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Blind Identification of Non-Minimum Phase ARMA Systems

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Abstract

This paper presents a blind identification algorithm for non-minimum phase single-input single-output (SISO) plants using an over-sampling technique with each input symbol lasting for several sampling periods. First, an SISO autoregressive moving average (ARMA) plant is converted into its associated single-input multi-output (SIMO) system by holding the system input and over-sampling the system output. A sufficient and necessary condition for coprime transfer functions of the SIMO system is provided. A new second-order statistics (SOS) based blind identification algorithm for the SIMO ARMA model is then presented, which exploits the dynamical autoregressive information of the model contained in the autocorrelation matrices of the system outputs. Further, the transfer function of the SISO system is recovered from its associated SIMO transfer functions. Finally, the effectiveness of the proposed algorithm is demonstrated by simulation results.

Key words: ARMA model, Multi-rate systems, System identification, Second-order statistics.

1 Introduction

Blind system identification is to estimate system parameters based on measurements of the system outputs under some conditions without accessing to the system inputs. In general, the identification of non-minimum phase single-input single-output (SISO) systems requires higher order statistics [6, 22], because second-order statistics (SOS) of the output observations do not have sufficient information for recovering the non-minimum phase dynamics [20]. Fortunately, over-sampling techniques can help to resolve this problem. If the original output signal is stationary, the over-sampled output data is a cyclostationary signal which can provide the phase information for identification of non-minimum phase SISO systems [5].

In the literature, several results have been reported on blind identification of linear infinite impulse response (IIR) models using over-sampling techniques [1, 8, 14, 15]. The identification results of linear systems have also been extended to systems with nonlinearities [2, 14, 18, 19]. By over-sampling the system output at a higher rate than the system input, the SISO system model is converted into a single-input multi-output (SIMO) system model. After all the SIMO transfer functions have been estimated, the original SISO transfer function can be recovered according to their connections. In [8, 15], the input to the SISO system is a white random process with known variance. One drawback of this method is that the variance value of the system input should be known as a prior knowledge for the identification of the denominator parameters. In [1, 14, 18, 19], the system input to the SISO system can be any deterministic informative signal. One common necessary identifiability condition for the above algorithms is that the over-sampling rate should be larger than the order of the numerator of its transfer function. In practical applications, if the numerator's order is large or uncertain, it may impose high computational burden and cost on hardware.

In this paper, we use the input-holding and output over-sampling technique to transform an SISO system into its associated SIMO system [14]. A new blind identification algorithm is presented based on second-order statistics, which exploits the dynamical autoregressive information of the model contained in the autocorrelation matrices of the system outputs. Different from the existing methods for identification of SIMO ARMA systems, which estimate the numerator polynomials

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and the denominator polynomial separately using two individual methods, our method estimates the numerator polynomials and the denominator polynomial in a more systematic way. Unlike the conditions on the over-sampling rate required by the identification algorithms for IIR models in [1, 14, 18, 19], the sufficient and necessary condition for the coprime SIMO transfer functions only requires that the over-sampling rate is larger than or equal to 2. In contrast to the methods in [8, 12, 15], our proposed method combines both the block Toeplitz structure of the convolution matrix and the dynamical autoregressive information of autocorrelation matrices for system identification. The proposed method in this paper only uses lag-0 and lag-1 autocorrelation matrices, while [3] uses higher-lagged autocorrelation matrices. The method in [4] uses lag-0 and lag-1 autocorrelation matrices, but it assumes some knowledge on these autocorrelation matrices.

The main contributions of this paper are stated as follows: (1) a novel SOS based blind identification method for SISO (or SIMO) ARMA systems is presented which estimates the numerator polynomials and the denominator polynomial in a more systematic way; (2) a sufficient and necessary condition for coprime SIMO transfer functions is provided without requiring a lower bound of the over-sampling rate.

The rest of this paper is organized as follows. Section 2 gives the problem formulation. Section 3 formulates an SIMO system model obtained by over-sampling an SISO system, and provides a coprime condition of the derived SIMO transfer functions. Section 4 provides a new blind identification algorithm for SIMO ARMA systems. Section 5 deals with the recovery of the original SISO transfer from its associated SIMO transfer functions. Section 6 presents numerical simulation results of the proposed method and performance comparisons with existing results, followed by Conclusion in Section 7.

2 Problem Statement

Consider an SISO discrete time-invariant system in the following state-space equation

$$\begin{aligned} x(t+1) &= \mathbf{A}x(t) + \mathbf{B}u(t) \\ y(t) &= \mathbf{C}x(t) + \mathbf{D}u(t) \end{aligned} \tag{1}$$

where $x(t) \in \mathbb{R}^N$, $u(t) \in \mathbb{R}$ and $y(t) \in \mathbb{R}$ are the system state, input and output, respectively, and $\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}$ and $\mathbf{D}$ are the corresponding state matrices and vectors of appropriate dimensions. The linear state-space equation can represent dynamical systems in different applications, such as an RLC circuit, a rocket ascending process and the dynamics of heat transfer. It is a popular tool for system analysis and design, especially for feedback control systems.

Without loss of generality, the above state-space realization is assumed to be minimal and its z-transfer function, which is an irreducible rational polynomial, can be written as

$$T(z^{-1}) = \frac{Y(z^{-1})}{U(z^{-1})} = \mathbf{C}(\mathbf{I} - z^{-1}\mathbf{A})^{-1}\mathbf{B} + \mathbf{D} = \frac{Q(z^{-1})}{P(z^{-1})} \tag{2}$$

where

$$\begin{aligned} P(z^{-1}) &= \det(\mathbf{I} - z^{-1}\mathbf{A}) \\ &= 1 + p_1 z^{-1} + \cdots + p_{N-1} z^{-N+1} + p_N z^{-N} \\ &= (1 - \gamma_1 z^{-1})(1 - \gamma_2 z^{-1}) \cdots (1 - \gamma_N z^{-1}) \\ Q(z^{-1}) &= \mathbf{D}\det(\mathbf{I} - z^{-1}\mathbf{A}) + \mathbf{C}\mathrm{adj}(\mathbf{I} - z^{-1}\mathbf{A})\mathbf{B} \\ &= q_0 + q_1 z^{-1} + \cdots + q_{N-1} z^{-N+1} + q_N z^{-N} \end{aligned}$$

where $\gamma_i \in \mathbb{C}$, $i = 1, 2, \cdots, N$, are real or complex poles of the system, $\det(\cdot)$ and $\mathrm{adj}(\cdot)$ denote the determinant and adjoint operators, respectively. In addition, it is assumed that the N-th order system (1) is stable, i.e. $|\gamma_i| < 1$ for $i = 1, 2, \cdots, N$. For the sake of simplicity, noise-free SISO models are concerned first, and additive noise contaminated models will be examined in the simulation part.

When the SISO system in (2) is non-minimum phase, the second-order stationary statistics of its output signals sampled at the normal sampling rate do not contain sufficient information for system identification. Thus, the problem of interest is to design a second-order statistics based blind identification algorithm for non-minimum phase SISO ARMA systems using the over-sampling technique, i.e. estimating the coefficients $\{p_i\}_{i=1}^N$ and $\{q_i\}_{i=0}^N$ using the second-order statistics of the over-sampled output signals.

