Abstract

Complex systems are composed of interconnected heterogeneous components. The interconnections between these components may be nonlinear and unknown. As the uncertainty and complexity of the nature of complex systems require a proper control approach for their applications, control of complex systems has been receiving a great deal of attention in many engineering disciplines. Given many control strategies developed for complex systems, simplicity, learning capability and robustness are the key design criteria to ensure excellent control performance against uncertainties and nonlinearities in practical implementations.

The designs of traditional control strategies are mainly designed based on linear system theory. However, the limitation is that the well-developed linear controllers are applicable only around the equilibrium points. It is primarily because the global stability of the complex systems cannot be guaranteed by the linear controllers. To address this limitation, this thesis is concerned with the control design strategies for complex systems via Takagi-Sugeno (T-S) fuzzy models. Since heterogenous applications are concerned with the area of precise control and control over wide operating ranges, it is no longer desirable to design the controller based on simple linear system theory. Instead, the developments of control strategies for the T-S fuzzy model-based systems are expected to not only guarantee control performance and local stability, but also the global stability of the complex systems.

The control of a class of complex systems represented by T-S fuzzy models has been an active topic of research for at least 20 years. The key technical problems such as conservative stability conditions, chattering and requiring information about the uncertain system dynamics associated with the controller designs remain challenging research questions due to the demands of practical implementation. In this thesis, a
number of sliding mode learning control algorithms are developed to address these problems such that the proposed control algorithms are less conservative and can be used for more complex systems. The proposed sliding mode learning control algorithms have three major advantages: (i) the information of the parameter variations and disturbances is no longer required in the proposed controller designs, (ii) the control input is chattering-free, and (iii) the sliding mode learning control system possesses a strong robust property against parameter variations and disturbances. Thus, the sliding mode learning control algorithms have significant advantages in industrial applications and technological advances compared with conventional control schemes.

In particular, the theory of sliding mode learning control is applied to three major types of prospective design. First, the control of T-S fuzzy models that represent a class of single-input single-output (SISO) complex systems is investigated. A sliding mode-like learning control algorithm is designed to ensure asymptotic state convergence. The sufficient conditions for the sliding mode-like learning control to stabilize the global fuzzy model are discussed in detail. Second, the dominant control principle is used to facilitate the sliding mode learning control scheme for a class of complex systems with their T-S fuzzy models. Without knowing any information about the system uncertainties, the proposed sliding mode learning control can secure the existence of sliding mode and hence the robustness. Finally, the tracking control problem for large-scale systems via T-S models is investigated. One to the existence of nonlinear interconnections in large-scale systems, an adaptive sliding mode learning control scheme will be developed to ensure the global stability of the each subsystem in a large-scale system with good tracking performance. The performance of all three proposed sliding mode learning control algorithms will be verified using the numerical simulation examples.
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Declaration

This is to certify that:

1. This thesis contains no material which has been accepted for the award to the candidate of any other degree or diploma, except where due reference is made in the text of the examinable outcome.

2. To the best of the candidate’s knowledge, this thesis contains no material previously published or written by another person except where due reference is made in the text of the examinable outcome.

3. The work is based on the joint research and publications; the relative contributions of the respective authors are disclosed.

_______________________
Feisiang Tay, 2013
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List of Abbreviations and Acronyms

CEF  – composite energy function
COE  – centroid of area
FIS  – fuzzy inference systems
ILC  – iterative learning control
LC   – learning control
LMI  – linear matrix inequality
LTI  – linear time-invariant
MIMO – multi-input multi-output
PDC  – parallel distributed compensation
SISO – single-input single-output
SMC  – sliding mode control
T-S  – Takagi-Sugeno
VSC  – variable structure control
Chapter 1
Introduction

Most of the real-world complex systems are nonlinear in nature. As the complexity of systems increases, the design of mathematical models and control strategies for a complex system becomes more crucial. Modelling and control methods for general complex and nonlinear systems have been described in [1]. The conventional approach involves linearizing the complex systems around some equilibrium points such that the linear control theory can be applied to the local region of complex systems [2]. However, the associated disadvantage is that the linearized systems fail to completely represent the complex systems that are highly nonlinear. Linear control theory is aimed at analysing and controlling complex systems through linearization about the equilibrium points. The constraint is that the local linear controllers are applicable only around the equilibrium points [3]. As heterogeneous applications nowadays are concerned with the area of robust control over a wide operating range, the quest for developing new control solutions for complex systems appears to be a major challenge of our times.

