The effect of surface skewness on the super/postcritical coefficient of drag of roughened cylinders

Franz Konstantin Fuss*a*

aSchool of Aerospace, Mechanical and Manufacturing Engineering, RMIT University, Melbourne VIC 3083, Australia

Received 22 March 2011; revised 28 April 2011; accepted 1 May 2011

Abstract

Surface skewness ($R_{sk}$) is a further roughness parameter in addition to maximal ($R_z$) and average ($R_a$) roughness. So far, the aerodynamics and flow transition of spherical and cylindrical bodies were tested at different $R_z$ and constant $R_{sk}$ or variable $R_z$ and variable $R_{sk}$. The purpose of this study is to test surfaces of different $R_{sk}$ and same $R_z$. Eight surfaces of constant relative roughness of 0.9% and variable $R_{sk}$ from –5 to +5 including infinite skew (smooth surface) were rapid prototyped and wrapped around a cylinder. The drag force of the cylinder was recorded in a wind tunnel at speeds from 20-140 kph at 10 kph increments. The larger $R_{sk}$, the “rougher” is the surface profile, i.e. the larger is the postcritical coefficient of drag, and the smaller is the critical Reynolds number.

© 2011 Published by Elsevier Ltd.

Keywords: Surface roughness; skewness; roughness skew; aerodynamics; flow regimes; flow transition; coefficient of drag

1. Introduction

The rougher the surface of a spherical [1] or cylindrical object [2,3,4,5] is, the smaller is the critical Reynolds number, $Re_{crit}$, and the larger is the minimal coefficient of drag, $C_{D_{min}}$. The degree of roughness is usually expressed by the relative roughness parameter, $\varepsilon$.

$$\varepsilon = \frac{k}{D}$$  \hspace{1cm} (I)

* Corresponding author. Tel.: +61 3 9925 6123; fax: +61 3 9925 6108.

E-mail address: franz.fuss@rmit.edu.au.
Where \( k \) is the roughness height and \( D \) is the diameter of the test sphere or cylinder. As the test bodies are usually covered with elements of approximately the same roughness (grain size, diameter of glass beads), the roughness height \( k \) corresponds to \( R_z \), the maximal height of surface profile. In addition to \( R_z \), the roughness of a surface profile is characterized by \( Ra \) (arithmetic mean), \( Rq \) (root mean square), \( Rsk \) (skewness), \( Rku \) (kurtosis), and \( Rd \) (fractal dimension).

The skewness parameter \( Rsk \), defined as

\[
Rsk = \frac{n}{(n-1)(n-2)}Rq^3 \sum_{i=1}^{n} z_i^3
\]

(2)

Where \( z_i \) is the height of the \( i \)th data point of the profile, \( n \) is the number of points measured, and \( Rq \), the root mean square, is

\[
Rq = \sqrt{\frac{\sum_{i=1}^{n} z_i^2}{n}}
\]

(3)

Haake [6] performed a theoretically analysis of literature data (comparison of golf and soccer ball surface roughness, taken from various literature sources, and Achenbach’s data [1]) and concluded that the influence of \( Rsk \) on \( Re_{crit} \) and \( C_{D_{min}} \) is in essence the same as for \( \varepsilon \): the higher \( Rsk \), the smaller is \( Re_{crit} \), and the larger is \( C_{D_{min}} \). Barber and Carré [7] tested various standard and modified dimpled hockey balls in a wind tunnel and compared their \( Rsk \) to golf ball data taken from the literature and to Achenbach’s data [1]. Their conclusion matches the one of Haake [6]. Both studies, [6] and [7], involved both different \( \varepsilon \) and different \( Rsk \): hockey balls: \( Rsk \) from –1.8 to +0.6, \( \varepsilon \) of 0.17%-2.29%; golf balls: \( Rsk \) from –0.7 to +0.54, \( \varepsilon \) of 0.69%-1.52%; soccer ball: \( Rsk \) of –2.5, \( \varepsilon \) of 3.28%; Achenbach data [1]: \( Rsk \) of –0.49, \( \varepsilon \) of 1.25%

The purpose of this study was to investigate the effect of surface skewness on the drag coefficient at constant relative roughness \( \varepsilon \), in order to isolate the specific skewness effect.

2. Method

2.1 Surface design and manufacturing

Eight surfaces were designed in SolidWorks® 2008 (Dassault Systèmes SolidWorks Corp., Concord MA, USA), with a profile of the following \( Rsk \) (Figure 1):
Roughness elements for positive skew were designed as equilateral triangles of a height of 2 mm, sitting on a supporting base layer of 1.5 mm thickness. The base \( b \) of the triangles was 2.31 mm (4 * tan 30). The width of the floor of the valleys was 0\( b \) for \( Rsk = 0 \), 1\( b \) for \( Rsk = +1 \), 4\( b \) for \( Rsk = +2 \), and 16\( b \) for \( Rsk = +5 \). For negative skew, the surfaces with positive \( Rsk \) were inverted (Figure 1).

