Adaptive Fuzzy Systems for Integrated Lateral and Longitudinal Control of Highway Vehicles

by

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Abstract

In this research, the problem of integrated lateral and longitudinal control of highway vehicles is addressed using a number of adaptive fuzzy control techniques separately. These adaptive fuzzy controllers are based on ‘Single-Model/Single-Controller’, and several Multiple-Model/Multiple-Controller (MM/MC) models with ‘blending’ and ‘switching’ characteristics.

Firstly, a robust ‘single-adaptive fuzzy control system’ is developed in the solution of the control problem. Another important feature of the developed controller is to provide a basis of comparison with the ‘high-end’ MM/MC systems that are to follow.

Having identified the highway vehicle ‘operation space’ as a ‘multiple-environment’ due to its complicated dynamics and variations of working environments, and external disturbances, the problem of integrated lateral and longitudinal control of vehicles is redefined as a ‘multiple-modal’ problem. Therefore, in order to address the control problem, MM/MC solutions are developed based on ‘blending’ and ‘switching’ of a bank of adaptive fuzzy controllers for improved control of highway vehicles.

The MM/MC models developed in this research are as follows. The first two controller types are based on ‘blending’. These methods provide the advantage of blending of adaptive fuzzy controllers in the bank for achieving a more prominent effect in multiple environments. However, the third controller type is based on ‘switching’. The resulting switching controller selects the best adaptive fuzzy controller from the bank of controllers while isolating the rest of the controllers. These MM/MC types are described as follows.

The first MM/MC, ‘robust multiple-adaptive fuzzy control with blending’ model uses a fuzzy system to carryout blending of individual adaptive fuzzy controllers in the bank to obtain a unique single control output. The blending of these adaptive fuzzy controllers is done based on the designed fixed parameters of the fuzzy blender. The system uses the advantage of adaptability of each fuzzy controller in the bank in producing the final control effect.

The second MM/MC with blending category is ‘robust multiple-model PDC-based (Parallel Distributed Compensation) multiple-adaptive fuzzy controller’, and it includes
a number of ‘local’ vehicle models in each IF-THEN fuzzy rule. In addition to this, there is a parallel set of relevant adaptive fuzzy controllers relating to each ‘local’ vehicle model in each corresponding fuzzy-rule in the controller setup.

The third controller system, a MM/MC category based on ‘switching’, is described as follows. This controller is designed first, and then tested with simulations by inclusion of different number of adaptive fuzzy controllers in the bank, in two separate studies. This particular controller design can be utilized where there is a specific need for a single controller at a time, as required by a specific scenario. Therefore, the most suitable adaptive fuzzy controller in the bank at a time can be selected based on the minimum value of a cost function.

All the controllers developed in this research are established with comprehensive stability proofs based on a Lyapunov-based method, i.e. KYP (Kalman-Yakubovich-Popov) lemma, leading to asymptotic stability at large even in the presence of extreme conditions. In addition to that, all developed integrated lateral and longitudinal vehicle controllers are validated using an industry-standard simulation platform, i.e., veDYNA® that is based on BMW 325i vehicle model of 1988, considering a number of scenarios of external disturbance cases, e.g., un-symmetric loading of vehicle, tyre-road friction changes and crosswind effects. In addition to that, two cases of sudden non-catastrophic subsystem failures, e.g., a flat-tyre case and a wheel brake-cylinder defect case, are also considered in simulations for validating the developed controllers.

Importantly, it is shown that the developed adaptive fuzzy control systems, especially the MM/MC systems, exhibit improved results even in the presence of external disturbances and some subsystem failures of a non-catastrophic nature.
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Finally, I would like to thank my family for being with me, with support and encouragement, throughout the course of my study.

SRR
Declaration

This is to certify that:

1. This thesis contains no material which has been accepted for the award to the candidate of any other degree or diploma, except where due reference is made in the text of this thesis;

2. To the best of my knowledge, this thesis contains no material previously published or written by another person except where due reference is made in the text of this thesis; and

3. Where the work is based on joint research or publications, the relative contributions of the respective authors are disclosed.

________________________
Samaranath Ravipriya Ranatunga (April, 2013)
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CHAPTER 1

Introduction

Full automation of highway vehicles has almost been achieved in the present day due to developments in microprocessors, advanced sensor technologies, and communication technologies. An unceasing trend is underway for further development of ‘smart’ or ‘intelligent’ vehicles towards fully-fledged automation that will result in improved passenger safety while making the road more efficient with improved traffic flow, to name a few advantages.

1.1 Research Problem and Scope of the Research

1.1.1 Automation of Highway Vehicles for Safety and Higher Road Throughput

Intelligent vehicle controller designs can realize improved safety for passengers by way of precision operation and manoeuvring of automated vehicular systems — an answer to the problem that significant proportion of highway fatalities are due to human error [1]. The large number of road accidents due to human error validated facts given by the World Health Organization, which has identified road accidents as the ninth leading cause of death throughout the world in 2004 [2]. It is an important fact that the attention of drivers is not fully dedicated to driving task all the time. Such diversion of attention from driving can no doubt have serious consequences for safety [3]. In addition to that, the nature of humans is to panic in an emergency situation. This nature of humans further distances the proper and safe decision making at such a critical time. Apart from that fact, it takes a certain amount of time for processing decisions in humans, i.e., feeding signals to the brain (time for sensor stimuli to reach brain), for
correct identification of the scenario in its correct perspective, the decision for action and then activating it through hands/limbs (travel times of impulses to muscles) [4], for performing driving related tasks, e.g. turning of steering wheel/pressing of brake pedals. This time can be on the higher side at a critical point in the decision making, for example during collision prevention, or during other emergency manoeuvres on the road. Therefore, for overcoming ‘human error’ in handling vehicles, automating the control of highway vehicles is an effective solution since the advanced communication and processing technologies reduce the time factors dramatically improving safety.

The other main advantage of automation is to improve the throughput or the number of vehicles that can use the road at a specific period of time. With automation and some advanced technologies such as inter-vehicle communication, and sensor and microprocessor technologies, the vehicles can adhere to minimum safe gaps between the vehicles. This can be in the form of vehicle ‘platoons’ as discussed in PATH (Partners for Advanced Transit and Highways, California) program [5], or it can be merely in normal driving circumstances, e.g. adaptive cruise control with integrated support of lateral control strategies for improved autonomous driving conditions.

Another important feature of using ‘smarter’ or ‘intelligent’ vehicles is that they reduce air pollution and minimize the use of fuel. This fact can no doubt guarantee to reduce the risk of climate change [6]. These advantages have been facilitated by advanced control technologies that can precisely measure the fuel amount that is required to have an optimum air-fuel ratio for combustion.

These facts show that automating highway vehicles leads to many benefits that have not been obtained through the manual driving option.

1.1.2 Integrated Lateral and Longitudinal Control of Vehicles

Many past studies on vehicular controllers have focused on either pure lateral (steering) [7], [8] or longitudinal (speed) control [9] as if they were independent of each other. Many such studies relating to PATH program can be found and few of them are included in [10], [11]. It is known, however that the longitudinal and lateral dynamical parameters are not decoupled particularly at higher speeds, accelerations, and at larger tyre forces, or at reduced road friction [12]. An integrated controller that has the ability to account for longitudinal and lateral ‘coupling effects’, can operate reliably in
emergency maneuvers and slippery conditions [13]. Therefore, many studies followed attempted to merge the two control tasks into an integrated control problem for addressing the issues of coupling effects that arise due to interacting lateral and longitudinal dynamics. Mostly, studies on sliding mode control area have been prevalent. A lateral velocity observer-based control has been used in order to compensate for coupling effects [13]. Studies based on integrated lateral and longitudinal control have also been around using sliding mode control [14]–[16]. Apart from that, a robust adaptive back-stepping controller was used for optimizing traction force distribution [17].

The notion of an explicit dynamic compensation method being used has been proposed for providing an improved solution to address the problem of coupling dynamics [14]. A radial-based functional neural network was used for such a case [18].

In this regard, it is important to consider usage of adaptive fuzzy controllers for improving the tracking problem of integrated lateral and longitudinal control of highway vehicles, since fuzzy systems can be designed with effectiveness in addressing uncertainties involved with model discrepancies, nonlinearities and coupling effects.

1.1.3 Complex Modes and Multiple-Environments of Operation of Highway Vehicles

Highway vehicle systems have highly complex dynamics. While vehicle systems are in operation, there are possibilities for variations in dynamics, amplitudes of disturbances, frequency of change of states and changes in operation of actuator status within ‘sub-catastrophic’ levels. These facts suggest that vehicle systems have multiple-modes of operation. Therefore, it is wise to consider the fact that due to these operating complexities, the highway vehicles operate in ‘multiple-environments’.

On the other hand, ‘single-model/single-controller systems’ are constructed to operate in a ‘single-environment’, or in more elaborative terms, it is permitted to have only slow changes with limited disturbance levels in the system environment [19]. With such a setup in a ‘single-model/single-controller’ system (the term ‘single-model’ is used throughout to capture the notion of ‘non-multiple-model/non-multiple-controller’ setups in this research), the major problem is that it creates high transient effects and declines in tracking effectiveness when the environment of operation changes. This is quite applicable to complex systems like highway vehicles.
On the other hand, Multiple-Model/Multiple-Controller (MM/MC) setups provide an effective solution to vehicles operating in multiple-environments. This is because each individual model/controller can be pre-defined or ‘embedded’ with capability to a specific scenario of operation. Thereby, a number of finite and practically identified cases can be collectively considered to define a whole system of operations. With such ‘model/controller’ units in place, the ‘adaptive’ capabilities of controllers play an ‘enhancing’ function of performance, as well. The usage of multiple-model techniques, hence, enables improved vehicle tracking control in lateral and longitudinal sense. Therefore, by way of improving controller capability, it improves ‘identification’, and thereby leads to exert effective control effort—the difference made from enhanced capability of MM/MC techniques.

1.1.4 Drawbacks of Single-Model/Single-Controller Systems for Vehicle Control

The drawbacks of using single-model (or single ‘modal’) control systems include ‘lapses’ in adaptation. Since adaptation takes a certain amount of time, the sudden changes that occur in a vehicle system cannot be catered for satisfactorily [20]. Therefore, ‘single-model controllers’ are not sufficient to address control concerns of highly complex systems of the nature of vehicle systems when a wide perspective of operations consisting of ‘multiple-environments’ is considered [21]. It is more likely that transient errors go high with such a setup in place leading to poor performance in tracking as well [22].

On the other hand, automated highway systems (AHS) require vehicle control systems to be highly reliable and versatile systems as far as their control requirements are concerned. These positive qualities of control systems are required due to safety reasons when high-speed moving vehicles in an AHS leave less room for error. Nevertheless, single-model control systems, due to their performance limitations, have more difficulty in addressing the concerns of AHS properly, when the complex operation of vehicles in multiple-domains is considered. Since MM/MC systems have more built-in capability to address control requirements with improved tracking with higher reliability, it is more relevant and applicable to use multiple-model controllers within an AHS. After all, it requires higher speeds of operation of vehicles with high
precision in an AHS. Moreover, MM/MC approach ensures that it addresses a much wider scope of the problem with effective overall control of intelligent vehicle systems.

1.1.5 Advantages of using Adaptive Fuzzy Control in Vehicle Controllers

Vehicle control is a complicated problem due to a number of uncertainties and variations of dynamics associated with the system. Several such facts regard to highway vehicles have already been discussed in this section. Elaborating further, the dynamics of the tyre/road interface and the dynamical changes due to road friction are less understood, and vary throughout [23], [24]. The changes due to nonlinearity effects are not so obvious, and therefore are unpredictable [25]. Apart from the above factors, the vehicle design-specific features and exterior conditions, changes in loading and crosswind effects add up to complicate the problem further. Due to these complexities related to the vehicle control problem, the issue desires a more rigorous approach for more reliability. Adaptive fuzzy control has been used to address such complications successfully, since it even has the facility to be configured to include different levels of information in terms of its rule base as knowledge [26].

In this regard, fuzzy systems are more suitable as adaptive systems and knowledge representation systems. Further to this point, fuzzy systems provide a number of advantages over other existing control solutions. Firstly, it provides a ‘stratified’ approach where different layers of knowledge can be accumulated. Thereby, a control system can be built upon selectively to address one specific context of a problem within individual rules of the rule-base [27]. Hence, it can accumulate these rules to provide a total effect when taken as a whole with ‘defuzzification’. Secondly, it ensures that human expertise can be built into such a system, if required, and thereby add more versatility and reliability to the system performance [28]. Thirdly, it provides a base system so that it can be effectively converted into an adaptive structure that can apply and use the benefits of the facts mentioned in the first and second points.

Furthermore, it has been proven that an adaptive fuzzy system has the capability for approximating any nonlinear smooth function to any degree of accuracy when suitably defined in a convex compact region [29]. In addition to that, the ability of fuzzy logic theory to deal with uncertainties when making decisions in complex domains has
favoured its use. Importantly, fuzzy control is guaranteed to operate properly under less restrictive assumptions and for more general continuous-time nonlinear systems [30].

In this research, all the developed adaptive fuzzy control systems are employed together with proportional-derivative (PD) components as a ‘basis’ system where it acts as a coarse-tuning controller. The usage of PD system also addresses the ‘linear’ components of the system. On the other hand, the usage of a PD system can replace fixed models in multiple-model/controller system usage. The use of fixed models in multiple-model systems has been a normal practice [31].

The adaptive fuzzy system is used on top of the PD controller component for addressing ‘uncertainty effects’ mainly, so that the overall system error would be decreasing. The so called ‘uncertainty effects’ are due to model discrepancies, un-modeled dynamics, nonlinearity and coupling effects related with vehicle dynamics, and external disturbances. In other words, the adaptive fuzzy system control methods are used to do fine or robust-tuning as part of the solution of the control problem. The method of using a ‘fine-tuning’ on top of a ‘coarse-tuning’ has been a common trend in some past studies on adaptive fuzzy control aimed at approaching an unknown function, or on robust compensation. Some common examples for such systems on which adaptive fuzzy systems being used are PD, $H^\infty$ control and variable structure control terms and this practice is described in the literature [32]–[34].

In order to further intensify the discussed features of adaptive fuzzy control, there is a requirement for effective MM/MC setups based on adaptive fuzzy control models. Thereby, advantages of fuzzy control as an effective nonlinear control solution for overcoming ‘uncertainties’, can be fully utilized. In this regard, the MM/MC fuzzy models are developed in the areas of ‘switching’ (normally ‘hard-switching’ where binary levels of selection involves, i.e. ‘on’ or ‘off’) and ‘blending’(normally ‘soft-switching’ where multiple degree levels of ‘mixing’ of control outputs). The notion of switching/blending is required for selecting the ‘best’ model(s)/controller(s) (or selecting the best combination) from a MM/MC setup after identification using a suitable criterion at a particular moment of time [31]. In such a setup, the additional requirement of ‘resetting’ can be considered to prevent all models from converging to the same value, i.e. the system parameter set throughout the span of operation. Otherwise, it could prevent previously learned ‘experience’ by way of an already undergone specific operation [22].
1.1.6 The Burden of Calculation with Complex Fuzzy Controllers

One point that is apparent with the proposed MM/MC models in this research is that it may increase the complexity of calculations in the controllers, and therefore constitutes a burden. In addressing this issue, the advent of contemporary fast-paced computer technology, or in other words microcontroller technology can be considered. The addition of multi-cores and multi-threading technologies in the processors will no doubt improve the computational capacity of a typical controller setup. Even, the advancement of processor speed is a plus factor considering an answer to the issue. There are fuzzy chips which can be embedded with fuzzy systems to improve processing power rather than having sole software based fuzzy systems.

Another plus factor for overcoming the issue is that the fuzzy functions can be further optimized considering the firing levels of rules. For instance, some non-critical firing rules can be eliminated, thereby lessening the computational burden in the processor.

1.1.7 Scope of the Research

In this research, several adaptive fuzzy based control models are designed for solving integrated throttle/brake and steering problem of vehicles. All these fuzzy systems are separately used to model ‘uncertainties’ in the vehicle control system, and thereby do robust or ‘fine’ tuning for improving tracking error. In this regard, a single-model adaptive fuzzy system and several MM/MC based control models, i.e. a ‘multiple-adaptive fuzzy control with ‘blending’ system, a PDC-based multiple-model multiple-controller adaptive fuzzy ‘blending’ system, and a multiple-adaptive fuzzy controller with ‘switching’, are developed.

First, a robust ‘single-model’ adaptive fuzzy system is developed. The performance of the single-adaptive fuzzy system is compared against the latter MM/MC adaptive fuzzy systems with regard to improving integrated lateral and longitudinal control of vehicles.

Developed two control models based on ‘blending’ or ‘soft-switching’ methods are as follows.
The first of the blending category, a robust multiple-adaptive fuzzy control system uses a fuzzy system to do blending of individual fuzzy controllers in a bank and obtain an output as a unique control signal.

As the second of the category of ‘blending’ based control systems, a robust multiple-model, fuzzy PDC (Parallel Distributed Compensation)-based adaptive fuzzy controller is developed. This controller includes a number of ‘local’ vehicle models in each IF-THEN fuzzy rule. There is a parallel set of ‘local’ adaptive fuzzy controllers relevant to each ‘locally oriented’ vehicle model in the controller setup, included in the fuzzy PDC setup.

Thirdly, a multiple-adaptive fuzzy controller based on ‘switching’, or more specifically ‘hard-switching’, is developed.

All above novel control models are comprehensively proved with Lyapunov-based stability analysis, i.e. KYP (Kalman-Yakubovich-Popov) lemma, as achieving asymptotic stability under robust conditions at global level. Furthermore, all these control models are tested using comprehensive simulation studies including a number of external disturbance cases and fault-tolerant cases, using an industry-standard simulation platform, i.e. veDYNA®, which is based on a high precision BMW 325i vehicle model of 1988. The simulation is extended for the MM/MC ‘switching’ based setup, separately for two cases, i.e. two’, and ‘four’ number of adaptive fuzzy controllers in the bank of MM/MC setup.
1.2 Outline of the Thesis

In Chapter 1 of the thesis, an introduction to the research problem is provided. Followed by this, in Chapter 2 with the literature review, various approaches to the vehicle control problem and the state-of-the-art of vehicle control are discussed in detail. In Chapter 3, the vehicle models used in the thesis study are explained in detail. These vehicle models include a simplified vehicle model for synthesis of the controllers and a complex, high-precision industry-standard vehicle model on veDYNA® for validation of the control systems developed. In Chapter 4, a single-model based adaptive fuzzy controller for solving the integrated lateral and longitudinal control of highway vehicles, is described. First, the controller synthesis details are explained. This synthesis is followed by a comprehensive stability proof. Finally, the developed controller is validated using a number of scenarios of external disturbances and a few cases of non-catastrophic failure-modes of subsystems. In Chapter 5, two MM/MC adaptive fuzzy control models based on ‘blending’, firstly, a design with ‘controller-signals as inputs in fuzzy’, and secondly, a design with ‘state-signals as inputs in fuzzy PDC-based’, are explained in detail. For each controller, the controller synthesis details are explained, first. This synthesis is followed by a comprehensive stability proof. Then the developed controller is validated using a number of scenarios of external disturbances and a few cases of non-catastrophic failure-modes of subsystems. In Chapter 6, an MM/MC adaptive fuzzy control model based on ‘switching’ is explained. First, the controller synthesis details are explained. This synthesis is followed by a comprehensive stability proof. Then the developed controller is validated with simulations using a number of scenarios of external disturbances, and a few cases of non-catastrophic failure-modes of subsystems, separately for ‘two’ and ‘four’ adaptive fuzzy controllers in the bank. Finally, in Chapter 7, the conclusions and future directions drawn from the work carried out are described.
1.3 Significance of the Research and Contributions

The significance of the findings of this research from the points of view of vehicle control technology and control system technology are described below.

1.3.1 The Significant Results Achieved Related to Control Systems Technology

Modelling, comprehensive establishment of robust Lyapunov-based stability analysis leading to asymptotic stability at global level, validation with comprehensive simulation studies, and analysis of novel adaptive fuzzy based control models:

1. Robust ‘single-model’ based adaptive fuzzy control (novelty applicable only to stability analysis method)
2. Robust multiple-adaptive fuzzy control based on fuzzy ‘blending’
3. Robust multiple-model fuzzy PDC (parallel distributed compensation)-based adaptive fuzzy control based on ‘blending’
4. Robust multiple-adaptive fuzzy control based on ‘switching’

1.3.2 The Significant Results Achieved Related to Vehicle Control Technology

Comprehensive development of the control concepts:

1. Novel control concepts (as described in Section 1.3.1) using adaptive fuzzy based control systems for solving the integrated lateral and longitudinal problem of highway vehicles
2. Novel MM/MC based control concepts for vehicle control applications using three control models (of 2-4 under Section 1.3.1) that can be used to solve vehicle control problem being re-defined as a ‘multiple-modal’ control problem (which is more realistic according to the facts explained throughout in this thesis) leading to effective tracking performance, especially with regard to improved lateral stability
CHAPTER 2

Literature Review

2.1 Introduction

In the following literature review, first, the past research studies carried out on vehicle control are described. This paves the way to highlight the state-of-the-art of vehicle control research. This description will be followed by studies on the control aspects related to ‘fuzzy’ and ‘adaptive fuzzy’ systems. Finally, research related to multiple-model control, in general, and with specific applicability to vehicles, is described.

2.2 Vehicle Control

2.2.1 Vehicle Control: Background

The notion of automated vehicle control developed with the requirement for improving passenger safety and road throughput of vehicles. Automation of vehicle control has been mainly necessitated for safety reasons and that will be elaborated further. When the speed of a vehicle becomes high, and therefore the time allowed for the driver to act within normal manoeuvres becomes quite low, safe decision making becomes difficult [4].

In terms of improving road throughput, roads can have higher flow of vehicles having shorter gaps between the vehicles. Therefore, automation of vehicle control is a solution for maximising road throughput, provided it is supported with suitable infrastructure—the so called automated highway system (AHS). This is the second main reason for automating highway vehicles [35].
With regard to research and development of automatic control of vehicles, there have been a few reputed world-wide research initiatives, some of which still continue to exist, e.g. PATH program of California. But, some have completed their scope, and therefore are not currently active. Examples of the research initiatives include the California, PATH (Partners for Automated Transit and Highways) program [5], PROMETHEUS (PROgraMme for a European Traffic with Highest Efficiency and Unprecedented Safety) project in Europe, DRIVE (Dedicated Road Infrastructure for Vehicle Safety in Europe) in the European Community [36], ALVIN (Autonomous Land Vehicle In a Neural Network) in the USA, and ARTS and ASV in Japan [37].

California PATH program is one of the most prominent among the above group, and therefore, it is one of the best sources of intelligent vehicle control applications research. PATH has been involved in setting up a standard for automated highway systems and pioneered the creation of the popular AHS architecture. In this five–layer PATH AHS normal mode of operation control architecture [35], the ‘regulation layer’ includes the standard requirements and technical details applicable to vehicle longitudinal and lateral control, which relates closely with this research. The AHS architecture (of PATH) is arguably considered as the most recognized standard in intelligent transportation systems. Therefore, AHS architecture of PATH is adopted as a norm in intelligent vehicle research. As part of the research work within the California PATH program, many issues in designing automated highway systems were mentioned [38].

When it comes to vehicle control, there are several hierarchical levels of control [39]. The first and the primary level includes the systems that provide an advisory/warning to the driver (collision warning systems), e.g. forward collision warning, blind spot warning, lane departure warning and lane change/merge warning etc. The second level includes the systems that take partial control of the vehicle. This can be either for steady-state driver assistant systems such as semi-automatic control systems (e.g. adaptive cruise control), or as an emergency intervention system to avoid a collision or to stabilize the vehicle (e.g. Electronic Stability Program, ESP®). The third, or the foremost level includes the systems those take full control of vehicle operation or vehicle automation, which includes low speed automation for congested traffic, autonomous driving and close-headway platooning etc. [39].
Vehicle control research initially progressed through two separate streams: longitudinal control and lateral control. These studies were without consideration of existing dynamic coupling. The longitudinal controllers were primarily aimed at design of decoupled controllers that maintain a particular vehicle to vehicle spacing in a ‘normal’ environment [40], or within a ‘platoon’ environment [41]–[43], a widely accepted notion in an AHS environment for effective vehicle motion.

Separately, the studies based on lateral control of vehicle systems addressed the problem of regulating vehicle position relative to lane centre [44], [45]. When examining the past, longitudinal and lateral control of highway vehicles progressed as decoupled control systems where there is no connection between the two control aspects of the vehicle [14].

2.2.2 Decoupled Longitudinal Vehicle Control

When it comes to longitudinal control of vehicles, there are two acceptable concepts associated with it: spacing control and headway control. The advantage of headway control is that the leader vehicle information is not required to be fed into other vehicles of the platoon [14]. In spacing control, each vehicle has to keep a specified length between itself and the front vehicle. This spacing can be defined according to the speed of motion or can be taken as a constant, throughout. The notion of spacing control came into being in early 1990s [46], [47].

On the other hand, in headway control, a defined ‘time gap’ that is in turn expressed in terms of a distance gap, has to be maintained between the controlled vehicle and the leading vehicle. The emergence of the concept, headway control, also occurred in early 1990s [48], [49].

Other examples of longitudinal control strategy, though they are not confined to an AHS environment, include ‘stop-and-go’ (a vehicle speed and distance control method), and cruise control algorithms [50]. Various other strategies of spacing control were added later in relation to the platoon concept [51]–[53]. Spacing control studies based on sliding mode control have been prevalent in the literature [54], [55]. Further improvements on safety margin of stability for spacing control, gave rise to smooth ride quality using adaptive observers [56].
Longitudinal vehicle control strategies using different feedback measurements can also be found. These feedback measurements include using a video system based on a stereovision technique for a second order sliding mode control at low speed [57], and based on a single camera vision system for an adaptive cruise control system [58]. A real-time kinematic phase differential Global Positioning System (GPS) (i.e., centimetric GPS) has also been used for adaptive cruise control (ACC) with Stop&Go manoeuvres [59]. A vehicle position control system using a GPS and Inertial Navigation Systems (INS) was also used [60]. Most of these control strategies have been carried out on real-scale experimental vehicles.

2.2.3 Switching between Throttle and Brake

One important aspect of longitudinal control is the requirement for switching between throttle and brake actuation. A well established criterion is required for successful switching functionality. The importance of using switching is to prevent simultaneous application of throttle and brake.

Some perspective on evolvement of switching criteria between brake and throttle is considered below. Among these studies there have been a number of practical solutions suggested and used.

One method that has been predominantly used in PATH program with bypassing of throttle at ‘closed-throttle’ condition, is considered first. The assumption ‘closed-throttle’ condition does not have an effect on the controller design. In this switching criterion, the desired acceleration that is greater than the closed-throttle engine torque and drag terms, has to be switched to throttle control. An acceleration below the closed-throttle engine torque and drag terms requires braking. This method of switching is a sliding mode based control solution, and has been mainly used in PATH based research literature [61]. Though this method of switching as a sliding control based control solution is appealing, when different controller methods are used, this would not be practical. Another fact is that this brake-throttle switching method is not applicable to scenarios that involve considerable tyre-slip such as emergency braking [61]. A simpler and computationally less demanding type of brake-throttle switching rule also exists [46], [54], [62] where switching between the throttle and the brake is based on throttle angle variation, i.e. based on ‘positive’ or ‘negative’ sign values of the throttle angle.
The simplicity and versatility this method provides cannot be underestimated for usage where ‘uncertainty’ effects can be overcome. There is an alternative brake-throttle switch criterion [63], which bases its decision on the distance and the closing rate between the vehicle and the preceding vehicle. Since this method has been mainly applicable to ‘headway control’, there are difficulties in adopting it in ‘spacing control’ of vehicles.

2.2.4 Obtaining Throttle and Brake Commands from Total Torque

When calculating the throttle angle signal and brake signal from the function of total torque that acts as the longitudinal control output, there have been two methods developed. First method is based on multiple sliding mode control [64] that has been used as a popular control method for avoiding direct differentiation of engine map data when calculating the control signals [62], [64]–[66]. This method has been mainly used to avoid differentiation of engine map errors as a result of direct differentiation of engine map data. The second method of working out the throttle signal and brake signal from the total torque command involves the ‘inverse mapping’ method [43]. This inverse mapping method uses an inverse vehicle model for calculation of desired throttle angle from the controller output. In this method, direct maps have been used for inverse calculation of required throttle angle. Rarely there have been some alternative throttle/brake control laws being used based on sliding control [67].

2.2.5 Decoupled Lateral Control of Vehicles

Many past studies have been there based on developing a lateral vehicle controller [44], [68]–[71]. Some developments of decoupled lateral control of vehicles appeared based on a specific type of controller, e.g. H-infinity [8], sliding mode control [45]. Some controller designs have extended their lateral control aspect even for lane keeping and lane change manoeuvres [7].

Other lateral control strategies that implemented with different feedback measurements can also be found. In some applications, computer vision has been utilized to obtain measurements for lateral tracking control with a lead-lag, full-state
linear control [72], input-output linearizing control [73], and with a PD control [74]. A DGPS (Differential Geological Positioning System) has also been used in lateral control of a vehicle with chained systems theory and fuzzy control [75]. Most of these control strategies have been carried out on real-scale experimental vehicles.

The problem with decoupled control studies is that they do not identify the presence of coupling effects. These coupling effects are highly nonlinear and become apparent and quite detrimentally influential with regard to safe and reliable operation of the vehicle. The coupling effects become obvious when the speeds of motions are very high, or when emergency manoeuvres are present. Therefore for safe and reliable operation, the consideration of these coupling effects is necessary for improved control needs of vehicles [14].

2.2.6 Integrated Lateral and Longitudinal Vehicle Control

Having found the shortcomings of using decoupled controllers either in longitudinal or lateral form, the combined longitudinal and lateral controllers were considered as an effective solution [14]. Combined controller studies have been used to address the consequences of unsafe and unrealistic conditions that prevail due to coupling effects with separate longitudinal and/or lateral control. These coupling effects become quite obvious during emergency manoeuvres, high speed/high acceleration motion etc., and therefore threaten safety conditions, if totally neglected.

The notion of dynamic coupling emerged for discussion in early 1990s [76]. First comprehensive treatment of modelling and controller design work on integrated lateral (steering) and longitudinal (throttle/brake) vehicle control appeared in 1995 [77]. Ever since, there have been some discussions on combined lateral and longitudinal control of vehicles. The studies based on sliding mode control have been popular [13], [78]. Online RBF neural network modules have also been used for addressing coupling effects and other nonlinearities in a combined lateral and longitudinal vehicle control study [18]. In some studies, back stepping control has been used [17]. Combined longitudinal and lateral control based studies have been extended to constant headway policy as well [79]. Some studies even linked combined control issues with platoon environment [80].
2.2.7 Sensor/Actuator Fault Detection, Identification and Control in AHS Environment

The scope of this research includes control of vehicles in the presence of system/subsystem failures to some extent. But the emphasis is not on identification or detection of system/subsystem failures.

Many studies have been carried out in identifying faults in vehicles in an automated highway system setup [81]. Detailed designs of fault detection filters for longitudinal dynamics in an AHS setup have been studied to a greater extent [82], [83]–[85]. In this regard much research has focused on sensor/actuator fault detection algorithms [64].

Extensive research has been carried out on fault-tolerant control systems with regard to longitudinal vehicle control [86]. Some control aspects involve supervisory controllers for looking after fault-tolerant control systems [87]. Even in fault-tolerant control, sliding mode control has been used for a certain class of faults [88], [89].

2.3 Adaptive Fuzzy Control Systems

2.3.1 Fuzzy Control in General (Non-Adaptive Techniques)

Fuzzy systems, having a versatile nonlinear system structure, can provide the basic components for arriving at resilient solutions in complex systems to handle uncertainty effects [90], [91]. There is no doubt that fuzzy systems can also be used for satisfactory implementation of control solutions for automated vehicle systems supported by cutting edge technology. The uncertainty in such a complex problem can also be addressed using adaptive fuzzy control. Fuzzy systems also have the ability to store specific ‘human expert’ knowledge in terms of its IF-THEN rules, if the requirement arises. This view is emphasized and supported by the following statement: “Fuzzy control techniques usually decompose the complex system into several subsystems according to the human expert’s understanding of the system and use a simple control law to emulate the human control strategy in each local operating region” [92].
As fuzzy systems have excelled in performance in applications where there is a requirement to address ‘uncertainty’ and non-linearity effects, it is important to investigate the past studies using fuzzy control for such usages, especially intending for vehicle control applications. The perspective of stability analysis of fuzzy systems is a challenging and important area to cope with, in this regard.

T–S (Takagi–Sugeno) fuzzy systems are considered as systems that are computationally efficient and relatively easy to manipulate analytically [93]. The basic T–S fuzzy system relies on the concepts established in fuzzy set theory [28]. Before investigating stability research on T–S based adaptive fuzzy systems, it is worthwhile to look into the stability analysis trends of non-adaptive T–S fuzzy systems. Many past stability analysis methods for T-S fuzzy systems (dynamic systems) have been based on finding a common positive definite matrix to satisfy the Lyapunov equation or the LMI (Linear Matrix Inequality) for every local model considered in each fuzzy rule [92], [94]. There have been instances of usage of a piecewise Lyapunov function for obtaining certain boundary conditions for establishing stability of T-S systems as well [92]. Sometimes, it has been a piecewise quadratic based Lyapunov function that has been used for establishing stability [95]. Some alternative methods that have been inspired from the piecewise Lyapunov function method can also be found [92]. Usage of fuzzy partitions with piecewise smooth quadratic Lyapunov functions for establishing stability of parallel distributed compensation (PDC) structure has been a popular approach [96]. Some alternative approaches to stability have also been used. One example is the usage of a linearized form of a non-linear fuzzy system transfer function for establishing closed loop system stability [97].

With more relaxed stability conditions based on the fuzzy Lyapunov function approach, usage of fuzzy blending of multiple quadratic Lyapunov functions can be found [98]–[100]. A Lyapunov function being formed into a line-integral of a fuzzy vector for establishing stability has been a novel approach [101].

### 2.3.2 Adaptive Fuzzy Control

When it comes to adaptive fuzzy control, it is important to consider the class of nonlinear functions applicable to adaptive fuzzy controllers developed in this research. With regard to vehicle control applications, the nonlinear functions that can be
addressed by fuzzy systems should be similar to functions having the form of a simplified model of a passenger car—the inputs are separable from the rest of the function (either constant multiplied or appear linearly varying with a nonlinear functions), and the rest of the function is having an order maximum of two (even three can be considered but it is complicated). It is also a requirement in these systems that the unknown parameters appear linearly with respect to the known nonlinear functions.

Another important feature of fuzzy systems is the universal approximation capability that suggests that any nonlinear function can be approximated when suitably designed in a general fuzzy system [28], or in a T–S fuzzy system [29]. In a fuzzy system, as far as membership function selection is concerned, Gaussian basis functions have the best approximation property in comparison to other membership function forms [102].

Establishing stability in a fuzzy control system is a very important step [103]. Some special and important cases of developing direct adaptive fuzzy control systems with stability are provided as following. A supervisory control term used for establishing global asymptotic stability of a general nonlinear fuzzy system appeared as an approach in a seminal work [32]. A slight drawback of this method was the inclusion of the assumption of squared-integrate of fuzzy approximation error as finite. The trend set by this approach continued into later studies based on fuzzy universal approximation theorem for unknown nonlinear systems [33]. The ‘drawback’ to assume ‘squared-integrate’ of fuzzy approximation error as finite, has been alleviated in some later studies where a variable structure control term was used successfully, instead [104], [105].

Both direct and indirect adaptive fuzzy problems have been included for comprehensive analysis of stability based on Lyapunov criterion in later studies for uncertain SISO systems with large uncertainties [34], [106], and for a class of uncertain continuous-time MIMO nonlinear systems [107]. In all these works, a ‘hybrid strategy’ was considered for designing control systems where adaptive fuzzy systems played an important part. In these works, adaptive fuzzy has been used for rough tuning while the ‘second’ control component has been used for robust or fine tuning. Such usages of hybrid components in adaptive fuzzy systems were a supervisory component [32], a $H^\infty$ based component [34], [104], [108], [109], and a sliding mode component [110], [111].
Some slight variations to these investigations in contemporary usage include an adaptive fuzzy controller for compensating functional approximation error for improving control performance [108], [112], [113]. On the other hand, a self-structuring adaptive fuzzy controller scheme for nonlinear affine SISO systems has also been used [114]. There have been studies which discuss a number of different perspectives with regard to adaptive fuzzy control developments in a single presentation [115].

2.4 Multiple-Model Control Systems

2.4.1 Single-Model/Single-Controller Methods and their Drawbacks

The term ‘single-model’ is used in this research as a relative term to identify all the control systems that do not fall into the category of ‘multiple-model’ control systems. There are a number situations where the limitations of ‘single-model’ or conventional adaptive control systems in general control area, and thereby in vehicle control, occur. These cases can be identified by usages below.

In globally stable algorithms of Linear Time Invariant (LTI) systems, it has been shown that tracking errors have been quite oscillatory with very large amplitudes when there were large errors in initial parameter estimates [116]. It is important to stress that it is not always possible to have accurate initial parameter estimates in an adaptive system, in general. High transient errors can also become more significant if there exist large and abrupt (discontinuous) variations in the system dynamics that make changes in parameters during tracking tasks, e.g. non-catastrophic subsystem failures [22]. Identification of such limitations with the conventional ‘single model’ control approach for highly complex systems has been considered before [117], [118].

There is another drawback when it comes to ‘single’ model or conventional adaptive systems. As it has been highlighted, the main drawback of adaptive control as stated by many authors, is its slow adaptation and poor transient response [22]. Under these circumstances, adaptive control using a single model control system is not satisfactory.
Another perspective with single-model control systems is that sudden and emergency changes in the plant (especially in a highway vehicle) are not always possible to address. The rationale behind this is that the system will have to adapt itself to the new situation before an adequate control action can be taken. On the other hand, an inadequate initialization of estimated plant parameters in the estimation algorithm (of the control system) may result in unacceptably large transient deviations [21].

When frequency of dynamics changes, single-model controllers are not in a position to capture the dynamical changes satisfactorily so that there will not be any tracking adversity. On the other hand, it is generally accepted that the adaptation might be made faster by choosing a larger adaptation gain. This is not a proper solution because having higher gains may lead to performance deterioration while increasing steady-state noise sensitivity [22].

As a general rule, the control system design has traditionally been based on a ‘single-model’, ‘fixed’ or ‘slowly adapting model’ of the system. The implicit assumption used is that the operating environment is either time invariant or varies slowly with time [20]. With the advancement and widening of control requirements such as changes in application environments for more demanding needs, more scope have been included in the control systems. This is done so that complicated situations can be considered.

It has been shown that transient errors with globally stable nonlinear systems are significantly larger than in the linear case [116]. This suggests that the situation becomes worse with nonlinear ‘single model’ control systems as it is more challenging to apply control principles in the nonlinear ‘domains’. With adaptive controllers, control performance worsens when the plant is time-varying unless the variation of its parameters is slower than the rate of the adaptive scheme [19].

MM/MC systems can potentially address the problems highlighted previously since there are a number of models/controllers working together at a time. These models/controllers are for identifying ‘variations’ in the system, and at the same time to provide some instances of control thereby successfully reducing the tracking error.
2.4.2 Vehicles Working in Multiple Environments: Multiple-Model/Multiple-Controller (MM/MC) Systems for Vehicle Control

It is important to note that highway vehicles have complex dynamics and therefore, they operate in ‘multiple environments’ [119].

It is necessary to consider some form of control methodologies that identify and capture the complexities associated with systems as complex as vehicles, which exhibit such ‘multiple-modal’ operating characteristics. If this multiple-environment characteristic is not covered properly, it is difficult to overcome the transient errors and other tracking problems like external disturbances that encounter in vehicle control operations. MM/MC systems exhibit such capabilities to cater for the multiple-modal or multiple-environment operation of vehicles. Therefore, multiple-model systems can be considered as successful contenders for vehicle control. With such a mechanism for addressing multiple working environments in place, it is possible for autonomous vehicles to reliably operate in AHS environment where high precision longitudinal and lateral tracking is required within fast and safe operation.

2.4.3 MM/MC Control Systems in Multiple Environments: Multiple-Model Control Concept

It is clear that complex systems like highway vehicles operate in multiple environments that may change abruptly from one context to another within short time periods. Therefore, the need of multiple-model control systems for controlling highway vehicles is quite relevant and applicable. On the other hand, if a single-model control system was used, it would lead to deterioration of the system performance in such environments.

More specifically, in practical applications a system can include uncertainties due to a number of causes. These causes can be faults in the system, changes in subsystem dynamics, sensor and actuator failures, external disturbances, and changes in system parameters [21]. For successfully addressing these concerns of complex systems, MM/MC systems can be considered.

The multiple-model (or multiple-estimation) based controllers can be used as a methodology to improve transient response of adaptive systems. Such an MM/MC system can be used as a set of parameter estimation algorithms running in parallel with
each one being initialized by a different set of plant parameter values [21]. On the other hand, multiple-model controllers can also be used to overcome tracking errors when sudden emergency situations occur, or during non-catastrophic failures of system/subsystems when each of the algorithm is suitably parameterized.

Apart from the ‘regular’ context, using MM/MC systems can be related to ‘intelligence’ in a system. It is given “the speed and accuracy with which a controller responds to sudden and large changes may be considered as a measure of its ‘intelligence’. From this point of view, ‘intelligent control’ is the efficient control of dynamical systems operating rapidly in time-varying environments” [20]. Thus, in this context, it can be shown that using MM/MC systems improve ‘intelligence’ of a system. Therefore, according to the above statement, MM/MCs add more precision and versatility to control systems in presence of sudden and large changes in its operating environment [20]. With addition of multiple-models, the performance of complex systems such as highway vehicles can be improved in different environments [20]. Further to this point, with a number of models initially placed at different selected positions on ‘control parametric space’, the transient errors can be reduced further with the usage of MM/MCs.

Another perspective to the operation environment of highway vehicles can be considered. When it comes to vehicle systems, their dynamics at low speed are faster than that at high speed. But, normally the control ‘gains’ are designed mainly for high speed. Due to this reason, throttle acceleration and jerk exhibit oscillations in low speed. It says “In order to achieve good performance in both low and high speeds, one possible way is to use gain scheduling or adaptive gains” [43]. The method of gain scheduling has been a precursor to the method of multiple-model control [120]. On the other hand, multiple-model control introduces a much more effective and versatile method than gain-scheduling method. Therefore, multiple-model control systems provide an effective way to address the above problem. By having different models and controllers with different adaptive rates, the problem of operation with two ‘speeds’ can be addressed successfully. This sort of approach provides another important perspective for using MM/MCs for integrated throttle/brake (longitudinal) and lateral (steering) vehicle control over the ‘single-model’ control systems [120]. The similar solution approaches of multiple-model control systems have been discussed before [117], [118], though, not particularly relating to vehicle control.
Multiple-model controllers are divided into two groups: classical multiple-model control that uses non-intelligent methods, and the group that uses multiple-model control with intelligent methods. The classical multiple-model type of controllers generally work based on two principles: switching and tuning [20]. On the other hand, intelligent multiple-model control uses the advantages of intelligent methods such as fuzzy logic [121], and neural networks [122], [123]. These multiple-model control systems with intelligent methods can provide a good solution to the problem of integrated throttle/brake and steering vehicle control. An intelligent multiple-model controller is regarded as a type of controller that can accommodate large values of disturbances, even the unexpected ones, with some built-in robustness [123].

In the context of multiple-model control, there is an advantage of using non-linear models instead of regular usage of linear models. The advantage of using nonlinear models to identify each ‘scenario/event’ can be highlighted because nonlinear models perform better with a system of complex nature like the vehicle with more accurate tracking, and with a larger domain of operations [124].

In multiple-model based approaches, there have been studies involving classical linear control theories, stochastic approaches, and fuzzy architectures [125], as well.

2.4.4 History of Multiple-Model Control Systems

The simplest and earliest form of application of the multiple-model control concept was in gain scheduling [120], [126]. It is important to state that gain scheduling cannot be applied to fast changing environments, and the slowly varying gain scheduling variable is suitable only for slow variations. This is where the limiting factor of gain scheduling occurs. Due to such limitations, gain scheduling can not be successfully applied in automotive applications where the driving conditions and disturbances quite abruptly change in highway environments. This applies to aerospace applications too where flight control problems are present [120].

The individual concepts of multiple-models, switching, or tuning are not new in control theory. Multiple Kalman filter-based models were studied in 1970’s to improve accuracy of the state estimate in estimation and control problems [127], [128]. This work was followed in later years by several practical applications [129], [130]. But, no switching was involved in early studies, and only a convex combination of the control
determined by different models was used. No stability results were reported either, in these early days.

Importantly, the scheme of blending control inputs first appeared as a Multiple Model Adaptive Control Method (MMAC) [131]. But, no theoretical stability results have been presented for MMAC methodology. All proposed MMAC compensators have been linear.

2.4.5 Switching and Tuning in Multiple-Model Control

When the environment of a system changes abruptly, the original model (and hence controller) cannot be considered valid any longer [20]. If models can be constructed for different environments identified with the system operation, controllers corresponding to them can be designed a priori. During system operation, it is required to identify the existing environment to determine the controller that fits best to the particular environment. This nature of identification can only be achieved if a model for each environment is known in advance. Based on these ideas, an ideal way is to use a control strategy to determine the best model for the current environment at every instant, and activate the corresponding controller by way of an effective switching methodology. On the other hand, when adaptive controllers are used, it is possible to adapt to different environments to a certain extent [20].

In practice, the number of models that can be considered is finite in a multiple-model system setup, and at the same time, the number of possible environments can be immense. Therefore, identification of the environment takes place in two stages. Assuming that the models and environments are parameterized suitably, the model with the smallest error, according to some previously defined criterion, is selected at a faster time scale (switching). And its parameters are adjusted over a slower time scale to improve accuracy (tuning). Thus, the two stages of identification of the correct environment can be further explained: switching, where the problem is to determine when the current parameter value is unsatisfactory (i.e. when to switch) whereas tuning, where the problem is to determine the rule by which the parameter value is to be adjusted at each instant [21].

The decision of switching can be based on different criteria. It can be time-based, operator-based or operating condition based. It is also important to note that some
switching sequences can destabilize the closed-loop dynamics, even if each controller globally stabilizes in case of LTI plant [132], [133]. The criticality of such a problem may get worse in the case of nonlinear control systems. This is an important point to be considered when designing switching criterion for a given set of multiple-model control systems.

2.4.6 Switching

In adaptive control, switching was first introduced in late 1980s [134]. Two types of switching schemes have been proposed for adaptive control in the literature. In direct switching, the choice of when to switch to the next controller in predetermined sequence that is directly based on the output of the plant [135]–[137]. It is to be stressed that this form of switching has a little practical value [20].

On the other hand, some systems in which multiple models are used to determine both, when and to which controller one should switch, are more useful for applications. This kind of switching can be identified as an indirect switching method. This switching approach, too, was first proposed in late 1980s [138]. Later, this work was adapted for different levels of applications [139], [140].

2.4.7 Multiple Model Switching and Tuning (MMST) Method

Multiple Model Switching and Tuning (MMST) methodology, a popular method proposed by Narendra et al, provides an important framework [31]. In this method, a number of controllers are switched based on the minimum identification error as decided from a criterion based on a predefined error function. MMST has been used both with fixed and adaptive models. For adaptive models, any arbitrary switching scheme where the interval between successive switches has an arbitrary small but nonzero bound, yields a globally stable system. For fixed models, as the error index can grow in an unbounded fashion within a finite time, the system is switched to a free running adaptive model (and stays there for the rest of the time). In other words, fixed models have been used for reducing the tracking errors and adaptive models for ensuring asymptotic convergence of errors.
Stability analysis for MMST on continuous-time linear systems has been comprehensively studied [20]. This MMST method has been extended to include some nonlinear control systems where unknown parameters occur linearly, widening the principles studied in the linear case [116]. The MMST method has been used in a discrete-time linear case as well [141]. Improving transient effects using MMST method has separately been studied [142]. Application of MMST for stochastic based control methods can also be found [143]. MMST method has also been used to identify and control a problem with time varying parameters [144]. A further improvement in this regard is the usage of an ‘adaptive time’ between switching for overcoming chattering with frequent mode switching in a similar MMST setup [145].

2.4.8 Other Switching Methods in Multiple-Model Control

Other switching methods that are different to the MMST method can also be found in the literature, for instance the switching methods for selecting the best model from an assembly of models, by voting [146] or by using a supervisor index [147]. A switching based updating of models using an iterative learning method [148], a similar type of switching method to MMST, has also been there in the literature. Another switching type similar to MMST was used on feedback linearized control of nonlinear systems [22].

A novel approach to switching where a metric called ‘Vinnicombe metric’ being used in a multiple-model adaptive control with supervisory switching has been presented in the literature [149], [150]. This is an entirely different approach to the MMST method.

Switching functions used for avoiding chattering while selecting the best model can also be found [151]. Fuzzy systems, especially T–S fuzzy have also been used in switching between multiple-model systems [152]. In some cases, localization method has been used to pick up the model that is closest to the true model [153]. Usage of output feedback matrix is a unique and rarely found method for selecting the best model in some multiple-model systems [154]. In some systems, selection of models has been done after exceeding a threshold value [155].
2.4.9 Blending Methods in Multiple-Model Control

In observing blending based methods in multiple-model control, a number of examples can be found. Probabilistic blending method of weights has been used for blending multiple-model controllers [120]. In such cases, the weighting factor is taken as the residual between the model-predicted measurements and actual sensor-based measurements. Predictive modelling error has also been used in some instances, for the same function [156]. Another such method has been to use recursive Bayes’ theorem to calculate the conditional probability of each model [157]. Based on these probabilities, suitable weights were assigned to individual control models providing a blending scheme. Soft switching or blending of multiple-models using a fuzzy system can also be found [158]. Another usage of fuzzy method has been to identify the best model using a decision mechanism [21]. T–S fuzzy systems have also been used to select the best model [159]. Some other fuzzy usages in switching of nonlinear multiple-model based control studies are reported in the literature [160]–[162]. It is worth stressing that the list of adaptive fuzzy control cases being used as multiple-model control systems as reported in the literature has been small.

