Seismic performance of lightly reinforced structural walls for design purposes

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Lightly reinforced concrete walls are commonly found in low-to-moderate seismic regions such as Australia. While many theoretical analyses on lateral load–displacement of structural walls have been proposed and widely used, not many have been developed for lightly reinforced concrete walls. The lateral load–displacement behaviour and failure mechanism of lightly reinforced structural walls differ to those of heavily reinforced concrete walls, particularly in terms of tension stiffening effects, possible failure mechanisms and drift capacities. An analytical study on lightly reinforced rectangular concrete walls is presented in this paper. A parametric study was conducted to provide initial insight into the effect of four design parameters (aspect ratio, axial load ratio, transverse reinforcement ratio and longitudinal reinforcement ratio) on the ultimate displacement capacity of reinforced concrete walls. Two analytical models were developed to predict the lateral load–displacement behaviour of lightly reinforced walls consisting of a detailed wall model and a simplified wall model that provides a quick and conservative estimate for initial design checking purposes using displacement-based principles. Both models are shown to provide good agreement with experimental results in the literature.

Introduction

A number of research studies and analytical models investigating the behaviour of well-detailed reinforced concrete (RC) wall structures have been undertaken for high-seismicity regions. Buildings supported by lightly reinforced structural walls (ρv = 0.2–2.0%) investigated in this study represent the great majority of building stock in low-to-moderate seismic regions such as Australia in both commercial and high-density residential sectors and buildings occupied by organisations with a post-disaster function such as hospitals and emergency services. In general, designers have a very good understanding of the strength characteristic of wall elements but have very little understanding of the corresponding drift behaviour, which is essential for assessing the earthquake performance of wall structures and is the focus of this paper.

Well-detailed RC walls subjected to lateral load are commonly believed to behave in a ductile manner with a high ultimate drift capacity compared with lightly reinforced concrete walls. Structural walls with minimum reinforcement requirements are considered to have very limited ductility and lateral displacement capacity. The displacement capacity values stipulated in design guidelines such as FEMA 356 (FEMA, 2000) for lightly reinforced walls are much lower than for beams and columns due to the high compressive and tensile strains expected to be developed at the wall extremities, particularly for non-symmetric wall sections. Experimental tests conducted by Greifenhagen and Lestuzzi (2005) showed that lightly reinforced concrete walls provide flexure-dominant behaviour and significant drift capacity. Therefore, this study aimed to investigate the behaviour of lightly reinforced rectangular concrete walls subjected to lateral load and to develop analytical models to estimate the lateral load–displacement relationship of such walls. The resulting capacity curves could be used in conjunction with the capacity spectrum method (ATC, 1996) to assist designers in assessing the seismic performance of low and medium-rise buildings (i.e. first mode dominant response), particularly for regions of lower seismicity where lightly reinforced concrete walls are common.

A literature review on the seismic performance of lightly reinforced rectangular concrete walls was undertaken and is described in the next section where the effects of four design parameters (axial load ratio, aspect ratio, transverse reinforcement ratio and longitudinal reinforcement ratio) are presented. The following section introduces the conceptual development of two analytical lateral load–drift models comprising a detailed wall model and a simplified wall model, which are then described comprehensively. The proposed models are then compared to experimental test results from a test database comprising squat to slender concrete walls with aspect ratio 0.5 ≤ a ≤ 4.0, axial load...
Parameters influencing wall behaviour

Parameters influencing the drift behaviour of RC walls have been investigated by the authors and collaborators (Althbee et al., 2012), comprising axial load ratio (n), aspect ratio (a), transverse reinforcement ratio ($\rho_v$) and longitudinal reinforcement ratio ($\rho_h$), as shown in Figure 1. The nominated design parameter is the only variable in each graph, all other design parameters being constant. The trends for each of the four design parameters are now summarised.

Aspect ratio (a)

The influence of the aspect ratio (ratio of wall height to length) of RC walls on the lateral load–displacement behaviour and failure modes can generally be observed by investigating the flexural-to-shear strength ratio ($\text{FSSR} = M/LV$). Generally, moderate ($1 < a < 2$) and slender RC walls ($a \geq 2$) possess a low FSSR and tend to develop flexure-dominant action characterised by a concentration of inelastic behaviour at the wall base (Chiou et al., 2003; Lestuzzi and Bachmann, 2007; Wood, 1989). In contrast, well-reinforced squat walls ($a \leq 1$) generally develop an in-plane strut-and-tie mechanism to resist lateral forces in which the failure mode is mainly dominated by a shear mechanism characterised by diagonal crack patterns (Li and Xiang, 2011).

The influence of the aspect ratio on the ultimate drift capacity of RC walls based on experimental testing is plotted in Figure 1(a) and shows some interesting trends.

- The drift capacity tended to increase with increasing aspect ratio for slender RC walls (Hines et al., 2002).
- The drift capacity was not well correlated with the aspect ratio for short and moderate RC walls ($a < 2$) (Kuang and Ho, 2007; Lefas et al., 1990; Liang et al., 2010).
- Squat RC walls do not necessarily develop shear-dominant behaviour, particularly if the walls are lightly reinforced. Kuang and Ho (2007) tested six squat walls with aspect ratios of 1.0 and 1.5 that failed in flexure despite a shear deflection component of about 20–50% of the total deflection. The reason for this behaviour is the inherent shear strength of concrete compared with the lateral force that could be developed in lightly reinforced and lightly loaded squat walls.

