gas flow and gas pressure, which is not reliable due to the reasons described above.

2.8 Summary

In this chapter, the fundamental knowledge of bubbles relating to this project has been briefly reviewed, including bubble formation, bubble plume profile, spout eye size, sound of bubbling, vibration caused by bubbling and the current bubble flow control systems. Bubble formation can be generally categorized into spontaneous and non-spontaneous based on the free energy of the system is decreased or increased by the appearance of the bubbles in the thermodynamic sense. Bubble stirring is common in metallurgical operations and the objectives are to achieve homogenization, accelerate chemical reactions and promote the removal of inclusions. The bubble plume profile can be classified into momentum region, transition region, buoyancy region and surface region based on the bubble characteristic such as gas holdup on the centerline of the bubbling jet $\alpha_{cl}$.

Controlling the spout eye size is crucial to ensure the product quality in industrial
practice and various semi-empirical correlations have been proposed to predict the size of the ladle eye area, however, none of them are successful as they all depend on parameters which can not be reliably measured in industry. It is generally accepted that the spout eye area size increases with gas flow rate and decreases with increasing upper phase thickness.

The sound of the bubbling is generated by the formation, coalesce and volume oscillation of the bubbles. The initial drop in the acoustic signal pressure is found to be due to the contraction of the tip of the bubble during the neck-breaking process, however, the peak in acoustic signal pressure is found to be caused by a jet of liquid that penetrates the bubble after neck breaking. The force of gas bubbles rising through a liquid in a container and breaking the surface causes the container to vibrate at one or more characteristic frequencies, and this vibration signals has been applied in industry to monitor the liquid-gas interactions inside the vessel.

The bubbling phenomena inside a bulk of liquid stirred at the bottom can lead to the following three effects:

1) The variation of the spout eye area at the top of the disturbed surface, which can be monitored by the image signals from the top of the vessel.

2) Sound of the bubbling.

3) The vibration on the wall of the vessel.

These concepts have been investigated individually, however, there is no research published in the literature so far, where the combined effects of all these three signals have been considered.
CHAPTER 3

Image, Sound and Vibration Analysis

During the process of bubble stirring in metallurgical operations, human senses play an important role. Even though extensive knowledge has been accumulated regarding to the understanding of key aspects of both the chemistry and physics of the ladle process, unfortunately, this understanding has not been translated into the development of sophisticated control systems and industrial practice is still largely dominated by manual control[71]. The ladle eye area region is of great interest in industrial practice, and it is important to achieve tight control of the ladle eye size, particularly for critical grades of product[15]. Experienced operators depend on his or her observations of the disturbed top surface and hearing of the sound during the controlling process, which could be replaced by online sensors[74]. The literature review in Chapter 2 demonstrated that the bubble stirring process could be monitored by three different types of signals, i.e. the image from the top of the disturbed surface, sound of the bubbling and the vibration on the wall of the vessel. In this chapter, the related digital image processing technique, sound and vibration analysis will be reviewed.

3.1 Digital Image Processing

Digital image processing technique can be traced back to 1920’s when digitalized newspaper pictures were sent by submarine cable between London and New York, which reduced the time required to transport a picture across the Atlantic from more than a week to less than three hours[75]. Digital image processing has found a wide range of applications such as biological research, materials research, defense intelligence, factory automation, remote sensing and space exploration.
Gonzalez published —Digital Image Processing” (1st edition) in 1977, and it has been printed six times before the second edition in 1987[77]. Several other books also cover the contents of digital image fundamentals, transforms, encoding, restoration, enhancement, segmentation and description[76, 78-82].

Digital Signal Processing (DSP), as the term suggests, is the processing of signals by digital means. The frequency ranges for different type of digital signal applications are summarized in Table 3.1[83].

Table 3.1 Typical digital signal processing systems[83]

<table>
<thead>
<tr>
<th>Application</th>
<th>Upper frequency limit $f_{\text{max}}$</th>
<th>Sampling frequency $f_s$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Geological exploration</td>
<td>500 Hz</td>
<td>1-2 kHz</td>
</tr>
<tr>
<td>Biology</td>
<td>1 kHz</td>
<td>2-4 kHz</td>
</tr>
<tr>
<td>Mechanical oscillation</td>
<td>2 kHz</td>
<td>4-10 kHz</td>
</tr>
<tr>
<td>Voice</td>
<td>4 kHz</td>
<td>8-16 kHz</td>
</tr>
<tr>
<td>Music</td>
<td>20 kHz</td>
<td>40-96 kHz</td>
</tr>
<tr>
<td>Video</td>
<td>4 MHz</td>
<td>8-10 MHz</td>
</tr>
</tbody>
</table>

A digital image is defined as a function of $f(x, y)$ which has been discretized both in spatial coordinates and in brightness[75, 84], and can be considered as a matrix whose row and column indices identify a point in a image and the corresponding matrix element value identifies the grey level at that point. The element of such a digital array are called image element, picture element or pixel[84].

Generally speaking, digital images can be categorized into three types, i.e. binary image, grey image and color image[80]. The pixels in a binary image can assume only two values, “0” or “1”. A grey image may be represented by a number of intensity levels through a process of quantization, which involves assigning a single discrete number to each pixel by conversion of the sampled analog pixel
intensities, and it is schematically shown in Fig 3.1. While a color image may be quantized in different color bands, each color appears in its primary spectrum components of red, green and blue. The other color space such as CMYK (Cyan, Magenta, Yellow, and key black) and HSI (Hue, Saturation, and Intensity) are well documented by Gonzalez, however, the RGB (Red, Green and Blue) color space is the most commonly used.

Modern digital technology has made it possible to manipulate multi-dimensional signals with different systems and the goal of this manipulation can be divided into three categories:

- **Image Processing** image in → image out
- **Image Analysis** image in → measurements out
- **Image Understanding** image in → high-level description out

In the first category, an input image may be manipulated mathematically and an image is put out. For example, a blurred image may be made clear; a “dark” image may become “lighter”. Most of the commercial image software has this kind of functions, such as Photoshop, Corel Painter etc. While, in the second category, an image may be treated by complex mathematical operations, and some kind of measurement can be worked out, such as the periphery of a special object, the area of a particular body and so on, in order to find out the interested object, some distinctions should be made first. This type of analysis is
especially useful in geology and mining. The most important and difficult application is the third category. People wish to find important information from an image, such as identification of new star in the outer space, confirmation of the existence of special elements (such as water) on a planet. In the following sections, the types of operation for digital image processing and their applications will be reviewed.

### 3.1.1 Types of Image Processing

When a process generates an output image from an input image, there will be a correspondence between points of the two images. Each pixel in the output image corresponds to one pixel or several pixels in the input image. Thus, when the operation is applied to one point or a neighborhood centered upon one point, the resulting grey-level value is stored in the corresponding point in the output image. The operations mentioned above may be summarized in the following table and they will be explained by the Fig 3.2, in which the meaning for the transformation of an input image \( a(u) \) into an output image \( b(m, n) \) (or another representation) is illustrated schematically.

Table 3.2 Types of image operations

<table>
<thead>
<tr>
<th>Operation</th>
<th>Characterization</th>
<th>Generic Complexity/Pixel</th>
</tr>
</thead>
<tbody>
<tr>
<td>Point</td>
<td>The output value at a specific coordinate is dependent only on the input value at that same coordinate.</td>
<td>Constant</td>
</tr>
<tr>
<td>Local</td>
<td>The output value at a specific coordinate is dependent on the input values in the neighborhood of that same coordinate.</td>
<td>( P^2 )</td>
</tr>
<tr>
<td>Global</td>
<td>The output value at a specific coordinate is dependent on all the values in the input image.</td>
<td>( N^2 )</td>
</tr>
</tbody>
</table>

In which, image size = \( N \times N \), neighborhood size = \( P \times P \). Note that the
complexity is specified in operations per pixel.

As for the neighborhood operation, there are various kinds of neighborhood configurations in the digital image processing, some of the most common neighborhoods are the 4-pixel-connected neighborhood and the 8-pixel-connected neighborhood in the case of rectangular sampling and the 6-pixel-connected neighborhood in the case of hexagonal sampling illustrated in Fig 3.3\cite{85, 88}. Neighborhood operations play a key role in modern digital image processing.

Various tools have been developed for digital image processing. These include histogram based operations, such as contrast stretching and equalization (histogram normalization); mathematics based operations, such as binary operations and arithmetic operations; convolution based operations; derivative based operations; and morphology based operations, such as translation, dilation and erosion. They were well documented by Young and his coworkers\cite{88}.
3.1.2 Image Analysis Techniques

Most reviews of image analysis are organized in the following three broad topic areas: background, pre-processing and analysis\cite{75, 77, 78, 84}. The background of image analysis addresses the fundamental topics, such as human visual perception, image acquisition and storage. Processing or pre-processing of image deals with various image processing techniques, such as image transform, image enhancement (ranging from noise reduction and contrast enhancement to sharpening and color processing), image restoration, which plays a fundamental role in recovering image information obscured by degradation, such as blurring, and image data compression. Image analysis covers the topics of image segmentation, image recognition and interpretation, which deals with extracting information from an image\cite{84}. However, in automatic controlling system, image analysis can be regarded as image inspection, which ascertains whether or not the visual appearance of objects is as it should be; image location, which specifies both position and orientation of an object and image identification\cite{89}.

Image segmentation is the first step in image analysis, and the objective of segmentation is to subdivide an image into its constitute parts or objects. The level to which this subdivision is carried depends on the problem being solved, and segmentation should be stopped when the objects of interest in an application have been isolated\cite{84}. In general, autonomous segmentation is one of the most difficult tasks in image analysis, and thresholding is one of the most important approaches to image segmentation\cite{84}.

Thresholding technique, in particular, grey level thresholding, is a simple region based operation. It may be viewed as a test involving some function of the grey level at a point, some local property of the point (e.g. the average grey level over some neighborhood), and the position of the point in the image\cite{89}. A threshold operation is defined as a test involving a function $T$ of the form\cite{75, 84}:
\[ T = T(x, y, N(x, y), g(x, y)) \]  

(3-1)

Where \( N(x, y) \) denotes some local property of the point \((x, y)\) and \( g(x, y) \) is the grey-level at the point \((x, y)\), which is defined as\(^{[84]}\):

\[
g(x, y) = \begin{cases} 
1 & \text{if } g(x, y) > T(x, y, N(x, y), g(x, y)) \\
0 & \text{if } g(x, y) \leq T(x, y, N(x, y), g(x, y))
\end{cases}
\]  

(3-2)

The thresholding operation was categorized into the following three classes on the bases of restrictions placed on this function\(^{[89, 90]}\).

- **Global thresholding**: the operation is dependent only on the grey level of the point.

\[
T = T(g(x, y))
\]

(3-3)

- **Local thresholding**: the operation is dependent on a neighborhood property of the point and on the grey level of the point.

\[
T = T(N(x, y), g(x, y))
\]

(3-4)

- **Dynamic thresholding**: the operation is dependent on the point coordinates, neighborhood property of the point and on the grey level of the point.

\[
T = T(x, y, N(x, y), g(x, y))
\]

(3-5)

It is pertinent to note that most systems utilize the simplest of these three approaches, which is global thresholding. The threshold operation is based exclusively on the global threshold value and on the grey level of a test point, irrespective of its position in the image or of any local context\(^{[89]}\).

The selection of an appropriate threshold is very important for reliable segmentation, and most approaches are based on the analysis of the grey level histogram\(^{[89]}\).
The average grey level of those pixels which are on the boundary between the object and the background can be estimated as the threshold value, as shown in Fig 3.4 and Fig 3.5\cite{Vernon}. As the grey level of this boundary pixel typically lies between those of the object and the background, it provides a good indication of the threshold value. However, it is not always valid to assume that the histogram is indeed bi-modal, with one mode corresponding to the grey level representing the object and the other to the grey level representing the background. The best approach is to use a reliable edge detector, such as Marr-Hildreth operator described by Vernon\cite{Vernon}, or local derivative operator documented by Gonzalez\cite{Gonzalez} and Acharya\cite{Acharya}.
3.1.3 Image Noise

Quite often, an image may get corrupted by noise, which may arise in the process of acquiring the image, such as the weather, relative movement of the camera, and sensors noise during its transmission, or during reproduction of the image. The characteristics of noise depend on its source, and the reduction of the noise effect is of great importance for the image analysis, however, modern technology has made it possible to reduce the noise to almost negligible levels[88]. Generally speaking, noise can be grouped into the following two classes[79, 85].

- Independent noise
- Noise which is dependent on the image data

Image independent noise can often be described by an additive noise model; where the recorded image \( f(i, j) \) is the sum of the true image \( s(i, j) \) and the noise \( n(i, j) \)[79]:

\[
f(i, j) = s(i, j) + n(i, j)
\]  

(3-6)

The noise \( n(i, j) \) is often zero-mean and described by its variance \( \sigma_n^2 \). The impact of the noise on the image is often described by the Signal to Noise Ratio (SNR), which is given by[79]:

\[
SNR = \frac{\sigma_s}{\sigma_n} = \sqrt{\frac{\sigma_f^2}{\sigma_n^2}} - 1
\]  

(3-7)

Where \( \sigma_s^2 \) and \( \sigma_f^2 \) are the variances of the true image and the recorded image, respectively. In many cases, additive noise is evenly distributed over the frequency domain (i.e. white noise), whereas an image contains mostly low
frequency information. Hence, the noise is dominant for high frequencies and its effects can be reduced using some kind of low pass filter. This can be done either with a frequency filter or with a spatial filter. Often a spatial filter is preferable, as it is computationally less expensive than a frequency filter\[79\]. In the case of data-dependent noise, for example, the noise arises when monochromatic radiation is scattered from a surface whose roughness is of the order of a wavelength, will cause wave interference which results in image speckle. It is possible to model this kind of noise with a multiplicative or non-linear model. These models are mathematically more complicated, therefore, if possible, the noise is assumed to be data independent\[85\]. In the following section, two kinds of special noise, which is frequently mentioned in the literature, are briefly introduced.

**Detector Noise**

This kind of noise occurs in almost all recorded images to a certain extent and is due to the discrete nature of radiation, *i.e.* the fact that each imaging system is recording an image by counting photons\[79\]. Allowing some assumptions which are valid for many applications, this noise can be modeled with an independent, additive model, where the noise \(n(i,j)\) has a zero-mean Gaussian distribution described by its standard deviation \((\sigma)\), or variance. The 1-D Gaussian distribution has the form shown in Fig 3.6\[85\]. This means that each pixel in the noisy image is the sum of the true pixel value and a random, Gaussian distributed noise value.

![1D Gaussian distribution with mean -0 and standard deviation 1](image)

**Fig 3.6** 1D Gaussian distribution with mean \(-0\) and standard deviation \(1\)\[85\]
Salt and Pepper Noise

Another common form of noise is data drop-out noise, which is commonly referred to as intensity spikes, speckle or salt and pepper noise. The noise is always caused by errors in the data transmission\textsuperscript{[80]}. The corrupted pixels are either set to the maximum value which looks like snow in the image or have single bits flipped over. In some cases, single pixels are set alternatively to zero or to the maximum value, giving the image a “salt and pepper” like appearance. Unaffected pixels always remain unchanged. The noise is usually quantified by the percentage of pixels which are corrupted, for this kind of noise, conventional low pass filtering, \textit{e.g.} mean filtering or Gaussian smoothing is relatively unsuccessful because the corrupted pixel value can vary significantly from the original and therefore the mean can be significantly different from the true value. However, a median filter and conservative smoothing is more effective\textsuperscript{[80]}.

In summary, digital image processing has been widely used in different fields of research, and digital image process can be generally categorized into three types. Thresholding technique is a region based operation, and is one of the effective and reliable image segmentation approaches, in which, the selection of an appropriate threshold is very important. Image may be contaminated by noise during various stages of operations, such as data acquiring process, data transmission, and data reproduction. However, they can be effectively eliminated by various filtering processes.

3.2 Sound Analysis

Alternative physical compression and expansion of medium (such as solids, liquids and gases) with certain frequencies are called sound waves\textsuperscript{[91, 92]}. The medium contents oscillate in the direction of wave propagation; hence these waves are called longitudinal mechanical waves\textsuperscript{[91]}. Acoustics refer to the study
of sound, namely, its production, transmission through solid and fluid media, and any other phenomenon engendered by its propagation through media\textsuperscript{[93]}. An acoustic signal can arise from a number of sources, \textit{e.g.}, turbulence of air or any other gas, the passage of a body through a fluid, and the impact of a solid against another solid\textsuperscript{[93]}. In recent years, the applications of acoustic analysis to monitor and control steelmaking process have increased rapidly\textsuperscript{[4]}. It has been found that the sound emitted during certain metallurgical operations reflects the corresponding operations and thus the acoustic response may be used to monitor and control the process\textsuperscript{[6]}. Acoustic studies of injected bubble and experimental study of the sound emitted from gas bubbles in a liquid have been comprehensively investigated by Plesset \textit{et al.}\textsuperscript{[56]}, Leighton and his coworkers\textsuperscript{[52, 94]}. The interaction of acoustic fields with bubbles in liquids were well documented by Leighton in the book: —The Acoustic Bubble\textsuperscript{[58]}, and the related investigation regarding to bubble and sound were reviewed in section 2.5. In this subsection, the sound fundamentals, sound wave digitalization and related analyzing techniques will be briefly reviewed.

\subsection*{3.2.1 Sound Fundamentals}

The concept of sound is associated with the hearing range of a human ear which is approximately from 20 Hz to 20,000 Hz. Longitudinal mechanical waves below 20 Hz are called infrasound and above 20 kHz are called ultrasound\textsuperscript{[91]}. When the medium is compressed, its volume changes from \( V \) to \( V - \Delta V \). The ratio of change in pressure, \( \Delta p \), to relative change in volume is defined the bulk modulus of elasticity of medium\textsuperscript{[91]}:

\[
B = \frac{\Delta p}{\Delta V / V} = \rho_0 v^2
\]

(3-8)

Where, \( \rho_0 \) is the density outside the compression zone and \( v \) is the speed of sound in the medium. Then the speed of sound can be defined as\textsuperscript{[91]}.
\[ v = \sqrt{\frac{B}{\rho_0}} \]  

(3-9)

Therefore, the speed of sound depends on the elastic \((B)\) and inertia \((\rho_0)\) properties of the medium.

A sound wave may be considered as a pressure wave, for example, a pure harmonic tone wave can be represented by the displacement of the particle from the equilibrium position as the following equation\[^{[91]}\]:

\[ y = y_m \cos \left( \frac{2\pi}{\lambda} (x - vt) \right) \]  

(3-10)

Where, \(x\) is the equilibrium position of a particle and \(y\) is a displacement from the equilibrium position, \(y_m\) is the amplitude, and \(\lambda\) is the wave length. However, it is more convenient to deal with pressure variation in sound wave rather than with displacements of the particles. The pressure exerted by the sound wave can be expressed as\[^{[91]}\]:

\[ p = (k\rho_0 v^2 y_m) \sin(kx - \omega t) \]  

(3-11)

Where, \(k = 2\pi / \lambda\) is a wave number, \(\omega\) is angular frequency, and the first parentheses represent an amplitude \(p_m\) of the sound pressure. It should be noted that \(\text{sine}\) and \(\text{cosine}\) in the above two equations indicate that the displacement wave is 90° out of phase with the pressure wave.

Pressure at any given point in media is not constant and changes continuously. Acoustic pressure is defined as the difference between the instantaneous and the average pressure\[^{[91]}\]. During the sound wave propagation, vibrating particles oscillate near a stationary position with the instantaneous velocity \(\xi\), and acoustic impedance is defined as the ratio of the acoustic pressure and the instantaneous velocity\[^{[91]}\]:
\[ Z = \frac{P}{\xi} \]  

(3-12)

Which is a complex quantity and can be characterized by amplitude and phase, for an idealized media (no loss), the acoustic impedance is real and is related to the wave velocity as:

\[ Z = \rho_0 v \]  

(3-13)

The sound wave intensity \( I \) is defined as the power transferred per unit area, and can be expressed through the acoustic impedance:

\[ I = P\xi = \frac{P^2}{Z} \] 

(3-14)

Sound level is defined with respect to a reference intensity \( I_0 = 10^{-12} \text{W/m}^2 \), which is more commonly used instead of sound intensity:

\[ \beta = 10\log_{10} \frac{I}{I_0} \]  

(3-15)

The unit of sound level (\( \beta \)) is decibel (dB), and the reference magnitude is the lowest ability of human ear. Pressure level can also be expressed in decibels according to the following equation:

\[ \Pi = 20\log_{10} \frac{p}{p_0} \]  

(3-16)

Where, \( p_0 = 2.0 \times 10^{-5} \text{N/m}^2 (0.0002 \text{\mu bar}) \)

### 3.2.2 Sound Wave Digitalization

Before a sound signal can be manipulated or analysed with a digital computer, the signal must be acquired or digitized by a hardware device called an analog-to-digital (A/D) converter, or digitizer. The digitizer repeatedly measures or samples the instantaneous voltage amplitude of a continuously varying (analog)
input signal at a particular sampling rate, typically thousands or tens of thousands of times per second\textsuperscript{[91]}. Fig 3.7 illustrates the converting process of a voltage signal to a PCM (Pulse Code Modulation) signal\textsuperscript{[95]}. Generally speaking, higher pressure corresponds to higher voltage, and vice versa. To convert an analog signal to a digital format, the voltage is sampled at regular intervals, and the value of each sample is rounded to the nearest integer on a scale that varies according to the resolution of the signal. The integers are then converted to binary numbers\textsuperscript{[96]}.

Some related terms regarding to the sound wave digitalization such as sampling rate, resolution, quantization, dynamic range and noise to signal ratio are briefly reviewed below.

1) Sampling rate

It is how many times per second the voltage of the analog signal is measured. According to the Nyquist Theorem, the sampling rate must be at least twice as high as the highest frequency to be reproduced\textsuperscript{[98]}. The range of human hearing is roughly from 20 to 20,000 $Hz$, so a sampling rate of at least 40 $kHz$ is needed to reproduce the full range for the sound to be sensitive by the human ears. Higher sampling rates allow a digital recording to accurately record higher frequencies of sound.

2) Resolution
The resolution of a digital signal is the range of numbers that can be assigned to each sample. CD audio uses 16 bits, which provides a range of binary values from 0 to 65,534 \( (2^{16}) \). The binary value of 0,000,000,000,000,000 (zero) corresponds to -32,768 (the lowest possible level), and the value 1,111,111,111,111,111 (65,535) corresponds to 32,767 (the highest possible level). Higher resolution increases the dynamic range and reduces quantization distortion and background noise. The effect of increased resolution and sampling rates is shown in Fig 3.8\[95,96\].

![Fig 3.8 Effect of Increased Resolution and Sampling Rates\[95\]]

3) Quantization

Quantization is the process of selecting whole numbers to represent the voltage level of each sample. The A/D converter may select a whole number that is closest to the signal level at the instant it is sampled. This produces small rounding errors that cause distortion. Fig 3.9 shows the quantization errors schematically\[96\]. Quantization distortion increases at lower levels because the signal is using a smaller portion of the available dynamic range, so any errors are a greater percentage of the signal. A key advantage of audio encoding schemes, such as MP3, is that more bits can be allocated to low-level signals to reduce quantization errors\[95\].
4) Dynamic Range

Dynamic range is the range of the lowest to the highest level that can be reproduced by a system\(^7\). Digital audio at 16-bit resolution has a theoretical dynamic range of 96 dB, but the actual dynamic range is usually lower because of overhead from filters. The dynamic range of vinyl records and cassette tapes is much lower than CDs and varies depending on the quality of the recording and playback equipment. The dynamic range of cassette tapes also varies depending on the type of tape\(^5\).

5) Signal-to-Noise Ratio

The signal-to-noise ratio is the ratio of the desired signal level to the background noise (hiss, hum and static) level. Each additional bit of resolution corresponds to an increase of 6 dB in signal-to-noise ratio. Audio CDs achieve about a 90 dB
signal-to-noise ratio. A schematic representation of dynamic range and signal to noise relation is shown in Fig 3.10[95].

### 3.2.3 Sound Analysis Techniques

Any acoustic signal can be graphically or mathematically depicted in either of the two forms, called the time domain and frequency domain representations[93]. In the time domain, instantaneous pressure is represented as a function of time, such as shown by Equation (3-10) and (3-11). However, a time varying signal in the time domain can also be represented in the frequency domain, which involves decomposing the signal into its spectral components. The amplitude and phase of a signal can be represented as a function of frequency[99]. There are two ways of transforming a signal from the time domain to the frequency domain. The first involves the use of band limited digital or analog filters. The second involves the use of Fourier analysis where the time domain signal is transformed using a Fourier series[100, 101]. This is implemented in practice using a very efficient algorithm known as the FFT (Fast Fourier Transform) and it is well documented by Mitra in —Digital signal processing”[98], and comprehensively explained by Naidu[102]. Fig 3.11 shows the time domain and frequency domain representations of an infinitely long sound consisting of two tones, with frequencies of 490 Hz and 800 Hz[103].

![Diagram of a sound wave representation](image-url)
Fig 3.11 Schematic representations of an infinitely long pure sinusoidal signal (a) Time domain, (b) Magnitude spectrum in frequency domain, (c) Phase spectrum in frequency domain. (The phase of the frequency component at 500 Hz is arbitrarily taken as a reference and assigned a phase value of 0)\textsuperscript{[103]}

The Fourier transform is a mathematical function that converts the time domain form of a signal (which is the representation directly produced by most measuring and recording devices) to a frequency domain representation, or spectrum. When the signal and spectrum are represented as a sequence of discrete digital values, a version of the Fourier transform called the discrete Fourier transform (DFT) is used. The input to the DFT is a finite sequence of values, the amplitude values of the signal which are sampled (digitized) at regular intervals. The output is a sequence of values specifying the amplitudes of a sequence of discrete frequency components, evenly spaced from 0 Hz to half the sampling frequency\textsuperscript{[98, 103]}. FFT algorithm is available in many software packages, such as Wavepad\textsuperscript{[104]}, K-Multimedia player\textsuperscript{[105]}, and Prism\textsuperscript{[106]} etc.

Sound signals could be analysed in both time domain and frequency domain, depending the applications and the tools available to collect the acoustic signals. Hald applied the time domain sound analysis techniques to solve a few noise source location problems in the automotive industry\textsuperscript{[107]}, while Chen and his
coworkers\textsuperscript{[108]} investigated the force output from muscles by employing the spectrum analysis of the acoustic signals from the skeletal muscles vibration. Cents and his coworker\textsuperscript{[109]} developed a novel technique for in situ measurement of size distributions and phase holdup of particles, droplets and bubbles based on the propagation speed and attenuation of ultra-sound.

### 3.2.4 Sound Measurement

Acoustic signals are usually measured by microphones or hydrophones. A microphone is a pressure transducer adapted for the transduction of sound waves over a broad spectral range\textsuperscript{[91]}, and hydrophones are essentially underwater microphones\textsuperscript{[110]}. The microphones differ by their sensitivity, directional characteristics, frequency bandwidth, dynamic range, sizes, \textit{etc.} and their designs are quite different depending on the medium from which sound waves are sensed \textsuperscript{[111]}.

The main difference between a pressure sensor and a microphone is that the latter does not need to measure constant or very slow changing pressures. Its operating frequency range usually starts at several Hertz (or as low as tens of millihertz for some applications), while the upper operating frequency limit is quite high-up to several megahertz for the ultrasonic applications\textsuperscript{[112]}. The microphone can be categorized into the following types: piezoelectric, condenser, electric or dynamic, and they were comprehensively explained by Malchaire in his \textit{“Sound Measuring Instruments”}\textsuperscript{[111]}, and were also well documented by Fraden\textsuperscript{[91, 112]}. When selecting a microphone, its characteristics must be known so that its technical performance (\textit{e.g.} frequency response, dynamic range, directivity, stability), in terms of accuracy and precision, meets the requirement of the measurement in question, taking into account the expected conditions of use, such as ambient temperature, humidity, \textit{etc.}\textsuperscript{[111]}.