Switching using neural networks in multiple-model control systems is also reported as a neural identification [124], and as a neural controller [163]. These systems can also be categorized as soft-switching systems.

2.4.10 Multiple-Model Systems used in Vehicle Control

Multiple-model control systems used in vehicle control field are limited in number. Though, there have been identified benefits of using multiple-model systems over single-model counterparts in control, vehicle control systems have been mainly synthesized using ‘single-model control methods’, or slowly adapting methods, in the past. It is therefore, very rare to find vehicle control systems being used with MM/MC applications. Only a few cases are available in this regard, which are discussed below.

A multiple-model longitudinal controller solution in adaptive cruise control has been reported [120]. Two models differing in the rate of change of adaptation have been used in this work. A drawback in this approach is that there has been no guarantee that the control system can maintain stability when disturbances of varying degree affect it.
In other words, robustness of the control system has not been established. Another drawback is that only simpler vehicle models have been used to validate the developed control algorithms.

A multiple driver model system constructed using an assembly of multiple-inverse models of vehicle dynamics has also been reported in relation to vehicle control [146]. This controller has been developed mainly as a driver support system in an obstacle avoidance scene. A clear drawback of this work is that stability of the control system has not been established.

The centre of gravity of vehicles in changing scenarios is estimated in a multiple-model switching method in another vehicle control application [164]. This study can be considered a multiple-model system identification method rather than a control study.

Even though, there are major benefits in using the enhanced capabilities of multiple-model control systems for operating under complex scenarios such as controlling highway vehicles, it is surprising to note that there is very little research on the topic.
2.5 Chapter Summary

Firstly, the literature review presented in this chapter describes research on vehicle control. Description includes studies on longitudinal and lateral decoupled controllers as well as integrated controllers used in lateral and longitudinal vehicle control.

Secondly, literature review addresses the areas of fuzzy control and adaptive fuzzy control with special emphasis placed on the stability analysis process.

In the final part of the literature review, the area of multiple-model control and its relevance to vehicle control are described. As for multiple-model control, the review also covers switching and blending techniques.
CHAPTER 3

Vehicle Models in Controller Synthesis

In this chapter, two vehicle models are described. The first vehicle model, a simplified version from a complex model, is described first. This simplified vehicle model is used for synthesis of the control systems carried out in this research. The second vehicle model, which is based on a high precision BMW 325i, 1988 model in the setup of veDYNA®, is used for simulation of the control systems developed in this research. In elaborating the usage of veDYNA®, all important GUIs, interfaces, and settings which are relevant to the simulation studies are discussed in detail.

3.1 Vehicle Model for Controller Design

3.1.1 Introduction

A simple vehicle model is used in the synthesis of controller models in the research. This simplified model was derived starting from a complex vehicle model using a number of assumptions [14]. These assumptions included a ‘no slip condition’ in previous designs with regard to longitudinal control [42], [62]. The validity of using such an assumption was confirmed experimentally [42]. Another assumption used in longitudinal control is that the slip produced by the torque converter between the engine and axle is taken as zero with locked ‘rotor’ condition [46].

A two stage engine model was used in formulating the complex vehicle model [14]. This two state engine model was reported before [165], [166]. In this model the engine torque production is continuous and is determined from a steady-state engine map (Ford Motor Company) [14].
3.1.2 Nomenclature: Vehicle Control Model

The following nomenclature is used in the formulation of the simplified vehicle:

- $x, \dot{x}$: Longitudinal position, longitudinal velocity
- $y, \dot{y}$: Lateral position, lateral velocity
- $\psi, \dot{\psi}$: Yaw angle, yaw rate
- $m$: Mass of the vehicle
- $C_x (C_y)$: Longitudinal (lateral) aerodynamic coefficients
- $C_{sf} (C_{sr})$: Cornering stiffness value for front (rear) tyres
- $\zeta_f (\zeta_r)$: Front (rear) velocity vector angles
- $F_{roll}$: Tyre rolling force
- $r_w$: Radius of the wheel
- $J_{\text{eng}}$: Inertia for engine
- $J_w$: Inertia for the wheel
- $l_f$: Distance from c.o.g. to vehicle front
- $l_r$: Distance from c.o.g. to vehicle rear
- $I_z$: Moment of inertia of the vehicle around z-axis through c.o.g.
- $r_{\text{drive}}$: Drive ratio
- $r_{\text{gear}}$: Gear ratio
- $\delta$: Steering angle
- $T_{\text{net}}$: Net engine torque
- $T_{\text{brk}}$: Brake torque
3.1.3 Vehicle Model for Control

In order for the adaptive fuzzy controllers to be designed, the simplified dynamic vehicle model developed by Pham is used [14]. Fig. 3.1 shows the schematics for the simple vehicle model.

The dynamic equations of the simplified vehicle can be written in the $x$, $y$ and yaw directions, as follows,

\[
\begin{align*}
\ddot{x} &= \frac{-(r^*)^2}{m(r^*)^2 + J_{mx} + 2J_{ly}r^2} \left[ C_x \dot{x}^2 + F_{roll} - m \dot{y} \dot{\psi} \right] \\
&+ \frac{(r^*)^2}{m(r^*)^2 + J_{mx} + 2J_{ly}r^2} \left[ \frac{1}{l_{r,t}} \left( T_{net} - \frac{3}{2} r^* \dot{T}_{brk} \right) - 2\delta C_{sf} \left( \delta - \xi_f \right) \right] + d_x \\
\ddot{y} &= -\frac{1}{m} \left[ C_y \dot{y}^2 + m \dot{y} \dot{\psi} + 2C_y \dot{\xi} + 2C_{sf} \dot{\xi} \right] + \frac{2}{m} C_y \delta + d_y \\
\ddot{\psi} &= l_t \left[ 2 \frac{L_s}{l_t} C_y \xi - 2C_{sf} \xi_f \right] + \frac{2l_t}{l_{r,t}} C_y \delta + d_{\psi}
\end{align*}
\]  

(3.1)

Here, $r^* = r_{\text{drive}} \times r_{\text{gear}}$ is the product of the ratios of the drive and the gear.

The input vector to the vehicle system can be defined as

\[
\mathbf{u}(q) = \begin{bmatrix} u_x \\ u_y \\ u_{\psi} \end{bmatrix} = \begin{bmatrix} \frac{-1}{r_{r,t}} \left( T_{net} - \frac{3}{2} r^* \dot{T}_{brk} \right) - 2\delta C_{sf} \left( \delta - \xi_f \right) \\ 2C_{sf} \delta \\ 2C_{sf} \delta \end{bmatrix}.
\]  

(3.2)
\( \mathbf{d} = \begin{bmatrix} d_x & d_y & d_\psi \end{bmatrix}^T \) indicates un-modeled dynamics, modeling uncertainties and bounded disturbances that can be demarcated as \( |d_i| \leq D_i, \ D_i(\in \mathbb{R}) > 0 \) where \( i = \dot{x}, \ \dot{y}, \ \psi. \)

The dynamic equations provided in (3.1), (3.2) are in terms of distance of the centre of gravity (c.o.g.) of the vehicle from the road centreline. It has been a common practice to describe the vehicle dynamics in terms of lateral displacement at the sensor [14, [18], and is therefore, taken as \( y_s \). The relationship between \( \dot{y} \) and \( \dot{y}_s \) is approximated as

\[ \dot{y}_s = \dot{y} + l_s \dot{\psi} \]  

(3.3)
where \( \dot{\psi} = \psi - \psi_{des} \), and \( \psi_{des} \) is the desired yaw angle, i.e. approximate tangential angle to the centreline of the lane. \( l_s \) is the distance from the vehicle centre of gravity to the vertical centre of the vehicle mounted sensor. Thereby, longitudinal, lateral and yaw dynamics of (3.1) can be written as follows, respectively, in the sensor space in concise form as

\[ \ddot{x} = h_x(\dot{q}_s) + b_x u_x + d_x \]  

(3.4)

\[ \ddot{y}_s = h_y(\dot{q}_s) + b_y u_y + d_y \]  

(3.5)

\[ \dot{\psi} = h_\psi(q) + b_\psi u_\psi + d_\psi, \]  

(3.6)
and in vector form as \( \dot{\mathbf{q}}_s = \mathbf{h}(\dot{q}_s) + \mathbf{b}(q_s) + \mathbf{d} \) where \( \mathbf{u}(q_s) = \mathbf{u}(q) \), \( \dot{\mathbf{q}}_s = [\dot{x}_s, \ \dot{y}_s, \ \dot{\psi}]^T \), \( \mathbf{b} = \text{diag}[b_x, b_y, b_\psi]^T = \text{diag}[\frac{(c_x)^2}{m(c_x)^2 + J_{xy} + 2J_2 \dot{\psi}^2}, \ \frac{1}{m} \frac{l_s}{T_s}]^T \) and

\[ \mathbf{h}(\dot{q}_s) = \begin{bmatrix} h_x(\dot{q}_s) & h_y(\dot{q}_s) & h_\psi(\dot{q}) \end{bmatrix}^T \]

\[ = \begin{bmatrix} \frac{-c_x \dot{x}_s}{m(c_x)^2 + J_{xy} + 2J_2 \dot{\psi}^2} 
\frac{C_x \ddot{x}^2 + F_{roll} - m(\dot{x}_s - l_s \dot{\psi}) \dot{\psi}}{m(c_x)^2 + J_{xy} + 2J_2 \dot{\psi}^2} 
\frac{\ddot{y}_s - l_s \dot{\psi}^2}{m} + m \dot{x}_s + 2C_s \ddot{\zeta}_r + 2C_{s\dot{r}} \dot{\zeta}_r \end{bmatrix} \frac{l_s}{T_s} \begin{bmatrix} 2 \frac{1}{T_s} C_{s\dot{r}} \dot{\zeta}_r - 2C_s \ddot{\zeta}_r \end{bmatrix}. \]  

(3.7)

The measurement of longitudinal speed, \( \dot{x} \) is carried out using a wheel speed sensor. The yaw rate of the vehicle, \( \dot{\psi} \) is obtained using a gyroscope [18]. The calculation of \( \dot{y}_s \) can be done using the measurements from a magnetometer and
magnetic markers on the road centreline or using a vision-based lane sensing system [18].

There is no separate control input to the yaw motion. It works as internal dynamics of the vehicle and is used to relate longitudinal and lateral motions [13], [14].

Using $u_x$ and $u_y$, the values of total torque, $T_{tot} = \left( T_{net} - \frac{3}{2} r T_{brk} \right)$ and steering angle, $\delta$ are calculated, first. With the usage of $T_{tot}$, the actual vehicle input values of throttle angle, $\alpha$ and applied brake force, $F_{pr}$ are calculated using an inverse mapping model based on (3.1), i.e. $f^{-1}(x) = \hat{f}^{-1}(x) + d_f$ where $|d_f| \leq D_f$, $D_f (\in \mathbb{R}) > 0$ is a bounded disturbance and it can be apportioned in the disturbance terms of the vehicle dynamic equations provided in (3.1) [167].

For example, an approximate throttle angle, $\alpha$ is calculated by inverting and approximating the given engine map in the BMW 325i 1988 model in veDYNA®, i.e.,

$$\alpha = f^{-1} \left[ \omega_{eng}, T_{net} \right]$$

where $\omega_{eng}$ is the engine speed. There have been previous studies that effectively compute the throttle and brake commands when a target engine torque is given [54].

The switching between brake and throttle signals is done based on the ‘sign’ of throttle angle. This method is used because it is a simplified form of switching and can be easily applied to practical controllers based on different control methods. A similar approach has been implemented within sliding mode control [54], [61].

$$\alpha = \begin{cases} 
\geq 0 & \text{throttle} \\
< 0 & \text{brake} 
\end{cases}$$

### 3.1.4 Control System Design Preliminaries

In this research, the control solution is developed for a general integrated lateral and longitudinal problem for a highway vehicle, i.e. following of a preceding vehicle at a prescribed safe spacing (constant spacing problem) while keeping the vehicle centred on the lane. In other words, this is the solution to address the so called ‘two-car platoon’ problem related to an Automated Highway System (AHS) [5]. But it is to be stressed that the concepts explained here can also be used in controlling lateral and longitudinal
vehicle control aspects under normal road conditions, e.g., adaptive cruise control coupled with lateral vehicle control.

### 3.1.5 Error Definitions

The longitudinal spacing error between the leading vehicle and the vehicle being controlled is taken as $e_x$, and the lateral deviation error from the centreline of the lane in the sensor space is taken as $e_y$. The error definitions can be expressed as

$$
\begin{align*}
    e_x &= x_p - x_{\text{spacing}} - x \\
    e_y &= y_d - y
\end{align*}
$$

(3.10)

Here, $x_p$ is the position of the leading vehicle. The desired inter-vehicular gap, $x_{\text{spacing}}$ can be chosen as a function of the vehicle speed, or simply as a constant depending on the prevailing traffic conditions [18]. These tracking error definitions are consistent with the previous lateral and longitudinal control designs [13], [14], [17], [18].

The desired lateral value, $y_d$ can be taken as zero as the aim is to follow the centreline of the lane. The calculation of lateral deviation, $y$ can be done using a magnetometer and magnetic markers [18].
3.2 veDYNA® Vehicle Model for Controller Validation

3.2.1 Introduction

veDYNA® is a simulation platform with a high precision model of vehicle dynamics based on BMW 325i, 1988 model. veDYNA® has been designed to simulate vehicle driving properties based on the characteristics of aerodynamics, steering, suspension, tyres, transmission, engine and drive train etc.

veDYNA® is implemented in MATLAB/Simulink® environment providing a standard interface ensuring modular and open-model architecture. This study used veDYNA® ‘standard’ version that offers extended possibilities to configure a simulation model, including the configuration of separate component models for drive train, wheel systems and tyres, engine, chassis and suspension etc. The model version used in this study is veDYNA® 3.10.

3.2.2 veDYNA® Main Graphical User Interface

veDYNA® main dialogue that gives access to control the entire simulation process is provided below in Fig. 3.2. This includes model data management and adjustments of the simulation environment, i.e. settings for simulation platform and Simulink® model. The GUI helps to configure the vehicle model: specify manoeuvre, driver and road settings; define trace variables; start the simulation and examine the results etc.
3.2.3 veDYNA® Accelerator Pedal Input (for Throttle Control)

The accelerator pedal input can be set as ‘external’, and therefore connected through the controller setup. This is through which the accelerator pedal input of the controllers developed in this research, is interfaced to the vehicle model. This GUI is shown in Fig. 3.3.
3.2.4 veDYNA® Steering Angle Input (for Steering Wheel Control)

The steering angle input also can be set as ‘external’ input, and thereby, connect through the controller setup. This is through which the steering input of the controllers developed in this research, is interfaced to the vehicle model. This GUI is shown in Fig. 3.4.

![Fig. 3.4 Steering angle input (set as ‘external’ input)](Parent figure → Fig. 3.2, ‘main GUI’)

3.2.5 veDYNA® Brake Pedal Input

Similar to the other two inputs, the brake pedal input also can be set as an ‘external’, and thereby connected through from the controller setup. This is through which the brake input of the controllers developed in this research, is interfaced to the vehicle model. The GUI is shown in Fig. 3.5, below.
Fig. 3.5 Brake pedal input (set as an ‘external’ input) (Parent figure \(\rightarrow\) Fig. 3.2, ‘main GUI’)

3.2.6 veDYNA® Vehicle Assembly GUI

The vehicle assembly GUI, illustrated in Fig. 3.6, allows one to select the required vehicle configuration from the list of subsystems. It also allows to interact with and change parameters of the provided subsystems.

It is possible to navigate through the vehicle component structure by selecting a vehicle component in the structure tree displayed on the left browser window. The selected parameter files can be further edited. A modified or newly generated configuration can then be saved (under a different name, if required). Some component lines have an additional button, which generates a meaningful visualisation of the data record, especially to verify tables or force characteristics (e.g. engine torque table, spring characteristic, axle kinematics etc.).
3.2.7 Road Settings

The road profile can be constructed using a combination of segments of ‘linear’, ‘circle’, ‘clothoid’, ‘polynomial’ or ‘cubic-splines’. This is carried out through the main GUI interface on Fig. 3.2.

3.2.8 Handling External Models

External models can be linked by deactivating the switches in the internal models that are to be replaced by user defined Simulink® blocks.

3.2.9 Animation of Simulation

A certain scenario of a simulation study is depicted in the animation window shown in Fig. 3.7. In the provided animation window, the motion of the controlled vehicle can be observed. In this figure, the lead vehicle does not appear visually, though (Note: this facility is not provided in veDyna®). But, it acts logically in the simulation setup as defined in the control problem. Thereby, the logical motion of the lead vehicle influences the motion of the controlled vehicle.
3.2.10 veDYNA® Simulink® Model

The veDYNA® Simulink® Model is shown in Fig. 3.8. In this figure, the block External I/O contains pre-defined input/output (I/O) interfaces for brake hydraulics of the vehicle as well as inputs from the engine. The block veDYNA Simulation Model is the core of the program. Data flows to the external I/O block are via the ports ‘Output to I/O’ and ‘Input from I/O’.

Fig. 3.8 veDYNA® Simulink® model setup
3.2.11 veDYNA® Simulation Model

veDYNA simulation model is implemented in Simulink® and is shown in Fig. 3.9. The main model components, i.e. vehicle, road, and manoeuvre control, are included in the Simulink® model as S-functions. Different vehicle configurations, e.g. with or without trailer, lead to different vehicle models in Simulink®. The Simulink® model also includes an interface to import external data and external I/O for real-time applications.

Each block depicted in Fig. 3.9 stands for a model part or a task. The model parts ‘Manoeuvre Control’ and ‘Vehicle System’ are again multi-layered structures. The user may modify these subsystems or include user-defined model parts for manoeuvre control and vehicle components.

Fig. 3.9 veDYNA® Simulink® vehicle model (Parent figure → Fig. 3.8, ‘veDYNA® Simulink® model setup’

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3.2.12 veDYNA® Vehicle Model Implementation: Triggered System

Since it has been found that inclusion of fuzzy systems causes the vehicle model to look ‘stiff’ to be solved by the default ordinary differential equation solver, i.e., ‘ode1’ (Euler), the vehicle model is enclosed in a triggered system, as a solution. This setup allows the control system to run at a sample time of 0.0005 [s] with the higher order solver, ‘ode4’ (Runge-Kutta), while the rest of the veDYNA® system is set to work with the original sample time (i.e. 0.01 [s]). This arrangement is shown in Fig. 3.10. This inclusion of a triggered system is a newly added feature in this simulation setup, and is not a part of the original setup. However, this is a recommended method to synchronise the integration schemes, i.e. by putting all veDYNA® modules into a triggered subsystem.

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Fig. 3.10 Triggered block enclosing veDYNA® vehicle model (Parent figure → Fig. 3.8, ‘main model setup’)

---
3.2.13 veDYNA® Manoeuvre Control Block

The veDYNA® ‘Manoeuvre Control’ block, grouped into the blocks ‘Manoeuvre Scheduler’ and ‘Manoeuvre Controller’, contains all S-functions required to perform a manoeuvre. This is shown in Fig. 3.11. Additionally, there are blocks ‘Manoeuvre’, ‘Controller Inputs’, ‘Manoeuvre Scheduler Inputs’ and ‘HCI Inputs’ that provide necessary manoeuvre data interfaces.

3.2.14 Manoeuvre Controller Inputs

The manoeuvre controller inputs can be fed through the relevant blocks in the ‘Manoeuvre Controller Inputs’, as shown below in Fig. 3.12.
3.2.15 Longitudinal Control Inputs

The longitudinal control inputs, ‘throttle angle’ and ‘brake pedal position’ are fed through the ‘longitudinal control inputs’ blocks, as shown below in Fig. 3.12.

![Fig. 3.13  Longitudinal control input (Parent figure → Fig. 3.12, ‘Manoeuvre controller input block’)](image)

3.2.16 Lateral Control Inputs

The lateral control input, ‘steering angle’, is fed through the ‘lateral control inputs’ block, as shown below in Fig. 3.13.

![Fig. 3.14 Lateral control input (Parent figure → Fig. 3.12, ‘Manoeuvre controller input block’)](image)
3.2.17 External Controller Interface to veDYNA®

The developed controller setup (‘blue coloured’) is interfaced to the veDYNA® Simulink® setup using the following block shown in Fig. 3.15.

![Fig. 3.15 Control input interface to veDYNA® blocks (Parent figure→ Fig. 3.8, ‘External I/O / veDYNA Simulink model’)](image)

3.2.18 Settings for External Disturbances for Vehicle System

The following external disturbances can be setup in the veDYNA® system using the interfaces provided as described below.

(i) Unsymmetrical Load Mass

With addition of an unsymmetrical load mass, the resulting ‘second moment of area’ and ‘centre of gravity’ values are needed to be specified in the veDYNA® system. Therefore, the particular new values are defined with respect to the reference axes of the vehicle, in the relevant m-file (with parameters on load mass) as accessed from the veDYNA® Vehicle Assembly GUI. This particular program script is shown below. The
schematic diagram providing the relative dimensions of the location of the load mass is shown in Fig. 3.16.

Fig. 3.16 Unsymmetrical load mass arrangement (Note: Figure dimensions are not to proportion)

The part of the program script of the m-file defining ‘centre of gravity’ and ‘second moment of area’ values, is shown below in (3.11).

```
mass1 = 200; % default 0.0; % [kg] mass
rblk(1) = -2.0; % default 0.0; % [m] x-value CG
rblk(2) = 0.5; % default 0.0; % [m] y-value
rblk(3) = 0.1; % default 0.0; % [m] z-value CG

(3.11)
ttlk(1) = 1.33; % default 0.0; % [kgm**2] xx
ttlk(3) = 3.33; % default 0.0; % [kgm**2] yy
ttlk(6) = 3.33; % default 0.0; % [kgm**2] zz
mainbody_flag = 1; % default 0;
```

The moment of inertia values, i.e., $I_{xx}$, $I_{yy}$ and $I_{zz}$ with respect to the $x$, $y$ and $z$ axes were calculated based on the dimensions and the mass of the load.

(ii) Crosswind Effects

In order to set a uniform crosswind speed of 50 [m/s] in the system, the relevant program file, an m-file (with parameters on aerodynamics of the vehicle), was adjusted in the veDYNA® Vehicle Assembly GUI. The specific program script is shown below in (3.12).
(iii) Tyre-Road Friction Change

In order to simulate a friction change event, the tyre-road friction coefficient was set to 0.5 (from normal 1.0 value) only for the segment of the road that starts at 100 [m] and runs through up to 200 [m] using the window of ‘z-profile setting’ under ‘Advanced Road’ section. This setting is shown below in Fig. 3.17.

![Fig. 3.17 z-profile setting window (Parent figure Fig. 3.2, 'veDYNA® main GUI')](image)

3.2.19 Settings for Failure Modes (Non-Catastrophic)

The following failure mode cases were set by using additional function blocks provided in the veDYNA® library. Some changes were done as and when necessary in order to obtain the particular simulation setting.

(i) Sudden Tyre Deflation Incident

In order to create a sudden tyre deflation incident, an external ‘flat tyre block’ (a veDYNA® library block) was added to the ‘veDYNA Simulink Vehicle Model’, as shown in Fig. 3.18 below. With the execution of the ‘script’ that follows, it is possible for the system to disable the internal tyre block arrangement, and thereby switch the control to the external tyre block.
The ‘External Flat Tyre’ model is meant to trigger a sudden pressure loss of a tyre (or tyres). The tyre ‘flatness’ is modelled by changing the vertical and the lateral tyre stiffness as well as the rolling resistances, at the specified time of the pressure loss. The other tyre parameters (e.g. the tyre radius) remained unchanged.

The following script, (3.13), was run in the MATLAB® command prompt in order to set the deflation only in the front-left tyre after 10 [s], with a decreased longitudinal roll coefficient of 0.5.

```
MDL.FLATTYRE.FL.time.x=10  % Front-left tyre set to deflate at 10s
MDL.FLATTYRE.RL.time.x=0   % No flat tyre event set
MDL.FLATTYRE.FR.time.x=0   % No flat tyre event set
MDL.FLATTYRE.RR.time.x=0   % No flat tyre event set
MDL.FLATTYRE.xroll.x=0.5   % Roll resistance coefficient for flat tyre
```

It is to be noted that, if the flat-tyre time is set to zero the tyre is always inflated.
(ii) Defective Wheel Brake Cylinder (front-left wheel)

In order to simulate a pressure drop in the brake line leading to front-left wheel cylinder, an external brake system block (a veDYNA® library block) was added to the ‘Vehicle System’ block of ‘veDYNA Simulink Vehicle Model’. This is shown in Fig. 3.19. With the execution of the ‘script’ that follows, it is possible to disable the internal tyre block arrangement, and thereby, switch the control to the external block.

![veDYNA Vehicle System](image)

Fig. 3.19 External brake system block (veDYNA® library block) (Parent figure → Fig. 3.9, ‘veDYNA Simulink vehicle model’)

The following script, (3.14) is run in the MATLAB® command prompt in order to disable the internal brake system while activating the external brake system.

```
MDL.EXTBRK.rbr.ft.v=0.105  % set with the same parameters as the internal
MDL.EXTBRK.rbr.rr.v=0.105  % set with the same parameters as the internal
MDL.EXTBRK.abrk.ft.v=1.8E-3 % set with the same parameters as the internal
MDL.EXTBRK.abrk.rr.v=1.8E-3 % set with the same parameters as the internal
MDL.EXTBRK.muebr.ft.v=0.35 % set with the same parameters as the internal
MDL.EXTBRK.muebr.rr.v=0.35 % set with the same parameters as the internal
MDL.EXTBRK.t0.v=0          % time delay to reaction time of the wheel brake
```
In the activated external brake system, the defect in the front-left wheel cylinder was modelled as shown in Fig. 3.20. The defect in the front-left wheel cylinder was simulated by reducing the pressure of the brake line by 90% from the normal value. The pressure lines of the other wheel cylinders were assumed to remain as normal during the simulation.

Fig. 3.20 External brake system-brake lines (with the front-left wheel cylinder set as defective with 90% pressure drop in the brake line) (Parent figure → Fig. 3.19 External brake system block)

### 3.2.20 Calculation of $h$ Value for veDYNA® Vehicle Model

The value of $h$ as indicated by (3.7) is required for the analysis of the simulation and for data collection in the design of the fuzzy systems. Therefore the value of $h$ of the veDYNA® model can be calculated using the following equation.

$$ h_x(\dot{q}_x) = \ddot{x} - b_x u_x $$
$$ h_y(\dot{q}_y) = \ddot{y} - b_y u_y $$

(3.15)

The values of $\ddot{x}$ and $\ddot{y}$ can be obtained from the vehicle model sensor feedback. This is possible since $b_x$ and $b_y$ are known, and the control inputs $u_x$ and $u_y$ can be calculated.
3.3 Chapter Summary

In the first part of this chapter, a simple vehicle model to be used in the design of all control systems in this research is described in detail. This description also includes details on throttle/brake switching criterion used in the design of the controllers.

In the second part of the chapter, a veDYNA® setup, a simulation platform providing a high-precision vehicle model that is used as the validation platform with simulations for all the controllers developed in this research, is described in detail. This description includes all GUIs, interfaces and setups related to veDYNA® under different conditions of working of the vehicle for simulating vehicle control operations.
In this chapter, the development of a single-model adaptive fuzzy controller for integrated lateral and longitudinal control of highway vehicles is described. The single-model adaptive fuzzy system is comprehensively designed and its parameters are tuned using a subtractive clustering method, first, and then the ANFIS (Adaptive Networks Fuzzy Inference Systems) method for fine adjustments. Following tuning, the system of single-adaptive fuzzy control is established with stability analysis using a Lyapunov-based method, i.e. KYP lemma, leading to asymptotic stability at global level. Finally, the developed controller is tested for its robust qualities with validation studies using simulations in veDYNA®, exposing the controller to a number of robust conditions, including a few external disturbances and a couple of non-catastrophic failure cases of vehicle subsystems.
4.1 Robust Single-Model Adaptive Fuzzy Control System

4.1.1 Introduction

Vehicle control, especially involving integrated lateral and longitudinal control, has to deal with significant nonlinearity effects. According to the details highlighted in the main introduction, vehicle systems require specialized controllers.

In order to address the nonlinearity effects, and other uncertainty effects in vehicle control problems, a single adaptive fuzzy controller is designed as described in this chapter. The control system also contains a proportional-derivative (PD) module, which is important for coarse tuning in the solution of control problem. On the other hand, adaptive fuzzy system is for robust tuning or fine tuning of the controller addressing non-linearity effects, coupling effects, and model uncertainties.

4.1.2 Synthesis of Single-Adaptive Fuzzy Controller

In this section, the synthesis of a single-model adaptive fuzzy controller system is described. The PD module in the control system is for tracking linear components of the vehicle system. The fuzzy system is employed to address nonlinear components of the control problem. These nonlinear effects involve coupling effects, model discrepancies, and un-modelled dynamics. In addition to these modules, a variable structure control term is used in the control system for specifically addressing fuzzy approximation errors, and to overcome disturbances to certain extent.

Instead of using a PD module, if an integral component (I) was included in the system, e.g. as a PI system or as a PID system, the control system might be more susceptible to what is called a ‘winding up’ problem. In other words, the closed loop system would become unresponsive, and therefore lead to look like an open loop system. This whole phenomenon where there is a discrepancy between the controller output and the plant input due to actuator saturation is called integrator or controller windup in literature [168]. This phenomenon is due to the fact that in real-world controlled engineering systems, actuator capacity is limited by the inherent physical
constraints. In other words, actuators are confined to operate between certain saturation limits. This windup problem could result in a significant performance degradation, large overshoots or un-decaying transients. There is a more tendency of the integral controllers (PI or PID) to ‘windup’ during input saturation. In order to prevent this happening with the inclusion of an integral component in the controller, it is required to use an anti-windup module. But, this addition would increase the nonlinearity of the system, and therefore would require more complex analysis for establishing stability. This becomes a much more challenging problem in a complex nonlinear system [169]. In order to avoid these complexities, a PD system was used in this research without including any integral component.

(i) Control Law

The certainty equivalence control laws based on adaptive fuzzy control and proportional-derivative components can be formulated for longitudinal and lateral cases, respectively, as follows.

\[ \hat{u}_s = \frac{1}{b_s} \left[ \dot{x}_p + k_{dx} \dot{e}_s + k_{px} e_s + \hat{V}_s + u_{sx} - \hat{h}_s(q_s) \right] \]  
(4.1)

\[ \hat{u}_y = \frac{1}{b_y} \left[ \dot{y}_p + k_{dy} \dot{e}_y + k_{py} e_y + \hat{V}_y + u_{sy} - \hat{h}_y(q_y) \right]. \]  
(4.2)

Here, \( \hat{h}_{s,y}(q_s) \) denotes the calculated value of the term \( h_{s,y}(q_s) \). \( \hat{V}_s \) and \( \hat{V}_y \) are the inputs from the adaptive fuzzy controller for longitudinal and lateral cases, respectively. \( k_{dx}, k_{px} \) are the constant PD (proportional-derivative) gains for the longitudinal control, and \( k_{dy}, k_{py} \) are the constant PD gains for the lateral control. \( u_s = \begin{bmatrix} u_{sx} & u_{sy} \end{bmatrix} \) is the variable structure control term.

(ii) Error Dynamics

The control inputs from (4.1) can be substituted in the vehicle model equation (3.4) of Chapter 3, and it can be rearranged to have the longitudinal error dynamics as

\[ \ddot{e}_s + k_{ds} \dot{e}_s + k_{ps} e_s = \hat{h}_s(q_s) - h_s(q_s) - \hat{V}_s - u_{sx} - d_s. \]  
(4.3)
Since the developed fuzzy controller is used to approximate model uncertainties, 
\( \hat{h}_{\alpha}(\hat{q}_{\alpha}) - h_{\alpha}(q_{\alpha}) \) etc., it can be replaced by the fuzzy functions as 
\[
\ddot{e}_{\alpha} + k_{d\alpha}\dot{e}_{\alpha} + k_p e_{\alpha} = \nu_{\alpha} + w_{\alpha} - \dot{\nu}_{\alpha} - u_{st} - d_{\alpha}
= \nu_{\alpha} - u_{st} - d_{\alpha}.
\]

In the same manner, using (4.2) and vehicle model equation (3.5) of Chapter 3, 
the lateral error dynamics will result in as 
\[
\ddot{e}_{\gamma} + k_{d\gamma}\dot{e}_{\gamma} + k_p e_{\gamma} = \hat{h}_{\gamma}(\hat{q}_{\gamma}) - h_{\gamma}(q_{\gamma}) - \dot{\nu}_{\gamma} - u_{sy} - d_{\gamma}
= \nu_{\gamma} + w_{\gamma} - \dot{\nu}_{\gamma} - u_{sy} - d_{\gamma}
= \dot{\nu}_{\gamma} - u_{sy} - d_{\gamma}.
\]

In this discussion, \( \nu^{*} = [\nu_{\alpha}^{*}, \nu_{\gamma}^{*}] \) is the optimal adaptive fuzzy output while 
\( \dot{\nu} = [\dot{\nu}_{\alpha}, \dot{\nu}_{\gamma}] \) is the actual adaptive fuzzy output. \( w_{\alpha}, w_{\gamma} \) are the adaptive fuzzy approximation errors, and are bounded by \( |w_{\alpha}| < W_{\alpha}, |w_{\gamma}| < W_{\gamma} \) where \( W_{\alpha}, W_{\gamma} \in \mathbb{R} \) are the practical bounds.

The universal approximation theorem with regard to a typical fuzzy system can be 
expressed as follows.

Lemma 4.1: [28] For any given continuous function, \( \varphi(x) \), on a compact set \( U \in \mathbb{R}^n \), 
with an arbitrary constant \( \varepsilon(\in \mathbb{R}) > 0 \), there exist a fuzzy system, \( \hat{\nu} \) such that 
\[
\sup_{x \in U} |\varphi(x) - \hat{\nu}| < \varepsilon. \quad \square
\]

By using the above universal approximation theorem, with the fuzzy adaptation 
process, the adaptive fuzzy controller parameters can be gradually brought to optimal 
values, and thus the right half of the equations, (4.4) and (4.5) can be made zero.

Before formulating the single-adaptive fuzzy controller, the following brief 
overview of fuzzy system principles is presented.
4.1.3 Fuzzy Control: Takagi-Sugeno (T–S) Fuzzy Systems

A brief description of general fuzzy systems and T–S fuzzy system is presented. This brief is almost reproduced here as described by Passino and Yurkovich in [28].

(i) Fuzzy System Elements

Generally, a fuzzy controller is composed of the following four main elements.

1. A rule-base that holds the knowledge (in the form of a set of IF-THEN rules) that contains a fuzzy logic quantification of the controller engineer’s linguistic description of how to achieve good control.

2. An inference mechanism (also called an “inference engine” or “fuzzy inference” module) that evaluates which control rules are relevant at the current moment, and also that emulates design engineer’s decision making in interpreting and applying knowledge about how best to control the vehicle.

3. A fuzzification interface that converts or modifies controller inputs into information by means of mapping the inputs to a defined so called ‘membership function’ so that they can be easily interpreted and compared to the rules in the rule base.

4. A defuzzification interface that converts the conclusions reached by the inference mechanism into the actual inputs to the plant (vehicle).

(ii) Fuzzy System Concepts

The concepts related to fuzzy systems are described below.

(a) Fuzzy System

A fuzzy system (normally) is a static nonlinear mapping between its inputs and outputs (i.e. it is not a dynamic system). It is assumed that the fuzzy system has inputs $u_i \in U_i$ where $i = 1, \ldots, n$ and outputs $y_i \in Y_i$ where $i = 1, \ldots, m$. The inputs and outputs are crisp values, i.e. real numbers as against being fuzzy sets. The fuzzification process converts the crisp inputs to fuzzy sets while the inference mechanism uses the fuzzy rules in the rule-base to produce fuzzy conclusions. Finally, the defuzzification block
converts these fuzzy conclusions into the crisp outputs so that they can be used by real-world systems.

(b) Universes of Discourse

The ordinary sets or crisp sets, \( U_i \) and \( Y_i \), are called the “universes of discourse” or domains for \( u_i \) and \( y_i \), respectively. In practical applications, most often the universes of discourse are simply the set of real numbers or some interval or subset of real numbers. Also, sometimes it is convenient to refer to an “effective” universe of discourse \([\alpha, \beta]\) where \( \alpha \) and \( \beta \) are the points at which the outermost membership functions saturate for input universes of discourse, or the points beyond which the outputs will not move for the output universe of discourse.

(c) Linguistic Variables

To specify rules for the rule-base, the design engineers will use a ‘linguistic description’, hence, linguistic expressions are needed for the inputs and outputs and the characteristics of the inputs and outputs. The ‘linguistic variables’ or constant symbolic descriptions of general time-varying quantities are used to describe fuzzy system inputs and outputs.

(d) Linguistic Values

Linguistic variables \( \tilde{u}_i \) and \( \tilde{y}_i \) take on ‘linguistic values’ that are used to describe characteristics of the variables. Let \( \tilde{A}_i^j \) denote the \( j \)th linguistic value of the linguistic variable \( \tilde{u}_i \) defined over the universe of discourse \( U_i \). If it is assumed that many linguistic values are defined over \( U_i \), then the linguistic variable \( \tilde{u}_i \) takes on the elements from the set of linguistic values as defined by

\[
\tilde{A}_i = \{ A_i^j : j = 1, 2, \ldots, N_i \}.
\] (4.6)

Linguistic values are generally descriptive terms such as “positive large”, “zero”, and “negative large” etc.
(e) Linguistic Rules

The mapping of the inputs to the outputs for a fuzzy system is in part characterized by a set of condition → action rules, or in *modus ponens* (If-Then) form,

\[ \text{If premise Then consequent.} \]

Usually, the inputs of the fuzzy system are associated with the premise, and the outputs are associated with the consequent. These If-Then rules can be represented in a number of forms, e.g. multi-input-single-output (MISO) or multi-input-multi-output (MIMO) etc. The MISO form of a linguistic rule for a standard system (Mamdani) is,

\[ \text{If } \bar{u}_1 \text{ is } \bar{A}_1 \text{ and } \bar{u}_2 \text{ is } \bar{A}_2 \text{ and } \ldots \text{ and } \bar{u}_n \text{ is } \bar{A}_n \text{ Then } y = \bar{B}_p \quad (4.7) \]

But, for a functional fuzzy system where T–S (Takagi-Sugeno) type is a special case, it becomes

\[ \text{If } \bar{u}_1 \text{ is } \bar{A}_1 \text{ and } \bar{u}_2 \text{ is } \bar{A}_2 \text{ and } \ldots \text{ and } \bar{u}_n \text{ is } \bar{A}_n \text{ Then } b_i = g_i(\bullet). \quad (4.8) \]

The difference between a standard fuzzy system and a T–S fuzzy system is in the consequent part. The premise parts are similar in construction. Here, \( b_i \) can be any function, but linear or affine functions are mostly used.

(f) Membership Functions

Let \( U_i \) denote a universe of discourse and \( \bar{A}_i \subset \bar{A}_i \) denote a specific linguistic value for the linguistic variable \( \bar{u}_i \). The function \( \mu(u_i) \) associated with \( \bar{A}_i \) that maps \( U_i \) to \([0, 1]\), is called a ‘membership function’. This membership function quantifies the “certainty” that an element of \( U_i \), denoted \( \bar{u}_i \), can be classified linguistically as \( \bar{A}_i \). Membership functions are subjectively specified in an ad-hoc (heuristic) manner from experience or intuition.
(g) Fuzzy Sets

Given a linguistic variable $\tilde{u}_i$ with a linguistic value $\tilde{A}_i^j$ defined on the universe of discourse $U_i$, and membership function $\mu_{A_i}(u_i)$ (membership function associated with the fuzzy set $A_i^j$) that maps $U_i$ to $[0, 1]$, a “fuzzy set” denoted with $A_i^j$ is defined as

$$A_i^j = \{ (u_i, \mu_{A_i}(u_i)) : u_i \in U_i \}$$

(Note: a fuzzy set is simply a crisp set of pairings of elements of the universe of discourse coupled with their associated membership values).

(h) Fuzzification

Fuzzy sets are used to quantify the information in the rule-base, and the inference mechanism operates on fuzzy sets to produce fuzzy sets, hence, it is required to specify how the fuzzy system will convert its numeric inputs $u_i \in U_i$ into fuzzy sets (a process called ‘fuzzification’) so that they can be used by the fuzzy system.

Let $U_i^*$ denote the set of all possible fuzzy sets that can be defined on $U_i$. Given $u_i \in U_i$, fuzzification transforms $u_i$ to a fuzzy set denoted by $\hat{A}_i^{fuz}$ defined on the universe of discourse $U_i$. This transformation is produced by the fuzzification operator $F$ defined by

$$F: U_i \rightarrow U_i^*$$

where $F(u_i) = \hat{A}_i^{fuz}$.

Quite often, ‘singleton fuzzification’ is used, which produces a fuzzy set $\hat{A}_i^{fuz} \in U_i^*$ with a membership function defined by

$$\mu_{\hat{A}_i^{fuz}}(x) = \begin{cases} 
1 & x = u_i \\
0 & \text{otherwise} 
\end{cases}$$

Any fuzzy set with this form for its membership function is called a ‘singleton’.

(i) The Inference Mechanism

The inference mechanism has two basic tasks:
(1) Determining the extent to which each rule is relevant to the current situation as characterized by the inputs \( u_i, \ i = 1, \ldots, n \) (this task is called ‘matching’), and

(2) Drawing conclusions using the current inputs \( u_i \) and the information in the rule-base (this is called an ‘inference step’). For matching, note that \( A_{j1} \times A_{j2} \times \cdots \times A_{jn} \) is the fuzzy set representing the premise of the \( i \)th rule \((j, k, \ldots, l; p, q)\) (there may be more than one such rule with this premise).

(j) Matching

If the inputs are taken as \( u_i, \ i = 1, \ldots, n \), and fuzzification produces \( \hat{A}_{1}, \hat{A}_{2}, \ldots, \hat{A}_{n} \) (the fuzzy sets representing the inputs), then there are two basic steps to matching.

**Step 1: Combine Inputs with Rule Premises:**

The first step in matching involves finding fuzzy sets \( \hat{A}_{j1}, \hat{A}_{j2}, \ldots, \hat{A}_{jn} \) with membership functions

\[
\mu_{\hat{A}_{j1}}(u_1) = \mu_{\hat{A}_{j1}}(u_1) \ast \mu_{\hat{A}_{j2}}(u_1) \\
\mu_{\hat{A}_{j2}}(u_2) = \mu_{\hat{A}_{j2}}(u_2) \ast \mu_{\hat{A}_{j3}}(u_2) \\
\vdots \\
\mu_{\hat{A}_{jk}}(u_k) = \mu_{\hat{A}_{jk}}(u_k) \ast \mu_{\hat{A}_{jl}}(u_k) \\
\mu_{\hat{A}_{jn}}(u_n) = \mu_{\hat{A}_{jn}}(u_n) \ast \mu_{\hat{A}_{j1}}(u_n) \\
\text{(for all } j, k, \ldots, l) 
\]

As with singleton fuzzification, it gives \( \mu_{\hat{A}_{jn}}(u_i) = 1 \) for all \( i = 1, \ldots, n \) for the given \( u_i \) inputs, so that

\[
\mu_{\hat{A}_{j1}}(u_1) = \mu_{\hat{A}_{j1}}(u_1) \\
\mu_{\hat{A}_{j2}}(u_2) = \mu_{\hat{A}_{j2}}(u_2) \\
\vdots \\
\mu_{\hat{A}_{jn}}(u_n) = \mu_{\hat{A}_{jn}}(u_n). 
\]
Step 2: Determine which rules are on:

In the second step, the membership values \( \mu_i(u_1, u_2, \ldots, u_n) \) can be formed for the \( i \)th rule’s premise that represents the certainty that each rule premise holds for the given inputs. Define

\[
\mu_i(u_1, u_2, \ldots, u_n) = \mu_{A_{i1}}(u_1) \ast \mu_{A_{i2}}(u_2) \ast \ldots \ast \mu_{A_{in}}(u_n),
\]

which is simply a function of the inputs \( u_i \). When singleton fuzzification is used, it can be reduced to have

\[
\mu_i(u_1, u_2, \ldots, u_n) = \mu_{A_{i1}}(u_1) \ast \mu_{A_{i2}}(u_2) \ast \ldots \ast \mu_{A_{in}}(u_n).
\]

Here, \( \mu_i(u_1, u_2, \ldots, u_n) \) is used to represent the certainty that the premise of rule \( i \) matches the input information when singleton fuzzification is used.

An implied fuzzy set can be defined for a standard fuzzy system. But this step is not considered for functional fuzzy systems, since the consequent is not a fuzzy linguistic function.

(k) Defuzzification

A number of defuzzification strategies exist for standard fuzzy systems, e.g. centre-average method. As far as the scope of this research is concerned, the defuzzification of T–S fuzzy systems can be considered with a similar ‘centre-average’ method, as follows,

\[
y = \frac{\sum_{i=1}^{R} b_i \mu_i}{\sum_{i=1}^{R} \mu_i}
\]

where \( \mu_i \) is as provided in (4.14).

(l) T–S Fuzzy Systems

In this research, mainly T–S fuzzy systems based on Gaussian input fuzzy functions are used to construct adaptive fuzzy controllers and fuzzy ‘blending’ (soft-
switching systems) structures. In adaptive controller structures, the input fuzzy function parameters are kept fixed while making the output membership function parameters, i.e., the functional members, as adaptive. Generally, the ‘blending’ fuzzy structures are non-adaptive in form as used in this research.
4.1.4 Design of Single-Adaptive Fuzzy Controller

In this section, the synthesis of a robust single-adaptive fuzzy controller in solving the integrated lateral and longitudinal control problem of highway vehicles is described. Fig. 4.1 explains the setup for each adaptive fuzzy controller with reference to the equations (4.1) and (4.2). The term, $\hat{\nu}_{x,y}$ is the adaptive fuzzy output.

In this adaptive fuzzy control system, in addition to the basic architecture of a fuzzy controller, an adaptation algorithm is used for updating the output membership function parameters. The input fuzzy set parameters are kept fixed throughout.

![Fig. 4.1 Adaptive fuzzy system and PD module for longitudinal (lateral) vehicle controller](image)

There are two inputs (lateral and longitudinal) for each adaptive fuzzy controller, i.e., the ‘error’, $e$ and the ‘error rate’, $\dot{e}$. The adaptive fuzzy function output, $\hat{\nu} \in \mathcal{R}$ is obtained after defuzzification as a crisp value, and is fed together with the inputs from the PD control module to the vehicle. A variable structure control term, $u_s$ is incorporated into the control system for addressing fuzzy approximation errors and thereby to ensure system stability.

The usage of fuzzy control instead of variable structure control alone can be further justified. There is a major drawback in the variable structure control approach, which is the phenomenon of chattering due to high frequency switching. These chattering can often excite undesired dynamics. But fuzzy control can provide an effective way to resolve this problem in the presence of variable structure control [170]. Moreover, fuzzy control can further provide the smoothness in operation by selecting smooth membership functions such as Gaussian functions [171].
(i) Justification of using error, error rate as Inputs to Fuzzy System Instead of States

As explained related to equations (4.3) and (4.4), the main objective of developing a fuzzy function is to minimize the system error and tend it zero.

When the two equations (4.3) and (4.4) are taken separately, for longitudinal dynamics and lateral dynamics, the term that gives the difference between the ‘vehicle dynamics’, i.e. \( h_s(\dot{q}_s) \) (or \( h_y(\dot{q}_y) \)) and the ‘model dynamics’, i.e. \( \hat{h}_s(\dot{q}_s) \) (or \( \hat{h}_y(\dot{q}_y) \)), can be defined as a function of error. This fact becomes obvious when reference is made to left hand side of each equation. This explains that the fuzzy function that is used to model uncertainty factor can use ‘error’ and ‘error rate’ as inputs, and thereby, generate a function to cancel out the ‘uncertainty’ term.

Additionally, by having error and error rate as inputs in the adaptive fuzzy system, it is possible to model the system in accordance with the tracking problem. On the other hand, error (thereby error rate) is itself dependent on the states of the system, though, in an indirect way. More specifically, error (also error rate) provides a measure of a state with respect to the reference. Therefore, error provides information of a state with respect to the ‘intended goal’ at every moment, and this is more useful to a fuzzy system to base itself to ‘behave’ in a manner so that it can lead ultimately to pave the way to minimize the gap between the ‘state’ and its ‘reference’.

In this regard, error rate provides a futuristic direction of the ‘state’ with respect to the reference, and further enhances the understanding of the overall system behavior by the fuzzy system.

In contrast to using error and error rate, if ‘states’ were straight away used as inputs to the fuzzy system, it is difficult to compare the states against a ‘benchmark value’ and thereby produce its functional input to further minimize the system error. In short, by having error and error rate as inputs in the adaptive fuzzy system, it effectively paves the gap between the ‘vehicle’ and the ‘vehicle model’ as provided in the error dynamics (referring to (4.4) and (4.5)).

This description justifies usage of error and error rate as inputs in the adaptive fuzzy system as against using ‘states’ as inputs in the same system.

