The thermal state of the intergalactic medium 9–12 billion years ago

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"Remembrance of things past is not necessarily the remembrance of things as they were."

Marcel Proust
Abstract

This thesis presents an investigation of the thermal state of the intergalactic medium in the redshift range spanning the He II reionization (1.5 \( \lesssim z \lesssim 3.8 \)). This event is expected to leave imprints in the thermal history of the cosmic gas, particularly a peak in the temperature at the mean gas density \( (T_0) \) and a possible trough in the parameter \( \gamma \) which defines the power-law slope of the temperature–density relation. Evidence for heating has been found at 3 \( \lesssim z \lesssim 4 \) but the evidence for a subsequent cooling below \( z \approx 2.8 \) is weaker and depends on the evolution of \( \gamma \) which is still poorly constrained.

Here we report new temperature measurements using the flux “curvature”, which characterizes the H I Lyman-\( \alpha \) forest absorption line shapes, in a sample of 60 quasar spectra from the Very Large Telescope which, for the first time, extend down to the lowest optically-accessible redshifts, \( z \approx 1.5 \). We find that the temperature of the cosmic gas, \( T(\Delta) \), increases for increasing overdensity (\( \Delta \)), from \( T(\Delta) \approx 22670 \) K to 33740 K in the redshift range \( z \approx 2.8 - 1.6 \). Assuming reasonable \( \gamma \) values, we translate the \( T(\Delta) \) measurements into values of \( T_0 \) and find evidence for at least a flattened \( T_0 \) evolution or, in the case of \( \gamma \approx 1.5 \), a cooling temperature at redshifts \( z \lesssim 2.8 \). This reversal of the heating trend seen at 2.8 \( \lesssim z \lesssim 4 \) in previous results, and in our new measurements, is likely to indicate the completion of the reheating associated with He II reionization.

An unequivocal statement about the thermal evolution requires a tight constraint on the evolution of \( \gamma \). Here we present a new method to obtain independent measurements of \( \gamma \) using the ratio of curvatures in the Lyman-\( \alpha \) and \( \beta \) forests, \( \langle R_\kappa \rangle \). This ratio appears robust against relevant sources of observational uncertainties and is relatively fast and simple to compute. Applying this method to a sample of 27 quasar spectra, we obtain preliminary results broadly consistent with a \( \gamma \sim 1.5 \) for 2 \( \lesssim z \lesssim 3.8 \) with little evolution over this redshift range. The statistical uncertainties are promising: our sample provides \( \sim 6\% \) statistical errors in \( \langle R_\kappa \rangle \) in \( \Delta z \sim 0.6 \) redshift bins, corresponding to \( \lesssim 10\% \) uncertainties in \( \gamma \). However, possible systematic uncertainties arising from assumptions about the evolution of the gas thermal state require refined future investigation.

Finally, the possibility of using the He II Lyman-\( \alpha \) forest curvature to measure \( \gamma \) is also explored. Currently, the quality of observed He II forest spectra is too low to apply this technique. However, we demonstrate, using a convergence analysis, that the available suite of hydrodynamical simulations must also be significantly improved to even begin testing the viability of this method.
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My family, for the inexhaustible patience and understanding, in good and bad times.
Declaration

This thesis contains no material which has been accepted for the award of any other degree or diploma. To the best of my knowledge, it contains no material previously published or written by another person, except where due reference is made in the text of the thesis. All work presented is primarily that of the author.

Chapter 2, 3 and Appendix A have been published as:

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The thermal history of the intergalactic medium down to redshift z=1.5: a new curvature measurement.

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The tests presented in Chapter 5 have been conducted with the collaboration of Edoardo Tescari.

Minor alterations have been made to this work in order to maintain consistency of style. I acknowledge helpful discussions and critical feedback that were provided by my co-authors.

Elisa Boera
August 2015
To my family
Abstract

Acknowledgements

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In modern cosmology, the study of the properties of the intergalactic medium is one of the key goals for a more comprehensive understanding of galaxy evolution and formation. Starting from a very hot plasma made of electrons and protons after the Big Bang, to the gas that now fills the space between galaxies, the intergalactic medium (IGM) has been one of the main “recorders” of the different phases of evolution of the Universe. In fact, it is the thermodynamic state and chemical composition of this gas, the main reservoir of baryons in the Universe, that determined the conditions for the formation of the structures that we can observe today. In particular, the IGM thermal history can be an important source of information about the processes that injected vast amounts of energy into the intergalactic medium on relatively short cosmological timescales: the reionization events. Within the broader aim of a better understanding of the properties of the IGM and of the mechanisms governing its evolution, this thesis focuses on constraining the thermal state of the cosmic gas in the low-redshift Universe (1.5 \( \lesssim z \lesssim 3.8 \)), principally addressing the following open science questions: What is the thermal state of the IGM at low redshift and what are the possible heating processes that could explain it? and How is the temperature–density relation of the IGM evolving in the recent Universe and what are the possible phenomena that have determined it?

While a more detailed motivational context can be found at the beginning of each Chapter, this Introduction gives a general background about the physical processes and the techniques relevant for a fuller understanding of the thesis.

1.1 The origin of the IGM

According to the Standard cosmological model, after the Big Bang the Universe was filled with an hot plasma made mainly of electrons and protons in rapid thermal motion.
Chapter 1. Introduction

For hundreds of thousands of years the radiation was coupled with matter, with visible and ultraviolet (UV) photons scattered inside this ionized medium. As the expansion of the Universe proceeded the temperature of the cosmic gas decreased until, at $z \sim 1100$, the temperature dropped down to few thousand degrees Kelvin, low enough to allow the protons and electrons to recombine into neutral hydrogen (H\textsc{i}), marking the cosmic recombination phase. As the particles recombined, the scattering of the photons became rarer and rarer and finally the radiation was let free to travel undisturbed, determining the origin of the Cosmic Microwave Background (CMB). After the recombination, the Universe entered the so-called Dark Ages: it was left filled by neutral gas mainly made of hydrogen and for a small quantity (i.e. $\sim$25 per cent by mass) of helium. Its evolution was driven by the continuous gravitational collapse of overdense regions that, after hundreds of millions of years, allowed the formation of the first galaxies and their stars. These new structures and their UV radiation subsequently determined dramatic changes in the properties of the cosmic gas, including the IGM.

1.2 The search for the IGM

The vast majority of all that is known about the properties and the structures of the IGM comes from the study of optical and UV spectra. While attempts to detect the IGM have started even before, the finding of new brilliant objects called Quasi-Stellar Objects or QSOs (e.g. Schmidt [1963]) allowed significant improvements in the measurements of the intergalactic H\textsc{i} density. Studying the decrease in flux blueward the Lyman-$\alpha$ (Ly-$\alpha$) emission line of a QSO at $z = 2.01$, it was suggested that the cosmic mass density of neutral hydrogen was much smaller than what expected from cosmological predictions, showing the first evidence that the IGM had been reionized (Gunn & Peterson [1965]). In subsequent years, many absorption features were detected in higher resolution spectra, but until 1971 these were associated with intervening galaxy halos intercepted through the lines of sight. The real nature of these spectral imprints, as Ly-$\alpha$ absorption lines arising from discrete H\textsc{i} clouds, was reported for the first time by Lynds [1971]. Today, the sum of these discrete absorption features is called the Ly-$\alpha$ forest and after the disclosure of its intergalactic origin (Sargent et al. [1980]) it has represented the best laboratory to study the IGM.
1.3 The IGM absorption lines

The diffuse intergalactic gas can be studied detecting the absorption imprints that it leaves on the spectrum along the line of sight to a quasar. Figure 1.1 shows an example of flux density as a function of wavelength ($\lambda$) from the optical spectrum of a QSO at $z_{em} = 3.11$. The densely distributed, apparently discrete absorption features that constitute the Ly-$\alpha$ forest spread bluewards of the Ly-$\alpha$ emission line ($\lambda = 1215.67$ Å) down to the Lyman-$\beta$ (Ly-$\beta$) emission line ($\lambda = 4215.71$ Å) shortwards of which the Ly-$\alpha$ absorption is accompanied by subsequent higher orders of Lyman transitions. Longwards of the Ly-$\alpha$ emission line, metal absorbers give rise to fewer, narrow absorption lines. The properties of all the absorption features produced in a spectrum are determined by the equation of radiative transfer that describes the propagation of the radiation emitted by a background source thorough a medium of interest (in this case the IGM).

1.3.1 The equation of radiative transfer

The equation of radiative transfer describes how the absorption features are produced by the intervening intergalactic gas in the spectrum of a background quasar and is defined as follows:

$$\frac{1}{c} \frac{\partial I_\nu(r, t, \hat{n})}{\partial t} + \hat{n} \cdot \nabla I_\nu(r, t, \hat{n}) = -\alpha_\nu(r, t, \hat{n})I_\nu(r, t, \hat{n}) + j_\nu(r, t, \hat{n}),$$

(1.1)

where $I_\nu(r, t, \hat{n})$ represents the specific intensity that characterizes the source of radiation, $c$ is the speed of light, $\alpha_\nu(r, t, \hat{n})$ is the attenuation coefficient of the intergalactic medium and $j_\nu(r, t, \hat{n})$ is the emission coefficient that describes the local specific luminosity per solid angle per unit volume emitted by the source. The specific intensity at any given time $t$ and position $r$ represents the rate at which the energy, carried by photons of frequency $\nu$ in the direction $\hat{n}$, crosses a unit area per unit solid angle per unit time (Meiksin 2009).

In the case of a single background source, such as a QSO, $j_\nu = 0$ and the solution of Equation 1.1 will depend only on the way in which the incident radiation is attenuated by absorption and scattering of the photons due to the intervening gas. The attenuation coefficient is defined as follows:

$$\alpha_\nu(r, t, \hat{n}) = \rho(r, t)\kappa_\nu(r, t, \hat{n}) + n(r, t)\sigma_\nu(r, t, \hat{n}),$$

(1.2)

where $\rho(r, t)$ is the mass density of the gas, $\kappa_\nu(r, t, \hat{n})$ is its opacity, $n(r, t)$ is the number density of scattering particles of a specific mean mass $\bar{m}$ and $\sigma_\nu(r, t, \hat{n})$ is the scattering
Figure 1.1 An example of a typical high resolution QSO spectrum at $z = 3.11$. The spectrum has been retrieved from the archive of the Ultraviolet and Visual Echelle Spectrograph (UVES) on the Very Large Telescope (VLT) and has a resolving power $R \simeq 50000$. The peak at $\lambda \simeq 4996 \text{Å}$ is the Ly-\(\alpha\) emission line of the quasar that divides the spectrum into two main regions: the Ly-\(\alpha\) forest, lying bluewards of this feature and the region where metal absorption systems give rise to rarer and narrower lines redwards of the emission line.
1.3. The IGM absorption lines

The absorption features arise from the scattering of photons traveling from the background quasar through an intergalactic medium with number density \( n(\mathbf{r}, t) \). The resonance line scattering cross section will depend on the specific characteristics of the transition considered (rest-wavelength \( \lambda_0 \) or frequency \( \nu_0 \) and oscillator strength \( f \)) but also on the thermodynamic condition of the ensemble atoms. While the Lorentz profile well describes the scattering cross-section for a scatterer at rest, the IGM atoms are generally affected by thermal motion and may have additional components due to peculiar velocity flows or turbulent motion if in shocked or collapsed regions. Taking into account the thermal motion of the particles and neglecting non-thermal velocities, the resonance line scattering cross-section is obtained using the Voigt profile that, found by convolving the Lorentz profile with a Gaussian, well incorporates the thermal broadening into the line profile:

\[
\sigma_{\nu} = \left( \frac{\pi e^2}{m_e c} \right) \frac{1}{4\pi\epsilon_0} f \varphi_V(p, \nu),
\]

(1.3)

where \( \varphi_V(p, \nu) = \frac{1}{\pi^{1/2} \Delta \nu_D} H(p, x) \) is the normalized Voigt profile defined by the Voigt function \( H(p, x) \) and the Doppler width \( \Delta \nu_D = \nu_0 b/c \). The Doppler parameter \( b \) is generally defined with its thermal component \( b_{th} = \left( \frac{2k_B T}{m} \right)^{1/2} \) where \( k_B \) is Boltzmann’s constant and \( m \) is the mass of the atoms, but a kinematic component (\( b_{kin} \)) can be added to take into account possible turbulence effects (e.g. Pradhan & Nahar 2011).

The optical depth to absorption at the frequency \( \nu \), due to the presence of intervening gas along a line of sight (at the position \( s' \) and time \( t' \)) from the source position \( (s_0, t_0) \) to a position \( s \), is then given by the solution of Equation 1.1:

\[
\tau_{\nu} = \int_{s_0}^{s} ds' n(s', t') \sigma_{\nu}
\]

(1.4)

where \( \nu' = \nu a(t)/a(t') \) according to the frequency redshift due to the expansion factor \( \nu \propto a(t)^{-1} \). From the optical depth the corresponding attenuation of the intensity of the background QSO can be obtained as \( e^{-\tau_{\nu}} \).

While the above description is valid for any atomic transition of interest, the work presented in this thesis will mainly be focused on the Lyman-\( \alpha \) transition of \( \text{H} \, \text{i} \) (Chapters 2-3-4). Nevertheless, the \( \text{H} \, \text{i} \) Lyman-\( \beta \) transition will also be considered in Chapter 4 and the \( \text{He} \, \text{ii} \) Lyman-\( \alpha \) transition will be used in Chapter 5.

\[H(p, x) = \frac{2}{p} \int_{-\infty}^{+\infty} dy \frac{e^{-x^2}}{\sqrt{(x-y)^2 + p^2}} \quad \text{with} \quad p = \frac{\Gamma_0}{(4\pi \Delta \nu_D)} \quad \text{and} \quad x = \frac{(\nu - \nu_0)}{\Delta \nu_D}. \quad \Gamma_0 \text{ is the radiation damping width of the transition (e.g.} \Gamma_0 = 6.262 \times 10^8 \text{s}^{-1} \text{ for the H} \, \text{i Ly-\( \alpha \)).}
1.3.2 The Lyman-α forest

Since the first discovery, the sum of the absorption lines arising from Ly-α absorption in diffuse H\textsc{i} gas along the line of sight to a quasar has proved to be the main laboratory to study the properties of the IGM at different redshifts. Observations have shown that the Ly-α absorption in high redshift quasar spectra creates a dense aggregation of discrete lines (Lynds 1971). Such a distribution, called Ly-α forest (Weymann et al. 1981), suggested that these features arise in distinct localized regions of the IGM, implying the presence of an inhomogeneous density field (Sargent et al. 1980). Currently, the physical nature of the Ly-α absorbers is commonly associated with fluctuations in the density field of the IGM that result in a continuum of absorption rather than in a sum of discrete absorbers (e.g. Miralda-Escude & Rees 1993; Zhang et al. 1998). To this day, the availability of new instruments and spectroscopic capabilities has led to a detailed analysis of the possible connection between galaxies and absorption lines, allowing a more detailed classification of the Ly-α absorption systems, based mainly (but not only) on the basis of their H\textsc{i} content (e.g. Weymann et al. 1981; Cowie et al. 1995; Rauch 1998; Wolfe et al. 2005; Prochaska et al. 2005). The properties of the Ly-α absorption systems and their classification are discussed below.

The Ly-α optical depth

As a first approximation, for $z \lesssim 4$ it is possible to describe the Ly-α forest as a sum of discrete absorbers with a total optical depth ($\tau_\nu$) given by the sum of the individual optical depths, $\tau_\nu(i)$, corresponding to locally overdense regions (Meiksin 2009):

$$\tau_\nu = \sum_i \tau_\nu(i) = \pi^{1/2} \tau_0 (\phi_V(p,x))$$

(1.5)

where $\phi_V = \frac{H(p,x)}{\pi^{1/2}}$ is averaged over the line of sight, weighted by the density and $\tau_0$ is the optical depth at line centre defined as:

$$\tau_0 = \frac{N \sigma \lambda}{\pi^{1/2} b} = \frac{e^2 \pi^{1/2}}{m_e c} \left[ \frac{1}{4 \pi \epsilon_0} \right] \frac{N \lambda_0 f}{b \lambda_0 f}$$

(1.6)

as a function of the column density $N$. For the H\textsc{i} Lyα ($\lambda_{0\alpha} = 1215.67\text{Å}$, $f_\alpha = 0.4164$) Equation 1.6 becomes:

$$\tau_0 \sim 0.38 \left( \frac{N(\text{H} \textsc{i})}{10^{13}\text{cm}^{-2}} \right) \left( \frac{b}{20\text{km s}^{-1}} \right)^{-1}$$

(1.7)
1.3. The IGM absorption lines

The Lyα absorption is highly sensitive to the presence of even small amounts of neutral hydrogen, for this reason at higher redshifts the distinction between individual lines becomes more difficult. The total Ly-α optical depth increases up to a redshift \( z \gtrsim 5.5 \) where the individual lines merge together forming the effect of a trough in the spectrum (Gunn & Peterson 1965).

Equivalent width and curve of growth

For the purpose of determining abundances and to study blended absorption features, it is useful to define a quantity corresponding to the area between the line profile and the continuum, the equivalent width. The equivalent width, \( W_\lambda \) is usually defined in wavelength notation as:

\[
W_\lambda = \frac{\lambda^2}{c} \int (1 - e^{-\tau_\nu})d\nu.
\]  

(1.8)

For a Voigt profile, \( W_\lambda \) depends on the different parameters that define the line shape, but particularly interesting is its correlation with temperature and column density, well represented through the curve of growth. In Figure 1.2 is presented a reproduction of the curve of growth for hydrogen Ly-α; it is possible to distinguish three different segments, related to different line profiles (Pradhan & Nahar 2011, Meiksin 2009):

- The linear part: in optically thin regions \( (\tau \ll 1) \) the equivalent width should increase linearly as the number of ions. The line deepens and broadens in direct proportion to the flux removed from the continuum by an increasing number of absorbers, i.e. \( W_\lambda \sim N(H_\text{i}) \).

- The logarithmic part: corresponds to saturated line profiles, when the density of ions is sufficient to absorb nearly all of the continuum photons at the line center wavelength. In this regime the increase in density results in a slow increase in \( W_\lambda \sim \sqrt{\ln N(H_\text{i})} \). For these features the equivalent width allows a precise measurement of the Doppler broadening.

- The square-root part: represents the case in which the line profile is dominated by the damping wings. The wings, on both the side of the line center are enhanced as the column density increases and \( W_\lambda \sim \sqrt{N(H_\text{i})} \).

While this distinction refers to equivalent widths computed for a single Voigt profile (corresponding to an isolated absorption line), it is important to note that in the real IGM, with a fluctuating optical depth of Ly-α absorption, there is no clear way of defining the
The classification of Ly-\(\alpha\) absorption systems

The classification of the different absorption systems seen in quasar spectra is mainly due to the physical properties of the gas in which they originate (e.g. neutrality and density). These characteristics are, in turn, tightly connected with physical process such as radiation self-shielding and ionizing continuum absorption. However, in practice, Ly-\(\alpha\) absorption systems are broadly separated into three categories corresponding to their different column density ranges, as shown in Figure 1.2. Historically, absorption systems with \(\log N(\text{H}^1) \lesssim 17.2\) are called Ly-\(\alpha\) forest, those with \(17.2 \lesssim \log N(\text{H}^1) < 20.3\) Lyman limit systems (LLSs) and those with \(\log N(\text{H}^1) \geq 20.3\) damped Ly-\(\alpha\) absorbers (DLAs) (e.g. Weymann et al. 1981; Rauch 1998; Charlton & Churchill 2000; Wolfe et al. 2005; Prochaska et al. 2005).

The number of systems per unit of redshift has been found to decrease as their column density increases. That is, the Ly-\(\alpha\) forest absorbers are the most common, they mainly contain ionised hydrogen and may be associated with metals absorbers (e.g. Songaila 1998; Tytler et al. 2004). The LLSs have similar characteristics but, due to their higher column

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Figure 1.2 Reproduction of Figure 3 from Charlton & Churchill (2000). The curve of growth for the Ly-\(\alpha\) transition, relating the equivalent width \(W\) of an absorption feature to its column density \(N(\text{H}^1)\). Different line-styles show the effect of different Doppler \(b\) parameters in the logarithmic part. Different background colours mark the distinction, based on the column density ranges, among different types of absorbers: Ly-\(\alpha\) forest (blue), LLS (yellow), DLA (red).
density they are defined to be optically thick at the Lyman limit (912Å) (e.g. O’Meara et al. 2007). The hydrogen contained in DLAs is, instead, mainly neutral and this makes them particularly interesting reservoirs of gas for star formation at high redshift (e.g. Wolfire et al. 2003, Nagamine et al. 2004, Wolfe et al. 2005). Moreover, DLAs are often associated with halos of intervening galaxies along the line of sight and the presence or the lack of significant amounts of associated metals have been used to obtain important information about galaxies formation and evolution (e.g. Wolfe et al. 2005, Fumagalli et al. 2011).

On the theoretical side, models to reproduce synthetic Ly-α absorption systems have progressed with the aim of a better understanding of their origin and their place within the context of the standard theory of structure formation. Using both semi-analytical and full hydrodynamical simulations, it was shown that the low density absorption features of the Ly-α forest arise naturally from the fluctuations of the continuous medium formed by the gravitational collapse of the initial density perturbations (e.g. Gnedin & Hui 1998, Theuns et al. 1998). Moreover, the simulations show a variety of morphologies in the absorbing structures that correspond to different physical densities and H\textsc{i} column densities in the Ly-α forest absorption systems (e.g. Zhang et al. 1998). According to these models, most of the volume of the Universe is occupied by low density gas with a $\rho/\bar{\rho} \lesssim 10$, where $\bar{\rho}$ is the mean baryon density of the IGM.

The broad aim of this work is to explore statistically the connection between the line shapes of the absorption features and the properties of the IGM. It is therefore clear that our objects of interest must mainly be the low column density Ly-α forest features, that incorporate information about the physics of the majority of the gas in which galaxies are embedded. While present along the lines of sight used in this work, higher column density systems – DLAs and LLSs – have been masked out of our analysis, as have the narrow metal lines which arise from these systems and which contaminate the forest region.

1.4 The IGM Reionizations

Given the high sensitivity of the Ly-α absorption to the amount of neutral hydrogen, the lack of full absorption in quasar spectra implies that the IGM has been highly reionized after the recombination phase, and that it has been kept ionized from high redshift to the present. In Figure 1.3 is presented a schematic overview of the three identifiable epochs of reionization, one of hydrogen and two of helium. Given the similar ionization potential and recombination rate for both H\textsc{i} and neutral helium (He\textsc{i}), likely they have been reionized by the same radiation sources and, therefore, only the epochs of hydrogen reionization and
full helium reionization to He\textsc{iii} are generally thought of as distinct.

**Figure 1.3** A summary of the main phases of evolution of the IGM, from the neutral gas that filled the Universe after the recombination to the reionized gas that fills the space between galaxies after the three reionization events driven by stars and quasars. The ionization potentials for the different transitions are also reported.

### 1.4.1 Hydrogen reionization

From observations of the cosmic microwave background (CMB) polarization, produced by the Thomson scattering between free electrons and CMB photons, it is possible to obtain constraints on the time when the UV radiation emitted by the first objects, mostly stars (Haehnelt et al. 2001), was able to photoionize the cosmic neutral hydrogen (and He\textsc{i}). Recent measurements from the three-year Planck mission suggest a value of the integrated Thomson scattering optical depth $\tau_{el} = 0.066 \pm 0.012$ (Planck Collaboration et al. 2015). This corresponds to an instantaneous reionization taking place at $z = 8.8^{+1.2}_{-1.1}$, representing an approximate intermediate epoch between the beginning and the end of an extended reionization process. The end of this first Epoch of Reionization (EoR) is still not really well constrained: studies of the H\textsc{i} Ly-\alpha absorption features of several quasars have shown that the intergalactic hydrogen was completely reionized by $z \simeq 6$ (e.g. Becker et al. 2001, Fan et al. 2006, McGreer et al. 2015). This redshift corresponds to a rapid rise of the
1.4. The IGM Reionizations

HI Ly-α optical depth ($\tau_{\text{HI}}$), from low to high redshifts, due to the increased amount of neutral HI before the completion of the reionization. However, because only a small neutral fraction is adequate for providing a large $\tau_{\text{HI}}$, this interpretation is not unique (Becker et al. 2007). Moreover, some evidence for the final stages of a patchy hydrogen reionization has been found recently in Becker et al. (2015) at $z \sim 6$, suggesting a later end of this process (at $z \sim 5$). If for $z \gtrsim 6$ the Ly-α forest becomes so thick that only lower limits on the optical depth and, therefore, on the end of the HI reionization can be obtained, the second Reionization Epoch, that of He II at $z \sim 3$, is potentially much more accessible to direct observations.

1.4.2 He II reionization

Because the ionization potential of He II (from He II to He III) is 54.4 eV and fully ionized helium recombines more than 5 times faster than hydrogen, the second helium reionization event began later, when quasars started to dominate the UV background (e.g. Miralda-Escude 1993; Miralda-Escudé & Rees 1994; Miralda-Escudé et al. 2000). Theoretically, their much harder photons would have been able to fully ionize He II at a redshift $3 \lesssim z \lesssim 4.5$, but these estimates change depending on assumptions about the abundance of QSOs and the hardness of their spectra (Meiksin 2005).

Direct observation

The most direct evidence for He II reionization derives from measurements of the He II Ly-α optical depth ($\tau_{\text{HeII}}$). The evolution of $\tau_{\text{HeII}}$ along a line of sight to a quasar traces the presence of intergalactic He II ions: if, after the He II reionization, these ions are located in discrete clumps they will produce discrete absorption lines; however, if they are diffused throughout the IGM (as expected before the reionization) they will smoothly depress the flux level of a quasar bluewards of its He II emission line (e.g. Heap et al. 1998; Miralda-Escude et al. 2000). The latter is known as the “Gunn–Peterson absorption” (Gunn & Peterson 1965).

There are some advantages in the physics of the He II Gunn–Peterson effect that make its observation potentially more reliable with respect to the hydrogen one. In fact, due to the later redshift of its reionization, the lower abundance of helium versus hydrogen, the shorter wavelength of the He II Lyman-α line (304 Å) and the density fluctuations in the IGM, the He II Gunn–Peterson trough is sensitive to ion fractions $x_{\text{HeII}} \gtrsim 0.01$, while the hydrogen one saturates at $x_{\text{HI}} \simeq 10^{-5}$ (Fan et al. 2006; McQuinn et al. 2009). In addition, luminous quasars create large fluctuations in the ionizing background, making the flux
transmission possible even during the early stages of He II reionization (Furlanetto & Oh 2009).