3 Multi-Rate System Model

3.1 State-Space Representation

Let the input be held constant such that $u(t) = u(nL)$ for all $t \in \{nL, nL+1, \cdots, nL+L-1\}$, where L denotes the over-sampling rate. The state-space equation of an SIMO system can be written as [7]:

$$\begin{aligned} x(nL+L) &= \bar{\mathbf{A}}x(nL) + \bar{\mathbf{B}}u(nL) \\ \bar{y}(nL) &= \bar{\mathbf{C}}x(nL) + \bar{\mathbf{D}}u(nL) \end{aligned} \tag{3}$$

or equivalently as (with simplified notation)

$$\begin{aligned} x(n+1) &= \bar{\mathbf{A}}x(n) + \bar{\mathbf{B}}u(n) \\ \bar{y}(n) &= \bar{\mathbf{C}}x(n) + \bar{\mathbf{D}}u(n) \end{aligned} \quad (4)$$

where $\bar{\mathbf{A}} = \mathbf{A}^L$, $\bar{\mathbf{B}} = \sum_{j=0}^{L-1} \mathbf{A}^j\mathbf{B}$,

$$\bar{\mathbf{C}} = \begin{bmatrix} \mathbf{C} \\ \mathbf{CA} \\ \vdots \\ \mathbf{CA}^{L-1} \end{bmatrix}, \quad \bar{\mathbf{D}} = \begin{bmatrix} \mathbf{D} \\ \mathbf{D} + \mathbf{CB} \\ \vdots \\ \mathbf{D} + \mathbf{C}\sum_{j=0}^{L-2} \mathbf{A}^j\mathbf{B} \end{bmatrix},$$

$$\bar{y}(n) = \begin{bmatrix} y(nL) \\ y(nL+1) \\ \vdots \\ y(nL+L-1) \end{bmatrix}.$$

It is shown that the state-space equation (3) represents a single-input L-output system. Its z-transfer matrix, with respect to L-dimensional output, is

$$\bar{T}(z^{-1}) = \frac{\bar{Y}(z^{-1})}{U(z^{-1})} = \bar{\mathbf{C}}(\mathbf{I} - z^{-1}\bar{\mathbf{A}})^{-1}\bar{\mathbf{B}} + \bar{\mathbf{D}}. \quad (5)$$

It can be transformed into rational transfer functions as follows

$$\bar{T}_i(z^{-1}) = \frac{\bar{Q}_i(z^{-1})}{\bar{P}(z^{-1})} \quad i = 1, 2, \cdots, L \quad (6)$$

where $\bar{T}_i(z^{-1})$ denotes the i-th channel function, $\bar{P}(z^{-1}) = 1 + \bar{p}_1 z^{-1} + \cdots + \bar{p}_N z^{-N}$, and $\bar{Q}_i(z^{-1}) = \bar{q}_{i,0} + \bar{q}_{i,1}z^{-1} + \cdots + \bar{q}_{i,N}z^{-N}$. The poles of the system are determined by the denominator of the transfer function, which is

$$\bar{P}(z^{-1}) = \det\left(\mathbf{I} - z^{-1}\bar{\mathbf{A}}\right).$$

It is clear that all transfer functions $\bar{T}_i(z^{-1})$, from the input to the outputs, have the same denominator $\bar{P}(z^{-1})$.

3.2 Rational Polynomial Representation

The polynomial expression of (6) can also be directly obtained from that in (2). Introduce

$$\tilde{P}(z^{-1}) = \prod_{i=1}^{N}(1 + \gamma_i z^{-1} + \cdots + \gamma_i^{L-1} z^{-(L-1)}). \quad (7)$$

Then the product of $P(z^{-1})$ and $\tilde{P}(z^{-1})$ can be written as

$$\bar{P}(z^{-L}) = P(z^{-1})\tilde{P}(z^{-1}) = \prod_{i=1}^{N}(1 - \gamma_i^L z^{-L}), \quad (8)$$

and the system transfer function $T(z^{-1})$ can be written as

$$T(z^{-1}) = \frac{Q(z^{-1})\tilde{P}(z^{-1})}{P(z^{-1})\tilde{P}(z^{-1})} = \frac{\bar{Q}(z^{-1})}{\bar{P}(z^{-L})} \quad (9)$$

where $\bar{Q}(z^{-1})$ is an NL-th order polynomial which can be represented as

$$\bar{Q}(z^{-1}) = \hat{q}_0 + \hat{q}_1 z^{-1} + \cdots + \hat{q}_{NL} z^{-NL}. \quad (10)$$

Since $u(t) = u(nL)$ is held constant for all $t \in \{nL, nL+1, \cdots, nL+L-1\}$, the numerator $\bar{Q}_i(z^{-L})$ of $\bar{T}_i(z^{-L})$ can be expressed by

$$\bar{Q}_i(z^{-L}) = \bar{q}_{i,0} + \bar{q}_{i,1} z^{-L} + \cdots + \bar{q}_{i,N} z^{-NL}, \quad (11)$$

where

$$\bar{q}_{i,k} = \begin{cases} \sum_{j=0}^{i-1} \hat{q}_j & k = 0, \\ \sum_{j=i}^{i+L-1} \hat{q}_{(k-1)L+j} & k = 1, 2, \cdots, N-1, \\ \sum_{j=i}^{L} \hat{q}_{(N-1)L+j} & k = N. \end{cases} \quad (12)$$

Since the expression $\frac{\bar{Q}_i(z^{-L})}{\bar{P}(z^{-L})}$ can be obtained from (8) and (11), the SIMO transfer function $\frac{\bar{Q}_i(z^{-1})}{\bar{P}(z^{-1})}$ in (6) can be obtained accordingly.

The above derivation gives the connection between the SISO transfer function and the associated SIMO transfer functions, which is instrumental for the derivation of the coprime condition of SIMO transfer functions and the identification of the original SISO transfer function.

3.3 Coprime Condition for SIMO Transfer Functions

One essential condition for blind identification of SIMO systems is that all channels are coprime [12, 16, 20], i.e. all channel functions do not share any common zeros. In this paper, an SIMO system is derived from an SISO system, so what conditions on the SISO system such that the derived SIMO transfer functions are coprime will be studied here.

Since $|\gamma_i| < 1$ for $i = 1, \cdots, N$, each pole γ_i^L of $\bar{T}_i(z^{-1})$ in (6) satisfies $|\gamma_i^L| < 1$ and (6) is a stable ARMA model.

The coefficient matrix of the moving average (MA) polynomials $\bar{Q}_i(z^{-1})$ for $i = 1, \cdots, L$ is written as

$$\mathbf{Q} = \begin{bmatrix} \bar{q}_{1,0} & \bar{q}_{1,1} & \cdots & \bar{q}_{1,N} \\ \bar{q}_{2,0} & \bar{q}_{2,1} & \cdots & \bar{q}_{2,N} \\ \vdots & \vdots & \ddots & \vdots \\ \bar{q}_{L,0} & \bar{q}_{L,1} & \cdots & \bar{q}_{L,N} \end{bmatrix}.$$

Let $\Sigma = \begin{bmatrix} 1 & \sigma & \cdots & \sigma^N \end{bmatrix}^T \in \mathbb{C}^{N+1}$, then $\{\bar{Q}_i(z^{-1})\}_{i=1}^L$ are coprime if and only if there exists no Σ such that $\mathbf{Q}\Sigma = 0$.

Lemma 1. The MA polynomials $\{\bar{Q}_i(z^{-1})\}_{i=1}^L$ of the ARMA model (6) are coprime if and only if the MA polynomial $Q(z^{-1})$ of the SISO model (2) has no factors of the form $1 - z^{-1}$ and $1 - \sigma z^{-L}$ for any $\sigma \in \mathbb{C}$.

Proof. Necessity: If $Q(z^{-1})$ has a factor $1 - z^{-1}$ such that $Q(1) = 0$, $\bar{Q}(z^{-1})$ in (9) satisfies $\bar{Q}(1) = Q(1)\tilde{P}(1) = 0$. This together with the formation of the coefficients of $\bar{q}_{i,k}$ in (12) results in

$$\sum_{k=0}^{N} \bar{q}_{i,k} = \bar{Q}_i(1) = \bar{Q}(1) = 0, \quad i = 1, 2, \cdots, L.$$

It follows that $Q\Sigma_{|\sigma=1} = 0$ and the MA polynomials $\{\bar{Q}_i(z^{-1})\}_{i=1}^L$, are not coprime.

