Secure Continuous Variable Teleportation and Einstein-Podolsky-Rosen Steering

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We investigate the resources needed for secure teleportation of coherent states. We extend continuous variable teleportation to include quantum teleamplification protocols that allow nonunity classical gains and a preamplification or postattenuation of the coherent state. We show that, for arbitrary Gaussian protocols and a significant class of Gaussian resources, two-way steering is required to achieve a teleportation fidelity beyond the no-cloning threshold. This provides an operational connection between Gaussian steerability and secure teleportation. We present practical recipes suggesting that heralded noiseless preamplification may enable high-fidelity heralded teleportation, using minimally entangled yet steerable resources.

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Quantum teleportation (QT) is a process where Alice sends an unknown quantum state to Bob at a different location by communicating only classical information and exploiting a shared Einstein-Podolsky-Rosen (EPR) entangled state [1]. QT has inspired much interest, both as a fundamental quantum information primitive and as an important ingredient for practical quantum technologies [2–21]. In particular, high-fidelity QT is an essential building block for the *quantum internet* [22], a distributed quantum communication network relying on QT-based quantum repeaters to enhance the distance over which quantum signals can be securely transmitted [21,23]. Huge international investments are being devoted towards realizing such a vision; see, e.g., [24,25]. Quantum gate teleportation plays also a central role for universal quantum computation [26]. It is therefore very timely to identify optimal resources and feasible recipes for efficient QT.

QT was first developed for the transfer of qubit states [1] and was extended to continuous variable (CV) spectra by Vaidman [4] and Braunstein and Kimble (BK) [5]. In the CV case, the entanglement shared between Alice and Bob is modeled after the original EPR paradox, where they share systems with perfectly correlated positions and anticorrelated momenta [27–30]. Gaussian states (defined as having a Gaussian Wigner function) [31] can then be useful as approximations of EPR resources [6,7]. Unconditional CV teleportation based on Gaussian states has been demonstrated in a variety of light- and matter-based setups [6–8,10–13]. Recent breakthroughs in CV teleportation include QT of coherent states with fidelity up to 0.83 [11], deterministic QT between macroscopic atomic ensembles separated by 0.5 m [13], and a fully integrated CV teleportation device on a photonic chip [15]. For reference, discrete variable teleportation of single-qubit states has been implemented probabilistically over a distance up to 143 km with fidelity up to 0.86 for photonic qubits [16,17] and deterministically over a distance of 3 m with fidelity up to 0.77 for solid state qubits [18]. Improving the realization of CV teleportation schemes would clearly be very beneficial.

One may ask: What type of EPR entanglement is required for CV teleportation [32]? CV teleportation of a coherent state originally focused on a subset of entangled resource states, where the entanglement can be certified by the Tan-Duan criterion which treats Alice and Bob symmetrically [33–35]:

$$\Delta_{\text{ent}} = \frac{1}{4}\{[\Delta(X_A - X_B)]^2 + [\Delta(P_A + P_B)]^2\} < 1. \quad (1)$$

Here X_A, P_A and X_B, P_B are the quadrature phase amplitudes, respectively, of Alice and Bob's systems, and $(\Delta X)^2$ denotes the variance of X [36]. Allowing for local operations at Alice and Bob's stations to optimize the protocol, one finds that all two-mode Gaussian entangled states are useful for CV QT [37,38] with fidelity exceeding the benchmark for input coherent states, $F > 1/2$ [39].

These results, however, do not resolve a second fundamental question, first posed by Grosshans and Grangier (GG) [40] (see [41] for some more recent progress): What type of entangled state is required to guarantee that no (nondegraded) copy of the transmitted state is obtained by an unwanted second receiver, Eve? This form of entanglement becomes the vital resource for *secure* teleportation (ST), which is a prerequisite for the quantum internet [21,22]. An analysis based on quantum cloning reveals that, for coherent inputs, ST is ensured once the teleportation fidelity F exceeds $2/3$ [40,42].