Axial load ratio (n)

The axial load ratio has a significant effect on the ultimate drift capacity of RC walls as shown in Figure 1(b). Most of the tests (Dazio et al., 2009; Greifenhagen and Lestuzzi, 2005; Lefas et al., 1990; Su and Wong, 2007) indicate that walls with a low axial load ratio exhibit ductile flexural failure, while walls with a higher axial load ratio demonstrate more brittle compressive failure due to the increase of neutral depth and hence the increase of compressive strains and decrease of ultimate curvature capacity.

Furthermore, Su and Wong (2007) showed that the rate of shear strength degradation of the walls increased with increasing axial load ratio, which is very similar to other research outcomes on lightly reinforced concrete columns (Wibowo, 2012).

Transverse reinforcement ratio ($\rho_v$)

The effect of the transverse reinforcement ratio on the ultimate drift capacity of RC walls is shown in Figure 1(c) with three different locations investigated: web region (dashed lines), boundary elements (dotted lines) and total area (solid lines). The ultimate drift of RC walls increases with increasing total transverse reinforcement ratio (as shown by solid lines), with a much more significant effect in the boundary elements (dotted lines) compared to the web regions (dashed lines).

The effect of effective confinement has been investigated by Su and Wong (2007) and Thomsen and Wallace (2004). Su and Wong (2007) demonstrated that an increase of the web transverse reinforcement ratio (using bundled bars to increase the ratio while maintaining a similar space between stirrups) from $\rho_v = 0.54\%$ (specimen W2) to $\rho_v = 1.08\%$ (specimen W3) did not alter the ultimate drift capacity. The effective confinement was largely dependent on the arrangement of transverse reinforcement, with the evenly separated and more closely spaced stirrups along the wall height providing higher confinement compared with bundled stirrups with a wide spacing but the same transverse steel ratio. Furthermore, Thomsen and Wallace (2004) tested two rectangular walls (RW1 and RW2) with identical configurations except for a different transverse reinforcement ratio at the boundary. Both specimens showed very similar behaviour, except that RW2 (with $\rho_v = 1.0\%$) was able to maintain complete cycles up to drift of 2.2% while RW1 (with $\rho_v = 0.7\%$) only managed to have a single cycle at 1.9% drift due to buckling of the longitudinal reinforcement. As expected, this demonstrated that the closer spacing of the hoops delayed the onset of buckling of the longitudinal reinforcement.

Longitudinal reinforcement ratio ($\rho_h$)

The effect of the longitudinal reinforcement distribution along the walls length is analysed in Figure 1(d). Cardenas and Magura (1975) observed that the walls with concentrated longitudinal reinforcement steel at the boundary edges have a higher ultimate curvature compared with the walls with uniformly distributed longitudinal reinforcement steel. The experimental results confirmed that concentrating the longitudinal reinforcement at the boundary increases the ultimate drift, as shown in Figure 1(d). However, a test study conducted by Kuang and Ho (2007) showed that walls with longitudinal steel concentrated at the boundaries have more diagonal shear cracks, more pinched hysteretic behaviour and less strain-hardening effect and hence are less likely to perform well compared to walls with uniformly distributed longitudinal reinforcement steel.
The effect of the overall longitudinal reinforcement ratio on the ultimate drift capacity of RC walls is shown in Figure 1(e), with mixed results observed. No significant changes on the ultimate drifts were apparent for the specimens tested by Gebreyohaness et al. (2011) and Cardenas and Magura (1975) despite the increase of longitudinal reinforcement ratio, while more significant effects on ultimate drifts were observed by Dazio et al. (2009), Pilakoutas and Elnashai (1995a, 1995b), Oesterle et al. (1979) and Cao et al. (2009). This mixed result is expected since walls are generally under-reinforced, resulting in tension failure.
unlabeled}

Summary

The overall trends of the different parameters on the ultimate drift capacity are mixed, indicating that the four parameters have some interdependence – particularly the aspect ratio and the longitudinal steel ratio. However, it was observed that the ultimate drift capacity increased with decreasing axial load ratio, increasing transverse steel ratio and increasing aspect ratio for slender walls.

Conceptual outline of theoretical models

Several models for predicting the lateral load–displacement relationship of RC walls have been proposed by many researchers. These range from simple bilinear models (Huang et al., 2011; Paulay, 2001; Wallace, 2007), simplified hysteretic models (Hidalgo et al., 2002), hysteretic models based on spring elements (Ghobarah and Youssef, 1999; Orakcal et al., 2004) to more complex finite-element models (Belmouden and Lestuzzi, 2007; Jalali and Dashki, 2010).

In this study, two models consisting of a detailed wall model and a simplified wall model were developed to estimate the lateral load–drift relationship for lightly reinforced rectangular concrete walls. In particular, the focus was to estimate the ultimate drift capacity of lightly reinforced rectangular walls for use in a displacement-based checking procedure in regions of low to moderate seismicity.

- The detailed wall model (outlined in the next section) was developed to provide a more comprehensive estimate of the lateral load–displacement behaviour of walls using a simple flexure-dominant approach and comprising four stages: cracking, yield, peak and ultimate. Walls are classified based on aspect ratio and axial load ratio.
- The simplified wall model was developed to provide a quick and conservative estimate for initial design checking purposes and comprised three stages: cracking, yield and ultimate.

The displacement at the top of cantilever walls consists of four components: flexural displacement ($\Delta_f$), yield penetration displacement ($\Delta_p$), shear displacement ($\Delta_s$) and sliding displacement between the wall and the foundation ($\Delta_d$). For a typical wall design, it can be reasonably assumed that the sliding displacement is sufficiently small to neglect, while the yield penetration displacement can be implicitly accommodated in the flexural displacement using a suitable plastic hinge length. The shear displacement for slender walls can be conservatively omitted in the analysis since the flexural component is dominant. Similarly, lightly reinforced squat walls tend to fail in flexure (i.e. low moment capacity compared with the concrete shear strength) with relatively high ductility and drift (Greifenhagen and Lestuzzi, 2005). Therefore, the lateral load–drift relationships for both the detailed wall model and the simplified wall model were developed based on flexure-dominant behaviour.