If $Q(z^{-1})$ has a factor $1-\sigma z^{-L}$, $\bar{Q}(z^{-1}) = Q(z^{-1})\tilde{P}(z^{-1})$ has a factor $1 - \sigma z^{-L}$. It follows from the formation of the coefficients $\bar{q}_{i,k}$ in (12) that each $\bar{Q}_i(z^{-L})$ has a factor $1 - \sigma z^{-L}$, for $i = 1, 2, \cdots, L$. Hence, the MA polynomials $\{\bar{Q}_i(z^{-1})\}_{i=1}^L$ are not coprime.

Sufficiency: Suppose that the L MA polynomials $\bar{Q}_i(z^{-1})$ are not coprime such that $Q\Sigma = 0$ is satisfied for a $\sigma \in \mathbb{C}$. If $\sigma = 1$, then $\sum_{k=0}^{N} \bar{q}_{i,k} = \sum_{k=0}^{NL} \hat{q}_k = \bar{Q}(1) = Q(1)\tilde{P}(1) = 0$ for all $i = 1, 2, \cdots, L$. Since $\bar{P}(z^{-L}) = \tilde{P}(z^{-1})P(z^{-1})$ and $\bar{P}(z^{-L})$ in (9) is the characteristic polynomial of a stable ARMA model, $\tilde{P}(z^{-1})$ has no factor of the form $1 - z^{-1}$. It follows that $1 - z^{-1}$ must be a factor of $Q(z^{-1})$.

If $Q\Sigma = 0$ is satisfied for $\sigma \neq 1$, by multiplying it from the left by

$$J = \begin{bmatrix} 1 & 0 & & \cdots & -\sigma \\ -1 & 1 & 0 & \cdots & 0 \\ 0 & -1 & 1 & 0 \cdots & 0 \\ & & \ddots & \ddots & \\ 0 & \cdots & 0 & -1 & 1 \end{bmatrix} \in \mathbb{C}^{L \times L}$$

and using the formation of the coefficients $\bar{q}_{i,k}$ in (12), it yields

$$\begin{aligned} &J\mathbf{Q}\Sigma \\ &= \begin{bmatrix} \bar{q}_{1,0} - \sigma\bar{q}_{L,0} & \bar{q}_{1,1} - \sigma\bar{q}_{L,1} & \cdots & \bar{q}_{1,N} - \sigma\bar{q}_{L,N} \\ \hat{q}_1 & \hat{q}_{L+1} - \hat{q}_1 & \cdots & -\hat{q}_{NL-L+1} \\ \vdots & \vdots & \ddots & \vdots \\ \hat{q}_{L-1} & \hat{q}_{2L-1} - \hat{q}_{L-1} & \cdots & -\hat{q}_{NL-1} \end{bmatrix} \begin{bmatrix} 1 \\ \sigma \\ \vdots \\ \sigma^N \end{bmatrix} \\ &= \begin{bmatrix} \hat{q}_0 & \hat{q}_L - \hat{q}_0 & \cdots & \hat{q}_{NL} - \hat{q}_{NL-L} & -\hat{q}_{NL} \\ \hat{q}_1 & \hat{q}_{L+1} - \hat{q}_1 & \cdots & -\hat{q}_{NL-L+1} & 0 \\ \vdots & \vdots & \ddots & \vdots & 0 \\ \hat{q}_{L-1} & \hat{q}_{2L-1} - \hat{q}_{L-1} & \cdots & \hat{q}_{NL-1} & 0 \end{bmatrix} \begin{bmatrix} 1 \\ \sigma \\ \vdots \\ \sigma^{N+1} \end{bmatrix} \\ &= (1 - \sigma) \begin{bmatrix} \hat{q}_0 & \hat{q}_L & \cdots & \hat{q}_{NL-L} & \hat{q}_{NL} \\ \hat{q}_1 & \hat{q}_{L+1} & \cdots & \hat{q}_{NL-L+1} & 0 \\ \vdots & \vdots & \ddots & \vdots & 0 \\ \hat{q}_{L-1} & \hat{q}_{2L-1} & \cdots & \hat{q}_{NL-1} & 0 \end{bmatrix} \begin{bmatrix} 1 \\ \sigma \\ \vdots \\ \sigma^N \end{bmatrix} \\ &= 0. \end{aligned} \tag{13}$$

Since $\sigma \neq 1$, the equation (13) implies that the polynomials

$$z^{-i+1} \sum_{k=0}^{N} \hat{q}_{kL+i-1} z^{-kL}, \quad i = 1, 2, \cdots, L,$$

have a common factor of the form $1 - \sigma z^{-L}$, where $\hat{q}_l = 0$ for $l > NL$. It follows that

$$\bar{Q}(z^{-1}) = \tilde{P}(z^{-1})Q(z^{-1}) = \sum_{i=1}^{L} z^{-i+1} \sum_{k=0}^{N} \hat{q}_{kL+i-1} z^{-kL}$$

has a factor of the form $1 - \sigma z^{-L}$. Since $\tilde{P}(z^{-1})$ in the form (7) has no factor of the form $1 - \sigma z^{-L}$ and $Q(z^{-1})$ shares no common factors with $P(z^{-1})$ which is guaranteed by the minimal realization (2), the polynomial $Q(z^{-1})$ must have a factor of the form $1 - \sigma z^{-L}$. The lemma is proven. □

4 Blind SIMO System Identification

The SIMO ARMA model in (5) can be described by

$$\bar{y}_i(n) = \frac{\bar{Q}_i(z^{-1})}{\bar{P}(z^{-1})}u(n) = \bar{Q}_i(z^{-1})s(n) \quad i = 1, 2, \cdots, L \tag{14}$$

where $s(n) = u(n)/\bar{P}(z^{-1})$ denotes a pseudo colored source signal. The above model can be further formulated into the following matrix-vector multiplication equation

$$\bar{\mathbf{y}}(n) = \bar{\mathbf{Q}}\mathbf{s}(n) \tag{15}$$

where $\bar{\mathbf{y}}(n) \in \mathbb{R}^{ML}$ and $\mathbf{s}(n) \in \mathbb{R}^{M+N}$ are defined by

$$\begin{aligned}
\bar{\mathbf{y}}(n) &= \left[\tilde{\mathbf{y}}^T(n) \quad \tilde{\mathbf{y}}^T(n-1) \quad \cdots \quad \tilde{\mathbf{y}}^T(n-M+1)\right]^T, \\
\tilde{\mathbf{y}}(n) &= \left[\bar{y}_1(n) \quad \bar{y}_2(n) \quad \cdots \quad \bar{y}_L(n)\right]^T, \\
\mathbf{s}(n) &= \left[s(n) \quad s(n-1) \quad \cdots \quad s(n-M-N+1)\right]^T,
\end{aligned}$$

and $\bar{\mathbf{Q}} \in \mathbb{R}^{ML \times (M+N)}$ is a block Toeplitz matrix in the form

$$\bar{\mathbf{Q}} = \begin{bmatrix} \bar{\mathbf{q}}_0 & \bar{\mathbf{q}}_1 & \cdots & \bar{\mathbf{q}}_N & & & \\ & \bar{\mathbf{q}}_0 & \bar{\mathbf{q}}_1 & \cdots & \bar{\mathbf{q}}_N & & \\ & & \ddots & \ddots & & \ddots & \\ & & & \bar{\mathbf{q}}_0 & \bar{\mathbf{q}}_1 & \cdots & \bar{\mathbf{q}}_N \end{bmatrix}, \tag{16}$$

with $\bar{\mathbf{q}}_i = [\bar{q}_{1,i} \quad \bar{q}_{2,i} \quad \cdots \quad \bar{q}_{L,i}]^T$.