The intergalactic He II Ly-α transition, at redshifts relevant for the reionization, is observed in the far UV, its detection in quasar spectra is particularly difficult because intervening low-redshift hydrogen absorption along the line of sight can severely attenuate the quasar flux. It is also observable from space only at $z > 2$ due to the Galactic H I Lyman limit. Most observations of the He II Lyman-α forest have come from the Hubble Space Telescope (HST), although some observations of a few brighter targets were possible with the Hopkins Ultraviolet Telescope and the Far Ultraviolet Spectroscopic Explorer. However, only a few percent of known $z \simeq 3$ quasar sightlines were found to be “clean” (free of intervening H I Lyman limit systems) down to the He II Lyman limit, and therefore usable for a possible detection of the Gunn–Peterson effect. It has been detected at $z \gtrsim 3$ [Heap et al. 1998, Zheng et al. 2004], whereas the absorption becomes patchy at $z \lesssim 3$, reflecting an intermediate phase of reionization, and evolves into He II Ly-α forest (discrete absorption lines) at $z \lesssim 2.7$ (e.g. Shull et al. 2010; Worseck et al. 2011). In recent years, the advent of the Galaxy Evolution Explorer (GALEX) UV maps and the installation of the Cosmic Origin Spectrograph (COS) on HST have allowed very high quality re-observations of such known “He II quasars” (Shull et al. 2010) and the discovery of 2 new ones at $z < 3$ (Worseck et al. 2011). But even if in these recent works the end of the He II reionization seems to be observed at $z \simeq 2.7$ (e.g. Worseck et al. 2011; Syphers & Shull 2013), there are strong variations between the sightlines and any current constraint on the physics of this phenomenon is limited by the cosmic variance among this small sample studied in detail.

Waiting for the promising cross-matching of new optical quasar catalogs (e.g. BOSS) with GALEX UV catalogs, that has been very effective in finding new He II quasars (e.g Syphers et al. 2012), and looking forward to new higher resolution observations with COS, other indirect methods have been developed to obtain a detailed characterization of the He II reionization.

**Indirect evidence**

Various indirect methods of observing the He II reionization have been used, either by examining metal systems or the H I Ly-α forest. Variation of metal-line ratios (like C IV/Si IV; Songaila 1998) above or below the He II Lyman limit (228 Å) can be used to determine changes in the ionizing UV background connected with changes in the helium opacity (Furlanetto & Oh 2009). These measurements generally agree with a $z \simeq 3$ reionization
1.5. The IGM thermal state

but can be affected by metallicity variation that can complicate understanding them. The majority of the indirect evidence for the He II reionization comes from the attempts to exploit the IGM heating associated with this epoch and its effect on the H I Ly-α forest. In particular, one of the effects of an injection of a substantial amount of energy could have been a lower average H I Ly-α opacity \cite{Bernardi2003, Faucher-Giguere2008}. However, a small “dip” in the H I Ly-α opacity observed at \( z \approx 3.2 \) \cite{Faucher-Giguere2008} does not have a straightforward interpretation \cite{Bolton2009, McQuinn2009} and has not been confirmed in recent and refined measurements \cite{Becker2013}. On the other hand, the best indirect evidence of the reionization period so far seems to be provided by the study of the IGM temperature evolution.

1.5 The IGM thermal state

One of the main impacts of the Epochs of Reionization is on the thermal state of the intergalactic medium: the IGM cooling time is long, so the low density gas retains some ‘memory’ of when and how it was reionized \cite{Hui1997}. At different redshifts, the thermal state of the intergalactic medium can be described through its temperature–density \((T-\rho)\) relation. In the simplest scenario, for gas at overdensities \( \Delta \lesssim 10 \) \((\Delta = \rho/\bar{\rho}, \) where \(\bar{\rho}\) is the mean density of the IGM\), cosmological simulations show a tight power-law relationship between temperature and density:

\[
T(\Delta) = T_0 \Delta^{\gamma-1},
\]

where \(T_0\) is the temperature at the mean gas density \cite{Hui1997, Valageas2002}. The reason why the same power-law holds for a vast range of overdensities \((\Delta \sim 0.1 - 10)\) has been explored in different works \cite{Theuns1998, Puchwein2014} and can be intuitively attributed to the balance between photoheating by the UV background and the adiabatic cooling due to Hubble expansion \cite{McQuinn2015}. It has been shown that this relation is likely to be established in \(\Delta z \sim 1 - 2\) after the end of each of the reionization events \cite{Hui1997, Trac2008, Furlanetto2009, McQuinn2009}. While the dynamics of the reionization processes are still partially unclear, constraining the evolution of the parameters \(T_0\) and \(\gamma\) as a function of redshift will describe, with high precision, the evolution of the IGM thermal state, and bring important insights about these events. Different cosmological simulations, with different prescriptions for heating and radiative...
mechanisms, predict that, during the reionization, the parameters $T_0$ and $\gamma$ can undergo variations that, if identified, can robustly characterize the timing and physical mechanisms that drove the evolution of the cosmic gas (e.g. Bolton et al. 2004; McQuinn et al. 2009; Compostella et al. 2013).

While in principle the evolution of the IGM $T-\rho$ relation can be studied for both the reionization epochs, in practice the strong blending of the absorption features and the lower quality of the quasar spectra do not yet allow a reliable analysis at $z \gtrsim 5$ (but see Lidz & Malloy 2014). For this reason the work presented here is focused on the redshift range spanning the He II reionization event ($1.5 \lesssim z \lesssim 3.8$).

### 1.5.1 The IGM thermal state during the He II reionization

It is predicted that the IGM is reheated to several $10^4$ K by photo-heating during the completion of the He II reionization at $z \simeq 3$, leaving an ‘imprint’ on its temperature evolution. This boost in temperature is predicted in the case in which the UV photons that ionize the neutral gas are absorbed with higher energy than the ionization potential of the atoms, and so the free electrons will be released with some energy to share with the surrounding baryons through Coulomb scattering (e.g. McQuinn et al. 2009).

**The evolution of the parameter $T_0$**

Figure 1.4 presents a qualitative prediction for the evolution of $T_0$ during the He II reionization driven by the UV radiation emitted by quasars, motivated by several models of thermal thermal state evolution (e.g. Schaye et al. 2000; McQuinn et al. 2009; Puchwein et al. 2014). Before the reionization, the condition of balance between photoheating and cooling, mainly due to the adiabatic expansion of the Universe, results in a thermal asymptote, completely determined by the shape of the ionizing spectrum (Hui & Haiman 2003). Generally, quasar ionizing spectra are harder than stellar spectra and this is reflected in a higher temperature asymptote. At the mean density the characteristic signature of reionization is expected to be a peak: a marked heating followed by a subsequent cooling due to adiabatic expansion at the end of which the temperature of the gas will return to follow a new asymptotic decrease. If quasars dominate the UV background (UVB) after the He II reionization, the expectation is to be able to distinguish a shift toward higher temperatures in the new thermal asymptote at $z \lesssim 2$. A precise constraint on the evolution of $T_0$ is therefore of fundamental importance because it provides timing of the reionization event and also possible information about its sources.
1.5. The IGM thermal state

The evolution of $T_\rho$ during the He II reionization

![Diagram showing the evolution of $T_\rho$ during He II reionization.](image)

Figure 1.4 Qualitative predictions for the possible evolution of the temperature at the mean gas density during the He II reionization. Top panel: schematic representation of the IGM reionization process, from the overdense regions, where we expect quasars (black symbols) to switch on, the ionizing bubbles (yellow patches) expand and overlap, completing the reionization of the entire medium. Bottom panel: expected evolution of the temperature at the mean gas density during the reionization process; after a temperature peak, the cosmic gas is expected to return to follow a thermal asymptote determined by the hardness of the ionizing spectrum of the sources dominating the UVB.

The evolution of the parameter $\gamma$

The evolution of the parameter $\gamma$ that defines the slope of the IGM temperature–density relation (Equation 1.9) can be related with the manner in which the photoionization fronts expand and so with the topology of the reionization process (e.g. Furlanetto & Oh 2008, Compostella et al. 2013, Puchwein et al. 2014). In the condition of balance between photo-heating and cooling due to adiabatic expansion, assumed to apply both before and after the reionization event, the value of $\gamma$ is predicted to assume the asymptotic value of $\gamma \sim 1.6$ (Hui & Gnedin 1997). A $T$–$\rho$ relation with a positive slope (i.e. $\gamma > 1$) corresponds to a situation in which higher overdensities present higher temperatures because they are more bounded against the expansion and experience higher recombination rates (that will result in more atoms for the photoheating).

Depending on the topology of the He II reionization $\gamma$ could assume different values. So far predictions from cosmological simulations about how and if this variation would
The evolution of $\gamma$ during the He II reionization

Figure 1.5 Qualitative predictions for a possible evolution of the parameter $\gamma$ during the He II reionization. Top panel: schematic representation of the IGM reionization process, from the overdense regions, where we expect quasars (black symbols) to switch on, the ionizing bubbles (yellow patches) expand and overlap, completing the reionization of the entire medium. Bottom panel: expected evolution of the parameter $\gamma$ during the reionization; changes from the asymptotic values of $\gamma \sim 1.6$ can be expected during the reionization depending on the topology of the process.

happen rely on specific prescriptions of photoheating and radiative transfer mechanisms (e.g. Furlanetto & Oh 2008; Compostella et al. 2013; Puchwein et al. 2014). One often-proposed possibility is presented in Figure 1.5 (e.g. Furlanetto & Oh 2008; Bolton et al. 2008). According to this picture, if the reionization proceeds from high density regions (where quasars switched on) to low density ones, then the voids will be the last to reionize and reheat and, immediately after the end of the reionization, they will suffer the least amount of cooling and hence they will contain hot gas. It is then possible that, for a small redshift range, before the underdense regions cool down again, the IGM $T-\rho$ relation approaches an isothermal value ($\gamma \sim 1$). The study of the evolution of $\gamma$ in the redshift range spanning the reionization is therefore particularly interesting for understanding the physical process governing this event.

1.5.2 Previous methods to measure the $T-\rho$ relation

Temperature variations affect the structures of the H I Ly-\(\alpha\) forest and observations of this region of quasar spectra have been considered the best method to obtain information
on the IGM evolution. The widths and depths of the Ly-α lines are mainly set by the column densities of the absorbers, Hubble broadening as light travels across the absorbing gas, peculiar velocities and thermal broadening (e.g. Meiksin et al. 2010). Furthermore, recently it was shown that the broadening of these features is not only affected by the instantaneous temperature of the gas at the time of absorption but also indirectly by its thermal history at earlier times. This effect is referred as Jeans smoothing and reflects changes in the density distribution of the IGM on small scales (Gnedin & Hui 1998; Theuns et al. 2000; Rorai et al. 2013; Puchwein et al. 2014; Kulkarni et al. 2015).

Temperature information can be extracted from the analysis of the absorption line shapes but this process is not straightforward and complex cosmological simulations are required to characterize the large-scale structure and bulk motion of the IGM. For this reason any measurement of the IGM thermal state from the Ly-α forest, has been always strictly entangled in state of the art cosmological simulations that, while allowing a better understanding of the physical processes involved, introduce possibly relevant systematic uncertainties that have to be taken into account. Previous efforts can be divided into two main approaches: the study of individual absorption features and the quantification of the absorption structures with a global statistical analysis of the entire forest.

**Line-profile analysis**

The first method consists of decomposing the Ly-α spectra into a set of Voigt profiles. Since the minimum line width (b-parameter) can be considered dominated by thermal broadening, and since the column density (N) correlates strongly with the physical density, it is possible to trace the $T–\rho$ relation (Equation 1.9) using the low-b edge of the $b(N)$ distribution (Schaye et al. 2000; Ricotti et al. 2000; McDonald et al. 2001; Bolton et al. 2010; Rudie et al. 2013; Bolton et al. 2014). Even if simulations are needed to properly calibrate the relationship between the temperature–density relation and the $b(N)$ distribution, this technique represents the most direct test to simultaneously constrain the value of both the parameters $T_0$ and $\gamma$. Nevertheless, the results obtained so far are difficult to interpret because of the large statistical uncertainties: Schaye et al. (2000) and Ricotti et al. (2000) found some evidence for an increase in the IGM temperature and a decrease in the value of $\gamma$ at $z \simeq 3$, consistent with some expectations of He ii reionization while, in contrast, using similar data, McDonald et al. (2001) found a constant temperature over $z \sim 2–4$. Recently, the most precise measurement with this method has been presented in Bolton et al. (2014) at $z \simeq 2.4$. In their work they recalibrated with hydrodynamical simulations the line-profile analysis of Rudie et al. (2013), computed using a large sample.
Statistical analysis of the transmitted flux

While line fitting can be a time consuming and somewhat subjective process, a characterization of the transmitted flux using global statistical approaches, without decomposing it into individual absorption features, represents a much easier way to analyze the forest and extract information from the comparison with theoretical models (e.g. Kim et al. 2007). Examples of this general approach are the study of the flux probability distribution (PDF) based on pixel statistics (e.g. Bolton et al. 2008 Calura et al. 2012) and the wavelet analysis method (e.g. Theuns et al. 2002 Lidz et al. 2010).

The Ly-\(\alpha\) flux probability distribution function represents the simplest pixel statistic sensitive to the density distribution and the thermal state of the IGM. However, previous measurements of the parameter \(\gamma\) using this method have provided contradictory results: while in the recent work of Lee et al. (2015) using a large sample of 3393 spectra from the Baryon Oscillation Spectroscopic Survey (BOSS) they found that a \(\gamma = 1.6\) best describes the data over a redshift range \(z = 2.3 - 3.0\), previous measurements seemed to favor an ‘inverted’ temperature–density relation (\(\gamma < 1\)).

The work of Becker et al. (2007) first suggested that a \(\gamma < 1\) may provide a better fit to the data at low redshifts (\(z < 3\)). In this scenario, lower density regions are hotter, an interpretation that seems at first to be counterintuitive because denser gas is actually expected to trace regions more bounded against the expansion of the Universe, in which the adiabatic cooling is suppressed and the recombination is faster, yielding more neutral atoms per unit time for photo-heating. Evidence of a possible inverted \(T–\rho\) relation was also reported in other analyses (Bolton et al. 2008 Viel et al. 2009 Calura et al. 2012 Garzilli et al. 2012) and a possible explanation was suggested by considering radiative transfer effects (Bolton et al. 2008). Although it appears difficult to produce this result considering only \(\text{He}^{\text{II}}\) photo-heating by quasars (Bolton et al. 2009), a new idea of volumetric heating from blazar TeV emission seems to naturally explain an inverted temperature–density relation at low redshifts (Chang et al. 2012 Puchwein et al. 2012 Lamberts et al. 2015).

The current uncertainties in the constraints on the \(T–\rho\) relation through the PDF can be related to the sensitivity of this technique to a range of systematic uncertainties, among which the contamination by metal absorption lines have the major impact in high resolution and high signal-to-noise (S/N) spectra (Kim et al. 2007). Indeed, the uncertainties remain substantial using the wavelet decomposition, that involves the filtering of spectra of \(\text{H}^{\text{i}}\) absorbers (\(\sim 6000\)), and found a value of \(\gamma = 1.54 \pm 0.11\).
1.6. Thesis aim

In the scenario described in this Introduction, a general lack of knowledge about the behavior of the $T-\rho$ relation in the redshift range spanning the He\textsc{ii} reionization emerges. A further investigation of the evolution of the IGM thermal state during this phase assumes extreme importance in order to find constraints on the physics of the reionization process and on its effects on the intergalactic gas. Indeed, a better understanding in this field will bring important information about the radiation sources, the heating mechanisms and the phenomena that characterize the structure of the intergalactic medium in the latest chapter of its evolution.

Therefore, this thesis intends to obtain a better constraints on the thermal state of the IGM in the redshift range $z \simeq 1.5 - 3.8$ using a combination of high quality observational quasar spectra and state of the art hydrodynamical simulations. The general approach adopted here aims to study, statistically, the shapes of the features arising from the absorption by the intergalactic gas and, using the comparison with synthetic spectra, extract information about the evolution of the IGM temperature-density relation.

The statistic extensively used in this work is the curvature, introduced recently in the Ly-$\alpha$ forest analysis by Becker et al. (2011). The curvature statistic is able to deliver the most precise measurements of the IGM temperature at the gas overdensities traced by the Ly-$\alpha$ forest. However, probing only a narrow density range at each redshift, the curvature so far has not been used to constrain the parameter $\gamma$ of the $T-\rho$ relation. For this reason,
the investigation presented in this work will not be limited in the use of the curvature to find temperature constraints but it will also explore a “companion” statistic that should in figure allow precise measurements of $\gamma$ in order to obtain a complete characterization of the IGM temperature–density relation.

1.7 Thesis Structure

A detailed description of the curvature method, and of the procedures necessary to apply this statistic to the available observational spectra and suite of hydrodynamical simulations is presented in Chapter 2. After the selection and preparation of a uniform sample of high quality Ly-\(\alpha\) forest sections, synthetic spectra have been created and calibrated to match as closely as possible the characteristics of the real ones. A correct match of the noise properties and optical depth values between real and simulated spectra is shown to be particularly important in order to obtain reliable values for the characteristic overdensities ($\bar{\Delta}$) probed by the Ly-\(\alpha\) transition, at each redshift (see also Appendix A).

Chapter 3 describes and discusses the new measurements of the IGM temperature at the characteristic overdensities ($T(\bar{\Delta})$) probed by the forest. The temperature values at the mean gas density, $T_0$, are also obtained from the $T(\bar{\Delta})$ under the assumption of different $\gamma$ values. Because the curvature method, applied only to the Ly-\(\alpha\) forest region, does not allow an independent $\gamma$ constraint, new statistics have to be used to obtain a complete characterization of the $T$–$\rho$ relation. The results of Chapter 2 and Chapter 3 have been published in Boera et al. (2014).

Chapter 4 presents a new development of the curvature statistic that allows to obtain constraints on the parameter $\gamma$. This new statistic, the curvature ratio, represents, at each redshift, the ratio between the curvature computed from the HI Ly-\(\beta\) forest (contaminated with lower redshift Ly-\(\alpha\) absorption) and the corresponding Ly-\(\alpha\) forest along the same line of sight to a quasar. Because the overdensities contributing to the Ly-\(\beta\) absorption are, on average, higher than that for the Ly-\(\alpha\), the curvature ratio incorporates, at each redshift, information about the temperatures of two different gas densities and so will be sensitive to $\gamma$. The curvature ratio method is applied to our observational and simulated samples and the strengths and weakness of this method are explored and discussed. The preliminary results of Chapter 4 have been submitted for publication to Monthly Notice Letters of the Royal Astronomical Society but a more substantial description of our analysis is presented in this thesis and will be characterized by the text preceded by the symbol “*”.

Chapter 5 describes the preliminary results of an investigation aiming to understand
the feasibility of future applications of the curvature statistic to the HeII Ly-α forest. Being aware of the current observational limitations in the number and quality of the available spectra, it is also shown that any HeII Ly-α forest curvature analysis is currently prevented by the requirements of hydrodynamical simulations necessary to accurately resolve the helium absorption.

The results of Chapter 3, 4, and 5 are summarized and combined in Chapter 6 for a final discussion about the thermal state of the IGM that is the main focus of this thesis. The results of this investigation will be the starting point for new projects that will be the natural extension of the research shown here.
The curvature method at low redshifts

According to the photo-heating model of the intergalactic medium, He\textsc{ii} reionization is expected to affect its thermal evolution. Evidence for additional energy injection into the IGM has been found at $3 \lesssim z \lesssim 4$, though the evidence for the subsequent fall-off below $z \sim 2.8$ is weaker and depends on the slope of the temperature–density relation, $\gamma$. Here we present, for the first time, an extension of the IGM temperature measurements down to the atmospheric cut-off of the H\textsc{i} Lyman-\(\alpha\) forest at $z \simeq 1.5$. Applying the curvature method on a sample of 60 UVES spectra we investigated the thermal history of the IGM at $z < 3$ with precision comparable to the higher redshift results.


As discussed in Section 1.5 the IGM thermal history can be an important source of information about reionizing processes that injected vast amounts of energy into this gas on relatively short timescales and, in particular, about the He\textsc{ii} reionization. While the direct observation, through the detection of the “Gunn–Peterson effect”, recently suggests the end of the He\textsc{ii} reionization at $z \sim 2.7$ (e.g. Shull et al. [2010], Worseck et al. [2011], Syphers & Shull [2013], Syphers & Shull [2014]), any current constraint on the physics of this phenomenon is limited by the cosmic variance among the small sample of “clean” lines of sight, those along which the He\textsc{ii} Ly-\(\alpha\) transition is not blocked by higher-redshift H\textsc{i} Lyman limit absorption. For this reason indirect methods have been developed to obtain a detailed characterization of the He\textsc{ii} reionization.

The reionization event is expected to reheat the intergalactic gas leaving the charac-
characteristic signature of a peak, followed by a gradual cooling, in the temperature evolution at the mean gas density (e.g. McQuinn et al. 2009). In the last decade, the search for this feature and the study of the thermal history of the IGM as a function of redshift have been the objectives of different efforts, not only to verify this basic theoretical prediction and constrain the timing of He II reionization, but also to obtain information on the nature of the ionizing sources and the physics of the related ionizing mechanisms.

To obtain measurements of the temperature of the IGM, studying the absorption features of the H I Ly-α forest has proven to be a useful method so far. The widths of the Ly-α lines are sensitive to the thermal broadening, among the other effects. Therefore, using the comparison with cosmological simulations, different approaches have been able to extract from them information about the “instantaneous” temperature of the gas at the moment of absorption. However, as summarized in Section 1.5.2, the observational picture drawn by the results of previous efforts does not have a straightforward interpretation.

Recently, Becker et al. (2011) developed a statistical approach based on the flux curvature. This work constrained the temperature over $2 \lesssim z \lesssim 4.8$ of an “optimal” or “characteristic” overdensity, which evolves with redshift. The error bars were considerably reduced compared to previous studies, partially at the expense of determining the temperature at a single density only, rather than attempting to constrain the temperature–density relation. Some evidence was found for a gradual reheating of the IGM over $3 \lesssim z \lesssim 4$ but with no clear evidence for a temperature peak. Given these uncertainties, the mark of the He II reionization still needs a clear confirmation. Nevertheless, the curvature method is promising because it is relatively robust to continuum placement errors: the curvature of the flux is sensitive to the shape of the absorption lines and not strongly dependent on the flux normalization. Furthermore, because it incorporates the temperature information from the entire Lyman-α forest, this statistic has the advantage of using more of the available information, as opposed to the line-fitting method which relies on selecting lines that are dominated by thermal broadening.

Moreover, an injection of substantial amounts of thermal energy may also result in a change in the temperature–density relation (Equation 1.9). The detailed study of this process has to take into consideration the effects of the IGM inhomogeneities driven by the diffusion and percolation of the ionized bubbles around single sources, and currently constitutes an important object of investigation through hydrodynamical simulations (e.g. Compostella et al. 2013). Some analyses of the flux PDF have indicated that the $T$–$\rho$ relation may even become inverted (e.g. Becker et al. 2007; Bolton et al. 2008; Viel et al. 2009; Calura et al. 2012; Garzilli et al. 2012). However, the observational uncertainties in this
measurement are considerable (see discussion in Bolton et al. 2014). A possible explanation was suggested by considering radiative transfer effects (Bolton et al. 2008). Although it appears difficult to produce this result considering only He\textsc{ii} photo-heating by quasars (McQuinn et al. 2009; Bolton et al. 2009), a new idea of volumetric heating from blazar TeV emission predicts an inverted temperature–density relation at low redshift and at low densities. According to these models, heating by blazar $\gamma$-ray emission would start to dominate at $z \simeq 3$, obscuring the “imprint” of He\textsc{ii} reionization (Chang et al. 2012; Puchwein et al. 2012). In the most recent analysis, with the line-fitting method (Rudie et al. 2013; Bolton et al. 2014), the inversion in the temperature–density relation has not been confirmed, but a general lack of knowledge about the behavior of the $T$–$\rho$ relation at low redshift ($z < 3$) still emerges, accompanied with no clear evidence for the He\textsc{ii} reionization peak. A further investigation of the temperature evolution in this redshift regime therefore assumes some importance for obtaining constraints on the physics of the He\textsc{ii} reionization and the temperature–density relation of the IGM.

The purpose of the work presented in this Chapter and in Chapter 3 is to apply the curvature method to obtain new, robust temperature measurements at redshift $z < 3$, extending the previous results, for the first time, down to the optical limit for the Lyman-$\alpha$ forest at $z \simeq 1.5$. By pushing the measurement down to such a low redshift, we attempt to better constrain the thermal history in this regime, comparing the results with the theoretical predictions for the different heating processes. Furthermore, the exploration of this new redshift regime allows to search for the end of the heating seen previously in Becker et al. (2011), such an evidence is necessary to help bolster the interpretation as being due to the He\textsc{ii} reionization. We infer temperature measurements by computing the curvature on a new set of quasar spectra at high resolution obtained from the archive of the Ultraviolet and Visual Echelle Spectrograph (UVES) on the Very Large Telescope (VLT). Synthetic spectra, obtained from hydrodynamical simulations used in the analysis of Becker et al. (2011), and extended down to the new redshift regime, are used for the comparison with the observational data. Similar to Becker et al. (2011), we constrain the temperature of the IGM at a characteristic overdensity, $\bar{\Delta}$, traced by the Lyman-$\alpha$ forest, which evolves with redshift. We do not attempt to constrain the $T$–$\rho$ relation, but we use fiducial values of the parameter $\gamma$ to present results for the temperature at the mean density, $T_0$.

While the actual temperature measurements and their discussion will be presented in Chapter 3, this Chapter intends to explain the curvature analysis procedure and the necessary preparation of the observational and synthetic spectra; it is organised as follows.
Chapter 2. The curvature method at low redshifts

In Section 2.1 we present the observational data sample obtained from the VLT archive, while the simulations used to interpret the measurements are introduced in Section 2.2. In Section 2.3 the curvature method and our analysis procedure are summarized. In Section 2.4 we present the data analysis and we discuss the strategies applied to reduce the systematic uncertainties. Finally, the calibration and the analysis of the simulations from which we obtain the characteristic overdensities is described in Section 2.5.

