Simplified Analysis of Near Electromagnetic Fields from a Dipole in Lossy Dielectric

Teddy Kurniawan, Andrew W. Wood  
Brain Science Institute  
Swinburne University of Technology  
Melbourne, Australia  
awood@swin.edu.au

and Robert L. McIntosh  
Australian Centre for RF Bioeffects Research  
Melbourne, Australia  
Robert.L.McIntosh@team.telstra.com

ABSTRACT
This paper will consider a case for a dipole immersed in a lossy dielectric medium. Previous research has yielded relatively complex non-closed form formulae that still need numerical computation. Here we consider the simplification of the analysis of near field exposure although the analysis converges in the far field region. The usefulness of the closed-form formulae are verified through comparison with simulation results from a Method of Moments solver and then implemented in MATLAB to enable further analysis. The overall accuracy was better than 11% overall in terms of normalized root mean square error.

Index Terms — Dipole antennas, dielectric materials, absorbing media, computer aided analysis.

1 INTRODUCTION

THE ubiquitous dipole has found its way into many theoretical analyses and practical applications. This is due to its simple form, yet it is still practical when assessing new technologies such as mobile communications and microwave hypothermia [1, 2]. Furthermore, due to reciprocity for antenna radiation patterns [3], one could also model a thin cylindrical metal inserted in a human body as a dipole in a lossy dielectric medium. This is relevant when studying the health implications for people, such as RF workers, with metallic implants who are exposed to significant RF fields. Increasingly, RF devices are being used in therapeutic (catheter-mounted) or diagnostic situations which are appropriately modeled as an insulated dipole in a conducting medium [4, 5].

Previously, King et al in [2], analysed the electromagnetic fields (EMF), and subsequently the power dissipated, within a lossy dielectric medium surrounding an insulated dipole (the insulation to prevent conduction current flowing in the surrounding lossy medium). This work was performed primarily to investigate the application of inserting an insulated dipole to produce hyperthermia in conjunction with radiation therapy. The work by King has then been extended by Casey and Bansal [6], through the use of direct numerical evaluation instead of numerical approximation. Nevertheless, both approaches yield relatively complex non-closed form formulae that still need numerical computations.

This paper will consider the simplification of the analysis of near field exposure due to an immersed dipole in the lossy dielectric medium, although the analysis can be extended to the far field region. The near field distance here is defined as the region in which the radial electric field component radiated by a dipole has a significant value [7]. This is approximated to be $\lambda/2\pi$ in free space, where $\lambda$ is the RF free space wavelength. It has been observed empirically through the work carried out in this paper that such a distance varies inversely with the square root of the relative permittivity. In this paper, the relative permittivity value of the lossy dielectric medium being considered is in the range 5 to 56. The conductivity of the medium is chosen to be less than 1 S/m. A higher value of relative permittivity tend to be associated with a higher values of conductivity [8-12]. These relative permittivity and conductivity values can be taken to represent fat, skin, skull and brain tissues. Nevertheless, the near field will be in the range of 10 – 50 mm for a dipole immersed in the medium, whose relative permittivity values are as above.

A set of analytical closed-form formulae and their usefulness will be presented in this paper to elucidate the
relations between relevant parameters and the value of induced EMF, which result in effective and efficient computations. This is performed heuristically by extending the use of the simple closed-form analytical formulae of Balanis in [3]. The formulae are extended from determining the EMF in free space from a bare thin finite length dipole, to determining the EMF in lossy homogeneous medium by using correction factors following a comparison with King et al. in [2]. These closed-form formulae are verified through comparison with simulation results from a Method of Moments integral formulation of Maxwell’s equations (FEKO: EM Software & Systems, Stellenbosch, South Africa). The formulae are then implemented in MATLAB (see appendix and 2.7 – 2.9 below).

2 METHODS

The investigation in this paper considers the scenario used by King et al, which consists of:

- a thin wire dipole of perfect conductor
- a centre-fed dipole
- thin insulated dipole, whose insulation has a relative permittivity between 1.4 and 3, with zero conductivity and relative permeability value of one
- sinusoidal current distribution,
- lossy homogeneous medium: its relative permittivity is larger than the relative permittivity of the insulator (between 5 – 56), with non-zero value of conductivity (between 0 and 1 S/m).