Here we solve such a long-standing question by proving that secure CV teleportation requires a stronger form of entanglement exhibiting *EPR steering* [43–45]. EPR steering refers to the correlations of the original 1935 EPR paradox [27], where one observer appears to adjust ("steer") the state of the other by local measurements. A useful criterion to certify the EPR paradox and a steering of B by A in a bipartite state is [29,46]

$$E_{B|A}(\mathbf{g}) = \Delta(X_B - g_x X_A)\Delta(P_B + g_p P_A) < 1. \qquad (2)$$

Here $\mathbf{g} = (g_x, g_p)$, where g_x and g_p are real constants, usually chosen so that $E_{B|A}(\mathbf{g})$ takes the minimum possible value, denoted $E_{B|A}$. Then, B is steerable by A if $E_{B|A} < 1$. This condition is necessary and sufficient for two-mode Gaussian states [43,47], and a measure of Gaussian steering can be defined as a decreasing function of $E_{B|A}$ [48]. For steering, as in the original EPR paradox, Alice and Bob are *not* equivalent: A system can be steerable in one way but not the other [48–52].

Motivated by the recent progress in EPR steering characterization and quantification [43–55], we revisit CV teleportation by considering asymmetric Gaussian resources and the whole class of protocols including those allowing any classical gain and local preamplification or postattenuation of coherent states. We then prove the main claim: Any Gaussian resource, identified by its degree of entanglement to be useful for high-fidelity ST ($F > 2/3$) via an optimal protocol, is necessarily two-way steerable. For certain protocols and resource states, we show that this condition becomes also sufficient.

We clarify the trade-off between required entanglement and achievable ST fidelity. While two-way steerability requires exceeding a threshold in entanglement, the latter becomes low for states of sufficient purity [48]. We provide explicit proposals to use such states, combined with a heralded noiseless preamplification of the coherent state [56], for realizing high-fidelity ST. Our work contributes then to the practical problem of how to enhance CV teleportation fidelity without increasing the entanglement (and energy) of the EPR resource [57].

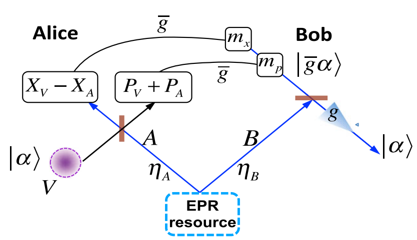

FIG. 1 (color online). Quantum teleamplification. A coherent state $|\alpha\rangle$ is teleported from Alice to Bob by using EPR entanglement. The fidelity is optimized for a given resource by adjusting the classical gain $\bar{g}$. To teleport the original state, Bob may postattenuate the state by using $g = 1/\bar{g}$ or Alice may preamplify the coherent state.

Quantum teleamplification.—We begin by considering the generalization of the BK protocol [5], which incorporates arbitrary classical gains (Fig. 1). As with conventional CV teleportation, Alice and Bob share an EPR entangled state, often modeled by the two-mode squeezed state (TMSS) $|\psi\rangle = (1-x^2)^{1/2}\sum_{n=0}^{\infty} x^n |n\rangle_A |n\rangle_B$ [28,29,36]. Here $x = \tanh(r)$, where r is the squeezing parameter that determines the amount of entanglement shared between Alice and Bob; the limit of maximal EPR entanglement is reached as $r \to \infty$. Current experiments have realized $r \approx 1.15$ (i.e., 10 dB of squeezing) for TMSS generated by mixing two individual squeezed beams at a balanced beam splitter [58] and $r \approx 0.97$ (i.e., 8.4 dB of squeezing) for TMSS generated by a single optical parametric amplifier [59]. Realistic conditions (e.g., losses) mean that the shared EPR resource is best described as a generally mixed two-mode Gaussian state [31,35,37,38,46]. A field V (with amplitudes X_V and P_V) is prepared by a third party, Victor, in the coherent state $|\psi_{\rm in}\rangle = |\alpha\rangle$ to be teleported to Bob. Alice performs a local Bell measurement of the combined quadratures $X_V - X_A$ and $P_V + P_A$, getting outcomes m_x and m_p, respectively. Finally, Bob displaces the amplitudes of his EPR field by an amount given by Alice's readout values m_x and m_p that are transmitted to him via classical communication.