2.1 The observational data

In this work we used a sample of 60 quasar spectra uniformly selected on the basis of redshift, wavelength coverage and S/N in order to obtain robust results in the UV and optical parts (3100–4870 Å) of the spectrum, where the Lyman-α transition falls for redshifts $1.5 < z < 3$. The quasars and their basic properties are listed in Table 2.1. The spectra were retrieved from the archive of the UVES spectrograph on the VLT. In general, most spectra were observed with a slit-width $\lesssim 1''$ wide and on-chip binning of $2 \times 2$, which provides a resolving power $R \simeq 50000$ (FWHM $\simeq 7$ km/s); this is more than enough to resolve typical Lyman-α forest lines, which generally have FWHM $\gtrsim 15$ km/s. The archival quasar exposures were reduced using the ESO UVES Common Pipeline Language software. This suite of standard routines was used to optimally extract and wavelength-calibrate individual echelle orders. The custom-written program UVES_POPLER\footnote{UVES_POPLER was written and is maintained by M. T. Murphy and is available at http://astronomy.swin.edu.au/~mmurphy/UVES_popler} was then used to combine the many exposures of each quasar into a single normalized spectrum on a vacuum-heliocentric wavelength scale. For most quasars, the orders were redispersed onto a common wavelength scale with a dispersion of 2.5 km/s per pixel; for 4 bright (and high S/N), $z \lesssim 2$ quasars the dispersion was set to 1.5 km/s per pixel. The orders were then scaled to optimally match each other and then coadded with inverse-variance weighting using a sigma-clipping algorithm to reject ‘cosmic rays’ and other spectral artefacts.

To ensure a minimum threshold of spectral quality and a reproducible sample definition, we imposed a “S/N” lower limit of 24 per pixel for selecting which QSOs and which spectral sections we used to derive the IGM temperature. A high S/N is, in fact, extremely important for the curvature statistic which is sensitive to the variation of the shapes of the Lyman-α lines: in low S/N spectra this statistic will be dominated by the noise and, furthermore, by narrow metal lines that are difficult to identify and mask.

The “S/N” cut-off of 24 per pixel was determined by using the hydrodynamical Lyman-α forest simulations discussed in Section 2.2. By adding varying amounts of Gaussian noise.
to the simulated forest spectra and performing a preliminary curvature analysis like that described in Sections 2.3 & 2.4, the typical uncertainty on the IGM temperature could be determined, plus the extent of any systematic biases caused by low S/N. It was found that a competitive statistical uncertainty of \( \simeq 10\% \) in the temperature could be achieved with the cut-off in “S/N” set to 24 per pixel, and that this was well above the level at which systematic biases become significant. However, in order for us to make the most direct comparison with these simulations, we have to carefully define “S/N”. In fact for the Lyman-\( \alpha \) forest the S/N fluctuates strongly and so it is not very well defined. Therefore, the continuum-to-noise ratio, \( C/N \), is the best means of comparison with the simulations. To measure this from each spectrum, we had first to establish a reasonable continuum.

The continuum fitting is a crucial aspect in the quasar spectral analysis and for this reason we applied to all the data a standard procedure in order to avoid systematic uncertainties due to the continuum choice. We used the continuum-fitting routines of \textsc{uves\_popler} to determine the final continuum for all our quasar spectra. Initially, we iteratively fitted a 5th-order Chebyshev polynomial to overlapping 10000-km/s sections of spectra between the Lyman-\( \alpha \) and Lyman-\( \beta \) emission lines of the quasar. The initial fit in each section began by rejecting the lowest 50\% of pixels. In subsequent iterations, pixels with fluxes \( \geq 3\sigma \) above and \( \geq 1\sigma \) below the fit were excluded from the next iteration. The iterations continued until the ‘surviving’ pixels remained the same in two consecutive iterations. The overlap between neighboring sections was 50\% and, after all iterations were complete, the final continuum was formed by combining the individual continua of neighboring sections with a weighting which diminished linearly from unity at their centers to zero at their edges. After this initial treatment of all spectra we applied further small changes to the fitting parameters after visually inspecting the results. In most cases, we reduced the spectral section size, the threshold for rejecting pixels below the fit at each iteration, and the percentage of pixels rejected at the first iteration to values as low as 6000 km/s, 0.8 \( \sigma \) and 40\% respectively. In Figure 2.1 we show examples of continuum fits for Lyman-\( \alpha \) forest regions at different redshifts obtained with this method. This approach allowed us to avoid cases where the fitted continuum obviously dipped inappropriately below the real continuum, but still defined our sample with specific sets of continuum parameters without any further manual intervention, allowing a reproducible selection of the appropriate sample for this analysis. Furthermore, as described in Section 2.4, to avoid any systematics due to the large-scale continuum placement, we re-normalized each section of spectra that contributed to our results.
Figure 2.1 Examples of continuum fit (green solid line) in Lyman-α regions for the quasar J112442-170517 with $z_{em} = 2.40$ (Top panel) and J051707-441055 at $z_{em} = 1.71$ (Bottom panel). The continuum fitting procedure used is described in the text.
Table 2.1 List of the QSOs used for this analysis. For each object we report the name (column 1) based on the J2000 coordinates of the QSO and the emission redshift (column 2). The redshift intervals associated with the Lyman-\(\alpha\) absorption are also reported with the corresponding C/N level per pixel (columns 3, 4 & 5). The sections of Lyman-\(\alpha\) forest obtained from this sample were required to have a minimum C/N of 24 per pixel.

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The redshift distribution of the Lyman-α forest ($z_{Ly\alpha}$) of the quasars in our selected sample is shown in Figure 2.2, where their distribution of C/N in the same region is also reported. Due to instrumental sensitivity limits in the observation of bluest part of the optical Ly-α forest, it is more difficult to collect data with high C/N for $z_{Ly\alpha} < 1.7$. The general lower quality of these data and the lower number of quasars contributing to this redshift region will be reflected in the results, causing larger uncertainties.
2.1. The observational data

Figure 2.2 Properties of our sample of quasar spectra with C/N > 24 in the Lyman-α forest region. Top panel: redshift distribution referred to the Lyman-α forest. Bottom panel: median value of the continuum to noise ratio (C/N) in the forest region per redshift bin. The redshift bins have been chosen for convenience of $\delta z = 0.025$. The vertical dashed lines mark the $\Delta z = 0.2$ redshift bins in which the measurements (at $z \lesssim 3.1$) will be collected in the following analysis.
2.2 The simulations

To interpret our observational results and extract temperature constraints from the analysis of the Lyman-α forest, we used synthetic spectra, derived from hydrodynamical simulation and accurately calibrated to match the real data conditions. We performed a set of hydrodynamical simulations that span a large range of thermal histories, based on the models of Becker et al. (2011) and extended to lower redshifts ($z < 1.8$) to cover the redshift range of our quasar spectra. The simulations were obtained with the parallel smoothed particle hydrodynamics code GADGET-3 that is the updated version of gadget-2 (Springel 2005) with initial conditions constructed using the transfer function of Eisenstein & Hu (1999) and adopting the cosmological parameters $\Omega_m = 0.26$, $\Omega_\Lambda = 0.74$, $\Omega_bh^2 = 0.023$, $h = 0.72$, $\sigma_8 = 0.80$, $n_s = 0.96$, according to the cosmic microwave background constraints of Reichardt et al. (2009) and Jarosik et al. (2011). The helium fraction by mass of the IGM is assumed to be $Y = 0.24$ (Olive & Skillman 2004). Because the bulk of the Lyman-α absorption corresponds to overdensities $\Delta = \rho/\bar{\rho} \lesssim 10$, our analysis will not be affected by the star formation prescription, established only for gas particles with overdensities $\Delta > 10^3$ and temperature $T < 10^5$ K.

Starting at $z = 99$ the simulations describe the evolution of both dark matter and gas using $2 \times 512^3$ particles with a gas particle mass of $9.2 \times 10^4 h^{-1} M_\odot$ in a periodic box of 10 comoving $h^{-1}$ Mpc (see also simulation convergence tests in Becker et al. 2011). Instantaneous hydrogen reionization is fixed at $z = 9$. From the one set of initial conditions, many simulations are run, all with gas that is assumed to be in the optically thin limit and in ionisation equilibrium with a spatially uniform ultraviolet background from Haardt & Madau (2001). However, the photoheating rates, and so the corresponding values of the parameters $T_0$ and $\gamma$ of Equation 1.9 were changed between simulations. In particular, the photo-heating rates from Haardt & Madau (2001) ($\epsilon_i^{HM01}$) for the different species ($i=\text{H}1, \text{He}1, \text{He} II$) have been rescaled using the relation $\epsilon_i = \zeta \Delta \epsilon_i^{HM01}$ where $\epsilon_i$ are the adopted photo-heating rates and $\zeta$ and $\Delta$ are constants that change depending on the thermal history assumed. Possible bimodality in the temperature distribution at fixed gas density, observed in the simulations of Compostella et al. (2013) in the early phases of the He II reionization, has not been taken into consideration in our models. We assume, in fact, that the final stages of He II reionization at $z < 3$, when the IGM is almost completely reionized, can be described in a good approximation by a single temperature–density relation and are not affected anymore by the geometry of the diffusion of ionized bubbles.
2.2. The simulations

Our models do not include galactic winds or possible outflows from AGN. However, these are expected to occupy only a small proportion of the volume probed by the synthetic spectra and so they are unlikely to have an important effect on the properties of the Lyman-α forest (see e.g. Bolton et al. 2008 and also Theuns et al. 2002 for a discussion in the context of the PDF of the Lyman-α forest transmitted fraction where this has been tested).

A summary of the simulations used in this work is reported in Table 2.2. We used different simulation snapshots that covered the redshift range of our quasar spectra \((1.5 \lesssim z \lesssim 3)\) and, to produce synthetic spectra of the Lyman-α forest, 1024 randomly chosen “lines of sight” through the simulations were selected at each redshift. To match the observational data, we needed to calibrate the synthetic spectra with our instrumental resolution, with the same H\(\text{I}\) Lyman-α effective optical depth and the noise level obtained from the analysis of the real spectra (see Section 2.5.1).

<table>
<thead>
<tr>
<th>Model</th>
<th>(\zeta)</th>
<th>(\xi)</th>
<th>(T_0^{z=3}) [K]</th>
<th>(\gamma^{z=3})</th>
</tr>
</thead>
<tbody>
<tr>
<td>A15</td>
<td>0.30</td>
<td>0.00</td>
<td>5100</td>
<td>1.52</td>
</tr>
<tr>
<td>B15</td>
<td>0.80</td>
<td>0.00</td>
<td>9600</td>
<td>1.54</td>
</tr>
<tr>
<td>C15</td>
<td>1.45</td>
<td>0.00</td>
<td>14000</td>
<td>1.54</td>
</tr>
<tr>
<td>D15</td>
<td>2.20</td>
<td>0.00</td>
<td>18200</td>
<td>1.55</td>
</tr>
<tr>
<td>E15</td>
<td>3.10</td>
<td>0.00</td>
<td>22500</td>
<td>1.55</td>
</tr>
<tr>
<td>F15</td>
<td>4.20</td>
<td>0.00</td>
<td>27000</td>
<td>1.55</td>
</tr>
<tr>
<td>G15</td>
<td>5.30</td>
<td>0.00</td>
<td>31000</td>
<td>1.55</td>
</tr>
<tr>
<td>D13</td>
<td>2.20</td>
<td>-0.45</td>
<td>18100</td>
<td>1.32</td>
</tr>
<tr>
<td>C10</td>
<td>1.45</td>
<td>-1.00</td>
<td>13700</td>
<td>1.02</td>
</tr>
<tr>
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<tr>
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<td>2.20</td>
<td>-1.60</td>
<td>17900</td>
<td>0.71</td>
</tr>
</tbody>
</table>

Table 2.2 Parameters corresponding to the different simulations used in this work. For each simulation we report the name of the model (column 1), the constants used to rescale the photo-heating rates for the different thermal histories (columns 2 & 3), the temperature of the gas at the mean density at \(z = 3\) (column 4) and the power-law index of the \(T-\rho\) relation at \(z = 3\) (column 4).
Chapter 2. The curvature method at low redshifts

2.3 The curvature method

The definition of curvature ($\kappa$), as used by Becker et al. (2011), is the following:

$$\kappa = \frac{F''}{1 + (F')^2}^{3/2}, \tag{2.1}$$

with the first and second derivatives of the flux ($F'$, $F''$) taken with respect to wavelength or relative velocity. The advantage of this statistic is that, as demonstrated in Becker et al. (2011), it is quite sensitive to the IGM temperature but does not require the forest to be decomposed into individual lines. In this way the systematic errors are minimised if this analysis is applied to high resolution and high S/N spectra. Its calculation is relatively simple and can be computed using a single b-spline fit directly to large regions of forest spectra. This statistic incorporates the temperature information from all lines, using more of the available information, as opposed to line-fitting which relies on selecting lines that are dominated by thermal broadening. If calibrated and interpreted using synthetic spectra, obtained from cosmological simulations, the curvature represents a powerful tool to measure the temperature of the IGM gas, $T(\bar{\Delta})$, at the characteristic overdensities ($\bar{\Delta}$) of the Lyman-$\alpha$ forest at different redshifts.

However, at low redshifts ($z \lesssim 3$), the IGM gas tends to show characteristic overdensities ($\bar{\Delta}$) much higher than the mean density of the IGM ($\bar{\rho}$). Estimating the temperature at the mean density of the IGM ($T_0$) is then not straightforward. In fact, in the approximation of a gas collected into non-overlapping clumps of uniform density that have the same extent in redshift space as they have in real space, the Lyman-$\alpha$ optical depth at a given overdensity ($\Delta$) will scale as:

$$\tau(\Delta) \propto (1 + z)^{4.5} \Gamma^{-1} T_0^{-0.7} \Delta^{2-0.7(1-\gamma)}, \tag{2.2}$$

where $\Gamma$ is the H I photoionisation rate and $T_0$ and $\gamma$ are the parameters that describe the thermal state of the IGM at redshift $z$ in Equation 1.9 (Weinberg et al. 1997). In general, if we assume that the forest will be sensitive to overdensities that produce a Lyman-$\alpha$ optical depth $\tau(\Delta) \simeq 1$, it is then clear from Equation 2.2 that these characteristic overdensities will vary depending on the redshift. At high redshift the forest will trace gas near the mean density while in the redshift range of interest here ($z \lesssim 3$) the absorption will be coming from densities increasingly above the mean. As a consequence, the translation of the $T(\bar{\Delta})$ measurements at the characteristic overdensities into the temperature at the mean density becomes increasingly dependent on the value of the slope of the temperature–
density relation (Equation 1.9). Due to the uncertainties related to a poorly constrained parameter \(\gamma\), a degeneracy is introduced between \(\gamma\) and the final results for \(T_0\) that can be overcome only with a more precise measurement of the \(T-\rho\) relation.

In this work we do not attempt to constrain the full \(T-\rho\) relation because that would require a simultaneous estimation of both \(T_0\) and \(\gamma\). Instead, following consistently the steps of the previous analysis of Becker et al. (2011), we establish empirically the characteristic overdensities (\(\bar{\Delta}\)) and obtain the corresponding temperatures from the curvature measurements. We define the characteristic overdensity traced by the Lyman-\(\alpha\) forest for each redshift as that overdensity at which \(T(\bar{\Delta})\) is a one-to-one function of the mean absolute curvature, regardless of \(\gamma\) (see Section 2.5). We then recover the temperature at the mean density (\(T_0\)) from the temperature \(T(\bar{\Delta})\) using Equation 1.9 with a range of values of \(\gamma\) (see Section 3.1).

We can summarize our analysis in 3 main steps:

- **Data analysis (Section 2.4):** from the selected sample of Lyman-\(\alpha\) forest spectra we compute the curvature (\(\kappa\)) in the range of \(z \approx 1.5 - 3.0\). From the observational spectra we also obtain measurements of the effective optical depth that we use to calibrate the simulations.

- **Simulations analysis (Section 2.5):** we calibrate the simulation snapshots at different redshifts in order to match the observational data. We obtain the curvature measurements from the synthetic spectra following the same procedure that we used for the real data and we determine the characteristic overdensities (\(\bar{\Delta}\)) empirically, finding for each redshift the overdensity at which \(T(\bar{\Delta})\) is a one-to-one function of \(\log\langle|\kappa|\rangle\) regardless of \(\gamma\).

- **Final temperature measurements (Section 3.1):** we determine the \(T(\bar{\Delta})\) corresponding to the observed curvature measurements by interpolating the \(T(\bar{\Delta})-\log\langle|\kappa|\rangle\) relationship in the simulations to the values of \(\log\langle|\kappa|\rangle\) from the observational data.

### 2.4 Data analysis

To directly match the box size of the simulated spectra, we compute the curvature statistic on sections of \(10h^{-1}\) Mpc (comoving distance) of “metal free” Lyman-\(\alpha\) forest regions in our quasar spectra. Metals lines are, in fact, a potentially serious source of systematic errors in any measure of the absorption features of the Lyman-\(\alpha\) forest. These lines tend
to show individual components significantly narrower than the Lyman-α ones ($b \lesssim 15 \text{ km s}^{-1}$) and, if included in the calculation, the curvature measurements will be biased towards high values. As a consequence, the temperature obtained will be much lower. For these reasons we need to “clean” our spectra by adopting a comprehensive metal masking procedure (see Section 2.4.2).

However, not only metals can affect our analysis and, even if effectively masked from contaminant lines, the direct calculation of the curvature on observed spectra can be affected by other sources of uncertainties, particularly, noise and continuum errors. To be as much as possible consistent with the previous work of Becker et al. (2011) we adopted the same strategies to reduce these potential systematic errors.

**Noise:** If applied directly to high resolution and high S/N spectra, the curvature measurements will be dominated by the noise in the flux spectra. To avoid this problem we fit a cubic b-spline to the flux and we then compute the curvature from the fit. Figure 2.3, top panel, shows a section of normalized Lyman-α spectra in which the solid green line is the b-spline fit from which we obtain the curvature. For consistency, we adopt the same specifics of the fitting routine of the previous work of Becker et al. (2011). We then use an adaptive fit with break points that are iteratively added, from an initial separation of 50 km s$^{-1}$, where the fit is poor. The iterations proceed until the spacing between break points reach a minimum value (6.7 km s$^{-1}$) or the fit converges. With this technique we are able to reduce the sensitivity of the curvature to the amount of noise in the spectrum as we can test using the simulations (see Section 2.5).

**Continuum:** Equation 2.1 shows a dependence of the curvature on the amplitude of the flux, which in turn is dependent on the accuracy with which the unabsorbed quasar continuum can be estimated. The difficulty in determining the correct continuum level in the Lyman-α region can then constitute a source of uncertainties. To circumvent this issue we “re-normalize” each 10$h^{-1}$ Mpc section of data, dividing the flux of each section (already normalized by the longer-range fit of the continuum) by the maximum value of the b-spline fit in that interval. Computing the curvature from the re-normalized flux, we remove a potential systematic error due to inconsistent placement of the continuum. While this error could be important at high redshifts, where the Lyman-α forest is denser, at $z \lesssim 3$ we do not expect a large correction. Figure 2.3, bottom panel, shows the value of the curvature computed from the b-spline fit of the re-normalized flux (applying Equation 2.1) for a section of forest.
2.4. Data analysis

Figure 2.3 Curvature calculation example for one section of Lyman-\(\alpha\) forest. Top panel: b-spline fit (green line) of a section of 10\(h^{-1}\) Mpc of normalized real spectrum. Bottom panel: the curvature statistic computed from the fit as defined in Equation 2.1.

We next measure the mean absolute curvature \(\langle |\kappa| \rangle\) for the “valid” pixels of each section. We consider valid all the pixels where the re-normalized b-spline fit \(F^R\) falls in the range 0.1 \(\leq F^R \leq 0.9\). In this way we exclude both the saturated pixels, that do not contain any useful information, and the pixels with flux near the continuum. This upper limit is in fact adopted because the flux profile tends to be flatter near the continuum and, as consequence the curvature for these pixels is considerably more uncertain. This potential uncertainty is particularly important at low redshift because increasing the mean flux also increases the number of pixels near the continuum (Faucher-Giguère et al., 2008).
Chapter 2. The curvature method at low redshifts

2.4.1 Observed curvature and re-normalized optical depth

The final results for the curvature measurements from the real quasar spectra are shown in the green data points in Figure 2.4. In this plot the values of $\langle |\kappa| \rangle$ obtained from all the $10h^{-1}$ Mpc sections of forest have been collected and averaged in redshift bins of $\Delta z = 0.2$. The error bars show the 1σ uncertainty obtained with a bootstrap technique generated directly from the curvature measurement within each bin. In all the redshift bins, in fact, the mean absolute curvature values of a large number of sections ($N > 100$) have been averaged and so the bootstrap can be considered an effective tool to recover the uncertainties. It is important to note that the smaller number of sections contributing to the lowest redshift bin ($1.5 \leq z \leq 1.7$; see Figure 2.2), is reflected in a larger error bar. For comparison, the results of the curvature from Becker et al. (2011) (black triangles) for redshift bins of $\Delta z = 0.4$ are also shown. In the common redshift range the results seem to be in general agreement even if our values appear shifted slightly towards higher curvatures. Taking into consideration the fact that each point cannot be considered independent from the neighbours, this shift between the results from the two different data samples is not unexpected and may also reflect differences in the signal-to-noise ratio between the samples. At this stage we do not identify any obvious strong departure from a smooth trend in $\langle |\kappa| \rangle$ as a function of redshift.

From each section we also extract the mean re-normalized flux ($F^R$) that we use to estimate the re-normalized effective optical depth ($\tau_{\text{eff}}^R = -\ln \langle F^R \rangle$) needed for the calibration of the simulations (see Section 2.5.1). In Figure 2.5 we plot the $\tau_{\text{eff}}^R$ obtained in this work for redshift bins of $\Delta z = 0.2$ (green data points) compared with the results of Becker et al. (2011) (black triangles) for bins of $\Delta z = 0.4$. Vertical error bars are 1σ bootstrap uncertainties for our points and 2σ for Becker et al. (2011). For simplicity we fitted our data with a unique power law ($\tau = A(1+z)^\alpha$) because we do not expect that a possible small variation of the slope as a function of redshift will have a relevant effect in the final temperature measurements. Comparing the least square fit computed from our measurements (green solid line) with the re-normalized effective optical depth of Becker et al. it is evident that there is a systematic difference between the two samples, increasing at lower redshifts: our $\tau_{\text{eff}}^R$ values are $\sim 10\%$ higher compared with the previous measurements and, even if the black triangles tend to return inside the $\pm 1\sigma$ confidence interval on the fit (green dotted lines) for $z \gtrsim 2.6$, the results are not in close agreement. However, the main quantities of interest in this paper (i.e. the curvature, from which the IGM temperature estimates are calculated) derive from a comparison of real and simulated spectra which have been re-normalized in the same way, so we expect that they will not
2.4. Data analysis

Figure 2.4 Curvature measurements from the observational quasar spectra. The curvature values obtained in this work (green points) for redshift bins of $\Delta z = 0.2$ are compared with curvature points from Becker et al. (2011) with $\Delta z = 0.4$ (black triangles). Horizontal error bars show the redshift range spanned by each bin. Vertical error bars in this work are $1\sigma$ and have been obtained from a bootstrap technique using the curvature measurements within each bin. In Becker et al. the errors are $2\sigma$, recovered using sets of artificial spectra. These errors from the simulations are in agreement with the direct bootstrap using the data from bins which contain a large number of data points. Curvature measurements obtained in this work from spectra not masked for metals are also shown (red points).

 depend strongly on the estimation of the continuum like the two sets of $\tau_{\text{eff}}$ results in Figure 2.5 do (we compare the effective optical depth prior the continuum re-normalization from the simulations calibrated with the $\tau_{\text{eff}}$ results in Section 2.5.1). The higher values shown by our re-normalized effective optical depth reflect the variance expected between different
samples: a systematic scaling, of the order of the error bar sizes, between our results and the previous ones may not be unexpected due to the non-independence of the data points within each set. These differences will reflect different characteristic overdensities probed by the forest (see Section 2.5.3). Fortunately, the correct calibration between temperature and curvature measurements will wash out this effect, allowing consistent temperature calibration as a function of redshift (see Section 3.1 and Appendix A).

Even if the scaling between the datasets could be smoothed, considering the fact that, according to Rollinde et al. (2013), the bootstrap errors computed from sections of 10 $h^{-1}$Mpc (and then $\lesssim 25\,\text{Å}$) could underestimate the variance, another possible cause could be differences in the metal masking procedures of the two studies. In the next section we
2.4. Data analysis

explain and test our metal masking technique, showing how our results do not seem to imply a strong bias due to contamination from unidentified metal lines.

2.4.2 Metal correction

Metal lines can be a serious source of systematic uncertainties for both the measures of the re-normalized flux ($F^R$) and the curvature. While in the first case it is possible to choose between a statistical (Tytler et al. 2004; Kirkman et al. 2005) and a direct (Schaye et al. 2003) estimation of the metal absorption, for the curvature it is necessary to directly identify and mask individual metals lines in the Lyman-α forest. Removing these features accurately is particularly important for redshifts $z \lesssim 3$, where there are fewer Lyman-α lines and potentially their presence could affect significantly the results. We therefore choose to identify metal lines proceeding in two steps: an “automatic” masking procedure followed by a manual refinement. First, we use well-known pairs of strong metal-line transitions to find all the obvious metal absorbing redshifts in the spectrum. We then classify each absorber as of high (e.g. C iv, Si iv) or low (e.g. Mg ii, Fe ii) ionisation and strong or weak absorption, and we evaluate the width of its velocity structure. To avoid contamination, we mask all the regions in the forest that could plausibly contain common metal transitions of the same type, at the same redshift and within the same velocity width of these systems. The next step is to double-check the forest spectra by eye, searching for remaining unidentified narrow lines (which may be metals) and other contaminants (like damped Lyman-α systems or corrupted chunks of data). Acknowledging that this procedure is in a certain way subjective, at this stage we try to mask any feature with very narrow components or sharp edges to be conservative.

Figure 2.6 presents the correction for the metal line absorption on the re-normalized effective optical depth measurements; from the raw spectra (red triangles), to the spectra treated with the first “automatic” correction (yellow stars), to the final results double-checked by eye (green points). For all the three cases we show the least square fit (solid lines of corresponding colors) and the 1σ vertical error bars. In Table 2.3 are reported the numerical values for our metal absorption compared with previous results of Schaye et al. (2003) and Kirkman et al. (2005) used in the effective optical depth measurements of Faucher-Giguère et al. (2008). The relative metal correction to $\tau^R_{\text{eff}}$ decreases with increasing redshift, as expected if the IGM is monotonically enriched with time (Faucher-Giguère et al. 2008) and in general is consistent with the previous results. In their work in fact, Faucher-Giguère et al. evaluated the relative percentages of metal absorption in their measurements of $\tau_{\text{eff}}$ when applying two different corrections: the one obtained with
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the direct identification and masking method by Schaye et al. (2003), and the statistical estimate of Kirkman et al. (2005) in which they used measurements of the amount of metals redwards the Lyman-α emission line. At each redshift, Faucher-Giguère et al. (2008) found good agreement between the estimates of their effective optical depth based on the two methods of removing metals. In individual redshift bins, applying our final relative metal absorption percentages, we obtain a $\tau_{\text{eff}}^R$ that agrees well within 1σ with the ones obtained after applying the corrections of these previous results, encouraging confidence that our metal correction is accurate to the level of our statistical error bar. However, our corrections are overall systematically larger than the previous ones. If we have been too conservative in removing potentially metal-contaminated portions of spectra in our second “by-eye” step, this will bias the effective optical depth to lower values.