In order to guarantee the identifiability of the SIMO system, the following assumptions are made:

A1: The SISO transfer function $T(z^{-1})$ is stable and irreducible;

A2: The MA polynomial $Q(z^{-1})$ has no factors of the form $1 - z^{-1}$ and $1 - \sigma z^{-L}$ for any $\sigma \in \mathbb{C}$;

A3: The source signal $u(t)$ is a persistently excited white noise without knowing its variance value;

A4: $ML \geq (M + N)$, i.e. $\bar{\mathbf{Q}}$ is a tall matrix.

Assumption A1 implies that the transfer functions $\{\frac{\bar{Q}_i(z^{-1})}{\bar{P}(z^{-1})}\}_{i=1}^{L}$ are stable, Assumption A2 hints that all the MA polynomials $\{\bar{Q}_i(z^{-1})\}_{i=1}^{L}$ are coprime, and Assumption A2 and A4 infer that the matrix $\bar{\mathbf{Q}}$ is of full column rank [12].

Under Assumptions A1 and A3, the pseudo source signal $s(n)$ and the output signals $\{\bar{y}_i(n)\}_{i=1}^{L}$ are wide-sense stationary. The associated autocorrelation matrices are connected by

$$\mathbf{R}_{\bar{\mathbf{y}}}(k) = \bar{\mathbf{Q}}\mathbf{R}_{\mathbf{s}}(k)\bar{\mathbf{Q}}^H, \tag{17}$$

where $\mathbf{R}_{\bar{\mathbf{y}}}(k) = E[\bar{\mathbf{y}}(n)\bar{\mathbf{y}}^T(n-k)]$, $\mathbf{R}_{\mathbf{s}}(k) = E[\mathbf{s}(n)\mathbf{s}^T(n-k)]$, E denotes the mathematical expectation operator, and the superscripts ^{T} and ^{H} denote transpose and Hermitian transpose, respectively. In addition, the autocorrelation matrices of different lags for the colored source signal $s(n)$ satisfy the following equation

$$\mathbf{R}_{\mathbf{s}}(k) = \begin{cases} \mathbf{\Gamma}_+{}^k\mathbf{R}_{\mathbf{s}}(0) & k \geq 0 \\ \mathbf{\Gamma}_-{}^{-k}\mathbf{R}_{\mathbf{s}}(0) & k < 0, \end{cases} \tag{18}$$

where

$$\mathbf{\Gamma}_+ = \begin{bmatrix} -\bar{p}_1 & \cdots & -\bar{p}_N & 0 & \cdots & 0 \\ 1 & & & & & \\ & 1 & & & & \\ & & \ddots & & & \\ & & & & 1 & 0 \end{bmatrix} \in \mathbb{R}^{(M+N)\times(M+N)},$$

$$\mathbf{\Gamma}_- = \begin{bmatrix} 0 & 1 & & & & \\ & & \ddots & & & \\ & & & 1 & & \\ & & & & 1 & \\ 0 & \cdots & 0 & -\bar{p}_N & \cdots & -\bar{p}_1 \end{bmatrix} \in \mathbb{R}^{(M+N)\times(M+N)}. \tag{19}$$

Based on Assumptions A1 and A3, we can obtain that the autocorrelation matrix $\mathbf{R}_{\mathbf{s}}(0)$ of any order is positive definite [11]. In (14), the AR polynomial $P(z^{-1})$ is unknown, so the second-order statistics of $s(n)$ and its autocorrelation matrices $\mathbf{R}_{\mathbf{s}}(k)$ are not available. Thus, the applied identification method should not depend on the values of the autocorrelation matrices $\mathbf{R}_{\mathbf{s}}(k)$. Let $\hat{\mathbf{R}}_{\bar{\mathbf{y}}}(0)$, $\hat{\mathbf{R}}_{\bar{\mathbf{y}}}(-1)$ and $\hat{\mathbf{R}}_{\bar{\mathbf{y}}}(1)$ denote autocorrelation matrices calculated from output observations. The relationships among $\hat{\mathbf{R}}_{\bar{\mathbf{y}}}(0)$, $\hat{\mathbf{R}}_{\bar{\mathbf{y}}}(-1)$, $\hat{\mathbf{R}}_{\bar{\mathbf{y}}}(1)$ and $\mathbf{R}_{\mathbf{s}}(0)$ showed in the following lemma are instrumental for solving the blind multi-channel identification problem.

Lemma 2. Associated with the matrices $\hat{\mathbf{R}}_{\bar{\mathbf{y}}}(0)$, $\hat{\mathbf{R}}_{\bar{\mathbf{y}}}(-1)$ and $\hat{\mathbf{R}}_{\bar{\mathbf{y}}}(1)$, there exists a nonsingular matrix $\mathbf{\Phi} \in \mathbb{C}^{(M+N)\times(M+N)}$ satisfying

$$\bar{\mathbf{Q}} = \mathbf{U}_1\mathbf{\Phi} \tag{20}$$

$$\mathbf{\Phi}\mathbf{\Gamma}_+\mathbf{\Phi}^{-1} = \mathbf{U}_1^H\hat{\mathbf{R}}_{\bar{\mathbf{y}}}(1)\mathbf{U}_1\mathbf{\Sigma}^{-1} \tag{21}$$

$$\mathbf{\Phi}\mathbf{\Gamma}_-\mathbf{\Phi}^{-1} = \mathbf{U}_1^H\hat{\mathbf{R}}_{\bar{\mathbf{y}}}(-1)\mathbf{U}_1\mathbf{\Sigma}^{-1} \tag{22}$$

where $\mathbf{U}_1 \in \mathbb{C}^{ML\times(M+N)}$ is a matrix constituted by $M + N$ orthogonal columns, $\mathbf{\Sigma} \in \mathbb{R}^{(M+N)\times(M+N)}$ is a nonsingular diagonal matrix, and $\mathbf{U}_1$ and $\mathbf{\Sigma}$ are determined by the singular value decomposition of the Hermitian matrix $\hat{\mathbf{R}}_{\bar{\mathbf{y}}}(0)$:

$$\hat{\mathbf{R}}_{\bar{\mathbf{y}}}(0) = \begin{bmatrix} \mathbf{U}_1 & \mathbf{U}_2 \end{bmatrix} \begin{bmatrix} \mathbf{\Sigma} & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \mathbf{U}_1^H \\ \mathbf{U}_2^H \end{bmatrix}. \tag{23}$$

Proof. In view of (17) and (23), combining the positive definite property of the matrix $\mathbf{R}_{\mathbf{s}}(0)$, the full rank matrix $\bar{\mathbf{Q}}$ has the same column space as that of $\hat{\mathbf{R}}_{\bar{\mathbf{y}}}(0)$. Thus, there exists a nonsingular matrix $\mathbf{\Phi}$ satisfying the equation (20).

Substituting (20) into (17) with $k = 0$ yields

$$\mathbf{U}_1\mathbf{\Sigma}\mathbf{U}_1^H = \mathbf{U}_1\mathbf{\Phi}\mathbf{R}_{\mathbf{s}}(0)\mathbf{\Phi}^H\mathbf{U}_1^H. \tag{24}$$

Consequently,

$$\mathbf{R}_{\mathbf{s}}(0) = \mathbf{\Phi}^{-1}\mathbf{\Sigma}\mathbf{\Phi}^{-H}. \tag{25}$$

Using (17) and (18), $\hat{\mathbf{R}}_{\bar{\mathbf{y}}}(1)$ can be written as

$$\begin{aligned}\hat{\mathbf{R}}_{\bar{\mathbf{y}}}(1) &= \bar{\mathbf{Q}}\mathbf{R}_{\mathbf{s}}(1)\bar{\mathbf{Q}}^H \\ &= \mathbf{U_1}\mathbf{\Phi}\mathbf{\Gamma}_+\mathbf{R}_{\mathbf{s}}(0)\mathbf{\Phi}^H\mathbf{U_1}^H \\ &= \mathbf{U}_1\mathbf{\Phi}\mathbf{\Gamma}_+\mathbf{\Phi}^{-1}\mathbf{\Sigma}\mathbf{U}_1^H.\end{aligned}$$

