Near-field Mie scattering in optical trap nanometry

Djenan Ganic, Xisaosong Gan, and Min Gu
Centre for Micro-Photonics,
School of Biophysical Sciences and Electrical Engineering,
Swinburne University of Technology,
PO Box 218, Hawthorn, Victoria 3122, Australia
mgu@swin.edu.au

ABSTRACT

A mathematical model for understanding near-field Mie scattering, as used in optical trap nanometry for single molecule detection, is developed. Both perpendicular and parallel polarization states of incident electromagnetic waves have been considered. Simulations under different incident angles, and refractive indices of trapped particle have been investigated. Half-space signal strength is studied on the base of the calculated three-dimensional scattered electromagnetic field.

Keywords: near-field, Mie scattering, optical trapping, evanescent wave scattering, optical trap nanometry, laser trapping, morphology dependent resonance.

1. INTRODUCTION

Biomolecules can be trapped and manipulated when they are attached to an optically trapped micro-sized bead. If a micro-sized bead is illuminated by near-field waves, the scattered signal provides information on displacement caused by an external force exerted on the bead. This is the principle of optical trap nanometry. This nanometric technique is also useful in near-field scanning optical microscopy with laser trapping.

Near-field waves are produced when an electromagnetic wave undergoes total internal reflection at an interface between two media with different refractive indices. To achieve total internal reflection, electromagnetic wave needs to be incident from optically denser medium and its angle of incidence must be larger than critical angle. Amplitude of such produced near-field waves, or sometimes called evanescent waves, drops off exponentially as it penetrates optically thinner medium. Decay constant of near-field wave amplitude depends on incident angle of the wave generating this near-field wave. For angles much larger than critical angles, decay constant is larger and near-field wave does not propagate much into optically thinner medium. To detect this near-field wave a small spherical particle positioned close to the interface between the two media could be used. Near-field waves are scattered by this particle and in this process, termed near-field Mie scattering, are promoted into propagating waves which could then be detected by a detector.

To enable us to study the signal strength as a function of particle displacement under near-field illumination, as used in optical trap nanometry, we need to investigate how near-field waves are scattered by spherical particles. Most of current work on near-field Mie scattering focuses on calculating scattering cross sections or use an approximate version of the analytical solution which is only valid for large distances from the particle in two principal planes XZ and YZ (Fig. 1). However, to apply near-field Mie scattering into an imaging process such as optical trap nanometry knowledge of scattered electromagnetic field at any point around the particle is needed. Therefore we have developed a mathematical model from the analytical solution for near-field Mie scattering which enables us to calculate scattered electromagnetic field at any...
point around the particle. Particle is assumed to be far from the interface where near-field is generated, so that surface effects are neglected.

![Diagram of scattering model](image)

**Fig. 1** Illustration of the scattering model used. \( n, n' \) and \( n_1 \) denote refractive indices of the substrate, surrounding media and the particle respectively.

In previous scattering cross section calculations,\(^5\)\(^-\)\(^9\) for near-field Mie scattering by dielectric particles, morphology dependent resonances (MDR) are evident when particle size is approaching and exceeding illumination wavelength. It is found that MDR structure depends strongly on the penetration depth of the incident near-field wave,\(^7\) but the peak positions and half widths are unaffected by the kind of excitation (plane wave or near-field wave).\(^8\)

### 2. SCATTERED INTENSITY DISTRIBUTION AROUND DIELECTRIC PARTICLES

Scattered electromagnetic field intensity distribution of near-field waves scattered by a dielectric particle of 2 \( \mu m \) in radius is shown in Fig. 2 for perpendicular (S) and parallel (P) polarization states of the incident electromagnetic wave. To demonstrate capability of calculating scattered electromagnetic field at any point around such a particle, the scattered intensity in the XZ plane and in the plane at 45\(^\circ\) anti-clockwise from the XZ plane containing the X axis is calculated (Fig. 2(a), 2(b)).

These plots give us an indication of what the scattered intensity distribution looks like in three-dimensional space under conditions from Fig. 2. The scattered intensity distribution is asymmetric in planes XZ and the plane at 45\(^\circ\) from the XZ plane, as is expected due to the illumination field being asymmetric in these planes. It can also be noticed that near-field waves, under conditions from Fig. 2, are most intensely scattered into certain regions around the particle. This effect is due to the fact that the particle size in this case is so large that any appreciable near-field strength is confined to the very bottom of the particle. Ray optics and Snell’s law, together with Fresnel reflection coefficients could be used to approximately interpret the interaction between the particle and the near-field distribution (Fig.3).
Fig. 2  Scattered intensity around the 2 µm dielectric particle, (a) XZ plane and (b) plane at 45° anti-clockwise from the XZ plane containing the X axis. Refractive index of the particle $n_1 = 1.6$, surrounding medium $n' = 1.0$ and substrate $n = 1.23$. Illumination wavelength $λ = 632.8$ nm.
The highest intensity rays, interacting at the very bottom of the particle (point A), are incident at such large angles that most of the rays is reflected rather than refracted. Refracted rays, after traversing through the particle, interact with the particle-medium boundary again (point B) and the largest amount emerges into the surrounding medium while a small portion is reflected. This reflected portion traverses through the particle again, interacts with the boundary (point C) and emerges into the surrounding medium while a negligible amount is reflected again etc. This process leads to the scattered intensity profile showing very strong scattered intensities into certain regions around the particle (most intensely into the region bellow 0°). These regions depend on several factors: refractive indices of the dielectric particle and the medium into which the particle is immersed, particle size and near-field wave decay rate.