Microphone sensitivity is defined by the following expression\textsuperscript{[111]}:
\[ Sensitivity = 20 \log_{10} \frac{p_{\text{in}}}{V_{\text{in}}} \text{ dB re } 1 \text{ V/Pa} \] (3-17)

in which, \( p_0 = 1\text{Pa}, \ V_0 = 1\text{V} \), thus, a microphone giving an output signal \( V \) of 10 mV for a pressure signal \( p \) of 94 dB has a sensitivity of 10 mV/pa, or -40 dB[111].

The unit for sensitivity is given in dB referenced to 1 V/m Pa (dB re 1 V/ m Pa). One is often tempted to choose a microphone with the largest bandwidth so it can be used in many applications. That choice can affect signal-to-noise ratio or even cause acoustic overload of the preamplifiers. Microphones have a maximum sound pressure level to which they can be exposed before the preamplifiers overload or start clipping the signals, and it is termed as acoustic overload pressure. This acoustic overload pressure is dependent on the microphone sensitivity and the magnitude of the preamplifier's supply voltage[110].

Good quality piezoelectric or condenser microphones have usually flat frequency response characteristics from 2 Hz to an upper limit which depends on their size. Below this limit, the frequency response is independent of the orientation of the microphone with respect to the noise source, and therefore the microphone can be held in any orientation. Above this limit, the frequency response will depend upon the direction of the sound wave on the microphone membrane[111].

The dynamic range of a microphone is limited by the internal noise of the transducer and the distortion resulting from high noise levels. The receiver to which the output signal of the microphone is fed will saturate if the signal is too high and will also give a false result if the signal is too low[111]. Therefore, high sensitivity microphones are needed to measure very low noise levels (lower than 30 dB), and low sensitivity ones have to be used for high noise levels such as for impact noise (above 130 dB). The dynamic range of typical good quality microphones are thus between 100 and 120 dB[111].

In summary, sound signal comes from the alternative physical compression and
expansion of medium with certain frequencies, and is associated with hearing range of human ears. Sound wave may be considered as a pressure wave, which may be represented by the displacement of medium particles from their equilibrium positions. Sound wave may be measured by sound intensities or sound levels, and may be converted into voltage signal and further converted into digital form. Sound analysis techniques can be generally separated into time domain and frequency domain analysis.

3.3 Vibration Analysis

Vibration concerns linear and angular motions of bodies that respond to applied disturbances in the presence of restoring forces\cite{113}. Acceleration is a dynamic characteristic of an object\cite{91}. Displacement, velocity and acceleration of an object are all related according to Newton’s 2nd law, velocity is a first derivative of displacement and acceleration is the second derivative. However, in a noisy environment, taking derivatives may result in extremely high errors, even if complex and sophisticated signal conditioning circuits are employed\cite{91}. Therefore, velocity and acceleration are not derived from position or proximity detectors, but rather measured by special sensors. Generally, in low frequency applications (on the order of 1 Hz), position and displacement measurements provide good accuracy; in intermediate frequency applications (less than 1 kHz), velocity measurement is usually favored; in measuring high frequency motions with appreciable noise levels, acceleration measurement is preferred. However, a basic idea behind any sensor for transduction of velocity or acceleration is a measurement of a displacing object with respect to some reference object\cite{91, 112}.

Velocity may be linear or angular, and shows how fast an object moves along a straight line or how fast it rotates. Vibration can occur in linear or rotational forms of motion, and they are termed as translational or torsional respectively \cite{114}. Many velocity or acceleration sensors contain components which are
sensitive to a displacement. Thus the position and displacement sensors described are the integral parts of velocity sensors and accelerometers. In some instance, however, velocity sensors and accelerometers do not use an intermediate displacement transducer as their motions can be directly converted into electric signals\textsuperscript{[112]}. For example, moving a magnet through a coil of wire will induce a voltage in the coil according to Faraday’s law; this voltage is proportional to the magnet’s velocity and the field strength\textsuperscript{[91]}.

Vibration is characterized by its frequency or frequencies, amplitude, and phases\textsuperscript{[113]}. Although the time history of vibrations encountered in practice usually does not exhibit a regular pattern, the sinusoidal oscillation serves as a basic representation, and irregular vibrations can be decomposed in several frequency components, each of which has its own amplitude and phase\textsuperscript{[115]}.

In the following sections, the vibration analysis is reviewed in vibration fundamentals, vibration measurement and applications, and vibration analysis techniques.

### 3.3.1 Vibration Fundamentals

Vibration in general is a periodic motion, which can be expressed as a sum of harmonic motion\textsuperscript{[116]}. A body in simple, undamped harmonic motion moves with a displacement $x$, can be represented by the following equation\textsuperscript{[116, 117]}:

$$x = x_0 \sin(\omega t) = x_0 \sin(2\pi f t)$$

(3-18)

Where, $x_0$ -amplitude of the displacement, $f$ -frequency in cycles per second

$\omega$ -angular frequency in radians per second

The period $T = 1/f = 2\pi/\omega$, the velocity $v = \dot{x}$ and acceleration $a = \ddot{x}$ are given by:
\[ v = x = x_0 \omega \cos(\omega t) = x_0 (2\pi f) \cos(2\pi ft) = v_0 \cos(\omega t) \]  

(3-19)

\[ a = \ddot{v} = -x_0 \omega^2 \sin(\omega t) = -x_0 (2\pi f)^2 \sin(2\pi ft) = a_0 \sin(\omega t) \]  

(3-20)

Where \( v_0 \) and \( a_0 \) are the velocity and acceleration amplitudes respectively\(^{[116]}\).

A mass \( m \), spring \( k \), and damper \( c \) translational motion system can be schematically shown in Fig 3.12\(^{[116]}\).

![Translational single degree of freedom system](image)

Fig 3.12 Translational single degree of freedom system\(^{[116]}\)

The general motion equation of the above system can be represented by the following equation\(^{[116]}\):

\[ m \ddot{x} + c \dot{x} + kx = P \]  

(3-21)

The solutions for the above equation were provided according to the following four cases, and they were well documented by Mobley\(^{[115]}\) and Pilkey\(^{[116]}\). In a free vibration, there is no energy supplied to the vibration system once the initial excitation is removed, an undamped system would continue to vibrate at its natural frequencies for ever. However, if a system is damped, it continues to vibrate until all the energy is dissipated. In contrast, energy is continuously supplied to the system for a forced vibration, and a forced vibration depends on the spectral form and spatial distribution of the excitation as well as on the dynamic characteristics of the system\(^{[115]}\).
### Free vibration without damping

\[ \ddot{x} + kx = 0 \]  \hspace{1cm} (3-22)

\[ x = C_1 \sin \left( \frac{k}{m} t + C_2 \cos \frac{k}{m} t = C_3 \sin (\omega_n t + \theta) \right) \]  \hspace{1cm} (3-23)

Where, \( C_3 = \sqrt{C_1^2 + C_2^2} \) and the phase angle \( \theta = \tan^{-1} \left( C_2 / C_1 \right) \)

\[ \omega_n = \sqrt{\frac{k}{m}} \] is termed as the natural frequency of the system\[^{[116]}\].

### Free vibration with viscous damping

\[ \dddot{x} + c \ddot{x} + kx = 0 \]  \hspace{1cm} (3-24)

The solution of the above equation depends on whether \( c \) is equal to, greater than, or less than the critical damping coefficient \( c_c \), which is defined as\[^{[116]}\]:

\[ c_c = 2\sqrt{km} = 2m\omega_n \]  \hspace{1cm} (3-25)

The ratio \( \zeta = c / c_c \) is termed as the fraction of critical damping, or the percentage of damping\[^{[116]}\]. If \( c = c_c (\zeta = 1) \), the case of critical damping, there is no oscillation and the solution is\[^{[116]}\]:

\[ x = (C_1 + C_2 t)e^{-\zeta \omega_n t/2m} \]  \hspace{1cm} (3-26)

If \( c / c_c = \zeta > 1 \), the system is over damped, so that the mass does not oscillate but returns to its equilibrium position, and the solution is\[^{[116]}\]:

\[ x = e^{-\zeta \omega_n t/2m} \left( C_1 e^{\zeta \omega_n \sqrt{\zeta^2 - 1} t} + C_2 e^{\zeta \omega_n \sqrt{\zeta^2 - 1} t} \right) \]  \hspace{1cm} (3-27)

If \( c / c_c = \zeta < 1 \), the system is under damped, and the solution is\[^{[116]}\]:

\[ x = e^{-\zeta \omega_n t/2m} (C_1 \sin \omega_d t + C_2 \cos \omega_d t) = C_3 e^{-\zeta \omega_n t/2m} \sin (\omega_d t + \theta) \]  \hspace{1cm} (3-28)
Where, \( C_3 = \sqrt{C_1^2 + C_2^2} \) and the phase angle \( \theta = \tan^{-1}(C_2 / C_1) \). The damped natural frequency \( \omega_d \) is related with the undamped natural frequency \( \omega_n \) by the following equation\(^{[116]}\), and can be shown schematically in Fig 3.13\(^{[116]}\):

\[
\omega_d = \sqrt{\frac{k}{m} + \frac{c^2}{4m^2}} = \omega_n \sqrt{1 - \zeta^2}
\]  
(3-29)

After being disturbed, an undamped system oscillates with continuous decreasing amplitude.

Fig 3.13 Damped natural frequency as it varies with critical damping\(^{[116]}\)

- **Forced vibration without damping**

\[
m x + kx = P_0 \sin \omega t
\]  
(3-30)

With the solution:

\[
x = C_1 \sin \omega_n t + C_2 \sin \omega_n t + \frac{\omega P_0 / \omega_n k}{1 - \omega^2 / \omega_n^2} \sin \omega t
\]  
(3-31)

The first two terms describe oscillation at the undamped natural frequency \( \omega_n \), and the third term gives the steady state oscillation\(^{[116]}\).

- **Forced vibration with viscous damping**
\[ \ddot{m} x + c \dot{x} + k x = P_0 \sin \omega t \]  

(3-32)

The solution is\(^{[116]}\):

\[
x = e^{-\alpha t / 2m} (C_1 \sin \omega_d t + C_2 \cos \omega_d t) + \frac{(p_0 / k) \sin(\omega t - \theta)}{\sqrt{1 - (\omega^2 / \omega_n^2)^2 + (2\zeta\omega / \omega_n)^2}}
\]

(3-33)

The phase angle is a function of \(\omega / \omega_n\) and \(\zeta\):

\[
\theta = \tan^{-1} \left( \frac{2\zeta \omega / \omega_n}{1 - \omega^2 / \omega_n^2} \right)
\]

(3-34)

The first term in Equation (3-31) involving \(C_1\) and \(C_2\) decays due to damping, leaving the steady state motion of amplitude:

\[
x_0 = \frac{p_0 / k}{\sqrt{1 - (\omega^2 / \omega_n^2)^2 + (2\zeta\omega / \omega_n)^2}}
\]

(3-35)

Where, \(p_0 / k\) is the static displacement \(x_{st}\) due to \(p_0\); the magnification factor ratio for a damped system is plotted in Fig. 3.14\(^{[116]}\).

![Fig 3.14 Magnification factor for a damped system\(^{[116]}\)](image)

From Fig. 3.14, the maximum magnification factor occurs at \(\omega / \omega_n = 1 - 2\zeta^2\).
3.3.2 Vibration Measurement and Applications

Vibration can be measured by direct comparison of instantaneous dimensional parameters relative to some adequately fixed datum point in space. The fixed datum point can be on an independent measurement framework (fixed reference method) or can be a part that remains stationary due to its high inertia (seismic system)\cite{118}. Accelerometers are commonly used for the measurement of vibration signals. They convert the mechanical motion into a voltage which corresponds to the surface acceleration\cite{91}. An accelerometer can be specified as a single degree of freedom device, which has some type of seismic mass, a spring like supporting system and a frame structure with damping properties\cite{112}. Mathematical models of an accelerometer are based on Equation (3-21), which has been reviewed in the previous section.

A correctly designed, installed, and calibrated accelerometer should have one clearly identifiable resonant (natural) frequency; and a flat frequency response where the most accurate measurement can be made, Fig 3.15 gives a schematic view of a typical frequency response of an accelerometer\cite{112}. In which, $f_n$ is a natural frequency and $f_{ref}$ is the reference frequency\cite{112}.

![Fig 3.15 A frequency response of an accelerometer\cite{112}](image)

Within the flat region in Fig 3.15, as the vibrating frequency changes, the output of the sensor will correctly reflect the change without multiplying the signal by
any variations in the frequency characteristic of the accelerometer. Viscous damping is used in many accelerometers to improve the useful frequency range by limiting effects of the resonant\textsuperscript{[112]}.

The following characteristics of an accelerometer should be determined after calibration\textsuperscript{[91]}.

- **Sensitivity**, it is the ratio of an electrical output to the mechanical input, and usually expressed in terms of volts per unit of acceleration under the specified condition. For example, the sensitivity may be specified as 1 V/g, the sensitivity is typically measured at a single reference frequency of a sine-wave shape.

- **Frequency response**, it is the outputs signal over a range of frequencies where the sensor should be operating, and it is specified with respect to a reference frequency which is where the sensitivity is specified.

- **Resonant frequency**, it shows as a clearly defined peak that can be 3-4 dB higher than the response at the reference frequency in a undamped sensor, however, in a near critically damped device, the resonant frequency may not be clearly visible and the phase shift is measured.

- **Linearity of the accelerometer**, it is specified over the dynamic range of the output signals.

There are different types of accelerometers and each has unique characteristics, advantages and disadvantages. The different types include: capacitive accelerometer, piezoresistive accelerometers, piezoelectric accelerometer, and thermal accelerometers. They were clearly documented by Fraden\textsuperscript{[91,112]}, Greene\textsuperscript{[119]}, Mauer and his coworkers\textsuperscript{[120]}.

Vibration monitoring and analysis are the primary diagnostic tools for most mechanical systems, and have been applied to maintain optimum operating
conditions and efficiency of critical plant systems\textsuperscript{[113]}. Vibration analysis can also be used to evaluate fluid flow through pipes or vessels, to detect leaks, and to perform a variety of non-destructive testing functions that improve the reliability and performance of critical plant systems\textsuperscript{[115]}.

In recent years, vibration analysis has found increasing application to monitor and control steelmaking processes\textsuperscript{[4, 121]}. Accelerometers are used to detect the vibrations generated by gas-solid interactions within the blast furnace, in torpedo cars during hot metal treatment, within steelmaking converters during decarburization and slag foaming\textsuperscript{[21]}. A vibration controlled argon gas ladle stirring system was developed at the Ladle Metallurgy Facility (LMF) in the Hiton Works of Stelco Inc., and it effectively minimized the concerns of the operators about temperature and chemistry stratification from reduced stirring levels. As a result, argon gas flow rates during reheating have been reduced significantly; reheating arc stability has also been improved; argon gas consumption has decreased, and operators can have more time for other LMF functions\textsuperscript{[7]}. Nupro have also developed an accelerometer system that is placed in contact with the ladle, either directly mounted or mounted on a structure that is pressed against the ladle, as shown in Fig 3.16\textsuperscript{[6]}. Nupro claimed that their TruStir\textsuperscript{TM} system\textsuperscript{[70]} which is based on the measurement of vibration of the ladle has a versatile measurement, control, achieving and process improvement tool\textsuperscript{[6]}. According to the industrial trial at BHP’s Newcastle Bloomcaster, an accelerometer attached to the manipulator arm of the ladle shroud can give an indication of steel flow rate and also inform the operator of the onset of slag entrainment which ensure less slag carryover and reduce the variability compared with visual detection method \textsuperscript{[5, 122]}. 
In a typical application, the signal from the accelerometer is amplified and pre-treated before it was transmitted to a PID control system\(^6\), however, the other signals such as sound of the bubbling and image from the top disturbed surface have not been considered and utilized simultaneously.

### 3.3.3 Vibration Analysis Techniques

Vibration analysis techniques can be generally separated into time domain and frequency domain. Vibration data plotted as amplitude versus time is referred to as a time domain data profile\(^{115}\). A signal from industrial machinery is shown in Fig 3.17\(^{115}\).

![Fig 3.16 Schematic view of accelerometer contact with ladle\(^6\)](image)

![Fig 3.17 A typical time domain vibration profile for a piece of machinery\(^{115}\)](image)
Time domain plots are usually useful in the overall analysis of a machine system to study changes in operating conditions. However, the time domain data is difficult to use, as it only represent the total displacement at any given time, and it is difficult to determine the contribution of any particular vibration source\textsuperscript{[115]}.

Frequency domain data are obtained by converting time domain data using a mathematical technique referred to as a Fast Fourier Transform (FFT). FFT allows each vibration component of a complex spectrum to be shown as a discrete frequency peak\textsuperscript{[115]}. The frequency domain amplitude can be the displacement per unit time related to a particular frequency, which is plotted as the Y-axis against frequency as the X-axis. This is opposed to the time domain spectrum, which sums the velocities of all frequencies and plots the sum as Y-axis against time as the X-axis. An example of a frequency domain plot or vibration signature is shown in Fig 3.18\textsuperscript{[115]}.

![Fig 3.18 A typical frequency domain vibration signature\textsuperscript{[115]}](image)

In summary, vibrations in general involve periodic motion, which can be expressed as a sum of harmonic motions. The dynamic characteristic of periodic
motion can be categorized into four types, i.e. free vibration without damping, free vibration with viscous damping, forced vibration without viscous damping, and forced vibration with viscous damping. Vibration analysis has been widely used as the primary diagnostic tools for most mechanical systems and a variety of non-destructive testing. Vibration analysis has also been effectively applied in metallurgical operations. Vibration signals can be measured by accelerometer which has a wide range of resonance frequencies, and the frequency response and sensitivity should be determined after calibration. Vibration analysis techniques can also be separated into time domain and frequency domains.

### 3.4 Signal Combination

Signal combination means making decisions based on all the relevant signals from different channels or understanding more details based on relevant information from different sources. Manasseh and his coworkers\[8\] investigated the precise mechanism by which a bubble creates sound on formation through the combination of sound and image signals. A hydrophone was used to monitor the acoustic signals emitted on a bubble formation, while the behavior of the bubble was studied by triggering a photograph at a series of different phases of the acoustic pulse. Though the sound and image signals were not collected simultaneously, and the series of the images is not the same bubble, but provided that the system is operated in the repeatable regime of bubble formation, the data was assumed to be the same. Thus precisely timed photographs were related to the acoustic signal produced on bubble formation to actual physical processes.

Griffith et al.\[123\] developed an ultrasound instrument that combines a real-time cross-sectional imaging system and a spectrum analyzer-based Doppler velocimeter. This combination allows the Doppler sample volume to be superimposed on the cross-sectional image of the heart so that the sample volume
can be located accurately.

In social science, the Combined Signal Approach (CSA) has been extensively used among market participants such as brokers, dealers, fund managers, speculators and individual investors in financial industry\cite{124}. CSA is based on the notion that information related to future price movements is somewhat dispersed among many trading signals, therefore, combining trading signals may generate a more informative signal than various trading rules\cite{124}. Lento and his coworkers\cite{125, 126} claimed that the combination of individual technical trading rules provides a synthesis whereby the whole is greater than the sum of the parts and excess profits can be generated. One of the major concerns with utilizing only one trading signals (trading rules) is that there is no theory to guide an investor when making decision amongst many different types of trading signals, however, a more informative signal may be generated by combining individual technical trading signals; combining the signals together and using the most agreed upon position reduces the risk of selecting and relying on a single signal at any given time\cite{126}.

The other type of signal combination is combining uncertain predictions and measurements in situations where there is some a priori knowledge, such as in the case of Structure Integrity Prognosis System (SIPS)\cite{127}. The model’s predictions and the sensor data are combined in a reasoning and prognosis system that makes informed predictions of the future state based on imperfect models and sparse sensor data\cite{128}.

However, no studies were reported so far which combined all the signals generated by the same bubbling flow in a bottom stirred cylindrical vessel. Though the image of the spout eye area, sound of the bubbling, and vibration on the wall of the vessel have been investigated individually, they have not been collected simultaneously and combined effectively to determine the stirring level.
inside the vessel.
CHAPTER 4

Multivariate Analysis Techniques

There are various methods in industrial control and a reliable control can be performed if the system can be described by a fixed physical law. For example, in a single bulb electrical circuit, the lightness of the bulb can be controlled by the current in the circuit or voltage at the two ends of the bulb according to Ohm’s law. However, most industrial processes are complicated and can not be described by a simple physical law. Measurements of the variables are required to make the control possible, the value of the variables are usually acting as feedback signals for the control system\[129\]. Conventional control systems based on the fixed-gain feedback signal have been widely applied in industry, mostly because their simplicity and can be described by linear functions\[130\]. However, most real systems exhibit nonlinear behavior from a control perspective, whereas, it is very difficult to model these systems using the law of physics, therefore, the conventional control techniques are not suitable for these complicated situations.

Another important issue for control is response time\[15\], or the speed for the control system to take actions while the process behaves against the expected procedures. This requires quick response from the measurement and feedback system. The response time may vary with different applications, however, an online analysis and feedback is ideal from the perspective of control\[10\].

Most metallurgical operations are related with complex chemical reactions and it is currently not possible to predict these processes with a single mathematical model\[72\], and in most cases, these processes can not be approximated as linear functions, therefore, it is difficult to provide an accurate holistic control by conventional control techniques\[131\]. It is clear that the bubble flow in metallurgical operations may cause the following effects, a disturbed top surface,
a sound spectrum generated from the bubbling and the vibrations on the wall of the vessel. These phenomena can be regarded as different types of signals generated from the same source, and there are a number of options to analyze signals from multivariate systems, such as neural networks, fuzzy logic and multivariate statistical techniques. In this chapter, the existing techniques which can be applied to analyze different types of data will be reviewed and compared.

### 4.1 Neural Networks

The field of neural networks is vast and interdisciplinary and was originally developed during inspiration from the human brain[^132]. Neural Networks (NN), as a branch of artificial intelligence, are computational models that consist of a number of simple processing units that communicate by sending signals to each other over a large number of weighted connections[^133]. Neural networks estimate a function without requiring a mathematical description of how the output functionally depends on the input[^134]. Instead, they learn from examples, or more precisely, they learn from input-output data samples, which is representative of the desired task, and this gives them a key advantage over traditional approaches to function estimation such as the statistical methods used in adaptive control[^134, 135]. Neural networks can be trained to store, recognize and associatively retrieve patterns or database entries, or solve combinational optimization problems to filter noise from measurement data or to control ill-defined plants.

The investigation of neural networks can be traced back to McCulloch and Pitts[^136, 137], and it was explored by a group of researchers during 1950s and 1960s such as Rosenblatt and Minsky[^138-140], McClelland and Fausett have comprehensively documented this development[^133, 141]. Recently, Bhadeshia has given a thorough review of the application of neural networks in material sciences[^142], and Singh and his coworkers applied neural networks approach to predict the effect of various raw materials (pellets, briquettes, hard lumps, friable
lumps, coke and quartzite) on the performance of a submerged arc furnace by incorporating a production capability index (PCI)\cite{143}. In the following sections, the fundamentals of neural networks and its applications are reviewed.

### 4.1.1 Fundamentals of Neural Networks

The interest in neural networks come from their ability to simulate human brains as well as their ability to learn and respond, the basic element of neural networks are termed as artificial neurons, which can be schematically shown in Fig.\ref{fig:4.1}\cite{138}. It consists of three basic components that include weights, thresholds and a single activation function.

- **Weights**, are the values ($W_1$, $W_2$, … $W_n$ in Fig 4.1) associated with each node to determine the strength of input row vector, which is $X = [x_1 \ x_2 \ x_3 \ ... \ x_n]^T$ in Fig 4.1. Each input is multiplied by the associated weight of the neuron connection $X^T W$. Depending upon the activation function, if the weight is positive, $X^T W$ commonly excites the node output; whereas, for negative weights, $X^T W$ tends to inhibit the node output\cite{138}.

![Fig 4.1 Basic element of an artificial neuron\cite{138}](image_url)
o **Thresholds**, is the magnitude offset that affects the activation of the node output, which can be represented as follows\[^{138}\]:

\[ y = \sum_{i=1}^{n} (X_iW_i) - \theta_k \]  

(4-1)

o **Activation function**, performs a mathematical operation on the signal output\[^{138}\]. Five of the most commonly applied activation functions are summarized in Table 4.1\[^{138}\].

Table 4.1 Common activation functions\[^{138}\]

<table>
<thead>
<tr>
<th>Functions</th>
<th>Forms</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linear function</td>
<td>[ y = f(u) = au ]</td>
</tr>
<tr>
<td>Threshold function</td>
<td>[ y = f(u) = \begin{cases} 0 &amp; \text{if } u &lt; 0 \ 1 &amp; \text{if } u \geq 0 \end{cases} ]</td>
</tr>
<tr>
<td>Piecewise linear function</td>
<td>[ y = f(u) = \begin{cases} -1 &amp; \text{if } u &lt; -1 \ u &amp; \text{if } -1 \leq u \leq 1 \ 1 &amp; \text{if } u \geq 1 \end{cases} ]</td>
</tr>
<tr>
<td>Sigmoidal (S shaped) function</td>
<td>[ f(x) = \frac{1}{1+e^{-\alpha x}}, \quad 0 \leq f(x) \leq 1 ]</td>
</tr>
<tr>
<td>Tangent hyperbolic function</td>
<td>[ f(x) = \frac{e^{\alpha x} - e^{-\alpha x}}{e^{\alpha x} + e^{-\alpha x}}, \quad 0 \leq f(x) \leq 1 ]</td>
</tr>
</tbody>
</table>

The architecture of each network is based on very similar building blocks which perform the processing, and a set of major aspects of a parallel distributed model can be distinguished, these features include\[^{135, 144}\]:

1) A set of processing units, also called neurons or nodes, which perform a relatively simple job, receiving inputs from neighbors or external sources and use them to compute output signals that is propagated to other units.
2) An activation state for each unit, which is equivalent to the output of the unit;

3) Connections between the units. Generally, each connection is defined by a weight $w_{jk}$ that determines the effect that the signal of unit $j$ has on unit $k$;

4) A propagation rule, which determines the effective input of the unit from its external inputs;

5) An activation function, which determines the new level of activation based on the effective input and the current activation;

6) An external input (bias, offset) for each unit;

7) A method for information gathering (learning rule);

8) An environment within which the system can operate, provide input signals and, if necessary, error signals.

There are essentially two types of NN learning models, supervised learning and unsupervised learning\textsuperscript{132}.

- **Supervised learning**, for each input pattern the value of the desired output is specified and the goal of the network is to minimize an error function. To train a network, an input vector is applied to the network and the output of the network is calculated and compared to the corresponding target vector with the difference (error) being fed back through the network to change the weights so that the error is minimized.

- **Unsupervised learning**, there is no feedback from the environment to indicate if the outputs of the network are correct\textsuperscript{144}. Neural networks do not rely on the use of target data, instead of trying to map the data input-output relationship, the goal is to find an underlying structure of the data,
called clusters, comparing with supervised learning, the unsupervised learning has the following advantages\cite{132, 135}:

a) It does not need the collection and labeling of output values.

b) If the characteristics or patterns of the data change over time, the network has the ability to change with the patterns.

c) In the early stages of an investigation, this method may provide valuable insight to a problem that can lead to a better end product.

Unsupervised learning does not require any feedback and theoretically shows potential for solving difficult problems, such as pattern recognizing and categorizing the input data into groups, however, it has not been widely applied in solving other complicated engineering problems \cite{132, 145}.