Table 2.3 Metal absorption correction to the raw measurements of $\tau_{\text{eff}}^R$. For each redshift (column 1) is reported the percentage metal absorption correction obtained in this work in the first, ‘automatic’ mask described in the text (column 2) and in the refinement by eye (column 3). For comparison, in the overlapping redshift range are presented the results of Faucher-Giguère et al. (2008) obtained applying the direct metal correction of Schaye et al. (2003) (column 4) and the statistical one of Kirkman et al. (2005) (column 5).

<table>
<thead>
<tr>
<th></th>
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</thead>
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<td>n.a.</td>
</tr>
<tr>
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<td>12.3%</td>
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<td>3.0</td>
<td>8.8%</td>
<td>14.0%</td>
<td>9.0%</td>
<td>6.0%</td>
</tr>
</tbody>
</table>

In Figure 2.4 is also shown the effect of our metal correction on the curvature measurements: red points are curvature values obtained from the raw spectra while the green points are the final measurements from masked sections, with vertical bars being the 1σ error. Metal contamination has important effects on the curvature measurements: after the correction, in fact, the curvature measurements decrease between $\sim 30\% - 40\%$ at each redshift even if the relative differences among redshift bins seem to be maintained. The potential effects of an inaccurate metal correction on the final temperature measurements will be considered in Section 3.1.
2.4. Data analysis

2.4.3 Proximity region

A final possible contaminant in the measurements of the effective optical depth is the inclusion in the analysis of the QSO proximity regions. The so-called “proximity regions” are the zones near enough to the quasars to be subjected to the local influence of its UV radiation field. These areas may be expected to show lower Lyman-α absorption with respect to the cosmic mean due to the high degree of ionisation. To understand if the proximity effect can bias the final estimates of $\tau_{\text{eff}}^R$, we compare our results with the measurements obtained after masking the chunks of spectra that are potentially effected by
Chapter 2. The curvature method at low redshifts

Figure 2.7 Effect of masking the proximity region on the re-normalized effective optical depth. The final results for $\tau_{\text{eff}}^R$ without the masking of the proximity zones are shown as green points and those with this correction are shown as light green circles. Solid lines represent the least square fit of the data and vertical error bars are the 1$\sigma$ statistical uncertainties.

The quasar radiation. Typically the ionizing UV flux of a bright quasar is thought to affect regions of $\lesssim 10$ proper Mpc along its own line of sight (e.g. Scott et al. 2000; Worseck & Wisotzki 2006). To be conservative, we masked the 25 proper Mpc nearest to each quasar Lyman-\(\alpha\) and Lyman-\(\beta\) emission lines and re-computed $\tau_{\text{eff}}^R$. The final comparison is presented in Figure 2.7, masking the proximity regions does not have any significant effect on $\tau_{\text{eff}}^R$. In fact, the results obtained excluding these zones (light green circles) closely match the results inferred without this correction (green points), well within the 1$\sigma$ error bars. We then do not expect that the inclusion in our analysis of the QSO proximity regions will affect significantly the temperature measurements.
2.5 Simulations Analysis

To extract temperature constraints from our measurements of the curvature we need to interpret our observational results using simulated spectra, accurately calibrated to match the real data conditions. In this Section we explain how we calibrate and analyse the synthetic spectra to find the connection between curvature measurements and temperature at the characteristic overdensities. We will use these results in Section 3.1 where we will interpolate the $T(\Delta)-\log(|\kappa|)$ relationship to the value of $\log(|\kappa|)$ from the observational data to obtain our final temperature measurements.

2.5.1 The calibration

To ensure a correct comparison between simulation and observational data we calibrate our synthetic spectra to match the spectral resolution and the pixel size of the real spectra. We adjust the simulated, re-normalized effective optical depth ($\tau_{\text{eff}}^{R}$) to the one extracted directly from the observational results (see Section 2.4.1) and we add to the synthetic spectra the same level of noise recovered from our sample.

Addition of noise

To add the noise to the synthetic spectra we proceed in three steps: first, we obtain the distributions of the mean noise corresponding to the $10h^{-1}\text{Mpc}$ sections of the quasar spectra contributing to each redshift bin. As shown in Figure 2.8 (top panel) these distributions can be complex and so to save computational time we simplify them by extracting grids of noise values with a separation of $\Delta \sigma = 0.01$ and weights rescaled proportionally to the original distribution (Figure 2.8 bottom panel). At each redshift the noise is finally added at the same levels of the corresponding noise distribution and the quantities computed from the synthetic spectra, with different levels of noise, are averaged with the weights of the respective noise grid.
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Figure 2.8 Top panel: an example of noise distribution for a $\Delta z = 0.2$ redshift bin of real quasar spectra (in this case one with $z_{\text{mean}} = 2.4$). On the x-axis of the histogram is presented the mean noise per section of $10h^{-1}\text{Mpc}$ while on the y-axis is shown the number of sections contributing to the particular bin. Bottom panel: the same distribution presented in the top panel but simplified, collecting the data in a noise grid of $\Delta \sigma = 0.01$. 
2.5. Simulations Analysis

Recovered optical depth

The simulated spectra are scaled to match the re-normalized effective optical depth, $\tau_{\text{eff}}^R$, of the real spectra. These can then be used to recover the corresponding effective optical depth ($\tau_{\text{eff}}$; prior to the continuum re-normalization). In fact, the re-normalized effective optical depth cannot be compared directly with the results from the literature and to do so we need to compute the mean flux and then $\tau_{\text{eff}}$ ($\tau_{\text{eff}} = -\ln\langle F \rangle$) from the synthetic spectra using the same procedure applied previously to the real spectra (see Section 5) but without the re-normalization. In Figure 2.9 is shown the trend of the recovered effective optical depth for three different simulations, A15, G15 and C15 in Table 2.2. For clarity we do not plot the curves for the remaining simulations but they lie in between the curves of simulations A15 and G15. Depending on the different thermal histories, the recovered effective optical depths vary slightly but the separation of these values is small compared with the uncertainties about the trend (e.g. green dotted lines referred to the simulation C15).

In Figure 2.9 we also compare our results with the previous studies of Becker et al. (2013), and Kirkman et al. (2005). The results of Becker et al. (2013), that for $z \lesssim 2.5$ have been scaled to the Faucher-Giguère et al. (2008) measurements, are significantly shifted toward lower $\tau_{\text{eff}}$, presenting a better agreement with Kirkman et al.. For $z < 2.2$ the effective optical depth of Kirkman et al. still shows values $\sim 30\%$ lower than ours. In this case, again, part of the difference between the results could be explained by the non-independence of the data points within each set. Such an offset could also be boosted by a possible selection effect: the lines of sight used in this work were taken from the UVES archive and, as such, may contain a higher proportion of damped Lyman-α systems; even if these systems have been masked out of our analysis, their presence will increase the clustering of the forest around them and so our sample will have higher effective optical depth as a consequence. The simulated $\tau_{\text{eff}}$ presented in Figure 2.9 were obtained by matching the observed $\tau_{\text{eff}}^R$ (see Figure 2.5) and do not represent one of the main results of this work, so we did not investigate further possible selection effects driven by our UVES sample. Being aware of this possibility, we decided to maintain the consistency between our curvature measurements and the simulations used to infer the temperature values, calibrating the simulated spectra with the effective optical depth obtained from our sample (see Appendix A). In the comparison between our results and the previous ones of Becker et al. (2011), the effect of a calibration with an higher $\tau_{\text{eff}}$ will manifest itself as a shift towards lower values in the characteristic overdensities traced by the Lyman-α forest at the same redshift (as we will see in Section 2.5.3).
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Figure 2.9 The effective optical depth, prior to the re-normalization correction, recovered from the simulations which matches the re-normalized effective optical depth, $\tau_{\text{eff}}^R$, of our real spectra. The effective optical depth for three different simulations is shown: A15 (blue solid line), G15 (pink solid line) and C15 (green solid line). The recovered $\tau_{\text{eff}}$ for the remaining simulations in Table 2 are not reported for clarity but they lie in between the trends of A15 and G15. The spread in values for different thermal histories is, in fact, small compared with the $1\sigma$ uncertainties about the trend of each of the effective optical depths (green dotted lines for simulation C15). Our results are compared with the effective optical depths of Becker et al. (2013) (red points) and Kirkman et al. (2005) (black points).
2.5. Simulations Analysis

2.5.2 The curvature from the simulations

Once the simulations have been calibrated we can measure the curvature on the synthetic spectra using the same method that we used for the observed data (see Section 2.4). In Figure 2.10 are plotted the values of $\log(|\kappa|)$ obtained from our set of simulations in the same redshift range and with the same spectral resolution, effective optical depth and mix of noise levels of the real spectra. Different lines correspond to different simulations in which the thermal state parameters are changed. We can preliminarily compare our data points with the simulations, noticing that the simulation that has the values of the curvature close to the real observations is C15, which assumes the fiducial parameter $\gamma = 1.54$ at redshift $z = 3$. The variation in the properties of the real data alters the trend of the simulated curvature to be a slightly non-smooth function of redshift.

As expected, the curvature values are sensitive to changes in the effective optical depth: as shown in Figure 2.11 for the fiducial simulation C15, the 1σ uncertainty about the fit of the observed $\tau_{\text{eff}}^R$ (see Figure 2.5) is in fact reflected in a scatter about the simulated curvature of about 10% at redshift $z \sim 1.5$, decreasing at higher redshifts. The next section shows how this dependence of the simulated curvature on the matched effective optical depth will imply differences in the recovered characteristic overdensities between our work and Becker et al. (2011).

2.5.3 The characteristic overdensities

The final aim of this work is to use the curvature measurements to infer information about the thermal state of the IGM, but this property will depend on the density of the gas. The Lyman-α forest, and so the curvature obtained from it, in fact does not always trace the gas at the mean density but, instead, at low redshift ($z \lesssim 3$) the forest lines will typically arise from densities that are increasingly above the mean. The degeneracy between $T_0$ and $\gamma$ in the temperature–density relation (Equation 1.9) will therefore be significant. For this reason at this stage we are not constraining both these parameters but we use the curvature to obtain the temperature at those characteristic overdensities ($\Delta$) probed by the forest that will not depend on the particular value of $\gamma$. We can in this way associate uniquely our curvature values to the temperature at these characteristic overdensities, keeping in mind that the observed values of $\kappa$ will represent anyway an average over a range of densities.
Figure 2.10 Curvature measurements: points with vertical error bars (1σ uncertainty) are for the real data and are compared with the curvature obtained from simulations with different thermal histories calibrated with the same spectral resolution, noise and effective optical depth of the observed spectra at each redshift.
2.5. Simulations Analysis

Figure 2.11 Dependence of the simulated $\log(|\kappa|)$ on the effective optical depth with which the simulations have been calibrated. The curvature recovered using the thermal history C15 (green solid line) is reported with the $1\sigma$ uncertainties about the trend (green dotted lines) corresponding to the $1\sigma$ uncertainties about the fit of the observed $\tau_{\text{eff}}^R$ in Figure 2.5. The variation generated in the curvature is about 10% for $z = 1.5$ corresponding to a scatter of $\pm 2 - 3 \times 10^3$ K in the temperature calibration, and decreases at higher redshift.

The method

We determine the characteristic overdensities empirically, finding for each redshift the overdensities at which $T(\bar{\Delta})$ is a one-to-one function of $\log(|\kappa|)$ regardless of $\gamma$. The method is explained in Figure 2.12: for each simulation type we plot the values of $T(\Delta)$ versus $\log(|\kappa|)$, corresponding to the points with different colors, and we fit the distribution with a simple power law. We change the value of the overdensity $\Delta$ until we find the one ($\bar{\Delta}$) for which all the points from the different simulations (with different thermal histories and $\gamma$ parameters) lie on the same curve and minimize the $\chi^2$. The final $T(\bar{\Delta})$ of our real data (see Section 3.1) will be determined by interpolating the $T(\bar{\Delta})$–$\log(|\kappa|)$ relationship in the simulations to the value of $\log(|\kappa|)$ computed directly from the real spectra.
Figure 2.12 Example of the one-to-one function between \( \log(\langle |\kappa|\rangle) \) and temperature obtained for a characteristic overdensity (\( \Delta = 3.7 \) at redshift \( z = 2.173 \)). Different colors correspond to different simulations. At each redshift we find the characteristic overdensity, \( \Delta = \bar{\Delta} \), for which the relationship between \( T(\bar{\Delta}) - \log(\langle |\kappa|\rangle) \) does not depend on the choice of a particular thermal history or \( \gamma \) parameter.
The results

The characteristic overdensities for the redshifts of our data points are reported in Table 3.1, while in Figure 2.13 is shown the evolution of $\bar{\Delta}$ as a function of redshift: as expected, at decreasing redshifts the characteristic overdensity at which the Lyman-$\alpha$ forest is sensitive increases. Note that Figure 2.13 also shows that while the addition of noise in the synthetic spectra for $z \lesssim 2.2$ has the effect of decreasing the values of the characteristic overdensities, that tendency is inverted for higher redshifts where the noise shifts the overdensities slightly towards higher values with respect to the noise-free results. In Figure 2.13 is also presented a comparison between the overdensities founded in this work and the ones obtained in Becker et al. (2011) in their analysis with the addition of noise. Even if the two trends are similar, the difference in values of the characteristic overdensities at each redshift is significant ($\sim 25\%$ at $z \sim 3$ and increasing towards lower redshift). Because we used the same set of thermal histories and a consistent method of analysis with respect to the previous work, the reason for this discrepancy lies in the different data samples: in fact, the effective optical depth observed in our sample is higher than the one recorded by Becker et al. (2011) (see Section 2.4.1). As we have seen in Section 2.5.2, the simulated curvature is sensitive to the effective optical depth with which the synthetic spectra have been calibrated and this is reflected in the values of the characteristic overdensities. It is then reasonable that for higher effective optical depths at a particular redshift we observe lower overdensities because we are tracing a denser universe and the Lyman-$\alpha$ forest will arise in overdensities closer to the mean density.
Figure 2.13 Evolution as a function of redshift of the characteristic overdensities obtained in this work with the addition of noise in the synthetic spectra (green solid line) and with noise-free simulations (green dotted line). As expected, the characteristic overdensities traced by the Lyman-α forest increase toward lower redshift. For comparison, the result for the characteristic overdensities from the previous work of Becker et al. (2011) (black solid line) is also presented. Our overdensities are lower and this can be associated with the higher effective optical depth observed in our sample that was used to calibrate the simulations.
Temperature measurements

We find that the temperature of the cosmic gas traced by the Lyman-α forest, $T(\bar{\Delta})$, increases for increasing overdensity from $T(\bar{\Delta}) \sim 22670$ K to 33740 K in the redshift range $z \sim 2.8 - 1.6$. Under the assumption of two reasonable values for the parameter $\gamma$, that defines the slope of the IGM temperature-density relation, the temperature at the mean density ($T_0$) shows a tendency to flatten at $z \lesssim 2.8$. In the case of $\gamma \sim 1.5$, our results are consistent with previous ones which indicate a falling $T_0$ for redshifts $z \lesssim 2.8$. Finally, our $T(\bar{\Delta})$ values show reasonable agreement with moderate blazar heating models.


The selection of the characteristic overdensities, $\bar{\Delta}$, and the associated one-to-one function between temperature and curvature presented in Chapter 2 allows to infer information about the temperature of the gas traced by the Lyman-α forest, $T(\bar{\Delta})$. This measurement is independent of the choice of the parameter $\gamma$ for the $T-\rho$ relation (Equation 1.9) and for this reason will represent the main result of this project. We also translate our temperature measurements to values at the temperature at the mean density, $T_0$, for reasonable values of $\gamma$. In this Chapter we present our results and compare them with those of Becker et al. (2011) at higher redshift. A broader discussion, taking into consideration theoretical predictions, along with the conclusions summarizing these measurements is also presented here.
Chapter 3. Temperature measurements

3.1 Temperature at the characteristic overdensities

The main results of this work are presented in Figure 3.1 where we plot the IGM temperature at the characteristic overdensities traced by the Lyman-\(\alpha\) forest as a function of redshift. The \(1\sigma\) errors are estimated from the propagation of the uncertainties in the curvature measurements. In fact, the uncertainties in the measured effective optical depth are reflected only in a small variation in the temperature measurements that falls well within the \(1\sigma\) uncertainties due to the errors in the curvature measurements. Our temperature measurements show good agreement with the previous work of Becker et al. (2011) at higher redshifts where they overlap. This accord is particularly significant because we analysed a completely independent set of quasar spectra, obtained from a different instrument and telescope.

The curvature method in fact demonstrates self-consistency: the lower overdensities recorded from our sample are in fact compensated by our higher values of observed curvature. In this way, interpolating at each redshift the \(T(\bar{\Delta}) - \log(\langle|\kappa|\rangle)\) relationship in the simulations to the \(\log(\langle|\kappa|\rangle)\) computed directly from the data, we obtained similar temperature values to the ones of Becker et al. (2011) in the overlapping redshift range. Differences in the characteristic overdensities, \(\bar{\Delta}\), at a particular redshift between the two studies will cause variation in the derived temperature at the mean density \(T_0\) because we will infer \(T_0\) using the \(T - \rho\) relation with the values of \(\bar{\Delta}\). However, this effect will be modest and will cause disparity at the level of the \(1\sigma\) error bars of our values (see Section 3.2 and Appendix A). For comparison, in Figure 3.1 we show the \(z = 2.4\) line-fitting result of Rudie et al. (2012), with their \(T_0\) and \(\gamma\) values recalibrated and translated to a \(T(\bar{\Delta})\) value by Bolton et al. (2014). Even if the line-fitting method is characterized by much larger \(1\sigma\) error bars, it represents an independent technique and its agreement with our temperature values gives additional confidence in the results.

In general, the extension to lower redshifts \((z \lesssim 1.9)\) that our new results provide in Figure 3.1 do not show any large, sudden decrease or increase in \(T(\bar{\Delta})\) and can be considered broadly consistent with the trend of \(T(\bar{\Delta})\) increasing towards lower redshift of Becker et al. (2011). The increasing of \(T(\bar{\Delta})\) with decreasing redshift is expected for a non-inverted temperature–density relation because, at lower \(z\), the Lyman-\(\alpha\) forest is tracing higher overdensities: denser regions are much more bounded against the cooling due to the adiabatic expansion and present higher recombination rates (and so more atoms for the photoheating process). We consider this expectation further in Section 3.3 after converting our \(T(\bar{\Delta})\) measurements to \(T_0\) ones, using a range of \(\gamma\) values.
3.1. Temperature at the characteristic overdensities

Figure 3.1 IGM temperature at the characteristic overdensities, $T(\Delta)$, as a function of redshift for this work (green points), for the line-fitting analysis of Bolton et al. (2014) (red triangle) and for the previous work of Becker et al. (2011) (black circles). Vertical error bars are $1\sigma$ for this work and $2\sigma$ for Becker et al. and are estimated from statistical uncertainties in the curvature measurements.
Table 3.1 Numerical values for the results of this work: the mean redshift of each data bin is reported (column 1) with the associated characteristic overdensity (column 2). Also shown are the temperature measurement with the associated $1\sigma$ errors obtained for each data bin at the characteristic overdensity (column 3) and at the mean density under the assumption of two values of $\gamma$ (column 4 & 5). Finally, the values of $\gamma$, recovered from the fiducial simulation C15, are presented (column 4).

<table>
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<tr>
<th>$z_{\text{mean}}$</th>
<th>$\Delta$</th>
<th>$T(\Delta)/10^4K$</th>
<th>$\gamma \sim 1.5$</th>
<th>$T_0^{\gamma=1.5}/10^4K$</th>
<th>$T_0^{\gamma=1.3}/10^4K$</th>
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<tr>
<td>1.63</td>
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<td>33.74±3.31</td>
<td>1.583</td>
<td>13.00±1.27</td>
<td>20.66±2.03</td>
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<tr>
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<td>4.55</td>
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<td>1.577</td>
<td>11.79±0.51</td>
<td>17.61±0.76</td>
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<tr>
<td>2.00</td>
<td>4.11</td>
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<td>1.572</td>
<td>12.60±0.38</td>
<td>18.08±0.55</td>
</tr>
<tr>
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<td>3.74</td>
<td>29.20±1.06</td>
<td>1.565</td>
<td>14.05±0.51</td>
<td>19.66±0.71</td>
</tr>
<tr>
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<td>1.561</td>
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<td>2.84</td>
<td>22.67±0.89</td>
<td>1.549</td>
<td>12.90±0.51</td>
<td>16.58±0.65</td>
</tr>
</tbody>
</table>

3.2 Temperature at the mean density

Using the $T-\rho$ relation, characterized by different $\gamma$ values, we translate our measurements of $T(\bar{\Delta})$ to values of temperature at the mean density ($T_0$). In Figure 3.2 we present $T_0$ under two different assumptions for $\gamma$: for $\gamma$ values measured from fiducial simulations (A15–G15) in the top panel and for a constant $\gamma = 1.3$ in the bottom panel. In both the plots our results are compared with those from Becker et al. [2011].

In the first case we use a $\gamma$ parameter that varies slightly with redshift, with $\gamma \sim 1.5$ at $z = 3$ (the exact values are reported in Table 3.1 and have been recovered from the fiducial simulation C15). These $\gamma$ values are close to the maximum values expected in reality and therefore correspond to the minimum $T_0$ case. Our results in the overlapping redshift range ($2.0 \lesssim z \lesssim 2.9$) are broadly consistent with the previous ones but show slightly higher values, a difference that can be attributed to the variation with respect to the previous work in the values of the characteristic overdensities from which we derived our $T_0$ values (see Section 2.5.3). The extension at lower redshifts suggests a tendency of flattening of the increase in $T_0$ that can be interpreted as an imprint for the completion of the reheating of the IGM by He II reionization. The least-square linear fit of our data presented in Figure 3.2 (top panel) for this particular choice of $\gamma$ shows, in fact, quantitatively an inversion in the slope of the temperature evolution at the mean density: the general trend of the temperature is therefore an increase from $z \sim 4$ to $z \sim 2.8$ with a subsequent flattening of $T_0$ around $\sim 12000K$ at $z \sim 2.8$. The evolution of the temperature for $z \lesssim 2.8$ is generally consistent with a linear decrease of slope $a = 0.80 \pm 0.81(1\sigma)$ generally in agreement with
the decrease registered in [Becker et al. (2011)] for the same choice of $\gamma$.

The situation is similar in the second case for a constant $\gamma = 1.3$. This choice, which is motivated by the numerical simulations of [McQuinn et al. (2009)], corresponds to a mild flattening of the temperature–density relation, as expected during an extended He II reionization process. The trend of $T_0$ again shows a strong increase in the temperature from $z \sim 4$ to $z \sim 2.8$ and then a tendency of flattening from $z \sim 2.8$ towards lower redshift. However, the temperature obtained in this case is higher, fluctuating around $\sim 17000$ K. The scatter between our data points and the ones from [Becker et al. (2011)] is also smaller for this choice of $\gamma$, even if ours are slightly higher on average. In this case the linear fit of our data points at $z \lesssim 2.8$ suggests a change in the slope of the temperature evolution but, while in the previous case we register a positive slope, for this $\gamma$ choice we see only a slowdown in the increasing temperature, with a slope that assumes the value $a = -1.84 \pm 1.06(1\sigma)$.

The exact redshift of the temperature maximum, reached by the IGM at the mean density approaching the tail-end of He II reionization, is then still dependent on the choice of $\gamma$, as already pointed out in [Becker et al. (2011)]. Nevertheless, the extension at lower redshift of our data points gives stronger evidence about the end of this event. In fact, while an increase in the temperature for $z \sim 4 - 2.8$ has been recorded in the previous work, if $\gamma$ remains roughly constant, a tendency to a temperature flattening at lower redshift is suggested for both the choice of $\gamma$. A particularly important result is the suggestion of a decrease in $T_0$ in the case of $\gamma \sim 1.5$. In fact, according to recent analysis with the line fitting method [Rudie et al. 2012; Bolton et al. 2014] at redshift $z = 2.4$ there is good evidence for $\gamma = 1.54 \pm 0.11(1\sigma)$ and, because we expect that at the end of He II reionization $\gamma$ will tend to come back to the asymptotic value of 1.6, indicating equilibrium between photoionization and cooling due to the adiabatic expansion, the possibility to have $\gamma \lesssim 1.5$ for $z \lesssim 2.4$ seems to be not realistic. Even if in this work we did not attempt to constrain the temperature–density relation, the scenario in the top panel of Figure 3.2 seems likely to reproduce the trend in the evolution of the temperature, at least at low redshift, with our results reinforcing the picture of the reheating of the IGM due to He II reionization being almost complete at $z \sim 2.8$, with a subsequent tendency of a cooling, the rate of which will depend on the UV background.
Figure 3.2 Temperature at the mean density, $T_0$, inferred from $T(\bar{\Delta})$ in this work (green points/stars) and in the work of Becker et al. (2011) (black point/stars) for different assumptions of the parameter $\gamma$: $T_0$ for $\gamma \sim 1.5$ (see table 4 for the exact values) (top panel) and for $\gamma = 1.3$ (bottom panel). The linear least square fit of our data points (green line) are presented for both the choices of $\gamma$ with the corresponding 1σ error on the fit (shaded region). The fits show a change of slope in the temperature evolution: from the increase of $T_0$ between $z \sim 4$ and $z \sim 2.8$ there is a tendency of a flattening for $z \lesssim 2.8$ with a decrease in the temperature for $\gamma \sim 1.5$ and a slowdown of the reheating for $\gamma = 1.3$. 
The UV background at low redshifts

The tendency of our $T_0$ results to flatten at $z \lesssim 2.8$ seems to suggest that at these redshifts the reheating due to the He II reionization has been slowed down, if not completely exhausted, marking the end of this cosmological event. In the absence of reionization’s heating effects the temperature at the mean density of the ionized plasma is expected to approach a thermal asymptote that represents the balance between photoionization heating and cooling due to the adiabatic expansion of the Universe. The harder the UV background (UVB) is, the higher the temperature will be because each photoionisation event deposits more energy into the IGM. In particular, under the assumption of a power-law ionizing spectrum, $J_\nu \propto \nu^{-\alpha}$, and that He II reionization no longer contributes any significant heating, the thermal asymptote can be generally described by (Hui & Gnedin 1997; Hui & Haiman 2003):

$$T_0 = 2.49 \times 10^4 K \times (2 + \alpha)^{-1} \left( \frac{1 + z}{4.9} \right)^{0.53}, \quad (3.1)$$

where the parameter $\alpha$ is the spectral index of the ionizing source. The observational value of $\alpha$ is still uncertain. From direct measurements of QSO rest-frame continua, this value has been found to range between 1.4 and 1.9 depending on the survey (e.g. Telfer et al. 2002; Shull et al. 2012), whereas for galaxies the values commonly adopted range between 1 and 3 (e.g. Bolton & Haehnelt 2007; Ouchi et al. 2009; Kuhlen & Faucher-Giguère 2012) even if, in the case of the emissivity of realistic galaxies, a single power law is likely be considered an oversimplification.