While the BK protocol takes $\bar{g} = 1$, we allow for nonunity classical gain factors $\bar{g}_x$ and $\bar{g}_p$ in the two classical channels. For simplicity, we consider equal gains: $\bar{g}_x = \bar{g}_p = \bar{g}$. This means that Bob's displacement is amplified or deamplified to $\bar{g}m_x$ and $\bar{g}m_p$. After feedback, Bob's field amplitudes are given by $X_B^f = \bar{g}X_V + (X_B - \bar{g}X_A)$ and $P_B^f = \bar{g}P_V + (P_B + \bar{g}P_A)$. We first compute the fidelity for the protocol $|\alpha\rangle \to |\beta_{\rm tele}\rangle = |\bar{g}\alpha\rangle$, called "quantum teleamplification" when $\bar{g} > 1$ [49,60]. The fidelity, defined as $F = \langle\beta_{\rm tele}|\rho_{\rm out}|\beta_{\rm tele}\rangle$, where $\rho_{\rm out}$ is Bob's output state [5,8,61], is $F = (2/\sigma_Q)\exp[-(2/\sigma_Q)|\beta_{\rm out} - \beta_{\rm tele}|^2]$, where $\sigma_Q = \sqrt{(1+\sigma_X)(1+\sigma_P)}$ and $\beta_{\rm out} = x_m + ip_m$. Here x_m, p_m and σ_X, σ_P are means and variances, respectively, of the quadratures X_B^f, P_B^f of Bob's output field. We find $\sigma_X = \bar{g}^2\sigma_{X,{\rm in}} + [\Delta(X_B - \bar{g}X_A)]^2$ and $\sigma_P = \bar{g}^2\sigma_{P,{\rm in}} + [\Delta(P_B + \bar{g}P_A)]^2$, where $\sigma_{X,{\rm in}}$ and $\sigma_{P,{\rm in}}$ are variances of the input state $|\alpha\rangle$. We then get ($\beta_{\rm out} = \bar{g}\alpha$)

$$F = 2/\sigma_Q = 2/[1 + \bar{g}^2 + E_{B|A}(\bar{g})], \qquad (3)$$

where $E_{B|A}(\bar{g})$ is the steering parameter of Eq. (2) with $\mathbf{g} = (\bar{g}, \bar{g})$. Note how the fidelity is sensitive to the steering parameter. In the special BK case ($\bar{g} = 1$), this reduces to $F^{\rm BK} = (1 + \Delta_{\rm ent})^{-1}$, where $\Delta_{\rm ent}$ is the entanglement parameter of Eq. (1). We have restricted to two-mode Gaussian states with equal position and momentum correlations [8], i.e., $|\langle X_A, X_B\rangle| = |\langle P_A, P_B\rangle|$, $\sigma_X = \sigma_P$, and $\Delta(X_B - \bar{g}X_A) = \Delta(P_B + \bar{g}P_A)$. This subclass, that we call

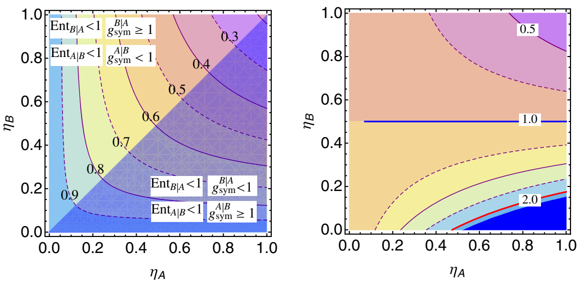

FIG. 2 (color online). Entanglement and steering of lossy two-mode squeezed states. Here $r = 0.85$ and $\eta_{A/B}$ are the channel efficiencies. Left: Contour plot of the entanglement parameter Ent. Higher loss on Alice's channel ($\eta_A < \eta_B$) implies $g^{B|A}_{\rm sym} > 1$ (note that $g^{A|B}_{\rm sym} = 1/g^{B|A}_{\rm sym}$ and $\eta_A = \eta_B$ gives $g^{B|A}_{\rm sym} = 1$). Right: Contour plot of the minimum value $E_{A|B}$ of $E_{A|B}(g)$, found by selecting $g = c/m$. EPR steering of Alice by Bob ($E_{A|B} < 1$) is possible only when $\eta_B > 1/2$.