This is an equivalent expression of (21). Similarly, (22) can also be established and the lemma is proven. □

4.1 AR Parameter Estimation

In view of (21) and (22), the matrices involved in the right-hand sides can be obtained from $\hat{\mathbf{R}}_{\bar{\mathbf{y}}}(1)$ and $\hat{\mathbf{R}}_{\bar{\mathbf{y}}}(-1)$ and the singular value decomposition of $\hat{\mathbf{R}}_{\bar{\mathbf{y}}}(0)$ showed in (23). Let

$$\hat{\bar{\mathbf{R}}}_{\bar{\mathbf{y}}}(1) = \mathbf{\Phi}\mathbf{\Gamma}_+\mathbf{\Phi}^{-1} = \mathbf{U}_1^H\hat{\mathbf{R}}_{\bar{\mathbf{y}}}(1)\mathbf{U}_1\mathbf{\Sigma}^{-1} \tag{26}$$

$$\hat{\bar{\mathbf{R}}}_{\bar{\mathbf{y}}}(-1) = \mathbf{\Phi}\mathbf{\Gamma}_-\mathbf{\Phi}^{-1} = \mathbf{U}_1^H\hat{\mathbf{R}}_{\bar{\mathbf{y}}}(-1)\mathbf{U}_1\mathbf{\Sigma}^{-1}. \tag{27}$$

Then the matrices $\hat{\bar{\mathbf{R}}}_{\bar{\mathbf{y}}}(1)$ and $\hat{\bar{\mathbf{R}}}_{\bar{\mathbf{y}}}(-1)$ can be used to estimate the parameters of the ARMA model (14) contained in the matrices $\mathbf{\Gamma}_+$, $\mathbf{\Gamma}_-$ and $\mathbf{\Phi}$.

It can be found that the matrices $\mathbf{\Gamma}_+$ and $\mathbf{\Gamma}_-$ are companion matrices [9] whose characteristic polynomial can be easily obtained as

$$\begin{aligned}\det(\lambda\mathbf{I} - \mathbf{\Gamma}_+) &= \det(\lambda\mathbf{I} - \mathbf{\Gamma}_-) \\ &= \lambda^{M+N} + \bar{p}_1\lambda^{M+N-1} + \cdots + \bar{p}_N\lambda^M.\end{aligned}$$

Obviously, the AR polynomial $\bar{P}(z^{-1})$ in (14) can be estimated by computing the characteristic polynomial of the observation matrix $\hat{\bar{\mathbf{R}}}_{\bar{\mathbf{y}}}(1)$ or $\hat{\bar{\mathbf{R}}}_{\bar{\mathbf{y}}}(-1)$ as follows

$$\begin{aligned}\hat{\bar{P}}(z^{-1}) &= z^{-(M+N)}\det\left(z\mathbf{I} - \hat{\bar{\mathbf{R}}}_{\bar{\mathbf{y}}}(1)\right) \\ &= z^{-(M+N)}\det\left(z\mathbf{I} - \hat{\bar{\mathbf{R}}}_{\bar{\mathbf{y}}}(-1)\right).\end{aligned} \tag{28}$$

4.2 MA Parameter Estimation

The MA parameter estimation are carried out according to the property of the matrix $\bar{\mathbf{Q}}$: (1) the matrix $\bar{\mathbf{Q}}$ is full column rank but without the Toeplitz structure; (2) the matrix $\bar{\mathbf{Q}}$ possess the Toeplitz structure. The first case only utilizes the inner structures of lag-0 and lag-1 autocorrelation matrices, which can be directly applied to the second case. Against the method applied in the first case, by exploiting the Toeplitz structure of the matrix $\bar{\mathbf{Q}}$ and the inner structures of autocorrelation matrices, an efficient and effective identification is given for the second case.

Case 1: The matrix $\bar{\mathbf{Q}}$ has full column rank but without the Toeplitz structure.

Partition the full rank square matrix $\mathbf{\Phi}$ as $\mathbf{\Phi} = [\phi_1, \phi_2 \cdots \phi_{M+N}]$ with ϕ_m being the mth column. In view of the structure of $\mathbf{\Gamma}_+$ and $\mathbf{\Gamma}_-$ in (19), the equations (26) and (27) can be equivalently written as

$$\begin{aligned}\hat{\bar{\mathbf{R}}}_{\bar{\mathbf{y}}}(1)\phi_1 + \bar{p}_1\phi_1 &= \phi_2, \\ &\vdots \\ \hat{\bar{\mathbf{R}}}_{\bar{\mathbf{y}}}(1)\phi_N + \bar{p}_N\phi_1 &= \phi_{N+1}, \\ \hat{\bar{\mathbf{R}}}_{\bar{\mathbf{y}}}(1)\phi_{N+1} &= \phi_{N+2}, \\ &\vdots \\ \hat{\bar{\mathbf{R}}}_{\bar{\mathbf{y}}}(1)\phi_{M+N-1} &= \phi_{M+N}, \\ \hat{\bar{\mathbf{R}}}_{\bar{\mathbf{y}}}(1)\phi_{M+N} &= 0,\end{aligned} \tag{29}$$

$$\begin{aligned}\hat{\bar{\mathbf{R}}}_{\bar{\mathbf{y}}}(-1)\phi_1 &= 0, \\ \hat{\bar{\mathbf{R}}}_{\bar{\mathbf{y}}}(-1)\phi_2 &= \phi_1, \\ &\vdots \\ \hat{\bar{\mathbf{R}}}_{\bar{\mathbf{y}}}(-1)\phi_M &= \phi_{M-1}, \\ \hat{\bar{\mathbf{R}}}_{\bar{\mathbf{y}}}(-1)\phi_{M+1} + \bar{p}_N\phi_{M+N} &= \phi_M, \\ &\vdots \\ \hat{\bar{\mathbf{R}}}_{\bar{\mathbf{y}}}(-1)\phi_{M+N} + \bar{p}_1\phi_{M+N} &= \phi_{M+N-1}.\end{aligned} \tag{30}$$

From the structure of the matrices $\mathbf{\Gamma}_+$ and $\mathbf{\Gamma}_-$ in (19), it can be verified that their rank is $M+N-1$, so is that of $\hat{\bar{\mathbf{R}}}_{\bar{\mathbf{y}}}(1)$ and $\hat{\bar{\mathbf{R}}}_{\bar{\mathbf{y}}}(-1)$. Thus, the dimension of the null space of $\hat{\bar{\mathbf{R}}}_{\bar{\mathbf{y}}}(1)$ and $\hat{\bar{\mathbf{R}}}_{\bar{\mathbf{y}}}(-1)$ is one. In view of (29), if ϕ_1 is determined, the rest of ϕ_ms can be represented in terms of $\hat{\bar{\mathbf{R}}}_{\bar{\mathbf{y}}}(1)$, $\bar{p}_m$s and ϕ_1. The vector ϕ_1 is, however, determined by the null space of $\hat{\bar{\mathbf{R}}}_{\bar{\mathbf{y}}}(-1)$ by (30) up to a constant scale factor. Similarly, using (29), the vector ϕ_{M+N} can be determined by the null space of $\hat{\bar{\mathbf{R}}}_{\bar{\mathbf{y}}}(1)$ up to a constant scale factor, which can further determine the rest ϕ_ms by (30).

After obtaining the estimate of the matrix $\mathbf{\Phi}$, the convolution matrix $\bar{\mathbf{Q}}$ can be computed according to the equation (20). Since the matrix $\bar{\mathbf{Q}}$ has the block Toeplitz structure as shown in (16), then all the MA parameters in (14) can be estimated accordingly.

Case 2: The matrix $\bar{\mathbf{Q}}$ possesses the Toeplitz structure.