The relative permeability value is unity for any dielectric of interest in this paper. Furthermore, assumption of infinitesimal gap for the centre of the dipole is additionally employed. The dipole length considered here is less than half of the free space wavelength at 900 MHz RF signals. Moreover, the analysis will emphasise the magnitude of current fed into the antenna instead of the input power, as well as focusing on the direct induction of EMF from the dipole, in line with the work of King et al [2].

Balanis in [3], has presented a relatively simple closed-form formulae, in determining the EMF of a centre-driven thin wire finite length bare dipole in the free space (see Appendix). Here, we extend the formulae use for insulated dipole in lossy homogeneous medium by using correction factors following a comparison with King et al in [2]. These closed-form formulae are verified through comparison with simulation results from a Method of Moments integral formulation of Maxwell’s equations (FEKO: EM Software & Systems, Stellenbosch, South Africa). The formulae are then implemented in MATLAB (see appendix and 2.7 – 2.9 below).

![2.1 SHORT DIPOLE](image)

Taking into account the existence of a ratio between the wave number for the insulator and the wave number for the lossy dielectric introduced in the work by King et al in [2], a correction factor $c_s$ is introduced to compute the induced near electric field $E$ in the free space is then introduced heuristically as:

$$c_s = \left| \frac{k_{L}}{k_{h}} \right| \left[ \frac{1}{\sin \left( \frac{|k_{L}| l_d}{2} \right)} \right]$$

(2.6)

where $l_d$ is the dipole length, $k_L$ and $k_h$ are the wavenumbers for the insulator (that accounts for the lossy dielectric medium surrounding it) and the ambient lossy dielectric medium, with $k_L < k_h$. The corrected $E$ value is found from the value computed by the Balanis formulae multiplied by $c_s$.

2.2 FINITE LENGTH DIPOLE

Assume a finite length dipole, whose length $l_d$ is $0.1\lambda < l_d < 0.5\lambda$. (with $\lambda$ being the free space wavelength). There exists a formula derived by King et al in [2] to determine the length of the dipole where sinusoidal current distributions are assumed, in which $l_d$ is given by:

$$l_d = \frac{\pi}{\beta_L}$$

(2.7)

where $\beta_L$ is the real component of complex wave number $k_L$. 

\[ \nabla \times \mathbf{H} = \sigma \mathbf{E} + j \omega \varepsilon \mathbf{E} \]

\[ = (\sigma + j \omega \varepsilon) \mathbf{E} \]

\[ = \left( \varepsilon - \frac{j \sigma}{\omega} \right) j \omega \mathbf{E} \]

or

\[ \nabla \times \mathbf{H} = \varepsilon_h j \omega \mathbf{E} \]

(2.5)
The introduction of the criterion in equation (2.7), although imposing a limitation on the varying length of the dipole that can be observed, still gives a high variation on discrete length of the dipole that is practical for usage. The dipole’s length can be varied from 43 to 82 mm, where the relative permittivity of the dipole’s insulator ranges from 1.4 to 3.0, for different lossy dielectric properties \( \varepsilon_r \) in 900 MHz case. Furthermore, the range of complex characteristic impedance can be obtained using a formula given in [2] (equation (4)) for the described conditions.