$(X - P)$-balanced, includes EPR resources such as the TMSS with phase-insensitive losses and noise.

Asymmetry and entanglement.—It is known that Gaussian steerable states not satisfying the Tan-Duan condition $\Delta_{\rm ent} < 1$ exist and can be created, e.g., from a TMSS by adding asymmetric losses or thermal noise to each of the EPR channels (Fig. 2) [48,49]. These asymmetric steerable states are *not* useful for standard BK teleportation, which requires a fidelity of $F > 1/2$ and hence a resource with $\Delta_{\rm ent} < 1$ [8].

The first point we make is that the generalization to nonunity classical gains allows all the $(X - P)$-balanced Gaussian entangled states to be useful for QT [49]. We clarify as follows: The threshold fidelity where one can rule out all classical measure-and-prepare strategies as in Ref. [39] and hence claim QT is $F > (1 + \bar{g}^2)^{-1}$ (for $\bar{g} \geq 1$) [39,62]. On examining (3), the condition on the resource to obtain QT reduces to

$$\mathrm{Ent}_{B|A}(\bar{g}) = [\Delta(X_B - \bar{g}X_A)\Delta(P_B + \bar{g}P_A)]/(1 + \bar{g}^2) < 1. \tag{4}$$

The inequality $\mathrm{Ent}_{B|A}(\bar{g}) < 1$, if satisfied, certifies entanglement between A and B ($\bar{g}$ is any real number) [63]. Defining the minimum $\mathrm{Ent} = \min_{\bar{g}} \mathrm{Ent}_{B|A}(\bar{g}) \equiv \mathrm{Ent}_{B|A}(g^{B|A}_{\rm sym})$ for some optimal gain $g^{B|A}_{\rm sym}$, the inequality $\mathrm{Ent} < 1$ is equivalent to Simon's necessary and sufficient condition for entanglement of Gaussian states [64]. Precisely, Ent coincides with the lowest symplectic eigenvalue of the partial transpose ν, which determines the logarithmic negativity, an entanglement monotone [37,38,48,65]. Maximum entanglement corresponds to $\mathrm{Ent} \to 0$. By analyzing the result Eq. (4), it is clear that, for any Gaussian entangled resource [within the $(X - P)$-balanced class] with $g^{B|A}_{\rm sym} \geq 1$, we can realize QT (from Alice to Bob) by using the classical gain set at $\bar{g} = g^{B|A}_{\rm sym}$ [49]. When $g^{B|A}_{\rm sym} < 1$, QT is obtained by switching the EPR channels A and B.

The value $g^{B|A}_{\rm sym}$ quantifies the asymmetry of the resource and is calculated as $g^{B|A}_{\rm sym} = x + \sqrt{x^2 + 1}$, where $x = (m - n)/2c$, $n = \langle X_A, X_A \rangle$, $m = \langle X_B, X_B \rangle$, and $c = \langle X_A, X_B \rangle = -\langle P_A, P_B \rangle$. The coefficients n, m, and c fully define the covariance matrix of Gaussian states in the $(X - P)$-balanced class. For a TMSS with losses at each channel A and B, so that η_A and η_B are the respective efficiencies, the covariances become $n = \eta_A \cosh(2r) + 1 - \eta_A$, $m = \eta_B \cosh(2r) + 1 - \eta_B$, and $c = \sqrt{\eta_A \eta_B} \sinh(2r)$. The entanglement and steering parameters for this resource are given in Fig. 2. We next present a useful result that holds for all Gaussian or non-Gaussian states with a covariance matrix of the $(X - P)$-balanced form.