The identification method in **Case 1** tries to estimate the nonsingular matrix $\mathbf{\Phi}$ followed by extracting the MA parameters from the Toeplitz matrix $\bar{\mathbf{Q}}$. It only makes use of the structure properties of lag-0 and lag-1 autocorrelation matrices. However, if the matrix $\bar{\mathbf{Q}}$ does possess the Toeplitz structure, an effective identification algorithm is developed here by utilizing the structure properties of autocorrelation matrices and the Toeplitz structure of the matrix $\bar{\mathbf{Q}}$.

Let $\mathbf{q} = \begin{bmatrix} \mathbf{q}_0^T & \mathbf{q}_1^T & \cdots & \mathbf{q}_N^T \end{bmatrix}^T$ denote the vector form of MA parameters and $\bar{\mathbf{Q}}(\mathbf{q})$ denote the parametric form of the Toeplitz matrix $\bar{\mathbf{Q}}$. According to the equation (20), the unknown matrix $\mathbf{\Phi}$ can be represented by $\mathbf{\Phi} = \mathbf{U}_1^T\bar{\mathbf{Q}}(\mathbf{q})$. Substituting this relationship into the equation (26), we have that

$$\hat{\bar{\mathbf{R}}}_{\bar{\mathbf{y}}}(1)\mathbf{U}_1^T\bar{\mathbf{Q}}(\mathbf{q}) = \mathbf{U}_1^T\bar{\mathbf{Q}}(\mathbf{q})\mathbf{\Gamma}_+. \tag{31}$$

The equation (31) is linear with respect to the parameter vector $\mathbf{q}$. After several trivial manipulations, it can be transformed into the equation as below

$$\mathbf{F}\mathbf{q} = \mathbf{0} \tag{32}$$

where $\mathbf{F} \in \mathbb{R}^{(M+N)(M+N)\times NL}$ is a tall matrix. Since the equation (32) is equivalent to the equation (26) which can determine the MA parameters uniquely up to a scalar constant, it can be inferred that the tall matrix $\mathbf{F}$ has reduced column rank. Then, the normalized estimation of $\mathbf{q}$ can be find by solving

$$\min_{\|\mathbf{q}\|_2=1} \quad \|\mathbf{F}\mathbf{q}\|_2^2. \tag{33}$$

The identification method in this case is efficient and effective in the following aspects: (1) it solves the MA parameters directly, thus avoiding the circumstance that the same parameter have different estimated values in the Toeplitz matrix $\bar{\mathbf{Q}}$; (2) the computational complexity is much lower than that in **Case 1** since less variables are to be estimated. The trivial point of this method is to transform the equation (31) into the equation (32).

5 SISO System Identification

In view of the formulation of the coefficients of the multiple MA polynomials in (12), another compact form of it will be derived here. We define a composite polynomial as below:

$$\begin{aligned} \mathcal{C}(z^{-1}) &= \mathbf{1}(z^{-1})\bar{Q}(z^{-1}) \\ &= c_0 + c_1 z^{-1} + \cdots + c_{(N+1)L-1} z^{1-(N+1)L} \end{aligned} \tag{34}$$

where $\mathbf{1}(z) = 1 + z^{-1} + \cdots + z^{-(L-1)}$, the order of $\mathcal{C}(z^{-1})$ is $(N+1)L-1$, and its coefficients are determined by

$$c_k = \begin{cases} \sum_{i=0}^{k} \hat{q}_i & 0 \le k \le L-1 \\ \sum_{i=k-L+1}^{k} \hat{q}_i & L \le k \le NL-1 \\ \sum_{i=k-L+1}^{NL} \hat{q}_i & NL \le k \le (N+1)L-1 \end{cases}.$$

It is evident that the coefficients of the MA polynomials $\bar{Q}_i(z^{-1})$ can be represented by

$$\bar{q}_{i,k} = c_{kL+i} \quad i = 1, 2, \cdots, L \text{ and } k = 0, 1, \cdots, N.$$

Then the polynomial $\mathcal{C}(z^{-1})$ can be represented by the MA polynomials $\{\bar{Q}_i(z^{-1})\}_{i=1}^L$ as follows

$$\mathcal{C}(z^{-1}) = \mathbf{1}(z^{-1})\bar{Q}(z^{-1}) = \sum_{i=1}^{L} z^{-i+1}\bar{Q}_i(z^{-L}) \tag{35}$$

Since $\bar{Q}_i(z^{-1})$ can be estimated using the proposed algorithm in the Section 4, the composite MA polynomial $\mathbf{1}(z^{-1})\bar{Q}(z^{-1})$ can also be estimated according to (35). In order to get the estimate of $\bar{Q}(z^{-1})$, a deconvolution operation should be carried out to remove the term $\mathbf{1}(z^{-1})$. From the equation (9), it can be found that the irreducible SISO transfer function $\frac{Q(z^{-1})}{P(z^{-1})}$ can be obtained by taking common factor cancelation on the rational function $\frac{\bar{Q}(z^{-1})}{\bar{P}(z^{-L})}$. In the literature, there are many greatest common divisor extraction algorithms [9,10,13]. We adopt the subspace method [13] in numerical simulations since it is robust to noise interference.

6 Numerical Simulations

In this section, simulation results are presented to evaluate the performance of our proposed algorithm. To examine the effectiveness of the proposed algorithm on dealing with the noise effect, the following system model is concerned

$$y(t) = T(z^{-1})u(t) + w(t) \tag{36}$$

where $w(t) \sim \mathcal{N}(0, \sigma^2)$, and σ^2 is the variance. To identify the noisy ARMA system model, some extra preprocessing operations on noisy observations should be included (see [21]).

For the sake of quantitative evaluations, two measurement criteria are adopted. The signal-to-noise ratio (SNR), used to measure the noise level, is defined as

$$\mathrm{SNR} = 10 \log_{10} \frac{E[\|T(z^{-1})u(t)\|^2]}{E[\|w(t)\|^2]}.$$

The normalized mean-square errors (nMSE), used to measure the identification accuracy, are defined for the estimates of the MA and AR parameters, respectively, as follows

$$\mathrm{nMSE}_1 = \frac{1}{N_m} \sum_{i=1}^{N_m} \left\{ \min_{c_i} \frac{\|c_i \hat{Q}^i(z^{-1}) - Q(z^{-1})\|^2}{\|Q(z^{-1})\|^2} \right\},$$

$$\mathrm{nMSE}_2 = \frac{1}{N_m} \sum_{i=1}^{N_m} \frac{\|\hat{P}^i(z^{-1}) - P(z^{-1})\|^2}{\|P(z^{-1})\|^2},$$

where N_m is the number of Monte Carlo runs, $\hat{Q}^i(z^{-1})$ ($\hat{P}^i(z^{-1})$) is the i-th estimation of the transfer function $Q(z^{-1})$ ($P(z^{-1})$), c_i is a factor to minimize $\|c_i \hat{Q}^i(z^{-1}) - Q(z^{-1})\|^2$ such that the scalar ambiguity for the blind system identification can be eliminated.

In order to have a comprehensive assessment of the performance, both the identifications of the SIMO transfer functions and the original SISO transfer function are evaluated in terms of nMSE. For the SIMO system identification, the evaluations of the MA polynomials and the AR polynomial are separated. To demonstrate the robustness of the proposed algorithm, the simulations are carried out under different noise levels. The nMSE curves, computed from 5000 and 10000 output observations, are plotted individually, and each nMSE value is calculated by averaging 500 Monte Carlo trials.

First, two examples are demonstrated to show the performance of our method against additive noises: one system model is minimum-phase but not proper, and the other is non-minimum phase but proper. Then, in the third example, a modified identification method presented in [15] is simulated for comparison purposes.