Because our data at $z \lesssim 2.8$ do not show any strong evidence for a rapid decrease or increase in the temperature, here we assume that this redshift regime already traces the thermal asymptote in Equation 3.1. Under this hypothesis we can then infer some suggestions about the expectation of a transition of the UV background from being dominated mainly by stars to being dominated mainly by quasars over the course of the He II reionization ($2 \lesssim z \lesssim 5$). In Figure 3.3 we show, as an illustrative example only, two models for the thermal asymptote: the first is the model of Hui & Haiman (2003) for the expected cooling in the absence of He II reionization, with $\alpha$ scaled to 5.65 to match the flattening of the Becker et al. (2011) data at $z \sim 4$–5, while in the second case $\alpha$ was scaled to 0.17 to match our results (for $\gamma \sim 1.5$) at $z \lesssim 2.8$. These values are at some variance with the quantitative expectations: in the first case our UVB spectrum is significantly softer compared to typical galaxies-dominated spectrum while, after He II reionization, our value is somewhat harder than a typical quasar-dominated spectrum. We also emphasise that any
Chapter 3. Temperature measurements

Figure 3.3 An example of thermal asymptotes before and after He II reionization with different UVB shapes. The evolution of the thermal asymptote for the model of Hui & Haiman (2003) with $\alpha$ (Eq. 4) scaled to match the high redshift ($z \sim 4-5$) data of Becker et al. (2011) (black dashed line) is compared with the same model with $\alpha$ scaled to match our results (assuming $\gamma \sim 1.5$ at $z \lesssim 2.8$ (green dashed line). The significant change in $\alpha$ required over the redshift range $2.8 \lesssim z \lesssim 4.5$ in this example suggests that the UV background has changed, hardening during the He II reionization.

such estimate of changes in the spectral index also involves the considerable uncertainties, already discussed, connected with the correct position of the peak in $T_0$ and the choice of $\gamma$, and so we cannot make firm or quantitative conclusions here. However, in general, the observed cooling at higher temperatures at $z \lesssim 2.8$ seems to suggest that the shape of the UV background has changed, hardening with the increase in temperature during to the reionization event.
3.3 Discussion

The main contribution of our work is to add constraints on the thermal history of the IGM down to the lowest optically-accessible redshift, \( z \sim 1.5 \). These are the first temperature measurements in this previously unexplored redshift range. In this Section we discuss the possible implications of our results in terms of compatibility with theoretical models.

Measuring the low-redshift thermal history is important for confirming or ruling out the photo-heating model of He\( ^{\text{II}} \) reionization and the new blazar heating models. According to many models of the former, the He\( ^{\text{II}} \) reionization should have left a imprint in the thermal history of the IGM: during this event, considerable additional heat is expected to increase the temperature at the mean density of the cosmic gas \( (T_0) \) at \( z \lesssim 4 \) (Hui & Gnedin 1997). The end of He\( ^{\text{II}} \) reionization is then characterized by a cooling of the IGM due to the adiabatic expansion of the Universe with specifics that will depend on the characteristics of the UV background. However, even if some evidence has been found for an increase in the temperature at the mean density from \( z \sim 4 \) down to \( z \sim 2.1 \) (e.g. Becker et al. 2011), the subsequent change in the evolution of \( T_0 \) expected after the end of the He\( ^{\text{II}} \) reionization has not been clearly characterized yet and remains strongly degenerate with the imprecisely constrained slope of the temperature–density relation \( \gamma \) (see Equation 1.9). This, combined with several results from PDF analysis which show possible evidence for an inverted temperature–density relation (Becker et al. 2007; Bolton et al. 2008; Viel et al. 2009; Calura et al. 2012; Garzilli et al. 2012), brought the development of a new idea of volumetric heating from blazar TeV emission (Chang et al. 2012; Puchwein et al. 2012). These models, where the heating rate is independent of the density, seem to naturally explain an inverted \( T-\rho \) relation at low redshift. Predicted to dominate the photo-heating for \( z \lesssim 3 \), these processes would obscure the change in the temperature evolution trend due to the He\( ^{\text{II}} \) reionization, preventing any constraint on this event from the thermal history measurements.

A main motivation for constraining the temperature at lower redshifts than \( z \sim 2.1 \) is to confirm evidence for a flattening in the already detected trend of increasing temperature for \( z \lesssim 4 \). A precise measurement of a change in the \( T_0(z) \) slope, in fact, could bring important information about the physics of the IGM at these redshifts and the end of the He\( ^{\text{II}} \) reionization event. It is therefore interesting that our new temperature measurements in Figure 3.2 which extend down to redshifts \( z \sim 1.5 \), show some evidence for such a change in the evolution for \( z \lesssim 2.8 \). However, in order to make a fair comparison with different heating models in terms of the temperature at the mean density, we must recognize the fact...
that we do not have yet strong constraints on the evolution of the $T-\rho$ relation slope as a function of redshift: assuming a particular choice of $\gamma$ for the translation of the temperature values at the characteristic overdensities to those at the mean density, without considering the uncertainties in the slope itself, could result in an unfair comparison. Furthermore, the blazar heating models’ $T-\rho$ relation at each redshift can be parametrized with a power-law (of the form of Equation 1.9) only for a certain range of overdensities that may not always cover the range in our characteristic overdensities \cite{Chang2012, Puchwein2012}. Therefore, to compare our results with the blazar heating model predictions, we decided to use directly the $T(\bar{\Delta})$ values probed by the forest.

In Figure 3.4 we compare the model without blazar heating contributions, and the weak, intermediate and strong blazar heating models of \cite{Puchwein2012}, with our new results for the temperature at the characteristic overdensities. The temperature values for all the models were obtained by computing the maximum of the temperature distribution function at the corresponding redshift-dependent characteristic overdensities ($\bar{\Delta}$) in Table 3.1 by E. Puchwein (private communication). Each of the three blazar heating models has been developed using different heating rates, based on observations of 141 potential TeV blazars, under the assumption that their locally-observed distribution is representative of the average blazars distribution in the Universe. The variation in the heating rates is due to the tuning of a coefficient in the model that corrects for systematic uncertainties in the observations: the lower this multiplicative coefficient, the weaker the heating rate \cite{Chang2012, Puchwein2012}. The observational constraints on the IGM thermal evolution in Fig. 3.4 seem to be in reasonable agreement with the intermediate blazar heating model, even if some fluctuations toward higher temperatures reach the range of values of the strong blazar model. This result in general reflects what was found by \cite{Puchwein2012} (their Figure 5) in their comparison with the temperature measurements of \cite{Becker2011}, even if in that case the models were tuned to different $\bar{\Delta}$ values than ours. This general agreement could be explained by the fact that our $T(\bar{\Delta})$ measurements closely match those of \cite{Becker2011} in the common redshift range $2.0 \lesssim z \lesssim 2.9$ (see Figure 3.1) and by the weak dependency on the density of the blazar heating mechanism.

According to Fig. 3.4, the model from \cite{Puchwein2012} without a blazar heating contribution, that is based on the UV background evolution of \cite{Faucher2009}, shows temperature values significantly lower than our $T(\bar{\Delta})$ measurements for $z \lesssim 3$. However, this model does not take into account the contribution of the diffuse hard X-ray background \cite{Churazov2007}. The excess energy of these ionizing photons
Figure 3.4 Comparison of blazar heating models: the temperature values at the redshift-dependent characteristic overdensities, \( T(\Delta) \), inferred in this work (green points) are compared with the model without a blazar heating contribution (black solid line) and the weak (green dashed line), intermediate (yellow dashed line) and strong (red dashed line) blazar heating models of Puchwein et al. (2012). The blazar heating predictions were computed at the corresponding \( \Delta(z) \) in Table 3.1 to allow a fair comparison with our \( T(\Delta) \) measurements. Our observational results seem to be in reasonable agreement with the intermediate blazar heating model. The vertical error bars represent the 1-\( \sigma \) errors on the temperature measurements.

could, in fact, contribute to the heating, shifting the range of temperatures towards higher values. Also, to definitely ruled out or confirm any of the different thermal histories, an interesting further test would be to compare the temperature at the mean density between observations and models.

That is, there is a strong need for model-independent measurements of \( T_0 \) that would allow a straightforward comparison between different \( T-\rho \) relations, and so different model predictions. In this context a promising prospect is the use of the He II Lyman-\( \alpha \) forest for
the identification of the absorption features useful for an improved line-fitting constraint of $\gamma$. The possibility to calculate directly the ratio between $b$ parameters of the corresponding H I and He II lines would, in fact, make possible an easier and more precise selection of the lines that are dominated by thermal broadening ($b_{HI}/b_{\text{HeII}} \approx 2$). Even if the low S/N of the UV spectra currently available make this identification difficult (Zheng et al. 2004), possible future, space-based telescopes UV with high resolution spectrographs (e.g. Postman et al. 2009) may offer the opportunity to improve the quality and the number of “clean” He II forests for future analyses (see also Chapter 5).

3.4 Conclusions

In this thesis so far we have utilized a sample of 60 VLT/UVES quasar spectra to make a new measurement of the IGM temperature evolution at low redshift, $1.5 \lesssim z \lesssim 2.8$, with the curvature method applied to the H I Lyman-\(\alpha\) forest. For the first time we have pushed the measurements down to the lowest optically-accessible redshifts, $z \sim 1.5$. Our new measurements of the temperature at the characteristic overdensities traced by the Lyman-\(\alpha\) forest, $T(\Delta)$, are consistent with the previous results of Becker et al. (2011) in the overlapping redshift range, $2.0 \lesssim z \lesssim 2.9$, despite the datasets being completely independent. They show the same increasing trend for $T(\Delta)$ towards lower redshifts while, in the newly-probed redshift interval $1.5 \lesssim z \lesssim 1.9$, the evolution of $T(\Delta)$ is broadly consistent with the extrapolated trend at higher redshifts.

The translation of the $T(\Delta)$ measurements into values of temperature at the mean density, $T_0$, depends on the slope of the temperature–density relation, $\gamma$, which we do not constrain in this first project (but see Chapter 4). However, for reasonable, roughly constant, assumptions of this parameter, we do observe some evidence for a change in the slope of the temperature evolution for redshifts $z \lesssim 2.8$, with indications of at least a flattening, and possibly a reversal, of the increasing temperature towards lower redshifts seen in our results and those of Becker et al. (2011) for $2.8 \lesssim z \lesssim 4$. In particular, for the minimum $T_0$ case, with $\gamma \sim 1.5$, the extension towards lower redshifts provided by this work adds to existing evidence for a decrease in the IGM temperature from $z \sim 2.8$ down to the lowest redshifts probed here, $z \sim 1.5$. This could be interpreted as the result of the completion of the reheating process connected with the He II reionization.

Following the additional hypothesis that our low redshift temperature measurements are already tracing the thermal asymptote, the cooling of $T_0$ inferred at $z \lesssim 2.8$ (assuming $\gamma \sim 1.5$) may suggest that the UV background has changed, hardening during the
3.4. Conclusions

He\text{II} reionization epoch. However, the expectation for the evolution of $T_0$ following He\text{II} reionization will depend on the evolution in $\gamma$ and on details of the reionization model.

We also compared our $T(\bar{\Delta})$ measurements with the expectations for the models of [Puchwein et al. (2012)] with and without blazar heating contributions. To allow a fair comparison with our observed values, the model predictions were computed at the corresponding (redshift-dependent) characteristic overdensities ($\bar{\Delta}$). Our observational results seem to be in reasonable agreement with a moderate blazar heating scenario. However, to definitely confirm or rule out any specific thermal history it is necessary to obtain new, model-independent measurements of the temperature at the mean density.

With the IGM curvature now constrained from $z \sim 4.8$ down to $z \sim 1.5$, the main observational priority now is clearly to tightly constrain the slope of the temperature–density relation, $\gamma$, and its evolution over the redshift range $1.5 \lesssim z \lesssim 4$. This is vital in order to fix the absolute values of the temperature at the mean density and to comprehensively rule out or confirm any particular heating scenarios. Therefore, in the second part of this Thesis we will focus on the constraint of the parameter $\gamma$, studying new applications of the curvature method that could allow a complete characterization of the IGM thermal state in this crucial redshift range.

Finally, we note that, even though our new measurements have extended down to $z \sim 1.5$, there is still a dearth of quasar spectra with high enough S/N in the 3000–3300 Å spectral range to provide curvature information in our lowest redshift bin, $1.5 < z < 1.7$. We have searched the archives of both the VLT/UVES and Keck/HIRES instruments for new spectra to contribute to this bin. However, the few additional spectra that we identified had relatively low S/N and, when included in our analysis, contributed negligibly to the final temperature constraints. Therefore, new observations of UV-bright quasars with emission redshifts $1.5 \lesssim z_{em} \lesssim 1.9$ are required to improve the temperature constraint at $1.5 < z < 1.7$ to a similar precision as those we have presented at $z > 1.7$. 
The post-reionization thermal state of the intergalactic medium is characterized by a power-law relationship between temperature and density, with a slope determined by the parameter $\gamma$. We describe a new method to measure $\gamma$ using the ratio of flux curvature in the Lyman-$\alpha$ and $\beta$ forests. At a given redshift, this curvature ratio incorporates information from the different gas densities traced by Lyman-$\alpha$ and $\beta$ absorption, thereby breaking the degeneracy between $\gamma$ and the temperature inferred at the gas mean density. It is relatively simple and fast to compute and appears robust against relevant sources of observational uncertainty. We apply this technique to a sample of 27 high-resolution quasar spectra from the Very Large Telescope, finding preliminary results broadly consistent with $\gamma \sim 1.5$ over the redshift range $z \sim 2.0$–3.8. However, while promising statistical errors appear to be achievable in these measurements, uncertainties in the assumptions about the thermal state of the gas and its evolution may complicate this picture.


The results presented in Chapter 3 highlight the necessity of a precise constraint of the parameter $\gamma$ of Equation [1.9] to completely characterize the IGM thermal evolution shown in Figure 3.2 in a model-independent way. Therefore, the second part of this Thesis will be dedicated to this aim.

During and immediately after the H$\textsubscript{i}$ and He$\textsubscript{II}$ reionisations (at $z = 6$–11 and 3–4, respectively, e.g. McGreer et al. 2015; Syphers & Shull 2014; Worseck et al. 2014) cosmolog-
ical simulations predict that $\gamma$ may vary, becoming multi-valued and spatially-dependent according to the dynamics, heating and radiative transfer mechanisms involved (e.g. [Bolton et al. 2004] [McQuinn et al. 2009] [Meiksin & Tittley 2012] [Compostella et al. 2013] [Puchwein et al. 2015]). Despite considerable recent improvements, accurately simulating the effect of reionisation events on the IGM remains an open challenge. Furthermore, current measurements offer a somewhat confusing observational picture.

The main laboratory to detect variations in the $T-\rho$ relation has been the HI Lyman-\(\alpha\) forest in quasar spectra. Efforts to infer the thermal state of the IGM and search for signals of reionisation have used either line-profile decomposition to measure gas temperature as a function of column density (e.g. [Schaye et al. 2000] [Ricotti et al. 2000] [McDonald et al. 2001] [Rudie et al. 2013] [Bolton et al. 2014]) and a variety of statistical approaches which are valuable at higher redshifts, $z > 3$, where line fitting is problematic (e.g. [Theuns et al. 2002] [Becker et al. 2007] [Bolton et al. 2008] [Lidz et al. 2010] [Becker et al. 2011] [Boera et al. 2014]). While these methods probe wide redshift and density ranges ($z \approx 1.6$–5, $\Delta \approx 8$–0.3), large uncertainties in the measurements of $T_0$ and $\gamma$ may be caused by strong degeneracies between the effects of temperature and density on Ly-\(\alpha\) forest absorption.

One way to reduce these degeneracies is to constrain the $T-\rho$ relation by comparing Ly-\(\alpha\) and higher-order Lyman-series transitions, such as Ly-\(\beta\). Ly-\(\beta\) lines of moderate optical depth ($\tau \sim 0.1$–1.0) arise from higher overdensities at which Ly-\(\alpha\) lines are saturated. That is, the Ly-\(\alpha\)-to-\(\beta\) optical depth ratio is $f_{\alpha}^{\lambda_{\alpha}}/f_{\beta}^{\lambda_{\beta}} = 6.24$ (proportional to the ratios of oscillator strengths and rest wavelengths). Statistically comparing Ly-\(\alpha\) and \(\beta\) absorption is therefore a promising approach for measuring $\gamma$. Indeed, using the Ly-\(\beta\) forest in IGM temperature measurements has been suggested in several theoretical works (e.g. [Dijkstra et al. 2004] [Furlanetto & Oh 2009] [Iršič & Viel 2014]). However, so far no practical attempt has been made to directly measure $\gamma$ from a joint Ly-\(\alpha\) and \(\beta\) forest analysis. One challenge is that the Ly-\(\beta\) forest (the region between the Ly-\(\beta\) and Ly-\(\gamma\) emission lines) is entangled with absorption from Ly-\(\alpha\) at lower redshifts and it is difficult to separate the two. Hereafter we will refer to the total Ly-\(\beta\) forest plus the foreground lower redshift Ly-\(\alpha\) absorption as the Ly-\(\beta\)+\(\alpha\) region. However, assuming that these Ly-\(\beta\) and \(\alpha\) lines arise from physically uncorrelated IGM structures, a possible strategy to overcome this problem is to statistically compare the properties of the Ly-\(\beta\)+\(\alpha\) and corresponding Ly-\(\alpha\) regions.

In this Chapter we present a new method to constrain the slope of the $T-\rho$ relation using the two forest regions (Ly-\(\alpha\) and Ly-\(\beta\)+\(\alpha\)) in 27 high resolution quasar spectra. We use a statistic based on the flux curvature analysis of [Becker et al. 2011] and Chapters
4.1. The observational data

2 & 3. These previous works demonstrated that the curvature method can measure the temperatures at the (redshift dependent) characteristic densities probed by the Ly-\(\alpha\) forest. However, as only a narrow density range is constrained, it has not yet been used to measure the slope of the \(T-\rho\) relation (but see Padmanabhan et al. 2015 for a recent theoretical analysis using the Ly-\(\alpha\) forest). Using hydrodynamical simulations, we show that, at each redshift, the ratio between the curvatures of corresponding Ly-\(\alpha\) and Ly-\(\beta+\alpha\) forest regions (where in the Ly-\(\beta+\alpha\) region the redshift always refers to the Ly-\(\beta\) absorption) is sensitive to differences in the IGM thermal state between the two density regimes. Averaged over many lines of sight, this curvature ratio appears to allow \(\gamma\) to be measured with little sensitivity to \(T_0\). We demonstrate the potential for this technique using 27 quasar spectra spanning the He\(\text{II}\) reionisation redshift range. That potential is currently limited by assumptions underpinning the available simulation suite; we propose how these limitations can be overcome with future simulations and a refined data analysis approach.

The Chapter is organized as follows. In Section 4.1 is presented the selection of the subsample of quasar spectra used for this analysis from the larger sample of Table 2.1. The production of synthetic forest spectra from hydrodynamical simulations is explained in Section 4.2. The curvature ratio statistic and the main steps of the analysis are introduced in Section 4.3. Possible systematic uncertainties in the evolution of the IGM thermal state are extensively discussed in Section 4.3.3. Our preliminary measurements of \(\gamma\) are presented in Section 4.4. Finally, the results are summarized and interpreted in Section 4.5.

4.1 The observational data

The 27 quasar spectra were originally retrieved from the archive of the Ultraviolet and Visual Echelle Spectrograph (UVES) on the Very Large Telescope (VLT). They were selected on the basis of quasar redshift, wavelength coverage and signal-to-noise ratio from the sample of 60 spectra used in Chapters 2 and 3 for measuring the \(z \simeq 1.5-3\) Ly-\(\alpha\) forest curvature. They have resolving power \(R \sim 50000\) and continuum-to-noise ratio \(\geq 24\) pix\(^{-1}\) in the Ly-\(\alpha\) forest region (see Section 2.1 for details). This level of spectral quality is necessary so that the curvature measurement is not dominated by noise and unidentified metal lines. Because our new method compares the curvature of the Ly-\(\alpha\) and \(\beta+\alpha\) forest regions, we extended these same criteria to the Ly-\(\beta+\alpha\) region at \(z \simeq 2.0-3.8\), reducing the sample from 60 to 27 spectra. The quasar sample details are provided in Table 4.1.
Table 4.1 List of the QSO subsample used for this analysis. Column 1 shows the quasar names and column 2 shows their emission redshifts.

<table>
<thead>
<tr>
<th>QSO</th>
<th>$z_{em}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>J110325−264515</td>
<td>2.14</td>
</tr>
<tr>
<td>J012417−374423</td>
<td>2.20</td>
</tr>
<tr>
<td>J145102−232930</td>
<td>2.21</td>
</tr>
<tr>
<td>J024008−230915</td>
<td>2.22</td>
</tr>
<tr>
<td>J212329−005052</td>
<td>2.26</td>
</tr>
<tr>
<td>J000344−232355</td>
<td>2.28</td>
</tr>
<tr>
<td>J045313−130555</td>
<td>2.30</td>
</tr>
<tr>
<td>J112442−170517</td>
<td>2.40</td>
</tr>
<tr>
<td>J220006−280323</td>
<td>2.40</td>
</tr>
<tr>
<td>J011143−350300</td>
<td>2.41</td>
</tr>
<tr>
<td>J033106−382404</td>
<td>2.42</td>
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<tr>
<td>J120044−185944</td>
<td>2.44</td>
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<td>J234628+124858</td>
<td>2.51</td>
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<td>J015327−431137</td>
<td>2.74</td>
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<td>J235034−432559</td>
<td>2.88</td>
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<tr>
<td>J040718−441013</td>
<td>3.00</td>
</tr>
<tr>
<td>J094253−110426</td>
<td>3.05</td>
</tr>
<tr>
<td>J042214−384452</td>
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<td>J103909−231326</td>
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<td>J114436+095904</td>
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<td>3.65</td>
</tr>
<tr>
<td>J014049−083942</td>
<td>3.71</td>
</tr>
</tbody>
</table>

To establish an initial continuum in the Ly-β+α region, we applied the same automatic, piece-wise polynomial fitting approach used in Section 2.1 for the Ly-α region, leaving the latter unchanged. While manual refitting was necessary in some spectra for particular parts of the Ly-β+α region, this initial continuum has little effect on the final curvature measurements because the spectra are subsequently “re-normalized” using a b-spline fit, as explained in Section 4.3.1 below, and as in Section 2.1. Similar to Figure 2.2, the redshift and the C/N distributions of the Ly-β+α region of the quasars in our sample is shown in Figure 4.1.
4.2 The simulations

We used the same hydrodynamical simulations used in Chapters 2 and 3 to produce synthetic spectra for Ly-β+α and Ly-α in the redshift range $2.0 \leq z \leq 3.5$. The GADGET-3 smoothed-particle hydrodynamics simulations include dark matter and gas, with $2 \times 512^3$ particles and a gas particle mass of $9.2 \times 10^4 h^{-1} M_\odot$ in a periodic box of 10 comoving
Chapter 4. The curvature ratio

$h^{-1}$ Mpc (see Section 2.2 for details). The gas, assumed to be optically thin, is in equilibrium with a spatially uniform UVB (Haardt & Madau 2001), but the photoheating rates have been rescaled so that the corresponding values of $T_0$ and $\gamma$ vary between different simulations. The parameters characterizing the different models are summarized in Table 2.2. From each model, synthetic Ly-\(\alpha\) forest spectra were generated for 1024 random lines of sight through each of 6 redshift snapshots over the range $z = 2.0–3.5$. The Ly-\(\beta\)+\(\alpha\) (optical depth) spectra were produced by scaling the Ly-\(\alpha\) optical depths by a factor $f_\alpha \lambda_\alpha/f_\beta \lambda_\beta = 6.24$ and contaminating them with Ly-\(\alpha\) absorption from lower-redshift snapshots ($z = 1.6–2.7$) with the same thermal history.

Finally, as in Section 2.5.1 we calibrated the synthetic spectra to match the properties of the real spectra (i.e. resolving power, pixel size and signal-to-noise ratio). The only difference is that, in this work, we scaled the effective optical depth of the synthetic spectra to match the recent results from Becker et al. 2013 rather than to a direct measurement from our spectra. As shown in Section 4.3.1 our measurements are relatively insensitive to this optical depth calibration.

4.3 The curvature ratio method

The curvature statistic is defined as in Equation 2.1 with the first and second derivatives of the flux ($F'$, $F''$) taken with respect to wavelength or relative velocity. As demonstrated in previous works, the Ly-\(\alpha\) forest curvature is directly related to the IGM temperature at the characteristic overdensities probed by this absorption, regardless of $\gamma$. Because the median overdensity contributing to Ly-\(\beta\) forest absorption is higher than that for Ly-\(\alpha\) (Furlanetto & Oh 2009), the mean absolute curvature computed from sections of Ly-\(\beta\)+\(\alpha\) forest will be, on average, a tracer of the IGM temperature in a higher density regime. Therefore, at each redshift $z$, the curvature ratio,

$$R_\kappa(z) \equiv \frac{\langle |\kappa_{\beta+\alpha}(z)| \rangle}{\langle |\kappa_\alpha(z)| \rangle},$$

(4.1)

will be sensitive to temperature differences between two different gas density regimes and, consequently, to $\gamma$. Here the redshift $z$ in the Ly-\(\beta\)+\(\alpha\) forest region always refers to the Ly-\(\beta\) absorption. In Equation 4.1 the mean absolute curvatures for Ly-\(\beta\)+\(\alpha\) and Ly-\(\alpha\) are averaged over corresponding spectral sections of 10 comoving $h^{-1}$ Mpc (corresponding to the simulation box size). We demonstrate in Appendix B the numerical convergence of our simulations for the curvature ratio in terms of box size and mass resolution.
4.3. The curvature ratio method

4.3.1 Analysis of simulated spectra: the $\gamma - \log \langle R_\kappa \rangle$ relation

We find the connection between the curvature ratio and $\gamma$ at the redshifts of the simulation snapshots ($z=[2.173, 2.355, 2.553, 2.760, 3.211, 3.457]$) by computing the mean $R_\kappa$ over the 1024 artificial lines of sight, for each thermal history, and fitting a simple function between $\log \langle R_\kappa \rangle$ and $\gamma$.