The problems of the conventional approach are mainly due to strongly nonlinear behaviour and it lacks the precise knowledge of the complex system. Therefore, advanced modelling and control techniques are needed to clearly describe the relationships among the system variables in terms of a mathematical expression, and then develop a robust controller to cope with the effects of uncertainty and nonlinearity in the complex systems. Since the fuzzy modelling technique can provide a sufficient approximation of complex systems, many researchers are mainly concerned with
developing control strategies for a global fuzzy model which describes the complex system dynamics.

1.1. Fuzzy Modelling

Fuzzy modelling has been a popular topic of research due to its ability to closely approximate the practical systems that are complex and nonlinear [4]-[7]. Similar to neural networks based modelling, the fuzzy modelling technique can be employed to approximate a class of complex systems to any desired degree of accuracy. Although neural networks can perform well in many cases, the fuzzy modelling technique effectively uses both qualitative and quantitative information about a complex system for the building and analysis of its mathematical model, for the design of controllers [8].

The Takagi-Sugeno (T-S) fuzzy modelling technique was proposed in an effort to develop a systematic approach to generate fuzzy rules from a given input-output data set [4]. This modelling technique is based on using a set of fuzzy rules to describe a global complex nonlinear system in terms of a set of local linear models which are smoothly connected by fuzzy membership functions. Compared with purely linearize the complex systems at equilibrium points, this approach leads to a more effective use of the information contained in its local linear models, and allows us to deal with the constraints between the linear models in a consistent way. It is well known that the global T-S fuzzy model not only has the capability of performing universal approximation of a class of complex systems, as the Mamdani model does, but it also reveals the internal dynamics of the system [9]. Such an advantage creates more convenience to the controller design for real-world complex systems where the systems may contain uncertainties, nonlinear interconnections and disturbances.

A challenging problem for research into T-S fuzzy modelling is how to design a robust and effective controller to stabilize T-S fuzzy model-based complex systems.

1.2. Control Design for T-S Fuzzy Model-based Complex System.
The control of a class of complex systems represented by T-S fuzzy models has become one of the most widely researched topics since 1985 [4]. One of the major researched control structures is the parallel distributed compensation (PDC) control structure in which each controller rule is designed according to the corresponding rule of the T-S model. The controller rules have linear state feedback control laws associated with them [10]-[13]. It is important to note that the PDC control scheme requires a common positive matrix, for all the local linear models must be found in order to satisfy all local Riccati equations. Finding the common positive matrix from the Linear Matrix Inequalities (LMI) might lead to a feasibility problem in which there is no solution for LMI expression and the process of finding the solution is complex [20]-[24]. In practice, this constraint has greatly limited the application of the control schemes for the control of T-S fuzzy model-based complex systems.

Shortly after researchers investigated the PDC control scheme, the H-infinity control, sliding mode control (SMC), guaranteed cost control and adaptive control were all studied extensively to deal with modelling uncertainties or external disturbances [14]-[19]. The majority of conventional control strategies can be categorized as T-S fuzzy robust controller designs for norm bounded uncertainties. In the control strategies, the upper bounds of the uncertainties are required in the controller designs. However, in practical situations, it is difficult to know the upper bounds of all the uncertainties.

Given the many effective control schemes developed for T-S fuzzy model-based complex systems, an SMC can ensure strong robustness and asymptotic convergence of closed-loop systems.

1.3. Sliding Mode Control

Variable structure control (VSC) systems evolved from the pioneering work in Russia of Emel’yanov and Barbashin in the 1960s, and then was further developed and investigated by researchers from both theoretical and applied aspects [25]. SMC is a particular type of VSC system designed to drive and then maintain the system state trajectory within a neighbourhood of the switching manifold for all subsequent time.
This method permits the dynamic behaviour of the system to be tailored by the particular choice of a switching manifold and exhibit an inherent robust property.

In [25]-[28], the conventional SMC techniques have been used to stabilize a closed-loop system and improve the robustness with respect to system uncertainties and external disturbances. However, its major drawback in practical applications is the chattering problem [29], [30]. Due to the discontinuous switching process, the undesired chattering in the control signal may excite a high-frequency system response and cause unpredictable instabilities. In addition, without knowing the information about uncertain system dynamics, it is impossible to design a robust SMC control scheme to ensure the stability of the system. These drawbacks largely restrict the application of SMC for the control of a complex system via T-S fuzzy models. Therefore, the design of a learning mechanism into SMC paradigm is needed to overcome these drawbacks.