Example 1: We consider a typical example of an SISO ARMA system simulated in [22] as follows

$$T(z^{-1}) = \frac{1.0000 - 1.4001z^{-1} + 0.9801z^{-2}}{1.0000 - 0.8000z^{-1} + 0.6500z^{-2}}.$$

It has poles at $0.4000 \pm j0.7000$, and zeros at $0.7000 \pm j0.7000$, where the prefix $j = \sqrt{-1}$ is the imaginary unit.

By setting the over-sampling rate to $L = 2$, the SIMO transfer functions can be computed using the state-space form described in Section 3, which are shown as follows:

$$\bar{T}_1(z^{-1}) = \frac{1.0000 + 1.4101z^{-1} + 0.3339z^{-2}}{1.0000 + 0.6600z^{-1} + 0.4225z^{-2}},$$
$$\bar{T}_2(z^{-1}) = \frac{1.6001 + 0.9360z^{-1} + 0.2079z^{-2}}{1.0000 + 0.6600z^{-1} + 0.4225z^{-2}}.$$

It can be easily verified that the above two transfer functions are coprime.

In the simulation, the system input is a sequence of a white Gaussian noise with each symbol lasting for two periods, and the output sequence is splitted into two sequences which corresponds to two output sequences of the SIMO system. Using the blind SIMO identification algorithm described in Section 4, the SIMO transfer functions $\bar{T}_1(z^{-1})$ and $\bar{T}_2(z^{-1})$ can be identified and their identification performances are shown on the left part of Fig. 1. Then, according to the SISO identification strategy stated in Section 5, the SISO transfer function $T(z^{-1})$ can be further identified, whose identification performance is shown on the right part of Fig. 1.

Fig. 1 shows that the identification performance improves little when the SNR is very high, but its estimation accuracy can still be improved by employing more output observations. Also, it can be found that the estimate of AR parameters is mainly affected by the factor of inadequate observation samples while the estimate of MA parameters is mainly affected by the additive noise. Note, from the derivation of the identification algorithm and the performance curves, we can find that both the AR and MA parameters can be accurately estimated with adequate measurement outputs in the absence of noise. However, in practical realizations, inadequate statistics may degrade the identification performance. Moreover, one can find that identification performance of the original SISO model is worse than that of the associated SIMO model since the SISO transfer function is estimated based on that of the SIMO model.

Example 2: We consider a continuous low-pass plant as follows:

$$\begin{aligned}
&\dot{x}(t) = \mathbf{A}_c x(t) + \mathbf{B}_c u(t)\\
&y(t) = \mathbf{C}_c x(t) + \mathbf{D}_c u(t)\\
&\mathbf{A_c} = \begin{bmatrix} -4 & -5 \\ 1 & 0 \end{bmatrix} \quad \mathbf{B_c} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}\\
&\mathbf{C_c} = \begin{bmatrix} 1 & -1 \end{bmatrix} \quad \mathbf{D_c} = 0.
\end{aligned}$$

In the above state-space realization, the time index t is continuous which is different from that in the model (1).

In order to obtain the corresponding discrete-time plant, the sampling period is set to $T = 0.3s$, which satisfies the Nyquist sampling theorem [17] where the system bandwidth is the frequency at half power point of its Bode plot. The discretized system can be obtained as follows:

$$\begin{aligned}
&\mathbf{A} = e^{\mathbf{A}_c T} = \begin{bmatrix} 0.1999 & -0.8109 \\ 0.1622 & 0.8487 \end{bmatrix},\\
&\mathbf{B} = \int_0^T e^{\mathbf{A}_c \tau} d\tau \mathbf{B}_c = \begin{bmatrix} 0.1622 \\ 0.03027 \end{bmatrix},\\
&\mathbf{C} = \mathbf{C}_c = \begin{bmatrix} 1.0000 - 1.0000 \end{bmatrix}, \quad \mathbf{D} = \mathbf{D}_c = 0.
\end{aligned} \tag{37}$$

The transfer function of the above discrete linear time-invariant system is

$$T(z^{-1}) = \frac{Q(z^{-1})}{P(z^{-1})} = \frac{0.1319z^{-1} - 0.1824z^{-2}}{1 - 1.049z^{-1} + 0.3012z^{-2}},$$

where it can be found that $T(z^{-1})$ is a non-minimum phase transfer function since one zero is outside the unit circle in the z-plane. By setting the over-sampling rate to $L = 2$, two ARMA transfer functions of the SIMO system are computed as follows:

$$\begin{aligned}
\bar{T}_1(z^{-1}) &= \frac{\bar{Q}_1(z^{-1})}{\bar{P}(z^{-1})} = \frac{0.0878z^{-1} - 0.2066z^{-2}}{1.0000 - 0.4972z^{-1} + 0.0907z^{-2}},\\
\bar{T}_2(z^{-1}) &= \frac{\bar{Q}_2(z^{-1})}{\bar{P}(z^{-1})} = \frac{0.1319 - 0.1957z^{-1} - 0.0550z^{-2}}{1.0000 - 0.4972z^{-1} + 0.0907z^{-2}}.
\end{aligned}$$

From Fig. 2, it can be found that the identification performance is quite similar to that of the previous example: the discrete-time SISO plant and its associated SIMO transfer functions can be accurately estimated when the noise level is low and the observation samples are adequate.

The proposed SIMO identification algorithm computes the AR parameters first followed by computing the MA parameters. Thus it seems that the AR parameter estimation performance can affect that of the MA parameters. In fact, the estimations of the MA parameters are fundamentally and uniquely determined by the autocorrelation matrices $\mathbf{R}_{\bar{\mathbf{y}}}(0)$ and $\mathbf{R}_{\bar{\mathbf{y}}}(1)$ up to a scalar constant. Thus the estimation outcomes of the MA parameters are little affected by the AR parameter estimates which can be considered as a set of intermediate data. This consideration has been verified by our numerous simulations.

Example 3: The considered SISO ARMA transfer function is

$$T(z^{-1}) = \frac{0.6000z^{-1} + 0.3300z^{-2}}{1.0000 - 1.7000z^{-1} + 0.7200z^{-2}}. \tag{38}$$

Here, we consider three representative identification methods for SISO IIR systems. In [1, 2], the input to the SIMO system is a discrete pulse sequence, which is a bit different from piece-wise constant input samples in our proposed method. In addition, it does not explicitly deal with noisy cases. In [8, 15], the input to the SIMO system is also a discrete pulse sequence and the variance value of the system input should be known as prior knowledge. In [14, 18, 19], the system model is different from that in (36). However, the proposed method can be applied to the model in (36) by some modifications. The AR parameters of the IIR transfer function are identified by the Instrumental-Variable method [11, Section 7.6], and the MA parameters are identified using the Subspace method [12] by considering $\frac{u(n)}{P(z^{-1})}$ as a common source of an SIMO FIR system. We call this method as IV-S method in the simulation below. For fair performance evaluation and comparison, only the IV-S method is simulated.