In line with the above arguments, in approximating the right half of (4.3) and (4.4), error and error rate are used as input variables in each lateral and longitudinal
adaptive fuzzy system in this study. It is also mentioned and clarified that ‘error’ and ‘error rate’ can be used in each fuzzy rule in order to partially define the ‘states’ of a dynamic system [28].

**Remark 4.1:** The advantage of taking $e$, $\dot{e}$ as fuzzy inputs allows effective design of the fuzzy system. This is because the causal relationship between the inputs and fuzzy rules are clear and straightforward. In contrast to this, if the system ‘states’ were used as inputs to the fuzzy system, there would not be any clear way to identify the relationship between them and the system error, even at least for initial parameters of the fuzzy system. □

**(ii) Adaptive Fuzzy Systems**

The detailed synthesis of the adaptive fuzzy controller is presented here. The $x$ and $y$ subscripts of the two adaptive fuzzy systems for longitudinal and lateral cases, respectively, are omitted for it to be a common construction for both lateral and longitudinal cases.

The set of T–S fuzzy IF-THEN rules for the adaptive fuzzy system can be expressed as

$$R^j: \quad \text{If } e \text{ is } A^j_i \text{ and } \dot{e} \text{ is } A^k_2 \quad \text{Then } z_j = a_{ij}e + b_{ij}\dot{e} \quad \vdots$$

$$R^n: \quad \text{If } e \text{ is } A^n_i \text{ and } \dot{e} \text{ is } A^n_k \quad \text{Then } z_n = a_{ni}e + b_{ni}\dot{e},$$

where $n$ is the total number of fuzzy rules. $A^j_i$ and $A^k_j$ are the $l$th and $k$th linguistic values associated with the linguistic variables of the inputs, $e$ and $\dot{e}$, respectively, in the $j$th rule. $z_j$ is the consequent of the $j$th rule where $j=1, \ldots, n \ (n(>1) \in \mathbb{N})$. The output membership function in the $j$th rule can be expressed as $z_j = [e \ \dot{e}]^T [a_{ij}\ b_{ij}]^T$.
Remark 4.2: In this subsection, $\bar{x}$ denotes the generic input element for the fuzzy system, i.e., $\bar{x} = \{e, \dot{e}\}$. $\bar{x}$ is used for differentiating it from the vehicle’s longitudinal distance values, $x$. □

The T–S fuzzy logic output with product inference, singleton fuzzifier and centre-average defuzzifier can be expressed as

$$
\hat{\nu} = \frac{\sum_{j=1}^{n} \left( \prod_{i=1}^{m} \mu_{A_i}^j(\bar{x}) \right) z_j}{\sum_{j=1}^{n} \left( \prod_{i=1}^{m} \mu_{A_i}^j(\bar{x}) \right)}.
$$

(4.17)

The term, $\mu_{A_i}^j(\bar{x})$, is the fuzzy inferred input membership function value with the $i$th input variable in the $j$th rule. It is obvious that $i = 1, 2$ (therefore, $m = 2$) for ‘error’ and ‘error rate’ variables.

In concise form, the adaptive fuzzy output, $\hat{\nu} \in \mathbb{R}$ can be stated as

$$
\hat{\nu} = \xi^T(\bar{x}) \theta
$$

(4.18)

where $\theta$ is the parameter vector and $\xi(\bar{x})$ is the regression vector of the fuzzy system.

Remark 4.3: Though fuzzy system in (4.18) is generally nonlinear, it is linear with respect to its unknown parameters. Hence, parameter adaptation algorithms applicable for linear systems, e.g. gradient-based algorithms, can be readily used in estimation of the unknown parameters in fuzzy systems in (4.18) [27]. □

The development of (4.18) is described as follows.

It can be assumed that $\hat{\nu}$, the mapping produced by the fuzzy system, is Lipschitz continuous [30].

The $j$th element of the parameter vector can be provided as

$$
\theta_j = [a_j, b_j]^T \quad \text{where} \quad \theta = [\theta_1, \theta_2, \ldots, \theta_n]^T \in \mathbb{R}^{n \times 2}.
$$

(4.19)

Let the error vector be $e = [e, \dot{e}]^T$, then $\xi_j(\bar{x}) = [\xi_j(\bar{x})e, \xi_j(\bar{x})\dot{e}] = [\xi_j e]^T$ is the $j$th element, for $j = 1, \ldots, n$, of the regression vector, $\xi(\bar{x}) = [\xi_1(\bar{x}), \xi_2(\bar{x}), \ldots, \xi_n(\bar{x})]^T \in \mathbb{R}^{n \times 2}$ [26]. Thereby, $j$th regression factor (for $j = 1, \ldots, n$ (where $n(>1) \in \mathbb{N}$)) can be denoted as
\[ \xi_j(\bar{x}) = \frac{\prod_{i=1}^{m} \mu_{A_i}^{j}(\bar{x}_i)}{\sum_{j=1}^{n}\left(\prod_{i=1}^{m} \mu_{A_i}^{j}(\bar{x}_i)\right)} . \] (4.20)

The adaptive fuzzy output membership function parameters of (4.18) can be tuned using an algorithm based on the adaptive law

\[ \dot{\theta} = -\gamma \xi(\bar{x})Ce \] (4.21)

\((C = [1 \quad 1])\), which is developed in the Section 4.2.4 in detail.

### 4.1.5 Training of Adaptive Fuzzy Controller Parameters

Tuning of input fuzzy membership function parameters were done as follows. The parameters included Gaussian centres and standard deviations of the fuzzy system as well the values *a priori* for the output membership functions. Data were collected by carrying out six runs of trials on veDYNA® vehicle model with activating only the PD components in the closed loop form. The six profiles included different configuration settings (different combinations in path and curvatures, i.e., straight line, left-right turn with maximum curvature of \( \pm 2.29e-3 \ [m^{-1}] \) and right-left turn with maximum curvature \( \mp 2.29e-3 \ [m^{-1}] \), and lead vehicle velocity variations, i.e., acceleration of \( 1 \ [ms^{-2}] \), fixed velocity of \( 10 \ [ms^{-1}] \)), with a sample rate of 0.05 \([s]\), and a period of 20 \([s]\) for each run.

The time period for collecting data sets was limited to 20 \([s]\). This is because the output membership function parameters that are adaptive in nature need only data of initial parts of the scenarios. But, in order to tune the fixed parts, (i.e. the centres and the standard deviations of Gaussian membership functions), it need fairly long runs. Nevertheless, these two parts of tuning are not possible to be done separately because it involves a single fuzzy structure. Therefore, these two ends have been compromised, and an initial period of 20 \([s]\) was considered sufficient for collecting data.

With the simulation runs, the data sets consisting of inputs, i.e. error and error rate, and the expected adaptive fuzzy output, were collected. The adaptive fuzzy output was taken as

\[ \hat{v}_j = \hat{h}(\dot{q}_j) - h_1(q_j), \] (4.22)
where $i = x, y$, for longitudinal and lateral cases, respectively. The equation (4.22) was obtained using (4.4) and (4.5) for longitudinal and lateral controllers, separately by making error terms zero. The expected output of fuzzy system, $h_i(\dot{q}_i)$ was calculated using output states of BMW 325i 1988 model of veDYNA®, and $\hat{h}_i(\dot{q}_i)$ was calculated using (3.7).

The design of fuzzy structure of each longitudinal and lateral adaptive controller was carried out as follows. First, subtractive clustering method was used in order to find the number of clusters [172], thereby the number of input membership functions. In order for that, cluster radii of 0.4 (provides a measure how close the clusters would be) for the lateral case, and 0.5 for the longitudinal case, were used separately. Subtractive clustering also provided the preliminary structure for the adaptive fuzzy systems. Once these membership functions were identified, IF-THEN rules for the fuzzy system were formed by considering all the combinations of these membership functions. The fuzzy systems were further tuned using an ANFIS (Adaptive Network Fuzzy Inference System) system written in C++ [173]. The ANFIS system, a neuro-fuzzy structure for tuning a given fuzzy system on a provided data set, is an offline tuning method and is used for obtaining the primary form of a fuzzy system. Subsequently, MATLAB/Simulink® based ‘ANFIS tool’ was used to visualize the rules and to generate control surfaces. The MATLAB/Simulink® based ANFIS tool was not used for further tuning purposes of the fuzzy system because it cannot be used for ‘linear functions’ as used in this study, and provided in (4.16).

(i) Data Collection

The Simulink® model shown in Fig. 4.2, was used to collect data from special runs on veDYNA®. The profiles used to collect the preliminary data are shown in detail.
The collected data sets were used for training (and checking) preliminary fuzzy systems for ‘single-model’ adaptive fuzzy controllers in lateral and longitudinal domains. In these data streams (error, error rate as inputs, and the fuzzy output), the data range from 1-1900 was used for ANFIS training while the range 1901-2000 was used for validation (or ‘checking’) purposes.

(ii) Profiles of Data Collection for Fuzzy System Training

The following road path profile and lead-vehicle velocity profile were used in different combinations to obtain data for preliminary training of adaptive fuzzy controller.

(a) Road Path Profiles

The road path profiles are shown in Fig. 4.3.

![Fig. 4.3 Road path profiles](image-url)
(b) Velocity Profiles (of Leading Vehicle)

The two velocity profiles of the leading vehicle used for obtaining data, are shown in Fig. 4.4. These profiles were used in combination with the road path profiles shown in Fig. 4.3.

![Velocity profile graph](image)

(a) Constant velocity 10[m/s] and acceleration 1[m/s²]

(b) Constant velocity 10[m/s]

Fig. 4.4 Velocity profiles of leading vehicle

(iii) Data for Adaptive Fuzzy System Training

(a) Longitudinal Data

The following Figs. 4.5(a)–4.5(b) provide input data for longitudinal fuzzy controller training, while Fig. 4.5(c) provides the expected output of the controller. These data were used to train the longitudinal adaptive fuzzy controller.
(a) Longitudinal control inputs: error [m] data profile

(b) Longitudinal control input: error rate [m/s] data profile

(c) Longitudinal control output: data profile

Fig. 4.5 Longitudinal data profile for training

(b) Lateral Data

Similar to the longitudinal case, the following Figs. 4.6(a)–4.6(b) provide input data for lateral fuzzy controller training, while Fig. 4.6(c) provides the expected output of the controller. These data were used to train the lateral adaptive fuzzy controller.
(iv) Classification and Clustering of Data Streams

The following identification of clusters is based on the fuzzy C-means clustering method, and it is presented only for illustrative purposes. The fuzzy C-means clustering was not used in the design process of the fuzzy controller in this research. Instead, the subtractive clustering method was used to identify the clusters as well as for generating...
preliminary fuzzy systems before being further trained with ANFIS to obtain the final form of the fuzzy controller.

(a) Longitudinal Data Plot (based on Fuzzy C-means Clustering)

The basic longitudinal data can be plotted as shown in Fig. 4.9 in order to identify clusters of data and their centres.

![Longitudinal Data Plot](image)

**Fig. 4.7 Data stream for longitudinal input data (Note: depiction purposes only)**

Data in Fig. 4.7 can be divided into 5 regions, and thereby, 5 clusters can be identified as shown in Fig. 4.8.

(b) Identification of Clusters using Fuzzy C-means Clustering Method - Longitudinal

![Identification of Clusters](image)

**Fig. 4.8 Location of cluster centres for longitudinal data using fuzzy C-means clustering method (Note: depiction purposes only)**
The subtractive clustering method was used to generate the initial fuzzy structure, and thereafter ANFIS method was used for fine tuning.

(c) Lateral Data Plot (based on Fuzzy C-means Clustering)

The basic lateral data can be plotted as shown in Fig. 4.9 in order to identify clusters of data and their centres.

![Fig. 4.9 Data stream for lateral input data](image)

(d) Identification of Clusters using Fuzzy C-means Clustering Method-Lateral

Data in Fig. 4.9 can be divided into 3 regions, and thereby, 3 clusters can be identified.

![Fig. 4.10 Location of cluster centres for lateral data using fuzzy C-means clustering method](image)
It is important to note that, in order to get good results from clustering methods, one condition to be considered is that the data used should conform to a specific type of distribution [93]. But, it seemed this condition was not possible to be met with above vehicle data since they did not follow a known distribution as such. Therefore, it was not appropriate to strictly follow the cluster centres that were produced following the fuzzy C-means clustering method. Instead, since subtractive clustering method had an advantage over fuzzy C-means clustering method, (because the former even generates a basic fuzzy structure on identification of cluster centres), the subtractive clustering method was chosen in this design process.

(v) ANFIS Training of Fuzzy Systems and Fuzzy Model Validation using Testing and Checking Data Sets

Neuro-adaptive learning in ANFIS system provides a technique for the fuzzy system models developed to learn information about the data sets. In this ANFIS testing method, the software system calculates best possible parameters for the fuzzy system so that it can accurately track the given data set [173].

In the process of fuzzy model validation, input/output vectors are chosen from collected data that has not been used to train the fuzzy system. This data is presented to the trained fuzzy system to observe how well the system predicts on the corresponding outputs of data sets. It is important to select a set of data that represents the data the trained fuzzy system is intended to emulate while the selected data being sufficiently distinct from the training data set so that the process of validation will not be trivial.

Testing or checking data is used to check the generalization capability of the developed fuzzy system.

The objective behind validation of fuzzy models is that after a certain point in training the model becomes over-fitting the training data set. As a principle, the model error with checking begins to decrease up to the point the over-fitting begins. At this point, the model error for checking starts to increase. Thereby, the fuzzy model parameters can be chosen at the point that associates with the minimum checking error [93].
(a) Training Procedure: Error Signals

The error signals due to training and checking during ANFIS tuning of fuzzy systems are shown below.

**Training Error: Longitudinal**

The training and checking error profiles for ANFIS training for the longitudinal controller are provided in Fig. 4.11, and Fig. 4.12, respectively.

![Fig. 4.11 Longitudinal controller-ANFIS training error profile](image1)

**Checking (Testing) Error: Longitudinal**

![Fig. 4.12 Longitudinal controller-ANFIS training: checking error profile](image2)

**Training Error: Lateral**

The training and checking error profiles for ANFIS training for the lateral controller are provided in Fig. 4.13, and Fig. 4.14, respectively.
Checking (Testing) Error: Lateral

(b) Longitudinal Input Fuzzy Membership Functions

Figs. 4.15–4.16 provide the input fuzzy membership functions, i.e. ‘error’ and ‘error rate’, respectively, for the longitudinal controller.
(c) Lateral Input Fuzzy Membership Functions

In a similar way to longitudinal case, Figs. 4.17–4.18 provide the input fuzzy membership functions, i.e. ‘error’ and ‘error rate’, respectively, for the lateral controller.

(vi) Trained Adaptive Fuzzy Functions

(a) Trained Adaptive Fuzzy Controllers - Longitudinal

The control surface for the tuned longitudinal adaptive fuzzy system *a priori* is illustrated in Fig. 4.19. This control surface provides the graphical representation of the
relationship between the two inputs, i.e., ‘error’ and ‘error rate’ and the preliminary output of the longitudinal adaptive fuzzy system. Obvisously, this control surface is bound to change with the adaptation process leading to a more ‘nonlinearized’ form.

![Control surface for longitudinal adaptive fuzzy controller setup](image)

**Fig. 4.19 Control surface for longitudinal adaptive fuzzy controller setup *a priori***

The ANFIS tuned ‘longitudinal’ Gaussian input membership functions, i.e., for ‘error’ and ‘error rate’, as well as T–S fuzzy output membership functions *a priori*, are provided in APPENDIX C.

**(b) Trained Adaptive Fuzzy Controllers - Lateral**

The control surface for the tuned lateral adaptive fuzzy system *a priori* is illustrated in Fig. 4.20.
In a similar way, the ANFIS tuned ‘lateral’ Gaussian input membership functions, i.e., for ‘error’ and ‘error rate’, as well as T–S fuzzy output membership functions \( a \) priori, are provided in APPENDIX C.

### 4.1.6 Robust Stability Analysis for Control System

In this section, the stability conditions of the developed single-adaptive fuzzy controller are analysed comprehensively. It is important to highlight that the following stability proof is carried out for the generic case irrespective whether it is lateral or longitudinal controller. Therefore, the subscripts that are specific to either longitudinal or lateral case are removed in this description. Nevertheless, the specific details will be mentioned as and when necessary.

Assumption 4.1: The fuzzy approximation error, \( w \) is bounded, i.e., \( |w| \leq W \) where \( W(>0) \in \mathbb{R} \). The external disturbances, \( d \) are bounded too, i.e., \( |d| \leq D \) where \( D(>0) \in \mathbb{R} \). Then the variable structure control gain is chosen as \( \sigma_f \geq W + D \).

The main feature of the developed fuzzy system can be summarized in the following theorem.
Theorem 4.1: Let the parameter vector, $\theta$ of the adaptive fuzzy system be adjusted by the adaptive law as provided in (4.21) and let Assumption 4.1 be true. The proposed adaptive fuzzy control laws, i.e. (4.1) and (4.2), can guarantee stability of the vehicle system with the following properties:

i) The closed loop vehicle system is stable, i.e., $\| e \| \in \mathcal{L}^\infty$ (considering lateral and longitudinal cases)

ii) The system errors and the parametric errors are asymptotically stable, i.e.,

$$\lim_{t \to \infty} e_x, e_y = 0 \quad \text{and} \quad \lim_{t \to \infty} \hat{\theta} = 0$$

(considering lateral and longitudinal cases).

□

Remark 4.4: $\| \cdot \|$ denotes the Euclidean norm for a vector or a matrix, throughout in this research. □

Proof 1: Positive Realness of Error System

First, as a precondition, the positive real property of the error system is proved for establishing stability in accordance with the KYP lemma. KYP lemma provides an effective method for establishing stability of a system. But due to the strictly positive real condition imposed on the transfer function, it makes fairly restrictive for application [174]. Once the strict positive realness of the transfer function is assured, it guarantees the existence of positive definite matrices related to the Lyapunov equation [175].

Positive realness of such an error system has been proved [18], [174]. The method provided by Kumarawadu and Lee is rather simpler, and therefore is almost exactly followed here [18].

Since it is $e = [e \quad \hat{e}]^T$, the error state vector, and thereby, equations (4.4) [or (4.5)] can be expressed in the standard form [18] as

$$\dot{e} = Ae + B\tilde{v} - Bd - Bu_s$$

$$= Ae + B(\tilde{v} - d - u_s) \quad (4.23)$$
where $A = \begin{bmatrix} 0 & 1 \\ -k_p & -k_d \end{bmatrix}$, and $B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$. A number of such error models have been comprehensively analysed [163]. The output error signal of the error dynamic system can be expressed as
\[ R = Ce = e + \dot{e}, \]
(4.24)
since it is earlier provided that $C = [1 \ 1]$. The equations (4.23) and (4.24) describe the state-space representation of the error dynamics. When $k_d \geq 1$, and $k_p \geq 0$ the real part of the transfer function $G(s) = R(s) / \nu(s) = C(sI - A)^{-1}B$ ($s$ is the Laplace operator), i.e. $\text{Re} G(s - \vartheta)$, is positive for $\vartheta > 0$ where $\vartheta \in \mathbb{R}$, if
\[ \sigma - \vartheta > -\left[ \frac{(k_d + 1)(\sigma - \vartheta)^2 + (k_d - 1)\omega^2 + k_p}{(\sigma - \vartheta)^2 + \omega^2 + k_p + k_d} \right]. \]
(4.25)
This implies that $G(s - \vartheta)$ is positive real for $\vartheta > 0$, and consequently $G(s)$ is strictly positive real [175], [176]. The re-verification of the positive-real property of the above transfer function can be done by drawing a Nyquist diagram with the chosen values of $k_d$ and $k_p$ [174].

**Proof 2: System Stability**

According to Kalman–Yakubovich–Popov (KYP) lemma [175], for a strictly positive real system, there exist two positive definite matrices $P$ and $Q$ (i.e., $P = P^T > 0$ and $Q = Q^T > 0$) satisfying
\[ A^T P + PA + Q = 0, \text{ and } PB = C^T. \]
(4.26)
Now, the following Lyapunov function can be defined as (for lateral or longitudinal system)
\[ V = e^T Pe + \frac{1}{\gamma} \tilde{\theta}^T \tilde{\theta}, \]
(4.27)
where $\gamma > 0$ is a real constant, and $\tilde{\theta} = \theta^* - \theta$ is the parameter error with respect to the optimal parameter set. Here, $\theta^* \in \mathbb{R}^{n \times 2}$ where $n$ is the total number of fuzzy rules, is the optimal parameter vector of the adaptive fuzzy controller.
The Lyapunov candidate, as provided in (4.27) quantifies both in tracking error and in parameter estimates. Differentiating the Lyapunov function along the trajectories of the error system will provide

$$
\dot{V} = (e^T Pe + e^T P \dot{e}) + \frac{1}{\gamma}(\dot{\theta}^T \dot{\theta} + \dot{\theta}^T \dot{\theta}).
$$

(4.28)

Substituting for $\dot{e}$ from the equation (4.23), in (4.28) provides

$$
\dot{V} = (Ae + B(\bar{v} - d - u_i))^T Pe + e^T P (Ae + B(\bar{v} - d - u_i))
+ \frac{1}{\gamma}(\dot{\theta}^T \dot{\theta} + \dot{\theta}^T \dot{\theta}).
$$

Further rearrangement will provide

$$
\dot{V} = e^T (A^T P + PA) e + (\bar{v} - d - u_i)^T B^T Pe + e^T PB (\bar{v} - d - u_i)
+ \frac{1}{\gamma}(\dot{\theta}^T \dot{\theta} + \dot{\theta}^T \dot{\theta}).
$$

(4.29)

From the argument, it can be expressed as

$$
\bar{v} = (v^* - \bar{v}) + w = \xi^T (\theta^* - \theta) + w = \xi^T (\bar{\theta}) + w
$$

where $w$ is the fuzzy approximation error which is bounded, $|w| \leq W$, i.e., $w \in \mathcal{C}^\infty$, and $W \in \mathcal{R}$ is a positive constant. The determination of fuzzy approximation error upper bound, $W$ has been discussed before [30].

Hence, (4.29) becomes

$$
\dot{V} = -e^T Qe + (\bar{\theta}^T \xi(\bar{\theta}) + w - d - u_i) B^T Pe
+ e^T PB (\xi^T (\bar{\theta}) + w - d - u_i)
+ \frac{1}{\gamma}(\dot{\theta}^T \dot{\theta} + \dot{\theta}^T \dot{\theta}).
$$

(4.30)

Rearranging (4.30) provides

$$
\dot{V} = -e^T Qe + \bar{\theta}^T \left( \frac{1}{\gamma} \dot{\theta} + \xi(\bar{\theta}) C e \right) + \left( \frac{1}{\gamma} \dot{\theta} + \xi(\bar{\theta}) C e \right)^T \dot{\theta}
+ (w - d - u_i)(C e + e^T C^T).
$$

(4.31)

If the fuzzy adaptive law is taken as $\dot{\theta} = -\gamma \xi(\bar{\theta}) C e$ as provided in (4.21), with $\gamma \in \mathcal{R} > 0$ as the adaptive parameter, (4.31) reduces to

$$
\dot{V} = -e^T Qe + (w - d - u_i)(C e + e^T C^T).
$$

(4.32)

The equation (4.32) can be further expressed as

$$
\dot{V} \leq -\lambda_{\min} \|e\|^2 + (w - d - u_i)(C e + e^T C^T).
$$

(4.33)

Here, $\lambda_{\min}$ is the minimum eigen value of $Q$. 

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Since fuzzy systems follow universal approximation theorem [29], \( |w| \) may be made arbitrarily small by a proper choice of the fuzzy system if \( \hat{\nu} = \xi^T (x) \theta \) is smooth (note that this may require an arbitrarily large number of rules) [30].

Since \( |w| \leq W \) and \( |d| \leq D \), (4.33) can be further written as

\[
\dot{V} \leq -\lambda_{\text{min}} \| e \|^2 + (W + D - u_x) (Ce + e^T C^T). \tag{4.34}
\]

Let the variable structure control term be defined as

\[
u = \sigma_j \text{sign} (Ce + e^T C^T). \tag{4.35}
\]

Thereby, (4.34) can be expressed as

\[
\dot{V} \leq -\lambda_{\text{min}} \| e \|^2 + (W + D - \sigma_j) \| Ce + e^T C^T \|. \tag{4.36}
\]

Further rearrangement of (4.36) will provide

\[
\dot{V} \leq -\lambda_{\text{min}} \| e \|^2 - \| Ce + e^T C^T \| (\sigma_j - W - D). \tag{4.37}
\]

If it is taken as

\[
\sigma_j \geq W + D, \tag{4.38}
\]

then (4.37) will establish that

\[
\dot{V} \leq 0, \quad \forall t \geq 0. \tag{4.39}
\]

Therefore, according to Lyapunov theory, it establishes the fact that \( e \) is bounded, i.e. \( e \in L^\infty \). Therefore, \( e, \dot{e} \) are bounded, too, i.e. \( e, \dot{e} \in L^\infty \). It also establishes that parameter errors are bounded, i.e., \( \theta \in L^\infty \).

Equation (4.37) also implies that

\[
\dot{V} \leq -\lambda_{\text{min}} \| e \|^2. \tag{4.40}
\]

Integrating (4.40) will provide

\[
\lambda_{\text{min}} \int_0^\infty \| e \|^2 dt \leq -\int_0^\infty \dot{V} dt = V(0) - V(\infty). \tag{4.41}
\]

Therefore, rearranging (4.41) will provide

\[
\int_0^\infty \| e \|^2 dt \leq \frac{1}{\lambda_{\text{min}}} (V(0) - V(\infty)). \tag{4.42}
\]
Equation (4.42) establishes that
\[ e \in L^2 \left\{ L^2 = \left\{ z(t) : \int_0^\infty z^2(t)dt < \infty \right\} \right\}, \]  
(4.43)
since it provides \( V(0), V(\infty) \in L^\infty \) according to (4.39) [30]. Since it has been obtained \( e \in L^2 \cap L^\infty \) (from (4.43) and (4.39)), and \( e \in L^\infty \) (from (4.40)), by Barbalat’s lemma [30], [176], it can be established with asymptotic stability of \( e \) (i.e. \( \lim_{t \to \infty} e = 0 \)). This result of asymptotic stability can be extended specifically to lateral and longitudinal cases, and consequently the lateral and longitudinal errors are asymptotically stable, i.e.
\[ \lim_{t \to \infty} e_x, e_y = 0. \]  
(4.44)

The equation (4.44) proves that when lateral and longitudinal cases are considered at the same time, with the above arguments, the system of errors can be proved asymptotically stable. Therefore, the result provided in (4.44) logically proves that the integrated lateral and longitudinal control system is asymptotically stable. This statement completes the proof of stability for robust single-adaptive fuzzy system.

The following definition is provided to formerly identify the variable vector spaces, and thereby to define the optimal parameter set.

**Definition 4.1:** The compact parameter spaces for \( \theta \) and \( \bar{x} \) variables can be defined as below. Let \( M_{\theta}(>0) \in \mathbb{R} \) and \( M_{\bar{x}}(>0) \in \mathbb{R} \) be finite
\[ \Omega_{\theta} = \left\{ \theta \in \mathbb{R} : \|\theta\| \leq M_{\theta} \right\}, \] and
\[ \Omega_{\bar{x}} = \left\{ \bar{x} \in \mathbb{R} : \|\bar{x}\| \leq M_{\bar{x}} \right\}. \]
Thereby, the optimal parameter set can be formally defined as
\[ \theta^* = \arg \min_{\theta \in \Omega_{\theta}} \left\{ \sup_{\bar{x} \in \Omega_{\bar{x}}} \left| v^* - \xi^T(\bar{x})\theta \right| \right\}, \]  
(4.45)
where \( v^*(\in \mathbb{R}) \) is the optimal adaptive fuzzy function output. □

**Remark 4.5:** The usage of the term \( \theta^* \) is only for analytical purposes in this proof. The value of it is not required in the design of the controller. □

**Remark 4.6** [176]: The above Lyapunov candidate (4.27) satisfies the following conditions
i) \( V \) is at least \( C^1 \).
\[ V(0) = 0 \text{ and } V(x(t)) > 0 \text{ for } x(t) \neq 0, \text{ and} \]
\[ \| x(t) \| \rightarrow \infty \Rightarrow V(x(t)) \rightarrow \infty \]

\((x(t))\) denotes a selected time-dependent state of the system in this remark). □

**Remark 4.7:** An update law with a projection operator can be defined to guarantee that the parameter matrices lie within the feasible parametric space. Details on using a projection operator were provided in [30], [32], and the references, therein. It is also important to note that the requirement of a projection operator is only to establish stability in the theoretical sense. □

### 4.1.7 Simulation Studies, Results and Discussion

The following simulations were done on veDYNA® (ver. 3.10) vehicle simulation software with a BMW 325i 1988 model. The simulation setup consists of front wheel-steered, front wheel-driven, automatic transmission settings. veDYNA® runs with an ordinary differential solver ‘ode1’ based on trapezoidal method with a sample time 0.001 [s]. But the developed controller and the inverse model were run with a solver ‘ode4’ viz. a 4-th order Runge-Kutta method, and with a sample time 0.0005 [s]. This new setting for the controller and inverse model was adopted because the embedded fuzzy systems looked ‘stiff’ with integration with a simpler differential solver of calibre ‘ode1’. In order to overcome this difficult issue, veDYNA® vehicle model was encapsulated in a triggered subsystem on Simulink/MATLAB®, as shown in Fig. 3.10, and thereby, was connected to the external control system. Hence, MATLAB® ordinary differential equation command ‘ode4’ was used with the fixed step size 0.0005 [s] for all vehicle controller modules.

While the simulation studies were aimed for solving the integrated lateral and longitudinal control problem of highway vehicles, the viability of the controller was also tested under simulated extreme parametric changes: changing of friction coefficient of tyre-road interface, presence of disturbances like crosswind forces, addition of an unsymmetrical external load. These disturbances were used to challenge the robust setting of the control system. Additionally, the performance of the developed controller was tested against some occurrences of faulty conditions at selected subsystems at non-catastrophic level. The considered faulty conditions included a flat-tyre case (front-left
tyre becoming flat at 10[s] after starting the simulation), and a defective front-left wheel brake cylinder (with a 90% pressure drop in the supply line, throughout).

(i) Simulation Setup

The PD gains for the controller were chosen as follows: for the longitudinal case; 
\[ k_{px} = 1e5, \quad k_{dx} = 1e5, \]  
and for the lateral case; 
\[ k_{py} = 1e6, \quad k_{dy} = 1e5. \]

The adaptive gains in the fuzzy controllers were chosen as \( \gamma_x = 0.03 \) and \( \gamma_y = 0.01 \) for longitudinal and lateral cases, respectively. And the gains for variable structure control terms were taken as \( \sigma_{px} = 275 \) and \( \sigma_{py} = 225 \) for longitudinal and lateral cases, respectively.

The inter-vehicular gap was taken as a constant value of 5 [m], for simplicity. The simulation profile used is shown in Figs. 4.21–4.23 below. The speed of the leading vehicle is varying from 0 [m/s] to 35 [m/s] with a maximum longitudinal acceleration/deceleration of \( \pm 1.5 \) [m/s\(^2\)], a value within the realistic capacity of vehicle operation [14].

As provided in Fig. 4.22, the radii of curvature of the highway path varies between [0 315] [m]. It is important to note that the controlled vehicle starts from zero velocity. This is important to testify the controller performance with more pronounced transient effects. In the meanwhile the leading vehicle cruises right on the highway centreline throughout.

(ii) Simulation Profile

The following figures, Figs. 4.21–4.23 describe the simulation profile used for validation of all the controllers developed in this research.

(a) Road Profile

The road path that was used in the simulation on veDYNA® is shown in Fig. 4.21. This road path consists of straight line segments as well as cubic-splines.
(b) Radii of Curvature Variation of the Road

Fig. 4.22 shows the curvature of the road profile provided in Fig. 4.21.

(c) Lead Vehicle Velocity Profile

The velocity profile of the lead vehicle is illustrated in Fig. 4.23. For comparison purposes, the controlled vehicle velocities with adaptive fuzzy controller at on and off (PD only) states are also provided.
(iii) Control Inputs to the Vehicle

The variations of control inputs to the vehicle, i.e. throttle angle/brake pedal position and steering angle with regard to single-adaptive fuzzy controller and ‘PD only’ controller are shown in Figs. 4.24–4.26.

(a) Throttle Angle

The throttle angle variations are provided in Fig. 4.24. This figure shows the comparison results of the single-adaptive fuzzy controller against that of the ‘PD only’ controller when the vehicle was cruising normally without any external disturbance.
(b) Brake Pedal Position

The brake pedal position variations are depicted in Fig. 4.25. In comparison with Fig. 4.24, which provides the variation of throttle angle, it is clear that when the brake pedal position becomes positive, the throttle angle stays at zero level.

![Fig. 4.25 Brake pedal position: single-adaptive fuzzy vs. PD only](image)

(c) Steering Angle

Fig. 4.26 provides the steering angle variation. In a similar way as before, this figure compares steering angle from single-adaptive fuzzy controller with that from ‘PD only’ controller when the vehicle was cruising normally without any external disturbances.

![Fig. 4.26 Steering angle: single-adaptive fuzzy vs. ‘PD only’](image)
(d) Vehicle Lateral Position

Fig. 4.27 provides the lateral deviation of the vehicle centre of gravity in global coordinates. This figure is mainly provided for comparison purposes with the other graphs for the movement of the vehicle.

![Fig. 4.27 Lateral position of vehicle centre of gravity](image)

(iv) Simulation Results and Discussion

In the following figures depicting simulation results, each performance parameter of single-adaptive fuzzy controller (solid blue line) is compared against that of ‘PD only’ controller (dotted red line).

(a) Normal Cruising Conditions: Single-Adaptive Fuzzy Control

The following simulation results provide the performance of the developed single-adaptive fuzzy controller against that of the ‘PD only’ controller under normal cruising conditions without any presence of external disturbances.

As shown in Fig. 4.28, there is hardly any improvement of longitudinal error of adaptive fuzzy over ‘PD only’ controller. It can be said that the change is slightly worse when it comes to response time initially, i.e. around the points 5–9 [s] on the time-scale. Therefore, ‘PD only’ controller is quite effective when compared to the single-adaptive fuzzy controller as far as longitudinal control is concerned in this case.
Fig. 4.28 Longitudinal error: single-adaptive fuzzy (RMS: 3.158) vs. ‘PD only’ (RMS: 3.078)

Fig. 4.29 shows that the lateral error has improved significantly with adaptive fuzzy controller over ‘PD only’ controller. Therefore, it can be concluded that the single-adaptive fuzzy controller contributes significantly to improve the lateral tracking of the vehicle under normal cruising.

Fig. 4.29 Lateral error: single-adaptive fuzzy (RMS: 0.003837) vs. ‘PD only’ (RMS: 0.01849)

The pitch angle variation due to the single-adaptive fuzzy controller is comparable to that with the ‘PD only’ controller as shown in Fig. 4.30. At some points, adaptive fuzzy controller has avoided reaching some ‘extreme’ values, e.g. around 5 [s] on the time-scale. It concludes that pitch angle variation effects are quite small, since the range of vertical movement of the suspension mass just above the front axle lies within [-0.03 0.02] [rad], which is similar to an approximate range of 3 [deg] variation. This is equivalent to approximately 4 [cm] maximum deviation from the normal level of the suspension mass just above the front axle through the longitudinal axis.
Roll angle, as shown in Fig. 4.31, exhibits some slight improvements with transient response with the adaptive fuzzy controller. These effects are most evidenced around the points 15 [s], and 45–47 [s], on the time-scale. Roll angle gives an indication of lateral stability of the vehicle.

Roll rate is another indication of lateral stability of the vehicle. As shown in Fig. 4.32, adaptive fuzzy controller performance is comparable to that of the ‘PD only’ controller as far as roll rate is concerned.
There are improvements in the transients with adaptive fuzzy controller over ‘PD only’ controller when it comes to side-slip angle, as shown in Fig. 4.33, when the vehicle makes a sudden change due to curvature of the road. Ensuring a low side-slip angle, especially during transient stages makes the vehicle more stable in the lateral context. In general case, under normal driving conditions with ensured safety, without any danger of losing road grip, the slide-slip angle for a car does not exceed ±2 [deg] [177]. The two controllers have performed well within this requirement of side-slip angle.

Since adaptive fuzzy controller has improved side-slip angle and roll angle responses while maintaining the roll rate at a competitive level, it can be said that the lateral stability of the vehicle has been improved by adaptive fuzzy controller.

The comfortable or low ranges of lateral accelerations can be categorized into different ranges: small-signal range, i.e., 0–0.5 [m/s²] where only straight-running behaviour can be included or ‘linear range’, i.e., 0.5–4 [m/s²] where sudden steering inputs, lane changes and load change reaction in turns [178]. As shown in Fig. 4.34, the variations of lateral acceleration do not go beyond 4 [m/s²]. Therefore, the two controllers have managed to contain lateral acceleration within low limits as far as design characteristics of today’s passenger vehicles are concerned [178].
In minimizing lateral acceleration, adaptive fuzzy controller has been quite competitive with the ‘PD only’ controller, and the former has even managed to avoid some transient effects that were present with the ‘PD only’ controller.

With comparison of side-slip angle and lateral acceleration, as shown in Figs. 4.33–4.34, the side-slip angle indicates an increase as the lateral acceleration curve increases (disregarding the ‘sign’). This fact is an indication of controllability of the vehicle provided that there have been less fluctuations in side-slip angle [178]. This is quite the case with the side-slip angle performance.

Overall, during normal cruising, lateral stability with adaptive fuzzy controller is quite competitive in comparison to the ‘PD only’ controller as suggested by the performances in the areas of side-slip angle, roll-angle and roll rate. Therefore, having a lower lateral tracking error is a significant result by the adaptive fuzzy controller in comparison to the ‘PD only’ controller, with a guaranteed lateral stability. Unlike lateral tracking, longitudinal tracking variation has not been better than that of ‘PD only’ controller.

(b) External Disturbances: Single-Adaptive Fuzzy Control

In the following set of simulations, the developed single-adaptive fuzzy controller was tested against the ‘PD only’ controller under a number of external disturbances. These disturbances included an unsymmetrical load mass, presence of crosswind forces and change of tyre-road friction.
1. Unsymmetrical Load Mass

The following simulation was carried out for comparison of the performances between the single-adaptive fuzzy controller and the ‘PD only’ controller, when there was an unsymmetrical load-mass on the vehicle as per the details described in Section 3.2.18.

As shown in Fig. 4.35, the longitudinal error has slightly decreased with adaptive fuzzy functions. Initially it has a longer response time towards the point 10 [s]. The second peak of the curves, around 23 [s] on the time-scale, is due to sudden change in the curvature of the road path. This response has been heightened due to an unbalanced force that is generated in the vehicle due to unsymmetrical loading.

As shown in Fig. 4.36, even in the presence of an unsymmetrical mass, the lateral tracking has shown an improvement with adaptive fuzzy controller as against that of the ‘PD only’ controller.

Fig. 4.35 Longitudinal error: single-adaptive fuzzy (RMS: 4.391) vs. ‘PD only’ (RMS: 4.375)

Fig. 4.36 Lateral error: single-adaptive fuzzy (RMS: 0.003957) vs. ‘PD only’ (RMS: 0.01741)
As shown in Figs. 4.37–4.39, pitch angle, roll angle and roll rate variations of adaptive fuzzy controller are not much different from that of the ‘PD only’ controller. Overall, it can be concluded that the adaptive fuzzy controller has improved the transient response in most instances, at least slightly, over the ‘PD only’ controller.

Overall, with an unsymmetrical load mass, the lateral stability of the vehicle with the adaptive fuzzy controller has not been excessive compared to that with the ‘PD
only’ controller. This can be concluded from the performances in the areas of side-slip angle, roll-angle and roll rate.

Side-slip angle is high with adaptive fuzzy when the speed of the vehicle is low. This is shown in Fig. 4.40. A slack of controllability initially would be the reason for such a display when the adaptive fuzzy controller was active [178]. In this instance, however, adaptive fuzzy controller has not been successful in eliminating the higher side-slip angle at low speed. But, this was only when the vehicle speed was very low. As the vehicle gathers speed adaptive fuzzy exhibits a comparatively improved response even with eliminating transient effects, over ‘PD only’ controller.

![Fig. 4.40 Side-slip angle: single-adaptive fuzzy (RMS: 0.008237) vs. ‘PD only’ (RMS: 0.008256)](image)

As shown in Fig. 4.41, the variations in lateral acceleration do not go beyond 4 \([m/s^2]\). Therefore, the two controllers have managed to contain the lateral acceleration within low limits according to the design characteristics of today’s passenger vehicles [178].

![Fig. 4.41 Lateral acceleration: single-adaptive fuzzy (RMS: 1.256) vs. ‘PD only’ (RMS: 1.262)](image)
Overall, in the presence of an unsymmetrical load mass, when considering lateral stability, the performance of the adaptive fuzzy controller is comparable to that with the ‘PD only’ controller. This fact can be observed with the overall performances in the areas of side-slip angle, roll-angle and roll rate. Once again, the adaptive fuzzy controller has exhibited that it can ensure a lower lateral tracking error compared to the ‘PD only’ controller, with guaranteed lateral stability. But, there has been no improvement in terms of longitudinal error compared to that with the ‘PD only’ controller.

2. Crosswind Effects

The following simulation results were obtained when there was a continuous crosswind with a speed 50 [m/s], from the ‘global’ y-direction. More details of this arrangement are provided in Section 3.2.18.

![Fig. 4.42 Longitudinal error: single-adaptive (RMS: 3.742) fuzzy vs. ‘PD only’ (RMS: 3.666)](image1)

![Fig. 4.43 Lateral error: single-adaptive fuzzy (RMS: 0.005914) vs. ‘PD only’ (RMS: 0.02073)](image2)
Fig. 4.44 Pitch angle: single-adaptive fuzzy (RMS: 0.01142) vs. ‘PD only’ (RMS: 0.01146)

Fig. 4.45 Roll angle: single-adaptive fuzzy (RMS: 0.01396) vs. ‘PD only’ (RMS: 0.01406)

Fig. 4.46 Roll rate: single-adaptive fuzzy (RMS: 0.02222) vs. ‘PD only’ (RMS: 0.02317)
Lateral acceleration with using the adaptive fuzzy controller has been comparable to that with the ‘PD only’ controller as shown in Fig. 4.48. Even though the variation of lateral acceleration shows some chattering, it has been contained with smaller amplitudes throughout. Therefore, the effect of chattering is quite localized, and is not significant in both the controllers. As shown in Fig. 4.48, the lateral acceleration has not gone beyond 4 \( [\text{m/s}^2] \). Therefore, the two controllers have managed to contain the lateral acceleration within comfortable limits to passengers [178].

Overall, in the presence of crosswind, the vehicle has shown a lower lateral tracking error with the adaptive fuzzy controller than with the ‘PD only’ controller, with guaranteed lateral stability. The performances of side-slip angle, roll-angle and roll rate with regard to the adaptive fuzzy controller suggest the level of lateral stability. Therefore, the adaptive fuzzy controller performs well in comparison to the ‘PD only’ controller in lateral domains. But, longitudinal error variation has not been better than with the ‘PD only’ controller.
3. Tyre-Road Friction Change

The following results were for the single-adaptive fuzzy controller in comparison with the ‘PD only’ controller performances when there was a change of tyre-road friction to a value of 0.5, between 100 – 200 [m] range on the road path.

![Fig. 4.49 Longitudinal error: single-adaptive fuzzy (RMS: 3.158) vs. ‘PD only’ (RMS: 3.08)]

![Fig. 4.50 Lateral error: single-adaptive fuzzy (RMS: 0.003996) vs. ‘PD only’ (RMS: 0.01848)]

![Fig. 4.51 Pitch angle: single-adaptive fuzzy (RMS: 0.01144) vs. ‘PD only’ (RMS: 0.01127)]
As far as roll rate is concerned, the adaptive fuzzy controller has shown some high values as compared to ‘PD only’ controller as shown in Fig. 4.53. But, it is not excessive.

With ensured consistency of side-slip angle variation with the single-adaptive fuzzy controller in comparison with the ‘PD only’ controller, most of the transient effects have been reduced, though slightly. This is shown in Fig. 4.54.
The variation of lateral acceleration, as shown in Fig. 4.55, is quite within the limits of that of the ‘PD only’ controller. Even single-adaptive fuzzy controller has gone into improving the transient effects slightly at some turning points. Most importantly, the variations of lateral acceleration have not gone beyond 4 [m/s^2]. Therefore, the two controllers have managed to contain lateral acceleration within low limits as far as the requirements of design of vehicles are concerned [178].

![Graph showing lateral acceleration](image)

Fig. 4.55 Lateral acceleration: single-adaptive fuzzy (RMS: 1.266) vs. ‘PD only’ (RMS: 1.272)

Overall, under changes of tyre-road friction, once again ensuring a lower lateral tracking error is a significant result from the adaptive fuzzy controller than with the ‘PD only’ controller. The lateral stability of the vehicle has been under control, throughout. The variation of longitudinal error with the adaptive fuzzy controller has not been improved once again in comparison to that with the ‘PD only’ controller.

(c) Failure Modes (non-catastrophic subsystem failures): Single-Adaptive Fuzzy Control

The following results compared the performances of the two controllers when some selected failure modes in the vehicle system were present. These failure modes included occurrence of a flat-tyre event in the front-left wheel after 10 [s] of starting the simulation, and a brake cylinder defective condition with a 90% pressure drop in the brake line in the front-left wheel.
1. Flat-Tyre

The following simulation results were obtained when an event of the front-left tyre becoming flat occurred at 10 [s] after starting the simulation. The details of the simulation arrangement are explained in Section 3.2.18.

Longitudinal error, once again is slightly high with the flat-tyre. This higher error, common to both controllers, is due to reduction in traction and other unbalanced effects with having a flat-tyre. But, what is expected from the controller in this kind of a situation is that it should have the ability to safely navigate the vehicle. In this regard, the performance of the adaptive fuzzy controller, as shown in Fig. 4.56, is slightly worse in comparison to that with the ‘PD only’ controller.

Fig. 4.56 Longitudinal error: single-adaptive fuzzy (RMS: 26.08) vs. ‘PD only’ (RMS: 24.84)

Lateral error as shown in Fig. 4.57 for the adaptive fuzzy control is more stable in comparison to the ‘PD only’ output, and it has even avoided some transient effects at some points.

Fig. 4.57 Lateral error: single-adaptive fuzzy (RMS: 0.02753) vs. ‘PD only’ (RMS: 0.04242)
During the flat tyre event, the roll angle performance was superior to that with the ‘PD only’ controller, as shown in Fig. 4.59. Especially, during transient effects, like at critical moments of change of curvature of road path, the single adaptive fuzzy controller has shown some improvements.

As illustrated in Fig. 4.60, as far as roll rate is concerned, the adaptive fuzzy controller has shown quite competitive behaviour to that with the ‘PD only’ controller, avoiding some extreme transient effects. This no doubt contributes to a non-declining lateral stability having acquired an improved lateral tracking with the adaptive fuzzy controller.
Fig. 4.60 Roll rate: single-adaptive fuzzy (RMS: 0.0212) vs. ‘PD only’ (RMS: 0.02158)

Side-slip angle resulting from the single-adaptive fuzzy controller, as shown in Fig. 4.61 has some improved response especially with regard to some extreme transient peak values, e.g. around 51 [s] on the time-scale. Therefore, the consistent values of side-slip angle using the adaptive fuzzy controller contribute to the consistency of the lateral stability of the vehicle.

Fig. 4.61 Side-slip angle: single-adaptive fuzzy (RMS: 0.007039) vs. ‘PD only’ (RMS: 0.007716)

There has been a similarity in the variation of values of lateral acceleration in the adaptive fuzzy control and the ‘PD only’ control cases. With either controller, as shown in Fig. 4.62, the variation of lateral acceleration is within the limits of 4 [m/s²]. This fact ensures that passenger comfort is not compromised beyond the design characteristic requirements of today’s passenger vehicles [178].
Fig. 4.62 Lateral acceleration: single-adaptive fuzzy (RMS: 1.248) vs. ‘PD only’ (RMS: 1.26)

Overall, with the flat-tyre case, the consistency of lateral tracking performance as shown by the adaptive fuzzy controller previously is not present in this case. In other words, its performance is inferior to that of the ‘PD only’ controller. But, the adaptive fuzzy controller has not compromised on lateral stability, though this is to its advantage. This is evidenced by the variations shown in side-slip angle, roll-angle and roll rate. Once again, the increased longitudinal error variation with the adaptive fuzzy controller in comparison to that with the ‘PD only’ controller, is evident.

2. Brake Cylinder Defect

A subsystem failure involving front-left wheel brake cylinder being defective is studied in this section (in the simulation, the pressure of the front-left wheel brake cylinder was kept reduced by 90% throughout the actual simulation time). More details on this simulation setup are provided in Section 3.2.18.

As shown in Fig. 4.63, the adaptive fuzzy control has ensured that longitudinal error has been maintained without going it into negative values. The ‘PD only’ controller has shown some weakness in this regard. The result achieved by the adaptive fuzzy controller is important in an event like brake cylinder failure where the braking system of the vehicle has been severely affected beyond the normal operating conditions.
Unlike, in the previous simulations, lateral error has shown some deterioration with the adaptive fuzzy control in comparison to the ‘PD only’ control, as shown in Fig. 4.64, i.e. around the point 5 [s]. But, apart from that point, the adaptive fuzzy controller has shown some positive trend in the rest of the time.