### Table 1. Complex antenna impedance for finite length dipole.

<table>
<thead>
<tr>
<th>Relative Permittivity of the Insulator</th>
<th>( \beta ) [m⁻¹]</th>
<th>Complex Antenna Impedance [Ω]</th>
<th>Dipole Length [m]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Real</td>
<td>Imaginary</td>
<td></td>
</tr>
<tr>
<td>1.40</td>
<td>53.75</td>
<td>55.29</td>
<td>-11.30</td>
</tr>
<tr>
<td>1.60</td>
<td>56.87</td>
<td>56.45</td>
<td>-11.68</td>
</tr>
<tr>
<td>1.80</td>
<td>59.70</td>
<td>57.62</td>
<td>-12.06</td>
</tr>
<tr>
<td>2.00</td>
<td>62.30</td>
<td>58.78</td>
<td>-12.43</td>
</tr>
<tr>
<td>2.20</td>
<td>64.71</td>
<td>59.94</td>
<td>-12.81</td>
</tr>
<tr>
<td>2.40</td>
<td>66.94</td>
<td>61.11</td>
<td>-13.19</td>
</tr>
<tr>
<td>2.60</td>
<td>69.01</td>
<td>62.27</td>
<td>-13.56</td>
</tr>
<tr>
<td>2.80</td>
<td>70.95</td>
<td>63.43</td>
<td>-13.94</td>
</tr>
<tr>
<td>3.00</td>
<td>72.78</td>
<td>64.60</td>
<td>-14.31</td>
</tr>
</tbody>
</table>

An example of values of \( \beta \) and characteristic impedance of the antenna, in relation with the recently mentioned parameters, are tabulated in Table 1 for the insulated dipole immersed in ambient medium of \( \varepsilon_r = 45.8 \) and \( \sigma_b = 0.77 \) S/m (values are close to skin or brain electrical properties). Longer dipoles can also be considered for lower values of the conductivity or permittivity of the insulator, which results in higher antenna’s characteristic impedance. Higher value of conductivity and permittivity of the insulator will require shorter length and lower value of the antenna’s characteristic impedance.

By comparing computation results from modified Balanis formulae (substituting \( \beta \) with \( k \)) with results from FEKO, the correction factor \( c_R \) can be heuristically approximated as:

\[
c_R = \begin{cases} \left( \frac{k_L}{k_o} \right) & \text{for } 0.75 < \left( \frac{k_L}{k_o} \right) < 0.95 \\ \exp \left( -0.002 \frac{k_L}{k_o} \sqrt{\frac{\lambda}{\rho}} \right) & \text{for } \left( \frac{k_L}{k_o} \right) < 0.75 \end{cases} \tag{2.8}
\]

For values of \( k_L / k_o > 0.95 \), the correction factor is set to one because the amount of correction was deemed insignificant.

Equation (2.8) is also obtained by considering varied length of the dipole, as well as the varied relative permittivity and conductivity of the ambient medium. The observed magnitude obtained by Balanis formulae of electric field components (multiplied by \( c_R \)), at various positions away from the dipole, is compared with results from FEKO. Afterwards, the ratio is profiled, and the correction factor as described by equation (2.8) is introduced to minimise the magnitude differences of results from Balanis formulae and FEKO. For the computation of the induced near magnetic field, use of the correction factor from equation (2.9) gives an error less than 11% - see the discussion in the next section.

### 3 VERIFICATION OF THE SIMPLIFIED ANALYSIS

The verification work is performed by showing the antenna’s current distribution \( I(z) \) from FEKO, as well as comparison between the results from FEKO and MATLAB on the induced electric field \( E \) and the induced magnetic field \( H \). This is for conditions of a dipole in lossy dielectric medium. The comparisons are made by observing the field magnitudes along the dipole axis, with variation of:

- the observation distance to the dipole,
- the dipole’s length, and
- relative permittivity and conductivity of the lossy dielectric

Furthermore, simulation results from FEKO were also investigated for different resolutions. It has been observed in FEKO that the various resolutions converge well with the 1 mm resolution that is used in the extensive verifications in this paper. Nevertheless, due to efficiency in the presentations, these were not shown in this paper.

Throughout this paper, the results from FEKO were obtained using FEKO Suite 5.3, which includes CADFEKO version 3.2 and EDITFEKO. The simulations using FEKO were run on a computer with Linux OS, with Intel Xeon Dual Processor 2.4 GHz and with memory of 4 GB. It could be observed that there was a contrast in time required for simulating the developed analysis in MATLAB and in FEKO. Computations in MATLAB only required a few seconds to tens of seconds using a PC with Intel Pentium 4 of 2 GHz processor and with 1 GB of memory; while simulations with FEKO required 5 – 30 minutes. This comparison was for the computation of the magnitudes and profile of the electric and magnetic field, for the same area and resolutions. Indeed, the comparison was observed only for the described investigation scenario in this paper.