The flexural displacement of cantilever RC walls can be commonly analysed the same way as cantilever columns, where the total flexural displacement is calculated as the sum of the elastic displacement based on the moment–curvature relationship and the inelastic displacement based on the plastic hinge mechanism (Park and Paulay, 1975)

1. $\Delta_d = \Delta_f + \Delta_p$

Detailed wall model

A model developed for lightly reinforced concrete columns (Wibowo, 2012; Wibowo et al., 2013) was modified to create the detailed wall model due to the similar behaviour of the column and rectangular wall specimens with both being dominated by flexural action. As noted earlier, the detailed wall model comprises four stages (cracking, yield, peak and ultimate displacement), as shown conceptually in Figure 2.

Point A (cracking strength)

The cracked lateral strength ($F_{cr}$) and corresponding drift limit ($\gamma_{cr}$) are calculated from

2a. $F_{cr} = \frac{M_{cr}}{H_w}$

2b. $\gamma_{cr} = \frac{M_{cr}H_w}{3E_cI_g}$

where $M_{cr}$ is the bending moment to initiate cracking of the concrete based on a flexural tensile strength $f_c$ (taken as

![Figure 2. Detailed wall model for lateral load–drift behaviour](image-url)
0.6(f_c/y)^{1/2} consistent with AS3600 (Standards Australia, 2001), H_w is the height of the wall (which is subject to a point force at the top), E_c is Young’s modulus of concrete and I_g is the gross second moment of inertia of the wall cross-section.

Point B (yield strength)
The yield point is estimated using classic yield moment strength (M_y) calculation and an effective stiffness ratio (I_{eff}/I_g). A wide range of models available for estimating the effective moment of inertia of RC walls is listed in Table 1. The detailed wall model uses the Paulay and Priestley (1992) effective moment of inertia formula, despite the model not considering the tension stiffening effect. The tension stiffening effect is largely dependent on the longitudinal reinforcement ratio and is more significant on lightly reinforced members than more heavily reinforced sections. However, for members subjected to axial load such as columns and walls, the axial load ratio provides a more significant effect on the stiffness and the ultimate drift capacity rather than the longitudinal reinforcement ratio as previously discussed. Moreover, the effect of tension stiffening decreases rapidly under cyclic loading (Park and Paulay, 1975). Consequently, the Paulay and Priestley (1992) formula was selected as follows.

For flexure-dominated walls

\[
I_{eff} = \left( \frac{100}{f_y} + \frac{P_u}{f'_cA_g} \right) I_g
\]

For shear-dominated walls

\[
I_{eff} = \left[ \left( \frac{100}{f_y} + \frac{P_u}{f'_cA_g} \right) + \frac{P_u}{f'_cA_g} \right] I_g\]

\[
C = \frac{30\left( \frac{100}{f_y} + \frac{P_u}{f'_cA_g} \right) I_g}{H_w^2t_wL_w}
\]

where P_u is the nominal axial load, f_y is the yield strength of vertical reinforcement, f'_c is the cylinder compressive strength of concrete, A_g is the gross cross-sectional area of the wall, H_w is the wall height, L_w is wall length and t_w is wall thickness.

The yield strength (F_y) and corresponding drift limit (\gamma_y) are hence calculated as

\[
F_y = \frac{M_y}{H_w}
\]

5. \[
\gamma_y = \frac{M_yH_w}{3E_cI_{eff}}
\]

Point C (peak strength)
The peak strength point is estimated using classic ultimate moment strength (M_u) calculations and flexural displacement calculation as described conceptually in an earlier section. The available plastic hinge length formulae are presented in Table 2. Most of the formulae were developed for general structural systems, except for the formula proposed by Priestley et al. (2007), which was developed especially for structural wall buildings and therefore was used in the detailed wall model.

The model was developed by investigating curvature within the plastic hinge region using the force equilibrium equation (\sum \{C_c + C_s/\sqrt{T}\}) with a nominal \epsilon_cu = 0.003 spalling strain used as a limit state for concrete strain.

For simplicity, by eliminating either the compression steel area for the case of a low axial load ratio (tension control) or eliminating the tension steel area for the case of a high axial load ratio (compression control), the peak flexural lateral load F_u and the drift at concrete fracture \gamma_u can be obtained using classic RC principles as follows

\[
F_u = \frac{M_u}{H_w}
\]

6. \[
\gamma_u = \gamma_y + \gamma_{pl,p}
\]

where

\[
\gamma_{pl,p} = (\phi_{peak} - \phi_y)L_p
\]

7. \[
\phi_{peak} = \frac{\epsilon_cu}{k_ud} = 0.003
\]

8. \[
\phi_y = \frac{\gamma_y}{H_w}
\]

9. \[
\phi_u = \frac{3\gamma_y}{H_w}
\]

10a. \[
k_u = \frac{N + A_s f_y}{0.85 f'_c \beta d t_w}
\]

10b. \[
N = \frac{0.85 f'_c \beta d t_w}{k_u}
\]

for a low axial load ratio
### Design parameters affecting effective inertia moment