To satisfy the identifiability condition of both the IV-S method and our method, the sampling rate is set to $L = 3$ and the system input of the SISO system is generated as a white Gaussian noise. From the left part of the Fig. 3, it can be found that our proposed method can provide superior performance in identifying the AR parameters of the SISO transfer function. In the Instrumental-Variable method, high lagged second-order statistics of the output observations are required in its identification procedure. On the contrary, low lagged statistics are used in our method, which results in higher estimation accuracy. Since the identification procedure in the

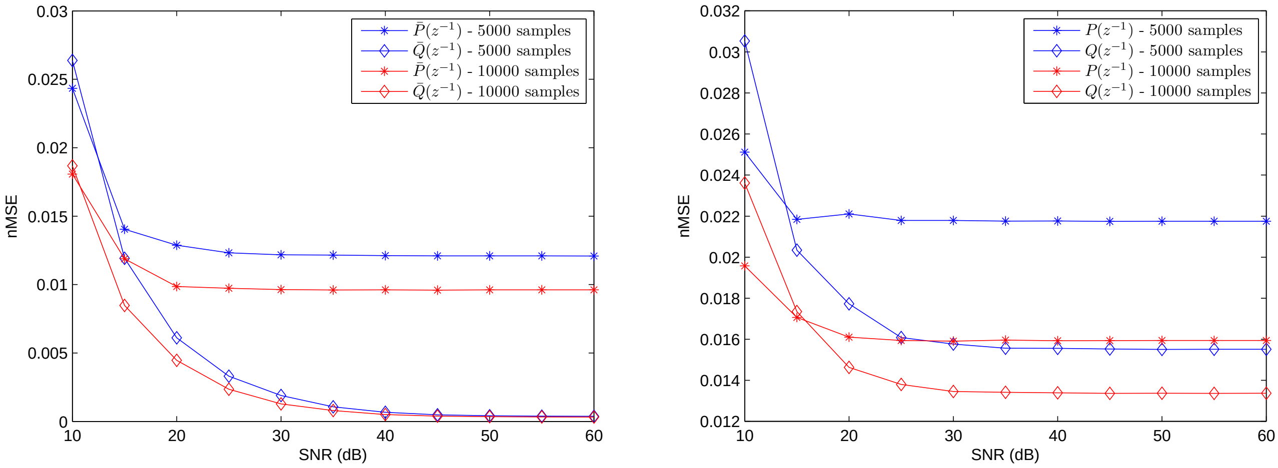

Fig. 1. Left: Performance of the SIMO system identification; Right: Performance of the SISO system identification. Blue curves are computed from 5000 output samples and red curves are computed from 10000 output samples. Star-solid curves denote the identification performances of denominator polynomials (Left: $\bar{P}(z^{-1})$; Right: $P(z^{-1})$) and diamond-solid curves indicate the identification performances of numerator polynomials (Left: $\bar{Q}(z^{-1})$; Right: $Q(z^{-1})$).

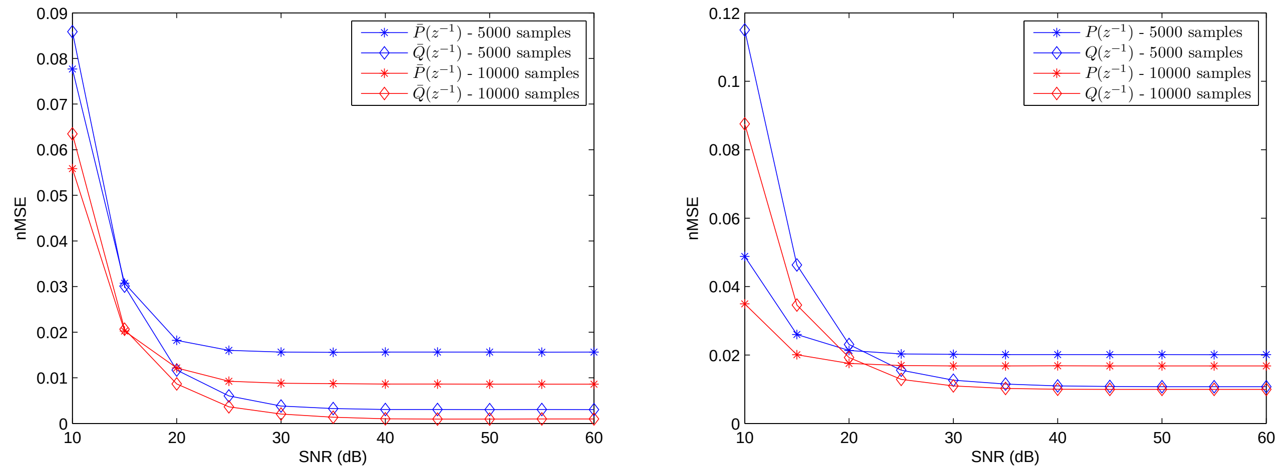

Fig. 2. Left: Performance of the SIMO system identification; Right: Performance of the SISO system identification. Blue curves are computed from 5000 output samples and red curves are computed from 10000 output samples. Star-solid curves denote the identification performances of denominator polynomials (Left: $\bar{P}(z^{-1})$; Right: $P(z^{-1})$) and diamond-solid curves indicate the identification performances of numerator polynomials (Left: $\bar{Q}(z^{-1})$; Right: $Q(z^{-1})$).

Instrumental-Variable method does not involve the variance of the additive noise, thus the nMSE curves are insensitive to the noise level, i.e. the nMSE curves are quite flat. From the right part of the Fig. 3, we can observe that the IV-S method and our method have similar performances on identifying the MA part of the SISO system at high SNRs. The Subspace method [12] does not involve the noise variance as well, thus its performance is less sensitive to noise than our method. Since our method has to remove the noise affect as a preprocessing operation, its identification performance is sensitive to the SNR criteria. This explains that the IV-S method performs better than our method on identifying the MA part of the SISO system at low SNRs.

7 Conclusion

In this paper, a second-order statistics based blind identification algorithm of non-minimum phase SISO ARMA systems has been presented using the input-holding output over-sampling technique. This method estimates the numerator polynomials and the denominator polynomial of the associated SIMO system in a more systematic way: (1) identify the denominator polynomial by exploiting the dynamical regressive information involved

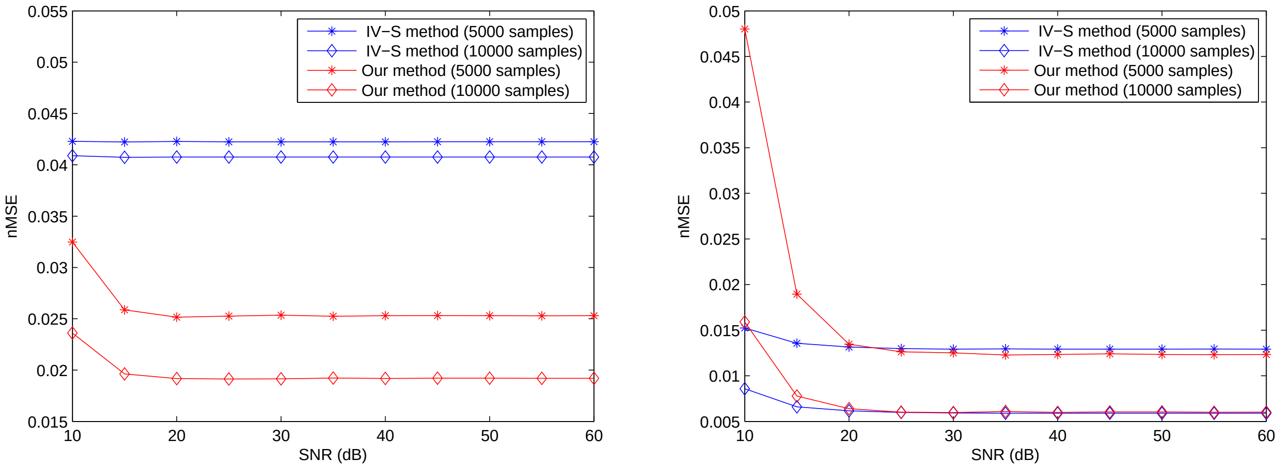

Fig. 3. Left: Identification performances of the AR part of the SISO transfer function; Right: Identification performances of the MA part of the SISO transfer function. Blue curves are estimated using the IV-S method and the red curves are estimated using our method. The star-solid curves are computed from 5000 output samples and the diamond-solid curves are computed from 10000 output samples.

in the lag-0 and lag-1 autocorrelation matrices of the system outputs; (2) identify the numerator polynomials by combining the block Toeplitz property of the convolution matrix of the SIMO model and the inner structures of autocorrelation matrices. A sufficient and necessary condition of coprime SIMO transfer functions has been provided. The effectiveness of the proposed SOS based identification algorithm has been demonstrated in numerical simulations.

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