For the investigations in this paper, FEKO simulations were performed utilising CADFEKO, as a visual tool in constructing:

- the geometry of the dipole and the dielectric object where the dipole is immersed.
- the dielectric properties of materials used in simulations
- the mesh for the geometry
- the port in the centre of the dipole as the voltage source with certain frequency of signals fed into the dipole

as well as in setting the output parameters, such as the region of observation and types of observation (current profile of the antenna, electric and magnetic field).
First, a line was built with perfect electrical conductor material, representing a thin dipole, where the line length (or the dipole length) was also set. Afterwards, a cube was built to enclose the line, with side’s length is of 1 free space wavelength. Dielectric properties of two materials, the lossy dielectric where the dipole is immersed, as well as the insulator coating the dipole, were also set. The mesh for the geometry was then set, where the edge length for the triangle structures in the cube was set for 0.025 m, and the segment length for the line was set between 0.001 and 0.004 m. The segment length is varied due to the different length of the dipole investigated, while also requiring:

- an odd number of segments
- the ratio of wire radius (which was set while setting the mesh) to the segment needs to be less than 0.25, in accordance with the FEKO manual

The number of segments was chosen to be from 21 – 29, with the wire radius set to 0.3 mm. This number of segments was able to produce appropriate current distribution of the antenna without losing the details of the profile nor excessive use of computing resource, while the wire radius is in the range of thin wire terminology according to [3]. The middle segment of the line was dedicated for the port of the voltage source, with the frequency and amplitude also specified. The value of the voltage source was set according to the estimate of the antenna impedance, which can be computed using a formula in [3], as well as trial and error process of running simulations in FEKO, to ensure that the specified peak current is obtained. It should be noted that FEKO has a number of built in constants, such as the permittivity and permeability of air. After setting up these inputs, the simulation was executed, and the results outputted into a file that was read into MATLAB. This information was, in turn, used for comparison with the simulation results determined using MATLAB.

It is necessary to obtain the range of applications for the developed analytical methods. This is performed by comparing the magnitude of induced electric and magnetic fields from analytical computations with results from FEKO. A normalized root mean square error (NRMSE) as a percentage is then introduced for each of the profiles that are presented here (N is typically 100).

For relative permittivity and conductivity of 45.8 and 0.77 S/m respectively for this type of lossy dielectric, the current distribution profile for a varied length of short dipole based on FEKO simulation is presented in Figure 1. The magnitude and frequency of signal fed into the dipole are respectively 0.01 A (peak) and 900 MHz, where the relative permittivity of the insulator is between 2.6 – 2.8 with zero conductivity. The diameter of the wire, including the insulator, is assumed to be 0.001 m. In Figures 2 – 3, the induced total electric field within the lossy dielectric medium, along the dipole axis at distances of 2, 3, 5, and 9 mm from the dipole, are observed. These are for varied length of the dipole of 10, 20 and 30 mm. Both results from FEKO simulations and the analytical based computations in MATLAB are shown.

Table 2 provides the NRMSE of the analytical approach, relative to the FEKO results, for the computed induced near EMF in lossy dielectric medium of $\varepsilon_r = 45.8$ and $\sigma = 0.77$ S/m for the short dipole discussed from Figures 1 – 3.
Verification is also performed for a finite length dipole. The magnitude and frequency of a signal fed into the dipole are respectively 0.1 A (peak) and 900 MHz, where the relative permittivity of the insulator is in the range of 1.4 – 2.4, with zero conductivity. The diameter of the wire, including the insulator, is assumed to be 1.0 mm. Two cases of relative permittivity and conductivity of 45.8 and 0.77 S/m, and 16.6 and 0.24 S/m are respectively observed as the ambient lossy dielectric. The current distribution profile based on a FEKO simulation, for two possible lengths of the dipole, is presented in Figures 4 and 5. The dipole length is chosen to be 52 and 50 mm respectively. Figures 6 – 7 show the induced total electric field within the lossy dielectric medium, observed along the dipole axis at distances of 2, 3, 5, 9 and 15 mm from the dipole. Both results of FEKO simulations and the analytical computations in MATLAB are shown.