The single adaptive fuzzy control has done well to keep the pitch angle low throughout, in comparison to the ‘PD only’ control, as shown in Fig. 4.65. In the meantime, as shown in Figs. 4.66, 4.67, the variations of roll angle and roll rate with the
single-adaptive fuzzy controller have some significant improvements, throughout in comparison to the ‘PD only’ controller. The former has contributed to eliminating transient effects as illustrated in both figures.

![Roll Angle](image1)

**Fig. 4.66** Roll angle: single-adaptive fuzzy (RMS: 0.01082) vs. ‘PD only’ (RMS: 0.01183)

![Roll Rate](image2)

**Fig. 4.67** Roll rate: single-adaptive fuzzy (RMS: 0.04617) vs. ‘PD only’ (RMS: 0.08575)

The variation of side-slip angle with the single-adaptive fuzzy controller, as shown in Fig. 4.68 shows some lapses in improvement, especially around 5 [s] on the time-scale, in comparison to that with the ‘PD only’ controller.

![Side-Slip Angle](image3)

**Fig. 4.68** Side-slip angle: single-adaptive fuzzy (RMS: 0.01052) vs. ‘PD only’ (RMS: 0.01012)
Unlike in the previous cases, the variation of lateral acceleration as exhibited by the adaptive fuzzy controller has been less satisfactory in comparison to that with the ‘PD only’ controller. This is shown in Fig. 4.69. The worst situation with the adaptive fuzzy controller occurs around 42 [s], at which point the value peaks around 15 [m/s$^2$]. Clearly, these values shown by the adaptive fuzzy controller exceed the normal design specifications of the passenger vehicles as far as lateral acceleration is concerned [178].

Fig. 4.69 Lateral acceleration: single-adaptive fuzzy (RMS: 1.334) vs. ‘PD only’ (RMS: 1.351)

Overall, in the presence of a brake cylinder pressure failure, the performance of the adaptive fuzzy controller provides some weaker responses with regard to lateral tracking. This is an unfamiliar response as far as the previous results of the adaptive fuzzy controller are concerned. On the other hand, there are some improvements with regard to lateral stability as shown in the performances of side-slip angle, roll-angle and roll rate. Quite strangely, the adaptive fuzzy control has shown some improved results with regard to longitudinal error variation in comparison to that with the ‘PD only’ control.

Considering the overall performance of the single-adaptive fuzzy controller in comparison to that with the ‘PD only’ controller, the following conclusions can be drawn. There have been significantly positive results as far as lateral tracking is concerned except in cases of failure modes. With regard to flat-tyre case, the results are mixed. But, the performance is worse in the case of brake cylinder failure.

The lateral stability has been consistent with the adaptive fuzzy controller. But, as far as longitudinal tracking is concerned, the performance of the adaptive fuzzy controller is worse. There is a slight change in the brake cylinder failure case where the adaptive fuzzy controller performance is slightly better.
The summary of performance of the single-adaptive fuzzy controller against that of the ‘PD only’ controller is provided in TABLE 4.1 below.

<table>
<thead>
<tr>
<th>Simulation</th>
<th>Longitudinal error</th>
<th>Lateral error</th>
<th>Pitch angle</th>
<th>Roll angle</th>
<th>Roll rate</th>
<th>Side-slip angle</th>
<th>Lateral acceleration</th>
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<td>similar</td>
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<td>similar</td>
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<td>similar</td>
<td>similar</td>
</tr>
<tr>
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<td>significantly improved</td>
<td>similar</td>
<td>slightly deteriorated</td>
<td>similar</td>
<td>similar</td>
<td>similar</td>
</tr>
<tr>
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<td>slightly improved</td>
<td>slightly improved</td>
<td>slightly improved</td>
<td>slightly improved</td>
<td>similar</td>
<td>similar</td>
</tr>
<tr>
<td>defective brake cylinder</td>
<td>slightly improved</td>
<td>slightly improved</td>
<td>similar</td>
<td>improved</td>
<td>improved</td>
<td>similar</td>
<td>significantly improved</td>
</tr>
</tbody>
</table>
4.2 Chapter Summary

In this chapter, error dynamics for the single-adaptive fuzzy controller design is described first. This is followed with a description of fuzzy system basics. Then a detailed design of the single-adaptive fuzzy controller is described.

The procedure for determining the fuzzy controller parameters of the system \textit{a priori} using data, is described. This description includes how a number of vehicle runs were conducted and data gathered. This is followed with a description on using the subtractive clustering method and ANFIS method for determining the parameters for the adaptive fuzzy controller using the gathered data.

A detailed robust stability analysis procedure is followed for ensuring asymptotic stability of the single-adaptive fuzzy system using Lyapunov based KYP lemma. The details of the developed controller being validated using a comprehensive set of simulations on veDYNA®, are described. These details on the simulations include testing of the controller for its robust qualities under a number of external disturbances as well as a couple of failure modes.
In this chapter, the design of two types of novel robust Multiple-Model/Multiple-Controllers (MM/MC) based on fuzzy systems is described. They are designed to address the integrated lateral and longitudinal control of highway vehicles. The first controller type discussed in this chapter is ‘robust multiple-adaptive fuzzy controller’, which is comprised of a number of adaptive fuzzy controllers tuned differently for addressing different dynamic situations. These adaptive fuzzy controllers are ‘blended’ using a ‘fixed-parameter’ fuzzy function to obtain a single control output. The second controller, ‘robust fuzzy PDC (Parallel Distributed Compensation) based MM/MC fuzzy controller’ is comprised of many local adaptive fuzzy controllers that are blended using a fuzzy PDC structure. The design of the fuzzy PDC system is based on the sector-nonlinearity method. Both controllers are established with stability based on a Lyapunov-based method, i.e. KYP lemma, leading to asymptotic stability at large. The two controller systems developed are validated with comprehensive set of simulation studies on veDYNA® and MATLAB/Simulink®, in the presence of many external disturbances and non-catastrophic subsystem failures.
5.1 Robust Multiple-Adaptive Fuzzy Control Systems with ‘Blending’ (Soft-Switching)

5.1.1 Introduction

It has already been highlighted in the main introduction that there are drawbacks in using single-model (or single ‘modal’) control systems in addressing the problem of integrated lateral and longitudinal control of highway vehicles.

Since MM/MC systems have more built-in capability to address control requirements with improved tracking with higher reliability, it is more relevant and applicable to use them within an Automated Highway System (AHS).

Therefore, in this section, the development of a robust multiple-adaptive fuzzy control system based on blending is described in order to address the problem of integrated lateral and longitudinal control of highway vehicles. By fuzzy blending of a number of inputs from a bank of suitably configured adaptive fuzzy controllers, the uncertainty between the vehicle and the vehicle-model is modelled. The design of the multiple-adaptive fuzzy controller is based on having ‘controller-signals as inputs’ in the fuzzy (blender) system. Having many adaptive fuzzy systems as multiple-controllers enables one to effectively define and model uncertainty of the system over which the control action is developed.

5.1.2 Synthesis of Multiple-Adaptive Fuzzy Controller with Blending

This section presents a comprehensive synthesis of a novel setup of a robust multiple-adaptive fuzzy control system. In this multiple-controller setup, a number of adaptive fuzzy controllers are configured specifically considering a variety of scenarios. These individual adaptive fuzzy controllers operate in parallel and their output control signals are blended using another ‘fixed-parameter’ fuzzy system. Blending of adaptive fuzzy controller in the fixed fuzzy system is according to pre-identified or tuned output membership parameters. This pre-identification of fixed parameters is done using a generic scenario of operation, e.g. normal cruising conditions where there are no
external disturbances. Since the system is quite versatile with the addition of multiple-adaptive fuzzy controllers, it can be used to cover different ‘scenarios’ or ‘modes’ in multiple-environments of vehicle operation.

The input variables of ‘fuzzy blender’ (a T–S fuzzy system) of the control system consist of outputs of individual adaptive fuzzy controllers. Though this inclusion makes the analysis of stability of the system complicated, it helps to track the input space more effectively. In this system each adaptive fuzzy controller is tuned using an online feedback signal from the blending fuzzy function. Therefore, the whole system creates a unique adaptable system of controllers where versatility is high in comparison to single-model control systems. Hence, the developed system produces a more reliable and accurate control system in comparison to a single-model controller.

Since the adaptive fuzzy controller outputs are blended using ‘IF – THEN’ rules of the fuzzy blender, such an input space can be considered as providing a measure of ‘identification’ of a state-space characteristic that is based on individual states or combination of states of the system. Therefore this arrangement improves identification of a particular scenario. This improvement can be compared against a few states of the system identifying a ‘characteristic’ or ‘condition’ of the control space before applying a control signal in the normal manner. But, the method adopted in this multiple-adaptive controller provides a more versatile ‘identification’. This process is part of mapping by individual adaptive fuzzy controllers that have more capability to identify a wider spectrum of ‘behaviour’ of the system.

(i) Design of Multiple-Adaptive Fuzzy Controllers with Fuzzy Blender

The following Fig. 5.1 shows the schematic diagram of the robust multiple-adaptive fuzzy controller with fuzzy blender. In this control system setup, the outputs of the bank of adaptive fuzzy controllers, \( \psi_j^j \theta \) where \( j = 1, \ldots, n \) (where \( n (>1) \in \mathbb{N} \) is the number total number of adaptive fuzzy controllers in the bank), are blended using a fuzzy blender. The resulting output from the fuzzy blender is used to complement the PD (proportional-derivative) output, and is used to model the ‘uncertainty’ between the vehicle and vehicle-model.
The following design of the multiple-adaptive fuzzy control system with fuzzy blender is done for the generic case irrespective of whether it is a lateral or longitudinal controller. In this generic setup there is no limitation in the number of adaptive fuzzy controllers that can be included in the multiple-controller system.

Fig. 5.1 Schematic diagram of multiple-adaptive fuzzy controller with fuzzy blender for longitudinal (lateral) control

The ‘fuzzy blender’ is a fuzzy system with fixed parameters. The input variables to the ‘fuzzy blender’ are the individual outputs of the bank of adaptive fuzzy controllers, i.e. $\psi_j \theta_j$, $j = 1, \ldots, n$, where $n (>1) \in \mathbb{N}$ is the total number of adaptive fuzzy controllers in the control system (separately in lateral and longitudinal control systems).

**Remarks 5.1:** The term $j$th adaptive fuzzy output, $\psi_j \theta_j$, where $j = 1, \ldots, n$, is equivalent to the single-adaptive fuzzy output, $\xi^T (\bar{x}) \theta$ in every sense as used in Chapter 4, except for the fact that in the former, the adaptive law of each adaptive fuzzy controller consists a relevant online feedback signal from the fuzzy blender. A different notation has also been used to avoid ambiguities with indices in the two cases. □

The output membership function parameters of the ‘fuzzy blender’, i.e. $a_{ij}$ where $i = 1, \ldots, m$ ($m (>1) \in \mathbb{N}$ is the total number of IF-THEN rules in the ‘fuzzy blender’), and $j = 1, \ldots, n$, are always kept ‘fixed’ in value unlike in individual adaptive fuzzy
controllers (in adaptive fuzzy controllers, only the output membership function parameters are ‘adaptive’).

Let the IF-THEN rules of ‘fuzzy blender’ (a T–S fuzzy system) be,

\[ \mathcal{R}^1: \]

\[ \text{IF } \psi_i^T \theta \text{ is } \mathcal{F}_i^q \text{ AND } \ldots \text{ AND } \psi_n^T \theta \text{ is } \mathcal{F}_n^r \text{ THEN} \]

\[ z_i = a_{i1}(\psi_1^T \theta) + \ldots + a_{in}(\psi_n^T \theta) \]

\vdots

\[ \mathcal{R}^m: \]

\[ \text{IF } \psi_i^T \theta \text{ is } \mathcal{F}_i^q \text{ AND } \ldots \text{ AND } \psi_n^T \theta \text{ is } \mathcal{F}_n^r \text{ THEN} \]

\[ z_m = a_{m1}(\psi_1^T \theta) + \ldots + a_{mn}(\psi_n^T \theta) \]

(5.1)

where \( \psi_i^T \theta \) is the control output from the \( j \)th adaptive fuzzy controller (\( \psi_j \) is the regression vector and \( \theta_j \) is the parameter set of the \( j \)th controller) where \( j = 1, \ldots, n \). \( z_i \) is the \( i \)th (\( i = 1, \ldots, m (m(>1) \in \mathbb{N}) \)) consequent membership function where \( m \) is the total number of IF–THEN rules in the ‘fuzzy blender’.

The tuning of each adaptive fuzzy controller, \( \psi_j^T \theta \) where \( j = 1, \ldots, n \) is done with an adaptive law (derived in the stability analysis process) consisting of two parts: ‘internal’ and ‘external’. The internal part of the adaptive law consists of the system error, and the external part consists of a relevant online ‘feedback signal’ from the ‘fuzzy blender’, i.e. \( a^T_j \zeta \) to the \( j \)th adaptive fuzzy controller where \( j = 1, \ldots, n \) (the term \( a^T_j \zeta \) is explained in the Lyapunov stability analysis). The input variables to each adaptive fuzzy controller are ‘error’ and ‘error rate’.

Having each adaptive law as a function of a relevant online feedback signal from the fuzzy blender is advantageous. This allows one to change each set of adaptive parameters in each adaptive fuzzy controller with respect to the fixed set of parameters of the fuzzy blender. Therefore, adaptation in each adaptive fuzzy controller becomes a ‘global’ phenomenon as against a purely ‘local’ one.
Let the regression factors of the ‘fuzzy blender’ be \( \mu_i (0 \leq \mu_i \leq 1) \in \mathcal{R} \) where \( i = 1, \ldots, m \), and \( m \) is the number of IF-THEN rules. Therefore the output from the ‘fuzzy blender’ will be (resulting as a defuzzification with the T–S system after taking individual outputs of the adaptive fuzzy controller bank as inputs to the fuzzy blender)

\[
\hat{v} = \frac{m \sum_{j=1}^{m} z_j (\prod_{j=1}^{n} \mu_{\lambda_j})}{\sum_{i=1}^{m} \prod_{j=1}^{n} \mu_{\lambda_j}}
\]

\[
= \frac{z_1 (\prod_{j=1}^{n} \mu_{\lambda_j})}{\sum_{i=1}^{m} \prod_{j=1}^{n} \mu_{\lambda_j}} + \ldots + \frac{z_m (\prod_{j=1}^{n} \mu_{\lambda_j})}{\sum_{i=1}^{m} \prod_{j=1}^{n} \mu_{\lambda_j}} \quad (5.2)
\]

where \( \mu_{\lambda_j}^{\prime} \) is the fuzzy inference of the input variable \( (\psi^T \theta)_j \) in the rule \( i \).

The \( r \)th regression factor of the ‘fuzzy blender’ can be expressed as

\[
\zeta_r = \frac{(\prod_{j=1}^{n} \mu_{\lambda_j})}{\sum_{i=1}^{m} \prod_{j=1}^{n} \mu_{\lambda_j}} , \quad (5.3)
\]

and therefore the normalized regression vector for the ‘fuzzy blender’ is

\[
\zeta = [\zeta_1 \quad \zeta_2 \quad \ldots \quad \zeta_m]^T . \quad (5.4)
\]

Now, the fuzzy blender output \((5.2)\) becomes

\[
\hat{v} = \zeta_1 z_1 + \zeta_2 z_2 + \ldots + \zeta_m z_m . \quad (5.5)
\]

Substituting for output membership function value in the expression \((5.5)\) provides

\[
\hat{v} = \zeta_1 [a_{11} (\psi^T \theta_1) + a_{12} (\psi^T \theta_2) + \ldots + a_{1n} (\psi^T \theta_n)] + \ldots + \\
\zeta_m [a_{m1} (\psi^T \theta_1) + a_{m2} (\psi^T \theta_2) + \ldots + a_{mn} (\psi^T \theta_n)] . \quad (5.6)
\]

Further rearrangement of \((5.6)\) provides

\[
\hat{v} = \left[ \zeta_1 a_{11} + \zeta_2 a_{21} + \ldots + \zeta_m a_{m1} \right] \psi^T \theta_1 + \ldots + \quad (5.7)
\]
\[ \begin{bmatrix} \zeta_1 a_{1n} + \zeta_2 a_{2n} + \cdots + \zeta_m a_{mn} \end{bmatrix} \psi_j^T \theta_i. \]

Separating the expression (5.7) into individual vectors provides

\[ \hat{\nu} = \left[ \begin{array}{c} \zeta_1 \\ \zeta_2 \\ \vdots \\ \zeta_m \end{array} \right] \begin{bmatrix} a_{11} \\ a_{21} \\ \vdots \\ a_{m1} \end{bmatrix} \psi_j^T \theta_i + \cdots + \left[ \begin{array}{c} \zeta_1 \\ \zeta_2 \\ \vdots \\ \zeta_m \end{array} \right] \begin{bmatrix} a_{1n} \\ a_{2n} \\ \vdots \\ a_{mn} \end{bmatrix} \psi_j^T \theta_n. \tag{5.8} \]

Once again, contracting the expression (5.8) with vector symbols provides

\[ \hat{\nu} = \zeta^T \left[ a_1 (\psi_1^T \theta_1) + a_2 (\psi_2^T \theta_2) + \cdots + a_n (\psi_n^T \theta_n) \right]. \tag{5.9} \]

Finally, it results in as

\[ \hat{\nu} = \zeta^T \left( \sum_{j=1}^n a_j (\psi_j^T \theta_j) \right). \tag{5.10} \]

\textbf{Remark 5.2:} Though (5.10) is generally nonlinear, it is linear with respect to its unknown parameters. Hence, parameter adaptation algorithms applicable for linear systems, e.g. gradient-based algorithms, can be readily used in estimation of the unknown parameters in (5.10) [27]. \( \Box \)

For the \( j \)th adaptive fuzzy controller, the regression vector can be expressed as

\[ \psi_j = \begin{bmatrix} \psi_{j1} & \psi_{j2} & \cdots & \psi_{jp} \end{bmatrix}^T. \]

Thereby the \( k \)th element of \( j \)th controller becomes

\[ \psi_{jk} = \begin{bmatrix} \psi_{jk} & \psi_{jk} \dot{e} \end{bmatrix} = \begin{bmatrix} \psi_{jk} e^T \end{bmatrix} \]

where \( j = 1, \cdots, n \) (\( n \) is the total number of controllers), and \( k = 1, \cdots, p \) where \( p (> 1) \in \mathbb{N} \) is the total number of IF-THEN fuzzy rules in each adaptive fuzzy controller.

Each output membership function parameter set of each adaptive fuzzy controller can be tuned using an algorithm based on an adaptive law. For the \( j \)th adaptive fuzzy controller, this adaptive law becomes

\[ \hat{\theta}_j = -\gamma_j \psi_j a_j^T \zeta Ce \tag{5.11} \]

where \( j = 1, \cdots, n \) (\( n \) is the number of adaptive fuzzy controllers).
(ii) Control Law

The certainty equivalence control laws for the longitudinal and lateral control of the vehicle are, respectively

\[
\begin{align*}
\hat{u}_x &= \frac{1}{b_x}\left[\ddot{x}_p + k_{ds}\dot{e}_x + k_{ps}e_x + \varsigma^T \left(\sum_{j=1}^{n} \alpha_j (\psi_j^T \theta_j)\right)\right] + u_{sx} - \hat{h}_x(\dot{q}_x) \\
\hat{u}_y &= \frac{1}{b_y}\left[\ddot{y}_p + k_{dy}\dot{e}_y + k_{py}e_y + \varsigma^T \left(\sum_{j=1}^{n} \alpha_j (\psi_j^T \theta_j)\right)\right] + u_{sy} - \hat{h}_y(\dot{q}_y).
\end{align*}
\]

(5.12) (5.13)

Here, \(\varsigma = [\varsigma_1, \varsigma_2, \ldots, \varsigma_m]^T\) is the regression matrix of the fuzzy blender where \(\varsigma_i (0 \leq \varsigma_i \leq 1) \in \mathbb{R}, \ i = 1, \ldots, m\) is the \(i\)th fuzzy regression factor. This fuzzy blender does the blending of each adaptive fuzzy controller output (for \(j\)th controller it is \(\psi_j^T \theta_j\) where \(j = 1, \ldots, n\)) together, and thereby generates a unique output. Each \(\theta_1, \theta_2, \ldots, \theta_n\) provides parameter matrices for each adaptive fuzzy controller.

Here, \(\hat{h}_{x,y}(\dot{q}_j)\) denotes the calculated value of the term \(h_{x,y}(\dot{q}_j)\). \(\hat{V}_x = \varsigma^T \left(\sum_{j=1}^{n} \alpha_j (\psi_j^T \theta_j)\right)\) and \(\hat{V}_y = \varsigma^T \left(\sum_{j=1}^{n} \alpha_j (\psi_j^T \theta_j)\right)\) are the outputs from the multiple-adaptive fuzzy controller for longitudinal and lateral cases, respectively. \(k_{ds}, k_{ps}\) are the constant PD (proportional-derivative) gains for the longitudinal controller, and \(k_{dy}, k_{py}\) are the constant PD gains for the lateral controller. \(u_x = [u_{sx}, u_{sy}]\) is the variable structure control term.

(iii) Error Dynamics

Equation (5.12) can be substituted in the vehicle model equation (3.4), and thereby it can be rearranged to have the longitudinal error dynamics as

\[
\begin{align*}
\ddot{e}_x + k_{ds}\dot{e}_x + k_{ps}e_x &= \hat{h}_x(\dot{q}_x) - h_x(\dot{q}_x) - \varsigma^T \left(\sum_{j=1}^{n} \alpha_j (\psi_j^T \theta_j)\right) - u_{sx} - d_x \\
&= \hat{V}_x + W_x - u_{sx} - d_x \\
&= \tilde{V}_x - u_{sx} - d_x.
\end{align*}
\]

(5.14)

In the same manner, using (5.13) and (3.5), lateral error dynamics will result in as
\[
\dot{e}_j + k_{ey} \dot{e}_y + k_{ey} e_y = \hat{h}_y(q_j) - h_y(q_j) - \xi^T \left( \sum_{j=1}^n \alpha_j (\psi_j^T \theta_j) \right) - u_{sy} - d_y
\]

\[
= v^*_y + w_y - \dot{v}_y - u_{sy} - d_y
\]

\[
= \dot{v}_y - u_{sy} - d_y.
\]

(5.15)

In this discussion, \( v^* = [v^*_x \ v^*_y] \) is the optimal adaptive fuzzy output while \( \dot{v} = [\dot{v}_x \ \dot{v}_y] \) is the actual adaptive fuzzy output. \( w_x, w_y \) are the adaptive fuzzy approximation errors. These errors are bounded by \( |w_x| < W_x \) and \( |w_y| < W_y \) where \( W_x, W_y \in \mathbb{R} \) are the practical bounds.

By using the universal approximation theorem (provided in Chapter 4), with the fuzzy adaptation process, the fuzzy controller parameters can be gradually brought to the optimal values. Thus the right half of the equations (5.14) and (5.15) can be made zero.

The variable structure term, \( u_s \) is used to balance the effects of fuzzy approximation errors. The overall uncertainty factor in the equation is made zero with the fuzzy function.

The detailed design of each adaptive fuzzy controller (e.g. \( j \)th adaptive fuzzy controller, \( \psi_j^T \theta_j \), where \( j = 1, ..., n \)) is provided in Chapter 4 under the topic of single-model adaptive fuzzy systems.

### 5.1.3 Training of Multiple-Adaptive Fuzzy Controller Parameters

(i) **Training of Adaptive Fuzzy Controllers**

The set of adaptive fuzzy controllers in the control system can be used in a number of different ways.

Firstly, the controllers can be trained with a data set (obtained from running a vehicle through a real scenario), similar to that with the ‘single model adaptive fuzzy controller’ discussed in Chapter 4. This data is for tuning the input and output
membership parameters of the adaptive fuzzy controllers. Then the adaptive rates in the adaptive law can be changed to differentiate each controller from the other. The adaptive fuzzy controllers having ‘higher adaptive rate’ can be used in the faster changing scenarios whereas the controllers having ‘lower adaptive rate’ can be used with low frequency changes in the scenarios. Since each controller is trained on the same set of ‘global’ data, each controller can act globally.

Instead of having a common data set for training each of adaptive fuzzy controllers, data space can be categorized into groups, e.g. according to the frequency of change of the variable ‘error’, i.e. error rate. Thereby, using each of the ‘local’ data sets, a single adaptive fuzzy controller can be trained. It is clear that blending each of the controllers will address the ‘global’ scenarios having full ‘spectrum’ of frequencies of operation. This is another option for tuning the adaptive fuzzy controllers. But, the first option will be followed in the design of the control system.

Using the subtractive clustering method, the fuzzy membership function centres and standard deviations (Gaussian) have been obtained. Then the basic fuzzy structure obtained using the subtractive clustering method is further tuned using ANFIS with the gathered data from veDYNA®. The details of the tuning process are explained below.

(ii) Training of Fuzzy Blender Function

The fixed parameter fuzzy blender of the multiple-adaptive fuzzy system was tuned as follows. The tuning of the fuzzy blender was done considering only two adaptive fuzzy controllers as inputs in each longitudinal and lateral case. The adaptive rates of the adaptive fuzzy controllers were chosen as 0.03 and 0.04 in the longitudinal controller, and 0.01 and 0.03 in the lateral controller.

The following tuning procedure of the fuzzy blender was adopted in general, irrespective of the lateral or longitudinal controller.

In each \( k \)th trial, the output for the fuzzy blender was calculated using the equation

\[
\hat{v}_k = \hat{h}_i(q_\chi_k) - h_i(q_\chi_k)
\]

where \( i = x, y \) for longitudinal and lateral cases respectively. The vehicle dynamics, \( h_i(q_\chi_k) \) was calculated using BMW 325i 1988 model of veDYNA®, and the vehicle
model dynamics, $\hat{f}(\dot{q}_i)$ was calculated using equation (3.7).

The tuning procedure of the fuzzy blender is described as follows:

i) First, by setting the output membership function parameters of the blender as ‘ones’ (i.e. ‘1’s), data was collected for the output of the blender, following a profile of vehicle simulation trial on veDYNA®. This data collection was similar to the case of the single adaptive fuzzy system, as described in Section 4.3.

ii) Using the output data obtained in (i), and together with the two adaptive fuzzy controller outputs taken as the two inputs to the fuzzy blender, a basic fuzzy structure was generated using the subtractive clustering method. The same set of data has been used further to obtain a more refined fuzzy structure for the fuzzy blender using the ANFIS method. The fuzzy blender structure from the first trial was thus obtained.

iii) Once the first set of parameters (parameters from the first trial) of the fuzzy blender was identified, taking these parameters as the current working setup for the next trial, a new data set was collected. The same profile of vehicle simulation trial on veDYNA® was followed. The data obtained was used to work out the new fuzzy blender structure in the same way as discussed in (i) and (ii). This structure was taken as the fuzzy blender for the second trial.

The trials of optimizing the fuzzy blender parameters were continued until there was no change of value that was more than 5% of the output of the fuzzy blender. Thereby, using the final set of data, the parameters of the fuzzy blender were identified.

(a) Training Data: Longitudinal

Figs. 5.2 and 5.3 show the inputs and the output for tuning the longitudinal fuzzy blender in the first trial. More details on validation of developed fuzzy models are provided in Section 4.1.5.
The ANFIS training and checking errors for tuning the longitudinal fuzzy blender in the first trial are shown in Figs. 5.4 and 5.5, respectively.

The fuzzy blender parameters that were associated with the lowest error in the checking (validation) were selected as the final values. This validation error is as shown in Fig. 5.5.
The longitudinal fuzzy blender control surface depicting the relationship between the output of the function and the inputs is shown in Fig. 5.6. It is to be noted that the input range of the fuzzy blender has been scaled down for minimizing calculation errors. The following was taken as the final form of the control surface of the longitudinal fuzzy bender after three trials (Fig. 5.6).

(b) Training Data: Lateral

Figs. 5.7 and 5.8 show the inputs and the output for tuning the lateral fuzzy blender in the first trial. It is important to note the details on validation of developed fuzzy models as provided in Section 4.1.5.
**Inputs**

Fig. 5.7 Train data (inputs) for ANFIS training of lateral fuzzy blender

**Output**

Fig. 5.8 Train data (output) for ANFIS training of lateral fuzzy blender

The ANFIS training and checking errors for tuning the lateral fuzzy blender in the first trial are shown in Figs. 5.9 and 5.10, respectively.

Fig. 5.9 Training error for ANFIS training of lateral fuzzy blender

The fuzzy blender parameters that were associated with the lowest error in the checking (validation), were selected as the final values for the lateral fuzzy blender. This validation error is as shown in Fig. 5.10.
The lateral fuzzy blender control surface depicting the relationship between the output of the function and the inputs is shown in Fig. 5.11. It is to be noted that the input range of the fuzzy blender has been scaled down for minimizing calculation errors. Fig. 5.11 shows the final form of the control surface of the lateral fuzzy bender after three trials.

The final tuned input and output membership function parameters of the fuzzy blender in longitudinal and lateral forms are provided in APPENDIX C.
5.1.4 Robust Stability Analysis of Control System

The following stability proof of the developed multiple-adaptive fuzzy control system with fuzzy blender is carried out for the generic case irrespective whether it is lateral or longitudinal controller. Therefore, the subscripts that are specific to either the longitudinal or lateral case are removed in this description. Nevertheless, the specific details will be mentioned as and when necessary.

Assumption 5.1: The sum of fuzzy approximation errors, $w$ is bounded, i.e., $|w| \leq W$ where $W(>0) \in \mathbb{R}$. The external disturbances, $d$ are bounded too, i.e., $|d| \leq D$ where $D(>0) \in \mathbb{R}$. Thereby the variable structure control gain is chosen as $\sigma_j \geq W + D$. □

The main features of the robust multiple adaptive fuzzy control system with ‘blending’ can be summarized in the following theorem.

Theorem 5.1: Let the parameter vectors, $\theta_j$ of the adaptive fuzzy systems be adjusted by the adaptive law as provided in (5.11) and let Assumption 5.1 be true. The proposed adaptive fuzzy control law as provided in (5.12) [and (5.13)] can guarantee stability of the vehicle system with the following properties:

i) The closed loop vehicle system is stable, i.e. $\|e\| \in L^\infty$, considering lateral and longitudinal cases separately.

ii) The system errors and the parametric errors are asymptotically stable, i.e. $\lim_{t \to \infty} e_x, e_y = 0$, and $\lim_{t \to \infty} \hat{\theta}_j = 0$ for $j = 1, ..., n$ where $n$ is the total number of adaptive fuzzy controllers in the control system (considering lateral and longitudinal cases separately). □
**Proof:**

Positive realness of the transfer function of the error dynamics equation, i.e., (4.23) and (4.24), related to vehicle dynamics has already been established in Section 4.2.6.

Assuming that, positive realness of the vehicle error dynamics can be established as shown in Section 4.2.6, according to Kalman–Yakubovich–Popov (KYP) lemma [175] for a strictly positive real system there exist two positive definite matrices $P$ and $Q$ (i.e., $P = P^T > 0$ and $Q = Q^T > 0$) satisfying

$$A^T P + PA + Q = 0, \text{ and } PB = C^T. \quad (5.17)$$

With KYP lemma, the following Lyapunov function candidate can be selected,

$$V = e^T Pe + \sum_{j=1}^{n} \gamma_j^{-1} \hat{\theta}_j^T \hat{\theta}_j \quad (5.18)$$

where $P = P^T > 0$, and $\gamma_j (>0) \in \mathbb{R}$ where $j = 1, \ldots, n$. Here $e$ is the overall system error, and $\hat{\theta}_j$ is the parametric error vector with respect to the optimal parameter error vector for the $j$th adaptive fuzzy controller where $j = 1, \ldots, n$.

The Lyapunov candidate quantifies both in tracking error and in parameter estimates. Differentiating the Lyapunov candidate along the trajectories of the error system will provide

$$\dot{V} = \dot{e}^T Pe + e^T P \dot{e} + \sum_{j=1}^{n} \left( \gamma_j^{-1} \hat{\theta}_j^T \dot{\hat{\theta}}_j + \gamma_j^{-1} \dot{\hat{\theta}}_j^T \hat{\theta}_j \right). \quad (5.19)$$

Substituting for $\dot{e}$ from (4.23) in the equation (5.19), it results in

$$\dot{V} = (A e + B \hat{v} - B d - B u_i)^T Pe + e^T P (A e + B \hat{v} - B d - B u_i)$$

$$+ \sum_{j=1}^{n} \left( \gamma_j^{-1} \hat{\theta}_j^T \dot{\hat{\theta}}_j + \gamma_j^{-1} \dot{\hat{\theta}}_j^T \hat{\theta}_j \right). \quad (5.20)$$

Rearranging (5.20) will provide

$$\dot{V} = e^T (A^T P + PA)e + \hat{v}^T B^T Pe + e^T PB \hat{v} - d B^T Pe - de^T PB - u_i B^T Pe - u_i e^T PB$$

$$+ \sum_{j=1}^{n} \left( \gamma_j^{-1} \hat{\theta}_j^T \dot{\hat{\theta}}_j + \gamma_j^{-1} \dot{\hat{\theta}}_j^T \hat{\theta}_j \right). \quad (5.21)$$
With the Lyapunov equation \( A^T P + PA + Q = 0 \) where \( Q = Q^T > 0 \), (5.21) can be further simplified as

\[
\dot{V} = -e^T Q e + \tilde{v}^T B^T P e + e^T P B \tilde{v} - (d + u_j) (B^T P e + e^T P B) + \sum_{j=1}^{n} \left( \gamma_j^{-1} \hat{\theta}_j^T \tilde{\theta}_j + \gamma_j^{-1} \hat{\theta}_j^T \dot{\theta}_j \right)
\]

(5.22)

From the argument, it can be expressed as \( \dot{V} = (\nu^* - \nu) + \tau \) where \( \nu \) is the input from the fuzzy blending function. \( \nu^* \) is the optimum fuzzy input, and \( \tau \) is the approximation error of the fuzzy blending function. It can be further elaborated that

\[
\dot{\nu} = \varsigma^T \left( \sum_{j=1}^{n} \alpha_j (\psi_j^T \hat{\theta}_j) \right) - \varsigma^T \left( \sum_{j=1}^{n} \alpha_j (\psi_j^T \dot{\theta}_j) \right) + \sum_{j=1}^{n} w_j
\]

(5.23)

where \( \theta_j^* \in \mathbb{R}^{m_2} \) is the optimal parameter matrix for the \( j \)th adaptive fuzzy controller. \( \dot{\theta}_j \) is the parameter set of the \( j \)th adaptive fuzzy controller, and \( w_j \) is approximation error of the \( j \)th adaptive fuzzy controller. This error is bounded, \( |w_j| \leq W \), i.e. \( w_j \in \mathcal{L}^{\infty} \) where \( W \in \mathbb{R} \) is a positive constant.

Equation (5.23) can be further simplified so that

\[
\dot{\nu} = \varsigma^T \left( \sum_{j=1}^{n} \alpha_j \psi_j^T (\theta_j^* - \dot{\theta}_j) \right) + \sum_{j=1}^{n} w_j
\]

(5.24)

Let \( w = \sum_{j=1}^{n} w_j \) where \( |w| < W (\in \mathbb{R}) \) is a positive constant, and let the parameter error be taken as \( \tilde{\theta}_j = \theta_j^* - \dot{\theta}_j \). The determination of fuzzy approximation error upper bound, \( W \) has been discussed before [30].

Therefore equation (5.24) now becomes

\[
\dot{\nu} = \varsigma^T \left( \sum_{j=1}^{n} \alpha_j \psi_j^T \tilde{\theta}_j \right) + w
\]

(5.25)

Substituting (5.25) in (5.22) provides

\[
\dot{V} = -e^T Q e + \left[ \varsigma^T \left( \sum_{j=1}^{n} \alpha_j \psi_j^T \tilde{\theta}_j \right) + w \right]^T B^T P e + e^T P B \left[ \varsigma^T \left( \sum_{j=1}^{n} \alpha_j \psi_j^T \tilde{\theta}_j \right) + w \right]
\]

\[-(d + u_j) (B^T P e + e^T P B) + \sum_{j=1}^{n} \left( \gamma_j^{-1} \dot{\theta}_j^T \tilde{\theta}_j + \gamma_j^{-1} \dot{\theta}_j^T \dot{\theta}_j \right) \]

(5.26)

Since \( PB = C^T \), it can be further simplified to provide
Further rearrangement of (5.27) will result in

\[ \dot{V} = -e^T Q e + \sum_{j=1}^{n} \left( (\tilde{\theta}_j)^T \psi_j \alpha_j^T \right) \zeta C e + e^T C^T \zeta^T \sum_{j=1}^{n} (a_j \psi_j^T (\tilde{\theta}_j)) + \sum_{j=1}^{n} \left( \gamma_j^{-1} \hat{\theta}_j \tilde{\theta}_j + \gamma_j^{-1} \hat{\theta}_j \tilde{\theta}_j \right) + (w - d - u_i) (Ce + e^T C^T). \] (5.27)

Now, (5.28) can be rearranged and written as

\[ \dot{V} = -e^T Q e + \sum_{j=1}^{n} \left( e^T C^T \zeta^T (a_j \psi_j^T) + \gamma_j^{-1} \hat{\theta}_j \right) \tilde{\theta}_j \]

\[ + \sum_{j=1}^{n} \left( \hat{\theta}_j \left( \psi_j \alpha_j^T \zeta C e + \gamma_j^{-1} \hat{\theta}_j \right) \right) + (w - d - u_i) (Ce + e^T C^T). \] (5.29)

If the adaptive law as provided in (5.11), i.e., \( \hat{\theta}_j = -\gamma_j \psi_j \alpha_j^T \zeta C e \), is substituted in the equation (5.29), it can be reduced to

\[ \dot{V} = -e^T Q e + (w - d - u_i) (Ce + e^T C^T). \] (5.30)

Equation (5.30) can be further expressed as

\[ \dot{V} \leq -\lambda_{q_{\text{min}}} \| e \|^2 + (w - d - u_i) (Ce + e^T C^T) \] (5.31)

where \( \lambda_{q_{\text{min}}} \) is the minimum eigen value of \( Q \). Since fuzzy systems follow universal approximation theorem [29], \( |w| \) may be made arbitrarily small by making each \( w_j \) small by proper choice of each fuzzy system if each \( \psi_j^T \hat{\theta}_j \) is smooth (note that this may require an arbitrarily large number of rules for each controller) [30].

Since disturbances are bounded, i.e., \( |d| < D \), and it is known that \( |w| < W \), it can be further expressed that

\[ \dot{V} \leq -\lambda_{q_{\text{min}}} \| e \|^2 + (D + W - u_i) (Ce + e^T C^T). \] (5.32)
Let the variable structure control term be \( u_s = \sigma_j \text{sign} \left( C e + e^T C^T \right) \). Thereby, (5.32) can be expressed as
\[
\dot{V} \leq -\lambda_{q_{\min}} \left\| e \right\|^2 + \left( \left\| D + W \right\| - \sigma_j \right) \left\| C e + e^T C^T \right\|.
\] (5.33)
Further arrangement of (5.33) will provide
\[
\dot{V} \leq -\lambda_{q_{\min}} \left\| e \right\|^2 - \left\| C e + e^T C^T \right\| \left( \sigma_j - \left\| D + W \right\| \right).
\] (5.34)
If it is taken as \( \sigma_j \geq \left\| D + W \right\| \), then (5.34) will establish that
\[
\dot{V} \leq 0, \quad \forall t.
\] (5.35)
Therefore, according to the Lyapunov theory, it establishes the fact that \( e \) is bounded, i.e. \( e \in \mathcal{L}^\infty \). Therefore, \( e, \dot{e} \) are bounded, i.e. \( e, \dot{e} \in \mathcal{L}^\infty \). It also establishes that parameter errors are bounded, i.e. \( \tilde{\theta} \in \mathcal{L}^\infty \).

Equation (5.34) also implies that
\[
\dot{V} \leq -\lambda_{q_{\min}} \left\| e \right\|^2.
\] (5.36)
Integrating (5.36) will provide
\[
\lambda_{q_{\min}} \int_0^\infty \left\| e \right\|^2 dt \leq - \int_0^\infty \dot{V} dt = V(0) - V(\infty).
\] (5.37)
Therefore, rearranging (5.37) will provide
\[
\int_0^\infty \left\| e \right\|^2 dt \leq \frac{1}{\lambda_{q_{\min}}} \left( V(0) - V(\infty) \right).
\] (5.38)
Equation (5.38) leads to
\[
e \in \mathcal{L}^2 \left( \mathcal{L}^2 = \left\{ z(t) : \int_0^\infty z^2(t) dt < \infty \right\} \right)
\] (5.39)
since it provides \( V(0), V(\infty) \in \mathcal{L}^\infty \) according to (5.35) [30]. Since it can be proved as \( e \in \mathcal{L}^2 \cap \mathcal{L}^\infty \) (from (5.39) and (5.35)), and \( \dot{e} \in \mathcal{L}^\infty \) (from (5.36)), by Barbalat’s lemma [30], [176], it leads to asymptotic stability of \( e \) (i.e. \( \lim_{t \to \infty} e = 0 \)). This can be extended specifically to lateral and longitudinal cases, and consequently the lateral and longitudinal errors are asymptotically stable, i.e.
\[
\lim_{t \to \infty} e_x, e_y = 0. \tag{5.40}
\]

This asymptotic stability result, as provided by (5.40) proves that when lateral and longitudinal cases are considered at the same time, with the given arguments, the system of errors can be proved \textit{asymptotically stable}. Therefore, this result logically proves that the integrated lateral and longitudinal control system is \textit{asymptotically stable}. This statement completes the proof of stability for the robust multiple-adaptive fuzzy controller system based on blending. ■

The following definition is provided to formerly identify the variable vector spaces, and thereby to define the optimal parameter set.

\textit{Definition 5.1:} It is possible to define the compact parameter spaces for \( \theta \) and \( \pi \) variables. Let \( M_\theta(>0) \in \mathbb{R} \) and \( M_\pi(>0) \in \mathbb{R} \) be finite, and

\[
\Omega_\theta = \left\{ \theta_j \in \mathbb{R} : \left\| \theta_j \right\| \leq M_\theta \right\}, \text{ and} \\
\Omega_\pi = \left\{ \pi \in \mathbb{R} : \left\| \pi \right\| \leq M_\pi \right\}.
\]

Thereby the optimal parameter set for each \( j \)th adaptive fuzzy controller, where \( j = 1, \ldots, n \), can be formally defined as

\[
\theta_j^* = \arg \min_{\theta_j \in \Omega_\theta} \left\{ \sup_{\pi \in \Omega_\pi} \left| V^* - \zeta^T \left( \sum_{j=1}^n \alpha_j (\psi_j \hat{\theta}_j) \right) \right| \right\}, \tag{5.41}
\]

where \( V^*(\in \mathbb{R}) \) is the optimal adaptive fuzzy function output. \( \pi \) is the generic input variable to each fuzzy system. □

\textit{Remark 5.3:} The Lyapunov candidate satisfies the following conditions:

i) \( V \) is at least \( C^1 \)

ii) \( V(0) = 0 \), and \( V(x(t)) > 0 \) for \( x(t) \neq 0 \)

and

iii) \( \left\| x(t) \right\| \to \infty \Rightarrow V(x(t)) \to \infty \). (\( x(t) \) denotes a selected time-dependent state of the system in this remark). □

\textit{Remark 5.4:} The definition of \( \theta_j^* \) is only for theoretical reasons, and it is not required for the implementation of the controller. □
5.1.5 Simulation Studies, Results and Discussion

(i) Simulation Setup

The simulation setup for this section was similar to the one provided in Chapter 4 for the single-adaptive fuzzy controller system, except for facts otherwise stated in this section. The simulation setup included veDYNA® setup, simulation profiles and other settings.

The adaptive gains of adaptive fuzzy controllers in the bank were taken as $\gamma_{x1} = 0.03$ and $\gamma_{x2} = 0.04$ for longitudinal: $\gamma_{y1} = 0.01$, and $\gamma_{y2} = 0.03$ for lateral cases, in multiple-adaptive fuzzy controller setting.

(ii) Simulation Results and Discussion

In the following simulation results, each performance parameter of multiple-adaptive fuzzy controller (solid blue line) is compared against that of the single-adaptive fuzzy (dotted red line) controller.

(a) Normal Cruising Conditions: Multiple-Adaptive Fuzzy Control with Blending

The following simulation results provide the performance of the developed multiple-adaptive fuzzy controller against the performance of single-adaptive fuzzy controller under normal cruising conditions without presence of any external disturbances.

The multiple-adaptive fuzzy controller with blending has shown an improved longitudinal tracking performance with quick response in comparison to the single-adaptive fuzzy system, i.e. around 5 – 7 [s] on the time-scale. This is shown in Fig. 5.12.
As far as lateral tracking is concerned, the multiple-adaptive fuzzy controller has shown a significant improvement over the single-adaptive fuzzy controller, as shown in Fig. 5.13. In the meantime, multiple-adaptive fuzzy controller has also reduced the transient effects, and is clear with the reduction of amplitudes of chattering effects, throughout.

Even though, pitch angle with multiple-adaptive fuzzy controller has shown some high peaks in the initial period, there are no major differences in the variations in the two curves of the controllers for the rest of the time. This is shown in Fig. 5.14 below.
As far as lateral stability is concerned, as evidenced by roll angle, roll rate, and side-slip angle as shown in Figs. 5.15–5.17, there are no major differences in the cases of single-adaptive or multiple-adaptive fuzzy controller performances. It can be said that lateral stability with multiple-adaptive fuzzy controller has been within the limits of its
counterpart. Nevertheless, multiple-adaptive fuzzy systems have shown a slightly better performance due to having slightly less transients with low amplitudes.

Side-slip angle variation, as shown in Fig. 5.17, has improved slightly in the case of multiple-adaptive fuzzy controller, especially when transient effects are concerned. Apart from that there are no major differences in the two performance graphs corresponding to the two controllers.

As shown in Fig. 5.18, the variations of lateral acceleration do not go beyond 4 [m/s$^2$], therefore the two controllers have managed to contain lateral acceleration within low limits as far as the design requirements of passenger vehicles are concerned [178]. Since variations of side-slip angle, as shown in Fig. 5.17, follows similarly with lateral acceleration (increase-decrease trends considering only the absolute magnitudes), the controllability exercised by both controllers stand at good level [178].
Overall, under normal cruising conditions, the multiple-adaptive fuzzy controller has shown good tracking both in lateral and longitudinal aspects in comparison to the single-adaptive fuzzy controller. As evidenced by variations of side-slip angle, roll-angle and roll rate, lateral stability of multiple-adaptive fuzzy system stands at a good level in comparison to the performances of its counterpart. In other words, while multiple-adaptive fuzzy controller shows good lateral and longitudinal tracking performance in comparison to its counterpart, it shows a level of lateral stability at least similar to that with its counterpart—this is a good performance.

(b) External Disturbances: Multiple-Adaptive Fuzzy Control with Blending

In the following set of simulations, the developed multiple-adaptive fuzzy controller was tested against the single-adaptive fuzzy controller under a number of external disturbances. These disturbances included an unsymmetrical load mass on the vehicle, presence of crosswind forces, and change of tyre-road friction.

1. Unsymmetrical Load Mass

The following simulation was carried out for the comparison of results between the multiple-adaptive fuzzy controller and the single-adaptive fuzzy controller, when there was an unsymmetrical load-mass on the vehicle as per the details described in Section 3.2.18.

As shown in Fig. 5.19, longitudinal error with the multiple-adaptive fuzzy controller is slightly less initially (approximately around 8 [s] on the time-scale) against that of the single-adaptive fuzzy controller. The peak value around 20 – 25 [s] is due to a sudden change of curvature with increasing speed.
Lateral error, as shown in Fig. 5.20, has been reduced with the multiple-adaptive fuzzy controller throughout as compared to that with the single-adaptive fuzzy controller. Especially, diminishing of transient effects can be seen throughout with the reduction of the amplitudes of chattering.
Variations of roll angle, roll rate and side-slip angle throughout the simulation have been slightly on the low side with the multiple-adaptive fuzzy controller. Therefore, it has performed slightly better (due to reduction in transient effects) in comparison to the single-adaptive fuzzy controller. This suggests lateral stability with multiple-adaptive fuzzy controller is slightly better. These parameter changes are illustrated in Figs. 5.22–5.24.
The variations of lateral acceleration fall within the ‘linear range’, i.e., 0.5 – 4 [m/s\(^2\)] [178], as shown in Fig. 5.25, for both controllers. This is a good sign shown by the two controllers to contain lateral acceleration within low limits as far as the design requirements are concerned [178]. Even, the ‘spikes’ at intermittent points with short existences in time will not have any drastic effects on the comfort of passengers.

Overall, with an unsymmetrical load mass in vehicle, lateral and longitudinal tracking control by the multiple-adaptive fuzzy controller have been significant in comparison to the single-adaptive-fuzzy controller. It is important to note that even lateral stability has not been compromised while showing the positive performance by the multiple-adaptive fuzzy controller.

2. **Crosswind Effects**

The following simulation results were obtained when there was a continuous crosswind with a speed of 50 [m/s] through the global y-direction. More details of this simulation arrangement are described in Section 3.2.18.
Once again, longitudinal and lateral tracking performances of multiple-adaptive fuzzy controller have been significantly better than that of the single-adaptive fuzzy controller, as shown in Figs. 5.26–5.27. Especially with lateral tracking, transient errors have been mostly suppressed throughout by the multiple-adaptive fuzzy controller as against that by the single-adaptive fuzzy controller.