The aim of this thesis is to develop a number of sliding mode learning control algorithms to address the problems that exist in the conventional control strategies for a class of complex systems with T-S fuzzy models, thus achieving an excellent robust property in the control system and asymptotic state convergence. It is expected that these control algorithms are less conservative and can be used for more complex control systems. Several numerical examples are provided to demonstrate the effectiveness of the proposed sliding mode learning control algorithms.

1.4. Motivation

The T-S fuzzy modelling technique attempts to solve complex system modelling problems by decomposing them into a number of simpler sub-problems. The theory of fuzzy sets offers an excellent tool for representing the uncertainties associated with the decomposition task, for providing smooth transitions between the individual local sub models [8]. However, most of the control strategies developed for the T-S fuzzy systems require a prior knowledge of the uncertain system dynamics. In practice, this is often impossible, since the information about uncertain system dynamics is not available. Such control strategies may not be applicable to large-scale systems.
Although various approaches have been developed to address this problem, there has not been a perfect solution. There is still an urgent need to focus on the development of a robust controller to deal with real-time complex systems without relying on the information related to the system uncertainties and disturbances. This has led to an intense interest in the development of a robust control scheme to overcome the following major issues:

- Convergence and robustness
- Lack of system information
- Ease of implementation

In order to develop an efficient control scheme for T-S model-based complex systems, all the above issues will be addressed appropriately in this thesis. Motivated by the SMC and learning control (LC) theory, a number of sliding mode learning control algorithms have been proposed to stabilize a class of complex systems with T-S fuzzy models. The proposed control algorithms not only guarantee asymptotic stability of the closed-loop systems, but also allow the close-loop systems to possess a strong robust property against system uncertainties and disturbances. The major advantage of the proposed control algorithms is that the controller designs do not require the information about uncertain system dynamics to be known, meanwhile, the chattering phenomenon that frequently appears in conventional SMC is also eliminated without deteriorating the robustness of the systems.

1.5. Objectives and Major Contributions

In this thesis, we address the issues of fuzzy dynamic modelling and investigate the various types of robust controller designs. However, we focus more on stability analysis and sliding mode learning controller design of continuous-time single-input single-output (SISO) T-S model-based complex systems. Specifically, the following list outlines the major contributions of the thesis:

- Address the limitations of conventional control schemes.
- Study SMC theory and solve existing problems of the theory.
• Utilize the Lipschitz-like condition in the proposed control scheme, to avoid requiring information about system uncertainties and disturbances.
• Develop a number of sliding mode learning control schemes to overcome the drawbacks of conventional control schemes.
• Develop a learning control scheme allowing the system designers to directly design a simple global sliding mode learning control instead of a complex global control by aggregating all local controllers using the fuzzy inference law.
• Develop a learning control scheme for a class of large-scale systems.
• Implement a few simulation examples to verify the proposed control algorithms through comparison with the conventional control.

In summary, the work of the thesis has the potential to significantly enhance the conventional control scheme to produce the required control performance in practice despite the discrepancies between the actual plant and the mathematical model.

1.6. Organization of the Thesis

Chapter 2 provides a brief survey of the basic SMC, LC and fuzzy control systems. Some important aspects in this area are discussed. Special attention is given to the SMC controller design methods, robustness analysis and key issues in SMC theory and applications.

Chapter 3 addresses the efficiency of sliding mode learning control algorithms. From an implementation perspective, we have designed a fuzzy sliding mode-like learning control scheme based on local T-S fuzzy models, by using SMC and LC techniques. The concept of PDC is used to determine the global control signal which aggregates the control signal from each fuzzy rule. The stability analysis and simulation results show that the proposed fuzzy sliding mode-like learning controller can drive the sliding variable to converge to the sliding surface asymptotically and the system states can also asymptotically converge to zero.

Chapter 4 investigates the control problems for the global T-S fuzzy model. The concept of the dominant control principle is employed to facilitate the sliding mode learning
control scheme for a class of complex systems with its T-S fuzzy models. Global control is determined by the control signal of the dominant linear model which dominates the entire dynamics of the global fuzzy system. A sliding mode learning control scheme is developed to guarantee the asymptotic convergence of the system state. In addition, the information about the system uncertainties is no longer required in the proposed sliding mode learning controller design.