<table>
<thead>
<tr>
<th>Author(s)</th>
<th>Formula</th>
<th>Description</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Paulay and Priestley (1992)</td>
<td>( I_e = \left( \frac{100}{f_y} + \frac{P_u}{f_c A_g} \right) I_g )</td>
<td>Flexure-dominated walls</td>
<td>Yield strength of main rebar, Axial load ratio</td>
</tr>
<tr>
<td>( A_g )</td>
<td>Gross cross-section area of walls</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Shear-dominated walls</td>
<td>( I_w = \frac{I_e}{1.2 + C}, C = \frac{30 I_e}{L^2 D} )</td>
<td>Wall height, Wall thickness, Wall length</td>
<td></td>
</tr>
<tr>
<td>Fenwick and Bull (2000)</td>
<td>( I_e = 0.267 \left( 1 + 4.4 \frac{P_u}{f_c A_g} \right) \left( 0.62 + \frac{190}{f_y} \right) \left( 0.76 + 0.005 f_y \right) I_g )</td>
<td>Yield strength of main rebar, Axial load ratio</td>
<td>Concrete strength</td>
</tr>
<tr>
<td>Fenwick et al. (2001)</td>
<td>For grade 500 reinforcement</td>
<td>Axial load ratio</td>
<td>Yield strength of main rebar, Creep and shrinkage</td>
</tr>
<tr>
<td>( I_e = (0.21 + \frac{P}{f_c A_g}) I_g ) (including creep and shrinkage)</td>
<td>For grade 300 reinforcement</td>
<td>( I_e = (0.26 + 1.2P/f_c A_g) I_g ) (including creep and shrinkage)</td>
<td></td>
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<tr>
<td>( I_e = (0.31 + \frac{P}{f_c A_g}) I_g ) (neglecting creep and shrinkage)</td>
<td>( I_e = (0.44 + 1.2P/f_c A_g) I_g ) (neglecting creep and shrinkage)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>ASCE (2006)</td>
<td>For ( \mu = 1.25 )</td>
<td>Fixed modifier</td>
<td></td>
</tr>
<tr>
<td>( I_e = 0.8I_g ) (uncracked)</td>
<td>For ( \mu = 3.00 )</td>
<td>( I_e = 0.7I_g ) (uncracked)</td>
<td></td>
</tr>
<tr>
<td>( I_e = 0.5I_g ) (cracked)</td>
<td>For ( \mu = 6.00 )</td>
<td>( I_e = 0.35I_g ) (cracked)</td>
<td></td>
</tr>
<tr>
<td>ACI (2008)</td>
<td>For ( \mu = 1.25 )</td>
<td>Fixed modifier</td>
<td></td>
</tr>
<tr>
<td>( I_e = 0.7I_g ) (uncracked)</td>
<td>For ( \mu = 3.00 )</td>
<td>( I_e = 0.5I_g ) (cracked)</td>
<td></td>
</tr>
<tr>
<td>SNZ (1995)</td>
<td>For ( \mu = 1.25 )</td>
<td>Fixed modifier</td>
<td></td>
</tr>
<tr>
<td>( I_e = 0.45I_g ) (axial load ratio ( n = 0.2 ))</td>
<td>For ( \mu = 3.00 )</td>
<td>( I_e = 0.25I_g ) (0)</td>
<td></td>
</tr>
<tr>
<td>( I_e = 0.15I_g ) (0)</td>
<td>At serviceability limit state</td>
<td>( I_e = 0.40I_g ) (0)</td>
<td></td>
</tr>
<tr>
<td>Expected inelastic ductility demand ( \mu )</td>
<td>|</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( I_e = 0.25I_g ) (0)</td>
<td>At ultimate limit state</td>
<td>( I_e = 0.45I_g ) (axial load ratio ( n = 0.2 ))</td>
<td></td>
</tr>
<tr>
<td>( I_e = 0.15I_g ) (0)</td>
<td>For ( \mu = 6.00 )</td>
<td>( I_e = 0.40I_g ) (0)</td>
<td></td>
</tr>
<tr>
<td>Li and Xiang (2011) (model intended for squat structural walls)</td>
<td>Upper bound</td>
<td>Yield strength of main rebar, Axial load ratio</td>
<td>Aspect ratio</td>
</tr>
<tr>
<td>( I_e = 0.19 \left( \frac{100}{f_y} + \frac{P_u}{f_c A_g} \right) \left( 0.53 + 0.37 \frac{L}{D} + 0.31 \frac{L^2}{D^2} \right) I_g )</td>
<td>Lower bound</td>
<td>Axial load ratio</td>
<td></td>
</tr>
<tr>
<td>Adebar et al. (2007)</td>
<td>Upper bound</td>
<td>Axial load ratio</td>
<td></td>
</tr>
<tr>
<td>( I_e = \left( 0.6 + \frac{P}{f_c A_g} \right) I_g \leq 1.0I_g )</td>
<td>Lower bound</td>
<td>Axial load ratio</td>
<td></td>
</tr>
<tr>
<td>( I_e = \left( 0.2 + 2.5 \frac{P}{f_c A_g} \right) I_g \leq 0.7I_g )</td>
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<td></td>
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<tr>
<td>Table 1. Effective moment of inertia models for RC walls</td>
<td></td>
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</tbody>
</table>
\[ k_u = \frac{N - A_{sc} f_y}{0.85 f_y \beta d L_w} \] for a high axial load ratio

\[ k_u = \frac{N - A_{st} f_y}{0.85 f_y \beta d L_w} \] for a low axial load ratio where tension steel failure is the limiting criteria

\[ k_u = \frac{N - A_{sc} f_y}{0.85 f_y \beta d L_w} \] for a high axial load ratio where compression concrete crushing failure is the limiting criteria

\[ k_u = \frac{N - A_{st} f_y}{0.85 f_y \beta d L_w} \] for a low axial load ratio where compression concrete crushing failure is the limiting criteria
where $c_{env}$ is the ultimate confined concrete strain, $f_{sh}$ is the steel strain-hardening strain, $f_{yh}$ is the confinement steel yield strength and $r_h$ is the volumetric confinement steel ratio.