![Ray optics interpretation of distinct high intensity regions for scattering of near-field waves by a large dielectric particle (2 µm radius). Substrate refractive index n = 1.23, electromagnetic wave in substrate incident at α = 60°. Relative intensity denoted by the arrow length.](image)

3. MDR EFFECTS

MDR effects are due to the rays propagating inside dielectric particle confined by an almost total internal reflection. As the ray propagates inside the particle, it returns to its starting position in phase, given an appropriate particle size. Due to this process, large energy densities can accumulate inside such particle.
MDR resonance has been observed in plane-wave Mie scattering for larger dielectric particles ~10 µm and they are evident in the calculations of differential scattering cross sections in near-field Mie scattering by other researchers.

To investigate how changes in particle morphology affect near-field Mie scattering, scattered intensity into upper half-space of the particle is calculated. Fig. 4 shows the calculated intensity scattered into upper-half space by a particle with refractive index of 1.5 immersed in air for both incident polarization states. MDR becomes evident as the particle radius increases. For large particle radii, resonances become more pronounced and sharper. This is understandable if the particle is thought of as a micro-cavity, which by becoming larger, interacts significantly with the near-field only at the very bottom. Due to this localized interaction, rays refracted into the particle are very close to the critical angle. These rays are then more efficiently reflected off the cavity walls, which increases the coefficient of finesse of such cavity giving sharper resonance peaks inside the cavity.

![Graph showing scattered intensity as a function of particle radius for S and P incident polarizations.](image)

**Fig. 4** Half-space scattered intensity as function of particle radius for S and P incident polarizations. Illumination wavelength $\lambda = 632.8$ nm, electromagnetic wave in substrate incident at $\alpha = 60^\circ$, particle’s refractive index $n_1 = 1.5$, surrounding media refractive index $n' = 1.0$ and refractive index of the substrate $n = 1.23$.

The average distance between neighboring resonance peaks for near-field Mie scattering under conditions from Fig. 4 is determined to be 75 nm ± 1 nm, which agrees well with prediction of 75.8 nm for distance between neighboring peaks for plane wave illumination under the same conditions.12
According to our calculations for a low refractive index of the particle MDR effects are not observed even for larger particle sizes. As the refractive index of the particle increases, MDR effects become more pronounced with sharper resonance peaks and decreasing separation between neighboring resonances. One may expect this decrease in resonance separation, as increase in refractive index shortens the wavelength inside the particle and decreases the speed of propagation of the electromagnetic wave inside the particle, effectively decreasing separation of micro-cavity modes. Increase in resonance sharpness is due to higher coefficient of finesse of such cavity for larger effective refractive index, caused by increased reflectivity at the particle-air interface.

Our calculations also indicate that MDR peaks positions are independent of the incident angle $\alpha$ of the electromagnetic wave inside the substrate that generates near-field wave. This suggests that MDR effects are mainly caused by rays interacting at the bottom edge of the particle. As incident angle $\alpha$ approaches critical angle, decay rate of the produced near-field wave decreases and the particle is immersed into near-field to a greater extent than in the case when incident angle is larger. However resonance peaks are at the same positions for different incident angles, indicating that these resonant effects are dominated by rays at the bottom region of the particle.

Another parameter that we can allow to change is the illumination wavelength. When the illumination wavelength is changed, it effectively changes the size parameter so the MDR effects are only re-scaled. This is because refractive index of a dielectric changes very little with wavelength in the visible part of the spectrum which we have investigated.

4. CONCLUSIONS

We have applied a non-approximated mathematical model to calculate electromagnetic field distribution, resulting from scattering of near-field waves by small spherical particles, at any point in space around the particle. The near-field Mie scattering distribution around a dielectric particle is calculated in a number of planes, as well as the integrated scattered intensity in the upper half space of the particle, for S and P incident polarization states.

The main contributions to the scattered electromagnetic wave in the large particle case are due to reflection and refraction of the strong portion of the near-field near the very bottom of the particle.

MDR effects are evident in near-field Mie scattering by dielectric particles and its period is the same as for plane wave scattering. MDR peak positions, sharpness and their separation depend on particle size and refractive index. As the particle size increases resonant peaks become more pronounced and sharper. Similarly, increase in refractive index causes sharper resonant peaks but also decreases the separation between neighboring resonances. It is also shown that MDR peak positions are independent of the incident angle $\alpha$, which indicates that the main contribution to MDR effects is from rays at the bottom region of the particle.

REFERENCES