Single $R_\kappa$ values for each line of sight and redshift were obtained as defined in Equation 4.1. To compute the mean absolute curvature of synthetic Ly-$\alpha$ and Ly-$\beta+\alpha$ sections, we adopt the same procedure described in Becker et al. (2011) and in Section 2.3, an example of which is shown in Fig. 4.2: in each section of artificial spectrum, the mean absolute curvature is computed from a b-spline fit which has been “re-normalized” by its maximum value in that interval. This approach is required to avoid systematic errors when determining the curvature from real spectra, so it must be applied to the synthetic spectra for consistency. The b-spline fit reduces the sensitivity of the curvature ($\kappa$) to noise and the re-normalization minimizes potential uncertainties arising from inconsistent continuum placement. Finally, only the pixels where the re-normalized b-spline fit falls in the range 0.1–0.9 are used to measure the mean absolute curvature of each section. In this way we exclude saturated pixels, which do not contain useful information, and pixels with little-to-no absorption whose curvature is near zero and uncertain.

In Figure 4.3 we present the $\gamma - \log \langle R_\kappa \rangle$ relationship obtained by averaging the curvature ratio computed from the synthetic spectra with different thermal histories, i.e. different $\gamma$ and $T_0$ values. At each redshift, the $\log \langle R_\kappa \rangle$ values computed from each of the simulations (coloured points) and plotted as a function of the $\gamma$ value characterizing the different thermal histories, lie on the same curve, with little sensitivity to differences in $T_0$. Therefore, for each redshift, it is possible to fit a simple function that connects the mean $\log \langle R_\kappa \rangle$ and $\gamma$ independently of $T_0$ (see also Appendix C for the complete set of plots). As redshift decreases, the relation shifts towards lower values of $\log \langle R_\kappa \rangle$ and a somewhat steeper relationship with $\gamma$. Given this tight correspondence, the curvature ratio represents an interesting tool to independently measure the slope of the IGM $T-\rho$ relation.

The sensitivity of this nominal $\gamma - \log \langle R_\kappa \rangle$ relation to the main observational uncertainties in the spectra are tested as follows:

- Noise: The synthetic spectra from which we obtain the nominal relationship need to include noise at the same level as in the real spectra. If, as an extreme example, no noise was added, Fig. 4.3 shows the effect on the $\gamma - \log \langle R_\kappa \rangle$ relation: this leads to a $\sim 5$ per cent decrease in $\langle R_\kappa \rangle$ (and hence a variation of $\sim 0.03$ dex in $\log \langle R_\kappa \rangle$).
Figure 4.2 Curvature calculation for simulated and real spectra. Top two panels: curvature (bottom) from b-spline fits of a simulated, $10\,h^{-1}\text{Mpc}$-wide spectrum (top) of Ly-$\alpha$ forest at $z=2.76$ (blue solid line) and Ly-$\beta+\alpha$ (green solid line). The latter is obtained by contaminating the corresponding Ly-$\beta$ forest (red dotted line) with a randomly chosen Ly-$\alpha$ section at lower redshift (black dashed line). Bottom two panels: same as above but for a real Ly-$\alpha$ and Ly-$\beta+\alpha$ spectrum. The spectra (black lines) are plotted behind the b-spline fits. Shading shows a Ly-$\beta+\alpha$ region contaminated by metal absorption (green dashed line); the corresponding part of the Ly-$\alpha$ spectrum is also masked (black dashed line).

This would cause a $\sim 8$ per cent underestimation of $\gamma$, comparable to the statistical errors in our sample. Therefore, any errors in how the noise properties are incorporated into the $\gamma$–log$\langle R_\kappa \rangle$ relation should cause relatively small systematic uncertainties in $\gamma$ measurements of the quality coming from a sample like ours.
4.3. The curvature ratio method

Figure 4.3 Relationship between the slope parameter of the temperature–density relation, $\gamma$, and the curvature ratio, $R_\kappa$, from our nominal simulations. Each solid line represents this relationship at a particular redshift, obtained from synthetic spectra with different thermal histories. For clarity, we show the values of $\log \langle R_\kappa \rangle$ for the different models, and the relation computed from spectra without noise only for $z = 2.760$ (coloured points and dashed line, respectively). But see Appendix C for the scatter plots corresponding to the other redshifts.

- Optical depth calibration: Changing the effective optical depth used to calibrate the simulations by 10 per cent alters $\langle R_\kappa \rangle$ by only $\sim 2$ per cent and, consequently, $\gamma$ by $\sim 4$ per cent, well within the statistical uncertainties in our sample. This point is particularly promising because the curvature for the Ly-\textalpha forest alone (and the Ly-\textbeta+\textalpha forest alone) is considerably more sensitive to this calibration, as explored in 2.5.

However, while the nominal $\gamma$–$\log \langle R_\kappa \rangle$ relation is robust to these observational aspects, uncertainties in the evolution of the IGM thermal state must also be considered (see Section 4.3.3).
4.3.2 The observed curvature ratio

To apply the method to the 27 real quasar spectra we compute the curvature ratio in sections of 10 comoving $h^{-1}\text{Mpc}$ of metal-free Ly-$\alpha$ and corresponding Ly-$\beta$+\alpha forest regions. Narrow metal lines ($b \lesssim 15\text{km}\text{s}^{-1}$) represent a potentially serious source of systematic errors in any measure of forest absorption and should be avoided. With this aim we “clean” the spectra, extending the metal masking procedure described in 2.4.2 to the Ly-$\beta$+\alpha region: metal absorbers redward of the Lyman-$\alpha$ emission line are identified and all strong metal transitions at their redshifts are masked out, followed by a by-eye check of the remaining forest.

While the metal correction produces spectra that are reasonably free of contaminants, this procedure reduces the quantity of information available in different sections in a non-uniform way, introducing a possible source of bias. Because the curvature ratio traces differences in the absorption features of two different regions of the same observed spectrum, avoiding systematic effects requires that they cover the same absorption redshift range. Therefore, before measuring $R_\kappa$ from the real data, we mask out regions of the Ly-$\beta$+\alpha forest corresponding to any range masked from the Ly-$\alpha$ forest, and vice versa. Figure 4.2 shows an example of this masking procedure. Finally, possible edge effects are avoided: we do not include the 4 pixels closest to the edge of any masked region in the curvature ratio calculation.

Figure 4.4 presents the curvature ratio results from our observational sample. The statistical uncertainty in the single log $R_\kappa$ measurement, computed from each useful pair of Ly-$\beta$+\alpha and Ly-$\alpha$ sections, is negligible compared to the much larger variance among different measurements. Therefore, we collected the measurements in three broad redshift bins which include $>30$ individual measurements. In each bin we then calculate the mean log $\langle R_\kappa \rangle$ and its uncertainty using a bootstrap technique over the individual measurements enclosed. The distribution of the log $R_\kappa$ values for each bin and the bootstrapped distribution of the log $\langle R_\kappa \rangle$ value are shown in Figure 4.5.

The width and roughly Gaussian shape of the log $R_\kappa$ distribution within each bin is reproduced by our simulated spectra, providing some confidence that the simulations adequately describe the statistical properties of the observed forest absorption. An example of this agreement is shown in Figure 4.6 where, for the lowest redshift bin, the observed distribution of log $R_\kappa$ values is compared with the distribution obtained from one of our simulation models (model F15 in Table 2.2) at $z = 2.355$.

Figure 4.4 shows evidence for a mild evolution in log $\langle R_\kappa \rangle$ as a function of redshift. A Spearman rank correlation test reveals a positive correlation ($r \approx 0.26$) with the associated
4.3. The curvature ratio method

Figure 4.4 Curvature ratio, $R_\kappa$, from each useful pair of Ly-β+α and Ly-α sections from 27 quasar spectra (black circles). The measurement of $\log \langle R_\kappa \rangle$, averaged within each redshift bin, is shown with 1-σ bootstrap errors both before (red squares) and after (green points) the metal masking procedure. The expected evolution of $\log \langle R_\kappa \rangle$, from our nominal simulations with relatively constant $\gamma$, is presented for $\gamma \sim 1.5$ (solid blue line) and $\gamma \sim 1.0$ (dashed blue line). The redshift bins are $(z_{\text{min}}, \bar{z}, z_{\text{max}}) = (2.0, 2.27, 2.5), (2.5, 2.86, 3.1), (3.1, 3.36, 3.74)$, and the (metal-masked) $\log \langle R_\kappa \rangle$ measurements are $-0.110 \pm 0.026$, $-0.019 \pm 0.030$ and $0.017 \pm 0.032$, respectively.

Probability (of an uncorrelated system producing the observed correlation by chance) of $\approx 0.001$. For a constant $\gamma$, this increase in $\log \langle R_\kappa \rangle$ is consistent with the expected increase in $\log \langle R_\kappa \rangle$ at increasing redshifts seen in Fig. 4.3. In particular, our $\log \langle R_\kappa \rangle$ measurements in Figure 4.4 show good agreement with the expected evolution for a model with $\gamma \sim 1.5$ (blue solid line in Figure 4.4) and this scenario would be in agreement with the recent results of Bolton et al. (2014) with $\gamma = 1.54 \pm 0.11$ at $z \sim 2.4$. However, the absence of a very large change in the curvature ratio in the redshift range considered is consistent with the assumption in the nominal simulations that $\gamma$ varies little ($\lesssim 0.05$) for redshifts $z = 2–3.5$ (see also Section 4.3.3).

Figure 4.4 also shows $\log \langle R_\kappa \rangle$ computed without first masking the metal absorption...
Figure 4.5 Distribution of the log $R_\kappa$ values for each bin and the bootstrapped distribution of the log $\langle R_\kappa \rangle$ value. For each of the three redshift bins in which we collected our measurements (figures (a), (b) and (c)) are plotted the distribution of the log $R_\kappa$ measurements (top panels) and the bootstrapped distribution of the log $\langle R_\kappa \rangle$ value (bottom panels). Bottom panels: the mean computed directly from the data, without bootstrapping, is indicated with the green solid line and the center value of the bootstrapped distribution of the mean (blue solid line) and its 1σ uncertainties (red solid lines) are also shown.

Even though the effect of metal contamination is important when measuring the curvature of the Ly-α forest alone (see Section 2.4.2), it is similar in the corresponding sections of Ly-β+α forest, so the curvature ratio is understandably less sensitive to this
4.3. The curvature ratio method

Figure 4.6 Comparison between the shape of the log $R_\kappa$ distribution observed in the lowest redshift bin (green histogram) and the distribution obtained from the simulation model F15 at $z = 2.355$ (black histogram). Both the distributions have been normalized for the comparison and show similar Gaussian shapes and widths. This agreement gives us some confidence that the dispersion of our measurement in Figure 4.4 is well reproduced by the simulations used in this work.

The results in Figure 4.4 show that, even without applying the metal correction, the bias introduced in $\langle R_\kappa \rangle$ is $\sim 8$ per cent (and hence a variation of $\sim 0.05$ dex in $\log \langle R_\kappa \rangle$). Therefore, possible errors in the metal masking procedure will introduce negligible uncertainties in $\gamma$ compared to the statistical ones.
4.3.3 Rapid evolution in the IGM thermal state

Our nominal simulations assume only mild evolution in $T_0$ ($\Delta T_0 \sim 2000$ K) and $\gamma$ ($\Delta \gamma \lesssim 0.02$) between the Ly-$\beta$ and the foreground Ly-$\alpha$ redshifts (see Figure 1 in Becker et al. 2011). However, even if these are reasonable starting assumptions, if these parameters vary more drastically on short timescales due to, for example, blazar heating (Puchwein et al. 2012) or non-equilibrium effects (Puchwein et al. 2015) these assumptions may have important consequences for the $\gamma$–$\log \langle R_\kappa \rangle$ relation and, therefore, preclude a final conversion of our $\log \langle R_\kappa \rangle$ measurements to $\gamma$ values with formal error bars.

We can construct a toy model to investigate how sensitive the $\gamma$–$\log \langle R_\kappa \rangle$ relation is to rapid evolution in $T_0$ and $\gamma$ by altering these parameters in the foreground Ly-$\alpha$ forest via a simple post-processing of the simulated spectra. For a given simulation model, at a given redshift, new synthetic spectra are extracted after imposing a new one-to-one power-law $T$–$\rho$ relationship. Note this is an approximation, as it removes the natural dispersion in the temperatures at a given overdensity from shock-heating and/or radiative cooling. However, in this way we can easily modify the $T_0$ and $\gamma$ parameters, and their evolution, to explore the effect on the $\gamma$–$\log \langle R_\kappa \rangle$ relation without running new hydrodynamical simulations.

The largest effects on the $\gamma$–$\log \langle R_\kappa \rangle$ relation were found in the following two tests:

- Rapid evolution in $T_0$: For the redshift range $z = 2–3.5$, $T_0$ evolves in our nominal simulations such that the temperature difference between the foreground and the Ly-$\beta$ redshift is small: $\Delta T_0 \equiv T_0(\text{Ly-}\beta) - T_0(\text{foreground}) \lesssim 2000$ K. To test the effect of much stronger variations in the temperature at the mean density, we modified the values of $T_0$ in each of our nominal simulations, for the foreground only, using the post-processing approach. We then computed the change in $\log \langle R_\kappa \rangle$ compared to the nominal values, $\Delta \log \langle R_\kappa \rangle$. Fig. 4.7 shows the direct relationship between $\Delta T_0$ and $\Delta \log \langle R_\kappa \rangle$ (black solid line). For example, if $T_0$ changes by a further $\approx 5000$ K between $z = 3.2$ (Ly-$\beta$) and $z = 2.6$ (foreground), we would expect a systematic error in our measurement of $\log \langle R_\kappa \rangle$ of $\approx 0.03$, similar to the statistical error per redshift bin derived from our 27 quasar spectra. Of course, if we alter $T_0$ by the same amount, with the same post-processing approach, for both the foreground and the Ly-$\beta$ redshift, the $\gamma$–$\log \langle R_\kappa \rangle$ relation does not change.
Figure 4.7 Expected relation between $\log\langle R_\kappa \rangle$ and the temperature change between the redshifts corresponding to the Ly-$\beta$ and foreground Ly-$\alpha$ forest (black line; see text for details). The green shading indicates the typical statistical uncertainty in our measured $\log\langle R_\kappa \rangle$ values, per redshift bin. The $\Delta T_0$ in our nominal simulations is typically $\lesssim 2000$ K (dashed line) and the specific results for the self-consistent ‘T15fast’ and ‘T15slow’ simulations with their statistical errors are shown for $z = 2.7$ and 3.2.

- Rapid evolution in $\gamma$: Similar to the $\Delta T_0$ test above, we emulated rapid changes in $\gamma$ over short timescales, $\Delta z \approx 0.6$, by post-processing the foreground Ly-$\alpha$ only. Figure 4.8 shows the relationship between the change in $\gamma$ at the redshift of the foreground Ly-$\alpha$ forest and $\Delta \log\langle R_\kappa \rangle$ (black solid line); for example, an decrease in the foreground $\gamma$ by 0.15 implies $\Delta \log\langle R_\kappa \rangle \approx 0.03$, equivalent to the statistical uncertainty in our observations.
Figure 4.8 Expected relation between \( \log\langle R_\kappa \rangle \) and the change in \( \gamma \) at the redshift of the foreground Ly-\( \alpha \) forest (black line; see text for details). The green shading indicates the typical statistical uncertainty in our measured \( \log\langle R_\kappa \rangle \) values, per redshift bin. \( \Delta \gamma = 0 \) corresponds to the foreground \( \gamma \) value of our nominal simulations (dashed line).
4.3. The curvature ratio method

While highlighting the potential importance of assumptions for the foreground Ly-\(\alpha\) forest, the above tests rely on a rather simplistic toy model that does not reproduce self-consistently the evolution of the complex relationships between physical parameters. Ideally, the sensitivity of the \(\gamma - \log\langle R_\kappa \rangle\) relation to different physical assumptions and thermal histories would need to be tested with additional self-consistent simulations in the redshift range of interest. Our simulation suite does offer one such self-consistent test in the case of strong \(T_0\) evolution: we used the ‘T15fast’ and ‘T15slow’ simulations of Becker et al. (2011) (see their Figure 8) to mimic a possible \(\sim 5000\) K heating event from He\(\text{II}\) reionisation at \(z > 3\) such that between the Ly-\(\beta\) and foreground redshifts there was a typical temperature decrease of \(\Delta T_0 \approx 2000–4000\) K. In both simulations we find that \(\log\langle R_\kappa \rangle\) varies by \(\lesssim 0.01\) compared to the nominal simulations (see Figure 4.7), which seems less sensitive to substantial evolution in \(T_0\) than implied by our simplistic toy model. However, differences in the pressure smoothing scale in these models (which may be acting to improve the agreement) prevent us from reliably estimating the systematic uncertainties involved without further self-consistent tests.

*Moreover, our results in Section 4.3.2 do not support strong \(\gamma\) or \(T_0\) evolution with redshift: considering the strong sensitivity, previously explored, to the variation of these parameters, a possible considerable departure from a small evolution, similar to the one assumed in our nominal simulations, would manifest itself in a detectable imprint in the evolution of \(\log\langle R_\kappa \rangle\) with redshift that our observations do not show. Using the correlation between the change in \(\gamma\) at the redshift of the foreground Ly-\(\alpha\) forest and \(\Delta \log\langle R_\kappa \rangle\) presented in Figure 4.8 we can attempt an indicative prediction for the expected \(\log\langle R_\kappa \rangle\) evolution in different scenarios of strong evolution in \(\gamma\) with redshift. Figure 4.9 shows how considerable departures from just a small evolution in \(\gamma\) would manifest themselves as detectable imprints on the evolution of \(\log\langle R_\kappa \rangle\) with redshift, that are inconsistent with our observations. Different toy models, for different scenarios of \(\gamma\) evolution in the redshift range \(z \sim 2.0–4\) are presented in Figure 4.9 (a). The corresponding predictions in the \(\log\langle R_\kappa \rangle\) are shown in Figure 4.9 (b) in comparison with our observational measurements. Our observational results are strongly inconsistent with the models (the “slow evolution” and the “fast evolution” models) which predict a strong decrease in \(\gamma\) (possibly due to the He\(\text{II}\) reionization) by more than 0.4. Given the current possible systematic uncertainties, we cannot exclude a “mild” flattening in the temperature–density relation with a \(\Delta \gamma \sim 0.2\).
Figure 4.9 Indicative predictions for the log$(R_\kappa)$ evolution with redshift in different scenarios of strong evolution in $\gamma$. Panel (a): toy models for different scenarios of $\gamma$ evolution as a function of redshift. The models are indicatively emulating a possible flattening (i.e. a reduction in $\gamma$) in the temperature–density relation due to the He II reionization. Panel (b): predictions for the log$(R_\kappa)$ evolution for the different scenarios of panel (a) (solid and dashed lines), compared with our observational results (green dots). Our observational results are consistent with a small evolution in $\gamma$ for $z = 2 - 4$ predicted by our nominal simulation (e.g. solid blue line). Strong evolutionary scenarios in this redshift range with a possible decrease in $\gamma$ by more than 0.4 (red and green dashed lines) are inconsistent with the observed evolution of log$(R_\kappa)$. Possible systematic uncertainties, arising from assumptions about the evolution of the IGM thermal state, do not allow to exclude possible “mild” flattening in $\gamma$ (black dashed line).
4.4. *The slope of the $T-\rho$ relation*

Despite the above complexities, which require future, more comprehensive simulations to resolve in full, the results of our preliminary analysis suggest that the curvature ratio is a promising alternative tool to measure the density dependence of the IGM thermal state. Our preliminary forward-modeling of the quasar spectra is consistent with $\gamma \sim 1.5$ and no strong evolution in this quantity at $2 \lesssim z \lesssim 3.8$.

4.3.4 *Jeans smoothing effect*

Another potential source of systematic uncertainties may be represented by modifications of the absorbers size during the IGM thermal evolution that can affect the broadening of the absorption lines. This is referred to as the Jeans smoothing effect. To test the sensitivity of the $\gamma-\log\langle R_\kappa \rangle$ relation to variations in the integrated thermal history we can apply modifications to the density distribution of the absorbers (i.e. overdensity distribution) in the simulations using again a post-processing approach. Firstly, for each simulation model of Table 2.2 we impose a one-to-one power-law $T-\rho$ relationship in which we maintain the original density fields of the nominal simulations. From the $\log\langle R_\kappa \rangle$ values computed from these models we then obtain a “reference” $\gamma-\log\langle R_\kappa \rangle$ relation.

Secondly, holding fixed the “instantaneous” values of $T_0$ and $\gamma$ at each redshift, we substitute the density fields of one nominal simulation to another, evolved with different thermal histories, i.e with different $T_0$ or $\gamma$. Any difference between the $\log\langle R_\kappa \rangle$ computed from the new extracted spectra and those from the reference ones will then only be due the differences in the overdensity distribution and, consequently, to the Jeans smoothing effect.

Interestingly, Figure 4.10 shows that the $\gamma-\log\langle R_\kappa \rangle$ relation is insensitive to variations in the integrated thermal history: if compared with the reference relation (dashed line), in all cases, the values of $\log\langle R_\kappa \rangle$ do not seem to be affected by variations in the overdensity distribution and consequently by the Jeans smoothing effect.

4.4 *The slope of the $T-\rho$ relation*

As shown in Section 4.3.3 the $\gamma-\log\langle R_\kappa \rangle$ relation may be sensitive to assumptions about the thermal state of the IGM and its evolution. For this reason we cannot yet present final measurements of $\gamma$ with formal uncertainties. However, from the analysis presented in this Chapter, the current simulation suite seems to well reproduce the properties of the observed spectra (see Section 4.3.2) and it is likely that a future refined analysis will provide similar results to the ones that we have currently obtained. In this Section we will
Chapter 4. The curvature ratio

Figure 4.10 Sensitivity of the $\gamma$–$\log\langle R_\kappa \rangle$ relation to variations in the integrated thermal history. The $\log\langle R_\kappa \rangle$ values are computed from synthetic spectra obtained after imposing on each simulation a different density field, evolved in models with different $\gamma$ values (red dots) or different $T_0$ (blue stars). In all cases, the values of $\log\langle R_\kappa \rangle$ do not seem to depart significantly from the reference relation (black dashed line; see text for details).

therefore present the preliminary measurement of $\gamma$ obtained using our nominal simulation $\gamma$–$\log\langle R_\kappa \rangle$ relations (in Figure 4.3) and the measurements of $\log\langle R_\kappa \rangle$ computed from our 27 quasar spectra (in Figure 4.4). Note that the uncertainties presented here are purely statistical and do not include any estimate of the systematic errors stemming from the effects discussed in Section 4.3.3.

Interpolating the observed values of $\log\langle R_\kappa \rangle$ with the $\gamma$–$\log\langle R_\kappa \rangle$ relation we obtain our first estimates of $\gamma$ for redshift $z \simeq 2.0$–3.8. These results are compared in Figure 4.11 with previous line-fitting measurements from Schaye et al. (2000), Ricotti et al. (2000), McDonald et al. (2001) and Bolton et al. (2014), and with the recent theoretical models for $\gamma$’s evolution presented in McQuinn et al. (2009) and Puchwein et al. (2014). Using the 27 lines of sight immediately available, our 1$\sigma$ uncertainties are comparable with that of the recent measurement by Bolton et al. (2014) (a recalibration of that by Rudie et al., 2013) from $\sim$6000 individual H I absorbers. Because the line-fitting technique is time-consuming
4.4. *The slope of the \( T - \rho \) relation

and potentially subjective, our measurement represents a promising improvement. Assuming that the statistical errors dominate the total error budget, combining our 3 binned measurements of \( \gamma \) with a simple weighted mean gives \( \gamma = 1.55 \pm 0.06 \) with little evolution \((\lesssim 15 \text{ per cent})\) over \( z \simeq 2.0\)–3.8. This result is consistent with the \( \gamma \) value adopted to translate \( T(\bar{\Delta}) \) measurements into \( T_0 \) values in our previous curvature analyses (see Section 3.2).

While these preliminary \( \gamma \) estimates are in good agreement with the Bolton et al. (2014) result, some tension exists with the low \( \gamma \) values measured by Schaye et al. (2000), and possibly Ricotti et al. (2000), in the redshift range \( z \simeq 2.5\)–3.0 (but see discussion in Bolton et al. 2014). An inverted \( T - \rho \) relation (i.e. \( \gamma \lesssim 1 \)) at similar redshifts has also been suggested by analyses of the flux probability distribution (Becker et al. 2007; Bolton et al. 2008) and would correspond to a temperature increase, independent of the density, during He\( \text{II} \) reionisation (but see discussion in Becker & Bolton 2013). According to this argument, regions with lower temperature (i.e. lower density) before He\( \text{II} \) reionisation would experience a larger temperature increase, manifesting itself as a flattening of the \( T - \rho \) relation (i.e. a decrease in \( \gamma \)). However, in the gas density range traced by the curvature ratio \((\Delta \gtrsim 2 \text{ for } z \lesssim 3.8)\) our results are inconsistent at more than 2.5-\( \sigma \) with \( \gamma \lesssim 1 \) over the redshift interval considered (again considering only the statistical errors in our \( \gamma \) estimates).

A flattening of the \( T - \rho \) relation (from the asymptotic \( \gamma \approx 1.6 \)) is also predicted in the non-equilibrium model of Puchwein et al. (2014), albeit with only a modest reduction in \( \gamma \) to \( \approx 1.4 \). In this simulation, based on the radiative transfer model of Haardt & Madau (2012), photoionization equilibrium is not assumed during the He\( \text{II} \) reionisation phase. A similar dip in \( \gamma \) is also visible in the radiative transfer simulations of McQuinn et al. (2009) with He\( \text{II} \) reionisation at \( z \sim 3 \); however, no significant flattening occurs in the case of an early He\( \text{II} \) reionisation at \( z \gtrsim 6 \). At present, the uncertainties in our measurements do not allow us to distinguish a clear preference for one of the models. We estimate that, according to the statistical uncertainties obtained in this work, a sample of a further \( \sim 100 \) quasar spectra covering absorption redshifts 2.7–4.0 would improve the overall statistical precision of the \( \gamma \) measurement to \( \approx 0.03 \), allowing a \( \approx 5-\sigma \) discrimination between the McQuinn et al. models. However, such a measurement will only be useful if the possible systematic errors discussed above are resolved and removed in future refinement of the curvature ratio technique.
Chapter 4. The curvature ratio

Figure 4.11 Preliminary measurements of $\gamma$ in the $T$–$\rho$ relation, obtained from the curvature ratio (green points) compared with previous line-fitting measurements from Schaye et al. (2000) (grey points), Ricotti et al. (2000) (azure triangles), McDonald et al. (2001) (light blue squares) and Bolton et al. (2014) (red circle). The non-equilibrium model of Puchwein et al. (2014) (red dashed line) and the radiative transfer simulations of McQuinn et al. (2009) with an early ($z \gtrsim 6$; blue dotted line) and late ($z \sim 3$; green dot-dashed line) He II reionisation are also presented. Note that the latter model is initialized with $\gamma = 1$ at $z = 6$. Our measured $\gamma$ values are $1.57 \pm 0.08$, $1.53 \pm 0.11$ and $1.49 \pm 0.15$ in the redshift bins defined in Fig. 4.4.
4.5 Conclusions

We have presented a new method to constrain the $T-\rho$ relation using the ratio of curvatures, $\langle R_\kappa \rangle$, in the Ly-\(\alpha\) and \(\beta\) forests. The technique allows an independent measurement of $\gamma$, appears robust against observational uncertainties in the noise level, metal contamination and effective optical depth, and is relatively simple and fast to compute. The $\langle R_\kappa \rangle$ measured in 3 redshift bins covering $z \simeq 2.0$–3.8 in 27 VLT/UVES quasar spectra, provides $\approx 6$ per cent statistical uncertainties and matches the redshift evolution derived from our nominal hydrodynamical simulations with $\gamma \sim 1.5$. In the absence of any other systematics, this statistical error in $\langle R_\kappa \rangle$ translates to a $\lesssim 10$ per cent uncertainty in $\gamma$ in $\Delta z \sim 0.6$ bins. This would be competitive with recent attempts to measure $\gamma$ using line decomposition [Rudie et al. 2013; Bolton et al. 2014].