4.1.2 Types of Neural Networks

The topology of a neural network is defined by the number of layers, the number of units per layer, and the interconnection patterns between layers\cite{134, 135}. The topology of neural networks is generally divided into two categories based on the pattern of connections\cite{144}.

- **Feed forward neural networks**, the data flow from input units to output units is strictly feed-forward. The data processing can extend over multiple layers of units, but no feedback connections are present, i.e. connections extending from outputs of units to inputs of units in the same layer or previous layers are not permitted\cite{135, 144}. This type of networks is easy to analyze theoretically because their outputs can be represented with explicit function of the inputs and the weights. Fig 4.2 schematically shows an example of a layered network with one hidden layer.
o **Recurrent neural network**, is schematically shown in Fig 4.3\textsuperscript{[135]}\textsuperscript{[135]}. It is also called sequential network and was proposed by Jordan\textsuperscript{[146]}\textsuperscript{[146]}. Contrary to feed-forward networks, the dynamical properties of the network are important\textsuperscript{[135, 144]}\textsuperscript{[135, 144]}. This network is particularly useful when modeling a continuous system in which the recent past history of a process is valuable in determining the process value at the next time step\textsuperscript{[132]}\textsuperscript{[132]}. The model consists of a sequence of actions, which are to be produced in the order of a “plan”. Additional inputs come from the current —stat” nodes that together with the plan nodes force the next state into a desirable state of action. This type of network would be useful, for example, when modeling a car's speed as a function of the accelerator's position. The recent past history of the car's speed is important in determining the speed of the next time step. An example of the input values provided by the plan might include wind speed and direction or the slope of the road in which the car is traveling\textsuperscript{[132]}\textsuperscript{[132]}.
4.1.3 Applications of Neural Networks

An important area of application of neural networks is in the field of robotics[^144]. For example, neural networks are designed to direct a manipulator, which is the most important form of the industrial robot, to grasp objects, based on sensor data, or to steer an autonomous robot vehicle depends on feedback signals from the environment. However, the traditional applications of neural networks can be roughly categorized into the following three types[^135, 138], classification, noise reduction or pattern recognition, and prediction.

Although scientific investigations on metallurgical operations have helped greatly in understanding the underlying phenomena, there remain many problems where quantitative treatments are lacking[^129, 147]. Neural networks are extremely useful in such circumstances wherever the complexity of the problem is overwhelming from a fundamental perspective and where simplification is unacceptable[^129], because neural networks can discover complex relationship between the dependent and independent variables without any priori assumption of the form of the relationship[^147]. The relationship between the variables is, in
fact, discovered during the training procedure. Once the model is trained, it can be tested to examine whether the predictions are consistent with the established qualitative metallurgical principles, and then the model can be used for prediction and process control\[147\].

As in the case of steel rolling process\[147, 148\], when a hot ingot or slab enters a rolling mill, its typical dimensions are so large that it has to be reduced to the required thickness in many separate passes. The purpose of this deformation is to refine the cast microstructure, to produce the steel in the required shape, and to achieve the optimum mechanical properties that depends on the chemical composition of the steel and the deformation schedule during rolling. A neural network model was developed to estimate the strength of the product as a function of a large number of rolling parameters and the chemical composition of the steel (108 variables), and the model has been shown to be consistent with established metallurgical trends and can be used to study the effect of each variable in isolation\[148\].

4.2 Fuzzy Logic

Fuzzy control is an attempt to make computers understand natural language and behave like a human operator\[149\]. The idea was originated from Zadeh in 1965\[150-152\]. However, it took two decades before fuzzy logic controller was applied firstly in a laboratory by Mamdain and Assilian\[149, 150\]. The first industrial application of fuzzy logic controller was for a cement kiln in Denmark\[149\], and it becomes a practical alternative for a variety of challenging control applications since it provides a convenient method for constructing nonlinear controllers via the use of heuristic information, which may come from an operator who has acted as a “human-in-the-loop” controller for a process\[153\]. Today, there is a tendency to combine the fuzzy logic control paradigm with other techniques such
as artificial neural networks, and provide both the natural language interface from fuzzy logic and learning capabilities of neural networks\textsuperscript{[149]}

### 4.2.1 Fundamental of Fuzzy Logic

Fuzzy logic starts from the idea of fuzzy sets, which is to solve the problem of sharp separation of conventional sets—complete membership or complete non-membership\textsuperscript{[152]}. Instead, fuzzy sets can handle partial membership. An element can be determined to what degree or extend it is a member of a fuzzy set, and a clear definition of a fuzzy sets can be found in several documentations\textsuperscript{[149-152]}. A typical fuzzy control which is embedded in a closed-loop control system is schematically shown in Fig 4.4\textsuperscript{[153]}, in which, $y(t)$ denotes the plant output, $u(t)$ denotes the plant input, and the reference input to the fuzzy controller is denoted by $r(t)$.

![Fig 4.4 Architecture of fuzzy control system\textsuperscript{[153]}](image)

As shown schematically in Fig 4.4, the fuzzy control system is composed of the following four elements\textsuperscript{[153]}:

- **A rule-base** (a set of If-Then rules). Holds the knowledge of how to best control the system in the form of a set of rules. It contains a fuzzy logic quantification of the expert’s linguistic description of how to achieve good control.
- **An inference mechanism** (also called an “inference engine” or “fuzzy inference” module). The inference mechanism evaluates which control rules are relevant at the current time and then decides what the input to the plant should be. It emulates the expert’s decision making in interpreting and applying knowledge about how best to control the plant.

- **A fuzzification interface**. Modifies the inputs so that they can be interpreted and compared to the rules in the rule-base. It converts controller inputs into information that the inference mechanism can easily use to activate and apply rules.

- **A defuzzification interface**, which converts the conclusions of the inference mechanism into actual inputs for the process.

### 4.2.2 Operations of Fuzzy System

A fuzzy system can be regarded as a static nonlinear mapping between its inputs and outputs, provided that the pre-processing is separated from the control system according to the convention. Fig 4.5 demonstrates how a fuzzy control system works schematically.
1) Pre-processing

It conditions the measurements before they enter into the controller, and examples of the operations are\textsuperscript{[149]}:

- Quantization in connection with sampling or rounding to integers
- Normalization or scaling onto a particular, standard range
- Filtering in order to remove noise
- Averaging to obtain long-term or short-term tendencies
- A combination of several measurements to obtain key indicators
- Differentiation and integration, or their approximations in discrete time

2) Fuzzification

Fuzzification is the process of decomposing a system input and/or output into one or more fuzzy sets. Many types of curves can be used, but triangular or trapezoidal shaped membership functions are the most common because they are easier to represent in embedded controllers\textsuperscript{[149]}. It converts the numeric inputs from the pre-processor into fuzzy sets\textsuperscript{[153]} and evaluates the input measurements according to the premises of the rules and specifies a membership grade expressing the degree of fulfillment of the premise. Quite often, ―singleton fuzzification‖ is used and it produces a fuzzy set with a membership function.

\[
\mu(iu) \text{ maps } u_i \text{ to } [0, 1] \text{ and is subjectively specified in}
\]

![Fig 4.6 Typical membership functions\textsuperscript{[153]}](image-url)
an ad hoc (heuristic) manner from experience or intuition, it describes the
—certain— of an element $u_i$ with a linguistic description $\tilde{u}_i$ into a number in
specific a range. There are many options for the shape of the membership
function, such as the shapes of triangular or trapezoidal, and each of them will
provide a different meaning for the linguistic values they quantify. A variety of
membership functions are graphically illustrated in Fig 4.6, the mathematical
characterization of the triangular and Gaussian membership functions are
summarized in Table 4.2 and Table 4.3[153], and other membership functions can
be characterized with mathematics using a similar approach[153].

3) Rule base

A rule base allow for several variables both in the premise and the conclusion,
therefore the controller can be multi input, and multi output (MIMO), or single
input and single output (SISO)[149]. A typical controller regulates a control signal
according to an error signal, which can be separated into error, change in error
and integral error respectively[149]. A linguistic controller contains rules in the if-
then format, for example, if error is negative, change in error is negative, then
control is negative big. However, they can also appear in other formats. Table 4.4
summarized the operations of a typical linguistic controller[149].

Table 4.2 Mathematical characterization of triangular membership functions[153]

<table>
<thead>
<tr>
<th></th>
<th>Triangular membership functions</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Left</strong></td>
<td>$\mu^L(u) = \begin{cases} 1 &amp; \text{if} \quad u \leq c^L \ \max\left{0, 1 + \frac{c^L - u}{0.5w^L}\right} &amp; \text{otherwise} \end{cases}$</td>
</tr>
<tr>
<td><strong>Centers</strong></td>
<td>$\mu^C(u) = \begin{cases} \max\left{0, 1 + \frac{u - c}{0.5w}\right} &amp; \text{if} \quad u \leq c^L \ \max\left{0, 1 + \frac{c - u}{0.5w}\right} &amp; \text{otherwise} \end{cases}$</td>
</tr>
</tbody>
</table>
In Table 4.4, there are two inputs (Error and Change in Error), and one output. The names Zero (Zero error), Pos (Positive), Neg (Negative) are labels of fuzzy sets as well as NB (Negative Big), NM (Negative Medium), PB (Positive Big) and PM (Positive Medium)\textsuperscript{[149]}.

Table 4.4 Operation of a typical linguistic controller\textsuperscript{[149]}

<table>
<thead>
<tr>
<th>Error</th>
<th>Change in error</th>
<th>Control</th>
</tr>
</thead>
<tbody>
<tr>
<td>Neg</td>
<td>Neg</td>
<td>NB</td>
</tr>
<tr>
<td>Neg</td>
<td>Zero</td>
<td>NM</td>
</tr>
<tr>
<td>Neg</td>
<td>Pos</td>
<td>Zero</td>
</tr>
<tr>
<td>Zero</td>
<td>Neg</td>
<td>NM</td>
</tr>
<tr>
<td>Zero</td>
<td>Zero</td>
<td>Zero</td>
</tr>
<tr>
<td>Zero</td>
<td>Pos</td>
<td>PM</td>
</tr>
<tr>
<td>Pos</td>
<td>Neg</td>
<td>Zero</td>
</tr>
<tr>
<td>Pos</td>
<td>Zero</td>
<td>PM</td>
</tr>
<tr>
<td>Pos</td>
<td>Pos</td>
<td>PB</td>
</tr>
</tbody>
</table>
There are two specific problems that need to be considered inevitably when designing a rule base, i.e. How to determine the shape of the sets? How many sets are necessary and sufficient? Since the choice of shape and width is subjective, a solution is to ask the plant operators to draw their personal preferences for the membership curves. However, quite often, operators find it difficult to settle on particular curves. A few “rules of thumb” apply in such circumstances\textsuperscript{[149]}.

- A term set should be sufficiently wide to allow for noise in the measurement.
- A certain amount of overlap is desirable; otherwise the controller may run into poorly defined states, where it does not return a well-defined output.
- If there is a gap between two neighbouring sets, no rules fire for values in the gap. Consequently the controller is undefined in that gap.
- The necessary and sufficient number of sets in a family depends on the width of the sets, and vice versa.

4) Inference engine

There are two standard alternative operations in the inference step, one involves the use of implied fuzzy sets and the other uses the overall implied fuzzy set\textsuperscript{[153]}. For the first option, the fuzzy set specifies the certainty level by considering only one rule, however, for the other option, the membership function represents the conclusion reached by considering all the rules in the rule base at the same time \textsuperscript{[153]}. The overall justification for using the above operations to represent the inference step lies in the fact that “we can be no more certain about our conclusions than we are about our premises”\textsuperscript{[153]}.

5) Defuzzification

The defuzzification is a process which converts the fuzzy output back to classical
output to the control objective\textsuperscript{150}. Corresponding to the implied fuzzy set and overall implied fuzzy set approaches, the defuzzification process can also be roughly categorized into two types. Centre of gravity, centre average, maximum criterion, centre of area mathematical operations have been applied respectively to convert the result of membership functions to classical output. The detailed process operations has been well documented by both Passino\textsuperscript{153} and Jantzen\textsuperscript{149}.

6) Post processing

The outputs from the defuzzification are required to be scaled to the appropriate range so that they can be applied directly by the actuators. For example, the outputs are usually represented in standard universe; however, the acceptable signals for the actuator are volts, meters, or tons per hour \textit{etc}.\textsuperscript{149}. Therefore, it is necessary to convert the outputs into a suitable range which can be applied by the actuators, this is process is termed as post processing.

4.2.3 Application of Fuzzy Logic System

The initial objective for fuzzy control is to make computers understand natural language and behave like a human operator. The first laboratory application (mid-1970s) was a two-input-two output steam engine controller by Mamdani and Assilian in UK, and the first industrial application was a controller for a cement kiln by Holmblad and Smidth in Denmark\textsuperscript{149, 150}. Today fuzzy logic has been widely used in image processing and pattern recognition and is regarded as a useful tool for handling the uncertainty in the images associated with vagueness and/or imprecision\textsuperscript{154}, and have been successfully applied to many industrial applications and medical diagnoses over the past decades, particularly in the field of automation\textsuperscript{143, 151, 155-159}.

Karakuzu and Ozturk\textsuperscript{130} compared the different controlling techniques based on
the same experimental application on water temperature control system. The results showed that the classical (P, PI, PID, Proportional-Integral-Differential) control techniques were good for the simple linear systems, since they are fixed-gain feedback controllers and can not compensate the parameter variations in the plant and can not adapt changes in the environment. However, from a perspective of control, most of the real systems exhibit nonlinear behavior, it is difficult to model these systems according to the law of physics, and sometimes it is impossible to make mathematical models for those complicated systems, classical control systems are not the suitable choice. Fuzzy control systems can be developed along with linguistic lines and can transform sets of structured information into the appropriate control actions, no mathematical representation of the plant is required if neural network systems can be well trained and expertise information about the plant were embedded to the systems\textsuperscript{130}, thus it provides a good option for the nonlinear circumstances.

However, Haack\textsuperscript{160} argues that there are only two area where fuzzy logic could possibly be demonstrated to be “needed”, and otherwise it can be shown that fuzzy logic is not necessary. The first area Haack defined is that of the nature of Truth and Falsity: if it could be shown that these are fuzzy values and not discrete ones, then a need for fuzzy logic would have been demonstrated. For example, “The sky is blue” is either true of false, any fuzziness to the statement arises from the imprecise definition of terms, not out of the nature of truth. The other area is that of fuzzy systems’ utility: if it could be demonstrated that generalizing classic logic to encompass fuzzy logic would aid in calculations of a given sort, then again a need for fuzzy logic would exist. Haack claimed\textsuperscript{160} that no area of data manipulation is made easier through the introduction of fuzzy calculus, therefore fuzzy logic is unnecessary. Fox\textsuperscript{161} responded the above objection by indicating that there are three areas in which fuzzy logic can be of benefit, such as 1) “requisite” apparatus (to describe real world relationships which are inherently
fuzzy); 2) —prescriptive‖ apparatus (because some data is fuzzy, and therefore requires a fuzzy calculus); and 3) —descriptive‖ apparatus (because some inference systems are inherently fuzzy).

Additionally, Fuzzy logic system has been successfully applied to search, identify and compare different metal alloys, based on their chemical composition and/or mechanical properties, or partial information thereof, the results and performance obtained match high profile human experts\textsuperscript{162}. Fuzzy modeling and control in BOF promise a solution to the strongly non-linear problems associated with the process, which have so far proven extremely difficult to be solved by conventional control methods, it can be effectively used to improved process control of BOF furnace\textsuperscript{163}. Combination of neural network and fuzzy logic models have been applied to optimize the control of electric arc furnace (EAF) and reduce the energy consumption\textsuperscript{164}, about 250 variables concerning the EAF process are collected, such as temperature (oxygen and carbon measurements), weight and layers of slag in each basket, sound amplitude at different frequency’s ranges and other process parameters, some of them have been continuous sampling along the process and other ones have only one value per casting. These models are very useful to optimize several aspects of the furnace operation, such as: determination of overall process characteristic and process control, modeling of the off-gas system or the heat transfer inside the furnace and modeling of meltdown and slag chemistry and slag foaming.

4.3 Multivariate Statistical Analysis

Multivariate statistical methods are applied to investigate the relationships between several variables simultaneously, and are usually applied to summarized data and reduce the number of variables necessary to describe it\textsuperscript{165}. Principal component analysis (PCA) is an important multivariate statistical technique, and
has been called one of the most valuable results from applied linear algebra\cite{166}. It is a standard tool in modern data analysis, because it is a simple, non-parametric method for extracting relevant information from confusing data sets. With minimum effort, PCA provides a roadmap for how to reduce a complex data set to a lower dimension to reveal the sometimes hidden, simplified structures that often underlie it\cite{167}. The other multivariate statistical methods includes factor analysis, discriminate function analysis, cluster analysis, and multidimensional scaling techniques, they will be reviewed below.

4.3.1 Principal Component Analysis

The technique of principal component analysis was first described by Karl Pearson in 1901\cite{168}, who supposed that the technique was the correct solution to some of the problems that were of interest to biometricians at that time. However, the variables were only a few because they had to be calculated by hand. It was not until computers became widely available that the technique achieved widespread use, because rapid treatment of a larger number of calculations became possible.

PCA involves a mathematical procedure that transforms a number of possibly correlated variables into a smaller number of uncorrelated variables which are called principal components. The first principal component accounts for as much of the variability in the data as possible, and each succeeding component accounts for as much of the remaining variability as possible. If a multivariate dataset is visualised as a set of coordinates in a high-dimensional data space (1 axis per variable), PCA supplies the user with a lower-dimensional picture, a “shadow” of this object when viewed from its (in some sense) most informative viewpoint\cite{169}. The goal of PCA is to identify the most meaningful basis to re-express a data set, and it has become a standard tool in modern data analysis,
because it is a simple, non-parametric method for extracting relevant information from confusing data set\cite{167}.

The following example may schematically illustrate the meaning of PCA. In an ideal spring motion system shown schematically in Fig 4.7\cite{167}, it consists of a ball of mass $m$ attached to a massless, frictionless spring. The ball is released a small distance away from equilibrium (i.e. the spring is stretched). The ball will oscillate indefinitely along the $x$-axis about its equilibrium at a set frequency due to the spring is ideal.

![Fig 4.7 Ideal spring ball motion system\cite{167}](image.png)

Provided that we are ignorant of which and how many axes and dimensions are important to measure, and decided to measure the ball’s position in a three-dimensional space, as we live in a three dimensional world. Three video cameras (camera A, B and C) were placed around the system and each camera records an image at 120 Hz indicating a two dimensional position of the ball (a projection). After several minutes recording, the position of the ball tracked by each camera is depicted in Fig 4.8\cite{167}
The position of the ball can be defined by a set of two dimensional data for each camera, suppose we collected the data simultaneously, then the position of the ball can be indicated by the following vector:

\[ \vec{X} = [x_A, y_A, x_B, y_B, x_C, y_C]^T \]  \hspace{1cm} (4-2)

Where each camera contributes a Two-dimensional projection of the ball’s position on the entire vector \( \vec{X} \). If 10 minutes were recorded, then the number of the vectors is \( 10 \times 60 \times 120 = 72000 \). Therefore, the position of the ball can be represented by an \( m \times n \) (\( m=6 \), \( n=72000 \)) matrix, which can be represented by a new matrix \( Y \) by the following linear transformation:

\[ PX = Y \]  \hspace{1cm} (4-3)

It is clear that the underlying dynamics can be expressed as a function of a single variable, or, the data sets of the three cameras are highly correlated. The transformation matrix \( P \) will provide a new basis vector which may re-express the original dataset into a simple form\[167\]. The detailed explanation of the process will be described afterwards in section 4.3.1.2.

Principal components analysis provides an objective way of finding indices so that the variation in the data can be accounted for as concisely as possible. It may well turn out that two or three principal components provide with a good summary of all the original variables. Consideration of the values of the principal
components instead of the values of the original variables may then make it easier to understand what the data have to say\cite{9}. In short, principal components analysis is a means of simplifying data by reducing the number of variables or dimensions.

The problem of dimensionality reduction is closely related to feature extraction, which refers to identifying the salient aspects or properties of data to facilitate its use in a subsequent task, such as regression or classification\cite{170}. Its features are a set of derived variables, functions of the original problem variables, which efficiently capture the information contained in the original data. In general, selection of features depends on the ultimate application. However, in cases where the ultimate applications of the data is not known in advance, a suitable objective for feature extraction is to reduce dimensionality with minimal loss of information\cite{171}.

It should be noted that a principal component analysis does not always work in a sense that a large number of original variables are reduced to a small number of transformed variables. If the original variables are uncorrelated then the analysis will achieve nothing. The best results are obtained when the original variables are very highly correlated, positively or negatively. If that is the case, then it is quite conceivable that 20 or 30 original variables can be adequately represented by two or three principle components\cite{9}.

The development of computer technology has revolutionized the multivariate statistics in several respects. The availability of modern computer facilities makes possible the analysis of large data sets and that ability permits the application of multivariate methods to new areas, such as image analysis, and more effective analysis of data such as meteorological.
4.3.1.1 Assumptions for Principal Component Analysis

It can be said that both the strength and the weakness of PCA is that it is a non-parametric analysis\[^{166}\]. The following assumptions are made before processing the PCA, and they are also constrains that limits its application.

- **Linearity.** Linearity frames the problem as a change of basis. Several areas of research have explored how applying a nonlinearity prior to performing PCA could extend this algorithm and this has been termed kernel PCA\[^{166}\].

- **Mean and variance are important parameters in statistics.** The formalism of sufficient statistics captures the notion that the mean and the variance entirely describe a probability distribution. The only class of probability distributions that are fully described by the first two moments are exponential distributions (e.g. Gaussian, Exponential, etc). In order for this assumption to hold, the probability distribution of \( x \) must be exponentially distributed. Deviations from this could invalidate this assumption. On the flip side, this assumption formally guarantees that the SNR (Signal to Noise Ratio) and the covariance matrix fully characterize the noise and redundancies\[^{166}\].

- **Large variances have important dynamics.** This assumption also encompasses the belief that the data has a high SNR. Hence, principal components with larger associated variances represent interesting dynamics, while those with lower variances represent noise\[^{166}\].

- **The principal components are orthogonal.** This assumption imposes an intuitive simplification that makes PCA solvable with linear algebra decomposition techniques\[^{166}\].

If the structure feature of a system is known a priori, then it makes sense to incorporate these assumptions into a parametric algorithm, or an algorithm with selected parameters. This prior non-linear transformation is sometimes termed a
−kernel transformation” and the entire parametric algorithm is termed “kernel PCA”.

One might envision situations where the principal components need not be orthogonal. Furthermore, the distributions along each dimension (\(x_i\)) need not be Gaussian. For instance, in a 2-D exponentially distributed data set, the largest variances do not correspond to the meaningful axes; thus PCA fails. This less constrained set of problems is not trivial and only recently has been solved adequately via Independent Component Analysis (ICA)\(^9\).

### 4.3.1.2 Procedures of Principal Component Analysis

A principal component analysis starts with data on \(p\) variables for \(n\) individual samples, as indicated in the Table 4.5\(^167\).

<table>
<thead>
<tr>
<th>Individual</th>
<th>(X_1)</th>
<th>(X_2)</th>
<th>(\ldots)</th>
<th>(X_p)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(x_{11})</td>
<td>(x_{12})</td>
<td>(\ldots)</td>
<td>(x_{1p})</td>
</tr>
<tr>
<td>2</td>
<td>(x_{21})</td>
<td>(x_{22})</td>
<td>(\ldots)</td>
<td>(x_{2p})</td>
</tr>
<tr>
<td>(\ldots)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(N)</td>
<td>(x_{n1})</td>
<td>(x_{n2})</td>
<td>(\ldots)</td>
<td>(x_{np})</td>
</tr>
</tbody>
</table>

Suppose that there are \(p\) variables, \(x_1, x_2, \ldots, x_p\) and the values of these for the \(i\) th individual in a sample are: \(x_{i1}, x_{i2}, \ldots, x_{ip}\), respectively. Assuming the principal component is the linear combination of the variables:

\[
Z_i = a_{i1}x_1 + a_{i2}x_2 + \ldots + a_{ip}x_p
\]

(4-4)

Step 1: Calculate the sample mean \(\bar{x}_j\) and the sample variance \(s_j^2\).
\[
\bar{x}_j = \frac{\sum_{i=1}^{n} x_{ij}}{n} \quad (4-5)
\]
\[
S_j^2 = \frac{\sum_{i=1}^{n} (x_{ij} - \bar{x}_j)^2}{n-1} \quad (4-6)
\]
then the following Table 4.6\cite{4-6} can be attained.

<table>
<thead>
<tr>
<th>Individual</th>
<th>(X_1)</th>
<th>(X_2)</th>
<th>(\ldots)</th>
<th>(X_p)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(X_{11})</td>
<td>(X_{12})</td>
<td>(\ldots)</td>
<td>(X_{1p})</td>
</tr>
<tr>
<td>2</td>
<td>(X_{21})</td>
<td>(X_{22})</td>
<td>(\ldots)</td>
<td>(X_{2p})</td>
</tr>
<tr>
<td>(\ldots)</td>
<td>(\ldots)</td>
<td>(\ldots)</td>
<td>(\ldots)</td>
<td></td>
</tr>
<tr>
<td>N</td>
<td>(X_{n1})</td>
<td>(X_{n2})</td>
<td>(\ldots)</td>
<td>(X_{n3})</td>
</tr>
<tr>
<td>Sample mean</td>
<td>(\bar{X}_1)</td>
<td>(\bar{X}_2)</td>
<td>(\ldots)</td>
<td>(\bar{X}_p)</td>
</tr>
<tr>
<td>Sample variance</td>
<td>(S_1^2)</td>
<td>(S_2^2)</td>
<td>(\ldots)</td>
<td>(S_p^2)</td>
</tr>
</tbody>
</table>

Step 2: Calculate the sample covariance between different variables and get the covariance matrix. The covariance between variable \(j\) and variable \(k\) is defined as:
\[
C_{jk} = \frac{\sum_{i=1}^{n} (x_{ij} - \bar{x}_j)(x_{ik} - \bar{x}_k)}{n-1} \quad (4-7)
\]
and the sample covariance matrix \(C\) can be attained:
\[
C = \begin{bmatrix}
    C_{11} & C_{12} & \cdots & C_{1p} \\
    C_{21} & C_{22} & \cdots & C_{2p} \\
    \vdots & \vdots & \ddots & \vdots \\
    C_{p1} & C_{p2} & \cdots & C_{pp}
\end{bmatrix} \quad (4-8)
\]
the diagonal element \(c_{ii}\) is the variance of \(x_i\) and \(c_{ij}\) is the covariance of \(x_i\) and \(x_j\), obviously, the matrix \(C\) is symmetrical.
Step 3: Calculate the eigenvalues of the matrix $C$ and the corresponding eigenvectors. The eigenvalue of a matrix is defined as:

$$Ax = \lambda x$$  \hspace{1cm} (4-9)

or

$$(A - \lambda I)x = 0$$  \hspace{1cm} (4-10)

Where $I$ is an $p \times p$ identity matrix and $0$ is an $p \times 1$ zero vector.