The surface cross sections were extruded in z-direction such that the roughness elements were converted to ridges. Per surface two test sheets (345x325 mm) were rapid-prototyped from TangoPlus FullCure®930 resin (TP930 by Proto3000 Inc., Vaughan, Ontario, Canada). TP930 is a flexible polymeric rapid prototyping material with a Shore A Hardness of 27.

![Surface profiles and skewness numbers](image)

2.2 Wind tunnel testing and data analysis

The experiments were conducted at the large industrial wind tunnel of RMIT University. The test rig (Figure 2) designed for wind tunnel experiments consisted of a horizontal cylinder located in the centre of the tunnel’s cross section and mounted on a 6DOF force balance (Type 9260AA6, Kistler, Winterthur, Switzerland). The length and diameter of the cylinder were 325 mm and 220 mm, respectively, resulting in an aspect ratio of 1.5. In order to reduce 3D effects, two endplates of three times the cylinder diameter
and 6 mm thickness, with a 20° semi-revolved chamfer slanting towards the outside edge, were mounted on either side of the cylinder. The TP930 test sheets were wrapped around the cylinder with the ridges aligned in spanwise direction. The sheets were glued to the surface of the cylinder and additionally secured with tape at the seams (stagnation points) and the cylinder end plates (Figure 2). Considering the supporting base layer of 1.5 mm thickness of each TP930 test sheet, the relative surface roughness $\varepsilon$ was 0.9%.

The wind speed was increased in increments of 10 kph from 20 kph to 140 kph, and at each velocity step, the wind speed was kept constant for 10 s. The data were recorded with Kistler Bioware (Kistler, Winterthur, Switzerland) at 20 Hz. Before analysis, the horizontal force data were tared, by subtracting the test rig’s drag force (without cylinder) from the experimental data (cylinder with TP930 test sheets). The blockage ratio of 1.17% was considered negligible and thus not corrected. On the data plots, the segments of constant wind speed were identified, and the average drag force calculated for each velocity step. The drag forces and wind speeds were converted to coefficient of drag $C_D$ and Reynolds number $Re$.

3. Results

Figure 3 shows the data of the smooth test sheet ($Rsk = \pm \infty$) compared to literature data ([3], [4], [5], [8]). Figure 4 shows the $C_D$ of all test sheets against $Re$. The profile of positively skewed test sheets was too “rough” for speeds larger than 20 kph such that $C_{D_{\text{min}}}$ at $Re_{\text{crit}}$ could not be recorded. The term “too rough” refers to the combination of relative roughness (0.9%) and the specific skewness value. $Re_{\text{crit}}$ of the zero-skewed surface can be assumed at $Re \approx 10^5$. $C_{D_{\text{min}}}$ increases, and $Re_{\text{crit}}$ decreases, with increasing skewness number (at least for $Rsk \leq 0$; Figure 4). The postcritical $C_D$ increases with the skewness number.
The data obtained in this study confirm the conclusions of Haake [6] drawn from literature data and of Barber and Carré [7] drawn from combining own experimental and literature data.

When applying the principle of relative roughness $\varepsilon$ ($C_{D\text{min}}$ increases, and $Re_{crit}$ decreases, with increasing $\varepsilon$) to $Rsk$, then a surface is the rougher the larger the skewness value is. This means that surfaces with large positive skew are roughest, with zero skew medium rough, and with large negative skew are almost smooth. This result seems counterintuitive as roughness is commonly associated with density, and from Figure 1, the zero-skew surface has the highest density of roughness elements. +5 and – 5 skewed surfaces are least dense. The maximally possible surface roughness thus is a single roughness element or trip wire, provided that it is located before the separation point of a smooth surface at the subcritical regime and not too close to the front stagnation point. This explains why single or very few trip wires, such as the seams of base- and cricket balls, and the Nagano strip (zig-zag strip of speed skating suits), are ideal $C_D$ reducers at small $Re$.

At small $Re$, the postcritical $C_D$ of large skewness profiles approaches the subcritical $C_D$ of smooth profiles. Figure 5 shows the postcritical $C_D$ at $Re = 5 \cdot 10^5$ against $Rsk$. 

4. Discussion
5. Conclusion

The larger the surface skewness $Rsk$, the “rougher” is the surface profile, i.e. the larger is the postcritical $C_D$ and $C_{D_{\text{min}}}$, and the smaller is $Re_{\text{cri}}$.

References