Chapter 5 investigates the control problem for a class of large-scale systems with T-S models, as the nonlinear interconnections exist in the large-scale T-S fuzzy systems, an adaptive sliding mode learning control scheme has been developed to ensure the global stability of the systems with good tracking performance.

Finally, in Chapter 6 we conclude the thesis with a summary, highlight the major contributions and suggest future work.

The author’s publications based on this thesis’ research are given at the end of this thesis. In addition, the Matlab codes for each of the new sliding mode learning algorithms developed in this thesis are provided in the Appendix.
Chapter 2
Literature Review

2.1. Introduction

In 1965, L. A. Zadeh invented a fuzzy set theory, which has found many applications in a wide variety of disciplines [31]. Shortly after that, the concept of complex system modelling and analysis by means of linguistic variables was introduced by Zadeh in 1973 [32]. Based on Zadeh’s paper, the first fuzzy rule-based control system was applied to a laboratory scale steam engine by E. H. Mamdani and his colleagues in 1975 [33], [34]. As many control systems are not amenable to conventional modelling approaches due to a lack of precise knowledge about the system, Mamdani’s work led to the emergence of fuzzy control systems. Today, fuzzy control systems are widely employed in a broad range of applications, which can be found not only in the process industry, chemical engineering, automotive engineering, but also consumer products or financial domains [8], [35], [36]. To meet the growing demands concerning quality and flexibility in production, fuzzy control systems offer a potential solution to the problem of ensuring high performance over a wide range of operating conditions [4].

The advantage of the fuzzy rule-based modelling technique is its ability to make use of human knowledge and deductive processes to approximate the inexact nature of the real-world. General speaking, there are two major types of fuzzy modelling techniques, they are, Mamdani fuzzy modelling and Takagi-Sugeno (T-S) fuzzy modelling. The main difference lies in the consequent of the fuzzy rules. Mamdani fuzzy modelling uses a fuzzy set as the rule consequents; while T-S fuzzy modelling
uses linear functions as the rule consequents [4]. Compared with Mamdani fuzzy modelling, T-S fuzzy modelling offers more precise description of the dynamic behaviour of complex systems. Furthermore, the T-S fuzzy modelling technique works well with linear control techniques and is well suited to mathematical analysis.

The T-S fuzzy systems are more computationally efficient and accurate in system modelling than Mamdani fuzzy systems. Therefore, T-S fuzzy systems have been more widely applied to fuzzy modelling and control of complex systems. However, the control problems of T-S fuzzy systems have not been fully solved. The key technical problems, such as conservative stability conditions and requiring information of the parameter variations associated with the controller designs remain, challenging research questions due to the demands of practical implementation.

SMC has been studied extensively for over 50 years and widely used in practical applications due to its simplicity and robustness against system uncertainties and disturbances [2], [26], [37]-[40]. It is one of the most important approaches to the design of robust controllers for both linear and nonlinear systems. However, the chattering phenomenon that frequently appears in the SMC systems is the main obstacle for the SMC implementation in practice. Although various techniques e.g. boundary layer control technique and fuzzy control technique have been developed to address this problem, there has not been a perfect solution [41]-[43]. In addition, a prior knowledge of both the upper and lower bounds of parameter variations and disturbances is required in sliding mode controller designs. These limitations have greatly restricted the applications of SMC in practice.

This chapter presents a brief literature review of fuzzy modelling, SMC and LC. The contents of this chapter are organized as follows. In Section 2.2, the basic concepts of fuzzy set theory are provided. In Section 2.3, the T-S fuzzy system and Mamdani fuzzy system are discussed in detail. In Section 2.4, the basic concepts of Lyapunov stability theory and Lyapunov’s direct method are summarized. In Section 2.5, a brief survey for the basic SMC theory is given and controller design methods for linear and nonlinear systems are reviewed. In Section 2.6, the basic concepts of iterative learning
control (ILC) are summarized. Finally, in Section 2.7, a conclusion for this chapter is given.

2.2. Basic Concepts of Fuzzy Set Theory

In this thesis, fuzzy set theory has been employed in the fuzzy modelling and control of a class of complex systems. Therefore in this section, the basic concepts of fuzzy set theory are summarized, which are necessary for understanding fuzzy modelling and control from a practical viewpoint.