Fig. 5.26 Longitudinal error: multiple-adaptive fuzzy with blending (RMS: 3.658) vs. single-adaptive fuzzy (RMS: 3.742)

Fig. 5.27 Lateral error: multiple-adaptive fuzzy with blending (RMS: 0.003626) vs. single-adaptive fuzzy (RMS: 0.005914)
Fig. 5.28 Pitch angle: multiple-adaptive fuzzy with blending (RMS: 0.01146) vs. single-adaptive fuzzy (RMS: 0.01142)

Fig. 5.29 Roll angle: multiple-adaptive fuzzy with blending (RMS: 0.01399) vs. single-adaptive fuzzy (RMS: 0.01396)

Fig. 5.30 Roll rate: multiple-adaptive fuzzy with blending (RMS: 0.01223) vs. single-adaptive fuzzy (RMS: 0.02222)
As shown by variations of roll angle, roll rate and side-slip angles, lateral stability has been maintained to a consistent level by the multiple-adaptive fuzzy controller. This level of lateral stability is almost in par with that of the single-adaptive fuzzy controller. These performances are shown in Figs. 5.29–5.31. Nevertheless, there are some slight over reactions by the multiple-adaptive fuzzy controllers, too, e.g. with side-slip angle during 5–10 [s] on the time-scale in Fig. 5.31. Since these variations are not excessive, they do not threaten lateral stability of the vehicle.

As shown in Fig. 5.28, pitch angle variations have been consistent with each controller, and therefore the longitudinal stability has been maintained with both controllers.

As shown in Fig. 5.32, the variation of lateral acceleration with the multiple-adaptive fuzzy controller has been slightly in excess over 4 [m/s²] around the point 42 [s] on the time-scale. This is said to be just above the ‘linear range’, and therefore slightly in the so called ‘transition stage’ [178]. Since the effective time period in relation to the slightly excessive values is quite limited, the effects felt on the vehicle are quite small. Therefore, the lateral acceleration variation as shown for the multiple-adaptive fuzzy controller is still acceptable.
Overall, in the presence of crosswind effects, the lateral and longitudinal tracking established by the multiple-adaptive fuzzy controller is satisfactory. Nevertheless, lateral stability specifically in the areas of side-slip angle, roll-angle and roll rate, has shown some slight lapses in initial periods. But, these lapses are not excessive and should be considered against the gains from the improved lateral tracking.

3. Tyre-Road Friction Change

The following simulation results show the performance of the multiple-adaptive fuzzy controllers compared against the single-adaptive fuzzy controller when there was a change of tyre-road friction in the range 100–200 [m], on the highway.

As shown in Figs. 5.33–5.34, longitudinal and lateral tracking performances with the multiple-adaptive fuzzy controller are better than that with the single-adaptive fuzzy controller.
The two controllers fared similarly on variation of pitch angle, except for a high ‘spike’ by the multiple-adaptive fuzzy controller towards the point 5 [s] on the time-scale. This is shown in Fig. 5.35 below.

Fig. 5.34 Lateral error: multiple-adaptive fuzzy with blending (RMS: 0.002091) vs. single-adaptive fuzzy (RMS: 0.003996)

Fig. 5.35 Pitch angle: multiple-adaptive fuzzy with blending (RMS: 0.01162) vs. single-adaptive fuzzy (RMS: 0.01144)

Fig. 5.36 Roll angle: multiple-adaptive fuzzy with blending (RMS: 0.01086) vs. single-adaptive fuzzy (RMS: 0.01085)
The variations of roll angle, roll rate and side-slip angle as shown in Figs. 5.36–5.38, indicate that lateral stability has been quite consistent with both types of controllers.

The variation of lateral acceleration, as shown in Fig. 5.39, with the multiple-adaptive fuzzy controllers has almost been below 4 [m/s$^2$] limit [178], except for a slight ‘spike’ at around 41 [s] on the time-scale. Therefore, it suggests that the multiple-adaptive fuzzy controllers have ensured that lateral acceleration is almost below the limit ensured by design requirements [178]. The only exception to the limit is quite negligible since it is not overly excessive and the duration of the spike is quite small.
Overall, in the presence of tyre-road friction changes, the lateral and longitudinal tracking performances have been impressive as shown with the multiple-adaptive fuzzy controller in comparison to the single-adaptive-fuzzy controller. These good performances have been ensured by good lateral stability as suggested by side-slip angle, roll angle and roll-rate.

(c) Failure Modes (non-catastrophic subsystem failures): Multiple-Adaptive Fuzzy Control with Blending

The following simulation results compare the performance of the multiple-adaptive fuzzy controllers with blending against that of the single-adaptive fuzzy controller when some failures in the vehicle components occur. The failures considered in this simulation included a flat-tyre in the front-left tyre, and a 90% drop of pressure in the brake line of the front-left wheel cylinder of the vehicle.

1. Flat-Tyre

The following simulation results were obtained when an event of flat-tyre in the front-left wheel occurred at 10[s], after starting the simulation. The details of the simulation arrangement is explained in Section 3.2.18

As shown in Fig. 5.40, the longitudinal tracking performance with the multiple-adaptive fuzzy controller is slightly better than that with the single-adaptive fuzzy controller.
Lateral tracking as shown in Fig. 5.41, provides some significant improvement with the multiple-adaptive fuzzy controllers compared to that with the single-adaptive fuzzy controller.
Roll angle, roll rate and side-slip angle variations, as shown in Figs. 5.43–5.45, with the multiple-adaptive fuzzy controller ensure that lateral stability has not been excessive compared to that of the single-adaptive fuzzy controller. The multiple-adaptive fuzzy controller has performed slightly better with improving some transients throughout those variations. Pitch angle variations, normally a longitudinal stability
measure, as shown in Fig. 5.42, on the other hand, provide almost similar performances for both controllers throughout.

Another point to observe is that the roll angle settles down with a steady angle around -0.02 [rad] in the latter half of the simulation on the straight road. This is due to having a flat-tyre on one side of the vehicle, and therefore becomes unbalanced, an obvious fact, as shown in Fig. 5.43.

Lateral acceleration, as shown in Fig. 5.46 does not go beyond 4 [m/s$^2$] for either controller. Therefore, the multiple-adaptive controller has managed to contain the lateral acceleration within low limits as suggested by the design characteristics for passenger vehicles [178].

![Graph showing lateral acceleration comparison between multiple-adaptive and single-adaptive fuzzy controllers]

Fig. 5.46 Lateral acceleration: multiple-adaptive fuzzy with blending (RMS: 1.248) vs. single-adaptive fuzzy (RMS: 1.248)

Overall, in the flat-tyre event, the multiple-adaptive fuzzy controller has shown improved longitudinal and lateral tracking throughout, in comparison to the single-adaptive fuzzy controller. This performance of tracking is while ensuring a fair level of lateral stability.

2. Brake Cylinder Defect

A subsystem failure of front-left wheel-brake cylinder being defective by a 90% pressure drop in the brake line is studied in this chapter. More details on this simulation setup are discussed in Section 3.2.18.
As shown in Figs. 5.47–5.48, the multiple-adaptive fuzzy controller has shown significant improvements in both types of tracking, i.e. lateral and longitudinal.

Pitch angle variations, normally a measure for longitudinal stability, as shown in Fig. 5.49 provide similar performances for both controllers.

As shown in Fig. 5.50, as far as roll angle is concerned, the two controllers show performances with some inconsistency, i.e., during some time intervals multiple-
adaptive controller has done well, e.g., $38 - 43$ [s], while during other time intervals single-adaptive fuzzy controller has done better, e.g., $16 - 17$ [s]. Therefore, it is difficult to compare one controller against the other in this regard. Therefore, it can be said that the performance of the multiple-adaptive fuzzy controllers is acceptable compared to the single-adaptive fuzzy controller as far as lateral stability is concerned.

![Fig. 5.50 Roll angle: multiple-adaptive fuzzy with blending (RMS: 0.01078) vs. single-adaptive fuzzy (RMS: 0.01082)](image)

When the variation of roll rate, as shown in Fig. 5.51, is considered, the multiple-adaptive fuzzy controller has shown some significant improvements over its counterpart during high-values, e.g. $37 - 43$ [s], but during periods with lower values of roll-rate, the single-adaptive fuzzy controller has outperformed its counterpart, e.g. throughout $22 - 37$ [s], $47 - 60$ [s].

![Fig. 5.51 Roll rate: multiple-adaptive fuzzy with blending (RMS: 0.03665) vs. single-adaptive fuzzy (RMS: 0.04617)](image)

Side-slip angle variations as shown in Fig. 5.52, have been similar most the time throughout, while multiple-adaptive fuzzy controller has shown some improvements compared to its counterpart over transients, i.e. especially during $5 - 10$ [s] and $38 - 43$ [s].
Considering Figs. 5.50–5.52, on roll angle, roll rate and side-slip angle, it can be said that the multiple-adaptive fuzzy controller ensures a fairly good level of lateral stability in comparison to the single-adaptive controller.

As shown in Fig. 5.53, lateral acceleration for the multiple-adaptive fuzzy controller does not go beyond 4 [m/s$^2$]. This is a good performance in comparison to some other performances by the single-adaptive fuzzy controller as shown in Fig. 5.53. Therefore, the multiple-adaptive fuzzy controller has performed well to contain the lateral acceleration within low limits as suggested by design requirements for passenger vehicles [178].

Overall, in the presence of a wheel brake cylinder failure, the multiple-adaptive fuzzy controller has managed to ensure a good level of lateral and longitudinal tracking performance in comparison to the single-adaptive fuzzy controller. These results have been achieved in the presence of a good level of lateral stability.
Considering the overall performance of the multiple-adaptive fuzzy controller, lateral and longitudinal tracking performances have been consistently good compared to that of the single-adaptive fuzzy controller. Lateral stability has been maintained at an acceptable level throughout by the multiple-adaptive fuzzy controller. Considering lateral acceleration, it has been a consistent and satisfactory performance as far as design requirements for vehicles are concerned.

The summary of performance of the multiple-adaptive fuzzy controller with blending against that of the single-adaptive fuzzy controller is provided in TABLE 5.1 below.

TABLE 5.1 Comparison of performance of multiple-adaptive fuzzy controller against single-adaptive fuzzy controller

<table>
<thead>
<tr>
<th>Simulation</th>
<th>Longitudinal error</th>
<th>Lateral error</th>
<th>Pitch angle</th>
<th>Roll angle</th>
<th>Roll rate</th>
<th>Side-slip angle</th>
<th>Lateral acceleration</th>
</tr>
</thead>
<tbody>
<tr>
<td>normal cruising</td>
<td>improved</td>
<td>significantly improved</td>
<td>similar</td>
<td>similar</td>
<td>similar</td>
<td>similar</td>
<td>similar</td>
</tr>
<tr>
<td>un-symmetrical loading</td>
<td>slightly improved</td>
<td>significantly improved</td>
<td>similar</td>
<td>similar</td>
<td>similar</td>
<td>similar</td>
<td>similar</td>
</tr>
<tr>
<td>crosswinds</td>
<td>improved</td>
<td>significantly improved</td>
<td>similar</td>
<td>similar</td>
<td>similar</td>
<td>slightly deteriorated</td>
<td>slightly deteriorated</td>
</tr>
<tr>
<td>tyre-road friction</td>
<td>improved</td>
<td>significantly improved</td>
<td>slightly deteriorated</td>
<td>similar</td>
<td>similar</td>
<td>similar</td>
<td>similar</td>
</tr>
<tr>
<td>flat-tyre</td>
<td>slightly improved</td>
<td>slightly improved</td>
<td>similar</td>
<td>similar</td>
<td>similar</td>
<td>slightly deteriorated</td>
<td>similar</td>
</tr>
<tr>
<td>defective brake cylinder</td>
<td>improved</td>
<td>significantly improved</td>
<td>similar</td>
<td>improved</td>
<td>slightly improved</td>
<td>similar</td>
<td>significantly improved</td>
</tr>
</tbody>
</table>
5.2 Robust Multiple-Model Fuzzy PDC-based Multiple-Adaptive Fuzzy Control System - Blending (Soft - Switching)

5.2.1 Introduction

As explained in the main introduction, there are drawbacks in using single-model (or single ‘modal’) control systems for addressing the problem of integrated lateral and longitudinal control of highway vehicles.

Multiple-Model/Multiple-Controller (MM/MC) systems have more built-in capability to address control requirements with improved tracking, and therefore are more suitable in an Automated Highway System (AHS) environment.

In this section, the design of a fuzzy PDC (Parallel Distributed Compensation) based multiple-adaptive fuzzy controller (as a multiple-model control method) is described. The MM/MC system designed in this section is to address the problem of integrated lateral and longitudinal control of highway vehicles. This novel design of fuzzy PDC system provides an effective MM/MC setup since it improves versatility of the system with the inclusion of nonlinear adaptive fuzzy controllers. In most of the previous PDC based fuzzy systems, local linear models and linear controllers were used. Due to the addition of complex adaptive fuzzy controllers in the control structure, the scope of the fuzzy PDC based controller has been widened. Therefore, it can successfully address the multiple-modal characteristics and other nonlinear issues and complexities of the integrated lateral and longitudinal control problem of vehicles. Importantly, adaptive fuzzy control functions enable one to define and model uncertainty of the system over which the control action is developed. The design of the fuzzy PDC system is based on having ‘state-signals as inputs’ in the fuzzy PDC system. In the design for implementation of the controller, nonlinear local models are generated out of the vehicle model (simplified) following the ‘sector nonlinearity’ method.
5.2.2 Synthesis of Multiple-Model Fuzzy PDC-based Multiple-Adaptive Fuzzy Controller

This section describes the synthesis of a novel robust fuzzy PDC based multiple-model/multiple-adaptive fuzzy controller where a number of adaptive fuzzy controllers are blended using a fuzzy PDC structure to obtain the control function. In the development of the proposed controller, non-linear local models are used. A parallel set of adaptive fuzzy controllers are used to address ‘uncertainty’ of the control problem of integrated lateral and longitudinal control of highway vehicles. In the meantime, proportional-derivative (PD) control module is used for coarse tracking in the lateral and longitudinal aspects of the vehicle control problem.

Having adaptive fuzzy controllers in the PDC structure widens the capability of the control system to handle uncertainty levels. Therefore, the proposed control system can effectively handle a larger ‘uncertainty’ gap between the real vehicle system and the models generated from the design process. With the PDC fuzzy blending, the capabilities of individual adaptive fuzzy controllers are combined to focus on an enhanced performance.

(i) Design of Robust Fuzzy PDC Multiple-Model/Multiple-Adaptive Fuzzy Control System

In this PDC system, there is no limitation in the number of local models that can be included, therefore the relevant number of adaptive fuzzy controllers. It is clear that both the number of models and adaptive fuzzy controllers are based on the number of IF-THEN rules used in the PDC fuzzy system.

Let the IF-THEN rules of the fuzzy PDC (Parallel Distributed Compensation) system (A T–S fuzzy system) be

\[ \mathcal{R}^1: \]
If \( f_1(e) \) is \( \mathcal{F}^q_1 \) AND \( f_2(e) \) is \( \mathcal{F}^r_1 \) THEN \( \ddot{x} = h_1(\dot{\varphi}) + b_1u_1 + d_i \)

\[ \vdots \quad \vdots \]  

\[ (5.42) \]
\[ \mathcal{R}^n: \]
If \( f_1(e) \) is \( \mathcal{F}_n^q \) AND \( f_2(e) \) is \( \mathcal{F}_n^r \) THEN \( \dot{x} = h_n(\dot{q}) + b_n u_n + d_n \)

where \( h_j(\dot{q}), \ j=1, \cdots, n \) are true localized dynamics of the vehicle. Here, \( n \) is the number of IF-THEN rules in the fuzzy PDC structure.

\( f_1(e) \) and \( f_2(e) \) are state-based functions in \( n \) IF-THEN rules (\( n \) number of localized dynamics for \( n \) modes of operations). For example, \( f_1(e) \) can be considered as a longitudinal velocity based function, and \( f_2(e) \) can be considered as a lateral velocity based function.

Let the fuzzy inference from each fuzzy input function of the PDC be \( \mu^j \) where \( i=1, \cdots, m \) (it is taken as \( m=2 \) in this design), and \( j=1, \cdots, n. \) \( n \) is the number of fuzzy IF-THEN rules. Thereby the T–S fuzzy logic output with product inference, singleton fuzzifier and centre-average defuzzifier can be expressed as

\[
\dot{x} = \frac{\sum_{j=1}^{n} (\prod_{i=1}^{m} \mu^i_j)(h_j(\dot{q}) + b_j u_j + d_j)}{\sum_{j=1}^{n} (\prod_{i=1}^{m} \mu^i_j)}. \tag{5.43}
\]

Let the normalized regression factor for each rule be

\[
\mu_j = \frac{(\prod_{i=1}^{m} \mu^i_j)}{\sum_{j=1}^{n} (\prod_{i=1}^{m} \mu^i_j)}. \tag{5.44}
\]

Then output of the fuzzy PDC system becomes

\[
\dot{x} = \sum_{j=1}^{n} \mu_j (h_j(\dot{q}) + b_j u_j + d_j). \tag{5.45}
\]

Let \( \rho_j(\dot{q}) = \sum_{j=1}^{n} \mu_j h_j(\dot{q}) \). Thereby the PDC system output as provided in (5.45) becomes

\[
\dot{x} = \rho_j(\dot{q}) + \sum_{j=1}^{n} \mu_j (b_j u_j + d_j). \tag{5.46}
\]
In the implementation of the fuzzy PDC structure, the schematics of the set of adaptive fuzzy controllers are provided in Fig. 5.54 below. In this setup, the designed vehicle model setup has been replaced by the vehicle itself.

![Fig. 5.54 Schematic diagram for fuzzy Parallel Distributed Compensation (PDC) scheme for implementation in integrated longitudinal (lateral) control of highway vehicles](image)

(ii) Control Law

The control output from the \( j \)th model (using certainty equivalence control law)

for longitudinal case is

\[
\hat{u}_j = \frac{1}{b_j} \left[ \dot{x}_p + k_{dx} \dot{x} + k_{px} e_x + \psi_j^T \theta_j + u_{sx} - \hat{h}_j(\dot{q}) \right],
\]

where \( \psi_1(\dot{q}) = \mu \) and \( \psi_i(\dot{q}) = 1 \) for \( i > 1 \).

\( \hat{h}_j(\dot{q}) \) is

\[
\hat{h}_j(\dot{q}) = \mu \nu + \mu_1 \nu_{1},
\]

and for lateral case is

\[
\hat{u}_j = \frac{1}{b_j} \left[ \dot{y}_p + k_{dy} \dot{y} + k_{py} e_y + \psi_j^T \theta_j + u_{sy} - \hat{h}_j(\dot{q}) \right]
\]

(5.47)
where \( b_x = \sum_{j=1}^{n} \mu_j b_{jx} \), and \( b_y = \sum_{j=1}^{n} \mu_j b_{jy} \). \( \psi_j \theta_j \) is the \( j \)th adaptive fuzzy controller output. \( u_x \) is the variable structure control term. \( \hat{h}_{jx}(\dot{q}) \) and \( \hat{h}_{jy}(\dot{q}) \) are the longitudinal and lateral local dynamic models of the vehicle, respectively.

Fuzzy blending of control output from each model in the PDC system will result in the overall control outputs from the PDC structure as

\[
\hat{u}_x = \frac{1}{b_x} \left[ \ddot{x}_p + k_d \dot{x}_{e_x} + k_p e_x + \sum_{j=1}^{n} \mu_j (\psi_j \theta_j) + u_x - \sum_{j=1}^{n} \mu_j \hat{h}_{jx}(\dot{q}) \right],
\]

\[
\hat{u}_y = \frac{1}{b_y} \left[ \ddot{y}_p + k_d \dot{y}_{e_y} + k_p e_y + \sum_{j=1}^{n} \mu_j (\psi_j \theta_j) + u_y - \sum_{j=1}^{n} \mu_j \hat{h}_{jy}(\dot{q}) \right]
\]

for longitudinal and lateral cases, respectively. It is provided that \( b_x = \sum_{j=1}^{n} \mu_j b_{jx} \), \( b_y = \sum_{j=1}^{n} \mu_j b_{jy} \), and thereby (5.49), (5.50) can be expressed as

\[
\hat{u}_x = \frac{1}{b_x} \left[ \ddot{x}_p + k_d \dot{x}_{e_x} + k_p e_x + \sum_{j=1}^{n} \mu_j (\psi_j \theta_j) \right] + u_x - \hat{\rho}_x(\dot{q}),
\]

\[
\hat{u}_y = \frac{1}{b_y} \left[ \ddot{y}_p + k_d \dot{y}_{e_y} + k_p e_y + \sum_{j=1}^{n} \mu_j (\psi_j \theta_j) \right] + u_y - \hat{\rho}_y(\dot{q})
\]

where \( \hat{\rho}_x(\dot{q}) = \sum_{j=1}^{n} \mu_j \hat{h}_{jx}(\dot{q}) \) and \( \hat{\rho}_y(\dot{q}) = \sum_{j=1}^{n} \mu_j \hat{h}_{jy}(\dot{q}) \).

**Remark 5.5:** Though fuzzy systems in (5.51), (5.52) are generally nonlinear, each is linear with respect to its unknown parameters. Hence, parameter adaptation algorithms applicable for linear systems, e.g. gradient-based algorithms, can be readily used in estimation of unknown parameters in the fuzzy systems in (5.51), (5.52) [27].

For the \( j \)th adaptive fuzzy controller \( (j = 1, \cdots, n) \), the regression vector can be expressed as \( \psi_j = [\psi_{j1}, \psi_{j2}, \cdots, \psi_{jp}]^T \) where \( p \) is the total number of IF-THEN fuzzy rules in each adaptive fuzzy controller. The \( k \)th element, where \( k = 1, \cdots, p \), of the \( j \)th
controller (where \( j = 1, \cdots, n \)) is \( \psi_{jk} = [\psi_{jk} e \ psi_{jk} \dot{e}] = [\psi_{jk} e^T] \). There are \( p \) IF-THEN rules in each adaptive fuzzy controller, i.e. \( k = 1, \cdots, p \).

The T–S fuzzy output membership function parameter set of \( j \)th adaptive fuzzy controller can be tuned using an algorithm based on the adaptive law

\[
\dot{\theta}_j = -\gamma \mu_j \psi_jCe
\]

(5.53)

where \( j = 1, \cdots, n \).

(iii) Error Dynamics

Substitution of equation (5.51) in (3.4) will result for the longitudinal dynamics

\[
\ddot{e} + k_{ae} \dot{e} + k_{pe} e = \hat{\rho}_x(q) - \rho_x(q) - \sum_{j=1}^{n} \mu_j \left( \psi_j^T \theta_j \right)_x - u_{sx} - d_x
\]

\[
= (\nu^* + \tau) - \hat{\nu} - u_{sx} - d_x
\]

(5.54)

and in the same way, substitution of (5.52) in (3.5) will result for the lateral dynamics

\[
\ddot{e} + k_{ae} \dot{e} + k_{pe} e = \hat{\rho}_y(q) - \rho_y(q) - \sum_{j=1}^{n} \mu_j \left( \psi_j^T \theta_j \right)_y - u_{sy} - d_y
\]

\[
= (\nu^* + \tau) - \hat{\nu} - u_{sy} - d_y
\]

(5.55)

where \( \hat{\nu} \) is the output from the fuzzy PDC system. \( \nu^* \) is the optimum fuzzy PDC output, and \( \tau \) is the approximation error of the fuzzy PDC system.

By using the universal approximation theorem (provided in Chapter 4), with the fuzzy adaptation process, the fuzzy controller parameters can be gradually brought to the optimal values, and thus the right half of the equations, (5.54) and (5.55) can be made zero.
5.2.3 Robust Stability Analysis of Control System

The following stability proof of the fuzzy PDC multiple-adaptive fuzzy control system is described for the generic case irrespective whether it is lateral or longitudinal controller. Therefore, the subscripts that are specific to either longitudinal or lateral case are removed in this description. Nevertheless, the specific details will be mentioned as and when necessary.

**Assumption 5.2:** The sum of fuzzy approximation errors, \( w \) is bounded, i.e., \(|w| \leq W\) where \( W(>0) \in \mathbb{R}\). The external disturbances, \( d \) are bounded too, i.e., \(|d| \leq D\) where \( D(>0) \in \mathbb{R}\). Thereby the variable structure control gain is chosen as \( \sigma_j \geq W + D. \Box \)

The main features of the robust multiple-model PDC-based multiple-adaptive fuzzy controller can be summarized in the following theorem.

**Theorem 5.2:** Let the parameter vectors, \( \theta_j \) of the adaptive fuzzy systems be adjusted by the adaptive law as provided in (5.53) and let Assumption 5.2 be true. The proposed adaptive fuzzy control law as provided in (5.51) [and (5.52)] can guarantee stability of the vehicle system with the following properties:

i) The closed loop vehicle system is stable, i.e. \( \| e \| \in \mathcal{L}^\infty \) (considering both lateral and longitudinal cases)

ii) The system errors and the parametric errors are asymptotically stable, i.e. \( \lim_{t \to \infty} e_x, e_y = 0 \), and \( \lim_{t \to \infty} \theta_j = 0 \) (considering both lateral and longitudinal cases) where \( j = 1, \ldots, n. \) \( \Box \)
Proof:

Positive realness of the transfer function of error dynamics equations, i.e., (4.23) and (4.24), related to vehicle dynamics has already been established in Section 4.2.6.

Assuming that positive realness of the vehicle error dynamics can be established as shown in Section 4.2.6, according to Kalman–Yakubovich–Popov (KYP) lemma [175], for a strictly positive real system there exist two positive definite matrices $P$ and $Q$ (i.e., $P = P^T > 0$ and $Q = Q^T > 0$) satisfying

$$A^TP + PA + Q = 0,$$

and $PB = C^T$. \hfill (5.56)

With KYP lemma, the Lyapunov function candidate can be selected as

$$V = e^TPe + \sum_{j=1}^n \gamma_j^{-1}\hat{\theta}_j^T\hat{\theta}_j \hfill (5.57)$$

where $P = P^T > 0$, and $\gamma_j(>0) \in \mathbb{R}$ where $j = 1, \ldots, n$. Here, $e$ is the overall system error, and $\hat{\theta}_j$ is the parametric error vector (with respect to optimal parameters) for the $j$th adaptive fuzzy controller where $j = 1, \ldots, n$. $n$ is the total number of controllers (and models) in the PDC system.

Lyapunov candidate provided in (5.57) quantifies both in tracking error and in parameter estimates. Differentiating the Lyapunov candidate (5.57) along the trajectories of the error system will provide

$$\dot{V} = \dot{e}^TPe + e^T\dot{P}e + \sum_{j=1}^n \gamma_j^{-1}\left(\hat{\theta}_j^T\dot{\hat{\theta}}_j + \dot{\hat{\theta}}_j^T\dot{\hat{\theta}}_j\right). \hfill (5.58)$$

Substituting the equation (4.23) in (5.58) will result in

$$\dot{V} = (Ae + B\varphi - Bd - Bu)\dot{e}^TPe + e^T(\dot{A}e + B\dot{\varphi} - B\dot{d} - B\dot{u}) + \sum_{j=1}^n \gamma_j^{-1}\left(\dot{\hat{\theta}}_j^T\dot{\hat{\theta}}_j + \dot{\hat{\theta}}_j^T\dot{\hat{\theta}}_j\right). \hfill (5.59)$$
Rearranging (5.59) will provide
\[
\dot{V} = e^T(A^TP + PA)e + \tilde{v}^TB^TPe + e^T\tilde{P}B\tilde{v} - dB^TPe - de^T\tilde{P}B - u_dB^TPe - u_d e^T\tilde{P}B \\
\sum_{j=1}^n \gamma_j^{-1}\left(\hat{\theta}_j^T \hat{\theta}_j + \hat{\theta}_j^T \hat{\theta}_j\right).
\] (5.60)

With the Lyapunov equation \(AP + PA + Q = 0\) where \(Q = Q^T > 0\), equation (5.60) can be further simplified as
\[
\dot{V} = -e^TQe + \tilde{v}^T B^TPe + e^T\tilde{P}B\tilde{v} - (d + u_d)(B^TPe + e^T\tilde{P}B) \\
+ \sum_{j=1}^n \gamma_j^{-1}\left(\hat{\theta}_j^T \hat{\theta}_j + \hat{\theta}_j^T \hat{\theta}_j\right).
\] (5.61)

From the argument, it can be expressed as \(\dot{V} = (\nu - \tilde{\nu}) + \tau\), and can be further elaborated as
\[
\dot{V} = \left(\sum_{j=1}^n \mu_j (\psi_j^T \theta_j)\right) - \left(\sum_{j=1}^n \mu_j (\psi_j^T \hat{\theta}_j)\right) + \sum_{j=1}^n w_j
\] (5.62)

where \(\theta_j^T \in \mathcal{R}^{n\times 2}\) is the optimal parameter matrix for the \(j\)th adaptive fuzzy controller. \(\hat{\theta}_j\) is the parameter set of the \(j\)th adaptive fuzzy controller. \(w_j\) is the approximation error of the \(j\)th adaptive fuzzy controller, and is bounded, \(|w_j| \leq \bar{W}\), i.e. \(w_j \in \mathcal{L}^\infty\) where \(\bar{W} \in \mathcal{R}\) is a positive constant.

Let \(w = \sum_{j=1}^n w_j\) where \(|w| < W (\in \mathcal{R})\) is a positive constant. The determination of fuzzy approximation error upper bound, \(W\) has been discussed before [30].

Thereby (5.62) can be further simplified as
\[
\dot{V} = \sum_{j=1}^n \mu_j \psi_j^T(\theta_j^* - \hat{\theta}_j) + w.
\] (5.63)

Let the parameter error be taken as \(\tilde{\theta}_j = \theta_j^* - \hat{\theta}_j\), and thereby (5.63) now becomes
\[
\dot{V} = \sum_{j=1}^n \mu_j \psi_j^T\tilde{\theta}_j + w.
\] (5.64)

Substituting (5.64) in (5.61) will provide
\[
\dot{V} = -e^TQe + \left[\sum_{j=1}^n (\mu_j \psi_j^T \theta_j) + w\right]^T B^TPe + e^T\tilde{P}B \left[\sum_{j=1}^n (\mu_j \psi_j^T \hat{\theta}_j) + w\right] \\
- (d + u_d)(B^TPe + e^T\tilde{P}B) + \sum_{j=1}^n \gamma_j^{-1}\left(\hat{\theta}_j^T \hat{\theta}_j + \hat{\theta}_j^T \hat{\theta}_j\right).
\] (5.65)

Since \(PB = C^T\) and, (5.65) can be further simplified to provide
\[
V = -e^T Q e + \sum_{j=1}^{n} \left( \mu_j (\hat{\theta}_j)^T \psi_j \right) Ce + e^T C^T \sum_{j=1}^{n} \left( \mu_j \psi_j (\hat{\theta}_j) \right)
+ \sum_{j=1}^{n} \gamma_j^{-1} \left( \hat{\theta}_j^T \hat{\theta}_j + \hat{\theta}_j^T \hat{\theta}_j \right) + \left( w - d - u \right) \left( Ce + e^T C^T \right).
\] (5.66)

Further rearrangement of (5.66) will result in
\[
V = -e^T Q e + \left( w - d - u \right) \left( Ce + e^T C^T \right)
+ \sum_{j=1}^{n} \left( \left( e^T C^T \left( \mu_j \psi_j^T \right) + \gamma_j^{-1} \hat{\theta}_j \right) \hat{\theta}_j \right) + \sum_{j=1}^{n} \left( \hat{\theta}_j^T \left( \left( \mu_j \psi_j \right) Ce + \gamma_j^{-1} \hat{\theta}_j \right) \right).
\] (5.67)

Now (5.67) can be written as
\[
V = -e^T Q e + \left( w - d - u \right) \left( Ce + e^T C^T \right)
+ \sum_{j=1}^{n} \left( \left( \mu_j \psi_j \right) Ce + \gamma_j^{-1} \hat{\theta}_j \right) \hat{\theta}_j \right) + \sum_{j=1}^{n} \left( \hat{\theta}_j^T \left( \left( \mu_j \psi_j \right) Ce + \gamma_j^{-1} \hat{\theta}_j \right) \right).
\] (5.68)

If the adaptive law of the \( j \)th fuzzy controller (as provided in (5.53)) is taken as
\[\hat{\theta}_j = -\gamma_j \mu_j \psi_j Ce\] where \( j = 1, \cdots, n \), equation (5.68) can be further simplified as
\[
V = -e^T Q e + \left( w - d - u \right) \left( Ce + e^T C^T \right).
\] (5.69)

Since \( \left| w \right| \leq W \) and \( \left| d \right| \leq D \), (5.69) can be further written as
\[
V \leq -\lambda_{Q_{\text{min}}} \left\| e \right\|^2 + \left( D + W - u \right) \left( Ce + e^T C^T \right)
\] (5.70)

where \( \lambda_{Q_{\text{min}}} \) is the minimum eigen value of \( Q \).

Since fuzzy systems follow the universal approximation theorem [29], \( \left| w \right| \) may be made arbitrarily small by making each \( w_j \) small by proper choice of each fuzzy system if each \( \psi_j^T \hat{\theta}_j \) is smooth (note that this may require an arbitrarily large number of rules for each controller) [30].

Let the variable structure control term be
\[u = \sigma_j \text{sign} \left( Ce + e^T C^T \right), \] (5.71)
and thereby (5.70) can be expressed as
\[
V \leq -\lambda_{Q_{\text{min}}} \left\| e \right\|^2 + \left( \left\| D + W \right\| - \sigma_j \right) \left\| Ce + e^T C^T \right\|.
\] (5.72)
Further arrangement of (5.72) will provide
\[
\dot{V} \leq -\lambda_{Q_{\text{min}}} \| e \|^2 - \| Ce + e^T C^T (\sigma_f - \| D + W \|) .
\] (5.73)

If it is taken as
\[
\sigma_f \geq \| D + W \|
\] (5.74)
then (5.73) will establish that
\[
\dot{V} \leq 0 , \forall t .
\] (5.75)

Therefore, according to the Lyapunov theory, (5.75) establishes that \( e \) is bounded, i.e. \( e \in \mathcal{L}^\infty \). Therefore, \( e, \dot{e} \) are bounded, i.e. \( e, \dot{e} \in \mathcal{L}^\infty \). It also establishes that parameter errors are bounded, i.e. \( \tilde{\theta} \in \mathcal{L}^\infty \).

Equation (5.72) also implies that
\[
\dot{V} \leq -\lambda_{Q_{\text{max}}} \| e \|^2 .
\] (5.76)

Integrating (5.76) will provide
\[
\lambda_{Q_{\text{max}}} \int_0^\infty \| e \|^2 dt \leq -\int_0^\infty \dot{V} dt = V(0) - V(\infty) .
\] (5.77)

Therefore, (5.77) becomes
\[
\int_0^\infty \| e \|^2 dt \leq \frac{1}{\lambda_{Q_{\text{max}}}} (V(0) - V(\infty)) .
\] (5.78)

It leads to
\[
e \in \mathcal{L}^2 \left( \mathcal{L}^\infty \right) = \left\{ z(\cdot) : \int_0^\infty z^2(\cdot) dt < \infty \right\} ,
\] (5.79)

since it provides \( V(0), V(\infty) \in \mathcal{L}^\infty \) according to (5.75) [30]. Since it is possible to establish that \( e \in \mathcal{L}^2 \cap \mathcal{L}^\infty \) (from (5.79) and (5.75)), and \( \dot{e} \in \mathcal{L}^\infty \) (from (5.76)), by Barbalat’s lemma [30], [176], it can be established asymptotic stability of \( e \) (i.e. \( \lim_{t \to \infty} e = 0 \)). This can be extended, specifically to lateral and longitudinal cases, and consequently the lateral and longitudinal errors are asymptotically stable, i.e.
\[
\lim_{t \to \infty} e_x , e_y = 0 .
\] (5.80)
This asymptotic stability result, as provided by (5.80), proves that when the lateral and longitudinal cases are considered at the same time, with the above arguments, the system of errors can be proved to be asymptotically stable. Therefore, this result logically proves that the integrated lateral and longitudinal control system is asymptotically stable. This statement completes the proof of stability for the robust fuzzy PDC-based multiple-model/multiple-adaptive fuzzy controller system.

The following definition is provided to formerly identify the variable vector spaces and thereby to define the optimal parameter set.

**Definition 5.2:** It is possible to define the compact parameter spaces for $\theta$ and $\bar{x}$ variables. Let $M_{\theta}(>0) \in \mathbb{R}$ and $M_{x}(>0) \in \mathbb{R}$ be finite,

$$\Omega_{\theta} = \left\{ \theta_j \in \mathbb{R} : \|\theta_j\| \leq M_{\theta} \right\}, \text{ and}$$

$$\Omega_{\bar{x}} = \left\{ \bar{x} \in \mathbb{R} : \|\bar{x}\| \leq M_{x} \right\}.$$  

Thereby the optimal parameter set for each $j$th adaptive fuzzy controller where $j = 1, \ldots, n$, can be formally defined as

$$\theta_j^* = \arg \min_{\theta_j \in \Omega_{\theta}} \sup_{x \in \Omega_{\bar{x}}} \left\{ \nu^* - \sum_{j=1}^{n} \mu_j (\psi_j^* \hat{\theta}_j) \right\}. \quad (5.81)$$

$\nu^*$ is the optimal adaptive fuzzy function output where $\nu^* \in \mathbb{R}$. $\bar{x}$ is the generic input variable to each fuzzy system.

**Remark 5.6:** The Lyapunov candidate satisfies the following conditions,

i) $V$ is at least $C^1$

ii) $V(0) = 0$ and $V(x(t)) > 0$ for $x(t) \neq 0$,

and

iii) $\|x(t)\| \to \infty \Rightarrow V(x(t)) \to \infty$. ($x(t)$ denotes a selected time-dependent state of the system in this remark).

**Remark 5.7:** The definition of $\theta_j^*$ is only for theoretical reasons and it is not required for the implementation of the controller.
5.2.4 Implementation Design of Fuzzy PDC Control System

(i) T – S Fuzzy Modelling of Vehicle System

In the implementation design of the fuzzy PDC system with multiple-adaptive fuzzy control method, the approach used is to utilize the idea of ‘sector nonlinearity’ method and local approximation to construct models for the fuzzy PDC structure [179]. An important observation is that this method guarantees an exact fuzzy model construction. The PDC based multiple-adaptive fuzzy system is designed using the sector nonlinearity method. Data is obtained for tuning the preliminary parameters of each adaptive fuzzy controller.

In this research, local models are developed for the simplified model of the vehicle as provided in Section 4.3. Thereafter, the adaptive fuzzy controllers are designed and used to generate the ‘uncertainty’ between blending of these models and the actual vehicle (in this case the high precision model of veDYNA®). In this regard, the adaptive fuzzy controllers are designed separately according to the generated data set using the relevant ‘local’ vehicle model. Therefore, in this implementation design, adaptive fuzzy controllers are defined to operate only in the ‘local model space’ as identified by the ‘sector nonlinearity’ method. It is clear that blending each of the controllers will address the ‘global’ scenarios having the full ‘spectrum’ of operation.

(a) Expression of Simplified Control Vehicle Model as a Non-Linear State-Space Model

The following assumption is made before commencing the design of the fuzzy PDC control system.

_Assumption 5.1:_ The lateral sensor is assumed to be mounted closer to the centre of gravity of the vehicle

The vehicle model provided in (3.1) can once again be mentioned as
\[ \ddot{x} = \frac{-(r_{e} \dot{r})^2}{m(r_{e} \dot{r})^2 + J_{mg} + 2 J_{a} \dot{r}^2} \left[ C_{x} \dot{x}^2 + F_{roll} - m \dot{y} \dot{\psi} \right] \\
+ \frac{(r_{e} \dot{r})^2}{m(r_{e} \dot{r})^2 + J_{mg} + 2 J_{a} \dot{r}^2} \left[ \frac{1}{r_{e}} (T_{net} - \frac{3}{2} r^2 T_{brk}) - 2 \delta C_{sf} (\delta - \dot{\zeta} f) \right] + d_x \]

\[ \ddot{y} = -\frac{1}{m} \left[ C_{y} \ddot{y}^2 + m \dot{x} \dot{\psi} + 2 C_{u} \dot{\zeta} r + 2 C_{u} \dot{\xi} f \right] + \frac{2}{m} C_{y} \delta + d_y \]

\[ \ddot{\psi} = \frac{l_{y}}{T_{c}} \left[ 2 \frac{1}{l_{y}} C_{u} \dot{\zeta} r - 2 C_{sf} \dot{\zeta} f \right] + \frac{2 l_{y}}{T_{c}} C_{y} \delta + d_{\psi} \] (5.82)

The equations (5.82) can be identified into different states as provided below:

\[ \dot{x}_1 = \dot{x} = x_2 \]
\[ \dot{x}_2 = \dot{x} = k_1 \left[ C_{x} \dot{x}^2 + F_{roll} - m \dot{y} \dot{\psi} \right] + b_x u_x \]
\[ \dot{x}_3 = \dot{y} = x_4 \]
\[ \dot{x}_4 = \dot{y} = k_2 \left[ C_{y} \dot{y}^2 + m \dot{x} \dot{\psi} + 2 \frac{(C_{u} - C_{y})}{x} \dot{y} + 2 \frac{(C_{u} l_{y} - C_{y} l_{y})}{x} \dot{\psi} \right] + b_y u_y \]
\[ \dot{x}_5 = \dot{\psi} = x_6 \]
\[ \dot{x}_6 = \dot{\psi} = k_3 \left[ 2 \left( \frac{1}{l_{y}} C_{u} \dot{\zeta} r - C_{sf} \dot{\zeta} f \right) + 2 \left( \frac{1}{l_{y}} C_{u} l_{y} - C_{sf} l_{y} \right) \dot{\zeta} f \right] + b_\psi u_y \] (5.83)

where the constants used can be defined as:

\[ k_1 = \frac{-(r_{e} \dot{r})^2}{m(r_{e} \dot{r})^2 + J_{mg} + 2 J_{a} \dot{r}^2} \]
\[ k_2 = -\frac{1}{m} \]
\[ k_3 = \frac{l_{y}}{T_{c}} \]

and

\[ b_x = \frac{(r_{e} \dot{r})^2}{m(r_{e} \dot{r})^2 + J_{mg} + 2 J_{a} \dot{r}^2} \]
\[ b_y = \frac{1}{m} \]
\[ b_\psi = \frac{l_{y}}{T_{c}} . \]

Substitution of state terms in (5.83) will result in the following state equations:

\[ \dot{x}_1 = \dot{x} = x_2 \]
\[ \dot{x}_2 = \dot{x} = k_1 \left[ C_{x} x_2^2 + F_{roll} - m x_3 x_6 \right] + b_x u_x \]
Expressing the state equations (5.84) in matrix form will result in

\[
\begin{bmatrix}
\dot{x}_1 \\
\dot{x}_2 \\
\dot{x}_3 \\
\dot{x}_4 \\
\dot{x}_5 \\
\dot{x}_6
\end{bmatrix} = \begin{bmatrix}
0 & 1 & 0 & 0 & 0 & 0 \\
0 & k_1C_x & 0 & 0 & 0 & -k_1mx_4 \\
0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & k_2\left[C_y + 2\frac{(C_{mu} + C_{md})}{x_2}\right] & 0 & 2k_2\frac{(C_{mu} - C_{md})}{x_2} + k_2mx_2 \\
0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 2k_3\frac{(\dot{C}_{mu} - C_{d})}{x_2} & 0 & -2k_3\frac{\ddot{C}_{mu} + C_{dl}}{x_2}
\end{bmatrix} + \begin{bmatrix}
b_x & 0 & 0 & 0 & 0 & 0 \\
b_y & 0 & 0 & 0 & 0 & 0 \\
b_{\psi} & 0 & 0 & 0 & 0 & 0
\end{bmatrix} \begin{bmatrix} u_x \\ u_y \\ u_{\psi} \end{bmatrix}.
\] (5.85)

Equation (5.85) provides the state-space model of the simplified vehicle system. Equation (5.85) can be expressed in concise form as \( \dot{x} = Ax + Bu \), with the following matrices being defined after calculating the relevant terms with vehicle constants.
when the longitudinal velocity of vehicle is zero, i.e., $x_2 = 0$, matrix $A$ in (5.85) becomes singular. But it is to be noted that the tracking controller is not defined or meant to be working when $x_2 = 0$. Even the lateral movements cannot be implemented when $x_2 = 0$ and therefore are not defined. Therefore, under all operative conditions, i.e. $x_2 \neq 0$, the matrix $A$ is non-singular. Further explanation can be provided on using the term $1/x_2$ in the matrix $A$. The term $1/x_2$ occurs as a result of defining the tyre velocity vector angles $\zeta_f$ and $\zeta_r$. When the longitudinal speed of the vehicle is zero, i.e. $x_2 = 0$ there is no meaning to velocity vector angle. Therefore, the velocity vector angles are undefined when $x_2 \neq 0$.

(b) Fuzzy PDC System Design

Longitudinal speed, i.e., $\dot{x}$ (or $x_2$) and lateral speed, i.e., $\dot{y}$ (or $x_4$) are taken as the input variables in the fuzzy T–S system. For forming fuzzy membership functions for the PDC system, the following limits are imposed for the input variables:
\[ \max(x_2(t)) = 35 \text{ [m/s]} \quad \min(x_2(t)) = -5 \text{ [m/s]} \]
\[ \max(x_4(t)) = 1 \text{ [m/s]} \quad \min(x_4(t)) = -1 \text{ [m/s]} \] (5.88)

From the maximum and minimum values, the input variables \( x_2(t) \) and \( x_4(t) \) can be defined in terms of the membership functions as

\[ x_2(t) = P_1(x_2(t)) \times 35 + P_2(x_2(t)) \times (-5) \]
\[ x_4(t) = Q_1(x_4(t)) \times (1) + Q_2(x_4(t)) \times (-1) \] (5.89)

where,
\[ P_1(x_2(t)) + P_2(x_2(t)) = 1 \]
\[ Q_1(x_4(t)) + Q_2(x_4(t)) = 1. \] (5.90)

The following fuzzy input membership functions can be formed satisfying the condition (5.90), according to the sector nonlinearity method [179]

\[ P_1(x_2(t)) = \frac{x_2 + 5}{40} \quad P_2(x_2(t)) = \frac{35 - x_2}{40} \] (5.91)
\[ Q_1(x_4(t)) = \frac{1 + x_4}{2} \quad Q_2(x_4(t)) = \frac{1 - x_4}{2}. \] (5.92)

The T–S fuzzy rules for forming the PDC fuzzy system for vehicle control system can be expressed as follows:

Rule 1:
IF \( x_2(t) \) is ‘\( P_1(x_2(t)) \)’ AND \( x_4(t) \) is ‘\( Q_1(x_4(t)) \)’
THEN \( \dot{x} = A_1 x + B_1 u \)

Rule 2:
IF \( x_2(t) \) is ‘\( P_1(x_2(t)) \)’ AND \( x_4(t) \) is ‘\( Q_2(x_4(t)) \)’
THEN \( \dot{x} = A_2 x_2 + B_2 u \) (5.93)
Rule 3:
IF $x_2(t)$ is ‘$P_2(x_2(t))$’ AND $x_4(t)$ is ‘$Q_1(x_4(t))$’
THEN $\dot{x} = A_3x_3 + B_3u$

Rule 4:
IF $x_2(t)$ is ‘$P_2(x_2(t))$’ AND $x_4(t)$ is ‘$Q_2(x_4(t))$’
THEN $\dot{x} = A_4x_4 + B_4u$

(c) Design of Local Models for Simplified Vehicle Model

In order to obtain solutions for $A_1, A_2, A_3$ and $A_4$ matrices, the following linear matrix equation is formed according to the fuzzy PDC principle and then solved, i.e.,

$$A = P_1(x_2(t))Q_1(x_4(t))A_1 + P_1(x_2(t))Q_2(x_4(t))A_2 + P_2(x_2(t))Q_1(x_4(t))A_3 + P_2(x_2(t))Q_2(x_4(t))A_4,$$  

(5.94)

The PDC equivalent system matrices for vehicle models are obtained as follows (with substitution of known values as provided in (5.88) in the linear matrix equation (5.94)):

$$A_1 = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & -0.0103 & 0 & 0 & 0 & 0.9527 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & \left(-0.000386 - \frac{183.2}{x_2}\right) & 0 & \left(-\frac{-20.44}{x_2} - 35\right) \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -\frac{15.14}{x_2} & 0 & -\frac{223.7}{x_2} \end{bmatrix} \tag{5.95}$$
To obtain $B_1, B_2, B_3$ and $B_4$ matrices, it is taken as

$$
B_1 = B_2 = B_3 = B_4 = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 \\
0.000735 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0.0007716 & 0 & 0 \\
0 & 0 & 0 & 0 & 0.0007143 & 0 \\
0 & 0 & 0 & 0 & 0 & 0.0007143 \\
0 & 0 & 0 & 0 & 0 & 0.0007143 \\
\end{bmatrix}. \tag{5.99}
$$
The defuzzification in the PDC system is carried out as follows, and obtained the system output as

\[
\dot{x} = \sum_{i=1}^{4} \eta_i(x(t))\{A_i x + B_i u\} \tag{5.100}
\]

where

\[
\eta_1(x(t)) = P_1(x_2(t)) \times Q_1(x_4(t))
\]
\[
\eta_2(x(t)) = P_2(x_2(t)) \times Q_2(x_4(t)) \tag{5.101}
\]
\[
\eta_3(x(t)) = P_3(x_2(t)) \times Q_1(x_4(t))
\]
\[
\eta_4(x(t)) = P_4(x_2(t)) \times Q_2(x_4(t)).
\]

According to the design method, i.e., sector nonlinearity method, this T–S fuzzy system exactly represents the vehicle model provided in (5.81) in the region \([-5, 35] \times [-1, 1]\) of \(x_2 - x_4\) space [179].

The fuzzy input membership functions of the developed PDC based T–S system are provided below in Fig. 5.55.