Lightly reinforced squat walls ($a \leq 1$)
The shear strength degradation mechanism developed for lightly reinforced concrete columns (Wibowo, 2012) was modified for lightly reinforced squat walls to calculate the ultimate drift. Lightly reinforced squat concrete walls and columns have a number of similarities, including the following.

- The rate of wall shear strength degradation increases with increasing axial load ratio.
- Experimental wall tests conducted by Beyer et al. (2008) and Vallenas et al. (1979) showed that the shear strains are concentrated in the plastic zones, particularly in cracked areas, and hence large tensile longitudinal steel strain occurred. This mechanism is similar to that of lightly reinforced concrete columns (Wibowo, 2012) in which a large portion of the shear deformations was concentrated in the plastic hinge region.

A summary of the methodology for assessing the ultimate drift capacity is provided in the following text and the reader is referred to Wibowo (2012) for further details. The shear strength ($V_u$) of RC walls consists of concrete strength ($V_c$) and steel strength ($V_s$) components

\[ V_u = V_c + V_s \]

The concrete shear strength in this model uses the formulae developed based on the principal tensile strength (Wibowo, 2012), while the steel shear strength component proposed by Wesley and Hashimoto (1981) is used as follows

\[ V_c = \frac{2}{3} A_{cr} \left( f_{cr}^2 + f_k^2 P \right)^{1/2} \]

\[ V_s = (c_h \rho_h + c_v \rho_v) f_j d_{lw} \]

The ultimate wall drift can be obtained using the formula developed by Wibowo (2012) for lightly reinforced columns

\[ \gamma_u = \frac{\gamma_y}{k} \left( 1 + k \alpha \right) - 0.8 \frac{F_y}{F_u} \]

and $\alpha$ is the drift ductility when the shear strength commences to decline and is taken as $a/n$ for squat walls ($a \leq 1$).

**Simplified wall model**
The simplified model aims to provide a simple and conservative procedure for estimating the lateral load–drift behaviour of lightly reinforced rectangular concrete walls. The method comprises three stages – cracking strength, yield strength and peak strength, as shown in Figure 3. The approach is considered reasonable for walls with an axial load ratio $n < 0.2$. 

![Figure 3. Simplified wall model for lateral load–drift behaviour](image)
Point A (cracking strength)
The cracked lateral strength and drift are directly calculated using the concrete modulus gross section properties and cracking strength (see Equation 2) and should result in a cracking drift \( \gamma_{cr} \) of the order of 0.05–0.10%.

Point B (yield strength)
The yield strength is estimated using classic yield moment calculations or approximated by the factored ultimate strength (assume \( \phi = 0.8 \) for \( n < 0.2 \))

27. \( F_y = \phi F_u \)

The yield drift is calculated using one of two methods – yield curvature or effective stiffness. The effective stiffness approach is recommended in particular for lightly reinforced concrete squat walls and walls where the cracking moment and ultimate moment strength are similar, such that distributed cracks up the height of the wall are unlikely to form.

Yield curvature approach
The yield drift can be calculated using the approximate yield condition shown in Figure 4, which is a reasonable representation for walls with a low axial load ratio (\( n < 0.2 \)). By substituting the approximate yield curvature (\( \phi_y = 2\varepsilon_y/L_w \)) into the elastic yield drift (\( \gamma_y = \Delta H_w = (\phi_y H_u)/3 \)), the yield drift can be estimated as a function of the aspect ratio (\( a \)) as follows

28. \( \gamma_y = \frac{2}{3} \varepsilon_y H_w \frac{L_w}{L_w} = \frac{2}{3} \varepsilon_y a \)

For typical walls, this will result in a yield drift \( \gamma_y \) in the order of 0.3–0.5% (i.e. \( f_y = 400 \text{ MPa}, a = 2–4 \)).

Effective stiffness approach
A wide range of effective stiffness models is available in the literature (see Table 1). The stiffness ratio \( I_{eff}/I_g \) is plotted in Figure 5 using these models for a rectangular wall with different axial load ratios. It is observed that the effective stiffness ranges between 0.2\( I_g \) and 0.8\( I_g \). For the simplified wall model, a value of \( I_{eff} = 0.5I_g \) is considered conservative and reasonable for estimating the yield drift and is supported by Wallace (2007) for axial load ratios up to \( n = 0.20 \).

Point C (ultimate strength)
The ultimate strength \( F_u \) is estimated to be equal to the normal design strength \( \phi F_u \) multiplied by an overstrength factor \( \Omega \) that accounts for strain-hardening and system effects. A default value of \( \Omega = 1.3 \) is recommended in the absence of more specific information for lightly reinforced walls, hence \( \Omega\phi = 1.3 \times 0.8 = 1.04 \) according to seismic design codes such as AS1170.4 (Standards Australia, 2007).

- Paulay and Priestley (1992)
- Fenwick and Bull (2000)
- Fenwick et al. (2001), including creep and shrinkage
- Fenwick et al. (2001), neglecting creep and shrinkage
- ASCE (2006)
- ACI (2008)
- SNZ (1995) \( u = 3 \) 0
- SNZ (1995) \( u = 6 \) 0
- Adebar et al. (2007), upper bound
- Adebar et al. (2007), lower bound

Figure 4. Yield curvature of RC wall

Figure 5. Comparison of the stiffness ratio of RC walls using different models
29. \( F_u = \Omega \phi F_u \)

The ultimate drift \( (\gamma_u) \) is estimated as the sum of the yield drift \( (\gamma_y) \) and the plastic drift \( (\gamma_{pl}) \) (see Figure 6).