Result (1).—The amount of EPR entanglement is limited by the asymmetry parameter, as $\nu \equiv \mathrm{Ent} \geq (g^2_{\rm sym} - 1)/(g^2_{\rm sym} + 1)$, where $g_{\rm sym} = \max\{g^{A|B}_{\rm sym}, g^{B|A}_{\rm sym}\}$. Hence, two-way steering ($\{E_{A|B}, E_{B|A}\} < 1$) is certified if $\mathrm{Ent} < 1/(1 + g^2_{\rm sym})$. Two-way steerable states are constrained to be reasonably symmetric, so that $g_{\rm sym} < \sqrt{2}$, and thus two-way steering is certified if

$$\nu \equiv \mathrm{Ent} < 1/3. \tag{5}$$

The condition is tight for $(X - P)$-balanced Gaussian states as shown in Fig. 3(a), and it agrees with that derived in Ref. [48] for arbitrary two-mode Gaussian states. The proofs are in Ref. [66].

Steering and secure teleportation.—We can now address the main question, namely, what the requirement is on the

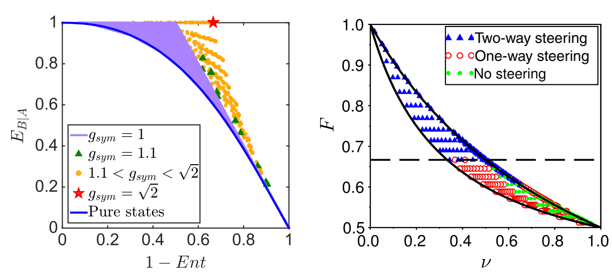

FIG. 3 (color online). Left: A two-mode Gaussian state is steerable (B by A) when $E_{B|A} < 1$ and entangled when $\mathrm{Ent} < 1$. Here we use $g^{B|A}_{\rm sym} > 1$ for which $E_{A|B} \leq E_{B|A}$. The purple continuous region is for symmetric states, $g_{\rm sym} = 1$, for which $\mathrm{Ent} < 1/2$ implies two-way steerability. The orange points are for asymmetric states, $g_{\rm sym} > 1$. The condition $\mathrm{Ent} < 1/3$ implies two-way steerability for all $(X - P)$-balanced states. Right: Optimal teleportation fidelity (using the BK, lsatt, or esa protocols, defined in the text) versus the parameter $\nu = \mathrm{Ent}$ of the resource. Black solid lines denote the MV bounds. Steering properties of lossy TMSS resources are plotted for all η_A, η_B and $r \geq 0.5$. Two-way steering is required for secure teleportation of coherent states, marked by $F > 2/3$.

resource to achieve no-cloning ST. Restricting first to lossy TMSS states, we see that a resource steerable A by B is required for ST. If there is no steering of A by B, the covariances imply that $\eta_B \leq 1/2$ (Fig. 2). Such a field B can be generated by using a 50-50 beam splitter that produces a second field B' satisfying $\langle X_A, X_{B'}\rangle = \langle X_A, X_B\rangle$, $\langle P_A, P_{B'}\rangle = \langle P_A, P_B\rangle$, etc. This implies that an observer Eve with access to B' can generate from the classical information (which is publicly accessible) the same teleported state as can Bob, who has access only to field B (see Fig. 1). This argument, while restricted to the lossy TMSS resource, is nonetheless quite powerful, applying to protocols with arbitrary $\bar{g}$ and local operations at Bob's station, and (similar to [55]) is not based on fidelity.

We now focus on the important case of conventional teleportation of the coherent state $|\alpha\rangle \to |\alpha\rangle$. Following GG [40], we consider *high-fidelity* ST, where the no-cloning threshold for security is established by $F > 2/3$. GG observed that, for the BK protocol, $F > 2/3$ requires a resource with $\Delta_{\mathrm{ent}} < 1/2$. We now know this implies two-way steering [49]. Furthermore, for symmetric resources, $g_{\mathrm{sym}} = 1$ and $\mathrm{Ent} \equiv \Delta_{\mathrm{ent}}$, and we see from result (1) and Fig. 3(a) that the GG condition $\Delta_{\mathrm{ent}} < 1/2$ is the *tight* condition on the entanglement parameter for two-way steerability. This motivates us to generalize the GG result, to include asymmetric Gaussian resources and protocols.