Fig. 5.55 Fuzzy input membership functions

(ii) Synthesis of Local Adaptive Fuzzy Controllers for PDC System

The subtractive clustering method was used to obtain the basic fuzzy systems for the adaptive fuzzy controllers similar to the procedure explained in Chapter 4. The final fuzzy membership function centres and standard deviations (Gaussian) were then obtained using the ANFIS method with further tuning.
(a) T–S Fuzzy Rules for Longitudinal and Lateral Vehicle Controllers in the PDC System

The fuzzy PDC rules that contain the adaptive fuzzy controllers are described as follows:

Rule 1:

IF \( x_2(t) \) is ‘\( P_1(x_2(t)) \)’ AND \( x_4(t) \) is ‘\( Q_1(x_4(t)) \)’

\[
\begin{align*}
\hat{u}_{x_1} &= \ddot{x}_p + k_{d_1} \dot{e} + k_{p_1} e + (\psi_i^T \theta)_{1x} + u_{sx} - \hat{h}_{1x}(\hat{q}) \\
\hat{u}_{y_1} &= k_d \dot{e} + k_{p_1} e + (\psi_i^T \theta)_{1y} + u_{sy} - \hat{h}_{1y}(\hat{q})
\end{align*}
\]

Rule 2:

IF \( x_2(t) \) is ‘\( P_1(x_2(t)) \)’ AND \( x_4(t) \) is ‘\( Q_1(x_4(t)) \)’

\[
\begin{align*}
\hat{u}_{x_2} &= \ddot{x}_p + k_{d_2} \dot{e} + k_{p_2} e + (\psi_i^T \theta)_{2x} + u_{sx} - \hat{h}_{2x}(\hat{q}) \\
\hat{u}_{y_2} &= k_d \dot{e} + k_{p_2} e + (\psi_i^T \theta)_{2y} + u_{sy} - \hat{h}_{2y}(\hat{q})
\end{align*}
\]

Rule 3:

IF \( x_2(t) \) is ‘\( P_2(x_2(t)) \)’ AND \( x_4(t) \) is ‘\( Q_1(x_4(t)) \)’

\[
\begin{align*}
\hat{u}_{x_3} &= \ddot{x}_p + k_{d_3} \dot{e} + k_{p_3} e + (\psi_i^T \theta)_{3x} + u_{sx} - \hat{h}_{3x}(\hat{q}) \\
\hat{u}_{y_3} &= k_d \dot{e} + k_{p_3} e + (\psi_i^T \theta)_{3y} + u_{sy} - \hat{h}_{3y}(\hat{q})
\end{align*}
\]

Rule 4:

IF \( x_2(t) \) is ‘\( P_2(x_2(t)) \)’ AND \( x_4(t) \) is ‘\( Q_2(x_4(t)) \)’

\[
\begin{align*}
\hat{u}_{x_4} &= \ddot{x}_p + k_{d_4} \dot{e} + k_{p_4} e + (\psi_i^T \theta)_{4x} + u_{sx} - \hat{h}_{4x}(\hat{q}) \\
\hat{u}_{y_4} &= k_d \dot{e} + k_{p_4} e + (\psi_i^T \theta)_{4y} + u_{sy} - \hat{h}_{4y}(\hat{q})
\end{align*}
\]

\((\psi_i^T \theta)_i\) where \( i = 1, \ldots, 4 \), are the control outputs from the adaptive fuzzy longitudinal controllers, and \((\psi_i^T \theta)_i\), where \( i = 1, \ldots, 4 \), are the control outputs from
the adaptive fuzzy lateral controllers. \( u_i \) and \( u_j \), where \( i = 1, \ldots, 4 \), are the longitudinal and lateral local control laws, respectively. \( \hat{h}_i(\dot{q}) \) and \( \hat{h}_j(\dot{q}) \) are the longitudinal and lateral dynamical models of the vehicle, respectively.

From the system matrices \( A_1, A_2, A_3 \) and \( A_4 \), the local vehicle model equations, i.e., for longitudinal, \( \hat{h}_i(\dot{q}) \), and lateral, \( \hat{h}_j(\dot{q}) \) where \( i = 1, \ldots, 4 \), in each fuzzy rule (of (5.102)), can be found as follows:

for longitudinal case
\[
\begin{align*}
\hat{h}_1(\dot{q}) &= -0.0103x_2 + 0.9527x_6, \\
\hat{h}_2(\dot{q}) &= -0.0103x_2 - 0.9527x_6, \\
\hat{h}_3(\dot{q}) &= 0.00147x_2 + 0.9527x_6, \\
\hat{h}_4(\dot{q}) &= 0.00147x_2 - 0.9527x_6,
\end{align*}
\]
(5.103)

and for lateral case:
\[
\begin{align*}
\hat{h}_1(\dot{q}) &= \left(-0.000386 - \frac{183.9}{x_2}\right)x_4 + \left(-\frac{20.44}{x_2} - 35\right)x_6, \\
\hat{h}_2(\dot{q}) &= \left(-0.000386 - \frac{183.9}{x_2}\right)x_4 + \left(-\frac{20.44}{x_2} - 35\right)x_6, \\
\hat{h}_3(\dot{q}) &= \left(-0.000386 - \frac{183.9}{x_2}\right)x_4 + \left(-\frac{20.44}{x_2} + 5\right)x_6, \\
\hat{h}_4(\dot{q}) &= \left(-0.000386 - \frac{183.9}{x_2}\right)x_4 + \left(-\frac{20.44}{x_2} + 5\right)x_6.
\end{align*}
\]
(5.104)

(b) Training of Adaptive Fuzzy Controllers

Longitudinal Adaptive Fuzzy Controllers

The inputs in each longitudinal adaptive fuzzy controller are error, \( e \) and error rate, \( \dot{e} \). The output in each longitudinal adaptive fuzzy control function is calculated by making the error terms zero starting from the model equations (as provided in (5.93)), and the controller equations (as provided in (5.102)) as follows:
\[
\begin{align*}
\left( \psi^T \theta \right)_{1x} &= h_{1x}(\hat{q}) - \hat{h}_{1x}(\hat{q}), \\
\left( \psi^T \theta \right)_{2x} &= h_{2x}(\hat{q}) - \hat{h}_{2x}(\hat{q}), \\
\left( \psi^T \theta \right)_{3x} &= h_{3x}(\hat{q}) - \hat{h}_{3x}(\hat{q}), \\
\left( \psi^T \theta \right)_{4x} &= h_{4x}(\hat{q}) - \hat{h}_{4x}(\hat{q}).
\end{align*}
\]

The values of \( h_i(\hat{q}) \), \( i = 1, \ldots, 4 \) are calculated as follows:

According to the PDC principle, it can be provided that

\[
\rho_x(\hat{q}) = P_1(x_2(t)) \times Q_1(x_2(t)) \times h_{1x}(\hat{q}) + P_1(x_2(t)) \times Q_2(x_3(t)) \times h_{2x}(\hat{q}) + P_2(x_2(t)) \times Q_1(x_4(t)) \times h_{3x}(\hat{q}) + P_1(x_2(t)) \times Q_2(x_4(t)) \times h_{4x}(\hat{q}).
\]

(5.106)

Using (5.106), the longitudinal vehicle dynamics can be obtained as

\[
\begin{align*}
 h_{1x}(\hat{q}) &= \rho_x(\hat{q}) \bigg|_{x_2 = 35, \ x_4 = 1} \\
 h_{2x}(\hat{q}) &= \rho_x(\hat{q}) \bigg|_{x_2 = 35, \ x_4 = -1} \\
 h_{3x}(\hat{q}) &= \rho_x(\hat{q}) \bigg|_{x_2 = -5, \ x_4 = 1} \\
 h_{4x}(\hat{q}) &= \rho_x(\hat{q}) \bigg|_{x_2 = -5, \ x_4 = -1}.
\end{align*}
\]

(5.107)

Thus, according to the sector nonlinearity method, the true local longitudinal dynamics of the vehicle can be obtained by specifying the longitudinal and lateral speeds accordingly.

Remark 5.8: Practically (as far as data collection is concerned), it was difficult for the veDYNA® vehicle model to function when the speed was set to -5 [m/s], because such speed range was not provided in veDYNA®. When such a speed is selected, the vehicle (true model) adopts a very slow forward speed, instead. Therefore, a forward speed of 5 [m/s] was set, and thereby data was obtained. This setting makes no difference to the
throttle control input, because in a reverse operation only the gear is changed, but the throttle action is similar to that at -5 [m/s]. Therefore, as far as throttle control input is concerned, data is valid according to the design. □

Remark 5.9: In order to set the lateral speed for obtaining data for calculating the adaptive fuzzy controller outputs (in (5.107)), the following procedure was used. Since there was no facility to use lateral speed as an input to the vehicle system in veDYNA®, a relevant steering angle value was calculated using an inverse model based on the provided lateral speed. This steering angle was used, instead, as the vehicle lateral control input. □

**Lateral Adaptive Fuzzy Controllers**

The inputs in each lateral adaptive fuzzy controller are error, $e$ and error rate, $\dot{e}$. The output of each lateral adaptive fuzzy control function is calculated by making the error terms zero starting from the model equations (as provided in (5.93)), and the controller equations (as provided in (5.102)) as follows:

\[
\begin{align*}
\left( y^T \theta \right)_{1y} &= h_{1y}(\dot{q}) - \hat{h}_{1y}(\dot{q}) \\
\left( y^T \theta \right)_{2y} &= h_{2y}(\dot{q}) - \hat{h}_{2y}(\dot{q}) \\
\left( y^T \theta \right)_{3y} &= h_{3y}(\dot{q}) - \hat{h}_{3y}(\dot{q}) \\
\left( y^T \theta \right)_{4y} &= h_{4y}(\dot{q}) - \hat{h}_{4y}(\dot{q}) .
\end{align*}
\]

(5.108)

The values of $h_{yi}(\dot{q}), i = 1, \ldots, 4$ are calculated as follows:

Once again, according to fuzzy PDC principle, it can be provided that

\[
\rho_y(\dot{q}) = P_1(x_2(t)) \times Q_1(x_4(t)) \times h_{1y}(\dot{q}) + P_1(x_2(t)) \times Q_2(x_4(t)) \times h_{2y}(\dot{q}) + P_2(x_2(t)) \times Q_1(x_4(t)) \times h_{3y}(\dot{q}) + P_2(x_2(t)) \times Q_2(x_4(t)) \times h_{4y}(\dot{q}).
\]

(5.109)

Using (5.109), the lateral vehicle dynamics are obtained as
\[ h_1(a) = \rho_1(a) \begin{cases} \begin{aligned} x_2 &= 35 \\ x_4 &= 1 \end{aligned} \end{cases} \]

\[ h_2(a) = \rho_2(a) \begin{cases} \begin{aligned} x_2 &= 35 \\ x_4 &= -1 \end{aligned} \end{cases} \]

\[ h_3(a) = \rho_3(a) \begin{cases} \begin{aligned} x_2 &= -5 \\ x_4 &= 1 \end{aligned} \end{cases} \]

\[ h_4(a) = \rho_4(a) \begin{cases} \begin{aligned} x_2 &= -5 \\ x_4 &= -1 \end{aligned} \end{cases} \]

(5.110)

Thus, according to the sector nonlinearity method, the true local lateral dynamics of the vehicle can be obtained.

Remarks 5.10: The difficulty of obtaining values at the speed setting -5 [m/s] is discussed in the previous Remark 5.8. □

Remarks 5.11: The procedure adopted for using the lateral input to the vehicle is discussed in the previous Remark 5.9. □

(iii) Training of Adaptive Fuzzy Controller for PDC Structure

Special vehicle runs on veDYNA® were conducted in order to obtain input and output data for tuning adaptive fuzzy controller initial parameters. In these runs, in the presence of a lead vehicle (the speed of the lead vehicle was chosen to keep the longitudinal error within an applicable range), the controlled vehicle was set to achieve the longitudinal speed, i.e. 35 [m/s], using the provided cruise control system (in veDYNA®) together and the lateral speed input (see Remark 5.9 for more details). The collection of data was started after reaching the specified longitudinal speed by the vehicle. The input data to each adaptive fuzzy controller were system error and error rate. The outputs of each adaptive fuzzy controller were calculated according to the provided equations, i.e. equations (5.105) for longitudinal controller, and (5.108) for lateral controller. The total length of the data set was 1000 for each four specified input ranges. This procedure was commonly applied for both longitudinal as well as lateral cases. In each of these cases, the first data set range, i.e. from 1–800 was used for training while the second data set range, i.e. from 801–1000 was used for validating
(checking) the training procedure, in the tuning process for each adaptive fuzzy controller.

Using the collected data sets, and using the subtractive clustering method, a basic fuzzy structure for each controller was developed. Using the same set of data, together with the ANFIS method, each final fuzzy (systems a priori) were developed. The following figures, i.e. Figs. 5.56 and 5.57, show the control output surfaces of the developed individual fuzzy controllers, for longitudinal and lateral cases. The set of parameters a priori for each adaptive fuzzy controller in longitudinal and lateral cases, are provided in APPENDIX C.

(a) Output Surface (a priori): Longitudinal Fuzzy Controllers

The control surfaces for the longitudinal adaptive fuzzy controllers a priori are illustrated below:

Fig. 5.56 Adaptive fuzzy longitudinal controller output surfaces
(b) Output Surfaces (*a priori*): Lateral Fuzzy Controllers

The control surfaces for the lateral adaptive fuzzy controllers *a priori* are illustrated below:

![controller-1](image1)

![controller-2](image2)

![controller-3](image3)

![controller-4](image4)

Fig. 5.57 Adaptive fuzzy lateral controller output surfaces

The figures illustrated below provide the training and checking (validation) error variations in developing each longitudinal and lateral fuzzy controller. The generalization capabilities of the developed fuzzy systems were checked with the validation or checking process. More details on validation of developed fuzzy models are provided in Section 4.1.5.
(c) Training and Checking Errors: Longitudinal Fuzzy Controllers

The longitudinal controller training and checking error variations are as follows:

Fig. 5.58 Longitudinal controller training and validation errors
(d) Training and Checking Errors: Lateral Fuzzy Controllers

The lateral controller training and checking error variations are as follows:

Fig. 5.59 Lateral controller training and validation errors
5.2.5 Simulation Studies, Results and Discussion

(i) Simulation Setup

The simulation setup for this section was similar to the one provided in Chapter 4 for the single-adaptive fuzzy controller system, except for facts otherwise stated in this section. The simulation setup included the veDYNA® setup, simulation profile details and other settings.

The gains in adaptive fuzzy controllers were taken as, \( \gamma_{1} = \gamma_{2} = \gamma_{3} = \gamma_{4} = 0.03 \) for longitudinal case, and \( \gamma_{y_{1}} = \gamma_{y_{2}} = \gamma_{y_{3}} = \gamma_{y_{4}} = 0.01 \) for lateral case, in the PDC based multiple-model adaptive fuzzy controller setup.

(ii) Simulation Results and Discussion

In the following figures depicting simulation results, each performance parameter of the multiple-model fuzzy PDC-based adaptive fuzzy controller (solid blue line) is compared against that of the single-adaptive fuzzy controller (dotted red line).

(a) Normal Cruising Conditions: Multiple-Model PDC-based Adaptive Fuzzy Control

The following simulation results provide the performance of the developed PDC-based multiple-model adaptive fuzzy controller against the results of the single-adaptive fuzzy controller under normal cruising conditions without the presence of any external disturbances.

In the variations of tracking performances in the lateral and longitudinal cases, PDC-based multiple-model controller has shown some improved display, as provided in Figs. 5.60–5.61, over the single-adaptive fuzzy controller. There is a clear indication of elimination of transient effects together with high amplitudes by the PDC fuzzy controller.
When it comes to variation of pitch angle, the two controllers fared similarly except for a high ‘spike’ by the PDC multiple-model fuzzy controller towards the point 5 [s] on the time-scale. This is shown in Fig. 5.62 below. But, this cannot be considered as overly excessive.
Roll angle variations, as shown in Fig. 5.63, for the two controllers have been similar to each other. But, the PDC multiple-adaptive fuzzy controller has shown some chattering effects slightly, e.g. 42 – 43 [s]. When the variation of roll rate, as provided in Fig. 5.64 is considered, the PDC multiple-adaptive fuzzy controller has shown some significant improvements in comparison to its counterpart during high-values, e.g. 37 – 43 [s]. But during periods with lower values of roll-rate, the single-adaptive fuzzy controller has outperformed its counterpart, slightly, e.g. throughout 22 – 37 [s], 47 – 60
As illustrated in Fig. 5.65, the PDC multiple-adaptive fuzzy control has mildly improved the response of side-slip angle especially with transients as against its counterpart. It can be said that lateral stability ensured by the PDC multiple-model adaptive fuzzy controller stands at a similar level to that by the single-adaptive fuzzy controller, as evidenced by similar variations of roll angle, roll rate and side-slip angle.

As shown in Fig. 5.66, the variations of lateral acceleration do not go beyond 4 [m/s²], therefore, the PDC based multiple-model adaptive fuzzy controller has managed to contain lateral acceleration within low limits as far as the design requirements of passenger vehicles are concerned [178].

Overall, in the presence of normal cruising conditions, the performance of the PDC-based multiple-adaptive fuzzy controller has been significantly improved with regard to lateral and longitudinal tracking in comparison to that of the single-adaptive-fuzzy controller. While these tracking performance levels are maintained, the PDC based multiple-model controller has ensured an acceptable level of lateral stability in comparison to the single-adaptive fuzzy controller. Lateral acceleration has also been maintained throughout at an acceptable standard level by both controllers.

(b) External Disturbances: Multiple-Model PDC-based Adaptive Fuzzy Control

In the following set of simulations, the developed PDC-based multiple-model adaptive fuzzy controller was tested against the single-adaptive fuzzy controller under a number of external disturbances. These disturbances included addition of an un-
symmetrical load mass in the vehicle, presence of crosswind forces, and change of tyre-road friction.

1. **Unsymmetrical Load Mass**

The following simulation was based on comparison of the results between the PDC based multiple-adaptive fuzzy controller and the single-adaptive fuzzy controller, when there was an unsymmetrical load-mass on the vehicle as per the details described in Section 3.2.18.

As shown in Figs. 5.67–5.68, lateral and longitudinal tracking as exercised by the PDC based multiple-model adaptive fuzzy controller is improved in comparison to that by the single-adaptive fuzzy controller.

When it comes to variation of pitch angle, the two controllers have displayed similar results. This is shown in Fig. 5.69 below.
Fig. 5.69 Pitch angle: PDC-based multiple-adaptive fuzzy with blending (RMS: 0.02009) vs. single-adaptive fuzzy (RMS: 0.01998)

Fig. 5.70 Roll angle: PDC-based multiple-adaptive fuzzy with blending (RMS: 0.01767) vs. single-adaptive fuzzy (RMS: 0.01766)

Fig. 5.71 Roll rate: PDC-based multiple-adaptive fuzzy with blending (RMS: 0.01443) vs. single-adaptive fuzzy (RMS: 0.015)
The PDC multiple-adaptive fuzzy controller has shown the results of variations of roll angle, roll rate and side-slip ratio as quite consistent in comparison to that of the single-adaptive fuzzy controller, as shown in Figs. 5.70–5.72. Therefore, it can be said that the PDC multiple-adaptive fuzzy controller has shown acceptable lateral stability results in comparison to its counterpart.

As shown in Fig. 5.73, variations of lateral acceleration with regard to the PDC based multiple-model adaptive fuzzy controller, have not gone beyond 4 [m/s$^2$]. Therefore, the PDC based multiple-adaptive fuzzy controller has managed to contain lateral acceleration within low limits as far as the design requirements of vehicles are concerned [178].

Overall, in the presence of unsymmetrical load mass, the PDC based adaptive fuzzy controller has managed to show significant improvements in lateral and longitudinal tracking, in comparison to the single-adaptive fuzzy controller.
Performances of the former in lateral stability have also been consistent throughout. Design requirements, too, are satisfied where lateral acceleration variation is concerned.

2. Crosswind Effects

The following simulation results were obtained when there was a presence of a continuous crosswind with a speed of 50 [m/s] through the global y-direction. More details of this arrangement are described in Section 3.2.18.

Once again, the PDC based multiple-model adaptive fuzzy controller has shown significant improvements over lateral and longitudinal tracking, as shown in Figs. 5.74–5.75, in comparison to the single-adaptive fuzzy controller.

![Fig. 5.74 Longitudinal error: PDC-based multiple-adaptive fuzzy with blending (RMS: 3.12) vs. single-adaptive fuzzy (RMS: 3.742)](image1)

![Fig. 5.75 Lateral error: PDC-based multiple-adaptive fuzzy with blending (RMS: 0.002672) vs. single-adaptive fuzzy (RMS: 0.005914)](image2)

Pitch angle, as shown in Fig. 5.76, of the PDC multiple-model control system shows a variation that is similar to the variation shown by the single-adaptive fuzzy controller.
As far as variations of roll angle, roll rate and side-slip angle go as shown in Figs. 5.77–5.79, the PDC multiple-adaptive fuzzy controller has shown some ‘spikes’. But these ‘spikes’ will not make the variations excessively different from that of the single-adaptive fuzzy controller. Therefore, the PDC multiple-adaptive fuzzy controller can be said to maintain lateral stability at a level not much different from that of the single-adaptive fuzzy controller.

![Pitch Angle Graph](image1)

**Fig. 5.76 Pitch angle: PDC-based multiple-adaptive fuzzy with blending (RMS: 0.0117) vs. single-adaptive fuzzy (RMS: 0.01142)**

![Roll Angle Graph](image2)

**Fig. 5.77 Roll angle: PDC-based multiple-adaptive fuzzy with blending (RMS: 0.01419) vs. single-adaptive fuzzy (RMS: 0.01396)**

![Roll Rate Graph](image3)

**Fig. 5.78 Roll rate: PDC-based multiple-adaptive fuzzy with blending (RMS: 0.03319) vs. single-adaptive fuzzy (RMS: 0.02222)**
As far as lateral acceleration is concerned, the PDC multiple-model system tries to keep the values within 4 \([m/s^2]\), thereby satisfying the design requirements of today’s passenger vehicles [178].

Overall, in the presence of crosswind effects, the PDC multiple-model adaptive fuzzy controller has shown some good performances of tracking in lateral and longitudinal domains, once again exceeding the capacity of the single-adaptive fuzzy system.

3. Tyre-Road Friction Change

The following simulation results show the performance of the PDC multiple-adaptive fuzzy controller and the single-adaptive fuzzy controller when there was a change of tyre-road friction in the range 100-200 \([m]\), on the highway.
Fig. 5.81 Longitudinal error: PDC-based multiple-adaptive fuzzy with blending (RMS: 3.065) vs. single-adaptive fuzzy (RMS: 3.158)

Fig. 5.82 Lateral error: PDC-based multiple-adaptive fuzzy with blending (RMS: 0.001217) vs. single-adaptive fuzzy (RMS: 0.003996)

Fig. 5.83 Pitch angle: PDC-based multiple-adaptive fuzzy with blending (RMS: 0.0116) vs. single-adaptive fuzzy (RMS: 0.01144)
Fig. 5.84 Roll angle: PDC-based multiple-adaptive fuzzy with blending (RMS: 0.01085) vs. single-adaptive fuzzy (RMS: 0.01085)

Fig. 5.85 Roll rate: PDC-based multiple-adaptive fuzzy with blending (RMS: 0.008594) vs. single-adaptive fuzzy (RMS: 0.01255)

Fig. 5.86 Side-slip angle: PDC-based multiple-adaptive fuzzy with blending (RMS: 0.007175) vs. single-adaptive fuzzy (RMS: 0.007268)
Overall, in the presence of tyre-road friction changes, the PDC-based multiple-adaptive fuzzy controller has demonstrated that it can perform better with longitudinal and lateral tracking than the single-adaptive fuzzy controller (as shown in Figs. 5.81–5.82). Also, as far as the variations of roll angle, roll rate and side-slip angle are concerned, as shown in Figs. 5.84–5.86, the performances of the two controller types are similar. This fact suggests that the PDC multiple-controller can ensure lateral stability at least similar to that of the single-model controller. It is also worth to consider similar characteristics of pitch angle variation regarding longitudinal characteristics.

As shown in Fig. 5.87, since variations of lateral acceleration do not go beyond 4 [m/s\(^2\)], it can be said that the two controllers are addressing the design requirements of passenger vehicles [178].

(c) Failure Modes (non-catastrophic subsystem failures): Multiple-Model PDC-based Adaptive Fuzzy Control

The following simulation results show the performance of the PDC multiple-model adaptive fuzzy controller and the single-adaptive fuzzy controller when some failures occurred in the vehicle components. The failures considered in this simulation included a flat-tyre case in the front-left tyre and a 90% drop of pressure in the brake line of the front-left wheel cylinder of the vehicle.
1. Flat-Tyre

The following simulation results were obtained when an event of front-left tyre becoming flat occurred at 10[s] after starting the simulation. The details of the simulation arrangement are explained in Section 3.2.18.

Longitudinal and lateral tracking performance with the PDC-based multiple-model adaptive fuzzy controller, as shown in Figs. 5.88–5.89, demonstrates an improved response in comparison to that with the single-adaptive fuzzy controller. This is a good performance by the PDC-based multiple-model controller.

![Fig. 5.88 Longitudinal error: PDC-based multiple-adaptive fuzzy with blending (RMS: 25.3) vs. single-adaptive fuzzy (RMS: 26.08)](image1)

![Fig. 5.89 Lateral error: PDC-based multiple-adaptive fuzzy with blending (RMS: 0.01158) vs. single-adaptive fuzzy (RMS: 0.02753)](image2)

The variations of roll angle, roll rate and side-slip angle, as shown in Figs. 5.91–5.93, of the PDC multiple-controller are quite similar to that of the single-adaptive fuzzy controller. This fact suggests that the PDC multiple-controller can ensure lateral stability to a level at least similar to that of the single-model controller. On pitch angle variation also, as shown in Fig. 5.90, one can draw a similar conclusion.
Fig. 5.90 Pitch angle: PDC-based multiple-adaptive fuzzy with blending (RMS: 0.01057) vs. single-adaptive fuzzy (RMS: 0.01052)

Fig. 5.91 Roll angle: PDC-based multiple-adaptive fuzzy with blending (RMS: 0.02224) vs. single-adaptive fuzzy (RMS: 0.02225)

Fig. 5.92 Roll rate: PDC-based multiple-adaptive fuzzy with blending (RMS: 0.01682) vs. single-adaptive fuzzy (RMS: 0.0212)
Fig. 5.93 Side-slip angle: PDC-based multiple-adaptive fuzzy with blending (RMS: 0.00719) vs. single-adaptive fuzzy (RMS: 0.007039)

Since variations of lateral acceleration, as shown in Fig. 5.94, do not go beyond 4 [m/s²], it can be concluded that the two controllers satisfy the design requirements of passenger vehicles [178].

Fig. 5.94 Lateral acceleration: PDC-based multiple-adaptive fuzzy (RMS: 1.252) with blending vs. single-adaptive fuzzy (RMS: 1.248)

Overall, in the presence of the tyre flatness event, the PDC-based multiple-adaptive fuzzy controller has managed to ensure a good level of longitudinal and lateral tracking performance compared to that of the single-adaptive fuzzy controller. It can also be concluded that lateral stability of the controller stands at an acceptable level. As far as the design is concerned, the controller exhibits good lateral acceleration levels.

2. Brake Cylinder Defect

A subsystem failure of the front-left wheel-brake cylinder being defective with a 90% pressure decline in the brake line of the vehicle is studied with the PDC multiple-model controller, in this section. More details on this simulation setup are discussed in Section 3.2.18.
The tracking performances of the PDC multiple-model adaptive fuzzy controller have been significant with regard to longitudinal and lateral tracking aspects as shown in Figs. 5.95–5.96, in comparison to that of the single-adaptive fuzzy controller.

Fig. 5.95 Longitudinal error: PDC-based multiple-adaptive fuzzy with blending (RMS: 3.062) vs. single-adaptive fuzzy (RMS: 3.159)

Fig. 5.96 Lateral error: PDC-based multiple-adaptive fuzzy with blending (RMS: 0.007504) vs. single-adaptive fuzzy (RMS: 0.03453)

Pitch angle variation, normally a longitudinal stability measure as shown in Fig. 5.97 for both controllers, has been similar.

Fig. 5.97 Pitch angle: PDC-based multiple-adaptive fuzzy with blending (RMS: 0.01574) vs. single-adaptive fuzzy (RMS: 0.01583)
The variations of roll angle, roll rate and side-slip angle, as shown in Figs. 5.98–5.100, for the PDC multiple-controller are within the same range as that for the single-adaptive fuzzy controller. There are some high-valued areas too, in relation to the PDC multiple-model controller. Since these values are still within the range suggested by the single-adaptive fuzzy controller, the lateral stability levels exercised by the PDC multiple-adaptive fuzzy controller are acceptable.

Fig. 5.98 Roll angle: PDC-based multiple-adaptive fuzzy with blending (RMS: 0.01069) vs. single-adaptive fuzzy (RMS: 0.01082)

Fig. 5.99 Roll rate: PDC-based multiple-adaptive fuzzy with blending (RMS: 0.04591) vs. single-adaptive fuzzy (RMS: 0.04617)
As far as lateral acceleration is concerned, as shown in Fig. 5.101, the variation exhibited by the PDC multiple-model adaptive fuzzy controller is nearly around 4 \([\text{m/s}^2]\) in the extreme. But, in comparison to the performance by the single-adaptive fuzzy controller, the PDC multiple-model controller shows an exceptional improvement. It can be said that the values of the PDC multiple-adaptive fuzzy controller remain within the design requirements of passenger vehicles, with a slight drift towards the ‘transition range’ [178], though.

Overall, in the presence of a brake cylinder pressure failure, the PDC multiple-model adaptive fuzzy controller has shown to exhibit significant improvements in lateral and longitudinal tracking performances in relation to the single-adaptive fuzzy controller. As suggested by variations of side-slip angle, roll-angle and roll rate, the lateral stability performance of the PDC controller can be said to be slightly superior to that of the single-adaptive fuzzy controller.
Considering the overall performance of the PDC-based multiple-adaptive fuzzy controller in relation to the single-adaptive fuzzy controller, it can be said that tracking performances in the domains of lateral and longitudinal, have been good. It is worth to be noted that such performances have been ensured while the vehicle exhibits good lateral stability.

The summary of performance of the fuzzy PDC based multiple-adaptive fuzzy controller against that of the single-adaptive fuzzy controller is provided in TABLE 5.2 below.

**TABLE 5.2 Comparison of performance of fuzzy PDC based multiple-adaptive fuzzy controller against single-adaptive fuzzy controller**

<table>
<thead>
<tr>
<th>Simulation</th>
<th>Longitudinal error</th>
<th>Lateral error</th>
<th>Pitch angle</th>
<th>Roll angle</th>
<th>Roll rate</th>
<th>Side-slip angle</th>
<th>Lateral acceleration</th>
</tr>
</thead>
<tbody>
<tr>
<td>normal cruising</td>
<td>slightly improved</td>
<td>significantly improved</td>
<td>similar</td>
<td>similar</td>
<td>slightly improved</td>
<td>similar</td>
<td>similar</td>
</tr>
<tr>
<td>un-symmetrical loading</td>
<td>slightly improved</td>
<td>significantly improved</td>
<td>similar</td>
<td>similar</td>
<td>similar</td>
<td>similar</td>
<td>similar</td>
</tr>
<tr>
<td>crosswinds</td>
<td>slightly improved</td>
<td>significantly improved</td>
<td>similar</td>
<td>slightly deteriorated</td>
<td>deteriorated</td>
<td>slightly deteriorated</td>
<td>similar</td>
</tr>
<tr>
<td>tyre-road friction</td>
<td>slightly improved</td>
<td>significantly improved</td>
<td>similar</td>
<td>similar</td>
<td>slightly improved</td>
<td>similar</td>
<td>similar</td>
</tr>
<tr>
<td>flat-tyre</td>
<td>slightly improved</td>
<td>improved</td>
<td>similar</td>
<td>similar</td>
<td>slightly improved</td>
<td>similar</td>
<td>similar</td>
</tr>
<tr>
<td>defective brake cylinder</td>
<td>slightly improved</td>
<td>significantly improved</td>
<td>similar</td>
<td>slightly deteriorated</td>
<td>slightly improved</td>
<td>similar</td>
<td>significantly improved</td>
</tr>
</tbody>
</table>

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5.3 Chapter Summary

In this chapter, the development of two robust multiple-model/multiple-adaptive controllers based on soft-switching (blending) is described.

Firstly, a robust multiple-adaptive fuzzy controller with blending is synthesized. A similar procedure to developing the single-adaptive fuzzy controller is followed in tuning the individual adaptive fuzzy controllers in the multiple-adaptive setup. In the design of the fuzzy blender, for determining the fixed parameters, a number of vehicle runs are conducted and data gathered. Using this data, with the subtractive clustering and ANFIS methods, parameters for the fuzzy blender are found. A detailed robust stability analysis procedure is followed for ensuring asymptotic stability of the system using a Lyapunov based KYP lemma. The developed controller is validated using a comprehensive set of simulations on veDYNA®. These simulations include testing of the controller under a number of external disturbances as well as a couple of failure modes.

Secondly, a robust PDC-based multiple-model adaptive fuzzy controller is synthesized comprehensively. In designing an implementation level PDC-based controller, a comprehensive sector nonlinearity method is used to construct the local models and local adaptive fuzzy controllers. Once again a detailed robust stability analysis procedure is followed for ensuring asymptotic stability of the general PDC-based system using the Lyapunov based KYP lemma. The developed controller is validated using a comprehensive set of simulations on veDYNA®. These simulations also include the testing of the controller under a number of external disturbances as well as a couple of failure modes.
CHAPTER 6

Robust Multiple-Adaptive Fuzzy Control Systems – Switching (Hard-Switching)

This chapter describes a design of a multiple-adaptive fuzzy controller based on ‘switching’ (more specifically termed as ‘hard-switching’) for addressing the problem of integrated lateral and longitudinal control of vehicles. The resulting controller is a ‘binary’ switching structure consisting of a bank of adaptive fuzzy controllers. In the design of the MM/MC based adaptive fuzzy controller with switching, a comprehensive investigation of stability is carried out using a Lyapunov based KYP lemma leading to asymptotic stability. In the validation of the designed ‘switching’ based control system, two separate simulations are carried out. The first simulation is included with ‘two’ number of adaptive fuzzy controllers (AFCs), while the second is included with ‘four’ adaptive fuzzy controllers, in the multiple-adaptive fuzzy setup. These two simulations are used to demonstrate the effects of having different number of adaptive fuzzy controllers in the switching based MM/MC structure.
6.1 Robust Multiple-Adaptive Fuzzy Control Systems with Switching (Hard-Switching)

6.1.1 Introduction

There are drawbacks of using single-model (or single ‘modal’) control systems in addressing the problem of integrated lateral and longitudinal control of highway vehicles. This issue has already been discussed in the main introduction.

But, MM/MC systems provide a solution in the multiple-environments of operation of vehicles, which overcomes the drawbacks that associate with the single-model control systems.

It is customary to define a particular generic scenario as a combination of sub-scenarios. For such occasions, there is little doubt that ‘blending’ based controllers are more appropriate. But, when the scenarios become more specifically defined, the most appropriate solution would be to have a ‘dedicated’ controller selected from a bank of controllers with ‘switching’ (binary selection). In this regard, the need of a ‘switching’ controller in the context of MM/MC setup arises when a particular scenario requires a specific controller without combining it with any other controller in the bank in addressing the problem of vehicle control. This chapter provides a design of a robust multiple-adaptive fuzzy controller based on switching for addressing the problem of integrated lateral and longitudinal control of vehicles.

This robust multiple-adaptive fuzzy controller (as a multiple-model method) with switching is more versatile in comparison to linear multiple-model control systems due to the inclusion of adaptive fuzzy controllers [20]. This inclusion substantially pays off with the fact that it is possible to program the adaptive fuzzy controllers in a versatile manner in accordance with complex scenarios. In addition, having adaptive fuzzy control functions enables one to define and model ‘uncertainty’ of the system over which the control action is executed.
6.1.2 Synthesis of Robust Multiple-Adaptive Fuzzy Controller with Switching

In this section, the design of a novel setup of robust multiple-adaptive fuzzy controllers based on switching is described. In this controller setup, a number of adaptive fuzzy controllers are included in the multiple-adaptive fuzzy system. These adaptive fuzzy controllers are designed based on possible scenarios the vehicle system is expected to be exposed to. The adaptive rates of each adaptive controller in the controller bank are made different from each other to allow for different scenarios. In other words, a higher adaptive rate allows for fast dynamical changes while a lower adaptive rate allows for slow changes. This is the general setup of the multiple-adaptive fuzzy controller that will follow.

In the normal setup, adaptive fuzzy controllers are trained in the normal operating conditions. The scope of the multiple-adaptive fuzzy controller with switching can be extended to include fault conditions for some selected vehicle sub-components. This extension will introduce more versatility to the system. Identification of different fault conditions can be done with predefined residuals [180]. But, this extended usage is not covered in this research.

In the designed controller setup, the switching system activates to select the particular adaptive fuzzy controller that is the most suitable to be connected to the vehicle at a time. The selection of one of these controllers is based on the minimum value of a pre-defined cost function. The switching is carried out in a similar way to the well-established MMST (Multiple-Model Switching and Tuning) method [31]. But, the main difference is that the proposed method in this section allows switching of more complicated and versatile adaptive fuzzy controllers rather than linear controllers (the mostly used form in the MMST method). The other difference from the MMST method is that the proposed method does not need different vehicle models being defined in the aid of switching.
Fig. 6.1 shows the schematics of the multiple-adaptive fuzzy controller with switching

![Schematics of the multiple-adaptive fuzzy controller with switching](image)

where \( \psi_j^T \theta_j \) is the control output from the \( j \)th controller (\( \psi_j \) is the regression vector and \( \theta_j \) is the parameter matrix of the \( j \)th controller) where \( j = 1, \ldots, n \) is the total number of adaptive fuzzy controllers in the bank.

**Remark 6.1:** The term \( j \)th adaptive fuzzy output, \( \psi_j^T \theta_j \) where \( j = 1, \ldots, n \), is equivalent to the 'single-model adaptive fuzzy controller' (as explained under ‘single-adaptive fuzzy’ controller), \( \xi^T (\bar{x}) \theta \), in every sense as used in Chapter 4. But, the adaptive rates have been selected differently for different controllers in the new setup in Chapter 6. The change of notation has been used to avoid ambiguities with indices.

Tuning of each adaptive fuzzy controller, \( \psi_j^T \theta_j \) where \( j = 1, \ldots, n \), is according to an adaptive law (to be derived in the stability analysis process). The input variables to each adaptive fuzzy controller are ‘error’ and ‘error rate’.

For the \( j \)th adaptive fuzzy controller, where \( j = 1, \ldots, n \), the regression vector can be expressed as \( \psi_j = [\psi_{j1}, \psi_{j2}, \ldots, \psi_{jp}]^T \), and thereby the \( k \)th element of each \( j \)th controller becomes \( \psi_{jk} = [\psi_{jk} e \quad \psi_{jk} \dot{e}] = [\psi_{jk} e^\tau] \), where \( j = 1, \ldots, n \) (\( n \) is the total
number of controllers in the bank), and \( k = 1, \cdots, p \). Here, \( p \) is the total number of IF-THEN fuzzy rules in each individual adaptive fuzzy controller.

The output membership function parameter set of each adaptive fuzzy controller can be tuned using an algorithm based on an adaptive law. For the \( j \)th adaptive fuzzy controller, the adaptive law can be expressed as

\[
\dot{\theta}_j = -\gamma_j \psi_j Ce
\]

(6.1)

where \( j = 1, \cdots, n \), for \( n \) number of adaptive fuzzy controllers.

(i) Control Law

The certainty equivalence control laws can be obtained for longitudinal control as

\[
\hat{u}_x = \frac{1}{b_x} \left[ \dot{x}_p + k_{dx} \dot{e}_x + k_{ex} e_x + \psi_j^T \theta_j \right]_x + u_{sx} - \hat{h}_x(\dot{q}_x).
\]

(6.2)

and for lateral control as

\[
\hat{u}_y = \frac{1}{b_y} \left[ \dot{l}_y \dot{\psi} + k_{dy} \dot{e}_y + k_{ey} e_y + \psi_j^T \theta_j \right]_y + u_{sy} - \hat{h}_y(\dot{q}_y).
\]

(6.3)

Remark 6.2: Though fuzzy systems in (6.2), (6.3) are generally nonlinear, each is linear with respect to its unknown parameters. Hence, parameter adaptation algorithms applicable for linear systems, e.g. gradient-based algorithms, can be readily used in estimation of the unknown parameters in fuzzy systems in (6.2), (6.3) [27].

The switching system to be described, selects the most suitable adaptive fuzzy controller from the bank of adaptive fuzzy controllers (for \( j \)th controller, it is \( \psi_j^T \theta_j \) where \( j = 1, \cdots, n \)), and connects it to the output. Each \( \theta_1, \theta_2, \cdots, \theta_n \) provides the set of parameter matrices for each adaptive fuzzy controller.
(ii) Error Dynamics

The equation (6.2) can be substituted in (3.4)–(3.6), and will result in

for longitudinal dynamics

\[ \ddot{e}_x + k_{dx} \dot{e}_x + k_{px} e_x = \hat{h}_x(\dot{q}_x) - h_x(\dot{q}_x) - \psi^T \theta_j \bigg|_{x} - u_{sx} - d_x \]

\[ = \nu_x^* + w_x^* - \dot{\nu}_x - u_{sx} - d_x \]

\[ = \nu_x - u_{sx} - d_x, \quad (6.4) \]

and for lateral error dynamics

\[ \ddot{e}_y + k_{dy} \dot{e}_y + k_{py} e_y = \hat{h}_y(\dot{q}_x) - h_y(\dot{q}_y) - \psi^T \theta_j \bigg|_{y} - u_{sy} - d_y \]

\[ = \nu_y^* + w_y^* - \dot{\nu}_y - u_{sy} - d_y \]

\[ = \nu_y - u_{sy} - d_y. \quad (6.5) \]

Here, \( \hat{h}_{x,y}(\dot{q}_i) \) denotes the calculated value of the term \( h_{x,y}(\dot{q}_i) \), true vehicle dynamics. \( \dot{\nu}_x \) and \( \dot{\nu}_y \) are the outputs from the selected adaptive fuzzy controller for longitudinal and lateral cases, respectively. \( k_{dx}, k_{px} \) are the constant PD (proportional-derivative) gains for the longitudinal control and \( k_{dy}, k_{py} \) are the constant PD gains for the lateral control. \( u_{sx} = \begin{bmatrix} u_{sx} & u_{sy} \end{bmatrix} \) is the variable structure control term.

In this discussion, \( \nu^* = \begin{bmatrix} \nu_x^* & \nu_y^* \end{bmatrix} \) is the optimal adaptive fuzzy output while \( \dot{\nu} = \begin{bmatrix} \dot{\nu}_x & \dot{\nu}_y \end{bmatrix} \) is the actual adaptive fuzzy output. \( w_x, w_y \) are the adaptive fuzzy approximation errors, and are bounded by \( |w_x| < W_x \) and \( |w_y| < W_y \). \( W_x, W_y (> 0) \in \mathbb{R} \) are the practical bounds.

By using the universal approximation theorem (provided in Chapter 4), with the fuzzy adaptation process, the fuzzy controller parameters can be gradually brought to optimal values, and thus the right half of the equations, (6.2) and (6.3) can be made zero.
In this case, the variable structure term, $u_s$, is used to dissipate the effects of fuzzy approximation errors, and to some extent the disturbances. The overall uncertainty factor in the equation is made zero with the fuzzy function.

(iii) Training of Multiple-Adaptive Fuzzy System

The set of adaptive fuzzy controllers can be used in two different ways with effectiveness. First, each adaptive fuzzy controller can be trained with the same data set (obtained from running a vehicle through a real scenario) to obtain its input and output membership parameters. Then the adaptive rates in the adaptive law can be changed to differentiate each controller from another. The adaptive fuzzy controllers having ‘higher adaptive rate’ can be used to make use of the faster changing scenarios while the controllers having ‘lower adaptive rate’ can be used to catch up with the low frequency changes in the scenarios. This is the procedure adopted in the rest of the section.

Even though it is not followed, another option can also be considered. Instead of having a common data set for the fuzzy system, the data space can be categorized into groups, e.g., according to the frequency of the variable ‘error’. Using each of the ‘local’ data set, each adaptive fuzzy controller can be trained. The ‘best’ adaptive fuzzy controller for the task can be selected according to the switching criteria.

The following procedure was adopted for tuning the multiple-adaptive fuzzy system based on switching:

(i) The subtractive clustering method was used to obtain the preliminary fuzzy membership function centres and standard deviations (Gaussian), and then the basic structure was further trained with ANFIS using gathered data from veDYNA®.

(ii) The individual adaptive fuzzy controller systems are based on the design of Section 4.3, and thereafter different adaptive rates are added for individual controllers.
6.1.3 Stability Analysis of Control System

(i) Switching Function Characteristics

The switching function characteristics of the controller can be described as follows. The magnitude of the adaptation function of the \( j \)th adaptive fuzzy controller can be taken as

\[
\dot{e}_j = \| \gamma_j \psi_j Ce \| \tag{6.6}
\]

where \( j = 1, \ldots, n \). This function can be described as an overall indicator for determining how much the particular adaptive fuzzy controller in the bank, i.e. \( j \)th controller, adapts. In other words, this ‘magnitude’ of the function provided by (6.6) is a measure of how much the particular controller drifts from the present status. The argument here is that the larger the amount of drift from the present status, the more likely it is away from the required present ‘state’. In other words, if a controller has to work out a larger change to adapt to the next stage, it can be taken as less suitable in the selection procedure. For more reliability of selection, the magnitude of the adaptation function is used to define a cost function, and the value of the cost function is used for determining the best controller for the task.

Definition 6.1: The cost function based on adaptation for the \( j \)th adaptive fuzzy controller can be defined as

\[
J_j = \kappa_1 \| \dot{e}_j \|^2 + \kappa_2 \| \int_0^t \dot{e}_j \|_1^2 , \text{ where } j = 1, \ldots, n . \quad \square
\]

(ii) Stability Analysis of Multiple-Adaptive Fuzzy Control with Switching

The following stability proof of the multiple-adaptive fuzzy control system with switching is done for the generic case irrespective whether it is lateral or longitudinal controller. Therefore, the subscripts that are specific to either longitudinal or lateral case are removed in this description. Nevertheless, the specific details will be mentioned as and when necessary.
Assumption 6.1: The sum of fuzzy approximation errors, $w$ is bounded, i.e., $|w| \leq W$ where $W(>0) \in \mathfrak{R}$. The external disturbances, $d$ are bounded too, i.e., $|d| \leq D$ where $D(>0) \in \mathfrak{R}$. Thereby the variable structure control gain is chosen as $\sigma_j \geq W + D$. □

The following assumptions are made to ascertain the switching, and thereby to ensure stability properties.

Assumptions 6.2: The assumptions made on switching are:

(C.1) The active controller, i.e., $k$th adaptive fuzzy controller, is selected at a particular time according to the criterion $J_k = \min \left\{ J_j \right\}, k \in \{1, \ldots, n\}$ and $j = 1, \ldots, n$.

(C.2) Let the timing sequence for switching be $S = \{ t_1, t_2, \ldots, t_m \}$, and let $t_q$ be any time of typical switching, where $q = \{1, 2, \ldots, \infty\}$. Then it is always ensured $t_{q+1} \geq t_q + t_0$ where $t_0 > 0$ is the minimum time gap for two consecutive switching cases enforced to avoid an infinite switching to occur at a time. The decision for choosing $t_0$ is based on optimum control performance of the system. □

The main features of the robust multiple-adaptive fuzzy control with switching can be summarized in the following theorem.

Theorem 6.1: Let the parameter vectors, $\theta_j$ of the adaptive fuzzy systems be adjusted by the adaptive law as provided in (6.1) and let Assumptions 6.1 and 6.2 be true. The proposed adaptive fuzzy control law as provided in (6.2) [and (6.3)] can guarantee stability of the vehicle system with the following properties:

i) The closed loop vehicle system is stable, i.e. $\| e \| \in L^\infty$ (considering both lateral and longitudinal cases).

ii) The system errors and all the control system parametric errors are asymptotically stable, i.e. $\lim_{t \to \infty} e_x, e_y = 0$ and $\lim_{t \to \infty} \tilde{\theta}_j = 0$ (considering both lateral and longitudinal cases) where $j = 1, \ldots, n$. □
**Proof:**

Positive realness of the transfer function of error dynamics equation, i.e., (4.23) and (4.24), related to vehicle dynamics has already been established in Section 4.2.6.

Assuming positive realness of the vehicle error dynamics can be established as shown in Section 4.2.6, according to Kalman–Yakubovich–Popov (KYP) lemma [175] for a strictly positive-real system, there exist two positive definite matrices $P$ and $Q$ (i.e., $P = P^T > 0$ and $Q = Q^T > 0$) satisfying

$$A^T P + PA + Q = 0, \text{ and } PB = C^T. \quad (6.7)$$

Now, the following Lyapunov function can be defined as (for lateral or longitudinal system)

$$V = e^T P e + \gamma_j^{-1} \tilde{\theta}^T j \tilde{\theta} j \quad (6.8)$$

where $P = P^T > 0$. And $\gamma_j (\in \mathbb{R}) > 0$ is a constant. Here $e$ is the overall system error vector and $\tilde{\theta} j$ is the parametric error vector for the $j$th adaptive fuzzy controller, where $j = 1, ..., n$, of the bank of controllers.