Thus attain the following equations:

$$c_{11}x_1 + c_{12}x_2 + \ldots + c_{1p}x_p = \lambda_1 x_1$$
$$c_{21}x_1 + c_{22}x_2 + \ldots + c_{2p}x_p = \lambda_2 x_2$$
$$\ldots$$
$$c_{p1}x_1 + c_{p2}x_2 + \ldots + c_{pp}x_p = \lambda_p x_p$$  \hspace{1cm} (4-11)

Solving the above equations simultaneously; and the eigenvalues of matrix $C$ can be attained. Assuming that they are ordered as $\lambda_1 \geq \lambda_2 \geq \ldots \geq \lambda_p$, and $\lambda_i$ corresponds to the $i$th principal component

$$Z_i = a_{i1}x_1 + a_{i2}x_2 + \ldots + a_{ip}x_p$$  \hspace{1cm} (4-12)

The constants: $a_{i1}$, $a_{i2}$, $\ldots$, $a_{ip}$ are the elements of the corresponding eigenvectors, scaled so that:

$$a_{i1}^2 + a_{i2}^2 + \ldots + a_{ip}^2 = 1$$  \hspace{1cm} (4-13)

An important property of the eigenvalues is that they added up to the sum of the diagonal elements (the trace) of $C$. i.e.,

$$\lambda_1 + \lambda_2 + \ldots + \lambda_p = c_{11} + c_{22} + \ldots + c_{pp}$$  \hspace{1cm} (4-14)

As $c_{ii}$ is the variance of $x_i$ and $\lambda_i$ is the variance of $Z_i$. This means that the sum
of the variances of the principal components is equal to the sum of the variances of the original variables. Therefore, in a sense, the principal components accounts for all the variation in the original data.

**Step 4: Choosing components and deriving the new data set**

Discard any components that only account for a small proportion of the variation in the data. For example, starting with 30 variables, there might be found that the first three components account for 90% of the total variance, on this basis, the other 27 components may be reasonably ignored. Then form the new data set with the eigenvectors corresponding to the important eigenvalues.

### 4.3.1.3 Extended PCA

The PCA reviewed above may be regarded as conventional PCA or linear PCA. It is based on linear algebra and aims to find the linear relationship of a database, and it can be used to identify a relatively small number of factors that represent the relationships among many inter-correlated variables. Principal component analysis was used for several years before its inherent limitations were fully realized, and it has been extended in several ways, such as complex principal component analysis, sparse principal component analysis, probabilistic principal component analysis, and nonlinear principal component analysis.

1) Complex Principal Component Analysis (CPC)

Principal component analysis has been extended in several ways, and complex principal component analysis (CPC) is the one which is particularly for detecting propagating phenomena, it is ideally suited for climatic records of sea level pressure or wind analysis, especially when the variance is spread over a number of frequencies. However, there are many drawbacks of CPC which must be considered.
CPC analysis is a mixture of principal component analysis and cross-spectral methods and it suffers from the problems of both techniques, e.g. the principal component may not be reproducible from one sample to another if successive eigenvalues are nearly identical and information at the end of the time series is lost.

The complex principal components and their eigenvectors are difficult to interpret in all but the simplest of cases because both amplitude and phase relationships must be considered, and reconstruction of the data can be misleading in regions which are poorly explained by the complex principal components.

Sudden transitions and noisy spikes are emphasized unless the data is low-pass filtered prior to CPC analysis.

CPC solutions from separate analysis are difficult to compare.

The primary benefit of CPC analysis is that it allows propagating features to be detected and dissected in terms of their spatial and temporal behavior. The CPC approach efficiently reduces the data set into the fewest number of modes possible[172].

2) Nonlinear Principal Component Analysis (NLPCA)

The nonlinear principal component analysis (NLPCA) is a general purpose feature extraction algorithm producing features that retain the maximum possible amount of information from the original data set, for a given degree of data compression. Information preservation assures that the selected features will be useful in most situations, independent of the ultimate application[175].

The NLPCA method applies artificial network (ANN) training procedures to generate nonlinear features. The networks are of a conventional type, featuring
feed forward connections and linear or sigmoidal nodal transfer functions, trained by back propagation. The typical network architecture employs three hidden layers, including an internal ―bottleneck‖ layer of smaller dimension than either input or output. The network is trained to perform the identity mapping, where the input is approximated at the output layer. Since there are fewer units in the bottleneck layer than the output, the bottleneck nodes must represent or encode the information in the inputs for the subsequent layers to reconstruct the input. If network training finds an acceptable solution, a good representation of the input must exist in the bottleneck layer. This implies that data compression caused by the network bottleneck may force hidden units to represent significant features in data\textsuperscript{[176]}.

The mean difference between PCA and NLPCA is that the latter involves nonlinear mapping between the original and the reduced dimension spaces. If nonlinear correlations between variables exist, NLPCA will describe the data with greater accuracy and/or by fewer factors than PCA, provided that there are sufficient data to support the formulation of more complex mapping functions\textsuperscript{[175]}.

### 3) Probabilistic Principal Component Analysis

Probabilistic principal component analysis (PPCA) is a latent variable model in which the maximum likelihood solution for the parameters is found through solving an eigenvalue problem on the data’s covariance matrix\textsuperscript{[174, 177]}. A probability interpretation of PCA is given by Lawrence\textsuperscript{[177]} and it is closely related with factor analysis, in which the relationship of the variables are linear. Tipping\textsuperscript{[174]} demonstrated how the principal axes of a set of observed data vectors may be determined through maximum likelihood estimation of parameters in a latent variable model.

The principal axes emerge as maximum likelihood parameters which may be computed by the usual eigendecomposition of the sample covariance matrix and
subsequently incorporated in the latent variable model. Alternatively, the latent variable formulation leads naturally to an iterative, and computationally efficient expectation maximization algorithm for effecting PCA. As the definition of a likelihood measure enables a comparison with other probabilistic techniques, while facilitating statistical testing and permitting the application of Bayesian methods. The advantages of PPCA may be described as follows\(^{174}\):

- The probability model offers the potential to extend the scope of conventional PCA.
- As well as its application to dimensionality reduction, PPCA can be utilized as a constrained Gaussian density model. The benefit for so doing is that maximum likelihood estimates for the parameters associated with the covariance matrix can be efficiently computed from the data principal components. Potential applications include classification and novelty detection.

4) **Sparse Principal Component Analysis (SPCA)**

The results of PCA are reduced dimension of original variables, which are linear combinations of a number of variables. One obvious drawback is that the loadings are typically nonzero, which makes it difficult to interpret the derived principal components\(^{173}\). To facilitate the interpretation, the resulting principal components are modified with sparse loadings which have very few nonzero elements; this manipulation is regarded as sparse principal components analysis (SPCA). SPCA is built on the fact that PCA can be written as a regression-type optimization problem, thus the lasso (elastic net, is a penalized least square method) can be directly integrated into the regression criterion so that the resulting modified PCA produces sparse loadings. Shen *et al.*\(^{178}\) proposed a new SPCA via regulated singular value decomposition (SVD) of the original data matrix and extracted the principal components through solving a low rank
approximation matrix by introducing regularization penalties to promote sparsity in loadings. Zou et al.\textsuperscript{[173]} introduced an algorithm of SPCA based on sparse approximation criterion and proposed an elastic net which shrinks the coefficient continuously towards zero, thus gaining its prediction accuracy via the bias variance trade-off; however, the number of selected variables by the elastic net is limited by the number of observations.

According to Zou\textsuperscript{[173]}, from a practical point of view, a good method to achieve the sparseness goal should (at least) possess the following properties.

- Without any sparsity constraint, the method should reduce to PCA
- It should be computationally efficient for both small and big number of principal component data set
- It should avoid mis-identifying the important variables.

### 4.3.2 Other kind of multivariate statistical techniques

Apart from PCA, there are some other kinds of multivariate statistical techniques, such as factor analysis, discriminate function analysis, cluster analysis, canonical correlation and multidimensional scaling techniques\textsuperscript{[168]}. A brief introduction of them is provided in the following section.

#### 4.3.2.1 Factor Analysis

Factor analysis attempts to account for the variation in a number of original variables using a small number of index variables or factors\textsuperscript{[168]}. It is assumed that each original variable can be expressed as a linear combination of these factors, plus a residual term that reflects the extent to which the variables is independent of the other variables.
One type of factor analysis starts by taking a few principal components as the factors in the data being considered. These initial factors are then modified by a special transformation process called "factor rotation" in order to make them easier to interpret. Other methods for finding initial factors are also used. A rotation to simpler factors is almost always done.

In simple terms, factor analysis is a technique for expressing in the language of mathematics hypothetical variables (factors) by using a variety of common indicators that can be measured. The analysis is considered exploratory when the concern is with determining how many factors (or latent variables) are necessary to explain the relationships among the indicators.

PCA and FA (Factor Analysis) are generally seen as competing strategies and there is a good deal of disagreement regarding which is the appropriate method to use\[^1\text{79}\]. The great difference between PCA and FA is that they constitute distinct models with different goals. PCA is a data reduction method, and its goal is to arrive at a relatively small number of components that will extract most of the variance of a relatively large set of variables. FA is aimed at explaining common variance, which is the variance shared by the observed variables.

### 4.3.2.2 Discriminate Function Analysis

Discriminate function analysis is concerned with the problem of seeing whether it is possible to separate different groups on the basis of the available measurements. Like principal components, discriminate function analysis is based on the idea of finding suitable linear combination of the original variables. Discriminate function analysis involves the predicting of a categorical dependent variable by one or more continuous or binary independent variables\[^1\text{2}\]. It is statistically the opposite of multivariate analysis of variance. It is useful in determining whether a set of variables is effective in predicting category membership. It is also a useful follow-up procedure to multivariate analysis of
variance. Instead of doing a series of one-way analysis of variance (ANOVAs), which is a collection of statistical models, for ascertaining how the groups differ on the composite of dependent variables.

4.3.2.3 Cluster Analysis

Clustering is the classification of objects into different groups, or more precisely, the partitioning of a data set into subsets which are called —clusters”, so that the data in each cluster share some common trait, which is often according to some defined distance measure\(^{[12]}\). Data clustering is a common technique for statistical data analysis, and it has been used in many fields, including machine learning, data mining, pattern recognition, image analysis and bioinformatics, etc. Generally speaking, clustering can be categorized into two types according to the algorithms, i.e. hierarchical or partitioned. Hierarchical algorithms find successive clusters using previously established clusters, whereas partitioned algorithms determine all clusters at once. Hierarchical algorithms can be agglomerative (—bottom-up”) or divisive (—top-down”). Agglomerative algorithms begin with each element as a separate cluster and merge them into successively larger clusters. Divisive algorithms begin with the whole set and proceed to divide it into successively smaller clusters\(^{[168]}\).

4.3.2.4 Multi-Dimensional Scaling

This technique begins with data on some measure of the distances apart from a number of objects. From these distances a “map” is constructed showing how the objects are related\(^{[168]}\). This is a useful facility as it is often possible to measure how far apart pairs of objects are without having any idea on how objects are related in a geometric sense. With a one-dimensional “map” the groups are placed along a straight line. With a two-dimensional “map” they are represented by points on a plane. With a three-dimensional “map” they are represented by points in cube. Four- and higher-dimensional solutions are also possible although
these have limited use because they can not be visualized.

4.4 Comparison of Different Analysis Techniques

There are several techniques available for analyzing different types of data concerning the bubbling phenomena inside the metallurgical vessels, such as neural networks, fuzzy logic and principal component analysis, which have been reviewed in this chapter. The strength of neural networks lies in its ability to simulate human brains as well as its ability to learn and respond, however, the neural networks need to be trained before it can be applied for any specific industrial process. Regarding to the scenario of bottom bubble stirring in a metallurgical vessel, many variables are concerned (e.g. heat transfer and mass transfer coefficients, physical properties of the liquid metal, slag and the injecting gas, operational parameters such as gas flow rate and pressure), it is difficult to define standard inputs and ideal outputs, therefore, training of the neural networks based on supervised learning would be difficult to perform. Though unsupervised learning has advantages and theoretically shows potential for solving such complicated problems, however, it has not been proved to be very successful in industry\textsuperscript{[132]}.

Fuzzy logic also attempts to simulate human operators during the control process, and its strength lies in transforming expert knowledge into automation. It can be regarded as a static nonlinear mapping between the inputs and outputs\textsuperscript{[180]}, therefore, specific expert knowledge is required to embed into the system, and a complete rule base is obliged to be formed for the system to make decisions. In the case of bottom bubble stirring, gas flow rate indicated by the volumetric flow meters may not be the actual gas which have been injected into the vessel due to the leakage between pipes, hook-ups and refractory components\textsuperscript{[6]}, it is difficult to design a complete rule base for the corresponding relations between the volume flow rate display and the exact values which has been actually injected
into the vessel. Neural networks might be required in this case however, it is difficult to train the network with limited industrial database.

The strength of PCA lies in its ability to extract useful information from correlated data base, and its simple mathematical operations which make it suitable for online analysis. Consequently it has been successfully applied for multivariate statistical analysis in industry\textsuperscript{[181-188]}. Concerning the scenario of bottom bubble stirring, there is a clear logic relation between the different phenomena, the stirring process excites the bubble flow, which emits acoustic signals as well as creates disturbed top surface and causes vibrations on the wall of the vessel. Therefore, the image signals from the top surface, the sound of bubbling and the vibrations on the wall of the vessel should be correlated as they all indicate the stirring process in some respect. Consequently, PCA is an appropriate tool to analyze the data base from these three types of signals. Neural network combined with fuzzy logic may also be an option to investigate this complex scenario, however, in this study, PCA has been identified as the most appropriate analysis tool.

\textbf{4.5 Summary}

The available techniques for multivariate analysis including neural networks, fuzzy logic and principal component analysis has been briefly reviewed and compared in this chapter. Both neural networks and fuzzy logic system can be listed as part of artificial intelligence. They attempt to simulate human operators during the control process and are suitable to analysis complex control problems; however, more efforts are required for their application in the scenario of bubble stirring at the bottom of the vessel. Principal component analysis is one of the important multivariate statistical analysis techniques, and it been proved to be a powerful tool to reduce the dimensions of variables and extract useful
information from a correlated data base, its calculation procedure, assumptions and limitations, other multivariate statistical techniques and extended PCA techniques have also been briefly reviewed.
CHAPTER 5
Research Issues

Bottom gas stirring is a common practice in metallurgical operations to achieve homogenization of temperature and chemical composition, and enhance chemical reactions. This process has been investigated extensively and there has been established understanding regarding the relationship between the gas stirring power and average mass transfer coefficient, which is strongly related with the rate of desulfurization and dephosphorization, and also related with the rate of inclusion removal. However, there is no reliable method to quantify the stirring power, particularly in the high temperature circumstances. The current control system is based on the measurement of gas line pressure and gas flow rate, however, the gas flow rate which is displayed by the volumetric meter can not indicate the gas which is exactly injected into the vessel due to various reasons such as leakage and partially plugged nozzle, nor can the gas line pressure due to the variation of the resistances inside the porous plug channel, which in turn leads to the gas line fluctuation. It is clear that the bubble flow which is induced by the gas injection at the bottom of the vessel causes the following effects: (1) image signal from the disturbed top surface, or more specifically, the variation of the spout eye size; (2) an acoustic signal from bubbling and (3) vibration signal on the wall of the vessel. The objective of this study is to quantify the complicated bubble flow phenomena using all the signals which can be reliably measured. Therefore, the key research issue of this study can be summarized into the following four questions.

1. The variation of the spout eye area size, and the sound of the bubbling are intuitively correlated with each other, for example, in the case of secondary steelmaking, an experienced operator based on these signals to make judgment of the stirring level. Vibrations on the wall of the vessel
are also caused by the same source, and have been applied in industry to indicate the stirring process. Can we rigorously prove that these three signals which are collected simultaneously, are correlated with each other through multivariate statistic approach? And if so, can they be simplified into one combined signal?

2. Is there any clear relationship between this combined signal and the stirring power?

3. The variation of the spout eye area size is an important effect of the stirring process, and the size of the spout eye area is crucial to the quality of the product. Current techniques take a relatively longer time to calculate the spout eye area size. Is it possible for us to develop a rapid image analysis technique which is suitable for online analysis?

4. The image of the disturbed top surface, the acoustic signal of the bubbling process and the vibration on the wall of the vessel can all be reliably quantified in the cold model scenario; however, it is difficult to collect all these signals simultaneously in industry. Is it possible to indicate the stirring process by just one or two channels of the signals? If then, what is the relationship between them and the stirring power?

Cold modeling experiments based on both dimensional and dynamical similarity criteria over a wide range of stirring conditions were performed in this study to investigate the effects of stirring process by five variables which were collected simultaneously. A rapid image analysis technique was developed in this study which could calculate the size of the spout eye area with a speed of about 0.1 second per frame in average, which can effectively quantify the dynamics of the disturbed top surface, industrial trials were performed to test the application of this technique in an industrial scenario. A multivariate statistical analysis approach, PCA (Principal Component
Analysis) was taken in this study to investigate the results of cold model experiments. In the following two chapters, the methodology of the cold modeling experiments and signal analysis will be described and the results are discussed in Chapter 8.
CHAPTER 6

Cold Model Experiments

Due to the harsh environmental conditions of the ladle metallurgical operations, cold models have been applied to study the various parameters which may affect the stirring conditions. In particular, the dynamic behavior of the spout eye area or the transient phenomena of the slag has been the topic of many researches\(^2, 10, 39, 42, 44-46, 189\). In this study, the bubbling phenomena was analyzed by the spout eye area size, sound of the bubbling and the vibration on the wall of the vessel which are all generated from the pressured gas stirring at the bottom of the vessel. These signals were collected simultaneously and pre-treated individually before they were organized into a state matrix to describe the stirring process. In this chapter, the cold modeling experiments and the methods of signal analysis are described.

6.1 Experimental Rig

A transparent cylindrical tank was selected as the geometry of the cold model vessel, and stirring was provided by compressed air from a nozzle at the centre of the bottom of the tank. Water was used to simulate liquid steel, and motor oil (SAE 20, density \(\rho=0.85 \times 10^3 \text{ kg/m}^3\) at 25°C) was used to simulate slag. The color of the oil was light brown and could be easily discriminated from the water by naked eyes.

The cylindrical geometry was selected for the following three reasons. Firstly, the bubbling phenomena can be easily observed and the signals can be reliably collected. For example, the disturbance at the top surface of the secondary layer (upper layer) or the spout eye area size can be captured by a camera installed
above the vessel, sound of the bubbling process can be picked up by a microphone; the vibration signal of the vessel can be collected by an accelerometer installed on the wall of the vessel. Secondly, this kind of geometry has been applied and investigated extensively by previous researchers [2, 3, 39, 44, 45, 190, 191], and the results can be easily compared. Thirdly, this geometry is very similar to the ladle furnace in the secondary steelmaking plant, thus it can be used as a cold model for the industrial process, allowing comparison of the cold model results with an actual industrial process data.

The cold model rig was based on 1/10 scale of a 200 ton steel ladle and the picture of the rig is shown in Fig 6.1.

![Picture of cold modeling experimental rig](image)

Fig 6.1 Picture of cold modeling experimental rig

The height and inner diameter of the cylindrical tank is 500 \textit{mm} and 420 \textit{mm} respectively, which is the same size of the cold model rig in McMaster University [2]. A square tank with a length of 500 \textit{mm} and height of 520 \textit{mm} was installed outside the cylindrical and shared the same base; it can help to prevent the noise from outside during the stirring process. A mirror was installed below the base of
the tank with an angle of $45^\circ$, which makes it easy to observe the bubble flow profile from the bottom. The nozzle was installed at the centre of the bottom and the internal diameter of the nozzle was fixed to $3.0\ mm$ during the cold modeling experiments.

### 6.2 Cold Model Experiments

A schematic diagram of the cold modeling apparatus is shown in Fig 6.2, and the experimental conditions were summarized in Table 6.1.

![Video camera](image)

Fig 6.2 Schematic diagram of cold modeling apparatus

<table>
<thead>
<tr>
<th>Fixed parameters</th>
<th></th>
<th>Varied parameters</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Water bath height</td>
<td>Compressed air pressure</td>
<td>Secondary layer (motor oil) thickness</td>
<td>Compressed Air flow rate</td>
</tr>
<tr>
<td>$210.0\ mm$</td>
<td>$200.0\kPa$</td>
<td>$5.0\ mm, 10.0\ mm, 15.0\ mm, 20.0\ mm$</td>
<td>$2.0 -20.0\ l/min, (2.0\ l/min$ interval)</td>
</tr>
<tr>
<td>Nozzle diameter</td>
<td>$3.0\ mm$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The height of the water inside the cylindrical tank was fixed to $210.0\ mm$, and the
thickness of the top motor oil layer varied from 5.0 \textit{mm} to 20.0 \textit{mm} at 5.0 \textit{mm} intervals. The pressure of the compressed air was fixed to 200.0 kPa at the exit of the compressed air tank, and the flow rate was controlled by a standard rotameter (error of 3%). The air flow rate values were selected based on the range of Froude number which is the ratio of inertia force and buoyancy force, and have been applied in previous studies\cite{2,39}, and covered all the flow regimes from low, medium and high gas flow rates. These Froude values were in the range of values relevant to industrial practices. As the compressed air was connected with a large tank, the air pressure was almost steady. The illumination was provided by a couple of 100 W tube light symmetrically installed at both sides of the tank, the distance between the light and the top surface of the vessel was approximately one meter. The average maximum light intensity at the top of the tank cover was 8653 lux and the average maximum light intensity at the secondary surface was 5935 lux. The average back ground noise level was 66.4 dB. The top cover of the cylindrical tank was covered by a black paper except the top surface of the secondary layer where the spout eye area exposed to the air. Such a procedure prevents the disturbance caused by the other lightening matters within the scope of the image during the calculation of the spout eye area, which was also confronted by Graham and his coworkers\cite{11,12}. The accelerometer (BK Type 4371) was installed on the outside of the wall of the vessel close to the level of water bath, and the vibration signals were observed by an oscilloscope (Tektronix TDS 2024), and were stored digitally by a portable CF (Compact Flash) card attached with the Oscilloscope.

Sound of the bubble flow and image of the disturbed top surface from above the vessel were collected by a common digital video camera (Canon MVX 330i) to assure synchronism, and the audio and video signals were separated by commercial software. The video was taken continuously, while the vibration signals were collected intermittently, with the time scales on the recorders, they
were edited to start at the same time, thus, the image, sound and vibration signals collected can be regarded as simultaneous.

The temperature of the bath was monitored by a digital thermometer and the temperature variance within one set of experiments (one top layer thickness and 10 flow rates range) was less than 0.8 °C, therefore, temperature was not regarded as a variable in this study.

The specification for the video, audio and vibration signals are shown in Table 6.2. The cold model data were categorized by the top layer thickness and flow rate conditions, *i.e.* for each top layer thickness, the gas flow rate varied from 2.0 l/min to 20.0 l/min, at 2.0 l/min intervals. At each stirring conditions, 3 samples were collected to obtain the average values. In total, 120 data sets (4 secondary layer depth × 10 flow rate stirring conditions × 3 samples) were collected.

Table 6.2 Specification of signals

<table>
<thead>
<tr>
<th>Image (each frame)</th>
<th>Audio signal</th>
<th>Video signal</th>
<th>Vibration signal</th>
</tr>
</thead>
<tbody>
<tr>
<td>Width 960 pixels</td>
<td>Sample size 16 bit</td>
<td>Sample size 24 bit</td>
<td>Sample rate 250 Hz</td>
</tr>
<tr>
<td>Height 720 pixels</td>
<td>Channels 1 (mono)</td>
<td>Frame rate 15 frames per second</td>
<td>mV/Unit out = 31.6</td>
</tr>
<tr>
<td>Horizontal resolution 96 dpi</td>
<td>Sample rate 44.1 kHz</td>
<td></td>
<td>Transducer sensitivity</td>
</tr>
<tr>
<td>Bit depth 24 bits</td>
<td></td>
<td></td>
<td>0.1-1 pC / m / s²</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>


6.3 Editing of Cold Model Data

Three digital video episodes (one-minute length for each episode) were selected from each sample according to the time scale of the vibration signals. The format of the video is *.wmv (Windows Media Audio/video file), it was converted into *.avi format by a free ware KMplayer[105] (version 2.9.4), and then separated into each individual frame by a commercial video transfer software (PFV version 2.4.3.8) according to the same frame rate at which they were taken, i.e. 15 frame per second. Each frame can be regarded as a separate image which carries the information about the size of the spout eye area.

The same digital video file was converted into *.mp3 format by a commercial software package WavePad[104] (master’s edition), and edited according to the time scale of the video frame and the vibration signals. The sampling rate for the audio was set to be 16 kHz, and the file was converted into *.wav format by Switch Sound Format Converter[192] (Plus edition), so that it may be read directly by Matlab programs.

The format of the vibration signal was *.csv, which can not be read directly by Matlab, more over, the information on the first 4 columns shows the conditions at which the signals were recorded, therefore, the files were edited individually and stored in the format of *.xls, which can be read directly by Matlab.

6.4 Image Analysis

It is widely accepted that the behavior of the slag surface is dynamic[3, 39, 41, 47, 193, 194], and the fluctuation of the spout eye area have been observed by previous researchers[2, 39]. Furthermore, the lateral movement of the spout are also found to be strongly fluctuated[41].
Image analysis is an important aspect of this project and a rapid method for analyzing the image should be established. In 2003, Multivariate Image Analysis (MIA) technique has been proposed by Subagyo and Brooks\textsuperscript{[10,15]}, however, the computational time is in the range of 30 seconds\textsuperscript{[10]}. Though the fluid slag and crusty slag can also be detected and quantified by their method, the time required to calculate the spout eye area size can not meet the requirement of online analysis. Various methods have been tried in this study to calculate the size of the spout eye area with the accuracy and speed being the two main criteria considered in selecting an appropriate method.

\subsection*{6.4.1 Physical Measurement}

Due to the dynamic behavior of the slag layer or the disturbed top surface of the cold model during stirring\textsuperscript{[2, 39, 41, 193]}, it is difficult to get an exact size of the spout eye area. Physical method has been approached by measuring the diameter of the spout eye area in different directions, and the average values were obtained.