30. \( \gamma_u = \gamma_y + \gamma_{pl} \)

The plastic drift is calculated one of two ways, depending on whether the cracking moment strength \( (M_{cr}) \) exceeds the ultimate moment capacity of the wall \( (M_u) \).

\( M_{cr} > M_u \)

If \( M_{cr} > M_u \), then only one crack is likely to form at the base of the wall and all inelastic action will be concentrated at this location, greatly reducing the effective plastic hinge length. The plastic drift can be estimated by assuming a maximum acceptable local peak strain in the reinforcement at a single crack at the wall base of the order of \( \varepsilon_s = 5.0\% \) for ductile reinforcement \( (2.5\% \) for low ductile steel), and taking a slightly more conservative approach than the Priestley and Paulay (2002) strain penetration length \( l_{yp} \) either side of the crack, producing an effective plastic hinge length of \( L_p = 15d_b \).

The resulting crack width \( W_{cr} \) and crack rotation, which is directly correlated with the plastic drift \( \gamma_{pl} \), are shown in Figure 7 and are given by

\[ W_{cr} = \phi_{pl}(\phi_s - \phi_{cr})L_p \]

\[ \theta_{pl} = \frac{W_{cr}}{L_w} = \frac{0.75d_b}{L_w} \]

\[ \gamma_{pl} = \frac{\phi_{pl} - \phi_s}{L_p} \]

\[ \varepsilon_{cu} = 0.4\% \]

\( M_{cr} < M_u \)

If \( M_{cr} < M_u \), multiple flexural cracks will develop up the height of the wall and an effective plastic hinge will develop at the base. The plastic hinge lengths presented in Table 2 range between \( 0.5L_w \) and \( 1.0L_w \) and, in this simplified wall, a conservative value of \( L_p = 0.5L_w \) is assumed. The ultimate curvature is calculated assuming concrete crushing \( (\varepsilon_{cu} = 0.4\%) \) and an average ultimate steel strain of \( 2.0\% \) for ductile reinforcement \( (1.0\% \) for low ductile steel) in the plastic hinge region occurs simultaneously, as shown in Figure 8.

The plastic drift \( \gamma_{pl} \) is estimated as

\[ \gamma_{pl} = (\phi_{pl} - \phi_s)L_p \]

Experimental test results

The proposed models were compared with results from the literature, which included 14 rectangular RC walls from four studies (Dazio et al., 2009; Greifenhagen and Lestuzzi, 2005; Su and Wong, 2007; Thomsen and Wallace, 2004). The database of wall specimens is summarised in Table 3 with the following range of properties:

- aspect ratio \( 0.5 \leq a \leq 4.0 \)
- axial load ratio \( 0.02 \leq n \leq 0.50 \)
- longitudinal reinforcement ratio \( 0.2\% \leq \rho_v \leq 2.0\% \)
- transverse reinforcement ratio \( 0.0\% \leq \rho_h \leq 1.0\% \).

The models proposed in this paper are intended to predict the lateral load–drift behaviour of rectangular RC walls and hence other cross-sectional shapes (e.g. H-shape, T-shape, core walls).
have not been included. The specimens in the database were selected to represent three categories – squat walls (\(a \leq 1\)), moderate walls (\(1 < a < 2\)) and slender walls (\(a \gg 2\)). The database consisted mostly of lightly reinforced concrete walls with \(\rho_v < 1.0\%\), although some RC walls with a combination of moderate longitudinal reinforcement and high axial load ratio (Su and Wong, 2007) were included to check the capability of the proposed models. A brief summary of results from each of the experimental studies is now given.

**Experimental test database**

**Greifenhagen and Lestuzzi (2005)**

Greifenhagen and Lestuzzi (2005) tested four squat RC walls with aspect ratios of \(a = 0.56\) (specimens M1 and M2) and \(a = 0.63\) (specimens M3 and M4) (Table 3). All specimens were lightly reinforced with a longitudinal reinforcement ratio of \(\rho_v = 0.34\%\) and a transverse reinforcement ratio of \(\rho_h = 0.3\%\) (except specimen M2, which had no transverse reinforcement), as shown in Figure 9. The axial load ratios varied between the specimens with values of \(n = 0.023\) (M1 and M2), \(n = 0.05\) (M4) and \(n = 0.10\) (M3).

Overall, the yield drifts for these squat walls were low with values in the range 0.11–0.23% and ultimate drifts in the range of 1.4–3.2%, with the lower values corresponding to the larger axial load ratios.

**Dazio et al. (2009)**

Six lightly reinforced moderate RC walls were tested by Dazio et al. (2009) with identical aspect ratios (\(a = 2.28\)) and transverse reinforcement ratios (\(\rho_h = 0.25\%\)) for all specimens (Table 3). The specimens differed in their axial load ratio (\(n = 0.05–0.13\)) and the layout, ductility and quantity of the longitudinal reinforcement as shown in Figure 10. All specimens had ductile reinforcement except specimen WSH1. Specimen WSH1 (\(n = 0.05, \rho_v = 0.54\%\)) had very low ductile steel (almost no strain-hardening and ultimate strain capacities of 4.6% and 2.3% for 10 mm and 6 mm bar diameters, respectively); hence, poor ductility behaviour was expected. Specimen WSH2 (\(n = 0.06, \rho_v = 0.54\%\)) had a very similar configuration to WSH1 but with normal ductile steel reinforcement (ultimate strain capacities of 7.7% and 5.8% for 10 mm and 6 mm bar diameters, respectively). Specimens WSH3 and WSH4 had an axial load ratio \(n = 0.06\) and almost twice the longitudinal reinforcement (\(\rho_v = 0.82\%\)) of WSH1, but specimen WSH4 was designed without proper confinement reinforcement in the boundary edges. Finally, two specimens (WSH5 and WSH6) had double the axial load ratios to that of WSH1 (\(n = 0.13\) and 0.11, respectively), while the longitudinal reinforcement ratio of WSH5 (\(\rho_v = 0.39\%\)) was half that of WSH6 (\(\rho_v = 0.82\%\)).