Lyapunov candidate as provided in (6.8) quantifies both in tracking error and in parameter estimates. Differentiating the Lyapunov function along the trajectories of the error system will provide

$$\dot{V} = e^T Pe + e^T Pe + \left( \gamma_j^{-1} \dot{\tilde{\theta}}^T j \tilde{\theta} j + \gamma_j^{-1} \tilde{\theta} j \dot{\tilde{\theta}} j \right). \quad (6.9)$$

Substituting for $\dot{e}$ from the equation (4.23), it will provide

$$\dot{V} = (Ae + B\tilde{v} - B\tilde{d} - Bu,)^T Pe + e^T P(Ae + B\tilde{v} - B\tilde{d} - Bu, + \left( \gamma_j^{-1} \dot{\tilde{\theta}}^T j \tilde{\theta} j + \gamma_j^{-1} \tilde{\theta} j \dot{\tilde{\theta}} j \right). \quad (6.10)$$

Rearranging (6.10) will provide

$$\dot{V} = e^T (A^T P + PA)e + e^T B^T Pe + e^T PB\tilde{v} - dB^T Pe - de^T PB - u, B^T Pe - u, e^T PB$$

$$+ \left( \gamma_j^{-1} \dot{\tilde{\theta}}^T j \tilde{\theta} j + \gamma_j^{-1} \tilde{\theta} j \dot{\tilde{\theta}} j \right). \quad (6.11)$$
By substitution of the Lyapunov equation \( A^T P + PA + Q = 0 \), where \( Q = Q^T > 0 \), equation (6.11) can be further simplified as
\[
V = -e^T Q e + v^T B^T P e + e^T P B v - (d + u_j)\left( B^T P e + e^T P B \right) + \sum_{j=1}^{n} \left( \gamma_j^{-1} \hat{\theta}_j^T \hat{\theta}_j + \gamma_j^{-1} \tilde{\theta}_j^T \tilde{\theta}_j \right).
\] (6.12)

From the argument, it can be expressed as \( \hat{v} = (v^* - \hat{v}) + \tau \). It can be further elaborated that
\[
\hat{v} = \psi_j^T \theta_j^* - \psi_j^T \hat{\theta}_j + w_j\] (6.13)
where \( \theta_j^* \in \mathbb{R}^{\rho \times 2} \) is the optimal parameter matrix. \( \rho \) is the total number of IF-THEN fuzzy rules in each individual adaptive fuzzy controller. \( \hat{\theta}_j \) is the parameter set of \( j \)th controller, and \( w_j \) is the fuzzy approximation error of the \( j \)th adaptive fuzzy controller. \( w_j \) is bounded, \( |w_j| \leq W \) where \( w_j \in \mathcal{C}^\infty \). And \( W \in \mathbb{R} \) is a positive constant. The determination of fuzzy approximation error upper bound, \( W \) has been discussed before [30].

Equation (6.13) can be further simplified so that
\[
\hat{v} = \psi_j^T \theta_j^* - \psi_j^T \hat{\theta}_j + w_j.
\] (6.14)
Let the parameter error be taken as \( \tilde{\theta}_j = \theta_j^* - \hat{\theta}_j \). Equation (6.14) now becomes
\[
\hat{v} = \psi_j^T \tilde{\theta}_j + w_j.
\] (6.15)

Substituting (6.15) in (6.12) will become
\[
\hat{V} = -e^T Q e + \left( \psi_j^T \hat{\theta}_j + w_j \right)^T B^T P e + e^T P B \left( \psi_j^T \hat{\theta}_j + w_j \right) - (d + u_j)\left( B^T P e + e^T P B \right) + \left( \gamma_j^{-1} \hat{\theta}_j^T \hat{\theta}_j + \gamma_j^{-1} \tilde{\theta}_j^T \tilde{\theta}_j \right).
\] (6.16)

Since \( PB = C^T \), (6.16) can be further simplified to provide
\[
\hat{V} = -e^T Q e + \theta_j^T \psi_j C e + e^T C^T \psi_j^T \tilde{\theta}_j + \left( \gamma_j^{-1} \hat{\theta}_j^T \hat{\theta}_j + \gamma_j^{-1} \tilde{\theta}_j^T \tilde{\theta}_j \right) + \left( w_j - d - u_j \right) \left( C e + e^T C^T \right).
\] (6.17)

Further rearrangement of (6.17) will result in
Now, (6.18) can be written as
\[
\dot{V} = -e^T Q e + \left( e^T C^T \psi_j + \gamma_j^{-1} \hat{\theta}_j \right) \hat{\theta}_j + \hat{\theta}_j^T \left( \psi_j C e + \gamma_j^{-1} \hat{\theta}_j \right) + \left( w_j - d - u_j \right) \left( Ce + e^T C^T \right).
\] (6.19)

At any moment of time, it is taken as \( \hat{\theta}_j = -\gamma_j \psi_j C e \) for \( \forall j \). Thereby (6.19) further simplifies to (with reference to the conditional assumptions C.1 and C.2)
\[
\dot{V} = -e^T Q e + \left( w_j - d - u_j \right) \left( Ce + e^T C^T \right).
\] (6.20)

Equation (6.20) can be further expressed as
\[
\dot{V} \leq -\lambda_{Q_{\text{min}}} \| e \|^2 + \left( w_j - d - u_j \right) \left( Ce + e^T C^T \right),
\] (6.21)

where \( \lambda_{Q_{\text{min}}} \) is the minimum eigen value of \( Q \).

Since fuzzy systems follow universal approximation theorem [29], \( |w_j| \) may be made arbitrarily small, thereby making each \( w_j \) small by proper choice of each adaptive fuzzy system if each \( \hat{v}_j = \psi_j \theta_j \) is smooth (note that this may require an arbitrarily large number of rules for each controller) [30].

Since \( |w_j| \leq W \), and \( |d| \leq D \), (6.21) can be further written as
\[
\dot{V} \leq -\lambda_{Q_{\text{min}}} \| e \|^2 + \left( D + W - u_j \right) \left( Ce + e^T C^T \right).
\] (6.22)

Let the variable structure control term be
\[
u_s = \sigma_j \text{sign} \left( Ce + e^T C^T \right),
\] (6.23)

and thereby (6.22) can be further expressed as
\[
\dot{V} \leq -\lambda_{Q_{\text{min}}} \| e \|^2 + \left( \| D + W \| - \sigma_j \| Ce + e^T C^T \| \right).
\] (6.24)
Further rearrangement of (6.24) will provide
\[ \dot{V} \leq -\lambda_{Q_{\text{min}}} \| e \|^2 - \| Ce + e^T C^T (\sigma_f - \| D + W \|) \]. \quad (6.25)

If it is taken as
\[ \sigma_f \geq \| D + W \|, \quad (6.26) \]
then (6.25) will establish that
\[ \dot{V} \leq 0, \quad \forall t. \quad (6.27) \]

Therefore, according to Lyapunov theory, it establishes the fact that \( e \) is bounded, i.e. \( e \in \mathcal{L}^{\infty} \). Therefore, \( e, \dot{e} \) are bounded, i.e. \( e, \dot{e} \in \mathcal{L}^{\infty} \). It also establishes that parameter errors are bounded, i.e. \( \hat{\theta}_j \in \mathcal{L}^{\infty} \) where \( j = 1, \ldots, n \).

Equation (6.25) also implies that
\[ \dot{V} \leq -\lambda_{Q_{\text{min}}} \| e \|^2. \quad (6.28) \]

Integrating (6.28) will provide
\[ \lambda_{Q_{\text{min}}} \int_0^\infty \| e \|^2 dt \leq -\int_0^\infty \dot{V} dt = V(0) - V(\infty). \quad (6.29) \]
Therefore, (6.29) becomes
\[ \int_0^\infty \| e \|^2 dt \leq \frac{1}{\lambda_{Q_{\text{min}}}} (V(0) - V(\infty)). \quad (6.30) \]

Equation (6.30) leads to
\[ e \in \mathcal{L}^2 \left( \mathcal{L}^{\infty} = \left\{ z(t) : \int_0^\infty z^2(t) dt < \infty \right\} \right), \quad (6.31) \]

since it provides \( V(0), V(\infty) \in \mathcal{L}^{\infty} \) according to (6.27) [30]. Since it is possible to establish as \( e \in \mathcal{L}^2 \cap \mathcal{L}^{\infty} \) (from (6.31) and (6.27)), and \( \dot{e} \in \mathcal{L}^{\infty} \) (from (6.28)) by Barbalat’s lemma [30], [176], it can be established with asymptotic stability of \( e \) (i.e. \( \lim_{t \to \infty} e = 0 \)). This result of asymptotic stability can be extended specifically to lateral and longitudinal cases, and consequently the lateral and longitudinal errors are asymptotically stable, i.e.
\[ \lim_{t \to \infty} e_x, e_y = 0. \quad (6.32) \]

This asymptotic stability result as provided by (6.32) proves that when lateral and longitudinal cases are considered at the same time, with the above arguments, the
system of errors can be proved to be *asymptotically stable*. Therefore, this result logically proves the integrated lateral and longitudinal control system is *asymptotically stable*. This statement completes the proof of stability for the robust multiple-adaptive fuzzy system with switching.

**Remark 6.3:** The Lyapunov candidate as provided in (6.8) satisfies the conditions
i) $V$ is at least $C^1$
ii) $V(0) = 0$ and $V(x(t)) > 0$ for $x(t) \neq 0$ and
iii) $\|x(t)\| \to \infty \implies V(x(t)) \to \infty$.
($x(t)$ denotes a selected time-dependent state of the system in this remark). □

The following definition is provided to formerly identify the variable vector spaces and thereby to define the optimal parameter set.

**Definition 6.2:** The compact parameter spaces for $\theta_j$ and $\bar{x}$ variables can be defined. Let $M_{\theta}(>0) \in \mathbb{R}$ and $M_x(>0) \in \mathbb{R}$ be finite, and it provide

$$\Omega_{\theta_j} = \left\{ \theta_j \in \mathbb{R} : \|\theta_j\| \leq M_{\theta} \right\},$$

$$\Omega_x = \left\{ \bar{x} \in \mathbb{R} : \|\bar{x}\| \leq M_x \right\}.$$ 

Thereby the optimal parameter set for each $j$th adaptive fuzzy controller where $j = 1, \ldots, n$, can be formally defined as

$$\theta_j^* = \arg \min_{\theta_j \in \Omega_j} \left\{ \sup_{x \in \Omega_x} \left| \nu^* - \psi_j^T \dot{\theta}_j \right| \right\},$$

(6.33)

where $\nu^*$ is the optimal adaptive fuzzy function output, and $\nu^* \in \mathbb{R}$. $\bar{x}$ is the generic input variable to each fuzzy system. □

**Remark 6.4:** The definition of $\theta_j^*$ is only for theoretical reasons, and therefore it is not required for implementation of the controller. □

The following features can be observed with regard to the switching scheme established for selecting the best adaptive fuzzy controller at a time. Since all adaptive fuzzy controller models have been proved as asymptotically stable, the switching scheme is not critical [116]. Since the overall system is ensured with finite switching, it is guaranteed to prevent from chattering [22]. Since there is a minimum timing gap
between every consecutive switching in the sequence, it can be called a permissible switching scheme [22]. These permissible switching schemes prevent infinite switching. The dynamics of the system during the switching period can be analysed as follows. In a typical switching setup when the controller switches from one controller to a different controller based on the provided conditions, there is a natural tendency to increase undesired transients or ‘bumps’ in the control output signal [181]. When the switching scheme of this research is considered while the adaptive fuzzy controllers are switched between, the PD module remains as the ‘coarse-tuning controller’ every time. What is really switched between is the module that addresses fine-tuning, i.e. adaptive fuzzy controller. In other words, the switched controllers occur in the ‘secondary-layer’, i.e. adaptive fuzzy controllers, while the ‘primary-layer’, i.e. PD module, remains in control every time. This is in contrast to that occurs in MMST method where the main controllers operate in the ‘primary-layer’ are switched [31]. Based on these facts it can emphatically say that the transients or ‘bumps’ that occur in the switching systems in this research are far less in comparison to that in the MMST method. In other words, the switching controllers proposed in this research can be said to be much closer to ‘bump-less’ switching controllers than that use the MMST method [182].
6.1.4 Simulation Studies, Results and Discussion for ‘Two’ Adaptive Fuzzy Controllers in the Bank (Multiple-Adaptive Fuzzy Controller with Switching)

In this simulation, the developed robust multiple adaptive fuzzy controller with switching was included with ‘two’ adaptive fuzzy controllers in the bank for each longitudinal and lateral controller.

(i) Simulation Setup

The simulation setup for this section was similar to the one provided in Chapter 4 for single-adaptive fuzzy controller system, except for the facts otherwise stated in this section. The simulation setup included veDYNA® setup, simulation profile details and other settings.

In this simulation, the multiple-adaptive fuzzy controller with switching was included with two adaptive fuzzy controllers in the bank. Therefore, the adaptive gains were taken as $\gamma_{1} = 0.03, \gamma_{2} = 0.04$ for longitudinal, and $\gamma_{1} = 0.01, \gamma_{2} = 0.03$ for lateral cases, for each adaptive fuzzy controller in the bank. This change in the range in adaptive gains is due to the fact that lateral dynamics changes at comparatively low rate than the longitudinal.

(ii) Simulation Results and Discussion

In the following figures depicting simulation results, each performance parameter of the multiple-adaptive fuzzy controller with switching (solid blue line) is compared against that of the single-adaptive fuzzy controller (dotted red line).
(a) Normal Cruising Conditions: Multiple-Adaptive Fuzzy Control with Switching (2-AFCs)

The following simulation results illustrate the performance of the developed multiple-adaptive fuzzy controller (with two adaptive controllers in the bank) with switching, as against the performance of single-adaptive fuzzy controller, under normal cruising conditions without the presence of any external disturbances.

The developed multiple-adaptive fuzzy controller with switching shows good performance with lateral tracking, as shown in Fig. 6.3. It shows equal longitudinal tracking capacity, as shown in Fig. 6.2, to the single adaptive fuzzy controller.

![Fig. 6.2 Longitudinal error: multiple-adaptive fuzzy with switching (2-AFCs in the bank) (RMS: 3.16) vs. single-adaptive fuzzy (RMS: 3.158)](image)

![Fig. 6.3 Lateral error: multiple-adaptive fuzzy with switching (2-AFCs in the bank) (RMS: 0.00205) vs. single-adaptive fuzzy (RMS: 0.003837)](image)

When it comes to variation of pitch angle, the two controllers performed similarly as shown in Fig. 6.4 below.
The variations of roll angle, roll rate and side-slip angle, as shown in Figs. 6.5–6.7, with the multiple-adaptive fuzzy controller follow a similar pattern to that with the single-adaptive fuzzy controller. The clear implication is that the multiple-adaptive fuzzy controller can ensure lateral stability in a similar fashion to the single-adaptive fuzzy controller.
Since variations of lateral acceleration, as shown in Fig. 6.8, are well within the limits of 4 [m/s$^2$], it suggests that the multiple-adaptive fuzzy controller with switching fulfils the design requirements of the passenger vehicles [178].

Overall, under normal cruising conditions, the multiple-adaptive fuzzy controller with switching has ensured that it follows good tracking performance in lateral and longitudinal aspects, in comparison to the single-adaptive fuzzy controller. In terms of lateral stability, it can be said that the multiple-adaptive fuzzy controller with switching performs as well as the single-adaptive fuzzy controller.

### (b) External Disturbances: Multiple-Adaptive Fuzzy Control with Switching (2-AFCs)

In the following simulations, the developed multiple-adaptive fuzzy controller with switching was tested against the single-adaptive fuzzy controller under a number of external disturbances. These disturbances included addition of an un-symmetrical load mass, presence of crosswind forces, and change of tyre-road friction.
1. Unsymmetrical Load Mass

The following simulation was based on comparison of the results between multiple-adaptive fuzzy controller with switching and the single-adaptive fuzzy controller, when there was an un-symmetrical load-mass on the vehicle as per the details described in Section 3.2.18.

Fig. 6.9 Longitudinal error: multiple-adaptive fuzzy with switching (2-AFCs in the bank) (RMS: 4.323) vs. single-adaptive fuzzy (RMS: 4.391)

Fig. 6.10 Lateral error: multiple-adaptive fuzzy with switching (2-AFCs in the bank) (RMS: 0.002768) vs. single-adaptive fuzzy (RMS: 0.003957)
Fig. 6.11 Pitch angle: multiple-adaptive fuzzy with switching (2-AFCs in the bank) (RMS: 0.01972) vs. single-adaptive fuzzy (RMS: 0.01998)

Fig. 6.12 Roll angle: multiple-adaptive fuzzy with switching (2-AFCs in the bank) (RMS: 0.01766) vs. single-adaptive fuzzy (RMS: 0.01766)

Fig. 6.13 Roll rate: multiple-adaptive fuzzy with switching (2-AFCs in the bank) (RMS: 0.01173) vs. single-adaptive fuzzy (RMS: 0.015)
The results of this section follow a similar pattern to the situation previously described, i.e., under normal cruising conditions. Therefore, it can be said that in the presence of unsymmetrical load mass, the multiple-adaptive fuzzy controller with switching provides good tracking performance in lateral and longitudinal aspects, as shown in Figs. 6.9–6.10, in comparison to the single-adaptive fuzzy controller. Considering lateral stability, as suggested by roll angle, roll rate and side-slip angle, as shown in Figs. 6.12–6.14, it can be said that the multiple-adaptive fuzzy controller with switching performs as well as the single-adaptive-fuzzy controller, if not better at some points.

As shown in Fig. 6.15, the variations of lateral acceleration do not go beyond 4 [m/s²], therefore, the multiple-adaptive fuzzy controller with switching has managed to contain lateral acceleration within acceptable limits as far as the design characteristics of passenger vehicles are concerned [178].
2. Crosswind Effects

The following simulation results were obtained when there was a presence of a continuous crosswind with a speed of 50 [m/s] through the global y-direction. More details of this arrangement are described in Section 3.2.18.

![Fig. 6.16 Longitudinal error: multiple-adaptive fuzzy with switching (2-AFCs in the bank) (RMS: 3.744) vs. single-adaptive fuzzy (RMS: 3.742)](image1)

![Fig. 6.17 Lateral error: multiple-adaptive fuzzy with switching (2-AFCs in the bank) (RMS: 0.002945) vs. single-adaptive fuzzy (RMS: 0.005914)](image2)

![Fig. 6.18 Pitch angle: multiple-adaptive fuzzy with switching (2-AFCs in the bank) (RMS: 0.01143) vs. single-adaptive fuzzy (RMS: 0.01142)](image3)
Fig. 6.19 Roll angle: multiple-adaptive fuzzy with switching (2-AFCs in the bank) (RMS: 0.01398) vs. single-adaptive fuzzy (RMS: 0.01396)

Fig. 6.20 Roll rate: multiple-adaptive fuzzy with switching (2-AFCs in the bank) (RMS: 0.0143) vs. single-adaptive fuzzy (RMS: 0.02222)

Fig. 6.21 Side-slip angle: multiple-adaptive fuzzy with switching (2-AFCs in the bank) (RMS: 0.01442) vs. single-adaptive fuzzy (RMS: 0.01438)
The results of this section can be summarised as follows. It can be said that in the presence of crosswind effects, the multiple-adaptive fuzzy controller with switching provides good tracking performance in lateral and longitudinal aspects, as shown in Figs. 6.16–6.17, in comparison to the single-adaptive fuzzy controller. It can be further said that lateral tracking has been better while longitudinal tracking follows a similar level to that of the single-adaptive fuzzy controller.

As suggested by roll angle, roll rate and side-slip angle, as shown in Figs. 6.19–6.21, it can be said that multiple-adaptive fuzzy controller with switching has ensured a good level of lateral stability that is on par with the single-adaptive fuzzy controller.

As shown in Fig. 6.22, the variations of lateral acceleration are well within 4 \([\text{m/s}^2]\). Therefore, the multiple-adaptive fuzzy controller with switching has managed to limit lateral acceleration to within specified design characteristics for passenger vehicles [178].

### 3. Tyre-Road Friction Change

The following simulation results show the performance of the multiple-adaptive fuzzy controller verses the single-adaptive fuzzy controller when there was a change of tyre-road friction in the range 100-200 [m], on the highway.
Fig. 6.23 Longitudinal error: multiple-adaptive fuzzy with switching (2-AFCs in the bank) (RMS: 3.161) vs. single-adaptive fuzzy (RMS: 3.158)

Fig. 6.24 Lateral error: multiple-adaptive fuzzy with switching (2-AFCs in the bank) (RMS: 0.002573) vs. single-adaptive fuzzy (RMS: 0.003996)

Fig. 6.25 Pitch angle: multiple-adaptive fuzzy with switching (2-AFCs in the bank) (RMS: 0.01143) vs. single-adaptive fuzzy (RMS: 0.01144)
Fig. 6.26 Roll angle: multiple-adaptive fuzzy with switching (2-AFCs in the bank) (RMS: 0.01092) vs. single-adaptive fuzzy (RMS: 0.01085)

Fig. 6.27 Roll rate: multiple-adaptive fuzzy with switching (2-AFCs in the bank) (RMS: 0.009368) vs. single-adaptive fuzzy (RMS: 0.01255)

Fig. 6.28 Side-slip angle: multiple-adaptive fuzzy with switching (2-AFCs in the bank) (RMS: 0.007194) vs. single-adaptive fuzzy (RMS: 0.007268)
Overall, quite similar to the last simulation, the results of this section can be described as follows. It can be said that in the presence of tyre-road friction change, the multiple-adaptive fuzzy controller with switching provides good tracking performance in the lateral aspect, as shown in Fig. 6.24, in comparison to the single-adaptive fuzzy controller. With regard to longitudinal tracking, the error curves are almost identical for both controllers, as shown in Fig. 6.23.

The multiple-adaptive fuzzy controller with switching has ensured a better level of lateral stability, as suggested by roll angle, roll rate and side-slip angle, as shown in Figs. 6.26–6.28, to that with the single-adaptive fuzzy controller. Such a conclusion can be drawn when comparing the results to that of the single-adaptive fuzzy controller.

The variations of lateral acceleration are well within 4 \([m/s^2]\), as shown in Fig. 6.29. Therefore, it can be concluded that the multiple-adaptive fuzzy controller with switching has managed to limit lateral acceleration within the design characteristics of passenger vehicles [178].

(c) Failure Modes (non-catastrophic subsystem failures): Multiple-Adaptive Fuzzy Control with Switching (2-AFCs)

The following simulation results show the performance of the multiple-adaptive fuzzy controller with switching against that of the single-adaptive fuzzy controller when some failures in vehicle components occur. The failures considered in this simulation included a flat-tyre in the front-left tyre, and a 90% drop of pressure in the brake line leading to front-left wheel brake cylinder of the vehicle.
1. Flat-Tyre

Fig. 6.30 Longitudinal error: multiple-adaptive fuzzy with switching (2-AFCs in the bank) (RMS: 26.31) vs. single-adaptive fuzzy (RMS: 26.08)

Fig. 6.31 Lateral error: multiple-adaptive fuzzy with switching (2-AFCs in the bank) (RMS: 0.01017) vs. single-adaptive fuzzy (RMS: 0.02753)

Fig. 6.32 Pitch angle: multiple-adaptive fuzzy with switching (2-AFCs in the bank) (RMS: 0.009741) vs. single-adaptive fuzzy (RMS: 0.01052)
Fig. 6.33 Roll angle: multiple-adaptive fuzzy with switching (2-AFCs in the bank) (RMS: 0.02216) vs. single-adaptive fuzzy (RMS: 0.02225)

Fig. 6.34 Roll rate: multiple-adaptive fuzzy with switching (2-AFCs in the bank) (RMS: 0.01546) vs. single-adaptive fuzzy (RMS: 0.0212)

Fig. 6.35 Side-slip angle: multiple-adaptive fuzzy with switching (2-AFCs in the bank) (RMS: 0.006872) vs. single-adaptive fuzzy (RMS: 0.007039)
Fig. 6.36 Lateral acceleration: multiple-adaptive fuzzy with switching (2-AFCs in the bank) (RMS: 1.232) vs. single-adaptive fuzzy (RMS: 1.248)

Overall, the results of the simulation in the presence of a flat-tyre event, a subsystem failure case, can be described as follows. As far as lateral tracking is concerned, it can be said that the multiple-adaptive fuzzy controller with switching provides better results compared to the single-adaptive fuzzy controller. This is shown in Fig. 6.31 in comparison to the single-adaptive fuzzy controller. With regard to longitudinal tracking, the error curves are quite similar throughout the time-span, as shown in Fig. 6.30.

Pitch angle variation, an indication of longitudinal stability provides some improved results with the multiple-adaptive fuzzy controller with switching. In comparison with the variations of roll angle, roll rate and side-slip angle, as shown in Figs. 6.33–6.35, it is shown that the multiple-adaptive switching controller has ensured lower values throughout, especially during transients. This suggests that the multiple-adaptive fuzzy controller with switching has achieved better lateral stability than its counterpart.

The variations of lateral acceleration are well within 4 [m/s²], as shown in Fig. 6.36. Therefore, the multiple-adaptive fuzzy controller with switching has managed to limit lateral acceleration to within the design requirements of passenger vehicles [178].

2. Brake Cylinder Defect

A subsystem failure with the front-left wheel brake cylinder being defective due to a 90% pressure drop in the brake line was investigated in this section. More details on this simulation setup are discussed in Section 3.2.18.
Fig. 6.37 Longitudinal error: multiple-adaptive fuzzy with switching (2-AFCs in the bank) (RMS: 3.156) vs. single-adaptive fuzzy (RMS: 3.159)

Fig. 6.38 Lateral error: multiple-adaptive fuzzy with switching (2-AFCs in the bank) (RMS: 0.02007) vs. single-adaptive fuzzy (RMS: 0.03453)

Fig. 6.39 Pitch angle: multiple-adaptive fuzzy with switching (2-AFCs in the bank) (RMS: 0.01483) vs. single-adaptive fuzzy (RMS: 0.01583)
Fig. 6.40 Roll angle: multiple-adaptive fuzzy with switching (2-AFCs in the bank) (RMS: 0.0109) vs. single-adaptive fuzzy (RMS: 0.01082)

Fig. 6.41 Roll rate: multiple-adaptive fuzzy with switching (2-AFCs in the bank) (RMS: 0.04623) vs. single-adaptive fuzzy (RMS: 0.04617)

Fig. 6.42 Side-slip angle: multiple-adaptive fuzzy with switching (2-AFCs in the bank) (RMS: 0.009507) vs. single-adaptive fuzzy (RMS: 0.01052)
Overall, the results of the simulation of a defective front-left brake cylinder, a subsystem failure case, are described as follows. It is seen that lateral tracking has been better, as shown in Fig. 6.38, with regard to the multiple-adaptive fuzzy controller. As far as longitudinal tracking is concerned, the performance of the two controllers is similar, as shown in Fig. 6.37.

Pitch angle variation, an indication of longitudinal stability, shows some improved response with the multiple-adaptive fuzzy controller with switching. Lateral stability with the multiple-adaptive fuzzy controller has achieved an improved level in comparison to that with the single-adaptive fuzzy controller. This conclusion is well supported by the behaviour of roll angle, roll rate and side-slip angle, as shown in Figs. 6.40–6.42, where it is shown that the multiple-adaptive switching controller has ensured lower values throughout, especially during transients.

The fact that variations of lateral acceleration are well within 4 [m/s$^2$], as shown in Fig. 6.43, suggests that the multiple-adaptive fuzzy controller with switching has managed to limit lateral acceleration to within acceptable design requirements for passenger vehicles [178].

Considering the overall performance of the multiple-adaptive fuzzy controller, the level of lateral tracking has been better in comparison to that of the single-adaptive fuzzy controller. The same cannot be said of longitudinal tracking, though. This is because longitudinal curves, almost in every case, show they are almost identical for both types of controllers. Another fact is that lateral stability has been maintained at a good level by the multiple-adaptive fuzzy controller in comparison to that by the single-adaptive fuzzy controller. It has also ensured that lateral acceleration is kept within
acceptable limits. Therefore, as a conclusion it can be said that the multiple-adaptive fuzzy controller with switching has maintained good lateral tracking with ensured lateral stability.

The summary of performance of the multiple-adaptive fuzzy controller with switching with ‘two’ adaptive fuzzy controllers in the bank against that of the single-adaptive fuzzy controller is provided in TABLE 6.1 below.

TABLE 6.1 Comparison of performance of fuzzy multiple-adaptive fuzzy controller (2 AFCs) against single-adaptive fuzzy controller

<table>
<thead>
<tr>
<th>Performance</th>
<th>Simulation</th>
<th>Longitudinal error</th>
<th>Lateral error</th>
<th>Pitch angle</th>
<th>Roll angle</th>
<th>Roll rate</th>
<th>Side-slip angle</th>
<th>Lateral acceleration</th>
</tr>
</thead>
<tbody>
<tr>
<td>normal cruising</td>
<td>similar</td>
<td>significantly</td>
<td>similar</td>
<td>similar</td>
<td>similar</td>
<td>similar</td>
<td>similar</td>
<td>similar</td>
</tr>
<tr>
<td>un-symmetrical loading</td>
<td>slightly</td>
<td>improved</td>
<td>similar</td>
<td>similar</td>
<td>similar</td>
<td>similar</td>
<td>similar</td>
<td>similar</td>
</tr>
<tr>
<td>crosswinds</td>
<td>similar</td>
<td>significantly</td>
<td>similar</td>
<td>similar</td>
<td>slightly</td>
<td>similar</td>
<td>similar</td>
<td>similar</td>
</tr>
<tr>
<td>tyre-road friction</td>
<td>similar</td>
<td>significantly</td>
<td>similar</td>
<td>similar</td>
<td>slightly</td>
<td>similar</td>
<td>similar</td>
<td>similar</td>
</tr>
<tr>
<td>flat-tyre</td>
<td>similar</td>
<td>significantly</td>
<td>slightly</td>
<td>improved</td>
<td>slightly</td>
<td>improved</td>
<td>similar</td>
<td>similar</td>
</tr>
<tr>
<td>defective brake cylinder</td>
<td>similar</td>
<td>improved</td>
<td>similar</td>
<td>similar</td>
<td></td>
<td></td>
<td>slightly</td>
<td>improved</td>
</tr>
</tbody>
</table>
6.1.5 Simulation Studies, Results and Discussion for ‘Four’ Adaptive Fuzzy Controllers in the Bank (Multiple-Adaptive Fuzzy Controller with Switching)

In this simulation, the developed robust multiple-adaptive fuzzy controller with switching, was included with ‘four’ adaptive fuzzy controllers in the bank for each longitudinal and lateral controller.

(i) Simulation Setup

The simulation setup for this section was similar to the one described in Chapter 4 for the single-adaptive fuzzy controller system, except for facts otherwise stated in this section. The simulation setup included veDYNA® setup, simulation profile details and other settings.

In this simulation, multiple-adaptive fuzzy controller with switching was included with four adaptive fuzzy controllers in the bank. Therefore, the adaptive gains were taken as, $\gamma_1 = 0.02$, $\gamma_2 = 0.03$, $\gamma_3 = 0.04$, and $\gamma_4 = 0.05$ for longitudinal case, and $\gamma_1 = 0.005$, $\gamma_2 = 0.01$, $\gamma_3 = 0.02$, and $\gamma_4 = 0.03$ for lateral case, in the multiple-adaptive fuzzy controller switching setup. This change in the range in adaptive gains is due to the fact that lateral dynamics changes at a comparatively low rate than the longitudinal.

(ii) Simulation Results and Discussion

In the following figures depicting simulation results, each performance parameter of the multiple-adaptive fuzzy controller (solid blue line) is compared against that of the single-adaptive fuzzy controller (dotted red line).

(a) Normal Cruising Conditions: Multiple-Adaptive Fuzzy Control with Switching (4-AFCs)

The following simulation results describe the performance of the developed multiple-adaptive fuzzy controller against the results of the single-adaptive fuzzy
controller under normal cruising conditions without the presence of any external disturbances.

Fig. 6.44 Longitudinal error: multiple-adaptive fuzzy with switching (4-AFCs in the bank) (RMS: 3.07) vs. single-adaptive fuzzy (RMS: 3.158)

Fig. 6.45 Lateral error: multiple-adaptive fuzzy with switching (4-AFCs in the bank) (RMS: 0.002759) vs. single-adaptive fuzzy (RMS: 0.003837)

Fig. 6.46 Pitch angle: multiple-adaptive fuzzy with switching (4-AFCs in the bank) (RMS: 0.01214) vs. single-adaptive fuzzy (RMS: 0.01195)
Fig. 6.47 Roll angle: multiple-adaptive fuzzy with switching (4-AFCs in the bank) (RMS: 0.01085) vs. single-adaptive fuzzy (RMS: 0.01086)

Fig. 6.48 Roll rate: multiple-adaptive fuzzy with switching (4-AFCs in the bank) (RMS: 0.008191) vs. single-adaptive fuzzy (RMS: 0.01)

Fig. 6.49 Side-slip angle: multiple-adaptive fuzzy with switching (4-AFCs in the bank) (RMS: 0.007226) vs. single-adaptive fuzzy (RMS: 0.007206)
Overall, the simulation results of this section can be detailed as follows. It can be said that under normal cruising conditions, the multiple-adaptive fuzzy controller with switching shows good lateral tracking performance, as shown in Fig. 6.45, in comparison to the single-adaptive fuzzy controller. It is important to note that this lateral tracking performance is slightly inferior to the performance under similar normal cruising conditions, displayed by the multiple-adaptive fuzzy controller with ‘two’ adaptive fuzzy controllers in the bank (2-AFCs), explained in Section 6.1.

With regard to longitudinal tracking, there is a slight improvement with the multiple-adaptive fuzzy controller in comparison to that with the single-adaptive fuzzy controller. This is shown in Fig. 6.44.

Pitch angle variation, a measure of longitudinal stability, stands at a competitive level compared to the single-adaptive fuzzy controller, as shown in Fig. 6.46.

The multiple-adaptive fuzzy controller with switching has ensured a good standing of lateral stability, as shown with variations of roll angle, roll rate and side-slip angle, provided in Figs. 6.47–6.49, compared to the single-adaptive fuzzy controller. This conclusion has been drawn when considered the results as shown by the single-adaptive fuzzy controller.

Importantly, the variations of lateral acceleration are well within $4 \text{ m/s}^2$, as shown in Fig. 6.50. Therefore, the multiple-adaptive fuzzy controller with switching has ensured that lateral acceleration is well within the specified design requirements of passenger vehicles [178].
(b) External Disturbances: Multiple-Adaptive Fuzzy Control with Switching (4-AFCs)

In the following set of simulations, the developed multiple-adaptive fuzzy controller with switching was tested against the single-adaptive fuzzy controller under a number of external disturbances. These disturbances included addition of an unsymmetrical load mass, presence of crosswind forces, and change of tyre-road friction.

1. Unsymmetrical Load Mass

The following simulation was based on comparison of the results between the multiple-adaptive fuzzy controller and the single-adaptive fuzzy controller, when there was an un-symmetrical load-mass on the vehicle as per the details described in Section 3.2.18.

![Longitudinal error graph](image1)

Fig. 6.51 Longitudinal error: multiple-adaptive fuzzy with switching (4-AFCs in the bank) (RMS: 3.587) vs. single-adaptive fuzzy (RMS: 4.391)

![Lateral error graph](image2)

Fig. 6.52 Lateral error: multiple-adaptive fuzzy with switching (4-AFCs in the bank) (RMS: 0.002848) vs. single-adaptive fuzzy (RMS: 0.003957)
Fig. 6.53 Pitch angle: multiple-adaptive fuzzy with switching (4-AFCs in the bank) (RMS: 0.01996) vs. single-adaptive fuzzy (RMS: 0.01998)

Fig. 6.54 Roll angle: multiple-adaptive fuzzy with switching (4-AFCs in the bank) (RMS: 0.01766) vs. single-adaptive fuzzy (RMS: 0.01766)

Fig. 6.55 Roll rate: multiple-adaptive fuzzy with switching (4-AFCs in the bank) (RMS: 0.01368) vs. single-adaptive fuzzy (RMS: 0.015)
Overall, the simulation results of this section can be detailed as follows. With an unsymmetrical load mass on the vehicle, the multiple-adaptive fuzzy controller with switching shows better lateral tracking performance, as shown in Fig. 6.52, in comparison to the single-adaptive fuzzy controller. But, it is important to note that the lateral tracking performance from the multiple-adaptive fuzzy with switching with ‘four’ adaptive fuzzy controllers in the bank (4-AFCs) (i.e. with regard to this simulation performance), is somewhat marred with transients with slightly high amplitudes in comparison to the performance of the multiple-adaptive fuzzy controller with ‘two’ adaptive fuzzy controllers in the bank (2-AFCs), as explained in Section 6.1.4 under similar conditions (see Fig. 6.10).

With regard to longitudinal tracking, there is an improvement in performance with the multiple-adaptive fuzzy controller with switching in comparison to that with the single-adaptive fuzzy controller, as shown in Fig. 6.51. This is a better performance compared with the performance of the multiple-adaptive fuzzy controller with switching with ‘two’ adaptive fuzzy controllers in the bank (2-AFCs). Considering the rest of the
performances of the multiple-adaptive fuzzy with switching, it can be said that lateral stability variation is acceptable in comparison to that of the single-adaptive fuzzy controller. Also the lateral acceleration performance stands at an acceptable level that is within the design specifications of vehicles.

2. Crosswind Effects

The following simulation results were obtained when there was a presence of a continuous crosswind with a speed of 50 [m/s] from the global y-direction. More details of this arrangement are provided in Section 3.2.18.

Fig. 6.58 Longitudinal error: multiple-adaptive fuzzy with switching (4-AFCs in the bank) (RMS: 3.234) vs. single-adaptive fuzzy (RMS: 3.742)

Fig. 6.59 Lateral error: multiple-adaptive fuzzy with switching (4-AFCs in the bank) (RMS: 0.003007) vs. single-adaptive fuzzy (RMS: 0.005914)
Fig. 6.60 Pitch angle: multiple-adaptive fuzzy with switching (4-AFCs in the bank) (RMS: 0.011) vs. single-adaptive fuzzy (RMS: 0.01142)

Fig. 6.61 Roll angle: multiple-adaptive fuzzy with switching (4-AFCs in the bank) (RMS: 0.01402) vs. single-adaptive fuzzy (RMS: 0.01396)

Fig. 6.62 Roll rate: multiple-adaptive fuzzy with switching (4-AFCs in the bank) (RMS: 0.01832) vs. single-adaptive fuzzy (RMS: 0.02222)
Overall, the simulation results of this section can be described as follows. With crosswind effects being present throughout, the multiple-adaptive fuzzy controller with switching performed better in lateral tracking, as shown in Fig. 6.59, in comparison to the single-adaptive fuzzy controller.

With regard to longitudinal tracking, there is an improvement in performance with the multiple-adaptive fuzzy controller with switching in comparison to that with the single-adaptive fuzzy controller, as shown in Fig. 6.56. This is a better performance compared with the performance of the multiple-adaptive fuzzy controller with switching with ‘two’ adaptive fuzzy controllers in the bank (2-AFCs).

Considering the rest of the performances of the multiple-adaptive fuzzy with switching, it can be repeated as follows as provided in the last description. The lateral stability variation is acceptable in comparison to that of the single-adaptive fuzzy controller. Also the lateral acceleration performance stands at an acceptable level that is within the design specifications of vehicles.
3. Tyre-Road Friction Change

The following simulation results compare the performance of the multiple-adaptive fuzzy controller with that of the single-adaptive fuzzy controller when there is a change of tyre-road friction in the range 100-200 \([\text{m}]\), on the highway.

![Diagram 1](image1.png)

**Fig. 6.65** Longitudinal error: multiple-adaptive fuzzy with switching (4-AFCs in the bank) (RMS: 3.073) vs. single-adaptive fuzzy (RMS: 3.158)

![Diagram 2](image2.png)

**Fig. 6.66** Lateral error: multiple-adaptive fuzzy with switching (4-AFCs in the bank) (RMS: 0.002864) vs. single-adaptive fuzzy (RMS: 0.003996)

![Diagram 3](image3.png)

**Fig. 6.67** Pitch angle: multiple-adaptive fuzzy with switching (4-AFCs in the bank) (RMS: 0.01143) vs. single-adaptive fuzzy (RMS: 0.01144)
Fig. 6.68 Roll angle: multiple-adaptive fuzzy with switching (4-AFCs in the bank) (RMS: 0.01087) vs. single-adaptive fuzzy (RMS: 0.01085)

Fig. 6.69 Roll rate: multiple-adaptive fuzzy with switching (4-AFCs in the bank) (RMS: 0.01416) vs. single-adaptive fuzzy (RMS: 0.01255)

Fig. 6.70 Side-slip angle: multiple-adaptive fuzzy with switching (4-AFCs in the bank) (RMS: 0.007222) vs. single-adaptive fuzzy (RMS: 0.007268)
Overall, the simulation results of this section can be elaborated as follows. With a tyre-road friction change, the multiple-adaptive fuzzy controller with switching shows better lateral tracking performance, as shown in Fig. 6.66, in comparison to the single-adaptive fuzzy controller. As far as lateral tracking performance of the multiple-adaptive fuzzy with switching (4-AFCs) is concerned, it can be considered as somewhat deteriorated with chattering in comparison to the performance of the multiple-adaptive fuzzy with switching (2-AFCs), under similar conditions as described in Section 6.1.4 (see Fig. 6.24).

Again, longitudinal tracking performance with the multiple-adaptive fuzzy controller with switching is better compared to that with the single-adaptive fuzzy controller.

According to the collective performance in the areas of roll angle, roll-rate and side-slip angle, lateral stability variation appears acceptable with the multiple-adaptive fuzzy controller with switching. As far as lateral acceleration is concerned, the multiple-adaptive fuzzy controller with switching can be accepted as performing within the design specifications of passenger vehicles.

(c) Failure Modes: Multiple-Adaptive Fuzzy Control with Switching (4-AFCs)

The following simulation results show the performance of the multiple-adaptive fuzzy with switching controller against the single-adaptive fuzzy controller when some failures occur in the vehicle components. The failures considered in this simulation included a flat-tyre in the front-left tyre, and a 90% drop of pressure in the brake line of the front-left wheel cylinder of the vehicle.
1. Flat-Tyre

The following simulation results were obtained when an event of front-left tyre becoming flat occurred at 10[s] after starting the simulation. The details of the simulation arrangement are explained in Section 3.2.18.

Fig. 6.72 Longitudinal error: multiple-adaptive fuzzy with switching (4-AFCs in the bank) (RMS: 24.76) vs. single-adaptive fuzzy (RMS: 26.08)

Fig. 6.73 Lateral error: multiple-adaptive fuzzy with switching (4-AFCs in the bank) (RMS: 0.01137) vs. single-adaptive fuzzy (RMS: 0.02753)

Fig. 6.74 Pitch angle: multiple-adaptive fuzzy with switching (4-AFCs in the bank) (RMS: 0.01078) vs. single-adaptive fuzzy (RMS: 0.01052)
Fig. 6.75 Roll angle: multiple-adaptive fuzzy with switching (4-AFCs in the bank) (RMS: 0.02222) vs. single-adaptive fuzzy (RMS: 0.02225)

Fig. 6.76 Roll rate: multiple-adaptive fuzzy with switching (4-AFCs in the bank) (RMS: 0.03229) vs. single-adaptive fuzzy (RMS: 0.0212)

Fig. 6.77 Side-slip angle: multiple-adaptive fuzzy with switching (4-AFCs in the bank) (RMS: 0.007065) vs. single-adaptive fuzzy (RMS: 0.007039)
Overall, the simulation results of this section can be detailed as follows. With the case of flat-tyre, the multiple-adaptive fuzzy controller with switching shows better lateral tracking performance, as shown in Fig. 6.73, in comparison to the single-adaptive fuzzy controller. The lateral tracking performance of the multiple-adaptive fuzzy with switching, i.e., with 4-AFCs, becomes slightly worse around 50–53 [s] on the time-scale, and the effect can be seen on most parameters affected by transverse motion of the vehicle. Such a deviation is clearly on the negative side of performance in comparison to that of the multiple-adaptive fuzzy controller with ‘two’ adaptive fuzzy controllers in the bank (2-AFCs) described in Section 6.1 (see Fig. 6.31).

With regard to longitudinal tracking, it provides some improved results with the multiple-adaptive fuzzy controller in comparison to the single-adaptive fuzzy controller, as shown in Fig. 6.72.

Pitch angle is at an improved level compared to the single-adaptive fuzzy controller, as shown in Fig. 6.74.

Lateral stability variation also appears acceptable with the multiple-adaptive fuzzy controller with switching. Conclusions on lateral stability have been inferred from the variations of roll angle, roll rate and side-slip angle as illustrated in Figs. 6.75–6.77. Such a conclusion can be drawn after comparing the results with that of the single-adaptive fuzzy controller.

Once again, the variation of lateral acceleration falls within 4 [m/s²], as shown in Fig. 6.78. Therefore, the multiple-adaptive fuzzy controller ensures that lateral acceleration is maintained at an acceptable level as required by the design specifications of passenger vehicles [178].
2. Brake Cylinder Defect

The investigation of a subsystem failure of front-left wheel-brake cylinder being defective due to a 90% pressure drop in the brake line is studied in this section. More details on this simulation setup are discussed in Section 3.2.18.

Fig. 6.79 Longitudinal error: multiple-adaptive fuzzy with switching (4-AFCs in the bank) (RMS: 3.062) vs. single-adaptive fuzzy (RMS: 3.159)

Fig. 6.80 Lateral error: multiple-adaptive fuzzy with switching (4-AFCs in the bank) (RMS: 0.02255) vs. single-adaptive fuzzy (RMS: 0.03453)

Fig. 6.81 Pitch angle: multiple-adaptive fuzzy with switching (4-AFCs in the bank) (RMS: 0.01479) vs. single-adaptive fuzzy (RMS: 0.01583)
Fig. 6.82 Roll angle: multiple-adaptive fuzzy with switching (4-AFCs in the bank) (RMS: 0.01128) vs. single-adaptive fuzzy (RMS: 0.01082)

Fig. 6.83 Roll rate: multiple-adaptive fuzzy with switching (4-AFCs in the bank) (RMS: 0.07434) vs. single-adaptive fuzzy (RMS: 0.04617)

Fig. 6.84 Side-slip angle: multiple-adaptive fuzzy with switching (4-AFCs in the bank) (RMS: 0.009719) vs. single-adaptive fuzzy (RMS: 0.01052)
Overall, the simulation results of this section can be discussed as follows. With a brake cylinder defect, the multiple-adaptive fuzzy controller with switching shows some improved lateral tracking performance, as shown in Fig. 6.80, in comparison to the single-adaptive fuzzy controller.

With regard to longitudinal tracking, there is some improvement with the multiple-adaptive fuzzy controller in comparison to the single-adaptive fuzzy controller, as shown in Fig. 6.79.

The variation of pitch angle again stands at an acceptable level when compared to the single-adaptive fuzzy controller, as shown in Fig. 6.81.

Once again, lateral stability variation appears acceptable with the multiple-adaptive fuzzy controller with switching. This result is inferred from the variations of roll angle, roll rate and side-slip angle shown in Figs. 6.82–6.84. Such a conclusion can be drawn when the results are compared to that shown by the single-adaptive fuzzy controller.

As far as lateral acceleration is concerned, it stands close to 4 [m/s$^2$] with the multiple-adaptive fuzzy controller with switching as shown in Fig. 6.85. This can be compared against the performances of the single-adaptive fuzzy controller, as shown in the same figure. Therefore, it can be said that the multiple-adaptive fuzzy controller performs exceptionally well in relation to the design requirements of passenger vehicles [178].

Considering the overall performance of the multiple-adaptive fuzzy controller with switching, the level of lateral tracking has been good in comparison to that of the single-adaptive fuzzy controller. But, it has to be mentioned that the level of
achievement with lateral tracking is somewhat lower in comparison to the results in Section 6.1.4 for the multiple-adaptive fuzzy controller with switching having ‘two’ adaptive fuzzy controllers in the bank (2-AFCs).

In contrast to the results obtained in Section 6.1, there have been some improved results with longitudinal tracking with regard to the multiple-adaptive fuzzy controller with switching (4-AFCs), though.

Another feature is that lateral stability has been maintained at an acceptable level by the multiple-adaptive fuzzy controller. According to the design requirements, the multiple-adaptive fuzzy controller with switching has ensured that lateral acceleration is kept within acceptable limits.

The summary of performances of the multiple-adaptive fuzzy controller with switching with ‘four’ adaptive fuzzy controllers in the bank against that of single-adaptive fuzzy controller is provided in TABLE 6.2 below.

TABLE 6.2 Comparison of performance of fuzzy multiple-adaptive fuzzy controller (4 AFCs) against single-adaptive fuzzy controller

<table>
<thead>
<tr>
<th>Simulation</th>
<th>Performance</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>longitudinal error</td>
</tr>
<tr>
<td>normal cruising</td>
<td>slightly improved</td>
</tr>
<tr>
<td>un-symmetrical loading</td>
<td>slightly improved</td>
</tr>
<tr>
<td>crosswinds</td>
<td>slightly improved</td>
</tr>
<tr>
<td>tyre-road friction</td>
<td>slightly improved</td>
</tr>
<tr>
<td>flat-tyre</td>
<td>improved</td>
</tr>
<tr>
<td>defective brake cylinder</td>
<td>improved</td>
</tr>
</tbody>
</table>
6.2 Chapter Summary

In this chapter, the development of a multiple-model/multiple-adaptive fuzzy controller based on hard-switching (on/off switching) is described. A detailed robust stability analysis is carried out for ensuring asymptotic stability in the developed control system using Lyapunov based KYP lemma.