A typical image of the cold modeling experiment is shown in Fig 6.3, and the diameters of the spout eye area were measured in six different directions. Several graphic edition softwares can provide such a platform, \textit{e.g.} Corel Painter\textsuperscript{[87]}, Matlab image processing toolbox, and GIMP\textsuperscript{[195]} (Version 2.4.7), etc. The unit can be in the form of mm, inch, and pixel. 10 images were randomly selected at each stirring condition and they were physically measured in this way. Totally, $5 \times 10 \times 10$ images were measured and the average spout eye area size is summarized in Table 6.3. The standard deviations calculated for the physical measurement is shown in Table 6.4. The spout eye areas were calculated from the mean of the measured diameters.
Fig 6.3 Physical measurement of spout eye area \((h=15\ mm, Q=\ 2.0\ l/min)\)

Table 6.3 Results of average spout eye area size measured by physical methods \((cm^2)\)

<table>
<thead>
<tr>
<th></th>
<th>(h = 5\ mm)</th>
<th>(h = 10\ mm)</th>
<th>(h = 15\ mm)</th>
<th>(h = 20\ mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Q=02\ l/min)</td>
<td>278.22</td>
<td>177.76</td>
<td>160.02</td>
<td>94.81</td>
</tr>
<tr>
<td>(Q=04\ l/min)</td>
<td>494.91</td>
<td>296.07</td>
<td>216.73</td>
<td>158.08</td>
</tr>
<tr>
<td>(Q=06\ l/min)</td>
<td>637.71</td>
<td>397.73</td>
<td>305.41</td>
<td>221.86</td>
</tr>
<tr>
<td>(Q=08\ l/min)</td>
<td>708.17</td>
<td>434.74</td>
<td>325.14</td>
<td>243.96</td>
</tr>
<tr>
<td>(Q=10\ l/min)</td>
<td>764.79</td>
<td>491.56</td>
<td>410.91</td>
<td>282.42</td>
</tr>
<tr>
<td>(Q=12\ l/min)</td>
<td>808.57</td>
<td>553.35</td>
<td>444.40</td>
<td>328.37</td>
</tr>
<tr>
<td>(Q=14\ l/min)</td>
<td>834.83</td>
<td>610.71</td>
<td>494.27</td>
<td>381.88</td>
</tr>
<tr>
<td>(Q=16\ l/min)</td>
<td>849.55</td>
<td>605.10</td>
<td>519.22</td>
<td>427.68</td>
</tr>
<tr>
<td>(Q=18\ l/min)</td>
<td>861.83</td>
<td>645.26</td>
<td>566.46</td>
<td>527.25</td>
</tr>
<tr>
<td>(Q=20\ l/min)</td>
<td>953.49</td>
<td>708.43</td>
<td>651.61</td>
<td>559.90</td>
</tr>
</tbody>
</table>

Table 6.4 Standard deviations calculated for physical measurement \((cm^2)\)

<table>
<thead>
<tr>
<th></th>
<th>(h = 5\ mm)</th>
<th>(h = 10\ mm)</th>
<th>(h = 15\ mm)</th>
<th>(h = 20\ mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Q=02\ l/min)</td>
<td>17.59</td>
<td>21.32</td>
<td>103.16</td>
<td>23.82</td>
</tr>
<tr>
<td>(Q=04\ l/min)</td>
<td>25.76</td>
<td>54.56</td>
<td>24.23</td>
<td>9.42</td>
</tr>
<tr>
<td>(Q=06\ l/min)</td>
<td>25.89</td>
<td>63.84</td>
<td>31.85</td>
<td>17.72</td>
</tr>
<tr>
<td>(Q=08\ l/min)</td>
<td>35.45</td>
<td>34.62</td>
<td>37.80</td>
<td>19.94</td>
</tr>
<tr>
<td>(Q=10\ l/min)</td>
<td>24.51</td>
<td>53.59</td>
<td>34.20</td>
<td>37.11</td>
</tr>
<tr>
<td>(Q=12\ l/min)</td>
<td>41.13</td>
<td>43.34</td>
<td>39.19</td>
<td>39.47</td>
</tr>
<tr>
<td>(Q=14\ l/min)</td>
<td>38.63</td>
<td>34.06</td>
<td>39.05</td>
<td>45.70</td>
</tr>
<tr>
<td>(Q=16\ l/min)</td>
<td>28.66</td>
<td>38.91</td>
<td>28.11</td>
<td>37.11</td>
</tr>
<tr>
<td>(Q=18\ l/min)</td>
<td>34.48</td>
<td>46.80</td>
<td>32.54</td>
<td>46.25</td>
</tr>
<tr>
<td>(Q=20\ l/min)</td>
<td>91.95</td>
<td>31.57</td>
<td>60.24</td>
<td>58.16</td>
</tr>
</tbody>
</table>
6.4.2 Threshold Technique

Various methods for calculation of the spout eye size were tried to meet the requirements of “accuracy” and “speed”, such as editing on image with and without morphological operations, using the original image with and without morphological operations and various threshold techniques. Due to the fact that the acrylic cold model vessel reflects light and results in pixels outside of the ladle having similar color intensity values as those within the ladle eye, a problem also confronted by Graham and his coworkers\[11\], the images were edited by trimming the boundary area out of the cylindrical intersection. The “accuracy” was increased, however, a large amount of human operations were involved, thus it can not meet the requirement of “speed”. Morphological operations were performed to reduce the noise of the images; however, the increase in “accuracy” was not significant. A threshold technique was developed which could meet the requirement of both “accuracy” and “speed”\[196\]. It separates the circular intersection and the spout eye area from the rectangular background and top layer by two thresholds and the average time required to treat one frame of image is 0.1 second. The detailed description of the technique is attached as Appendix 2.

6.4.3 Initial Results of Image Analysis

All the images (frames) of the cold model experiments were analyzed by the above described threshold technique, and the size of the spout eye area was represented by a dimensionless number \( A^* \) or relative spout eye area, which is defined as:

\[
A^* = \frac{A_e}{A_0} \quad (6-1)
\]
where \( A_0 \) is the intersection area of the tank, and \( A_e \) is the physical spout eye area. Fig 6.4 gives an example which shows the result of image analysis on a series of video frames, the upper layer oil depth \( h \) is 20 mm, and the gas flow rate \( Q \) is 4.0 \( l/min \).

The average relative spout eye area of 120 frames was analyzed for each stirring condition, and the results are shown in Fig 6.5. It demonstrates that the spout eye area increases with gas flow rate and decrease as the upper layer depth increase, which is in agreement with previous investigations\(^{[2,11,12]}\).

![Fig 6.4 Image analysis results of frames from a continuous video (\( h=20mm, Q=4.0l/min \))](image-url)
Fig 6.5 Relative spout eye area for various stirring conditions

Since the spout eye area is a dynamic or stochastic behavior\cite{2, 39, 41, 193}, and the cumulative average stabilized after a certain number of readings. In order to get a meaningful statistic understanding of the spout eye area size, a series of consecutive video frames were analyzed by threshold techniques. Fig 6.6 shows an example of the cumulative average and standard deviation of a certain stirring condition, in which, the depth of the upper layer phase is 20\text{mm} and the flow rate is 16.0\text{l/min}.

Fig 6.6 Cumulative average of relative spout eye area (h=20\text{mm}, Q=16.0\text{l/min})
From Fig 6.6, it can be seen that the relative spout eye area stabilized after approximately 60 frames, which was taken within 4.0 seconds. The same effects were found for the other stirring conditions. Therefore, 4.0 seconds was determined as the minimum time required for collecting the images.

6.5 Sound Analysis

Sound signals in the format of *.wav can be read directly by Matlab and a typical sound signal (time domain) is shown in Fig 6.7, the corresponding spectrum which was obtained by FFT approach is shown in Fig 6.8. As described in the literature review in section 3.2, the sound signal may reflect the related operating conditions, and they can be analyzed in both time domain and frequency domain.

![Sound intensity graph](image1)

Fig 6.7 A typical sound intensity ($h=5 \text{ mm}, Q=8.0 \text{ l/min}$)

![Spectrum analysis graph](image2)

Fig 6.8 Spectrum analysis of acoustic signal ($h=5 \text{ mm}, Q=8.0 \text{ l/min}$)
6.5.1 Sound Intensity Analysis

Various methods have been tried to investigate the sound intensity signals, such as selecting the intensity at certain fixed time steps, averaging the sound intensity at different time steps, and averaging the absolute value of the sound intensity at fixed time steps. It was found that the average sound intensities (absolute value) increase with the gas flow rates and varies with different upper layer depths, which could be used to indicate the stirring conditions. The results of average sound intensities within 4.0 seconds at different stirring conditions are shown in Fig 6.9. Though the sound intensity at different upper layer thickness (for the same gas flow rate) was not significant, particularly when the upper layer thickness \( h \) changes from 5 \( \text{mm} \) to 15 \( \text{mm} \), however, there was a significant difference while the upper layer thickness was 20 \( \text{mm} \). This has not been investigated further in this study.

![Fig 6.9 Average sound intensities at different stirring conditions](image-url)
6.5.2 Sound Amplitudes at Certain Frequencies

Sound signal can also be analyzed by its spectrum. According to Fig 6.8, the dominant frequency range falls in 100 Hz to 1500 Hz, and the lower frequency spectrum of the same signal are shown in Fig 6.10 and Fig 6.11 respectively.

![Fig 6.10 Spectrum analysis of acoustic signal (h=5 mm, Q=8.0 l/min)](image)

![Fig 6.11 Spectrum analysis of acoustic signal (h=5 mm, Q=8.0 l/min)](image)

The peaks at different frequency range might indicate different stages of the bubble, such as bubble formation, bubble volume oscillation, bubble detachment, shape distortion and bubble coalescence *etc.* which have been described in
section 2.5. However, the amplitude summation over the range of 100 Hz to 1500 Hz was found to be able to indicate the corresponding operating conditions, i.e. the different depth of the upper layer and the pressured gas flow rate. The summation of amplitude over the range of 100 Hz to 1500 Hz is shown in Fig 6.12. The amplitudes out of the range were assumed as noise. Like the sound intensity analysis, the differences of the amplitude summation for the upper layer depth from 5 mm to 15 mm at the same gas flow rate were not significant; however, there was a distinctive difference between the amplitude corresponding to the same gas flow rate while the upper layer was 20 mm.

![Fig 6.12 Amplitude summation of the sound signal within the range of 100 Hz to 1500 Hz](image)

### 6.5.3 Initial Results of Sound Analysis

The average sound intensity and summation of sound amplitude over the range of 100 Hz to 1500 Hz can indicate the stirring process to some extent and the sampling period was fixed to 4.0 seconds. Even though there is no significant difference for the summation at different upper layer depths, for example, when
the upper layer depths were 5 mm to 15 mm, the variation of the summation for both sound intensity and amplitude over the range of 100 Hz to 1500 Hz is not significant. However, the variation becomes large when the upper layer depth increased to 20 mm. Both the average sound intensity and summation of sound amplitude over the ranges of 100 Hz to 1500 Hz increase with the gas flow rate.

6.6 Vibration Analysis

Vibration signals were measured by the accelerometer and recorded in a digital form by a portable CF card in a specific format. The vibration signals can also be processed in both time domain and frequency domain. According to the manual of the accelerometer and the experiment set up\cite{197}, the acceleration value can be calculated by the following equation:

\[
A_{vz} (m/s^2) = \frac{V_{\text{amplitude}}}{mV/\text{Unit}} = \frac{V_{\text{amplitude}}}{31.6 \times 10^{-3}}
\]  

(6-2)

The data set on the original file was in the unit of voltage, and should be converted into the unit of acceleration. A typical vibration signal is shown in Fig 6.13.

Fig 6.13 Vibration signal (h=5 mm, Q=16.0 l/min)
6.6.1 Vibration Magnitude Analysis

Various methods were tried to indicate the stirring process by the vibration magnitude, it as found that the average of the absolute value of the magnitude could be used to characterize the operating parameters, such as upper layer depths and gas flow rates. The average value of the absolute magnitude at different stirring conditions is shown in Fig 6.14.

![Average vibration magnitudes at different stirring conditions](image)

**Fig 6.14** Average vibration magnitudes at different stirring conditions

6.6.2 Vibration Spectrum Analysis

The detailed analysis of the spectrum of the vibration signal shows that the dominant frequencies of the vibration signal varies with different stirring conditions, and the spectrum diagrams of the vibration signals can be roughly categorized into two types.

The first type of the spectrum diagram shows a dominant peak range, corresponding to the dominant frequencies. For example, when the upper layer...
depth \((h)\) is 10 mm, gas flow rate \((Q)\) is 2.0 l/min, which is shown in Fig 6.15, the dominant frequency lies in approximately 80 Hz.

For the second type of vibration spectrum, there are no clear dominant peaks, however, the peaks are widely distributed within a large range, for example, when the upper layer depth \((h)\) is 5 mm, and the gas flow rate \((Q)\) is 12.0 l/min, which is shown in Fig 6.16. It is difficult to select the dominant peak and find the corresponding dominant frequency range. Table 6.5 shows the detail of first and second dominant frequency range of the vibration signals at different stirring conditions. Table 6.6 summarizes the content of Table 6.5.
Table 6.5 Distribution of dominant frequency ranges (Hz)

<table>
<thead>
<tr>
<th>H=05 mm</th>
<th>Q(l/min)</th>
<th>2.0</th>
<th>4.0</th>
<th>6.0</th>
<th>8.0</th>
<th>10.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>1&lt;sup&gt;st&lt;/sup&gt; peaks</td>
<td>90-110</td>
<td>60-70</td>
<td>70-80</td>
<td>40-50</td>
<td>40-50</td>
<td></td>
</tr>
<tr>
<td>2&lt;sup&gt;nd&lt;/sup&gt; peaks</td>
<td>40-50</td>
<td>40-50</td>
<td>90-100</td>
<td>60-70</td>
<td>60-100</td>
<td></td>
</tr>
<tr>
<td>H=10 mm</td>
<td>Q(l/min)</td>
<td>12.0</td>
<td>14.0</td>
<td>16.0</td>
<td>18.0</td>
<td>20.0</td>
</tr>
<tr>
<td>1&lt;sup&gt;st&lt;/sup&gt; peaks</td>
<td>40-50</td>
<td>40-50</td>
<td>60-70</td>
<td>60-70</td>
<td>60-70</td>
<td></td>
</tr>
<tr>
<td>2&lt;sup&gt;nd&lt;/sup&gt; peaks</td>
<td>60-90</td>
<td>60-94</td>
<td>40-90</td>
<td>40-90</td>
<td>40-100</td>
<td></td>
</tr>
<tr>
<td>H=15 mm</td>
<td>Q(l/min)</td>
<td>12.0</td>
<td>14.0</td>
<td>16.0</td>
<td>18.0</td>
<td>20.0</td>
</tr>
<tr>
<td>1&lt;sup&gt;st&lt;/sup&gt; peaks</td>
<td>70-80</td>
<td>60-70</td>
<td>40-50</td>
<td>40-50</td>
<td>60-70</td>
<td></td>
</tr>
<tr>
<td>2&lt;sup&gt;nd&lt;/sup&gt; peaks</td>
<td>40-110</td>
<td>40-110</td>
<td>60-80</td>
<td>38-100</td>
<td>40-110</td>
<td></td>
</tr>
<tr>
<td>H=20 mm</td>
<td>Q(l/min)</td>
<td>12.0</td>
<td>14.0</td>
<td>16.0</td>
<td>18.0</td>
<td>20.0</td>
</tr>
<tr>
<td>1&lt;sup&gt;st&lt;/sup&gt; peaks</td>
<td>60-68</td>
<td>40-50</td>
<td>42-48</td>
<td>40-48</td>
<td>56-64</td>
<td></td>
</tr>
<tr>
<td>2&lt;sup&gt;nd&lt;/sup&gt; peaks</td>
<td>86-100</td>
<td>58-78</td>
<td>60-98</td>
<td>60-88</td>
<td>38-48</td>
<td></td>
</tr>
<tr>
<td>Q(l/min)</td>
<td>12.0</td>
<td>14.0</td>
<td>16.0</td>
<td>18.0</td>
<td>20.0</td>
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<tr>
<td>1&lt;sup&gt;st&lt;/sup&gt; peaks</td>
<td>40-50</td>
<td>76-88</td>
<td>40-50</td>
<td>40-50</td>
<td>50-60</td>
<td></td>
</tr>
<tr>
<td>2&lt;sup&gt;nd&lt;/sup&gt; peaks</td>
<td>60-88</td>
<td>10-20</td>
<td>70-80</td>
<td>58-78</td>
<td>70-80</td>
<td></td>
</tr>
</tbody>
</table>

Table 6.6 Summary of dominant frequency range

<table>
<thead>
<tr>
<th>Frequency range (Hz)</th>
<th>Number of cases</th>
<th>Percentage (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>40-50</td>
<td>17</td>
<td>42.5</td>
</tr>
<tr>
<td>60-70</td>
<td>7</td>
<td>22.5</td>
</tr>
<tr>
<td>0-120</td>
<td>40</td>
<td>100</td>
</tr>
</tbody>
</table>
The summation of the amplitudes within the range of 40-50 Hz, 60-70 Hz, and the amplitudes of the full ranges (0-120Hz) were compared and it was found that the summation of the amplitudes at full range gives more distinction between the various stirring conditions, and the pretreatment of the vibration amplitudes is fixed to the summation of amplitudes at all frequency ranges. It should be noted that only the horizontal vibration was considered in this study, the vibration components in the other two directions (transverse, and vertical) should be further investigated.

![Graph showing summation of amplitudes](image)

Fig 6.17 Summation of the amplitudes within the range of 1-120Hz for all stirring conditions

**6.6.3 Initial Results of Vibration Analysis**

According to the previous investigation, it was found that the average vibration magnitude increase with gas flow rate and decrease with the thickness of the upper layer. The summation of the amplitude at the frequency range from 1-120 Hz increases with the gas flow rate and the depth of the upper layer.
6.7 Summary

This chapter described the cold modeling experiments, including the cold model rig, experimental conditions and data collection procedures. The techniques for analyzing the different signals and their initial results have been described. The initial results showed that the average relative spout eye area, the average sound intensity, the summation of sound amplitude over the range of 100 Hz to 1500 Hz, the average magnitude of the vibration and the summation of amplitude over the range of 1-120 Hz show distinct variations at different stirring conditions, and they can be used to quantify the stirring effect indirectly.
CHAPTER 7

Signal Analysis

Bubbling phenomena in metallurgical operations have been studied extensively and these studies have shown that it is highly unlikely that the details of the bubbling process can ever be represented by a simple physical law or linear function of some operating parameters. For example, from a critical review of the existing semi-empirical correlations between the non-dimensional spout eye area and the Froude numbers, which is attached as Appendix 1, none of the correlations can successfully predict the spout eye area for the full range of stirring conditions\textsuperscript{[48]}. This, in part, reflects the difficulty in obtaining accurate relationship between the spout eye area size and the operating parameters because of the unreliable measurement of the metal depth, slag depth and operating parameters such as gas flow rate in industry. Previous studies have shown that the bubble flow in metallurgical operations can be monitored by the following three channels of signals indirectly: 1) image signals from the disturbed top surface, 2) acoustic signals of the bubbling and 3) vibration signals on the wall of the vessel.

Cold modeling data regarding the three channels of signals in all stirring conditions (Bottom gas injection rate from 2.0 \textit{l/min} to 20.0 \textit{l/min}) were collected simultaneously and pre-treated individually, which has been described in the previous chapter. There are several available techniques to analyze this large data set, such as neural network, fuzzy logic and multivariate statistical analysis, which were reviewed and compared in Chapter 4. PCA was selected as a suitable tool to investigate the combined effect of all the three channels of signals due to its strength in extracting useful information from large database and simple mathematic operations, which make it suitable for online analysis, and also
because it has been proved to be successful in industrial applications\textsuperscript{[181-188]}.

In this chapter, the three channels of signal which had been collected simultaneously during our cold modeling experiments are analyzed through the PCA approach. The pre-treatment of the signals are introduced and the results of PCA are presented, the cold model data were analyzed from various perspectives such as the influence of sampling period, the effect of removing one channel of signal and indicating the stirring process by just one channel of signal were investigated. Relationships between the dominant principal component and Froude number are studied and two approaches of analyzing sound and vibration signals are compared.

### 7.1 Analysis Strategy

Since the image of the disturbed top surface (size of the spout eye area), sound of the bubbling and vibration on the wall of the vessel are all generated from the same stirring process; each of them indicates the stirring process in a particular aspect, and they should be related with each other. Therefore, the principal component analysis of these signals may provide simplified information which shows the characteristics of the process. Fig 7.1 gives a schematic diagram of the analysis strategy used in this study.

During the cold modeling process, image of disturbed top surface, or the size of spout eye area, sound of the stirring process the vibration on the wall of the cylindrical tank were all collected at the same time. They can be regarded as measuring the process in three different dimensions, and the average of measured values of the same time period can be regarded as the records of these dimensions at the same time step. The collection of these signals was described in detail in Chapter 6. The signals from these three dimensions can be expressed by the following five variables, and the detailed definition of the variables will be explained later in section 7.3.1.
- Relative spout eye area
- Sound intensity
- Sound spectrum
- Vibration magnitude
- Vibration spectrum

![Diagram of the process](image)

Fig 7.1 Schematic diagram of the process

The state matrix which is the record of the stirring process for all the stirring conditions was organized by the values of the above 5 variables at the same time steps. This means, at a certain moment, the state of the bubble flow can be described by the values of all these five variables averaged for one specific period. As they were all applied to represent the same process, they were intuitively correlated closely, and a simplified signal can be expected to be extracted from them. This simplified signal can be regarded as the combined “measurement” of the stirring process and may be obliged to act as a feedback signal for the control system. A program written in MATLAB was developed to analyze the signals of image, sound and vibration, and the flow chart of the program is shown in Fig 7.2. The code and the results of the program are provided in Appendix 3.

The raw data from the cold modeling experiments were pre-treated before they
were organized into the state matrix. Pre-treatment of the data was necessary, because firstly, the data carries different types of information. For example, the relative spout eye area indicates the dynamics of the disturbed surface, the sound indicates the sound intensities and also different spectrum of the bubbling, and the vibration indicates the effect of the bubbling process on the wall of the vessel. Secondly, the values of the raw data are not in the same units. The unit for relative spout eye area is percentage; the unit for sound intensity is w/m^2, and the spectrum is dB at different frequencies (HZ); the unit for vibration magnitude is m/s^2, and the unit for vibration wave spectrum is the same as sound spectrum, however, they are at different frequency ranges. Thirdly, even though the three channels of signals were collected simultaneously, however, they were collected with different sampling rates due different instrumentation and different properties of the signals. For example, the sampling rate for image was 15 frame/s; for vibration was 250 Hz, and for sound, the sampling rate was 16 kHz. They should be averaged with the same time period before they were paralleled into the state matrix.

After pre-treatment, the values of the five variables at different time steps were organized in each column; and each row gives the value of the five variables averaged at that same time period for each stirring conditions. The dimension of this state matrix is 40×5, and it was analyzed by the standard procedures of PCA summarized by Smith[^198], which was reviewed in Chapter 4. As the decomposition of the covariance matrix was listed in the ascending order, they were rearranged in the descending order according to the absolute value of the eigenvalues, which indicates the degree of variations of the variables. The program was validated by tracing back the original data through the inverse operations of PCA, and by comparing the difference between the derived data sets and the original ones. If the difference is undetectable, it means that the results are reliable. However, if the difference is significant, that means the data
have not been processed successfully.

Fig 7.2 Flow chart of the PCA program

The program was also validated by various matrixes (such as a random matrix) and was proved to be successful in implementing the PCA procedure described in section 4.3.1.2. The detailed validation of the program is described in Appendix 4.

7.2 Pre-Treatment of Signals

7.2.1 Pre-treatment of Image

It is widely accepted that the spout eye area is a dynamic or stochastic behavior[12, 39, 41, 193], and the cumulative average stabilized after a certain number of readings.
From the initial results on image analysis in section 6.4.3, it revealed that 60 frames (images) are required to get a meaningful statistical understanding of the spout eye area size, which was taken within 4.0 seconds. Therefore, 4.0 seconds were determined as the sampling period in this study. Vibration and sound signals were also sampled within 4.0 seconds. The image signal was pre-treated by averaging the relative spout eye area size within 4.0 seconds for each stirring condition.

7.2.2 Pre-treatment of Sound

The sound signals were investigated in section 6.5 and the initial results showed that the average sound intensity and summation of sound amplitude over the range of 100 Hz to 1500 Hz can indicate the stirring process to some extent. The sound signals were pre-treated by both time domain and frequency domain. In the time domain, the absolute sound intensity was averaged over the period of 4.0 seconds for each stirring condition. As for the frequency domain, the 4.0 seconds period signal was transformed by FFT (Fast Fourier Transform) and the spectrums of each stirring condition were obtained. The summation of the amplitude over the range of 100 Hz to 1500 Hz for each stirring condition was calculated to indicate the particular circumstance.

7.2.3 Pre-treatment of Vibration

Vibration signals were investigated in Section 6.6 and they were pre-treated in time domain and frequency domain respectively. In the time domain, the absolute magnitude in each stirring condition was averaged over the period of 4.0 seconds. While in the frequency domain, 4.0 period vibration signals at each stirring conditions were transformed by FFT(Fast Fourier Transform) and the
corresponding spectrum were obtained. The summation of the amplitude over the range of 1-120 Hz was applied to indicate the stirring process at the corresponding moment.

7.3 Principal Component Analysis (PCA) of Image, Sound and Vibration

7.3.1 Definition of variables

The three types of signals can be described by five variables which were briefly introduced in section 7.1. The definitions of these five variables are explained below.

Average relative spout eye area (or “a” in the Matlab program)

The relative spout eye area is defined by equation (6-1). It is defined by the average value of the relative spout eye area of 60 consecutive frames (collected within 4.0 seconds period).

\[ a = \frac{1}{60} \sum_{i=1}^{60} A_i \]  

Where, \( A_i \) -relative spout eye area at \( t_i \) instant

Average sound intensity (or “s” in the Matlab program)

Average sound intensity is defined by the average absolute value of sound intensities collected at a sampling rate of 16 kHz within 4.0 seconds, which was the average of 4×16,000 records of sound signals in time domain.

\[ s = \frac{1}{4 \times 16000} \sum_{i=1}^{4 \times 16000} |s| \]  

(7-2)
Where, $s_i$ is the sound intensity collected at a sampling rate of 16 kHz, W/m$^2$

**Summation of sound amplitude (or “$sf$” in the Matlab program)**

Summation of sound amplitudes is defined by equation (7-3). It was the summation of amplitudes corresponding to the frequency ranges from 100-1500 Hz within a period of 4.0 seconds, which was the sum of 1400 records.

$$sf = \sum_{i=100}^{1500} l_i$$  \hspace{1cm} (7-3)

where, $l_i$ is the amplitude of the spectrum of sound wave within 4.0 seconds corresponding to each integer Hz from 100 to 1500.

**Average vibration magnitude (or “$v$” in the Matlab program)**

Average vibration magnitude is defined by the average absolute value of the vibration magnitude collected at a sampling rate of 250 Hz within 4.0 seconds, which was the average of 4×250 records of vibration signals in time domain.

$$v = \frac{1}{4 \times 250} \sum_{i=1}^{4 \times 250} |v_i|$$  \hspace{1cm} (7-4)

Where, $v_i$ -acceleration collected on the wall of the vessel at a sampling rate of 250 Hz, m/s$^2$

**Summation of vibration amplitude (or “$vf$” in the Matlab program)**

Summation of vibration amplitude is defined by equation (7-5). It was the summation of vibration amplitudes corresponding to the frequency ranges from 1-120 Hz within a period of 4.0 seconds, which was the sum of 120 records.

$$vf = \sum_{i=1}^{120} l_i$$  \hspace{1cm} (7-5)
where, \( l_i \) is the amplitude of the spectrum of vibration wave within 4.0 seconds corresponding to each integer \( Hz \) from 1 to 120.

It should be stated that any linear operations on the above definitions have no influence on the results of PCA on the cold model data set due to the properties of PCA\(^{[167]}\), which were reviewed in Chapter 4.

The relationships between the signals and 5 variables and their description are summarized in Table 7.1, and the state matrix was organized by the values of the five variables at all the 40 states, which included four different secondary layer depths or slag heights at 10 gas flow rates respectively. In summary, the state matrix (Matrix “MI” in Matlab program attached in Appendix 3) covered all the data base of our cold modeling experiments, the dimensions of the state matrix is \( 40 \times 5 \), which described the values of the five variables over 40 stirring conditions.

Table 7.1 Summary of signals and variables in the state matrix

<table>
<thead>
<tr>
<th>Signals</th>
<th>Variable</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Image</strong></td>
<td>( a )</td>
<td>Average of relative spout eye area of 60 consecutive frames</td>
</tr>
<tr>
<td><strong>Sound</strong></td>
<td>( s )</td>
<td>Average of absolute sound intensity of 4.0 seconds (( 4 \times 16 ) k values)</td>
</tr>
<tr>
<td></td>
<td>( sf )</td>
<td>Summation of sound amplitude of frequency range 0.1-1.5 kHz</td>
</tr>
<tr>
<td><strong>Vibration</strong></td>
<td>( v )</td>
<td>Average absolute magnitude of 4.0 seconds (( 4 \times 250 ) values)</td>
</tr>
<tr>
<td></td>
<td>( vf )</td>
<td>Summation of absolute amplitude of frequency range 1-120Hz</td>
</tr>
</tbody>
</table>

7.3.2 PCA Results

The flow chart of the PCA program is illustrated in Fig 7.2, and the program code based on Matlab is provided in Appendix 3. Assuming that all the five variables have the same significant level and they are equally important; the variables were
normalized by each column and organized into the state matrix, which was applied to indicate the bubble flow at different stirring conditions. Performing PCA to the state matrix (Matrix “MI” in Matlab program), which carried all of the information of the cold modeling experiments and recorded by the five variables described in the previous sections. The covariance matrix which shows the correlation between the variables is shown below:

\[
\begin{bmatrix}
0.0020 & 0.0026 & 0.0016 & 0.0023 & 0.0023 \\
0.0026 & 0.0053 & 0.0027 & 0.0049 & 0.0035 \\
0.0016 & 0.0027 & 0.0021 & 0.0024 & 0.0027 \\
0.0023 & 0.0049 & 0.0024 & 0.0045 & 0.0030 \\
0.0023 & 0.0035 & 0.0027 & 0.0030 & 0.0041 \\
\end{bmatrix}
\]

The relationship between the state matrix and its components can be represented by the following equation\cite{168}:

\[
MI = \sum_{i} t_{i} p_{i}^{T} + E = TP^{T} + E
\]

(7-6)

Where, \(MI\) is the state matrix, \(T\) is score matrix, \(P\) is loading matrix, and \(E\) is residual matrix.