However, we do not present here the $\gamma$ measurements with formal uncertainties, with any attempted estimate of systematic errors, because the $\gamma$–$\log \langle R_\kappa \rangle$ relation may also be sensitive to assumptions about the thermal state of the IGM and its evolution. Our nominal simulations predict a decrease of $\Delta T_0 \sim 2000$ K between the Ly-$\beta$ and foreground Ly-\(\alpha\) redshifts. If, in reality, there was a further $\Delta T_0 \sim 5000$ K it would result in a systematic uncertainty in $\gamma$ which is comparable to the statistical uncertainty. A change of $\Delta \gamma = 0.15$ (cf. $\Delta \gamma \approx 0.02$ in the nominal simulations) results in a similar uncertainty. Changes of this magnitude may be expected for non-equilibrium photo-heating [Haardt & Madau 2012; Puchwein et al. 2015] or more exotic models incorporating for example blazar heating [Puchwein et al. 2012]. Therefore, future efforts should aim to explore the full parameter space for these assumptions. This will likely require a much more comprehensive simulate suite than available for this work so far. Alternatively, these systematics may be largely mitigated if the foreground Ly-\(\alpha\) lines that contaminate the Ly-$\beta$ absorption can be identified and masked (or divided) out. If this can be achieved and/or the major foreground uncertainties are marginalised over, the curvature ratio promises a novel technique for constraining the evolution of $\gamma$ over a large redshift range. Distinguishing between models of the IGM’s thermal history could then be improved by utilizing the large samples of high signal-to-noise ratio quasar spectra available in the archives of several high-resolution spectrographs.
Resolving the curvature of the He II Ly-\(\alpha\) forest

While the curvature ratio, presented in Chapter 4, seems to represent a valid method to constrain the parameter \(\gamma\) of the IGM temperature–density relation (Equation 1.9), the contamination of the Ly-\(\beta\) forest with lower redshift foreground Ly-\(\alpha\) absorption may necessitate further refinement in the data analysis which may not be straightforward. In this Chapter we propose and explore the idea of using the He II Ly-\(\alpha\) forest, together with the corresponding H\(\text{I}\) Ly-\(\alpha\) forest to measure \(\gamma\). The aim of this Chapter is to understand the applicability of the curvature method to future He II high-resolution forest spectra. In particular, we use hydrodynamical simulations to understand the specifics in terms of resolution and box size necessary to resolve the underdense gas traced by the He II forest and statistically analyse the shape of its absorption features. The simulations used in Chapter 2, 3 and 4 are optimized in terms of resolution and box size (\(2 \times 512^3\) particles per \(10h^{-1}\)Mpc box) to accurately reproduce the absorption features derived from the typical overdensities probed by the H\(\text{I}\) Ly-\(\alpha\) absorption, but so far have not been used to study the helium counterpart. We show here, through a convergence analysis, that the specific parameters of the simulations previously adopted are insufficient to apply the curvature statistic to the He II forest.

5.1 Motivation & proposal

The He II Ly-\(\alpha\) absorption \((\lambda_0 = 303.78\ \text{Å})\) is accessible in the far UV (FUV) from space at \(z > 2\), due to the Galactic Lyman limit and, being free from H\(\text{I}\) Ly-\(\alpha\) contamination, could represent a valid alternative to constrain \(\gamma\) in the observable redshift range \(2 \lesssim z \lesssim 3\). The cross section for He II Ly-\(\alpha\) is 4 times smaller than for H\(\text{I}\). Hence, from Equation 1.6
Chapter 5. Resolving the curvature of the $\text{He}^{\text{II}}$ Ly-α forest

The ratio of optical depths can be defined as:

$$\frac{\tau_{\text{He}^{\text{II}}}}{\tau_{\text{H}^{\text{I}}}} = \frac{1}{4} \frac{b_{\text{H}^{\text{I}}}}{b_{\text{He}^{\text{II}}}} \eta,$$

where $\eta \equiv \frac{n_{\text{He}^{\text{II}}}}{n_{\text{H}^{\text{I}}}}$ is the ratio of $\text{He}^{\text{II}}$ and $\text{H}^{\text{I}}$ number densities. For $z \lesssim 2.8$, after the end of the $\text{He}^{\text{II}}$ reionization, where the absorption can be decomposed into distinct absorption features, $\eta$ can be approximated to the ratio of $\text{He}^{\text{II}}$ to $\text{H}^{\text{I}}$ column densities (e.g. Heap et al. 1998; Kriss et al. 2001; Fechner & Reimers 2007; Muzahid et al. 2011). The value of $\eta$ is connected with the hardness of the UVB and, if quasars dominate the ionizing background, at $z \sim 3$ it is expected to assume a value of $\eta \sim 80 - 100$ (e.g. Madau & Meiksin 1994; Haardt & Madau 1996). Given the higher opacity of the $\text{He}^{\text{II}}$ Ly-α transition with respect to the $\text{H}^{\text{I}}$ one, absorption systems which are optically thin in $\text{H}^{\text{I}}$ Ly-α will form saturated $\text{He}^{\text{II}}$ Ly-α lines (e.g. Meiksin 2009). That is, the $\text{He}^{\text{II}}$ Ly-α forest at moderate optical depths is sensitive to lower gas densities.

In principle, by applying a curvature analysis to coeval $\text{H}^{\text{I}}$ and $\text{He}^{\text{II}}$ Ly-α forests, it should be possible to trace the cosmic gas in different density regimes and obtain a measurement of $\gamma$ independent of $T_0$. Assuming a density-independent $\gamma$ and the same value of $T_0$ for the $\text{He}$ and $\text{H}$ in Equation 1.9, at each redshift it is then possible to compute $\gamma$ from the following expression:

$$\frac{T(\Delta_H)}{T(\Delta_{\text{He}})} = \left( \frac{\Delta_H}{\Delta_{\text{He}}} \right)^{\gamma - 1},$$

where $T(\Delta_H)$ and $T(\Delta_{\text{He}})$ are the temperatures at the characteristic densities traced by the $\text{H}^{\text{I}}$ and $\text{He}^{\text{II}}$ Ly-α forests, respectively.

However, in practice, there is currently only a small sample of “clean” sightlines, i.e. those free from intervening higher redshift $\text{H}^{\text{I}}$ Lyman limit absorbers that would severely attenuate the quasar flux, making them unavailable for $\text{He}^{\text{II}}$ Ly-α absorption studies. Moreover, the lower quality of the FUV spectra of the known $\text{He}^{\text{II}}$ quasars ($R \lesssim 20000$ and $S/N \lesssim 10$ per resolution element; e.g. Zheng et al. 2004; Syphers et al. 2012; Worseck et al. 2014) currently does not allow this kind of constraint. Nevertheless, over the past 5 years great improvements have been possible in increasing the number of available $\text{He}^{\text{II}}$ quasars, thanks to the discovery of new quasars at $z_{\text{em}} > 2.7$ by the Sloan Digital Sky Survey, BOSS and the FUV and near UV imaging of almost the entire extragalactic sky by GALEX (e.g. Syphers et al. 2012; Worseck et al. 2014). Moreover, it is possible that future space-based UV telescopes with high resolution spectrographs may provide higher quality spectra for future analyses (Postman et al. 2009). Exploring
5.2 The simulations

We performed new GADGET-3 hydrodynamical simulations with varied box size and mass resolution to extend the suite used in Appendix B for the curvature ratio convergence tests. The general parameters adopted for the cosmology and initial conditions are described in Section 2.2 and the helium fraction (by mass) of the IGM is assumed to be $Y = 0.24$ (Olive & Skillman 2004). All the runs assume the same thermal history (model C15 in Table 2.2) and are summarized in Table 5.1.

Table 5.1 Parameters corresponding to the different simulations used in the HeII Ly-$\alpha$ curvature convergence tests. For each simulation we report the name of the model (column 1), the box size (column 2), the total number of particles and their mass (column 3 & 4). All the simulations are run with the same thermal history assumption as in the model C15.

<table>
<thead>
<tr>
<th>Model</th>
<th>$L[h^{-1}\text{Mpc}]$</th>
<th>Particles</th>
<th>$M_{\text{gas}}[h^{-1}M_{\odot}]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>N1</td>
<td>2.5</td>
<td>$2 \times 512^3$</td>
<td>$1.4 \times 10^9$</td>
</tr>
<tr>
<td>N2</td>
<td>2.5</td>
<td>$2 \times 256^3$</td>
<td>$1.2 \times 10^4$</td>
</tr>
<tr>
<td>N3</td>
<td>2.5</td>
<td>$2 \times 128^3$</td>
<td>$9.2 \times 10^4$</td>
</tr>
<tr>
<td>N4</td>
<td>10</td>
<td>$2 \times 992^3$</td>
<td>$1.3 \times 10^4$</td>
</tr>
<tr>
<td>C15</td>
<td>10</td>
<td>$2 \times 512^3$</td>
<td>$9.2 \times 10^4$</td>
</tr>
<tr>
<td>R1</td>
<td>10</td>
<td>$2 \times 256^3$</td>
<td>$7.4 \times 10^5$</td>
</tr>
<tr>
<td>R2</td>
<td>10</td>
<td>$2 \times 128^3$</td>
<td>$5.9 \times 10^6$</td>
</tr>
<tr>
<td>R3</td>
<td>20</td>
<td>$2 \times 256^3$</td>
<td>$5.9 \times 10^6$</td>
</tr>
<tr>
<td>R4</td>
<td>40</td>
<td>$2 \times 512^3$</td>
<td>$5.9 \times 10^6$</td>
</tr>
</tbody>
</table>
From each simulation, synthetic He\textsuperscript{II} Ly-\textalpha forest spectra are extracted for 1024 random sightlines through each of 4 redshift snapshots over the redshift range available for the real observations, $z \simeq 2.0$–2.8. The He\textsuperscript{II} Ly-\textalpha effective optical depth has been rescaled using Equation 5.1 where $\tau_{\text{HI}}$ has been scaled to match the results of Becker et al. (2013), a constant $\eta = 80$ is assumed at all redshifts and the Doppler parameters for hydrogen and helium are assumed to be the same as a first approximation ($b_{\text{HI}} \sim b_{\text{HeII}}$). The value of $\eta = 80$ seems to fit well the observed optical depth for $z \lesssim 2.8$ (e.g. Worseck et al. 2014) but we are aware that the current uncertainties in the measurement of $\tau_{\text{HeII}}$ from observations do not allow a precise match between real and simulated spectra. No noise has been added at this stage to the synthetic spectra.

An example of a synthetic section of H\textsc{i} forest compared to the corresponding He\textsuperscript{II} Ly-\textalpha forest is shown in Figure 5.1 for all the redshifts considered in this work. From Figure 5.1 it is clear the difference between the absorption structures of the two ions and their evolution from redshift $z = 2.760$ to 2.173. While the absorption lines are always well-defined in the H\textsc{i} absorption, from the lowest to the highest redshift snapshot, the He\textsuperscript{II} absorption features tend to saturate as redshift increases, reaching almost entire absorption of the flux at $z = 2.760$. The shapes of the He\textsuperscript{II} absorption features will be then mainly dominated by the lowest flux range ($F \lesssim 0.7$) of each section of spectra. An accurate curvature analysis would then necessitate an appropriate selection of the flux range corresponding to “valid pixels” over which the mean curvature is computed as, for example, saturated pixels do not contain any useful information.
5.2. The simulations

Figure 5.1 H I and He II Ly-α forest synthetic spectra from the highest resolution 10 $h^{-1}$ Mpc simulation (model N4 in Table 5.1). Each panel shows the corresponding H I (blue line) and He II (yellow line) Ly-α forest sections at each of the redshift snapshots considered ($z = [2.173, 2.355, 2.553, 2.760]$). While the H I absorption features are always well defined, the corresponding He II forest appears almost completely saturated at $z = 2.760$. 
5.3 Convergence tests

We obtained the mean absolute curvature statistic, $\langle |\kappa| \rangle$, over the 1024 sections of HeII Ly-α forest for each of the models of Table 5.1, to study the convergence with box size and resolution in the 4 different redshift snapshots. The mean curvature has been computed as in Section 2.4 with the only difference that we did not exclude at this stage any pixels from the calculation and so we did not impose a “valid” normalised flux range (previous calculations used $0.1 \leq F^R \leq 0.9$). The main purpose here is to understand if the specifics of the available simulations are enough to ensure the convergence of the curvature statistic. Therefore, at this stage we take into consideration the entire absorption region of each synthetic spectral chunk.

Figure 5.2 presents the difference in log$\langle |\kappa| \rangle$ measured from our different simulations relative to the highest resolution model (N1). The results are presented as a function of gas particle mass for each of the redshifts considered. There are significant relative differences at all redshifts. In terms of mass resolution, the models with $2.5 \, h^{-1} \, \text{Mpc}$ box size (N1, N2 and N3) converge at all redshifts but only because these models are not sampling a large enough spectral range, and for this reason they are not the preferred ones: they could not be used in practice for measuring the curvature. The vertical separation between the data points allows us to judge the box size convergence and shows that the decrease in log$\langle |\kappa| \rangle$ computed from the models N4 and C15 (with the same mass resolution of N2 and N3, respectively, but four times their box size) is more than 0.3, even in the best case at $z = 2.173$. Similarly, at the same redshift the simulation R4 (with $40 \, h^{-1} \, \text{Mpc}$ box size) shows a $\sim 0.2$ decrease from the value computed using the model R2 at the same resolution but with 4 times smaller box size. The box size differences become larger for increasing redshift. However, the differences in mass resolution among models with the same $10 \, h^{-1} \, \text{Mpc}$ box size (N4, C15, R1 and R2) remain similar at all redshifts. In particular, at the lowest redshift, there is a marginal convergence between the $10 \, h^{-1} \, \text{Mpc}$ highest mass resolution models (N4 and C15), but this convergence worsens for higher redshifts, showing a constant relative difference of $\sim 0.1$ between the two values of log$\langle |\kappa| \rangle$.

Figure 5.2 makes it clear that a $10 \, h^{-1} \, \text{Mpc}$ box size simulation with a mass resolution of $9.2 \times 10^4 M_\odot$ would be enough to study the HeII forest curvature at $z \sim 2.2$, though probably a significant box size correction would still be required. However, for higher redshifts, the mass resolution and box size requirements are stricter and necessitate much larger corrections.
5.3. Convergence tests

Figure 5.2 Difference in $\log(|\kappa|)$ relative to the highest resolution value (from model N1), as a function of gas particle mass for the simulations in Table 5.1. The four panels show the results at the redshifts $z = [2.173, 2.355, 2.553, 2.760]$. The relative difference becomes larger at higher redshifts and for decreasing mass resolution and box size.
Chapter 5. Resolving the curvature of the He\textsubscript{II} Ly-\alpha forest

5.4 Conclusions

From the convergence tests of the previous Section we understand that the curvature statistic applied to the He\textsubscript{II} Ly-\alpha absorption is not well converged in the redshifts of interest. The simulations previously used to study the H\textsubscript{i} counterpart (e.g. model C15 in Table 5.1) are not enough to analyse the absorption shapes in possible future high resolution spectra. Ideally, at all redshifts a larger box size ($20h^{-1}$ Mpc) would be useful to better resolve the underdense regions probed by the helium absorption but, at the same time, the high mass resolution required to populate these regions in the simulations (with $M_{\text{gas}} \lesssim 9.2 \times 10^4 M_\odot$) would require new and very time-consuming simulations. Another possibility is to apply mass resolution and box size corrections to the available suite of hydrodynamical simulations. The marginal convergence, observed for models with mass resolution $M_{\text{gas}} \lesssim 9.2 \times 10^4 M_\odot$ (N4 and C15) at $z = 2.173$, and the fact that for higher redshift the differences between the values of $\log\langle|\kappa|\rangle$ computed from these models is maintained at about 0.1, give some confidence in an eventual mass resolution correction. However, in terms of box size, the size of the corrections necessary is much more relevant and increases significantly for higher redshifts.

Currently, the curvature analysis of the He\textsubscript{II} Ly-\alpha forest is not only limited by the low quality of observed quasar spectra, but would also require a refinement of the suite of hydrodynamical simulations required for the method. For this reason we leave to future work a fuller explanation of the possibility of utilizing the He\textsubscript{II} forest for measuring $\gamma$. 
This thesis investigates the thermal state of the intergalactic medium in the redshift range $z \simeq 1.5 - 3.8$. This phase of the IGM evolution is particularly interesting because it is characterized by the second reionization event, the He II reionization, that may have left detectable imprints in the evolution of the temperature at the mean gas density ($T_0$) and in the steepness of the power-law that defines the relation between the IGM temperature and density ($\gamma$). The picture drawn by previous observations of the thermal state of the IGM is still confusing, with some indication of the imprints of the He II reionization but still much room for more definitive evidence. The findings presented in the previous Chapters can help to improve the understanding of the evolution of the IGM in this complex phase of its thermal history. While detailed descriptions of our measurements can be found in the Conclusions of Chapters 3, 4 and 5, in this final Chapter we summarise the main results, discuss them in the context of the status of the field, and describe new prospects for future follow-up work.

6.1 Summary of the main results

The analyses presented in the previous Chapters aimed mainly to provide new constraints and methods to study the evolution of $T_0$ and $\gamma$ as a function of redshift. In particular, in Chapters 2 and 3 we provided new measurements of the IGM temperature at the characteristic gas overdensities probed by the H I Ly-\alpha forest, $T(\tilde{\Delta})$ (see Figure 3.1). With a sample of 60 high quality UVES spectra, for the first time we extended the curvature measurements (previously obtained for $2 \lesssim z \lesssim 4$ by Becker et al. 2011) to the lowest optically-accessible redshifts, down to $z \sim 1.5$. We found that the values of $T(\tilde{\Delta})$ increase for increasing overdensities probed by the forest towards lower redshifts, from $T(\tilde{\Delta}) \sim 22670$ K to 33740 K in the redshift range $z \sim 2.8 - 1.6$. These results are broadly
consistent with the trend found by others at higher redshifts. Moreover, in the overlapping redshift range $2.0 \lesssim z \lesssim 2.9$, our results (from a completely independent dataset) are in excellent agreement with the measurements of Becker et al. (2011), demonstrating the self-consistency of the curvature method.

Using the temperature–density relation (Equation 1.9) we translated the $T(\bar{\Delta})$ values to temperatures at the gas mean density $T_0$ to investigate the heating trend observed previously by others at redshifts $2.8 \lesssim z \lesssim 4$. While, later in Chapter 4 we attempted to constrain the parameter $\gamma$, in Chapter 3 we assumed two reasonable values of $\gamma$ to obtain the measurements in $T_0$ (see Figure 3.2). For both the choices of $\gamma$ we observed some evidence for a change in the tendency of the temperature evolution for $z \lesssim 2.8$, with indications of at least a flattening of the increasing temperature towards lower redshifts. In particular, for the case of a broadly constant $\gamma \sim 1.5$, the extension towards lower redshifts provided by our work shows the evidence for a decrease in the IGM temperature from redshifts $z \sim 2.8$ down to the lowest redshift $z \sim 1.5$. This supports tentative evidence for the same from Becker et al. (2011). Such a reversal of the heating trend can be interpreted as the end of the IGM reheating process driven by He II reionization, with a subsequent tendency of a cooling of a rate depending on the UV background.

In Chapters 4 and 5 we proposed two new possible applications of the curvature method to constrain the parameter $\gamma$. In particular, in Chapter 4 we presented and tested with hydrodynamical simulations the possibility of using the ratio of curvatures, $\langle R_\kappa \rangle$, in the Ly-α and β forests to trace simultaneously two different gas density regimes and obtain measurements of the slope of the $T–\rho$ relation. We applied this technique to a subsample of 27 UVES spectra selected for their high S/N (> 24 per pixel) in both the Ly-α and β regions and we analysed the strengths and weakness of this method. The technique, relatively fast and simple to compute, appears robust against observational uncertainties in the noise level, metal contamination and effective optical depth. The evolution of $\langle R_\kappa \rangle$, measured from our observational spectra in 3 redshift bins covering $z \simeq 2.0–3.8$, matches that derived from our nominal simulations with $\gamma \sim 1.5$. The statistical errors obtained from the 27 spectra are promising: in the absence of any systematic errors, the $\approx 6$ per cent statistical uncertainties in our $\langle R_\kappa \rangle$ measurements would translate to a $\lesssim 10$ per cent uncertainty in $\gamma$ in $\Delta z \sim 0.6$ bins. This would be competitive with the most precise results available in the literature derived using line decomposition (Rudie et al. 2013; Bolton et al. 2014). However, we did not present $\gamma$ measurements with formal uncertainties because the nominal $\gamma–\log\langle R_\kappa \rangle$ relation, used to translate $\langle R_\kappa \rangle$ measurements to $\gamma$ values, may also be sensitive to assumptions in the simulations about the evolution of the IGM thermal state.
6.2 What is the thermal state of the IGM at low redshift?

The thermal state of the intergalactic medium is defined by its temperature–density relation that, in the simplest scenario, is represented by a power-law (Equation 1.9). Understanding the evolution of the IGM thermal state therefore relies on measuring the evolution of the parameters $T_0$ and $\gamma$ of Equation 1.9. For more than a decade, significant efforts have been invested in developing methods that would allow the two parameters to be determined independently of each other using quasar absorption lines. However, the necessity of using complex simulations and large statistical samples of high quality observational spectra, makes it particularly difficult. Nowadays, the main challenge is represented by the requirement to reduce the systematic and statistical uncertainties in the measurements to a level that would allow to clarify the confusing picture, drawn so far by previous estimations.

Including the constraints obtained in this work, Figure 6.1 provides a summary of the state of the field to date. Panel (a) presents the observed evolution of $T_0$ with redshift while panel (b) shows the constraints on the evolution of $\gamma$ (reproduction of Figure 4.11). Note that for clarity, in both the panels we could not include all the measurements available in
the literature; rather, the comparison presented here is intended to provide a representative idea of the size of the error bars, obtained using different approaches, and of the dispersion of the values of temperature and $\gamma$ measured in the broad redshift range $z \simeq 1.5 - 5$.

This summary figure allows us to put our results in context and address the following main questions.

Is there any evidence for a peak in $T_0$ due to the He\textsubscript{II} reionization?

The results of Schaye et al. (2000), presented in Figure 6.1 panel (a), were historically the first to show a large scatter, and so a possible variability, of the temperature at the mean gas density, measured through a Voigt profile decomposition of H\textsubscript{i} Ly-\alpha absorption lines at redshifts $z \simeq 2 - 4.5$. However, the clearly large uncertainties characterizing these results do not allow any conclusion about the real presence of a temperature peak: a heating of the IGM during reionization followed by a gradual cooling as the temperature, at the end of the reionization, returns to a thermal asymptote. Similarly, large uncertainties characterized subsequent statistical analysis, such as the wavelength decomposition obtained by Lidz et al. (2010). These large error bars are partially driven by the strong degeneracy between the effects of temperature and density on defining the shapes of the H\textsubscript{i} absorption lines.

In its attempt to reduce the error bars characterizing these measurements, the curvature method shows a considerable improvement. The first results obtained with this method by Becker et al. (2011) in the redshift range $z \simeq 2 - 5$, present significantly smaller error bars that allow to trace the evolution of the temperature in a much clearer way. In particular, the curvature method measures with high precision (typically $1\sigma \lesssim 5$ per cent) the temperature at the characteristic overdensity probed by the Ly-\alpha forest, $T(\Delta)$ (see Figure 3.1), but it does not simultaneously constrain $\gamma$ because it only accesses a small range of densities at each redshift. The translation of the $T(\Delta)$ measurements into $T_0$ values therefore involves an assumption about the value of the parameter $\gamma$ which is itself independently very poorly constrained. In the work of Becker et al. (2011), while strong evidence for an increase in $T_0$ between redshift $z \sim 4$ and $z \sim 2.8$ was found, the uncertainties in the value of $\gamma$ prevented an unambiguous identification of an actual peak: $T_0$ could continue to increase for $z \lesssim 2.8$ or could start to flatten, depending on the $\gamma$ assumed in the translation.

Our curvature analysis, presented in Chapters 2 and 3, investigated the temperature evolution of the IGM down to the lowest optically-accessible redshift ($z \sim 1.5$), providing further evidence for at least a flattening, and possibly a reversal, of the increasing IGM temperature for $z \lesssim 2.8$. In particular, for the $T_0$ case with $\gamma \sim 1.5$ (shown in Figure 6.1),
6.2. What is the thermal state of the IGM at low redshift?