Overall, the yield drifts for these moderate walls were low with values in the range 0.14–0.25%, while the ultimate drifts were reasonable and in the range 1.0–2.0%. The lowest drift of 1.0% corresponded to specimen WSH1 with the low ductile reinforcement, as expected.

**Thomsen and Wallace (2004)**

Experimental tests on two slender rectangular walls (RW1 and RW2) with an aspect ratio \(a = 3.0\) were conducted by Thomsen and Wallace (Table 3). Both specimens had identical axial load ratios (\(n = 0.10\)) and very low longitudinal reinforcement ratios.

---

**Table 3. Wall properties of previous experimental work**

<table>
<thead>
<tr>
<th>Wall</th>
<th>(a)</th>
<th>(n)</th>
<th>(H_w:)</th>
<th>(L_w:)</th>
<th>(t_w:)</th>
<th>(\rho_v:)</th>
<th>(\rho_h:)</th>
<th>Drift: %</th>
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<tbody>
<tr>
<td>Greifenhagen and Lestuzzi (2005)</td>
<td></td>
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<tr>
<td>M1</td>
<td>0.56</td>
<td>0.02</td>
<td>565</td>
<td>1000</td>
<td>100</td>
<td>0.34</td>
<td>0.31</td>
<td>0.11</td>
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<tr>
<td>M2</td>
<td>0.56</td>
<td>0.02</td>
<td>565</td>
<td>1000</td>
<td>100</td>
<td>0.34</td>
<td>0.00</td>
<td>0.23</td>
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<tr>
<td>M3</td>
<td>0.63</td>
<td>0.10</td>
<td>565</td>
<td>900</td>
<td>80</td>
<td>0.39</td>
<td>0.26</td>
<td>0.12</td>
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<tr>
<td>M4</td>
<td>0.63</td>
<td>0.05</td>
<td>565</td>
<td>900</td>
<td>80</td>
<td>0.39</td>
<td>0.26</td>
<td>0.21</td>
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<td>Dazio et al. (2009)</td>
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<td></td>
<td></td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>WSH1(^a)</td>
<td>2.28</td>
<td>0.05</td>
<td>4560</td>
<td>2000</td>
<td>150</td>
<td>0.54</td>
<td>0.25</td>
<td>0.18</td>
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<tr>
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<td>0.06</td>
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<td>2000</td>
<td>150</td>
<td>0.54</td>
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<tr>
<td>WSH3</td>
<td>2.28</td>
<td>0.06</td>
<td>4560</td>
<td>2000</td>
<td>150</td>
<td>0.82</td>
<td>0.25</td>
<td>0.25</td>
</tr>
<tr>
<td>WSH4</td>
<td>2.28</td>
<td>0.06</td>
<td>4560</td>
<td>2000</td>
<td>150</td>
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<td>0.25</td>
<td>0.25</td>
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<tr>
<td>WSH5</td>
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<td>2000</td>
<td>150</td>
<td>0.39</td>
<td>0.25</td>
<td>0.14</td>
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<tr>
<td>WSH6</td>
<td>2.26</td>
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<td>4520</td>
<td>2000</td>
<td>150</td>
<td>0.82</td>
<td>0.25</td>
<td>0.22</td>
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<td>Thomsen and Wallace (2004)</td>
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<tr>
<td>RW1</td>
<td>3.00</td>
<td>0.10</td>
<td>3658</td>
<td>1219</td>
<td>102</td>
<td>0.17</td>
<td>0.47</td>
<td>0.77</td>
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<tr>
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<td>0.10</td>
<td>3658</td>
<td>1219</td>
<td>102</td>
<td>0.17</td>
<td>0.47</td>
<td>0.74</td>
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<td>Su and Wong (2007)</td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>W1</td>
<td>4.00</td>
<td>0.25</td>
<td>1600</td>
<td>400</td>
<td>80</td>
<td>1.96</td>
<td>0.52</td>
<td>0.72</td>
</tr>
<tr>
<td>W2</td>
<td>4.00</td>
<td>0.50</td>
<td>1600</td>
<td>400</td>
<td>80</td>
<td>1.96</td>
<td>0.54</td>
<td>0.61</td>
</tr>
</tbody>
</table>

\(^a\) Non-ductile reinforcement used in this test
Figure 9. Reinforcement layout of wall specimens (a) M1 and M2 and (b) M3 and M4 (Greifenhagen and Lestuzzi, 2005) (dimensions in mm)
(ρv = 0.17%), while the transverse reinforcement ratio of RW1 (ρh = 0.47%) was smaller than that of RW2 (ρh = 0.70%), as shown in Figure 11.

The yield drifts for these slender walls were relatively large with values of the order of 0.75%, while the ultimate drifts were reasonable and in the range of 1.9–2.2%. Interestingly, with such low reinforcement ratios and corresponding ultimate moment capacity, the flexural behaviour was dominated by a single crack at the base of the wall.