To teleport an unknown coherent state with optimal fidelity for a given resource, local operations are needed at Alice and Bob's stations (Fig. 1). Optimizing over all local Gaussian channels, Mari and Vitali (MV) [38] derived fidelity bounds for a given entanglement value $\nu \equiv \mathrm{Ent}$, expressed by

$$(1+\nu)/(1+3\nu) \leq F \leq 1/(1+\nu). \qquad (6)$$

The next result tells us that, once one allows asymmetric Gaussian protocols, the set of resources enabling ST is expanded on those satisfying the GG condition $\Delta_{\mathrm{ent}} < 1/2$.

Result (2).—All Gaussian entangled resources satisfying $\nu \equiv \mathrm{Ent} < 1/3$ are useful for ST of a coherent state. This entanglement threshold is the same tight entanglement threshold to certify two-way steering, Eq. (5); see Fig. 3(b). For symmetric resources (where $g_{\mathrm{sym}} = 1$) the condition weakens, and all entangled resources with $\mathrm{Ent} < 1/2$ are useful for ST.

Proof.—The MV bounds imply that, for Gaussian states with $\nu \equiv \mathrm{Ent} < 1/3$, an optimal protocol exists that achieves $F > 2/3$. The rest follows from result (1) and the GG condition.

We remark that two-way steerable Gaussian entangled states exist that satisfy $\nu \equiv \mathrm{Ent} < 1/3$ but do not satisfy the GG condition $\Delta_{\mathrm{ent}} < 1/2$. For these states, the optimal protocol is not the BK one. It is hence useful to understand how the optimal protocols can be carried out in practice (Fig. 4). We present two simple protocols that are readily achievable experimentally and that together with the BK

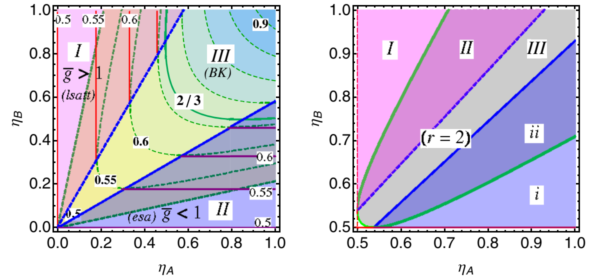

FIG. 4 (color online). Optimizing the teleportation fidelity using a lossy resource. Left: ($r = 1.0$). Contours show optimized fidelity values: We optimize via the lsatt protocol (region I), via the esa (region II), or via the BK protocol (central coned region III). For higher $r > 0.89$, ST ($F > 2/3$) is possible by using the asymmetric protocol. Right: ST can be achieved via the lsatt protocol (area I + II) or via the esa (area i + ii). The green curve corresponds to $\Delta_{\mathrm{ent}} < 1/2$ so that regions II, III, and ii give ST using the BK protocol. Note that for area III, ST can be achieved only by the BK. Regions I and i require asymmetric protocols for ST.

protocol allow, for any lossy TMSS resource, CV teleportation with a fidelity spanning the whole range within the MV bounds, Eq. (6). In fact, our study shows that, for any such entangled Gaussian resource with $\mathrm{Ent} < 1/3$, high-fidelity ST can be carried out by using one of the three protocols that we call lsatt, esa, or BK.