In the validation of the designed controller, two sets of simulations are performed. In the first, ‘two’ number of adaptive fuzzy controllers (AFCs) are included in the bank of the multiple-adaptive fuzzy controller while in the second, ‘four’ number of adaptive fuzzy controllers are included in the multiple-controller setup. These comprehensive sets of simulations are done on veDYNA®. Each of the simulation includes testing of the controller under a number of external disturbances as well as a couple of failure modes.
This research introduced four novel designs of robust adaptive fuzzy controllers: one controller on single-adaptive fuzzy where the novelty lies only for Lyapunov based stability analysis; two controllers on multiple-adaptive fuzzy with blending; one controller on multiple-adaptive fuzzy with switching.

In this research, all designed controllers were established with stability using a Lyapunov based KYP lemma leading to asymptotic stability at global level. In order to validate the designed controllers, simulation studies were conducted. All these simulations were performed in veDYNA®, which provided an industry-standard simulation platform. The following simulations were carried out to validate the developed controllers in this research: single-adaptive fuzzy controller; multiple-adaptive fuzzy controller with blending; PDC-based multiple-model adaptive fuzzy controller with blending; and multiple-adaptive fuzzy controller with switching (two sets of simulations with ‘two’ and ‘four’ adaptive fuzzy controllers in the bank).

As far as the results of the simulations are concerned, the following conclusions can be drawn.

Where overall results are concerned, MM/MC based adaptive fuzzy controllers have shown exceptional lateral tracking capabilities in comparison to the single-adaptive fuzzy controller in all simulation setups described in this thesis. These performances have been achieved without compromising the lateral stability. Comfort of passengers was also not compromised. Additionally, in longitudinal tracking, multiple-adaptive fuzzy systems have shown improved performance in comparison to the single-adaptive fuzzy controllers.

It is also important to mention that the PD controller module (without any adaptive fuzzy controller in aid) has shown some exceptional results in longitudinal tracking control, especially in comparison to the single-adaptive fuzzy controller. In most of these cases, the single-adaptive fuzzy controller has been slow, and higher in
length of settling time. But, where lateral tracking was concerned, ‘PD only’ controller performed worse.

The following conclusions can be drawn when comparing the groups of controllers developed in this research and described in this thesis.

When the performances of the multiple-adaptive fuzzy controller with blending is compared against the multiple-adaptive fuzzy controller with switching, the overall results suggest that the former is better with lateral tracking, and is also slightly better with longitudinal tracking over its counterpart.

When the PDC based adaptive fuzzy controller is compared against the multiple-adaptive fuzzy controller with blending, the former is slightly better with lateral tracking over the latter. But, this better result of the former is achieved at the expense of lateral stability. Where longitudinal tracking is concerned, the PDC based controller is slightly better than the latter in length of response time.

As far as the number of adaptive fuzzy controllers included in the controller bank is concerned, in multiple-adaptive fuzzy controllers with switching, the results are as follows: MM/MC with two adaptive fuzzy controllers (AFCs) shows exceptional lateral tracking whereas the controller with four AFCs shows a lower degree of lateral tracking. The latter shows results of lateral tracking marred with high-amplitude chattering, to some extent. The former, i.e., controller with two AFCs, as far as longitudinal tracking is concerned, shows less improvement compared to the single-adaptive fuzzy controller. On the other hand, the controller with four AFCs shows considerable improvement in terms of longitudinal tracking. Therefore, a better combination, also with effective tracking, would be to include 2-AFCs in the lateral control setup while including 4-AFCs in the longitudinal control setup in the multiple-adaptive fuzzy controller with switching.

Overall, in solving the integrated problem of lateral and longitudinal control of highway vehicles, the MM/MCs have shown some better results in comparison to the single-adaptive fuzzy controllers. On the other hand, in solving the same problem, ‘blending’ based MM/MCs showed slightly better performance over the ‘switching’ based MM/MCs.

As for future directions, a number of perspectives can be identified as far as MM/MCs developed in the research are concerned.
During the implementation stage, the designed controllers in the research have been somewhat simplified for the simulations by confining the consideration of the rate of change of dynamics with the changes in adaptive rate. This form of ‘simplification’ was acceptable since the simulations carried out in the research served to prove the viability of the controllers developed. But, the controllers developed in this research, especially the MM/MCs, have significantly more capacity with their multiple-adaptive fuzzy capabilities. Therefore, the scope of the controllers can be extended too. Tuning of adaptive fuzzy controllers based on specific scenarios can be a better option in this regard.

On the other hand, in order to fully use the capabilities of the controller, it needs to properly identify a particular scenario the vehicle undergoes. A combination of ‘functions’ based on a number of states can be developed and deployed to identify a given ‘scenario’ of the vehicle so that a specifically designed controller can be used more effectively. Thereby, such individual systems of functions can be used in multiple-adaptive fuzzy controllers that have been developed in this research. Such a scheme of effective identification allows tuning differently oriented individual adaptive fuzzy controllers in addressing multiple-modal based problems providing a much more versatile performance.
References


Publications List from the Research

Journal Publications

• Ravipriya Ranatunga, Zhenwei Cao, Romesh Nagarajah and Ahmad Rad, “Robust adaptive fuzzy systems for integrated lateral and longitudinal control of highway vehicles”, *Journal of Control and Intelligent Systems*, ACTA Press, IASTED Canada, submitted for publication.


Conference Publications

• Ravipriya Ranatunga, Zhenwei Cao, Romesh Nagarajah and Ahmad Rad, “Robust multiple-adaptive fuzzy control with switching for highway vehicles”, to be in *Proceedings of the 8th IEEE Conference on Industrial Electronics and Applications (ICIEA2013)*, Melbourne, Australia, June 2013 – accepted for publication.

• Ravipriya Ranatunga, Zhenwei Cao, Romesh Nagarajah and Ahmad Rad, “A Novel Fuzzy PDC Paradigm with Multiple-Adaptive Fuzzy Control for Highway Vehicles”, to be in *Proceedings of the 8th IEEE Conference on Industrial Electronics and Applications (ICIEA2013)*, Melbourne, Australia, June 2013 – accepted for publication.


## Appendix A

### Vehicle Model Parameters

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_x$</td>
<td>Longitudinal aerodynamic coefficient</td>
<td>0.4</td>
<td>-</td>
</tr>
<tr>
<td>$C_y$</td>
<td>Lateral aerodynamic coefficient</td>
<td>0.5</td>
<td>-</td>
</tr>
<tr>
<td>$C_{sf}$</td>
<td>Cornering stiffness value for front tyres</td>
<td>66366</td>
<td>N</td>
</tr>
<tr>
<td>$C_{sr}$</td>
<td>Cornering stiffness value for rear</td>
<td>52812</td>
<td>N</td>
</tr>
<tr>
<td>$F_{roll}$</td>
<td>Tyre rolling force</td>
<td>274.7</td>
<td>N</td>
</tr>
<tr>
<td>$J_{eng}$</td>
<td>Inertia for engine</td>
<td>0.319</td>
<td>kg m$^2$</td>
</tr>
<tr>
<td>$J_w$</td>
<td>Inertia for the wheel</td>
<td>1.2825</td>
<td>kg m$^2$</td>
</tr>
<tr>
<td>$I_z$</td>
<td>Moment of inertia of the vehicle around z-axis through c.o.g.</td>
<td>1750</td>
<td>kg m$^2$</td>
</tr>
<tr>
<td>$l_f$</td>
<td>Distance from c.o.g. to vehicle front</td>
<td>1.25</td>
<td>m</td>
</tr>
<tr>
<td>$l_r$</td>
<td>Distance from c.o.g. to vehicle rear</td>
<td>1.32</td>
<td>m</td>
</tr>
<tr>
<td>$m$</td>
<td>Mass of the vehicle</td>
<td>1296</td>
<td>kg</td>
</tr>
<tr>
<td>$r_w$</td>
<td>Radius of the wheel</td>
<td>0.286</td>
<td>m</td>
</tr>
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<td>$r_{drive}$</td>
<td>Drive ratio</td>
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</tr>
<tr>
<td>$r_{gear}$</td>
<td>Gear ratio</td>
<td>1.0</td>
<td>-</td>
</tr>
</tbody>
</table>
Appendix B

Control Structure Models in Simulink/MATLAB®

1. Single-Adaptive Fuzzy Control Structures

![Control Structure Models in Simulink/MATLAB®](image)

Fig. B.1 Main control block (Single-adaptive fuzzy controller)

![Lateral error calculation](image)

Fig. B.2 Lateral error calculation (Parent figure → Fig. B.1 Main block)

![Variable structure control](image)

Fig. B.3 Variable structure control (Parent figure → Fig. B.1 Main block)
Fig. B.4 Inverse model for vehicle control input calculation (Parent figure → Fig. B.1 Main block)

Fig. B.5 Calculation of combined longitudinal and steering inputs (Parent figure → Fig. B.4 Inverse model for vehicle inputs)

Fig. B.6 Calculation of engine speed (Parent figure → Fig. B.4 Inverse model for vehicle control input calculation)
Fig. B.7 Calculation of combined longitudinal inputs (Parent figure \(\rightarrow\) Fig. B.5 Calculation of combined longitudinal inputs and steering input)

Fig. B.8 Calculation of front slip angles (Parent figure \(\rightarrow\) Fig. B.4 Inverse Model for Vehicle Control Input Calculation)

Fig. B.9 Calculation of states of vehicle for slip angle calculation (Parent figure \(\rightarrow\) Fig. B.4 Inverse model for vehicle control input calculation)
Fig. B.10 Vehicle model calculations for control inputs (Parent figure → Fig. B.1 Main block)

Fig. B.11 Longitudinal model calculations (Parent figure → Fig. B.10 Vehicle model calculations for control inputs)
Fig. B.12 Lateral model calculations (Parent figure → Fig. B.10 Vehicle model calculations for control inputs)

Fig. B.13 Adaptive fuzzy controller-longitudinal (Parent figure → Fig. B.1 Main block)

Fig. B.14 Adaptive fuzzy controller-lateral (Parent figure → Fig. B.1 Main block)
2 Multiple-Adaptive Fuzzy Control with Blending

Fig. B.15 Main control block (Multiple-adaptive fuzzy controller)

Fig. B.16 Lateral control block (Parent figure → Fig. B.15 Main control block)
Fig. B.17 Longitudinal control block (Parent figure → Fig. B.15 Main control block)

Fig. B.18 Fuzzy blender- lateral (Parent figure → Fig. B.16 Longitudinal control block)

Fig. B.19 Fuzzy blender-longitudinal (Parent figure → Fig. B.17 Longitudinal control block)
3 Multiple-Model PDC-based Multiple-Adaptive Fuzzy Control with Blending

Fig. B.20 Main control block (Multiple-model PDC controller)

Fig. B.21 Longitudinal control block (Parent figure → Fig. B.20 Main control block)
Fig. B.22 Control inputs for fuzzy blending – longitudinal (Parent figure → Fig. B.21 Longitudinal controller block)

Fig. B.23 Vehicle local models – longitudinal (Parent figure → Fig. B.21 Longitudinal control block)
Fig. B.24 Lateral control block (Parent figure → Fig. B.20 Main control block)

Fig. B.25 Control inputs for fuzzy blending – lateral (Parent figure → Fig. B.24 Lateral control block)
4. Multiple-Adaptive Fuzzy Control with Switching (two AFCs in the bank)
Fig. B.28 Multiple-adaptive fuzzy control with switching – lateral (Parent figure \(\rightarrow\) Fig. B.27 Main control block)

Fig. B.29 Multiple-adaptive fuzzy control with switching – longitudinal (Parent figure \(\rightarrow\) Fig. B.27 Main control block)
Fig. B.30 Cost function comparator for switching (lateral/longitudinal) (Parent figure ➔ Fig. B.28/B.29 Multiple-adaptive fuzzy control with switching-lateral/longitudinal)

Fig. B.31 Controller switcher (longitudinal/lateral) (Parent figure ➔ Fig. B.28/B.29 Multiple-adaptive fuzzy control with switching-longitudinal/lateral)

Fig. B.32 Cost function for controller switching (w1 and w2) (Parent figure ➔ Fig. B.30 Multiple-adaptive switcher – lateral/longitudinal)
5. Multiple-Adaptive Fuzzy Control with Switching (four AFCs in the bank)

Fig. B.33 Main control block (Multiple-adaptive fuzzy control with switching: 4-AFCs)

Fig. B.34 Multiple-adaptive fuzzy control with switching – longitudinal (Parent figure → Fig. B.33 Main control block)
Fig. B.35 Cost function comparator for switching – longitudinal (Parent figure ➔ Fig. B.34 Multiple-adaptive fuzzy control with switching – longitudinal)

Fig. B.36 Controller switcher – longitudinal (Parent figure ➔ Fig. B.34 Multiple-adaptive fuzzy control with switching – longitudinal)
Fig. B.37 Multiple-adaptive fuzzy control with switching – lateral (Parent figure → Fig. B.33 Main control block)

Fig. B.38 Cost function comparator for switching – lateral (parent figure → Fig. B.37 Multiple-adaptive fuzzy control with switching–lateral)

Fig. B.39 Controller switcher–lateral (Parent figure → Fig. B.37 Multiple-adaptive fuzzy control with switching–lateral)
Appendix C

Trained Parameters of Fuzzy Systems

1. Single-Adaptive Fuzzy Control Structure

The trained T–S fuzzy membership function parameters are as follows

(i) Lateral Control:

The tuned Gaussian fuzzy input membership functions are as follows:

‘error’ variable:

\[ \mu_a^1(\bar{x}) = \exp(-(\bar{x} - 0.002)^2 / 0.021^2), \bar{x} \geq 0.002 \text{ or } \mu_a^1(\bar{x}) = 1, \bar{x} < 0.002 \]

\[ \mu_a^2(\bar{x}) = \exp(-(\bar{x} - 0.002511)^2 / 0.0003198^2) \]  \hspace{1cm} (C.1)

\[ \mu_a^3(\bar{x}) = \exp(-(\bar{x} - 0.006)^2 / 0.018^2), \bar{x} \leq 0.002 \text{ or } \mu_a^3(\bar{x}) = 1, \bar{x} > 0.002 \]

‘error rate’ variable:

\[ \mu_a^4(\bar{x}) = \exp(-(\bar{x} + 0.095)^2 / 0.039^2), \bar{x} \geq -0.095 \text{ or } \mu_a^4(\bar{x}) = 1, \bar{x} < -0.095 \]

\[ \mu_a^5(\bar{x}) = \exp(-(\bar{x} - 0.02323)^2 / 0.03457^2) \]  \hspace{1cm} (C.2)

\[ \mu_a^6(\bar{x}) = \exp(-(\bar{x} - 0.11)^2 / 0.032^2), \bar{x} \leq 0.11 \text{ or } \mu_a^6(\bar{x}) = 1, \bar{x} > 0.11 . \]

The tuned T–S fuzzy output membership functions \textit{a priori} are as follows,

\[ z_1 = -2.999e+006 - 2.972e+005 \hat{e} \]

\[ z_2 = -2.999e+006 - 2.972e+005 \hat{e} \]

\[ z_3 = -3.001e+006 - 2.98e+005 \hat{e} \]

\[ z_4 = -3.002e+006 - 2.987e+005 \hat{e} \]

\[ z_5 = -3.002e+006 - 2.986e+005 \hat{e} \]

\[ z_6 = -2.997e+006 - 2.978e+005 \hat{e} \]  \hspace{1cm} (C.3)

\[ z_7 = -25.63e + 317.3 \hat{e} \]

\[ z_8 = -7843e - 3.422e+004 \hat{e} \]

\[ z_9 = -305.2e - 898.4 \hat{e} . \]
(ii) Longitudinal Control:

The tuned Gaussian input fuzzy membership functions are as follows:

‘error’ variable:

\[ \mu_A^{(2)}(\bar{x}) = \exp(- (\bar{x} + 5.066)^2 / 2.14^2), \quad \bar{x} \geq -5.066 \quad \text{or} \quad \mu_A^{(1)}(\bar{x}) = 1, \quad \bar{x} < -5.066 \]

\[ \mu_A^{(3)}(\bar{x}) = \exp(- (\bar{x} + 2.748)^2 / 2.573^2) \]

\[ \mu_A^{(4)}(\bar{x}) = \exp(- (\bar{x} + 0.2756)^2 / 2.505^2) \]

\[ \mu_A^{(5)}(\bar{x}) = \exp(- (\bar{x} - 4.663)^2 / 0.5623^2) \]

\[ \mu_A^{(6)}(\bar{x}) = \exp(- (\bar{x} - 9.370)^2 / 2.406^2), \quad \bar{x} \leq 9.370 \quad \text{or} \quad \mu_A^{(8)}(\bar{x}) = 1, \quad \bar{x} > 9.370 \]

‘error rate’ variable:

\[ \mu_A^{(7)}(\bar{x}) = \exp(- (\bar{x} + 7.822)^2 / 0.352^2), \quad \bar{x} \geq -7.822 \quad \text{or} \quad \mu_A^{(9)}(\bar{x}) = 1, \quad \bar{x} < -7.822 \]

\[ \mu_A^{(10)}(\bar{x}) = \exp(- (\bar{x} + 1.394)^2 / 1.336^2) \]

\[ \mu_A^{(11)}(\bar{x}) = \exp(- (\bar{x} - 1.006)^2 / 1.313^2) \]

\[ \mu_A^{(12)}(\bar{x}) = \exp(- (\bar{x} - 3.210)^2 / 1.1^2) \]

\[ \mu_A^{(13)}(\bar{x}) = \exp(- (\bar{x} - 8.524)^2 / 0.6871^2), \quad \bar{x} \leq 8.524 \quad \text{or} \quad \mu_A^{(14)}(\bar{x}) = 1, \quad \bar{x} > 8.524 \]

The tuned output T–S fuzzy membership function parameters a priori are as follows:

\[ z_1 = 8.941 \times 10^4 - 3.172 \times 10^4 \bar{e} \]

\[ z_2 = -2.997 \times 10^5 - 3.001 \times 10^5 \bar{e} \]

\[ z_3 = -2.981 \times 10^5 - 3.005 \times 10^5 \bar{e} \]

\[ z_4 = -2.99 \times 10^5 - 2.997 \times 10^5 \bar{e} \]

\[ z_5 = 4.848 \times 10^4 - 8.951 \times 10^4 \bar{e} \]

\[ z_6 = -8.107 \times 10^4 - 3.329 \times 10^5 \bar{e} \]

\[ z_7 = -2.964 \times 10^5 - 3 \times 10^5 \bar{e} \]

\[ z_8 = -2.869 \times 10^5 - 2.998 \times 10^5 \bar{e} \]

\[ z_9 = -2.861 \times 10^5 - 3.002 \times 10^5 \bar{e} \]

\[ z_{10} = -2.252 \times 10^4 - 2.678 \times 10^5 \bar{e} \]

\[ z_{11} = -2.648 \times 10^5 - 2.522 \times 10^5 \bar{e} \]

\[ z_{12} = -2.987 \times 10^5 - 3 \times 10^5 \bar{e} \]

\[ z_{13} = -2.957 \times 10^5 - 3.001 \times 10^5 \bar{e} \]

\[ z_{14} = -2.948 \times 10^5 - 2.999 \times 10^5 \bar{e} \]
\[ z_{15} = -2.798e + 005 \hat{e} - 3.246e + 005 \hat{e} \]
\[ z_{16} = -2.987e + 005 \hat{e} - 3.006e + 005 \hat{e} \]
\[ z_{17} = -6.671e + 004 \hat{e} - 8.401e + 004 \hat{e} \]
\[ z_{18} = -1.508e - 12.57 \hat{e} \]
\[ z_{19} = -408.4 e - 368.6 \hat{e} \]
\[ z_{20} = -2.99e + 005 \hat{e} - 2.973e + 005 \hat{e} \]
\[ z_{21} = -2.991e + 005 \hat{e} - 2.999e + 005 \hat{e} \]
\[ z_{22} = -3e + 005 \hat{e} - 3e + 005 \hat{e} \]
\[ z_{23} = -3.001e + 005 \hat{e} - 3e + 005 \hat{e} \]
\[ z_{24} = -3e + 005 \hat{e} - 3e + 005 \hat{e} \]
\[ z_{25} = -2.969e + 005 \hat{e} - 2.947e + 005 \hat{e} . \]

2. Multiple-Adaptive Fuzzy Controller with Fuzzy Blender

Fuzzy Blender Parameters for Final Trial

(i) Longitudinal Controller:

(a) Input Membership Function Parameters:

Input variable: Controller \#1

\[ \mu_{A_1}(\bar{x}) = \exp(-(\bar{x} + 71.11)^2 / 13.45^2) \]
\[ \mu_{A_2}(\bar{x}) = \exp(-(\bar{x} + 41.13)^2 / 13.44^2) \]
\[ \mu_{A_3}(\bar{x}) = \exp(-(\bar{x} + 2.023)^2 / 13.61^2) \]
\[ \mu_{A_4}(\bar{x}) = \exp(-(\bar{x} + 0.006218)^2 / 13.44^2) \]  \hfill (C.7)

Input variable: Controller \#2

\[ \mu_{A_1}(\bar{x}) = \exp(-(\bar{x} + 71.13)^2 / 13.45^2) \]
\[ \mu_{A_2}(\bar{x}) = \exp(-(\bar{x} + 41.11)^2 / 13.45^2) \]
\[ \mu_{A_3}(\bar{x}) = \exp(-(\bar{x} + 1.816)^2 / 13.44^2) \]
\[ \mu_{A_4}(\bar{x}) = \exp(-(\bar{x} + 0.00376)^2 / 13.45^2) \] \hfill (C.8)
(b) Output Membership Function Parameters:

\[
\begin{align*}
  z_1 &= 4.238C_1 - 4.246C_2 \\
  z_2 &= 19.49C_1 + 15.91C_2 \\
  z_3 &= -1.07C_1 - 1.074C_2 \\
  z_4 &= -3.291C_1 - 3.293C_2 \\
  z_5 &= -15.95C_1 - 19.52C_2 \\
  z_6 &= 2.191C_1 - 2.428C_2 \\
  z_7 &= 28.54C_1 + 28.57C_2 \\
  z_8 &= -2.447C_1 - 2.379C_2 \\
  z_9 &= 0.1276C_1 + 0.1227C_2 \\
  z_{10} &= -21.14C_1 - 21.37C_2 \\
  z_{11} &= 4.091C_1 - 3.964C_2 \\
  z_{12} &= 7.113C_1 - 1.022C_2 \\
  z_{13} &= -3.333C_1 - 3.336C_2 \\
  z_{14} &= -4.117C_1 - 4.31C_2 \\
  z_{15} &= 1.995C_1 - 7.515C_2 \\
  z_{16} &= 4.498C_1 - 5.066C_2
\end{align*}
\] (C.9)

(ii) Lateral Controller:

(a) Input Membership Function Parameters:

Input variable: Controller #1

\[
\begin{align*}
  \mu_{A_1}^I(\bar{x}) &= \exp(-(\bar{x} + 36.11)^2 / 20.6^2) \\
  \mu_{A_2}^I(\bar{x}) &= \exp(-(\bar{x} - 27.77)^2 / 20.6^2)
\end{align*}
\] (C.10)

Input variable: Controller #2

\[
\begin{align*}
  \mu_{A_1}^I(\bar{x}) &= \exp(-(\bar{x} + 42.78)^2 / 20.5^2) \\
  \mu_{A_2}^I(\bar{x}) &= \exp(-(\bar{x} - 35.33)^2 / 20.5^2)
\end{align*}
\] (C.11)

(b) Output Membership Function Parameters:

\[
\begin{align*}
  z_1 &= 0.05C_1 + 0.076C_2 \\
  z_2 &= 0.034C_1 + 0.043C_2 \\
  z_3 &= -0.1129C_1 + 0.1133C_2 \\
  z_4 &= -0.09751C_1 + 0.09795C_2
\end{align*}
\] (C.12)
3. Multiple-Model PDC-based Multiple-Adaptive Fuzzy Structure

Parameters for the Individual Adaptive Fuzzy Controllers

(i) Lateral Control:

1. controller #1 (data obtained when \( x_2 = 35 \), \( x_4 = 1 \))

Input Gaussian function parameters:

‘error’ variable

\[
\mu_{\alpha'}(\bar{x}) = \exp(-\frac{(\bar{x} - 0.0724)^2}{0.2031^2}) \\
\mu_{\alpha''}(\bar{x}) = \exp(-\frac{(\bar{x} - 0.441)^2}{0.1974^2}) 
\]

‘error rate’ variable

\[
\mu_{\alpha'}(\bar{x}) = \exp(-\frac{(\bar{x} - 0.01726)^2}{0.0195^2}) \\
\mu_{\alpha''}(\bar{x}) = \exp(-\frac{(\bar{x} - 0.05816)^2}{0.06497^2}) 
\]

T–S Fuzzy Output Membership Function Parameters:

\[
\begin{align*}
z_1 &= -4671e + 2.732e + 004\dot{e} \\
z_2 &= 8724e - 4.857e + 004\dot{e} \\
z_3 &= -1851e + 4228\dot{e} \\
z_4 &= 3001e - 2588\dot{e} 
\end{align*} 
\]

(C.15)
2. **controller #2** (data obtained when $x_2 = 35, x_4 = -1$)

Input Gaussian function parameters:

‘error’ variable

\[ \mu_{\alpha}(\bar{x}) = \exp\left(-\left(\bar{x} + 0.4425\right)^2 / 0.198^2\right) \]  
\[ \mu_{\alpha}(\bar{x}) = \exp\left(-\left(\bar{x} + 0.0727\right)^2 / 0.2038^2\right) \]  
(C.16)

‘error rate’ variable

\[ \mu_{\alpha}(\bar{x}) = \exp\left(-\left(\bar{x} - 0.01725\right)^2 / 0.0195^2\right) \]  
\[ \mu_{\alpha}(\bar{x}) = \exp\left(-\left(\bar{x} - 0.05817\right)^2 / 0.06497^2\right) \]  
(C.17)

T–S Fuzzy Output Membership Function Parameters:

\[ z_1 = -1865e - 4226\dot{\bar{e}} \]  
\[ z_2 = 2988e + 2583\dot{\bar{e}} \]  
\[ z_3 = -4644e - 2.731e + 004\dot{\bar{e}} \]  
\[ z_4 = 8692e + 4.857e + 004\dot{\bar{e}} \]  
(C.18)

3. **controller #3** (data obtained when $x_2 = -5, x_4 = 1$)

Input Gaussian function parameters:

‘error’ variable

\[ \mu_{\alpha}(\bar{x}) = \exp\left(-\left(\bar{x} - 0.007666\right)^2 / 0.01307^2\right) \]  
\[ \mu_{\alpha}(\bar{x}) = \exp\left(-\left(\bar{x} - 0.03459\right)^2 / 0.04349^2\right) \]  
(C.19)

‘error rate’ variable

\[ \mu_{\alpha}(\bar{x}) = \exp\left(-\left(\bar{x} - 0.03945\right)^2 / 0.044414^2\right) \]  
\[ \mu_{\alpha}(\bar{x}) = \exp\left(-\left(\bar{x} - 0.0789\right)^2 / 0.02361^2\right) \]  
(C.20)
T–S Fuzzy Output Membership Function Parameters:

\[
\begin{align*}
z_1 &= 5175e - 6.181e+004e \\
z_2 &= -7424e - 1.364e+004e \\
z_3 &= 7165e - 4.996e+004e \\
z_4 &= 7432e + 1.133e+004e \\
\end{align*}
\]  (C.21)

4. controller #4 (data obtained when \( x_2 = -5 \), \( x_4 = -1 \))

Input Gaussian function parameters:

‘error’ variable

\[
\mu_{x_1}(\bar{x}) = \exp(-(\bar{x} - 0.0724)^2 / 0.2031^2) \\
\mu_{x_2}(\bar{x}) = \exp(-(\bar{x} - 0.441)^2 / 0.1974^2) \\
\]  (C.22)

‘error rate’ variable

\[
\mu_{x_3}(\bar{x}) = \exp(-(\bar{x} - 0.05816)^2 / 0.06497^2) \\
\mu_{x_4}(\bar{x}) = \exp(-(\bar{x} - 0.01726)^2 / 0.0195^2) \\
\]  (C.23)

T–S Fuzzy Output Membership Function Parameters:

\[
\begin{align*}
z_1 &= -4671e + 2.732e+004e \\
z_2 &= 8724e - 4.857e+004e \\
z_3 &= -1851e + 4228e \\
z_4 &= 3001e - 2588e \\
\end{align*}
\]  (C.24)
(ii) Longitudinal Control:

1. **controller #1** (data obtained when \( x_2 = 35, x_4 = 1 \))

Input Gaussian function parameters:

- ‘error’ variable
  \[
  \mu_{\delta^i}(\bar{x}) = \exp\left(-\frac{(\bar{x} + 23.19)^2}{22.83^2}\right)
  \]
  \[
  \mu_{\delta^i}(\bar{x}) = \exp\left(-\frac{(\bar{x} + 7.211)^2}{23.02^2}\right)
  \]

- ‘error rate’ variable
  \[
  \mu_{\delta^i}(\bar{x}) = \exp\left(-\frac{(\bar{x} + 12.14)^2}{4.44^2}\right)
  \]
  \[
  \mu_{\delta^i}(\bar{x}) = \exp\left(-\frac{(\bar{x} - 0.4308)^2}{3.797^2}\right)
  \]

T–S Fuzzy Output Membership Function Parameters:

\[
z_1 = \ -0.0424e \ - \ 0.09198\dot{e} \\
z_2 = \ 0.2779e \ + \ 0.3019\dot{e} \\
z_3 = \ -0.07343e \ + \ 0.1432\dot{e} \\
z_4 = \ 0.01449e \ - \ 0.1136\dot{e}
\]

2. **controller #2** (data obtained when \( x_2 = 35, x_4 = -1 \))

Input Gaussian function parameters:

- ‘error’ variable
  \[
  \mu_{\delta^i}(\bar{x}) = \exp\left(-\frac{(\bar{x} + 38.25)^2}{23^2}\right)
  \]
  \[
  \mu_{\delta^i}(\bar{x}) = \exp\left(-\frac{(\bar{x} - 4.77)^2}{23.03^2}\right)
  \]

- ‘error rate’ variable
  \[
  \mu_{\delta^i}(\bar{x}) = \exp\left(-\frac{(\bar{x} + 12.05)^2}{7.35^2}\right)
  \]
  \[
  \mu_{\delta^i}(\bar{x}) = \exp\left(-\frac{(\bar{x} + 7.566)^2}{8.532^2}\right)
  \]
T–S Fuzzy Output Membership Function Parameters:

\[
\begin{align*}
z_1 &= -0.3259 \epsilon + 3912 \dot{\epsilon} \\
z_2 &= 0.4104 \epsilon + 0.2188 \\
z_3 &= -0.03978 \epsilon + 0.02231 \dot{\epsilon} \\
z_4 &= 0.07179 \epsilon + 0.08603 \dot{\epsilon}
\end{align*}
\]  
(C.30)

3. **controller #3** (data obtained when \(x_2 = -5\), \(x_4 = 1\))

Input Gaussian function parameters:

‘\(error\)’ variable

\[
\begin{align*}
\mu_{\mu_3}(\bar{x}) &= \exp(-(\bar{x} - 35.29)^2 / 38.56^2) \\
\mu_{\mu_4}(\bar{x}) &= \exp(-(\bar{x} - 90.9)^2 / 38.54^2)
\end{align*}
\]  
(C.31)

‘\(error rate\)’ variable

\[
\begin{align*}
\mu_{\mu_5}(\bar{x}) &= \exp(-(\bar{x} - 8.994)^2 / 2.301^2) \\
\mu_{\mu_6}(\bar{x}) &= \exp(-(\bar{x} - 13.98)^2 / 1.649^2)
\end{align*}
\]  
(C.32)

T–S Fuzzy Output Membership Function Parameters:

\[
\begin{align*}
z_1 &= -0.01506 \epsilon + 1.464 \dot{\epsilon} \\
z_2 &= 0.405 \epsilon - 9.919 \dot{\epsilon} \\
z_3 &= -0.04345 \epsilon - 2.491 \dot{\epsilon} \\
z_4 &= -0.5207 \epsilon + 3.48 \dot{\epsilon}
\end{align*}
\]  
(C.33)

4. **controller #4** (data obtained when \(x_2 = -5\), \(x_4 = -1\))

Input Gaussian function parameters:

‘\(error\)’ variable

\[
\begin{align*}
\mu_{\mu_3}(\bar{x}) &= \exp(-(\bar{x} + 38.25)^2 / 23^2) \\
\mu_{\mu_4}(\bar{x}) &= \exp(-(\bar{x} - 4.77)^2 / 23.03^2)
\end{align*}
\]  
(C.34)
‘error rate’ variable

\[
\mu_{\alpha^1}(\bar{X}) = \exp\left(-\left(\bar{X} + 12.05\right)^2 / 7.35^2\right) \\
\mu_{\alpha^2}(\bar{X}) = \exp\left(-\left(\bar{X} + 7.566\right)^2 / 8.532^2\right)
\]

(C.35)

T–S Fuzzy Output Membership Function Parameters:

\[
z_1 = -0.3259e + 0.3912\dot{e} \\
z_2 = 0.4104e + 0.2189\dot{e} \\
z_3 = -0.03977e + 0.0223\dot{e} \\
z_4 = 0.07178e + 0.08601\dot{e}
\]

(C.36)
Appendix D

Pseudo-Codes of MATLAB® Programs Embedded in Simulink® Block Diagrams

Adaptive Fuzzy Controller

% Adaptive Fuzzy Function
% Integrated Vehicular Control
% Lateral/Longitudinal Controller n rule

% S R Ranatunga, 27/02/12

function [f] = afuzzycontroller(u,v,t)

%% Initialization

persistent mux;
persistent gx;
persistent Ax;
persistent Apastx;
persistent Cpastx;
persistent xi;

if isempty(gx)
    gx = zeros(n,1);
end
if isempty(mux)
    mux = zeros(n,1);
end
if isempty(Ax)
    Ax = zeros(n,2);
end
if isempty(Apastx)
    Apastx = zeros(n,2);
end
if isempty(xi)
    xi = zeros(n,1);
end
if isempty(Cpastx)
    Cpastx = zeros(1);
End

%% Initialize variables
x=[0.1 0.1];
xeta= [0 0];
xix=ones(n,1);
mu=ones(n,1);
g=zeros(n,1);
z=zeros(n,1);
% Initialise output membership function parameters

A=[ 1.5  1.5;  
    1.5  1.5;  
    .    .;  
    .    .;  
    1.5  1.5;  
    1.5  1.5];

Sigmapastx=0.8;
Sigmapast=0.1;

% Set number of inputs and rules

n=2;

gain=0.1;

% Output membership function parameters \textit{a priori}: example values only

Apast=[0.51  -0.35;  
       1.57   0.5;  
       .    .;  
       .    .;  
      -0.01 0.002;  
       4.31   7.21];

lambda=0.01;

%%%%%%%%%%%%%%%%%%%%
if (t<0.0002)

  Ax=A;
  Apastx=Apast;
  mux=mu;
  gx=g;
  Cpastx=Cpast;
  Sigmapastx=Sigmapast;
  xi=xix;

end
%%%%%%%%%%%%%%%%%%%%

x(1)=u;
x(2)=v;

% For error variable

c11= 0.001;
c12= 0.0035;
c13= 0.00058;
% For error rate variable
c21= 0.0017;
c22= 0.0038;
c23= 0.0006;

q21= 0.02;
q22= 0.018;
q23= 0.021;

%% Fuzzy IF-THEN Rules

% Rule 1
for j=1:n
    if (j==1) Sigmapastx=q11; Cpastx=c11;
        if (x(1)<c11)
            xeta(1)=1;
        else
            xeta(1)=exp(-.5*((x(j)-Cpastx)/Sigmapastx)^2);
        end
    end

    if (j==2) Sigmapastx=q21; Cpastx=c21;
        if (x(2)<c21)
            xeta(2)=1;
        else
            xeta(2)=exp(-.5*((x(j)-Cpastx)/Sigmapastx)^2);
        end
    end

    mux(1)=mux(1)*xeta(j);
gx(1)=gx(1)+Apastx(1,j)*x(j); % Sums up the consequent
end

for j=1:n
    Ax(1,j)=Apastx(1,j)-lambda*xi(1)*x(j)*(x(1)+x(2));
% Update consequent parameters
    Apastx(1,j)=Ax(1,j);
end

% Rule n
for j=1:n
    if (j==1) Sigmapastx=q13; Cpastx=c13;
        if (x(1)>c13)
            xeta(1)=1;
        else
            xeta(1)=exp(-.5*((x(j)-Cpastx)/Sigmapastx)^2);
        end
    end
if (j==2) Sigmapastx=q23; Cpastx=c23;
   if(x(2)>c23)
      xeta(2)=1;
   else
      xeta(2)=exp(-.5*((x(j))-Cpastx)/Sigmapastx)^2);
   end
end

mux(n)=mux(n)*xeta(j);
gx(n)=gx(n)+Apastx(n,j)*x(j); % Sums up the consequent
end

for j=1:n
   Ax(n,j)=Apastx(n,j)-lambda*xi(n)*x(j)*(x(1)+x(2));
   % Update consequent parameters
   Apastx(n,j)=Ax(n,j);
end

den=sum(mux(:)); % Denominator of xi for kth training data
if(den ~= 0)
   xi=mux(:)/den; % Compute xi vector for kth training data pair
else
   xi=zeros(n,1);
end
f=gx(:)'*xi(:); % Compute output for current training data
end

Adaptive Fuzzy Controller in blender setup

function [f] = afuzzycontroller(u,v,t,F)

% Adaptive Fuzzy Function
% Integrated Vehicular Control
% Multiple Adaptive Fuzzy Blending Scheme% Lateral/Longitudinal
Controller n rule

% S R Ranatunga

persistent mux;
persistent gx;
persistent Ax;
persistent Apastx;
persistent Cpastx;
persistent xi;

if isempty(gx)
    gx = zeros(n,1);
end
if isempty(mux)

mux=zeros(n,1);
end
if isempty(Ax)
Ax = zeros(n,2);
end
if isempty(Apastx)
Apastx = zeros(n,2);
end
if isempty(xi)
xi = zeros(n,1);
end
if isempty(Cpastx)
Cpastx = zeros(1);
end

%% Initialize variables

x=[0.1 0.1];
xeta= [0 0];

xix=ones(n,1);
mu=ones(n,1);
g=zeros(n,1);
z=zeros(n,1);

% Initialise output membership function parameters

A=[ 1.5 1.5;
    1.5 1.5;
    1.5 1.5;
    1.5 1.5;
    1.5 1.5;
    1.5 1.5;
    1.5 1.5;
    1.5 1.5;
    1.5 1.5];

Sigmapastx=0.8;
Sigmapast=0.1;

% Set number of inputs and rules

n=2;

gain=0.1;

Apast=[1.5 1.5;
    1.5 1.5;
    1.5 1.5;
    1.5 1.5;
    1.5 1.5;
    1.5 1.5;
    1.5 1.5;
    1.5 1.5;
    1.5 1.5];

lambda=0.1;

g1=0.2;
g2=0.2;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
if (t<0.0002)
    Ax=A;
    Apastx=Apast;
    mux=mu;
    gx=g;
    Cpastx=Cpast;
    Sigmapastx=Sigmapast;
    xi=xix;
end
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

dx(1)=u;
dx(2)=v;

%% For error variable
c11=0.0035;
c12=0.0005;
c13=0.15;

d11= 7.23;
d12= 6.93;
d13= 7.05;

%% For error rate variable
c21=-0.1;
c22=0.005;
c23=0.1;

d21= 13.23;
d22= 12.98;
d23= 12.67;

%% Fuzzy IF-THEN Rules
%% ‘F’ is the feedback signal from blender/switcher

% Rule 1
for j=1:n
    if (j==1) Sigmapastx=q11; Cpastx=c11;
        if(x(1)<c11)
            xeta(1)=1;
        else
            xeta(1)=exp(-.5*((x(j)-Cpastx)/Sigmapastx)^2);
        end
    end
    if (j==2) Sigmapastx=q21; Cpastx=c21;
        if(x(2)<c21)
            xeta(2)=1;
        else
            xeta(2)=exp(-.5*((x(j)-Cpastx)/Sigmapastx)^2);
        end
    end
    mux(j)=mux(1)*xeta(j);
    gx(j)=gx(1)+Apastx(1,j)*x(j);  % Sums up the consequent
end
for j=1:n
   Ax(1,j)=A_pastx(1,j)-lambda*xi(1)*x(j)*F*(x(1)+x(2));
   % Update consequent parameters
   A_pastx(1,j)=Ax(1,j);
end.
for j=1:n
   % Rule n
   if (j==1) Sigmapastx=q13; Cpastx=c13;
      if (x(1)>c13)
         xeta(1)=1;
      else
         xeta(1)=exp(-.5*((x(j)-Cpastx)/Sigmapastx)^2);
      end
   end
   if (j==2) Sigmapastx=q23; Cpastx=c23 ;
      if (x(2)>c23)
         xeta(2)=1;
      else
         xeta(2)=exp(-.5*((x(j)-Cpastx)/Sigmapastx)^2);
      end
   end
   mux(1)=mux(1)*xeta(j);
gx(1)=gx(1)+A_pastx(n,j)*x(j);  % Sums up the consequent
end
for j=1:n
   Ax(n,j)=A_pastx(n,j)-lambda*xi(n)*x(j)*F*(x(1)+x(2));
end
% Defuzzification
if (den ~= 0)
   xi=mux(:)/den;  % Compute xi vector for kth training data pair
else
   xi=zeros(n,1);
end
f=gx(:)'*xi(:);  % Compute output for the current training data

Fuzzy Blending for Controllers
% Adaptive Fuzzy Function_final
% Integrated Vehicular Control
% Multiple Adaptive Fuzzy Controllers with Blender
% Lateral/Longitudinal Blender -4 Rule

% S R Ranatunga, 10/03/12

function [f,f1,f2,Z] = fuzzyblend(u,v,t)

persistent mux;
persistent gx;
persistent Cpastx;
persistent xi;

f11=0;
f22=0;

if isempty(gx)
gx = zeros(4,1);
end

if isempty(mux)
mux=zeros(4,1);
end

if isempty(xi)
xi = zeros(4,1);
end

if isempty(Cpastx)
Cpastx = zeros(1);
end

%% Initialize variables

x=[0.1 0.1 ]';
xeta=[0 0];
mu=ones(4,1);
g=zeros(4,1);
z=zeros(4,1);

% Set number of inputs and rules

n=2;

Apast=[
-0.5056 0.5032 ;
-2.001 2.032
1.717 -1.748
-0.1122 0.1125];

Sigmapastx= 0.8;
Sigmapast=0.1;

mux=mu;
gx=g;
% 1st Fuzzy Controller
c11 = -3.362e+004;
c12 = -9.133;
q11 = 7811;
q12 = 7811;

% 2nd Fuzzy Controller
c21 = -3.334e+004;
c22 = -9.014;
q21 = 7795;
q22 = 7794;

% Fuzzy Blender IF-THEN Rules
% 1 Rule
for j=1:n
    if (j==1) Cpastx=c11; Sigmapastx=q11;
        if (x(1)<c11)
            xeta(1)=1;
        else
            xeta(1)=exp(-.5*((x(j)-Cpastx)/Sigmapastx)^2);
        end
    end
    if (j==2) Cpastx=c21; Sigmapastx=q21
        if (x(2)<c21)
            xeta(2)=1;
        else
            xeta(2)=exp(-.5*((x(j)-Cpastx)/Sigmapastx)^2);
        end
    end
    mux(1)=mux(1)*xeta(j);
gx(1)=gx(1)+ Apast(1,j)*x(j); % Sum up the consequent
end.
.
.
% 4
for j=1:n
    if (j==1) Cpastx=c12; Sigmapastx=q12;
        if (x(1)>c12)
            xeta(1)=1;
        else
            xeta(1)=exp(-.5*((x(j)-Cpastx)/Sigmapastx)^2);
        end
    end
    if (j==2) Cpastx=c22; Sigmapastx=q22;
        if (x(2)>c22)
            xeta(2)=1;
        else
            xeta(2)=exp(-.5*((x(j)-Cpastx)/Sigmapastx)^2);
        end
    end
end
\[ \text{mux}(4) = \text{mux}(4) \times \text{xeta}(j) \]
\[ \text{gx}(4) = \text{gx}(4) + \text{Apast}(4,j) \times x(j); \quad \% \text{Sum up the consequent} \]

end

\% Defuzzification

for \ j=1:4
    \[ z(j) = \text{mux}(j) \times \text{gx}(j); \]
end
\[ Z = z(:); \]
\[ \text{den} = \text{sum} (\text{mux}(:)); \quad \% \text{Denominator of } x_i \text{ for } k\text{th training data} \]

if (\text{den} == 0)
    \[ x_i = \text{mux}(:)/\text{den}; \quad \% \text{Compute } x_i \text{ vector for } k\text{th training data pair} \]
else
    \[ x_i = \text{zeros}(4,1); \]
end

\[ f = \text{gx}(:)' \times x_i(:); \quad \% \text{Compute the output for the current training data} \]

for \ j=1:4
    \[ f11 = f11 + (\text{mux}(j)/\text{den}) \times \text{Apast}(j,1); \]
    \[ f22 = f22 + (\text{mux}(j)/\text{den}) \times \text{Apast}(j,2); \]
end
\[ f1 = f11; \]
\[ f2 = f22; \]
\[ \text{mux} = \text{ones}(4,1); \]
\[ \text{gx} = \text{zeros}(4,1); \]

end

**Fuzzy PDC Based Blender**

\% Adaptive Fuzzy Function_final
\% Integrated Vehicular Control
\% PDC Structure for vehicle control

\% S R Ranatunga, 16/06/12

function \[ [x1, x2, x3, x4] = \text{pdc}(x2, x4) \]

persistent \ k1;
persistent \ k2;
persistent \ k3;
persistent \ k4;

if isempty(k1)
    \[ k1 = \text{ones}(1); \]
end
if isempty(k2)
    \[ k2 = \text{ones}(1); \]
end
end
if isempty(k3)
k3 = ones(1);
end
if isempty(k4)
k4 = ones(1);
end

%% Initialize variables
xeta = 0;

% Set number of inputs and rules
n = 2;

%% Fuzzy Blender IF-THEN Rules

%% 1 Rule
for j=1:n
    if (j==1)
        if (x2<-5 || x2>35)
            xeta = 0;
        else
            xeta = (5+x2)/40;
        end
    end
    if (j==2)
        if (x4<-1 || x4>1)
            xeta = 0;
        else
            xeta = (1+x4)/2;
        end
    end
    k1 = k1 * xeta;
end

%% 4
for j=1:n
    if (j==1)
        if (x2<-5 || x2>35)
            xeta = 0;
        else
            xeta = (35-x2)/40;
        end
    end
    if (j==2)
        if (x4<-1 || x4>1)
            xeta = 0;
        else
            xeta = (1-x4)/2;
        end
    end
end

    k4 = k4 * xeta;
end

den = (k1 + k2 + k3 + k4);
if (den == 0)
    xi1 = k1 / den;
    xi2 = k2 / den;
    xi3 = k3 / den;
    xi4 = k4 / den;
else
    xi1 = zeros(1);
    xi2 = zeros(1);
    xi3 = zeros(1);
    xi4 = zeros(1);
end

k1 = ones(1);
k2 = ones(1);
k3 = ones(1);
k4 = ones(1);
end

---

Multiple-Adaptive Switcher (2-AFCs in the bank)

% Switching Function
% Integrated Vehicular Control
% Multiple Adaptive Fuzzy Control with Switching (2-AFCs)

% S R Ranatunga, 16/06/12

function [xi1, xi2] = mas2(sq1, sq2, t)

    persistent k1;
    persistent k2;
    persistent tck;

    if isempty(tck)
        tck = zeros(1);
    end
    if isempty(k1)
        k1 = zeros(1);
    end
    if isempty(k2)
        k2 = zeros(1);
    end

    t0 = 0.1e-2;
    if (t < t0)
        xi1 = 1; xi2 = 0;
    end

    if (sq1 == 1)
        if (t >= tck + t0)
            k1 = sq1, k2 = 0, tck = t;
        end
    end

end
elseif (sq2==1)
    if (t>=tck+t0)
        k1=0, k2=sq2, tck=t;
    end
end

%% Output values
xi1=k1;
xi2=k2;
end

% Switching Function
% Integrated Vehicular Control
% Multiple Adaptive Fuzzy Control with Switching (4-AFCs)

% S R Ranatunga, 16/06/12
function [xi1,xi2,xi3,xi4] = mas4(sq1,sq2,sq3,sq4,t)
persistent k1;
persistent k2;
persistent k3;
persistent k4;
persistent tck;

if isempty(tck)
    tck = zeros(1);
end
if isempty(k1)
    k1 = zeros(1);
end
if isempty(k2)
    k2 = zeros(1);
end
if isempty(k3)
    k3 = zeros(1);
end
if isempty(k4)
    k4 = zeros(1);
end

%% Time gap between switching
T0=0.1e-2;

%% Initial setting
if (t<T0)
    xi1=1, xi2=0, xi3=0, xi4=0;
end

%% Switching
if (sq1==1)
    if (t>=tck+T0)
        k1=sq1, k2=0, k3=0, k4=0, tck=t;
    end
elseif (sq2==1)
    if (t>=tck+t0)
        k1=0, k2=sq2, k3=0, k4=0, tck=t;
    end
elseif (sq3==1)
    if (t>=tck+t0)
        k1=0, k2=0, k3=sq3, k4=0, tck=t;
    end
elseif (sq4==1)
    if (t>=tck+t0)
        k1=0, k2=0, k3=0, k4=sq4, tck=t;
    end
end

%% Output values
xi1=k1;
xi2=k2;
xi3=k3;
xi4=k4;
end