Due to space limitation, the complete NRMSE tables will not be provided. However, it is found that the maximum NRSME for Figure 7 is less than 11%.

### Table 2. Values for the Short Dipole from Figures 1 – 3.

<table>
<thead>
<tr>
<th>Dipole Length [mm]</th>
<th>Distance from the dipole [mm]</th>
<th>Induced electric field NRMSE [%]</th>
<th>Induced electric field Peak [V/m]</th>
<th>Induced magnetic field NRMSE [%]</th>
<th>Induced magnetic field Peak [A/m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>2</td>
<td>2.21</td>
<td>51.64</td>
<td>1.13</td>
<td>0.60</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>2.32</td>
<td>25.89</td>
<td>1.47</td>
<td>0.35</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>2.65</td>
<td>9.45</td>
<td>2.54</td>
<td>0.17</td>
</tr>
<tr>
<td></td>
<td>9</td>
<td>3.25</td>
<td>3.01</td>
<td>5.09</td>
<td>0.07</td>
</tr>
<tr>
<td>20</td>
<td>2</td>
<td>6.02</td>
<td>34.37</td>
<td>1.83</td>
<td>0.73</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>5.62</td>
<td>21.36</td>
<td>2.23</td>
<td>0.46</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>5.43</td>
<td>10.96</td>
<td>3.32</td>
<td>0.25</td>
</tr>
<tr>
<td></td>
<td>9</td>
<td>5.73</td>
<td>4.85</td>
<td>5.65</td>
<td>0.12</td>
</tr>
<tr>
<td>30</td>
<td>2</td>
<td>11.34</td>
<td>26.82</td>
<td>3.12</td>
<td>0.78</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>11.05</td>
<td>17.87</td>
<td>3.53</td>
<td>0.51</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>10.76</td>
<td>10.72</td>
<td>4.61</td>
<td>0.30</td>
</tr>
<tr>
<td></td>
<td>9</td>
<td>10.64</td>
<td>5.97</td>
<td>6.74</td>
<td>0.16</td>
</tr>
</tbody>
</table>

**Figure 4.** Observed current distribution from FEKO simulations for insulated finite length dipole in lossy dielectric medium whose relative permittivity and conductivity is 16.6 and 0.24 S/m.

**Figure 5.** Observed current distribution from FEKO simulations for insulated finite length dipole in lossy dielectric medium whose relative permittivity and conductivity is 45.8 and 0.77 S/m.

**Figure 6.** Observed induced total electric field from FEKO simulation results and analytical computations in MATLAB simulation results for insulated finite length dipole of 52 mm length in lossy dielectric medium whose relative permittivity and conductivity is 16.6 and 0.24 S/m.

**Figure 7.** Observed induced total electric field from FEKO simulation results and analytical computations in MATLAB simulation results for insulated finite length dipole of 50 mm length in lossy dielectric medium whose relative permittivity and conductivity is 45.8 and 0.77 S/m.

### 4 DISCUSSION

It can be seen that for a higher relative permittivity (>45.8) of the ambient medium, the use of proposed analytical approach for finite length dipole field analysis has its limitation. Nevertheless, the proposed analytical approach investigation of the peak and location remains relevant. Furthermore, a maximum NRMSE less than 11% for the brain or skin like ambient lossy dielectric, is well below the recommended margin of error by Australian Radiation
Protection And Nuclear Safety Agency (ARPANSA) and the International Electrotechnical Commission (IEC) [13, 14]. The proposed simplified analytical approach in this paper is thus useful for RF dosimetry assessment, which is able to give further insight on the characterization of near field from a dipole antenna.

**APPENDIX**

MATLAB™ routines were developed to code the following analytical formulae. Solutions of fields around the dipole in lossy media are then given by multiplying the field values by the appropriate correction factors (c_s or c_fl) appropriate to the circumstance (2.7 and 2.9).