Figure 6.1 Summary of observational status of the IGM thermal evolution from this work and selected literature. Panel (a): Temperature at the mean density, $T_0$, inferred under the assumption of $\gamma \simeq 1.5$ in this work (green points) and in the work of Becker et al. (2011) (black points) using the curvature method. The results of the line-profile analysis of Schaye et al. (2000) (grey points) and wavelet analysis of Lidz et al. (2010) (light green triangles) are also presented for comparison. Note that the error bars quoted by different authors are either 1- or 2-$\sigma$ (shown in the symbol key). Panel (b): $\gamma$ measurements and statistical uncertainties obtained from the curvature ratio in this work (green points) compared with the line-fitting measurements of Schaye et al. (2000) (grey points), Ricotti et al. (2000) (azure triangles), McDonald et al. (2001) (light blue squares) and Bolton et al. (2014) (red circle). The non-equilibrium model of Puchwein et al. (2014) (red dashed line) and the radiative transfer simulations of McQuinn et al. (2009) with an early ($z \gtrsim 6$; blue dotted line) and late ($z \sim 3$; green dot-dashed line) HeII reionisation are also presented.
our results provide evidence for a decrease in the IGM temperature that could be interpreted as an imprint of the completion of the reheating process connected with the He II reionization. Note that, using an independent observational dataset, our results in the overlapping redshift range \(2.0 \lesssim z \lesssim 2.9\) are broadly consistent with the previous curvature measurement of \cite{Becker2011} but show slightly higher values; this difference can be attributed to the different optical depth calibration, as demonstrated in Appendix A. The curvature ratio statistic, developed in this work, shows preliminary results that are consistent with \(\gamma \sim 1.5\) in the redshift range of interest. If this value is confirmed by future refinements of the method (see Section 6.3), the actual evolution of \(T_0\) presented in Figure 6.1 will be finally known to high precision.

Is there any evidence for a flattening in the \(T-\rho\) relation at low redshifts?

While a tight constraint on the evolution of the parameter \(\gamma\) of the \(T-\rho\) relation would be of vital importance to measure the absolute value of \(T_0\) and obtain the redshift of the peak with substantial precision, the actual observations presented in panel (b) of Figure 6.1 show an even more confused scenario than for \(T_0\). Large error bars dominate again the line-fitting measurements and, while in some cases some statistical evidence for a decrease in the value of \(\gamma\) at \(z \sim 3\) has been presented (e.g. \cite{Schaye2000}), this scenario has not been supported by other analyses (e.g. \cite{McDonald2001}). Moreover, several flux PDF analyses (not reported in Figure 6.1) suggested that the temperature–density relation could become inverted, with a \(\gamma < 1\) at \(z \lesssim 3\) (e.g. \cite{Becker2007}). A mild flattening, with a variation in \(\gamma\) from its asymptotic value (\(\gamma \sim 1.6\)) of \(\Delta \gamma \lesssim 0.2\) has been predicted by photo-heating models which include radiative transfer (e.g. \cite{McQuinn2009}), but an inverted \(T-\rho\) relation is difficult to explain without assuming more exotic models such as blazar heating (\cite{Puchwein2012}). An improvement in the statistical uncertainties has been recently demonstrated at \(z = 2.4\) by the measurement of \cite{Bolton2014} (which was a recalibration of that by \cite{Rudie2013}). However, that involved the profile fitting analysis of \(\sim 6000\) individual \(\text{H}I\) absorbers, which is a large undertaking.

In Chapter 4 we tested a new approach to obtain statistically competitive \(\gamma\) measurements over a large redshift range (\(z \simeq 2 - 3.8\)). This statistic, the curvature ratio, incorporates information from the two gas density regimes traced by the Ly-\(\alpha\) and \(\beta\) forest along the same line of sight to a quasar. Thus, if averaged over a large sample of spectra, it allows measurement of \(\gamma\), this is largely independent of \(T_0\) and different to all previous techniques. Relatively simple to compute, the curvature ratio appears robust against observational uncertainties in the noise level, effective optical depth and metal
6.2. What is the thermal state of the IGM at low redshift?

Testing this application of the curvature statistic for the first time, a sample of 27 quasar spectra provided a \( \lesssim 10 \) per cent statistical uncertainty in \( \gamma \) in \( \Delta z \sim 0.6 \) redshift bins. Such error bars are competitive with the recent results of [Bolton et al. (2014)]. However, we do not present here the \( \gamma \) measurements with final error bars: the error bars presented in Figure 6.1 do not incorporate yet the possible systematic uncertainties related to the sensitivity of our method to the assumptions about the thermal state of the IGM and its evolution. These assumptions have been necessarily adopted in the simulations used to calibrate our curvature ratio measurements and could be overcome with some future refinements of the analysis technique (see Section 6.3). Nevertheless, the evolution of the curvature ratio computed from the real spectra matches that derived from our nominal simulations which assume a \( \gamma \sim 1.5 \) with very little evolution in the redshift range \( z = 2 - 3.8 \). That is, the same model used in Chapter 3 to translate the \( T(\bar{\Delta}) \) measurements into \( T_0 \) values (shown in Figure 6.1) seems to reproduce correctly the observational value of our new statistic, giving some further confidence in our temperature measurements.

Similar to the previous measurement of \( \gamma \) by [Bolton et al. (2014)], our preliminary \( \gamma \) measurements do not support an inverted temperature–density relation. However, mild decreases in \( \gamma \) at redshifts \( z \simeq 2 - 4 \) cannot be excluded at this stage. Distinguishing between models of the IGM thermal history (e.g. the late and early reionization models of [McQuinn et al. 2009]) could be improved in the future by using larger samples of high quality spectra. Future, refined curvature ratio analyses could then represent a valid companion statistic to the curvature method itself, allowing to combine \( T(\bar{\Delta}) \) and \( \gamma \) measurements for a precise, first order characterization of the thermal evolution of the IGM.

Is there any evidence for blazar heating mechanisms at low redshifts?

The evidence for a possible inverted \( T-\rho \) relation obtained in previous flux PDF observations (e.g. [Becker et al. 2007]), suggested the idea that other, more exotic heating mechanisms could dominate the photo-heating at \( z \lesssim 3 \) (e.g. [Chang et al. 2012, Puchwein et al. 2012]). According to these models, blazar \( \gamma \)-ray emission would produce volumetric heating of the IGM. Because these models have a heating rate that can be considered independent of the density, if it dominates the photo-heating at \( z \lesssim 3 \), it could explain naturally an inverted \( T-\rho \) relation at low-redshift and obscure the “imprint” of the He\( \text{II} \) reionization in the IGM temperature evolution. The blazar heating models’ \( T-\rho \) relations, at each
redshift, can be parametrized with a power-law only for a limited range of overdensities that may not always cover the range of characteristic overdensities probed by curvature measurement. Therefore, the only fair, model-independent comparison between our temperature results and the blazar heating model predictions is directly with the $T(\hat{\Delta})$ values at the same redshift-dependent characteristic overdensities. As presented in Figure 3.4 from direct comparison, our $T(\hat{\Delta})$ measurements are in agreement with the intermediate blazar heating model of Puchwein et al. (2012). However, in contrast, our preliminary measurements of $\gamma$ and the evolution as a function of redshift of the curvature ratio statistic are in considerable tension with possible scenarios of $\gamma < 1$. Given these results, we cannot currently confirm or rule out any specific thermal history, as model-independent measurements of the temperature at the mean density would be necessary.

Nevertheless, more recently, new simulations that take into account the clustering of the sources, showed that the variability in the blazar heating is significant and leads to an “important scatter” in the temperature at $z \gtrsim 2$ (Lamberts et al. 2015). If this scatter is confirmed by future analyses of the Ly-\(\alpha\) forest properties, these more sophisticated blazar heating models could take into account both the increase in the temperature observed in our results and a non-inverted $T-\rho$ power-law.

6.3 Future work

Despite the considerable improvements over the last decade, a precise characterization of the IGM thermal state at low (and high) redshift has still to be completed. Such a goal, if achieved, would be of great importance for understanding the physics of the IGM, the reionization epochs, their sources and in general the mechanisms of structure formation. In this Section we present some short-term and long-term ideas to improve IGM thermal state measurements using the information coming from the shapes of quasar absorption lines.

The curvature ratio statistic refinement

The curvature ratio statistic, $\langle R_\kappa \rangle$, was demonstrated in Chapter 4 to be a promising tool for obtaining independent constraints on $\gamma$. It is relatively easy to compute, can be applied to a vast range of redshifts ($z \simeq 2-5$) and is robust against the main observational uncertainties. However, due to the foreground Ly-\(\alpha\) absorption at lower redshifts which contaminates the Ly-\(\beta\) forest, the simulations used to calibrate and translate curvature ratio measurements in $\gamma$ values necessarily rely on assumptions about the IGM thermal
evolution between the Ly-β and the contaminant Ly-α redshifts. As demonstrated in Chapter 4, if the $\gamma - \log \langle R_\kappa \rangle$ relation is sensitive to these assumptions, it could produce systematic uncertainties in the final $\gamma$ estimates.

Reliable sensitivity tests can only be conducted using new, self-consistent hydrodynamical simulations that assume different scenarios of $T_0$ and $\gamma$ evolution. Similar to the ‘T15fast’ and ‘T15slow’ simulations of Becker et al. (2011), used in Section 4.3.3 for self-consistently testing the effects of rapid changes in the evolution of $T_0$, the new models need to incorporate possible rapid variations of $\gamma$ between the Ly-β and the foreground Ly-α redshifts. The effects on the $\gamma - \log \langle R_\kappa \rangle$ relation of simultaneous rapid changes in the value of $\gamma$ and $T_0$ also need to be investigated. While the exploration of the entire possible parameter space would require a large undertaking, we estimate that a suite of $\sim 10$ new models would be sufficient to establish the magnitude of these effects in reasonable scenarios of $\gamma$ and $T_0$ evolution (with maximum variations of $\Delta \gamma \simeq 0.4$ and $\Delta T_0 \simeq 10000K$ in a $\Delta z \simeq 0.6$ redshift range). Once fully explored, the formal magnitude of possible systematics can be added to the statistical uncertainties for a final error budget, yielding reliable $\gamma$ measurements from this curvature ratio approach.

Alternatively, these systematics may be largely mitigated by identifying and masking out the contaminant foreground Ly-α absorption from the Ly-β forest spectra. One possible approach to do so is to automatically fit the “pure” Ly-α forest and then the contaminant Ly-α lines in the Ly-β forest after masking or dividing out the Ly-β lines. Then, one could divide out these contaminant Ly-α profiles to allow a refined joint analysis of a relatively “clean” Ly-β forest and its corresponding Ly-α forest. Some specific fitting problems have to be carefully taken into account (e.g. saturated Ly-α lines or metal contaminants) but this procedure, executed over large statistical sample, could help to reduce the possible systematic uncertainties analysed in Section 4.3.3.

Given the potential of the curvature ratio method in obtaining competitive statistical uncertainties on $\gamma$, its refinement represents a high priority in the short term.

**Global $\gamma$ constraint using a large sample of quasar spectra**

After its refinement, the curvature ratio method could be applied to a larger sample of quasar spectra. Given the statistical uncertainties obtained for the sample of 27 UVES spectra used in this work, we estimate that a sample of a further $\sim 100$ quasar spectra, covering absorption redshifts $z \sim 2.7 - 4$, would improve the precision of the $\gamma$ measurements to $1\sigma \simeq 0.05$ in $\Delta z \simeq 0.5$ redshift bins. This precision would allow a $\simeq 3\sigma$ discrimination between the McQuinn et al. (2009) models of Figure 6.1, finally clarifying
much of the physics behind the evolution of $\gamma$ during the He\textsc{ii} reionization. Theoretically, the curvature ratio can be also used to measure $\gamma$ at higher redshift (up to $z \sim 5$). A constraint on the evolution of $\gamma$ up to $z \sim 5$ represents the highest redshift limit for this kind of measurement, due to the rapid increase of the H\textsc{i} Ly-\(\alpha\) optical depth close to the end of the hydrogen reionization.

The large sample of high signal-to-noise ratio spectra, necessary for these measurements, is already available in the archives of the high resolution spectrographs VLT/UVES, Keck/HIRES and SUBARU/HDS, albeit mostly in un-reduced form. Moreover, further fully-reduced spectra can now be retrieved from the publicly available sample of 170 quasars at $0.29 \lesssim z_{em} \lesssim 5.29$ of the KODIAQ Survey \cite{O'Meara2015}. And, so far, high quality spectra from the High Dispersion Spectrograph (HDS) on the SUBARU Telescope have not been used for constraining the IGM thermal history. Furthermore, the new spectrograph, ESPRESSO (Echelle Spectrograph for Rocky Exoplanet and Stable Spectroscopic Observations) on the VLT will soon be available for new observations, providing further high-quality Ly-\(\alpha\) and $\beta$ spectra at $z \gtrsim 1.8$. Finally, an entirely new sample of much higher S/N spectra will be accessible in future with the new generation of telescopes, such as the Giant Magellan Telescope (GMT), the Thirty Meter Telescope (TMT) and the European Extremely Large Telescope (E-ELT). However, even if the new echelle spectrographs mounted on these future telescopes, such as G-CLEF commissioned for GMT, will be able to provide high quality data in shorter exposure times, it will still take a considerable amount of time on such highly oversubscribed facilities to collect a sample of quasar spectra as large as that currently available from HIRES, UVES and HDS.

Clearly, the existing spectra also offer new $T(\Delta)$ measurements as well. Therefore, they can be combined with the new $\gamma$ constraints to provide more precise estimates of $T_0$'s evolution from $z \sim 5$ down to $z \sim 1.5$.

**Independent constraint on $\gamma$ from the He\textsc{ii} Ly-\(\alpha\) curvature**

In Chapter 5 we introduced an alternative possibility to measure $\gamma$ at redshifts $z \sim 2 - 3$ using the coeval H\textsc{i} and He\textsc{ii} Ly-\(\alpha\) absorption. Even if the quality and the number of far UV spectra available today is not sufficient for the curvature analysis, new space telescopes, such as the Advanced Technology Large-Aperture Space Telescope (ATLAST) or the High Definition Space Telescope (HDST), with high resolution spectrographs \cite{Postman2009, Dalcanton2015} may be available in future, providing high quality spectra to be used for an independent constraint on the IGM thermal state. It is therefore important to explore the potential of such a constraint based on the curvature statistic,
as the findings may be of some influence for the design of such future telescopes and spectrographs.

However, as described in Chapter 5, new hydrodynamical simulations are necessary to properly sample the underdense regions probed by the He\textsc{II} Ly-\textalpha forest and obtain synthetic spectra usable for calibrating the curvature analysis. New simulations with a box size of at least $20h^{-1}\text{Mpc}$ and mass resolution $M_{\text{gas}} \lesssim 9.2 \times 10^{4}M_{\odot}$ would be required, but with today’s technologies these are very time-consuming. However, it could be possible that, with advancements in supercomputing and new generations of supercomputers, this task will be feasible within the next $\sim 5$ years.
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The effect of the optical depth calibration on the temperature measurements

In this Section we demonstrate that, in the curvature analysis of a particular sample of sight-lines, calibrating the Lyman-α forest simulations with the effective optical depth of the sample provides robust measurements of the temperature at the characteristic overdensities. We will show how, even if different $\tau_{\text{eff}}$ calibrations produce different characteristic overdensities $\bar{\Delta}(z)$; see Section 2.5.3, the temperature measurements, $T(\bar{\Delta})$, at each redshift will not be affected significantly by systematic effects related to possible biases in the sample selection. Instead, discrepancies in the characteristic overdensities will shift the derived temperature at the mean density. Nevertheless, this effect in the $T_0$ values will be modest, causing a disparity at the level of the observational 1-$\sigma$ error bars.

The test can be summarized as follows. We randomly select two sub-samples of 300 spectral sections from the suite of simulations of one thermal history. One of the sub-samples is selected in a biased way to result in a higher effective optical depth than the other; this difference is designed to be similar to that observed between the UVES sample used in this work and the sample adopted in the previous work of Becker et al. (2011). Treating the two sub-samples as observational data, we analyse them separately with the curvature method presented in Section 2.3 and obtain the corresponding $T(\bar{\Delta})$ and $T_0$ measurements. The two sets of $T(\bar{\Delta})$ are found not to differ significantly, while a modest shift in the $T_0$ values is observed due to discrepancies in the recovered $\bar{\Delta}$ values at each redshift. The details of this test are described below.
Appendix A. The effect of the optical depth calibration

A.1 Selection of synthetic sub-samples

We chose the 1024 synthetic sections of our fiducial simulation C15 (see Table 2.2) as the “global” sample from which to select, at each redshift, two sub-samples of ~ 300 sections with two slightly different mean $\tau_{\text{eff}}$ that would simulate two random observational samples. We deliberately biased the mean optical depth of the second sub-sample towards higher values using the method explained in Figure A.1: we fit a Gaussian function to the global distribution of mean fluxes (at $z = 1.75$ in the example shown in the Figure A.1) from all 1024 sections of the C15 simulation and used this, and a shifted version of it, as the probability distributions for selecting sections randomly for the two sub-samples of 300 sections each. By construction, the first subsample – which we call the ‘standard sub-sample’ for clarity – will have a mean $\tau_{\text{eff}}$ very close to the global mean. However, the mean optical depth of the second sub-sample – called the ‘biased sub-sample’ – is selected from the same probability distribution shifted slightly to lower mean fluxes, so it results in a higher mean $\tau_{\text{eff}}$. The shift in the probability distribution was tuned so that difference in the mean $\tau_{\text{eff}}$ at each redshift reflected the difference observed between our real UVES sample and the data used by Becker et al. (2011).

The results of the sub-sample selection are presented in Figure A.2 where we show the positions on the absolute curvature–mean $\tau_{\text{eff}}$ plane of all the sections from simulation C15 at $z = 1.75$. As expected, the distribution of mean $\tau_{\text{eff}}$ and curvature in the standard sub-sample is very similar to the parent distribution. Also, while noting that, by construction, the biased sub-sample has a higher mean $\tau_{\text{eff}}$ than the standard sub-sample, we also see that the mean curvature of the biased sub-sample is very similar to the parent distribution. The sub-sample selection therefore should allow a test of the effect of selecting an observational sample with a higher mean $\tau_{\text{eff}}$ on the measured $T(\bar{\Delta})$ values.

A.2 Parallel curvature analysis and results

After the selection of the two sub-samples we treated them as two separate observational datasets and we analysed the curvature following the steps presented in Section 2.3. That is, at each redshift and for each sub-sample, we computed the $\langle |\kappa| \rangle$ values and measured $T(\bar{\Delta})$ after calibrating all the simulations (from all thermal histories) with the mean $\tau_{\text{eff}}$ found in that particular sub-sample. Finally, using the $T-\rho$ relation, we derived the values of $T_0$ under the assumption of $\gamma = 1.54$ (corresponding to the chosen thermal history C15).

As expected, calibrating the simulations with the two different effective optical depths gave slightly different values for the $\bar{\Delta}$ at each redshift of the two sub-samples. However,
A.2. Parallel curvature analysis and results

Figure A.1 Global distribution of the mean fluxes at $z = 1.75$ for all the 1024 sections of the simulation C15. Two sub-samples have been selected following the probability distributions of the two Gaussian curves. The solid red curve was fit directly to the flux distribution and was used to select the standard sub-sample. The dashed green curve has the same FWHM but its mean value was shifted toward lower fluxes to allow the selection of the biased sub-sample with a higher mean optical depth.

We find excellent agreement between the $T(\bar{\Delta})$ values for the two sub-samples, as shown in the top panel of Fig. A.3. There we also plot the difference between the $T(\bar{\Delta})$ values, $\Delta T$, from the two sub-samples at each redshift in the lower panel. This difference is $\Delta T \lesssim 1100$ K at all redshifts and typically less than 800 K. Considering that the temperature measurements at the characteristic overdensities presented in this work have a minimum 1σ error bar of $\sim 1800$ K, these $\Delta T$ all fall inside the current statistical uncertainty budget. Also, we expect small, non-zero values of $\Delta T$, and small variations with redshift, due to the sample variance connected with the selection of the sub-samples. In general, we can conclude that calibration of the simulations with the effective optical depth of the particular observational sample being analysed results in a self-consistent measurement of $T(\bar{\Delta})$. 
Figure A.2 Example of the $\langle |\kappa| \rangle$ distribution as a function of the effective optical depth of the spectral sections at $z = 1.75$ of the simulation C15. The open black circles represent synthetic sections and the different colored solid symbols show the sub-sample selection results: red points represent the standard sub-sample while the green triangles represent the biased sub-sample with higher mean optical depth. Sections that fall in both sub-samples are also indicated as yellow squares.

In terms of the temperature values at the mean density, we find that the biased sub-sample produces a systematically higher temperature, as expected (top panel of Fig. A.4). This discrepancy is due to the slightly different values of $\Delta$ at each redshift in the two different sub-samples. However, it is still modest and is generally below the minimum 1-$\sigma$ uncertainty of the measurements presented in this work (lower panel in Fig. A.4).
A.2. Parallel curvature analysis and results

Figure A.3 Upper panel: $T(\Delta)$ values computed from the curvature analysis of the two synthetic sub-samples. Lower panel: Difference in $T(\Delta)$ between the standard and biased sub-samples, $\Delta T$. The discrepancy between the temperature values is shown as a function of redshift (blue squares) and the minimum 1σ error bar observed in the UVES sample in this work (see Table 4) is given by the red dashed line for comparison.

Figure A.4 Upper panel: $T_0$ values computed from the $T(\Delta)$ measurements of the two synthetic sub-samples under the assumption of $\gamma = 1.54$. Lower panel: Difference in $T_0$ between the standard and biased sub-samples, $\Delta T_0$. The discrepancy between the temperature values is shown as a function of redshift (blue squares) and the minimum 1-σ error bar observed in the UVES sample in this work (see Table 4) is given by the red dashed line for comparison.
Numerical convergence for the curvature ratio

Sampling the overdensity regions probed by the Ly-α and the Ly-β forests at $2 \lesssim z \lesssim 3.5$ represent an important challenge for cosmological simulations. In the absence of computational constraints, it would be ideal to use a both a large simulation box ($\gtrsim 40h^{-1}\text{Mpc}$) and high mass resolution ($M_{\text{gas}} \lesssim 10^5h^{-1}M_\odot$; e.g. Bolton et al. 2009; Becker et al. 2011). However, here the most stringent requirement is to verify that the specifics of the available simulations are enough to ensure the convergence of the curvature ratio statistic in the redshift range considered in this work.

To test the convergence with mass resolution and box size we varied box size and mass resolution values in the thermal history model C15 of Table 2.2. The comparison simulations (runs C15, R1-R4) are presented in Table B.1. All the simulations were calibrated and synthetic Ly-α and Ly-β+α forest sections were produced using the procedure illustrated in Section 4.2 with the only difference that no noise was added to the spectra. The curvature ratio was then computed as in Section 4.3.1.

Table B.1 Parameters corresponding to the different simulations used in the curvature ratio convergence tests. For each simulation we report the name of the model (column 1), the box size (column 2), the total number of particles and their mass (column 3 & 4). All the simulations are run with the same thermal history assumption as in the model C15.

<table>
<thead>
<tr>
<th>Model</th>
<th>$L[h^{-1}\text{Mpc}]$</th>
<th>Particles</th>
<th>$M_{\text{gas}}[h^{-1}M_\odot]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>C15</td>
<td>10</td>
<td>$2 \times 512^3$</td>
<td>$9.2 \times 10^4$</td>
</tr>
<tr>
<td>R1</td>
<td>10</td>
<td>$2 \times 256^3$</td>
<td>$7.4 \times 10^5$</td>
</tr>
<tr>
<td>R2</td>
<td>10</td>
<td>$2 \times 128^3$</td>
<td>$5.9 \times 10^6$</td>
</tr>
<tr>
<td>R3</td>
<td>20</td>
<td>$2 \times 256^3$</td>
<td>$5.9 \times 10^6$</td>
</tr>
<tr>
<td>R4</td>
<td>40</td>
<td>$2 \times 512^3$</td>
<td>$5.9 \times 10^6$</td>
</tr>
</tbody>
</table>
The convergence results for the redshifts relevant for this work are shown in Figure B.1 where the change in $\log \langle R_\kappa \rangle$ is plotted with respect to the model C15. The tests show that $\log \langle R_\kappa \rangle$ is nearly convergent with mass resolution and box size at all redshifts. The most significant departure in terms of box size convergence appears at $z < 2.5$: in the lowest redshift snapshot, at $z = 2.173$, the value of $\log \langle R_\kappa \rangle$ decreases by a maximum of $\sim 0.04$ when increasing the box size from $10h^{-1}$Mpc to $40h^{-1}$Mpc. For the nominal relationship between curvature ratio and $\gamma$ values, presented in Figure 4.3, this $\delta \log \langle R_\kappa \rangle$ corresponds to a difference in $\gamma$ of less than 0.05 (and so $\lesssim 5$ per cent; well within the statistical uncertainties presented in the current analysis). Given these results, a small box size correction can be applied to the $\log \langle R_\kappa \rangle$ values at low redshift when fitting the relationship of Figure 4.3. However, the possibly much more relevant systematic uncertainties discussed in Section 4.3.3 make this correction irrelevant for the measurements shown in this work.
Figure B.1 Convergence of the mean curvature ratio with box size and resolution. Curvature ratio values were computed from noise-free spectra, using the same procedure described in Section 4.3.1. The change in $\log \langle R_{\kappa} \rangle$ is plotted with respect to the model with the highest mass resolution. The results show that this statistic is well converged with mass resolution at all redshifts. The decrease in $\log \langle R_{\kappa} \rangle$ when increasing the box size from $10h^{-1}$Mpc to $40h^{-1}$Mpc at $z = 2.173$ would correspond to a small change ($\lesssim$5 per cent) in the final $\gamma$ measurements.
As explained in Section 4.3.1, we can obtain the relationship between $\gamma$ and $\log\langle R_\kappa \rangle$, at each redshift, by averaging the curvature ratio computed from the synthetic spectra with different thermal histories. The plots presented below show, at each redshift, the $\log\langle R_\kappa \rangle$ values obtained from the different models, plotted as a function of the corresponding $\gamma$ value. Differences in the values of $T_0$ in the simulations do not seem to affect relevantly the $\log\langle R_\kappa \rangle$ measurement and it is then possible to fit a simple function that connects the mean $\log\langle R_\kappa \rangle$ and $\gamma$ independently of $T_0$. 

Relationship between $\gamma$ and the curvature ratio
Appendix C. Relationship between $\gamma$ and the curvature ratio

Figure C.1 Relationship between the slope parameter of the temperature–density relation, $\gamma$, and the curvature ratio, $R_\kappa$, from our nominal simulations. Each panel shows the relationship at a particular redshift, obtained from synthetic spectra with different thermal histories. At each redshift, the values of $\log\langle R_\kappa \rangle$ for different models (coloured points) lie on the same same curve, with little sensitivity to differences in $T_0$. Each solid line represents the least square fit of the $\log\langle R_\kappa \rangle$ values.
Publications

The following publications contain material that appears in this thesis:

Refereed publications:

**Boera Elisa**, Murphy Michael T., Becker George D., Bolton James S.

*The thermal history of the intergalactic medium down to redshift z=1.5: a new curvature measurement.* (Chapters 2 & 3)


Submitted for publication:

**Boera Elisa**, Murphy Michael T., Becker George D., Bolton James S.

*Constraining the temperature-density relation of the intergalactic medium with the Lyman-\(\alpha\) and \(\beta\) forests.* (Chapter 4)