Su and Wong (2007) tested three slender RC walls subjected to high axial load ratios. All specimens were designed with identical aspect ratios of a = 4.0 and large longitudinal reinforcement ratios of ρv = 1.96%, while the axial load ratio and the transverse reinforcement ratio were varied. Specimen W1 was designed with the smallest axial load ratio of n = 0.25, which was half that of specimens W2 and W3 (n = 0.50). Specimens W2 and W3 differed only in the transverse reinforcement ratios (ρh = 0.54% and ρh = 1.08%, respectively) with the higher transverse reinforcement ratio of W3 achieved using bundled bars rather than spreading the spacing evenly along the specimen height. Only specimens W1 and W2 are included in Table 3 and Figure 12 since the lateral load–drift behaviour of wall W3 was similar to W2.

Overall, the yield drifts for these slender, heavily loaded and heavily reinforced walls were relatively large with values in the range 0.6–0.7%; the ultimate drifts were reasonable, in the range 1.4–2.7%. The lowest drift of 1.4% corresponded to specimen W2 with the high axial load ratio of n = 0.50.

Comparison between experimental results and proposed model
The experimental test data are compared with the proposed models using test results from Greifenhagen and Lestuzzi (2005), Dazio et al. (2009), Thomsen and Wallace (2004) and Su and Wong (2007) in Figures 13–16, respectively. The following points may be noted.

- Good agreement was found between the squat wall test data (a ≪ 1.0) of Greifenhagen and Lestuzzi (2005) and the predicted results, as shown in Figure 13. This confirms that...
Figure 11. Reinforcement layouts of wall specimens RW1 and RW2 (Thomsen and Wallace, 2004) (dimensions in inches, 1 inch = 25.4 mm)

Figure 12. Details of wall specimens W1 and W2 (Su and Wong, 2007) (dimensions in mm)
the lightly reinforced concrete squat walls were flexure-dominant despite the low aspect ratio. The simplified model tended to underestimate the ultimate drift capacities, but provides a useful quick lower bound estimate for designers.

Both the detailed wall model and simplified wall model are generally in good agreement with the moderate wall specimens ($a = 2.0$) of Dazio et al. (2009), as shown in Figure 14. It should be noted that the low ductile steel
Figure 14. Lateral load–drift behaviour: comparison between theoretical models (left, detailed wall model; right, simplified wall model) and experimental data of Dazio et al. (2009)
Figure 15. Lateral load–drift behaviour: comparison between theoretical models (left, detailed wall model; right, simplified wall model) and experimental data of Thomsen and Wallace (2004)

Figure 16. Lateral load–drift behaviour: comparison between theoretical models (left, detailed wall model; right, simplified wall model) and experimental data of Su and Wong (2007)
properties were used in the detailed and simplified wall models.

Both wall models are in good agreement with the test results for slender walls \((a = 3.0)\) with very light longitudinal reinforcement ratio (Thomsen and Wallace, 2004) and slender walls \((a = 4.0)\) with heavier longitudinal reinforcement ratio and higher axial load ratio (Su and Wong, 2007). This demonstrates that the models are capable of predicting the lateral load–displacement behaviour of slender walls with an extended range of longitudinal reinforcement ratio \((\rho_n > 2.0\%)\) and axial load ratio \((n > 0.20)\).

Overall, the detailed wall model prediction of the lateral load–drift behaviour shows good agreement with the experimental results. The simplified wall model provides a more conservative approach in estimating the drift capacity of walls, which can be very useful for an initial design check. It should also be noted that the simplified wall model is more suitable for moderate and slender walls rather than squat walls since it tends to underestimate the ultimate drift of RC walls with aspect ratios less than 1:0.

Interestingly, the yield drifts for all walls varied widely between 0.1 and 0.7\%, and the ultimate drifts were reasonable and in the range 1.3–3.2\%, except specimen WSH1 where the low ductile longitudinal reinforcement resulted in a maximum drift of 1.0\%.

Conclusions

In general, designers have a very good understanding of the strength characteristic of wall elements but have very little understanding of the corresponding drift behaviour, which is essential for assessing the earthquake performance of wall structures.

A literature study was undertaken to investigate parameters affecting the lateral load–displacement behaviour of RC rectangular walls, particularly lightly reinforced RC walls. Four parameters were investigated – aspect ratio, axial load ratio, transverse reinforcement ratio and longitudinal reinforcement ratio. The overall trends of the different parameters on the ultimate drift capacity are mixed, indicating that these parameters have some interdependence (particularly the aspect ratio and the longitudinal steel ratio). However, it was observed that the ultimate drift capacity increased with decreasing axial load ratio, increasing transverse steel ratio and increasing aspect ratio for slender walls.

Two models – a detailed wall model and a simplified wall model – were developed to predict the lateral load–displacement relationship of RC rectangular walls. Both models were compared with experimental results in the literature, consisting of 14 rectangular RC walls from four studies with aspect ratio \(0.5 < a < 4.0\), axial load ratio \(0.02 < n < 0.50\), longitudinal reinforcement ratio \(0.2\% < \rho_n < 2.0\%)\) and transverse reinforcement ratio \(0.0\% < \rho_t < 1.0\%\).

The yield drifts for all walls varied widely between 0.1 and 0.7\%. The ultimate drifts were reasonable, in the range 1.3–3.2\%, except specimen WSH1 where the low ductile longitudinal reinforcement resulted in a maximum drift of 1.0\%.

Overall, the detailed wall model provided good correlation with the experimental results, while the simplified wall model provided a conservative and quick guide for the initial checking of wall drift capacities, particularly for walls with an axial load ratio \(n < 0.20\). The simplified wall model is considered a very useful tool for initially assessing the seismic performance of buildings in regions of lower seismicity, using the capacity spectrum method and displacement principles.

Acknowledgement

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