Analyzing the principal component of the state matrix according to the standard procedure of PCA\cite{167, 199}, the obtained loading vector, \(p_{i}\) and eigenvalues are presented in Table 7.2. As shown in the table, the cumulative total variance of the first two principal components is 95.78% (85.92% and 9.86% respectively). Therefore, it is reasonable to assume that the majority of information of the state matrix (or, the stirring process) is retained in the first two principal components. The relationship between the first two principal components (PC1# against PC2#) is shown in Fig 7.3. No clear relationship could be found between the first two principal components, as would be expected by the analysis, because the two “latent” variables are uncorrelated with each other\cite{167}. Furthermore, the first principal component (“PC1#” or the dominant principal component) explained
almost 86% of the variation in the state matrix, it is also reasonable to assume that PC1 can represent the majority of the information of the state matrix.

![PCA analysis of image, sound and vibration signals](image)

**Fig 7.3 Result of PC1 against PC 2**

**Table 7.2 Loading vectors and eigenvalues of the cold modeling experiments**

<table>
<thead>
<tr>
<th>Score</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Score</td>
<td>0.312</td>
<td>-0.147</td>
<td>-0.935</td>
<td>0.080</td>
<td>-0.030</td>
</tr>
<tr>
<td>Score</td>
<td>0.568</td>
<td>0.418</td>
<td>0.124</td>
<td>-0.239</td>
<td>-0.656</td>
</tr>
<tr>
<td>Loading vector</td>
<td>0.333</td>
<td>-0.320</td>
<td>0.239</td>
<td>0.836</td>
<td>-0.175</td>
</tr>
<tr>
<td>Loading vector</td>
<td>0.514</td>
<td>0.455</td>
<td>0.086</td>
<td>0.095</td>
<td>0.716</td>
</tr>
<tr>
<td>Loading vector</td>
<td>0.453</td>
<td>-0.703</td>
<td>0.216</td>
<td>-0.478</td>
<td>0.159</td>
</tr>
<tr>
<td>Eigenvalue</td>
<td>0.01552</td>
<td>0.00178</td>
<td>0.000502</td>
<td>0.00021</td>
<td>0.00002</td>
</tr>
<tr>
<td>Total variance (%)</td>
<td>85.92</td>
<td>9.86</td>
<td>2.95</td>
<td>1.15</td>
<td>0.11</td>
</tr>
</tbody>
</table>

**7.3.3 Relationship between PC 1 and Stirring Power**

Our main concern is to identify or quantify the actual stirring effect by measuring related variables which reflect the stirring effect in different aspects, such as the disturbed top surface, sound of the bubbling and vibration on the wall of the vessel. It has been illustrated in section 2.7 that the gas flow rate, which is indicated by the rotameter and pressure gauges, can not describe the exact
stirring process due to various reasons such as leakage and erosion. The relationship between this combined signal and stirring power was investigated. Regarding to the definition of stirring power, the following equation was applied.

$$\varepsilon = 14.23 \left( \frac{\dot{V} T}{M} \right) \log \left( \frac{1 + H}{1.48 P_0} \right)$$

(7-7)

Where, $\varepsilon$ -stirring power (W/ton), $\dot{V}$ -gas flow rate (Nm$^3$/min)

T -bath temperature (K), M -bath weight (ton)

H -depth of gas injection (m)

$P_0$ -gas pressure at the bath surface (atm)

The relationship between the dominant principal component (―latent variable‖ or PC1) and stirring power, which was calculated from equation (7-7) according to our cold model data, is plotted in Fig 7.4.

Fig 7.4 Relationship between PC1 and stirring power
From Fig 7.4, it is clear that there is a strong linear relationship ($R^2=0.96$) between the dominant principal component and stirring power, and the relationship between the stirring power and the variables defined in this study can be de correlated by the following equations.

$$PC1=0.312a + 0.568s + 0.333v + 0.514sf + 0.454vf$$  \(7-8\)

$$\varepsilon = -0.57PC1 + 0.65$$  \(7-9\)

Therefore:

$$\varepsilon = c_0 + c_1a + c_2s + c_3v + c_4sf + c_5vf$$  \(7-10\)

In which, $c_0=0.65$, $c_1=-0.178$, $c_2=-0.324$, $c_3=-0.190$, $c_4=-0.293$, $c_5=-0.258$

Numerous experimental investigations on slag-metal gas reactions in gas stirred ladle system under a variety of experimental conditions have been performed, and it is well recognized that for most slag metal reactions the rates are controlled primarily by mass transfer of the reactants and the products across the slag-metal interface\[201\]. However, the slag-metal interfacial area is affected by the degree of agitation in a bath which, in turn, is determined by the stirring power. There has been established knowledge regarding the relationship between the desulfurization rate, dephosphorization rate, inclusion removal rate and stirring power\[200\]. Therefore, these aspects of the process can be predicted by the stirring power, which can also be quantified by this latent variable derived by all the three channels of signals.

### 7.4 Influence of Sampling Period

According to section 6.4.3 and 7.2.1, 60 frames were collected to get a meaningful idea on the size of the spout eye area, which lead to the determination
of the sampling period as 4.0 seconds. In this section, the same cold modeling data was investigated by different sampling period; and the dominant principle component which picked up a certain percentage of the total variations were compared. The relationship between the dominant principal component and the stirring power were also investigated in order to establish whether the dominant principal component can indicate the total variation of the stirring process clearly.

### 7.4.1 Sampling Period Set to Be 5.0 Seconds

When the sampling period was set to be 5.0 seconds, more information about the disturbed top surface, longer time of bubbling sound and vibration signals were pre-treated and organized into the state matrix.

Applying PCA to the new state matrix, the loading vector, eigenvalues and the variances picked up by each component were shown in Table 7.3. The relationship between the stirring power and the dominant principal component are plotted in Fig 7.5.

Table 7.3 Loading vectors and eigenvalues of the cold model data (Sampling period: 5.0 seconds)

<table>
<thead>
<tr>
<th>Score</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.304</td>
<td>-0.322</td>
<td>-0.880</td>
<td>-0.174</td>
<td>0.013</td>
<td></td>
</tr>
<tr>
<td>0.559</td>
<td>0.432</td>
<td>0.032</td>
<td>0.069</td>
<td>0.703</td>
<td></td>
</tr>
<tr>
<td>Loading vector</td>
<td>0.315</td>
<td>-0.401</td>
<td>0.405</td>
<td>-0.757</td>
<td>0.052</td>
</tr>
<tr>
<td>0.534</td>
<td>0.464</td>
<td>0.017</td>
<td>-0.063</td>
<td>-0.704</td>
<td></td>
</tr>
<tr>
<td>0.460</td>
<td>-0.578</td>
<td>0.246</td>
<td>0.623</td>
<td>-0.082</td>
<td></td>
</tr>
<tr>
<td>Eigenvalue</td>
<td>0.01641</td>
<td>0.00123</td>
<td>0.00049</td>
<td>0.00025</td>
<td>0.00002</td>
</tr>
<tr>
<td>Total variance (%)</td>
<td>89.22</td>
<td>6.67</td>
<td>2.68</td>
<td>1.34</td>
<td>0.09</td>
</tr>
</tbody>
</table>
From Table 7.3 and Fig 7.5, it was found that there was a linear relationship between stirring power and the dominant principal component ($R^2 = 0.96$), and PC1 can capture 89.22% of the total variation of the cold model data.

### 7.4.2 Sampling Period Set to Be 6.0 Seconds

When the sampling period was set to be 6.0 seconds, signals from disturbed top surface, bubbling sound and vibration were pre-treated and organized into the new state matrix. Applying PCA to the new state matrix, the loading vector, eigenvalues and the variances picked up by each component were shown in Table 7.4. The relationship between the stirring power and the dominant principal component were plotted in Fig 7.6.
Table 7.4 Loading vectors and eigenvalues of the cold model data (Sampling period: 6.0 seconds)

<table>
<thead>
<tr>
<th>Score</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.307</td>
<td>-0.332</td>
<td>-0.866</td>
<td>-0.214</td>
<td>0.020</td>
</tr>
<tr>
<td></td>
<td>0.557</td>
<td>0.433</td>
<td>0.028</td>
<td>0.079</td>
<td>0.703</td>
</tr>
<tr>
<td>Loading vector</td>
<td>0.313</td>
<td>-0.386</td>
<td>0.444</td>
<td>-0.743</td>
<td>0.056</td>
</tr>
<tr>
<td></td>
<td>0.532</td>
<td>0.468</td>
<td>0.009</td>
<td>-0.066</td>
<td>-0.703</td>
</tr>
<tr>
<td></td>
<td>0.463</td>
<td>-0.578</td>
<td>0.229</td>
<td>0.625</td>
<td>-0.090</td>
</tr>
<tr>
<td>Eigenvalue</td>
<td>0.01636</td>
<td>0.00115</td>
<td>0.00050</td>
<td>0.00025</td>
<td>0.00001</td>
</tr>
<tr>
<td>Total variance (%)</td>
<td>89.55</td>
<td>6.30</td>
<td>2.73</td>
<td>1.35</td>
<td>0.07</td>
</tr>
</tbody>
</table>

From Table 7.4 and Fig 7.6, it was found that there was a linear relationship between stirring power and the dominant principal component ($R^2=0.96$), and PC1 can capture 89.55% of the total variation of the cold model data.

### 7.4.3 Sampling Period Set to Be 7.0 Seconds

When the sampling period was set to be 7.0 seconds, signals from disturbed top surface, bubbling sound and vibration were pre-treated and organized into the new state matrix. Applying PCA to the new state matrix, the loading vector, eigenvalues and the variances picked up by each component were shown in Table 7.5. The relationship between the stirring power and the dominant principal
component were plotted in Fig 7.7.

Table 7.5 Loading vectors and eigenvalues of the cold model data (Sampling period: 7.0 seconds)

<table>
<thead>
<tr>
<th>Score</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.305</td>
<td>-0.365</td>
<td>-0.857</td>
<td>-0.197</td>
<td>0.024</td>
</tr>
<tr>
<td></td>
<td>0.557</td>
<td>0.436</td>
<td>0.016</td>
<td>0.067</td>
<td>0.704</td>
</tr>
<tr>
<td>Loading vector</td>
<td>0.316</td>
<td>-0.394</td>
<td>0.451</td>
<td>-0.734</td>
<td>0.053</td>
</tr>
<tr>
<td></td>
<td>0.531</td>
<td>0.467</td>
<td>-0.011</td>
<td>-0.080</td>
<td>-0.702</td>
</tr>
<tr>
<td></td>
<td>0.463</td>
<td>-0.550</td>
<td>0.249</td>
<td>0.642</td>
<td>-0.093</td>
</tr>
<tr>
<td>Eigenvalue</td>
<td>0.01644</td>
<td>0.00106</td>
<td>0.00052</td>
<td>0.00023</td>
<td>0.00001</td>
</tr>
<tr>
<td>Total variance (%)</td>
<td>90.06</td>
<td>5.79</td>
<td>2.82</td>
<td>1.25</td>
<td>0.08</td>
</tr>
</tbody>
</table>

Fig 7.7 Relationship between PC1 and stirring power

From Table 7.5 and Fig 7.7, it was found that there was a linear relationship between stirring power and the dominant principal component (R²=0.96), and PC1 can capture 90.06% of the total variation.

7.4.4 Sampling Period Set to Be 8.0 Seconds

When the sampling period was set to be 8.0 seconds, signals from disturbed top surface, bubbling sound and vibration were pre-treated and organized into the new state matrix. Applying PCA to the new state matrix, the loading vector, eigenvalues and the variances picked up by each component were shown in Table 7.6. The relationship between the stirring power and the dominant principal
component were plotted in Fig 7.8.

Table 7.6 Loading vectors and eigenvalues of the cold model data (Sampling period: 8.0 seconds)

<table>
<thead>
<tr>
<th>Score</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.306</td>
<td>-0.379</td>
<td>-0.853</td>
<td>-0.185</td>
<td>0.025</td>
</tr>
<tr>
<td>Loading vector</td>
<td>0.558</td>
<td>0.436</td>
<td>0.010</td>
<td>0.079</td>
<td>0.701</td>
</tr>
<tr>
<td></td>
<td>0.319</td>
<td>-0.375</td>
<td>0.444</td>
<td>-0.747</td>
<td>0.057</td>
</tr>
<tr>
<td></td>
<td>0.531</td>
<td>0.466</td>
<td>-0.022</td>
<td>-0.073</td>
<td>-0.704</td>
</tr>
<tr>
<td></td>
<td>0.459</td>
<td>-0.556</td>
<td>0.272</td>
<td>0.630</td>
<td>-0.095</td>
</tr>
<tr>
<td>Eigenvalue</td>
<td>0.01633</td>
<td>0.00108</td>
<td>0.00050</td>
<td>0.00022</td>
<td>0.00001</td>
</tr>
<tr>
<td>Total variance (%)</td>
<td>89.97</td>
<td>5.96</td>
<td>2.78</td>
<td>1.21</td>
<td>0.08</td>
</tr>
</tbody>
</table>

Fig 7.8 Relationship between PC1 and stirring power

From Table 7.6 and Fig 7.8, it was found that there was a linear relationship between stirring power and the dominant principal component ($R^2 = 0.96$), and PC1 can capture 89.97% of the total variation.

From the analysis of the last four subsections, it was found that the relationship between the dominant principal component and stirring power were almost steady, however, when the sampling period is 7.0 seconds, the dominant principal component picked up more of the variations of the stirring process, i.e. 90.06%. The comparison of the effects on different sampling period is shown in Table 7.7.
Table 7.7 Comparison of the PCA results for different sampling period

<table>
<thead>
<tr>
<th>Sample period (s)</th>
<th>PC1 pick up the total variation (%)</th>
<th>R^2 for PC1 and ε</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.0</td>
<td>85.92</td>
<td>0.96</td>
</tr>
<tr>
<td>5.0</td>
<td>89.22</td>
<td>0.96</td>
</tr>
<tr>
<td>6.0</td>
<td>89.55</td>
<td>0.96</td>
</tr>
<tr>
<td>7.0</td>
<td>90.06</td>
<td>0.96</td>
</tr>
<tr>
<td>8.0</td>
<td>89.97</td>
<td>0.96</td>
</tr>
</tbody>
</table>

Therefore, the best sampling period can be set as 7.0 seconds according to this study and our scope of cold modeling experiments. The dominant principal component can pick up 90% of the total variations of the stirring process, and their relationship between the dominant principal component and the five variables can be correlated by the following equations:

\[
PC1 = 0.305a + 0.557s + 0.316v + 0.531sf + 0.463vf
\]

(7-11)

\[
ε = -0.56PC1 + 0.64
\]

(7-12)

Substitute PC1 into equation (7-12). The stirring power can be correlated by the following equation:

\[
ε = c_0 + c_1a + c_2s + c_3v + c_4sf + c_5vf
\]

(7-13)

In which,

\[c_0 = 0.64, c_1 = -0.177, c_2 = -0.312, c_3 = -0.177, c_4 = -0.298, c_5 = -0.260\]

Therefore, equation (7-13) can be used to indicate the stirring power by monitoring the disturbed top surface, sound of the bubbling and vibration of the wall of the vessel, which in turn may be used to predict the mass transfer rate inside the vessel.
7.5 Relationship between Dominant Principal Component and Froude Number

The Froude number, which is the ratio of inertia force and the gravitational force, has been applied to predict the spout eye size in both cold models and industrial scenarios; however, its definition varies with different research groups\[^{2,3,39,42,47,193}\]. Table 7.8 provides the definitions for Froude number and comparison on existing semi-empirical models against the same data base collected by Yonezawa and Schwerdtfeger\[^{39}\].

<table>
<thead>
<tr>
<th>Groups of researchers</th>
<th>Mazumdar \textit{et al.}</th>
<th>Krishnapisharody \textit{et al.}</th>
<th>Subagyo \textit{et al.}</th>
</tr>
</thead>
<tbody>
<tr>
<td>( R^2 ) for industrial data</td>
<td>0.499</td>
<td>0.713</td>
<td>0.502</td>
</tr>
<tr>
<td>( R^2 ) for cold model data</td>
<td>0.499</td>
<td>0.619</td>
<td>0.796</td>
</tr>
<tr>
<td>Definition for Froude number</td>
<td>( \frac{U_p^2}{gH} )</td>
<td>( \frac{U_p^2}{gh} )</td>
<td>( \frac{Q^2}{gh^5} )</td>
</tr>
<tr>
<td>Definition for non-dimensional area</td>
<td>( \frac{A_e}{(H+h)^2} )</td>
<td>( \frac{A_e}{H^2} )</td>
<td>( \frac{A_e}{(H+h)^2} )</td>
</tr>
</tbody>
</table>

(The meaning for the symbols are explained in Nomenclature)

According to the statistical analysis of the discrepancies between the models and the data, the semi-empirical correlation from Krishnapisharody \textit{et al.} model was the best in predicting the industrial results, while Subagyo \textit{et al.} model predicated the cold model experiment more successfully than other studies (See Appendix 1). As all the \( R^2 \) values are less that 80\%, none of the models can accurately predict the size of the spout eye area successfully for the full range of stirring conditions\[^{48}\]. This, in part, reflects the difficulty in obtaining accurate relationships between the size of spout eye area and operating parameters.
The relationship between dominant principal component and Froude numbers defined by different research groups were investigated in this section.

(A) Froude number defined by Subagyo et al. \(^{[42]}\).

\[ Fr = \frac{Q^2}{gh^5} \]  

(7-14)

Where,

- \( Fr \) - Froude number
- \( Q \) - gas flow rate (m\(^3\)/s)
- \( g \) - gravitational constant
- \( h \) - upper layer (slag/oil) depth (m)

The results of PC1 against Froude number are shown in Fig 7.9. The data seems to scattered along two lines, however no clear definite relationship can be found between the Froude number and the dominant principal component PC1.

(B) Froude number defined by Mazumdar et al. \(^{[3]}\).

\[ Fr = \frac{U^2}{gH} \]  

(7-15)
Where, $U_p$ - plume velocity (m/s), g - gravitational constant (m/s)

$H$ - height of the bulk fluid (liquid steel/water) (m)

As the plume velocity can not be directly measured in this study, the empirical equation (7-16)\textsuperscript{[44, 45]} from Krishnapisharody was applied.

\begin{equation}
U_p = 2.57Q^{0.31}z^{-0.28}
\end{equation}  

Where, $Q$ - gas flow rate (m3/s), $z$ - axial height (m)

Regarding to the top surface of the vessel, the axial height almost equals to the height of the water bath, which keeps a constant in this study, therefore, the variations of the results were not significant, and this issue should be further investigated. It was found that there was a clear linear relationship between the dominant principal component (PC1) and the Froude number which was defined by Mazumdar \textit{et al.}\textsuperscript{[3]}. The results of PC1 against Froude number against were shown in Fig 7.10.

\begin{center}
\includegraphics[width=0.8\textwidth]{fig710.png}
\end{center}

Fig 7.10 Relationship between PC1 and Froude number defined by Mazumdar \textit{et al.}

(C) Froude number defined by Krishnapisharody \textit{et al.}\textsuperscript{[2]}

150
\[ Fr = \frac{U_p^2}{gh} \]  

(7-17)

Where,  
- \( Fr \) - Froude number  
- \( U_p \) - plume velocity (m/s)  
- \( h \) - upper layer (slag/oil) depth (m)  
- \( g \) - gravitational constant (m/s)

Equation (7-16) was also applied to calculate the velocity of the plume. The results of PC1 against Froude number were plotted in Fig 7.11. No clear relationship could be found from the latent variable of the combined signal and the Froude number defined by Krishnapisharody \textit{et al.}\cite{2}.

![Plot of PC1 vs. Froude number](image)

Fig 7.11 Relationship between PC1 and Froude number defined by Krishnapisharody \textit{et al.}.

In summary, no clear relationship can be found between the dominant component of the combined signal and the Froude number defined by Subagyo \textit{et al.}\cite{42} and Krishnapisharody \textit{et al.}\cite{2}. However, there was a clear linear relationship between the latent variable and the Froude number defined by Mazumdar \textit{et al.}\cite{3}. This means that the latent variable of the combined signal can be correlated with the Froude number specifically defined by Mazumdar \textit{et al.}\cite{3}.
7.6 Indicating the Stirring by Limited Signals

Due to the harsh environmental conditions in the industrial scenario, such as high temperature, dusty and noisy surroundings; it may be difficult to collect all the three channels of signals simultaneously and PCA results of the cold model data by removing one channel of signal were investigated in this section. The influence of the signals was investigated by removal of each channel of the signals out of the state matrix and comparing the resulting dominant principal component and the relationship between this PC1 and the stirring power.

7.6.1 Removal of Vibration Signal

The resulting principal component was obtained by deleting $v$ and $v_f$ in the state matrix or setting their values to any constant (except “0”, otherwise, the program can not perform the calculation) and performing PCA on the new state matrix. For the convenience of comparing, the state matrix was kept the same dimension, and setting the values of $v$ and $v_f$ in the state matrix as a constant “1”, thus the variation of the variables $v$ and $v_f$ will be “zero”. Applying the PCA to the state matrix, the resulting eigenvalues and corresponding loading vectors are shown in Table 7.9, and the relationship between stirring power and the dominant principal component is plotted in Fig 7.12.

From Fig 7.12 and Table 7.9, it is difficult to get a clear relationship between the stirring power and PC1, though the first principal described 93.9% of the total variations of the cold model data (except for the vibration signal of the stirring process). Therefore, it is difficult to characterize the stirring process without the signal from the vibrations on the wall of the vessel.
Table 7.9 Loading vectors and eigenvalues without vibration signals

<table>
<thead>
<tr>
<th>Score</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.357</td>
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<td>0.000</td>
<td>0.000</td>
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<td>0.243</td>
<td>0.682</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
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<td>0.000</td>
<td>0.000</td>
<td>-1.000</td>
<td>0.000</td>
<td>0.000</td>
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<td>-0.731</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>1.000</td>
<td>0.000</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Loading vector</th>
<th>Total variance (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>93.85</td>
<td>5.86</td>
</tr>
</tbody>
</table>

Eigenvalue

| 0.011116 | 0.000694 | 0.000035 | 0.000000 | 0.000000 |

Total variance (%)

| 89.93 | 7.27 | 2.80 | 0.00 | 0.00 |

Fig 7.12 Relationship between PC1 and stirring power

7.6.2 Removal of sound signal

Setting the values of $\rightarrow$ and $\rightarrow f$ in the state matrix values to any non-zero constant, and performing PCA to the state matrix, the resulting eigenvalues and corresponding loading vectors are shown in Table 7.10. The relationship between the dominant principal component and stirring power is plotted in Fig 7.13.

Table 7.10 Loading vectors and eigenvalues without sound signals

<table>
<thead>
<tr>
<th>Score</th>
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<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.457</td>
<td>0.887</td>
<td>0.064</td>
<td>0.000</td>
<td>0.000</td>
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<td>0.000</td>
<td>0.000</td>
<td>-1.000</td>
<td>0.000</td>
</tr>
<tr>
<td>0.505</td>
<td>-0.200</td>
<td>-0.840</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>1.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>0.732</td>
<td>-0.416</td>
<td>0.539</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Loading vector</th>
<th>Total variance (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.007412</td>
<td>0.000599</td>
</tr>
</tbody>
</table>

| 89.93 | 7.27 | 2.80 | 0.00 | 0.00 |
From Fig 7.13 and Table 7.10, it was found that there was an approximate linear relationship between the stirring power and PC1 ($R^2=0.84$), and the first principal component described 89.9% of the total variation of all the cold model data except for the sound of the stirring process.

### 7.6.3 Removal of image signal

Setting all the values of $-\pi$ in the state matrix as any non-zero constant, and performing PCA to the state matrix, the resulting eigenvalues and corresponding loading vectors are shown in Table 7.11. The relationship between the dominant principal component and stirring power is plotted in Fig 7.14.

#### Table 7.11 Loading vectors and eigenvalues without image signals

<table>
<thead>
<tr>
<th>Score</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Load</td>
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<td></td>
</tr>
<tr>
<td></td>
<td>0.000</td>
<td>0.000</td>
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<td>0.000</td>
<td>1.000</td>
</tr>
<tr>
<td></td>
<td>0.600</td>
<td>0.391</td>
<td>0.223</td>
<td>0.662</td>
<td>0.000</td>
</tr>
<tr>
<td></td>
<td>0.349</td>
<td>-0.345</td>
<td>-0.853</td>
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<td></td>
<td>0.543</td>
<td>0.432</td>
<td>-0.100</td>
<td>-0.713</td>
<td>0.000</td>
</tr>
<tr>
<td></td>
<td>0.473</td>
<td>-0.736</td>
<td>0.461</td>
<td>-0.150</td>
<td>0.000</td>
</tr>
<tr>
<td>Eigenvalue</td>
<td>0.014062</td>
<td>0.001751</td>
<td>0.000211</td>
<td>0.000021</td>
<td>0.000000</td>
</tr>
<tr>
<td>Variance (%)</td>
<td>87.65</td>
<td>10.91</td>
<td>1.31</td>
<td>0.13</td>
<td>0.00</td>
</tr>
</tbody>
</table>
From Fig 7.14 and Table 7.11, it was found that there was still a linear relationship between the stirring power and PC1 ($R^2=0.95$), and this principal component described 87.7% of the total variations except for the disturbance at the top surface of the vessel.

The results of dominant principal components and stirring power were summarized in Table 7.12.

Table 7.12 Comparison of the results by removal of one channel of signal

<table>
<thead>
<tr>
<th>Removal of signal</th>
<th>Dominant principal</th>
<th>Relationship between dominant principal component and stirring power</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vibration</td>
<td>93.9%</td>
<td>No clear relationship</td>
</tr>
<tr>
<td>Sound</td>
<td>89.3%</td>
<td>Linear ($R^2=0.84$)</td>
</tr>
<tr>
<td>Image</td>
<td>87.6%</td>
<td>Linear ($R^2=0.95$)</td>
</tr>
</tbody>
</table>

From the above analysis and Table 7.12, it can be concluded that though removing the vibration signal may lead to a higher percentage of the dominant component which picks up most of the variation of the cold model data (93.9%), however, this combined signal can not reflect the stirring effect clearly. Removing the image signal from the state matrix led to higher linear correlation-ship between the dominant principal component and stirring power ($R^2=0.95$)
while the latent variable still catches up most of the variation of the process (87.6%). Therefore, it is possible to indicate the level of stirring in the vessel without the image signal.

7.6.4 Predicting Stirring Power by One Channel of Signal

Since there has been established knowledge regarding the prediction of mass transfer coefficient and inclusion removal rate by stirring power\textsuperscript{[200]}, the correlation between stirring power and one channel of signals collected in our cold model data were investigated. The relationship between stirring power and average relative spout eye size was plotted in Fig 7.15. From Fig 7.15, weak linear relationship can be observed between the relative spout eye area and the stirring power (R\textsuperscript{2}=0.73), however, no clear linear relationship were found between sound and vibration signal with stirring power, therefore, it is difficult to describe the stirring process successfully using just one signal. This result can explain why the operators in the steelmaking stations not only need to observe the spout eye area but also listen to the bubble flows, in order to make judgments regarding the level of stirring\textsuperscript{[71]}, and can also explains the limited predictions obtained by just measuring the vibrations on the wall of the vessel\textsuperscript{[48]}.