Balantis analytical formulae, to estimate the near field of a thin finite length dipole, were developed based on the following assumptions of the dipole:

- free space surroundings
- sinusoidal current distributions
- centre-fed dipole with infinitesimal gap
- perfect conductor material for the dipole
- fed by radio frequency band signals

The sinusoidal current distribution is a close approximation for a thin dipole, whose length is in practice usually less than 0.5 wavelength [3, 15]. The geometrical reference used in developing the model is as follows:

![Diagram](image)

**Figure A.1.** The geometry of the investigated finite length dipole (the x-axis is penetrating the paper).

The formulae are represented mathematically as:

\[
\mathbf{H} = \mathbf{a}_\phi \mathbf{H}_\phi = \text{Re}(e^{j\omega t}) \mathbf{a}_\phi \frac{-jI_0}{4\pi} \times \left[ e^{-j\beta_1 R_1} + e^{-j\beta_2 R_2} - e^{-j\beta_2 R_1} 2\cos \frac{\beta l_{d}}{2} \right]
\]

\[
\mathbf{E}_x = \text{Re}(e^{j\omega t}) \left( \mathbf{a}_z \frac{-j\mu_0 I_0}{4\pi} \right) \times \left\{ \frac{e^{-j\beta_1 R_1}}{R_1} + \frac{e^{-j\beta_2 R_2}}{R_2} - 2 \frac{e^{-j\beta_1 R}}{r} \cos \frac{\beta l_{d}}{2} \right\}
\]

where:

- \( \omega \) = angular frequency of the RF signal employed by the dipole
- \( t \) = time
- \( I_0 \) = amplitude of the signal current fed into the dipole
- \( \beta_0 \) = free space wave number
- \( l_{d} \) = the length of the dipole
- \( \eta \) = intrinsic impedance = \( \sqrt{\frac{-j\omega \mu_0}{\sigma_0 + j\omega \cdot \text{Re}(\varepsilon_0)}} \)
- \( \varepsilon_0 \) = permittivity of the free space
- \( \sigma_0 \) = conductivity of the free space
- \( j = \sqrt{-1} \)
- \( \rho^2 = x^2 + y^2 \)

**REFERENCES**


[14] IEC, "International Standard 62209-1: Procedure to determine the Specific Absorption Rate (SAR) for hand-held devices used in close proximity to the ear (frequency range of 300 MHz to 3 GHz)”, Vol. 62209-1: International Electrotechnical Commission, 2005.


Teddy Kurniawan was born in March 1979 in Jakarta, Indonesia. He completed his BE (Hons) in electrical and electronic engineering at Atma Jaya Catholic University in Jakarta in 2001. He then studied for the Master degree by research at The University of Melbourne, performing optimization work on the performance of fiber radio links in multichannel environments, and consequently completed his M.Eng.Sc. in 2004. In 2005, he started his Ph.D. candidature with Swinburne University of Technology, developing analytical methods to estimate near field exposure in dielectrics to represent human tissues exposure with RF band of near electromagnetic field. He completed his Doctorate in March 2009.

Andrew W. Wood received the B.Sc. degree in physics from Bristol University, Bristol, U.K., and the Ph.D. degree in biophysics from London University, London, U.K. He is currently Professor at the Brain Science Institute at Swinburne University, Hawthorn, Australia, with a research interest in possible health effects of electromagnetic radiation. He is a Research Director with the Australian Centre for Radiofrequency Bioeffects Research.

Robert L. McIntosh received the Ph.D. degree in mathematics from the Australian National University, Canberra, Australia, in 1989, in the area of PDEs. He has been a member of the Electromagnetic Energy (EME) Safety Research team at Telstra Research since 1999, developing and applying a numerical modeling environment for the study of RF dosimetry, and human body absorption and thermal modeling. He is an Associate Investigator at the Australian Centre for Radio-Bioeffects Research (ACRBR). He has worked at the BHP Research Laboratories on the development of electromagnetic levitation, pumping, and braking devices for liquid metal, and new techniques in noise reduction in electromagnetic geophysics (1989–1999).