The simplest protocol is *late-stage attenuation (lsatt)*: To teleport the original state $|\alpha\rangle \to |\alpha\rangle$ when $\bar{g} > 1$, Bob locally attenuates his output field (Fig. 1). Bob may attenuate by using a beam splitter which yields a new output variance of $\sigma_X = \eta\sigma_X^T + 1 - \eta$, where $\eta = 1/\bar{g}^2 = g^2 < 1$ and σ_X^T is the variance for the original output. Then, from Eq. (3), the overall fidelity is $F_{A,B}^{\mathrm{lsatt}}(g) = 2/[3 - g^2 + E_{A|B}(g)]$ (using $\beta_{\mathrm{tele}} = \alpha$). Standard QT with $F > 1/2$ requires a resource satisfying Eq. (4), as for quantum tele-amplification. Setting the classical gain $\bar{g}_{\mathrm{opt}} = (m-1)/c$, the overall maximized fidelity becomes $F_{A,B}^{\mathrm{lsatt}} = 2/[3 + n - c^2/(m-1)]$, provided $\bar{g}_{\mathrm{opt}} > 1$ (requires $m > c + 1$) (see Fig. 4).

Alternatively, Alice may choose to amplify the input coherent state at her station by a factor of $g > 1$, prior to a teleportation protocol that uses a classical attenuation factor of $\bar{g} = 1/g < 1$. We call this *early-stage amplification (esa)*. Suppose Alice uses a TMSS amplifier [56]. Then the final amplified state at her station is Gaussian with mean $g\alpha$ and variance $\sigma_{X/P,\mathrm{in}} = 2g^2 - 1$. The final Gaussian output after teleportation to Bob has variance $\sigma_X = \bar{g}^2\sigma_{X,\mathrm{in}} + E_{B|A}(\bar{g})$. Substitution into Eq. (3) reveals the fidelity for the overall teleportation process to be $F_{A,B}^{\mathrm{esa}}(\bar{g}) = F_{B,A}^{\mathrm{lsatt}}(\bar{g})$. QT requires a resource satisfying Eq. (4). Hence, for an entangled Gaussian resource [in the $(X-P)$-balanced class] with $g_{\mathrm{sym}}^{B|A} < 1$, the esa protocol with $\bar{g} = g_{\mathrm{sym}}^{B|A}$ will give QT with $F > 1/2$. The overall fidelity $F_{A,B}^{\mathrm{esa}}(\bar{g})$ is maximized for $\bar{g}_{\mathrm{opt}} = c/(n-1)$ provided $\bar{g}_{\mathrm{opt}} < 1$ (requires $n > c + 1$), then yielding $F_{A,B}^{\mathrm{esa}} = F_{B,A}^{\mathrm{lsatt}}$.

We finally give the connection with two-way steering.

Result (3).—From the MV bounds, in order to achieve $F > 2/3$, the entanglement of the resource must satisfy $\text{Ent} < 1/2$. Not all resources with $\text{Ent} < 1/2$ will allow $F > 2/3$. Restricting to the three protocols (lsatt, esa, and BK), the requirement on the resource to achieve the ST fidelity $F > 2/3$ is exactly for *two-way steering* [see Fig. 3(b)]. The result is proved in Ref. [66].

Discussion.—We conclude by suggesting a further application of EPR steering to enhance the fidelity. The esa protocol relies on preamplification of a coherent state by a factor of $g > 1$, which has a limited maximum fidelity of $1/g^2$. Recent methods propose heralding to overcome this limitation: Heralded noiseless amplification of $|\alpha\rangle$ to $|g\alpha\rangle$ potentially allows fidelities approaching 1 [56]. The teleportation deamplification $|g\alpha\rangle \rightarrow |\alpha\rangle$ has fidelity given by Eq. (3) with $\bar{g} = 1/g < 1$. We show in Ref. [66] that, for the TMSS resource with the optimal choice of $\bar{g}$ [$\bar{g}_{\text{opt}} = \tanh(r)$], we get $F \rightarrow 1$. This holds for all $r > 0$; hence, it does not require significant shared entanglement. The overall fidelity becomes limited by the fidelity of the heralded amplification at Alice's site, suggesting a promising experimental route for achieving high CV teleportation fidelities. Note, however, that there remains the need for EPR steerability of the resource, which for low entanglement requires states of sufficient purity [48]. Last, we remark that two-way steerable states have been realized experimentally [46,67] and used for secure CV teleportation [10].

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