![Fig 7.15 Relationship between average relative spout eye area and stirring power](image)

Fig 7.15 Relationship between average relative spout eye area and stirring power
7.7 Relationship of Time and Frequency Domain

It is well known that sound and vibration signals can be analyzed in both time domain and frequency domain, and they were calculated in both two domains in this study. The influence of time domain and frequency domain for both sound and vibration signals are investigated in this section.

The relationship between $s''$ and $sf''$, $v''$ and $vf''$ are plotted in Fig 7.16 and Fig 7.17 respectively. Fig 7.16 shows that the two variables ($s''$ and $sf''$) which were defined to measure the sound signal was found to be highly correlated ($R^2=0.99$), thus either of them can be applied to indicate the sound signal of the bubbling. Fig 7.17 shows that the two variables ($v''$ and $vf''$) which were used to measure vibration signal, and was also found to be highly correlated ($R^2=0.84$), thus either of them can be applied to indicate the vibration signal of the bubbling.

The correlation between vibration magnitude and amplitude is weaker than that of the sound signal, possibly due to that fact that $sf''$ is summed over a wider range of frequency range (100-1400 Hz), while $vf''$ summed over a short range of the frequencies (1-120 Hz).

![Graph showing the relationship between sound intensity and sound spectrum]

Fig 7.16 Relationship between two approaches of sound signal measurement
Fig 7.17 Relationship between two approaches of vibration signal measurement

### 7.8 Combination of Sound and Vibration Signals

The above analysis revealed that the stirring effect can be indicated by the combination of sound and vibration signals and both of them can be analyzed in either time domain or frequency domain. 4.0 seconds were determined due to the fact that the size of spout eye area stabilized after 4.0 seconds. The accumulated average of the absolute value of both sound and vibration signals were investigated and they are plotted in Fig 7.18 and Fig 7.19 respectively.

It was found that the sound intensity stabilized after about 2.0 seconds while the thickness of the upper layer was 10 mm, and the gas flow rate was 12.0 l/min. The vibration magnitude also stabilized after approximately 2.0 seconds while the thickness of the upper layer was 15 mm, and the gas flow rate was 10.0 l/min. The same trend was found for the other stirring conditions. Therefore, the sample period could be reduced to 2.0 seconds. Performing PCA on the combined signal of sound and vibration in the time domain sampled within a period of 2.0 seconds, and the resulting loading vectors and the eigenvalues of the cold model data set is
shown in Table 7.12. The difference between the original data set and the new data set calculated by the inverse operation of PCA is undetectable, which ensured that the calculation process was correct.

Table 7.13 Loading vectors and eigenvalues of the cold modeling experiments measured by sound intensity and vibration magnitude within a 2.0 seconds period

<table>
<thead>
<tr>
<th>Score</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Score</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>Loading vector</td>
<td>-0.8770</td>
<td>0.4805</td>
</tr>
<tr>
<td></td>
<td>-0.4805</td>
<td>-0.8770</td>
</tr>
<tr>
<td>Eigenvalue</td>
<td>0.0069</td>
<td>0.0007</td>
</tr>
<tr>
<td>Total variance (%)</td>
<td>91.33</td>
<td>8.67</td>
</tr>
</tbody>
</table>
From the figures presented in Table 7.13, it demonstrates that the sound intensity and vibration magnitude are highly correlated, and they can be reduced to a new variable, which explained 91.3% of the total variation of our cold model data. This variable is named the “bubbling factor” or “BF” is defined by the following equation:

\[ BF = -0.877s - 0.481v \quad (7-18) \]

In which, “s” and “v” were defined previously, except that the sampling time should be reduced to 2.0 seconds. The relationship between “BF” and the corresponding stirring power was plotted in Fig 7.20.

Fig 7.20 Relationship between BF and stirring power

Fig 7.20 demonstrated that there was a clear linear relationship between the stirring power and the combined signal from sound intensity and vibration magnitude \((R^2=0.92)\) given that the sampling time reduced to 2.0 seconds, and the latent variable could be described by the following regression equation:

\[ \epsilon = -0.84BF + 0.31 \quad (7-19) \]
It should be noted that the bubbling factor defined in this study is not likely to be valid for all circumstances, particularly for industrial cases. As the vibration response of the ladle varies with the inertial mass, physical properties of the liquid, the gas and the ladle\cite{6}, the sound signals from the cold model also vary with that in the industrial scenario. The expressions for stirring factor and the correlation between stirring factor and stirring power should be different. However, this approach to monitoring the stirring process by the combination of sound intensity and vibration magnitude should be applicable to any bubbling process.

7.9 Summary

In this chapter, analysis of the cold model data has been provided. Five variables are defined by the three channels of signals collected simultaneously and pre-treated individually, the PCA results demonstrate that they are highly correlated and can be reduced to one combined signal, which explained most of the variation of the cold model data sets collected over a wide range of stirring conditions and there is a strong linear relationship between this combined signal and the stirring power ($R^2=0.96$). The influence of sampling period was investigated and the effects of removing one channel of signals were compared. When the sampling period is longer than 5.0 seconds, the influence of the sampling time is trivial regarding to capturing most of the variation of the stirring process. It is possible to monitor the stirring process by just combining the signals from sound of the bubbling and vibrations on the wall of the vessel. However, there is no certain way of monitoring the stirring process successfully using just one channel of these signals. The relationship between the dominant component and Froude number defined by different group of researchers were investigated and there is a clear linear relationship between the combined signal and Froude number defined by Mazumdar \textit{et al.} The comparison of the two
approaches of analysing sound and vibration signals demonstrated that they can be measured in either time domain or frequency domain. A bubbling factor ($BF$) based on the combination of sound intensity and vibration magnitude was defined and there is a clear linear relationship between $BF$ and stirring power ($R^2=0.92$). Therefore, the bubble stirring process can be monitored by the bubbling factor calculated by the sound of the bubbling and vibration on the wall of the vessel, which in turn can be used to predict the mass transfer coefficient and inclusion removal rate inside the vessel.
CHAPTER 8

Discussion

Bubble stirring is common in chemical engineering and metallurgical operations and its main objective is to homogenize temperature and chemical compositions by enhancing heat and mass transfer, and promote chemical reactions\textsuperscript{[201, 202]}. Bubble stirring is usually provided through tuyeres or porous plugs at the bottom of the vessel. The behavior of bubbles within metallurgical vessels is important to crucial aspects of their operations, such as, mass transfer, heat transfer, splash generation, reaction of phases with atmosphere and rate of inclusion removal. Therefore, the stirring process should be under precise control to ensure the quality of the product and optimize the operation of the process.

The current control system is based on the feedback signals from the gas line back pressure and gas flow rate\textsuperscript{[6]}, however, the signals can not indicate the stirring effect accurately because of erosion, leakage from connections between pipes, hook-ups and refractory components\textsuperscript{[6]} and partially plugged nozzle which were reviewed in section 2.7. Therefore, the control of stirring process is still largely dominated by manual operations\textsuperscript{[71]}. The operators depend on his observation of the disturbed surface of slag and listening to the process to make judgments about whether the required stirring level is obtained or not.

Cold modeling experiments were performed in this study to establish techniques which can accurately indicate the stirring process and provide reliable feedback signals for the control system. Previous investigation demonstrated that all of the following signals are caused by the same stirring process and can indicate the effect of stirring in some aspect, \textit{i.e.}, the image signal from the top disturbed surface, sound of the bubbling and vibration on the wall of the vessel. They have been investigated in industry individually. For example, various empirical models
were proposed to correlate the non-dimensional spout eye area and the operating parameters \cite{2, 3, 10, 15, 39, 41, 43, 47, 49}, however, none of them can provide accurate predictions as they depend on parameters which can not be reliably measured \cite{48}. Sound signals emitted by the bubbles were correlated with the image of bubbles \cite{8, 53, 59, 62, 67} and sound signals can provide indirect measurement of the bubbling as well, as the sound generation is related with the behavior of the bubbles inside the vessel, such as the formation \cite{67}, detachment \cite{53}, and coalescence \cite{59} which were reviewed in section 2.5. Vibration signals on the wall of the vessel can also reflect the behavior of the bubble groups, as they were caused by the forces of bubble flow rising through the liquid and breaking the surface \cite{6}. Vibration detectors have been used to determine the degree of stirring in a ladle \cite{6, 7, 71, 203} in industry, for example Stelco and Nupro Corporation have used vibration sensors on the wall of ladles as a basis for ladle control systems and positive results have been reported by using vibration analysis to control gas flow into the ladle \cite{6, 7}. However, all these three channels of signals have not been considered holistically or collected and analyzed simultaneously to provide understanding of the bubble behavior inside the vessel so far.

It is clear that the direct measurement of the stirring effect relies on the measurement of temperature gradient and the variation of chemical compositions over time. However, it is difficult to quantify these parameters accurately, particularly in the industrial scenario. It usually takes several minutes to quantify the chemical compositions of the melts and can not meet the requirement of real time control. Therefore, it would be very useful to quantify this stirring process by the indirect effects, such as the image signal from the disturbed top surface, sound of the bubbling and the vibration signals on the wall of the vessel.

It is conceivable that the combination of all these indirect signals can provide a full picture of the behavior of the bubbles inside the vessel, and each of them can be measured and analyzed by established techniques. The three types of signals
were investigated by five variables defined in this study, *i.e.* average relative spout eye area, average sound intensity, summation of sound amplitude over the range of 100 to 1500 Hz, average vibration magnitude, and summation of vibration amplitude over the range of 1 to 120 Hz. Alternative pretreatment of these original signals may be tried in the future work. Various methods for analyzing the multivariate data were compared, such as neural networks, fuzzy logic and multivariate statistical methods. PCA (Principal Component Analysis) approach was selected due to its powerful ability to extract useful information from correlated data sets\(^9\), simplicity(easy and quick to implement)\(^{167, 198}\) and its successful application in industry\(^{181-188}\). The other techniques may also be applied to investigate the data base; however, the results of this study confirmed the strength of PCA for this type of analysis.

PCA results of the five variables collected in our cold model data proved that the five variables are highly correlated, which was as expected, as they are all indirect measurements of the same process. The analysis in section 7.3 showed that the latent variable (or the dominant principal component, or \(\text{―PC1‖}\)) of the five variables can explain about 86% of the total variation of the cold model data, and there is a clear linear relationship between PC1 and stirring power \((R^2=0.96)\) which were applied to indicate the stirring effect inside the vessel\(^{200}\). As there is established knowledge\(^{201}\) regarding the relationship between the stirring power and the overall heat transfer coefficient, desulfuration rate, dephosphoration rate and the rate of inclusion removal, theoretically, it is possible to predict the heat transfer and mass transfer rate by this combined signal. Four seconds were determined initially as the sampling period, due to the understandings that the size of spout eye area is dynamic and will stabilize after a certain period of time\(^{2, 39, 41, 193}\), which was described in section 6.4.3. Different sampling periods were investigated and 7.0 seconds were determined as the shortest time that could describe the most of the variations of the cold model data (90.1% of the total
variations), however, the difference was trivial as long as the sampling period is longer than five seconds (89.6% of the total variations).

The relationship between the latent variable and Froude number, which is the ratio of inertia force and buoyancy force, was investigated in section 7.5. It was found that there is a strong linear relationship between this latent variable (dominant principal component calculated by the combination of image, sound and vibration signals of our cold model data) and the Froude number defined by Mazumdar et al ($R^2=0.97$).

The influence of monitoring the stirring process by limited signals were investigated by two approaches, i.e. by removing one channel of signals (in other words, by the combination of two channels of signals) and by only one channel of the signal. They were presented in section 7.5 by correlating the resulting signals against the stirring power. The results demonstrated that monitoring the stirring process by the combined signal from sound and vibration leads to some promising results (i.e., the dominant principal component can pick up approximate 86% of the total variation of the cold model data) and there was a clear linear relationship between PC1 and stirring power ($R^2=0.95$). However, no strong linear correlation could be found between the stirring power and each of the individual signals, e.g. the square of the correlation between the average relative spout eye area and stirring power was only 73%.

Both sound and vibration are wave signals and they can be analyzed in both time-domain and frequency-domain, the results of the two approaches on the same data were compared in section 7.7. The results demonstrated that representation of sound signal in time domain and frequency domain are almost equivalent ($R^2=0.99$). There was some difference between the two presentations of the vibration signal ($R^2=0.84$) due to the summation of vibration amplitude over a very limited range of frequency, and might be improved by quantifying the
vibration signals from transverse and vertical directions.

A new variable $BF^{''}$ (Bubbling Factor) was defined in this study based on the combined signal of sound intensity and vibration magnitude. There was a clear linear relationship ($R^2=0.92$) between bubbling factor defined in this study and stirring power. The sampling period can be reduced to 2.0 seconds to collect sufficient information about the stirring process inside the vessel, which means that the bubbling factor can give a feedback signal every two seconds. These results may lead to the development of an online sensor that can monitor the stirring process effectively.

Fig 8.1 Schematic diagram of the PCA approach on complicated phenomena

The multivariate statistical approach performed in this study is summarized in Fig 8.1, and provides an example of how to investigate complicated phenomena
when all the variables can be reliably collected. Theoretically, this approach should be applicable for the investigation of other complicated phenomena.

However, due to the time and equipment limitations, the following items have not been further investigated:

1. Viscosity and surface tension have not been considered in this study. It has been assumed that viscous and surface tension forces are negligible in comparison with inertia forces, as the gas density is negligible relative to liquid density\[^{14}\], however, at higher gas flow rate circumstances, it is more appropriate to consider the force balances involving drag and surface tension terms, in addition, to inertia and buoyancy\[^{204}\]. In the cold modeling experiments, this situation can be simulated by the solutions with different relative densities. As this study was focused on the possibility of quantifying the stirring process by combined effect of different signals, more cold modeling experiment are required to quantify the other effects of bubbling phenomena, particularly at higher gas flow rate conditions.

2. Different dimensional scale models have not been investigated. Only a 1/10 scale cold model rig was set up in this study, and the thickness of the wall (made from acrylic material) was fixed at 10 mm. The properties of refractory on the wall of the ladle will affect the resistance to the sound and vibration signals, which, in turn, will cause variations to the sound and vibration signals. However, the influence of the refractory could not be quantified from the current cold model.

3. The bubbling flow inside a vessel will lead to the vibration of the wall in all directions, however, only the horizontal component was quantified in this study, the variation of the vibration signals of the other two components have not been considered. The results of vibration signals analyzed in time domain and spectrum domain shows an evident difference, this is partially because
only a limited frequency range was calculated, and partially because only part of the components was considered. More complete quantification of the vibration signals would be desirable for future investigations.

Before transferring the current results into real industrial application, the following work needs to be carefully considered:

1. Verification of the present results with cold models of different scale and liquids of different density ratios.

2. Collecting all the three types of signals in an actual industrial environment, and pre-treated them in the same way as the cold model signals to verify the current conclusions in an industrial setting.

3. Considering the place and direction to install the microphone, or microphones, and how to filter out the background noise.

4. Quantifying the vibration signals in three directions, and correlate each of them, or all of them, with the other signals collected simultaneously, considering the place and method to install the accelerometer or accelerometers.

5. Establishing a method to calibrate the combined signal or bubbling factor in reference to the actual working conditions with industrial data.
CHAPTER 9

Industrial Trials

Cold modeling experiments have been performed in laboratory, and a rapid image analysis technique based on light intensity differences of the pixels have been developed for the cold model data. It was also demonstrated in Chapter 2 that the stirring process at the bottom of the metallurgical vessel may lead to the following three effects, *ie.* disturbed top surface of the upper layer, sound of bubbling and vibration on the wall of the vessel. In terms of secondary steel making, argon stirring is performed to achieve homogenization, accelerate chemical reactions and promote the removal of inclusions from the liquid steel. Controlling the ladle eye area is also crucial to ensure the product quality in industrial practice\cite{71}. Industrial trials performed to test the applicability of the established rapid image analysis technique for the industrial scenarios will be described in this chapter. Unfortunately, it was not possible to arrange installation of an accelerometer on the wall of the ladle and only limited images were taken from the side door of the ladle furnace.

9.1 Outline of the Industrial Trial

A Ladle Furnace (LA) located at the Laverton Steel Mill (LSM) in Melbourne, Australia, was selected for our industrial trial. The components of the ladle furnace are schematically shown in Fig 9.1\cite{200}.

The Furnace charged with a metallic charge of \sim 93\ tonnes, depending on the grade of steel to be made, it is divided into two buckets (about 58 + 35 tonnes respectively), which are charged into the furnace and melted one at a time.
Besides metal scrap, the furnace is charged with coke, calcined lime and magnesia. The charge is then melted by means of electrical power through three graphite electrodes and chemical energy supplied by the injection system injecting natural gas, oxygen and carbon into the furnace. Refining operations are then carried out following meltdown when a flat bath is achieved, to help remove most impurities such as phosphorous, sulphur, aluminium and chromium by transfer to slag. Once the right temperature and carbon content are reached, the melt is tapped into a ladle of 85 ton nominal capacity, which will then undergo the trimming process at the Ladle Furnace Station. The exhaust fumes generated during the melting process are conveyed into the fume extraction system for environmental control\textsuperscript{205}. 

The objective of the industrial trial was to test the effectiveness of image analysis on the ladle eye. The images were taken through a safety glass (UVEX SKYPER, Standard No.AS/NZS1337) by a common digital camera (Canon MVX 330i) outside the slid door of the ladle furnace during two heats. Three samples were
collected for quality check which tell the chemical compositions of the steel at
different stages for the first heat (Heat number 138479) and four samples were
collected for the second heat (Heat number 138480). The operational information
during the trials was obtained from the Mill’s Log Book after the trials were
finished.

A period of video was captured during the bubbling process, and the sound
signals were separated from the video file. The thickness of the slag was
measured manually through the dissolved tube length approach. A steel tube was
inserted to the bath until its extremity had been melted by the molten steel, and
the thickness of the slag was assumed to be the length of the red-hot portion after
the bar was pulled out. The solidified metal and slag on the poles showed 74 mm
for the first heat, but, unfortunately, the second one was not successful, as the
slag did not adhere to the tube when it was taken out.

The images were analyzed by the threshold technique\cite{196} which was developed
in our cold modeling, as described in Appendix 2. The threshold was set to 230
subjectively, the real size of the intersection area of the furnace were regarded as
the reference.

Four typical images were post treated in this way and the pixel percentages were
calculated which showed the ratio of the exposed eye area to the total area of the
surface, and they are shown in Fig 9.2 to Fig 9.5. Because the images were taken
through the door of the cover, not directly from the top, only part of the surface
were exposed. Therefore, the area calculation is only an approximation.

According to the drawing of the ladle, the diameter of the ladle surface was
estimated to be 2.5 m, which has an area of $4.91 m^2$. Then the eye areas were
estimated by the product of the total area and the percentage of the calculation.
9.2 Initial Results

Four images and their related operating parameters were demonstrated in Fig 9.2 to Fig 9.5 respectively. The blue region on the right side shows the pixels which have elemental value larger than the set value, thus was regarded as the liquid steel which is exposed to the air, or ladle eye area.

Taken at 20 Minutes in Stirring (7/2/08 13:26, Heat No. 138479)
Eye area: 5.76% of image area, approximately 0.28 $m^2$
Argon flow rate: 45 $Nm^3/hr$, slag height (est.) 74 $mm$,
Temperature: 1607°C
Metal: N: 0.0043, S: 0.029, Al: 0.002, C: 0.12, Mn: 0.73, Si: 0.23

Taken at 32 Minutes in Stirring (7/2/08 13:38, Heat No.138479)
Eye area: 7.04% of image area approximately 0.35 $m^2$
Argon flow rate: 45 $Nm^3/hr$, slag height (est.) 74 $mm$,
Temperature: 1608°C
Metal: N: 0.05735, S: 0.025, Al: 0.009, C: 0.12, Mn: 0.74, Si: 0.21
Fig 9.4(a) Original Image         Fig 9.4(b) Post Treated Image

Taken at 20 Minutes in Stirring (7/2/08 14:14 Heat No.138480)
Eye area: 16.54% of image area approximately 0.81 m²
Argon flow rate: 52 Nm³/hr, slag height (unsuccessfully),
Temperature: 1609°C
Metal: N: 0.0066, S: 0.029, Al: 0.002, C: 0.12, Mn: 0.66, Si: 0.21

Fig 9.5(a) Original Image         Fig 9.5(b) Post Treated Image

Taken at 29 Minutes in Stirring (7/2/08 14:33 Heat No.138480)
Eye area: 5.65% of image area approximately 0.28 m²
Argon flow rate: 52 Nm³/hr, slag height (unsuccessfully),
Temperature: 1596°C
Metal: N: 0.0124, S: 0.028, Al: 0.002, C: 0.19, Mn: 0.85, Si: 0.23

The sound signals were separated from the video file and analyzed by the same method as the cold model signals. It was found that the sound intensity was almost four times higher than the cold model data, however, the spectrum
analysis showed that the dominant frequencies were in the range of low frequency, quite similar to the result of cold model experiments. The sound intensity and spectrum analysis results are shown in Fig 9.6 and Fig 9.7 respectively.

![Sound intensity of industrial trial data](image1)

**Fig 9.6 Sound intensity of industrial sound signal during stirring process**

![Single sided Amplitude Spectrum of industrial trial data](image2)

**Fig 9.7 Sound spectrum analysis of industrial sound signal during stirring process**

### 9.3 Discussion

The initial industrial trial showed that the image analysis technique which was
established in cold modeling experiments can be applied to the image from industry and it may rapidly calculate the ladle eye area. If a digital camera were installed at the top of the furnace, it is possible to monitor the ladle eye area online by applying the established image analysis technique, however, the installation of a digital camera at the top of the ladle is a challenging work, many factors need to be considered, such as cooling system for the camera and dust elimination system for the lens, as the ladle eye may be blurred by the smoke, particularly for some mild carbon steel product. The sound signal was separated from the video files and was analyzed by both time and frequency domain. The results showed that the sound intensity for the industrial case was four times higher than the cold model scenario and the sound frequency was mainly distributed in the range of 100 Hz to 7 kHz, which was also different from the cold model case, future work is clearly required to quantify the image, sound and vibration signals in an industrial environment.
CHAPTER 10

Conclusions and Further Research

The objective of this study was to develop multivariate statistical techniques to analyze the different signals from bubbling vessels in metallurgical processes simultaneously. The main aims of this study have been achieved with the following significant results.

1. Cold modeling experiments based on bottom stirred cylindrical vessel were performed and images from the disturbed top surface, sound of the bubbling and vibration on the wall of the vessel have been measured simultaneously over a wide range of stirring conditions.

2. A threshold image analysis technique for calculating the spout eye area from the images of the disturbed top surface was developed, and the average time required to analyze one frame of image was reduced to 0.1 second, which makes it suitable to provide online image analysis.

3. Industrial trials were conducted and the images from an actual ladle furnace were analyzed using the new technique developed in this study.

4. Multivariate statistical methods were established to analyze the three different types of signals by five variables defined in this study. The PCA results proved that the image signals of the disturbed top surface, the sound of bubbling and the vibration on the wall of the vessel are highly correlated, and can be reduced to just one latent variable which captures most of the variations of all the cold model date sets. This latent variable can be expressed by the linear combination of the five variables, and it has a clear linear relationship with stirring power, which has the potential to
be used to indicate the stirring level within the vessel, which, in turn can be used to predict the mass transfer coefficient and inclusion removal rate using by established knowledge.

5. It was statistically proved that the combined signals from sound and vibration can capture most of the variation of the stirring process, and they can be measured in either time domain or frequency domain. This combined signal could be used as a feed back signal for process control.

6. Cold model experiment results showed that the image signal provides trivial contributions to the total variation of the process, and the process can be monitored without it. This finding is particularly important for the pyrometallurgical operations where it is difficult to install a digital camera above the vessel.

The current research can also be extended to other industrial process such as quantifying the height of the lance into the slag foam and predicting the chemical reactions inside the ladle.

The following topics are recommended for further research and development:

1. Verification of the current results with cold modeling installation of different size, different material (for the cold model rig) and different liquids, particularly the wall of the vessel, as the sound and vibration signals may be attenuated through the wall.

2. Pre-treating the signals in alternative ways. Analyzing the cold model data through other approaches, such as neural network and fuzzy logic, etc. and comparing them with the results obtained from this study.

3. Full scale industrial trials. Collecting, quantifying and calibrating all the three channels of signals, and investigating effective ways to fill out the noise.
4. Performing PCA on the variables calculated from the industrial data and defining the corresponding latent variables, correlating these latent variables with stirring power, which may be used to predict the mass transfer coefficient and inclusion removal rates using established correlations.
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Appendix 3: PCA code on analysing cold model data

This program is based on MATLAB platform, and is composed by the following modules:

(1) Data collection
(2) Image analysis
(3) Sound intensity analysis
(4) Sound spectrum analysis
(5) Vibration magnitude analysis
(6) Vibration spectrum analysis
(7) PCA on image, sound and vibration signals

The first six modules input and pre-treat all the signals collected into five variables, and the last module organizes the value of the five variables into a state matrix, and analyze this state matrix through the standard Principal Component Analysis Approach. The results are shown behind the program. The results of the program are validated by the inverse operation of the PCA, the difference between the original state matrix and the derived data set is stored in the matrix: “Difference”, an undetectable difference assured the correct calculation.

The “Fixed data” is all the original signals collected simultaneously in our cold modeling experiments and is stored in a well organized directory system.

Principal component Analysis of Total Cold Model Data Base
(Original Code and Results)

% This Program was applied to analyze all the data in the cold modeling
% Sampling Period was set as 4.0 seconds
% Data collection and pre-treatment
clear all
clc;
for dat=1:40

%% Data Collection
path (path, 'C:\Fixed data\total data')
ImageSeriesNoStart=(dat)*1000000; % Input the frame start No.
data=wavread([num2str(dat),'.wav']); % Input of sound signals
sheet=xlsread([num2str(dat),'.xls']); % Input of vibration signals

%% Image Analysis
path (path,' C:\Fixed data\image')
for i=1:60  % 15 frames per second, 4.0 seconds
    num=ImageSeriesNoStart+i;
    I=imread([num2str(num),'.jpg']);
    [m,n,k]=size(I);
    numb=0; % Accumulation of boundary pixels
    numb=length(find(I(:,:,1)+I(:,:,2)+I(:,:,3)<=50));
    nums=0; % Accumulation of the spout area
    nums=length(find(I(:,:,1)+I(:,:,2)-I(:,:,3)<=130)); % Spout eye size
    cir=0.25*3.14*686^2;  % Size of the circular intersection
    p(i)=(nums-numb)/cir*100;  % Relative spout eye area
    p=p';
end
a(dat)=sum(p)/60;  % Average relative spout eye area

%% Sound Intensity Analysis
soundintensity=data(1:64000); % Audio sample rate 16kHz, 4.0 seconds
s(dat)=sum(abs(soundintensity))/1000; % Average of sound intensity

%% Sound Spectrum Analysis
y=data(1:64000); % Input of sound signals
Fs=16000; % Sampling frequency
T=1/Fs; % Sample time
L=length(y); % Length of the signal
t=(0:L-1)*T; % Time vector

NFFT=2^nextpow2(L); % Next power of 2 from length of y
Y=fft(y, NFFT)/L; % Amplitude of sound signal at different frequency
f=Fs/2*linspace(0,1,NFFT/2); % Sound frequencies

% Summation of the amplitude of a certain range of frequency
% Summation of amplitudes within frequency range of 100 Hz to 1500 Hz
Y1=Y(100:1500);
sf(dat)=sum(abs(Y1));

% Vibration Magnitude Analysis
v(dat)=sum(abs(sheet(1:1000)))/1000; % Sample rare 250 /s, 4.0 seconds
% Average of vibration magnitude within 4.0 seconds
% Vibration Spectrum Analysis
y=sheet(1:1000); % Input of vibration signal
Fs=250; % Sampling frequency
T=1/Fs; % Sample time
L=length(y); % Length of the signal
t=(0:L-1)*T; % Time vector

NFFT=2^nextpow2(L); % Next power of 2 from length of y
Y=fft(y, NFFT)/L; % Vibration magnitudes
f=Fs/2*linspace(0,1,NFFT/2); % Vibration frequencies

% Summation of amplitude of a certain range of frequency
% Summation of vibration amplitude over the range of 1 Hz to 120 Hz
Y1=Y(1:120);
vf(dat)=sum(abs(Y1));
end

%% PCA of Image, Sound and Vibration signals
disp('The original signal matrix is:');
Rawdatamatrix0=[a', s', v', sf', vf'],
disp('Normalization of the raw data set');
Rawdatamatrix=normc(Rawdatamatrix0),
MI=Rawdatamatrix;

MDMI=mean(MI);  % Scale matrix MDMI by its mean
[m,n]=size(MI);
for i=1:1:n
    for j=1:1:m
        MI(j,i)=MI(j,i)-MDMI(i);
    end
end
disp('Substracting the mean in column is:');
MIadjust=MI;

% Calculation of covariance matrix CDMI
disp('Covariance matrix is:');
CDMI=cov(MI),
% Calculation of Eigen Vector (U) and Eigen Values (L) of matrix CDMI
[u,l]=eig(CDMI);
% change U and L order into descending order
disp ('U L discompose of the covariance matrix is:');
L=[l(5,5) l(4,4) l(3,3) l(2,2) l(1,1)],
U=[u(:,5) u(:,4) u(:,3) u(:,2) u(:,1)],

% Calculation of Principal Components
% disp('Transpose of Z is:');
disp('The components of state matrix is: ')
Z=MI*U,

disp('The sum of eign value is:');
Lsum=L(1)+L(2)+L(3)+L(4)+L(5),

ppc1=L(1)/Lsum*100; % Percentage of 1st principal component
ppc2=L(2)/Lsum*100; % Percentage of 2nd principal component
ppc3=L(3)/Lsum*100; % Percentage of 3rd principal component
ppc4=L(4)/Lsum*100; % Percentage of 4th principal component
ppc5=L(5)/Lsum*100; % Percentage of 5th principal component

% Percentage=[pc1;pc2;pc3;pc4;pc5],
disp(['The principal components for all the data set is: ']);
disp([num2str(L(1)/Lsum*100,'%6.2f'),'%']);
disp([num2str(L(2)/Lsum*100,'%6.2f'),'%']);
disp([num2str(L(3)/Lsum*100,'%6.2f'),'%']);
disp([num2str(L(4)/Lsum*100,'%6.2f'),'%']);
disp([num2str(L(5)/Lsum*100,'%6.2f'),'%']);

% Draw a figure of PC#1 Vs PC#2
x=Z(:,1);
y=Z(:,2);
plot(x,y,'bo'),
axis([-0.20,0.25,-0.10,0.07]),
title('PCA analysis of image, sound and vibration signals'),
xlabel('PC#1'), ylabel('PC#2'), grid
% Validation of the program
FeatureVector=U,

disp('The Final data or the components of the state matrix is: ')
Finaldata=MIadjust*FeatureVector,

disp('Inverse operation of dissociation: ')
Originaldata=Finaldata*FeatureVector'+ones(40,1)*MDMI,

disp('The difference between the raw data and the derived data is: ')

Difference=Rawdatamatrix-Originaldata,

**The calculation results are as follows:**

The original signal matrix is:

Rawdatamatrix0 =

```
  13.3828  1.6382  0.1250  0.8584  0.1624
  23.5673  2.8475  0.2470  1.5837  0.3267
  30.4425  3.8672  0.3240  2.0479  0.6123
  33.3839  4.2249  0.3506  2.3850  0.5785
  37.9493  5.1973  0.3662  2.9769  0.6628
  44.5626  6.3142  0.4280  3.3013  0.7663
  46.1230  7.7910  0.4352  3.9380  1.0012
  50.2246  8.1605  0.4730  4.0724  0.9153
  50.6917  9.7754  0.4995  4.7343  0.8838
  55.6795 11.2072  0.5316  5.0263  1.1965
  11.0856 1.5702  0.1763  0.9312  0.2570
  20.6522  2.9685  0.3410  1.6592  0.4613
  27.6464  3.7994  0.4170  2.1313  0.7112
  27.2921  4.2940  0.4264  2.4435  0.7435
  34.6955  5.5679  0.4769  2.9123  0.8685
  39.2088  7.0321  0.4756  3.5102  0.9477
  41.9754  8.3588  0.4950  4.0696  1.0781
  43.8529  8.9614  0.5203  4.2633  1.2201
  46.9818 10.0034  0.6094  4.6957  1.3642
  46.7099 11.8968  0.6218  5.4723  1.3090
  9.3106  1.2665  0.1971  0.7071  0.1827
```
| 33.4414 | 2.7806 | 0.4231 | 1.5064 | 0.5076 |
| 32.9826 | 4.0059 | 0.5251 | 2.2846 | 0.8836 |
| 29.7406 | 4.7501 | 0.4866 | 2.5723 | 0.7352 |
| 36.9580 | 5.8115 | 0.5326 | 2.9310 | 0.8999 |
| 33.4269 | 7.0743 | 0.5400 | 3.5074 | 0.8534 |
| 38.4391 | 8.2671 | 0.5642 | 4.2346 | 0.9596 |
| 35.5930 | 8.7020 | 0.5889 | 4.2820 | 1.2052 |
| 38.6478 | 9.8610 | 0.5519 | 4.5839 | 1.2280 |
| 41.3416 | 11.1264 | 0.6872 | 5.2761 | 1.5256 |
| 22.7488 | 0.9875 | 0.1427 | 0.6010 | 0.1863 |
| 37.8739 | 1.9309 | 0.3464 | 1.0341 | 0.6297 |
| 28.7360 | 2.5929 | 0.4140 | 1.4341 | 0.8964 |
| 40.4042 | 3.2154 | 0.4504 | 1.7280 | 0.9734 |
| 38.5089 | 3.6531 | 0.4836 | 1.8114 | 1.1206 |
| 38.8244 | 4.5882 | 0.4893 | 2.2633 | 1.0938 |
| 41.0827 | 5.5208 | 0.5401 | 2.7220 | 1.4713 |
| 43.1503 | 6.3785 | 0.5358 | 2.9372 | 1.3556 |
| 46.8209 | 6.7468 | 0.6159 | 3.0220 | 1.7086 |
| 47.7139 | 8.7533 | 0.6588 | 3.7470 | 1.7750 |

Normalization of the raw data set

Rawdatamatrix =

0.0563  0.0395  0.0419  0.0424  0.0259
0.0992  0.0686  0.0827  0.0783  0.0522
0.1281  0.0932  0.1085  0.1013  0.0978
0.1405  0.1018  0.1174  0.1179  0.0924
0.1597  0.1253  0.1226  0.1472  0.1059
0.1876  0.1522  0.1433  0.1632  0.1224
0.1941  0.1878  0.1457  0.1947  0.1599
0.2114  0.1967  0.1583  0.2014  0.1462
0.2134  0.2356  0.1672  0.2341  0.1412
0.2344  0.2701  0.1779  0.2485  0.1911
0.0467  0.0378  0.0590  0.0460  0.0410
0.0869  0.0715  0.1142  0.0820  0.0737
0.1164  0.0916  0.1396  0.1054  0.1136
0.1149  0.1035  0.1427  0.1208  0.1187
0.1460  0.1342  0.1596  0.1440  0.1387
0.1650  0.1695  0.1592  0.1736  0.1514
0.1767  0.2015  0.1657  0.2012  0.1722
0.1846  0.2160  0.1742  0.2108  0.1949
0.1977  0.2411  0.2040  0.2322  0.2179
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<td>0.2205</td>
<td>0.1853</td>
<td>0.2835</td>
</tr>
</tbody>
</table>

Subtracting the mean in column is:

Covariance matrix is:

CDMI =

0.0020  0.0026  0.0016  0.0023  0.0023
0.0026  0.0053  0.0027  0.0049  0.0035
0.0016  0.0027  0.0021  0.0024  0.0027
0.0023  0.0049  0.0024  0.0045  0.0030
0.0023  0.0035  0.0027  0.0030  0.0041

U L discompose of the covariance matrix is:

L =

0.0155  0.0018  0.0005  0.0002  0.0000

U =

0.3125  -0.1471  -0.9346  0.0801  -0.0295
The components of state matrix is:

\[
Z = \\
\begin{bmatrix}
-0.2296 & 0.0443 & 0.0160 & -0.0281 & -0.0029 \\
-0.1558 & 0.0350 & -0.0019 & -0.0066 & -0.0006 \\
-0.0917 & 0.0112 & -0.0079 & -0.0082 & 0.0016 \\
-0.0739 & 0.0215 & -0.0160 & 0.0023 & 0.0051 \\
-0.0317 & 0.0306 & -0.0244 & -0.0010 & 0.0113 \\
0.0149 & 0.0269 & -0.0372 & 0.0057 & 0.0033 \\
0.0711 & 0.0279 & -0.0275 & -0.0151 & 0.0079 \\
0.0830 & 0.0378 & -0.0419 & 0.0019 & 0.0019 \\
0.1232 & 0.0693 & -0.0351 & 0.0057 & -0.0026 \\
0.1829 & 0.0487 & -0.0358 & -0.0144 & -0.0095 \\
-0.2192 & 0.0306 & 0.0325 & -0.0210 & 0.0004 \\
-0.1358 & 0.0145 & 0.0224 & 0.0081 & -0.0016 \\
-0.0767 & -0.0070 & 0.0141 & 0.0101 & 0.0030 \\
-0.0591 & 0.0006 & 0.0202 & 0.0088 & 0.0066 \\
-0.0054 & -0.0000 & 0.0052 & 0.0108 & 0.0023 \\
0.0414 & 0.0166 & -0.0030 & 0.0003 & 0.0019 \\
0.0890 & 0.0242 & -0.0015 & -0.0083 & 0.0025 \\
0.1177 & 0.0148 & 0.0007 & -0.0140 & 0.0017 \\
0.1674 & 0.0074 & 0.0054 & -0.0029 & -0.0014 \\
0.2101 & 0.0489 & 0.0145 & -0.0026 & -0.0059 \\
-0.2344 & 0.0297 & 0.0368 & -0.0095 & -0.0056 \\
-0.1130 & -0.0127 & -0.0210 & 0.0322 & -0.0092 \\
-0.0385 & -0.0357 & 0.0090 & 0.0285 & 0.0026 \\
-0.0403 & 0.0011 & 0.0169 & 0.0251 & -0.0001 \\
0.0099 & -0.0080 & 0.0026 & 0.0234 & -0.0036 \\
0.0346 & 0.0243 & 0.0217 & 0.0233 & -0.0044 \\
0.0868 & 0.0349 & 0.0131 & 0.0203 & 0.0031 \\
0.1103 & 0.0120 & 0.0374 & 0.0051 & 0.0031 \\
0.1354 & 0.0300 & 0.0280 & -0.0112 & -0.0021 \\
0.2105 & 0.0088 & 0.0452 & 0.0009 & 0.0016 \\
-0.2290 & 0.0216 & -0.0216 & -0.0193 & -0.0033 \\
-0.1305 & -0.0401 & -0.0448 & 0.0056 & -0.0055 \\
-0.0964 & -0.0560 & 0.0094 & -0.0008 & 0.0021 \\
-0.0555 & -0.0629 & -0.0278 & 0.0052 & 0.0011 \\
\end{bmatrix}
\]
-0.0355 -0.0755 -0.0110 0.0005 -0.0009
-0.0121 -0.0537 -0.0080 0.0010 -0.0007
0.0483 -0.0832 0.0050 -0.0160 0.0071
0.0593 -0.0575 -0.0040 -0.0116 -0.0018
0.1058 -0.1024 0.0016 -0.0166 -0.0008
0.1624 -0.0784 0.0129 -0.0175 -0.0078

The sum of eigenvalue is:

Lsum = 0.0181

The principal components for all the data set is:

85.92%
9.86%
2.95%
1.15%
0.11%

FeatureVector =

0.3125 -0.1471 -0.9346 0.0801 -0.0295
0.5677 0.4184 0.1242 -0.2387 -0.6560
0.3326 -0.3199 0.2388 0.8363 -0.1752
0.5137 0.4548 0.0857 0.0953 0.7161
0.4535 -0.7030 0.2163 -0.4777 0.1588

The Final data or the components of the state matrix is

Finaldata =

-0.2296 0.0443 0.0160 -0.0281 -0.0029
-0.1558 0.0350 -0.0019 -0.0066 -0.0006
-0.0917 0.0112 -0.0079 -0.0082 0.0016
-0.0739 0.0215 -0.0160 0.0023 0.0051
-0.0317 0.0306 -0.0244 -0.0010 0.0113
0.0149 0.0269 -0.0372 0.0057 0.0033
0.0711 0.0279 -0.0275 -0.0151 0.0079
0.0830 0.0378 -0.0419 0.0019 0.0019
0.1232 0.0693 -0.0351 0.0057 -0.0026
0.1829 0.0487 -0.0358 -0.0144 -0.0095
-0.2192 0.0306 0.0325 -0.0210 0.0004
-0.1358 0.0145 0.0224 0.0081 -0.0016
Inverse operation of dissociation:

Original data =

\[
\begin{array}{cccccc}
0.0563 & 0.0395 & 0.0419 & 0.0424 & 0.0259 \\
0.0992 & 0.0686 & 0.0827 & 0.0783 & 0.0522 \\
0.1281 & 0.0932 & 0.1085 & 0.1013 & 0.0978 \\
0.1405 & 0.1018 & 0.1174 & 0.1179 & 0.0924 \\
0.1597 & 0.1253 & 0.1226 & 0.1472 & 0.1059 \\
0.1876 & 0.1522 & 0.1433 & 0.1632 & 0.1224 \\
0.1941 & 0.1878 & 0.1457 & 0.1947 & 0.1599 \\
0.2114 & 0.1967 & 0.1583 & 0.2014 & 0.1462 \\
0.2134 & 0.2356 & 0.1672 & 0.2341 & 0.1412 \\
0.2344 & 0.2701 & 0.1779 & 0.2485 & 0.1911 \\
\end{array}
\]
The difference between the raw data and the treated data is:

\[
\text{Difference} = 1.0\times 10^{-15} \times \\
0.1041 \ 0.0833 \ 0.0625 \ 0.1180 \ 0.0173 \\
0.0416 \ 0.0555 \ 0.0278 \ 0.0694 \ 0.0278 \\
0.0278 \ 0.0278 \ 0.0139 \ 0.0416 \ 0.0139 \\
0.0278 \ 0.0139 \ 0.0139 \ 0.0278 \ 0.0139 \\
0 \ 0 \ 0.0139 \ 0 \ 0.0139 \\
-0.0278 \ -0.0278 \ 0 \ -0.0278 \ 0.0139 \\
\]
-0.0278 -0.0555  0 -0.0555  0  
-0.0555 -0.0555  0 -0.0833  0  
-0.0555 -0.0833  0 -0.1110  0  
-0.0833 -0.1110  0 -0.1388 -0.0278  
0.0833  0.0694  0.0486  0.1457  0.0416  
0.0694  0.0833  0.0139  0.0694  0.0278  
0.0416  0.0416  0  0.0555  0  
0.0416  0.0139  0  0.0278  0.0139  
0  0  0  0  0  
0  -0.0278  0 -0.0278  0  
-0.0278 -0.0555  0 -0.0833  0  
-0.0278 -0.0833  0 -0.0833 -0.0278  
-0.0555 -0.0833 -0.0278 -0.0833 -0.0278  
-0.0833 -0.1110 -0.0278 -0.2220 -0.0278  
0.1110  0.0798  0.0278  0.1180  0.0312  
0.0278  0.0694  0  0.0833  0.0139  
0.0278  0.0278 -0.0278  0.0416  0  
0.0278  0.0139  0  0.0278  0.0139  
0  0  0  0  0  
0  -0.0278  0 -0.0278  0  
-0.0278 -0.0555  0 -0.0833  0  
0  -0.0555 -0.0278 -0.0833 -0.0278  
-0.0278 -0.0833  0 -0.1110  0  
-0.0278 -0.0555 -0.0278 -0.1110  0  
0.0694  0.0867  0.0347  0.1561  0.0312  
0  0.0555  0.0139  0.1110  0  
0.0416  0.0555  0  0.0971  0  
0  0.0416  0  0.0971  0  
0  0.0555 -0.0278  0.0694 -0.0278  
0  0.0278 -0.0278  0.0416 -0.0278  
-0.0278  0  -0.0278  0.0278 -0.0555  
-0.0278  0  -0.0278  0 -0.0278  
-0.0555  0  -0.0278  0.0278 -0.0555  
-0.0555 -0.0555 -0.0555 -0.0555 -0.0555
Figure: The relationship between PC1 and PC2
Appendix 4: PCA program validation

This program is based on MATLAB platform, and was designed according to the procedures introduced by Smith[^], which was reviewed in section 4.3.1.2. This program demonstrates that the columns of any random matrix can not be correlated, and there is no dominant principal component is a random matrix which can explain most of the variation of the total data set. For example, for a $40 \times 5$ random matrix, the first principal component can explain about 32% of the total variation, for a $200 \times 5$ random matrix, the first principal component can explain about 24% of the total variation, with the increase of data base size, the components tend to be evenly distributed. However, for a correlated data set, such as the sate matrix in this study, the first principal component can explain about 86% of the total variation, which proved that the original data set (state matrix) is highly correlated.

Validation of PCA program

```matlab
%% Validation of PCA program
% Input a random matrix
disp('The original signal matrix is:');
Rawdatamatrix=rand(40,5),
disp('Normalization of the raw data set');
Rawdatamatrix=normc(Rawdatamatrix);
MI=Rawdatamatrix,
MDMI=mean(MI);  % Scale matrix MDMI by its mean
[m,n]=size(MI);
for i=1:1:n
   for j=1:1:m
      MI(j,i)=MI(j,i)-MDMI(i);
   end
end
% MI=normc(MI); % normalize the matrix in column
disp('Subtracting the mean in column is:');
MIAjust=MI,
%Calculate covarian matrix DMI
disp('Covariance matrix is:');
CDMI=cov(MI),
```

% Calculate Eigen Vector (U) and Eigen Values (L) of matrix CDIM
[u,l]=eig(CDMI);
% change U and L order into descending order
disp('U L decompose of the covariance matrix is:');
L=[l(5,5) l(4,4) l(3,3) l(2,2) l(1,1)],
U=[u(:,5) u(:,4) u(:,3) u(:,2) u(:,1)],
disp('The components of state matrix is: ');
Z=MI*U,
disp('The sum of eigen values is:');
Lsum=L(1)+L(2)+L(3)+L(4)+L(5),
ppc1=L(1)/Lsum*100; % Percentage of the 1st principal component
ppc2=L(2)/Lsum*100; % Percentage of the 2nd principal component
ppc3=L(3)/Lsum*100; % Percentage of the 3rd principal component
ppc4=L(4)/Lsum*100; % Percentage of the 4th principal component
ppc5=L(5)/Lsum*100; % Percentage of the 5th principal component

% Percentage=[pc1;pc2;pc3;pc4;pc5],
disp(['The principal components for all the date set is:']);
disp(num2str(L(1)/Lsum*100,'%6.2f','%'));
disp(num2str(L(2)/Lsum*100,'%6.2f','%'));
disp(num2str(L(3)/Lsum*100,'%6.2f','%'));
disp(num2str(L(4)/Lsum*100,'%6.2f','%'));
disp(num2str(L(5)/Lsum*100,'%6.2f','%'));

% Draw a figure of PC#1 Vs PC#2
x=Z(:,1); y=Z(:,2); plot(x,y,'bo'),
axis([-0.20,0.25,-0.10,0.07]),
title('PCA analysis of image, sound and vibration signals'),
xlabel('PC#1'), ylabel('PC#2'), grid

% Validation of the program by detecting the difference between the
% original data set and the derived data set which was calculated by
% the inverse operation of PCA
FeatureVector=U;
disp('The Final data or the components of the state matrix is: ')
Finaldata=MIadjust*FeatureVector,
disp('Inverse operation of dissociation: ')
Originaldata=Finaldata*FeatureVector'+ones(40,1)*MDMI,
disp('The difference between the original data set and the derived
data set: ')

Difference=Rawdatamatrix-Originaldata,
Pc=[ppc1,ppc2,ppc3,ppc4,ppc5];

Results for 40×5 data set:
The original signal matrix is:

Rawdatamatrix =

0.5038 0.3037 0.9437 0.4504 0.5154
0.4896 0.0462 0.5492 0.2057 0.6575
0.8770 0.1955 0.7284 0.8997 0.9509
0.3531 0.7202 0.5768 0.7626 0.7223
0.4494 0.7218 0.0259 0.8825 0.4001
Covariance matrix is:

\[
\begin{bmatrix}
0.0074 & -0.0004 & -0.0005 & 0.0005 & 0.0013 \\
-0.0004 & 0.0053 & -0.0004 & -0.0004 & -0.0011 \\
-0.0005 & -0.0004 & 0.0039 & -0.0001 & -0.0005 \\
0.0005 & -0.0004 & -0.0001 & 0.0044 & 0.0004 \\
0.0013 & -0.0011 & -0.0005 & 0.0004 & 0.0068
\end{bmatrix}
\]

U L discompose of the covariance matrix is:

\[
L =
\begin{bmatrix}
0.0089 & 0.0062 & 0.0050 & 0.0043 & 0.0035
\end{bmatrix}
\]

\[
U =
\begin{bmatrix}
0.7016 & -0.6766 & -0.1698 & 0.1332 & -0.0587
\end{bmatrix}
\]
The principal components for all the data set is:

31.99%
22.16%
17.96%
15.35%
12.53%

The difference between the original data set and the derived data set:

Difference =

1.0e-015 *

0.0278  0.0278  0.0278  0  -0.0278
0  0.0226  -0.0833  0  -0.0555
-0.1110  0.0902  0.0555  -0.1110  -0.1110
0.0278  0.0278  0  -0.0278  -0.0555
-0.0833  -0.0278  -0.0477  -0.0555  0.0278
-0.1665  0  0.0278  -0.0139  0
0.1266  -0.0278  -0.0278  0.0278  0.0694
-0.1665  -0.0173  0.0139  -0.0833  0.0729
0.0833  -0.1110  -0.0694  0.0763  0.1284
-0.0278  -0.0278  0.0555  0.0416  0.1041
0  0  0  0.0278  0.0278
-0.0555  0  0.0833  -0.0278  0.0555
0.0139  -0.1110  -0.0416  0.0555  0.1136
-0.0833  0  0.0555  -0.0555  0
-0.1110  -0.0278  0.1110  -0.0278  0.1145
0.1344  -0.0139  -0.0278  0.0833  0.0208
0.1284  0.0139  -0.0278  0.0278  -0.0555
-0.1665  0  0.0347  -0.0833  -0.0555
-0.0555  0.0833  0.0278  -0.0833  -0.1665
0.0416  -0.0278  -0.0729  -0.0278  0.0278
0  0.0555  -0.0278  -0.0278  -0.1110
0.1249  -0.0278  -0.0278  0.0555  -0.0278
0.0833  0.0416  0  0.0278  -0.0278
0.0555  0.0208  0  -0.0278  0
0.0139  0.0139  0  -0.0278  0
0.0902  0.0416  0  0  -0.0555
0.0416  -0.1110  -0.0694  0.0720  0.1318
0.1665  0.0555  0.0278  0.0555  -0.0833
-0.1110  -0.0555  0.0833  0.0416  0.1561
0.1388  0.0416  -0.0555  0.0416  -0.0416
-0.1110  0.0130  -0.0278  -0.0486  -0.1110
-0.0139  0  -0.0139  -0.0278  0.0139
0.1266  0.0139  -0.1318  0  -0.0555
0.0833  0.0278  0.0278  0.0694  -0.0278
-0.0278  0  0.0555  0.0416  -0.0278
-0.0555  0.0694  0.0555  -0.0139  -0.1110
-0.0555  -0.0555  0.0278  0  0.0971
Results for 200×5 data set:

The original signal matrix is:

Rawdatamatrix =

\[
\begin{bmatrix}
0.5038 & 0.6723 & 0.1982 & 0.7449 & 0.0560 \\
0.6128 & 0.4315 & 0.1951 & 0.8923 & 0.8169 \\
0.8194 & 0.6944 & 0.3268 & 0.2426 & 0.5289 \\
0.5319 & 0.2568 & 0.8803 & 0.1296 & 0.6944 \\
0.2021 & 0.0098 & 0.4711 & 0.2251 & 0.2124 \\
0.4539 & 0.5323 & 0.4040 & 0.3500 & 0.5433 \\
0.4279 & 0.2794 & 0.1792 & 0.2871 & 0.7025 \\
0.9661 & 0.9462 & 0.9689 & 0.9275 & 0.9564 \\
0.6201 & 0.9064 & 0.4075 & 0.0513 & 0.4445 \\
0.6954 & 0.3927 & 0.8445 & 0.5927 & 0.0854 \\
0.7202 & 0.0249 & 0.6153 & 0.1629 & 0.0573 \\
0.3469 & 0.6714 & 0.3766 & 0.8384 & 0.6295 \\
0.5170 & 0.8372 & 0.8772 & 0.1676 & 0.7962 \\
0.5567 & 0.9715 & 0.7849 & 0.5022 & 0.6912 \\
0.1565 & 0.0569 & 0.4650 & 0.9993 & 0.3453 \\
0.5621 & 0.4503 & 0.8140 & 0.3554 & 0.9468 \\
0.6948 & 0.5825 & 0.8984 & 0.0471 & 0.5202
\end{bmatrix}
\]
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0.0001 & 0.0001 & 0.0000 & 0.0000 & 0.0013
\end{bmatrix}
\]

The principal components for all the data set is:
24.05%
21.36%
20.81%
17.69%
16.10%

The difference between the original data set and the derived data set:

\[
1.0e-016 * \\
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\]
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| 0 | -0.1952 | -0.1388 | 0.2776 | 0.1388 |
| 0.0694 | 0 | 0 | 0.1388 | 0 |
| -0.0694 | 0 | 0.1041 | 0.2082 | -0.1388 |
| 0 | 0.2776 | -0.1388 | -0.4163 | -0.1388 |
| 0.1388 | 0.1388 | 0 | 0.3383 | -0.0694 |
| 0 | 0.0694 | -0.2776 | -0.1388 | 0.0087 |
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| 0.0347 | -0.0347 | 0 | -0.2776 | 0.0694 |
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| 0.0694 | 0 | 0.0694 | 0.2776 | 0 |
| 0.1388 | 0.1388 | 0 | 0.0694 | -0.0173 |
| 0.1388 | 0.1388 | -0.1388 | 0.0694 | 0 |
| 0.0694 | 0 | -0.1388 | 0.2429 | 0 |
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PCA analysis of image, sound and vibration signals
List of presentations

2. “Project introduction and recent progress”, at CSIRO Mineral (Clayton), August 30th, 2007
6. “Progress on multivariate statistical analysis of ladle eye phenomena”, at Onesteel company, November 10th 2008
10. “Measuring the stirring process”, at Onesteel company, June 25th, 2009