2.2.1. Fuzzy Sets

A fuzzy set $\mathcal{F}$ on universe (domain) $\mathcal{U}$ is defined by the membership functions $\mu_{F}(x)$ which is a mapping from the universe $\mathcal{U}$ into the unit interval $[0,1]$. A fuzzy set may be viewed as a generalization of the concept of a crisp set whose membership function only takes two values $\{0,1\}$. Fuzzy set theory allows for partial membership of an element in a set. If the value of the membership function equals one, $x$ belongs completely to the fuzzy set. If it equals zero, $x$ does not belong to the set. If the membership degree is between 0 and 1, $x$ is a partial member of the fuzzy set. It is important to note that in fuzzy sets, an element can reside in more than one set for different degrees of similarity. However, this cannot occur in a crisp set theory.

![Figure 2.1 Typical membership functions of linguistic values “Slow”, “Moderate” and “Fast”](image-url)
Figure 2.1 illustrates the fuzzy concepts of slow, moderate and fast for a car speed which has speeds which are in the range of 0 to 100 km/h. It is important to note that the membership values vary from 0 to 1, and each fuzzy membership function corresponds to a fuzzy set.

In this thesis, the following form of membership functions is used:

Sigmoidal (“s-shaped”) membership function:

\[
\mu_{\text{sig}}(x, \alpha, \beta) = \frac{1}{1 + e^{-\alpha(x-\beta)}}
\]  \hspace{2cm} (2.2.1)

The sigmoidal membership function is the most commonly used function in fuzzy set theory. Compared with the trapezoidal membership function, it has the advantage of being smooth and nonzero at all points. Although the Gaussian membership function and bell membership function both can achieve similar smoothness, they are unable to specify an asymmetric membership function. The sigmoidal membership function can synthesize two sigmoidal functions to form an asymmetric and closed membership function, which is important in this thesis. The simulation example of the sigmoidal function is shown in Figure 2.2, the function is defined with the chosen parameters \(\alpha = 3\) and \(\beta = 1\).

![Figure 2.2 Sigmoidal membership function](image-url)
2.2.2. Operations on Fuzzy Sets

Let fuzzy sets $A$ and $B$ in $U$ be described with their membership functions $\mu_A(x)$ and $\mu_B(x)$. The definitions of fuzzy intersection, union and complement for fuzzy sets are defined as follows:

**Intersections (Conjunction):** The intersection of $A$ and $B$ is a fuzzy set $C$, denoted $C = A \cap B$, such that for all $x \in U$:

$$\mu_C(x) = \min\{\mu_A(x), \mu_B(x)\} \quad (2.2.2)$$

**Union (disjunction):** The union of $A$ and $B$ is a fuzzy set $C$, denoted $C = A \cup B$, such that for all $x \in U$:

$$\mu_C(x) = \max\{\mu_A(x), \mu_B(x)\} \quad (2.2.3)$$

**Complement (negation):** The complement of $A$ is a fuzzy set, denoted $\overline{A}$, such that for all $x \in U$:

$$\mu_{\overline{A}}(x) = 1 - \mu_A \quad (2.2.4)$$

It is important to note that, other consistent definitions for fuzzy intersection and fuzzy union have been proposed in the literature under the function known as T-norm and T-conorm respectively [8], [36], [44]-[46].

**T-norm (fuzzy intersection):** a t-norm, denoted by $\ast$, is a two-place function from $[0,1] \times [0,1]$ to $[0,1]$, which includes standard intersection, algebraic product, bold intersection (bounded product) and drastic product.

- standard intersection: $x \ast y = \min\{x, y\}$
- algebraic product: $x \ast y = xy$
- bold intersection: $x \ast y = \max\{0, x + y - 1\}$
drastic product: \[ x \times y = \begin{cases} x & \text{if } y = 1 \\ y & \text{if } x = 1 \\ 0 & \text{if } x, y < 1 \end{cases} \] (2.2.5)

where \( x, y \in [0,1] \).

**T-conorm (fuzzy union):** a t-conorm, denoted by \( \oplus \), is a two-place function from \([0,1] \times [0,1]\) to \([0,1]\), which includes standard union, algebraic sum, bold union (bounded sum) and drastic sum.

- **standard union:** \( x \oplus y = \max\{x, y\} \)
- **algebraic sum:** \( x \oplus y = x + y - xy \)
- **bold union:** \( x \oplus y = \min\{1, x + y\} \)
- **drastic sum:** \( x \oplus y = \begin{cases} x & \text{if } y = 0 \\ y & \text{if } x = 0 \\ 1 & \text{if } x, y > 0 \end{cases} \) (2.2.6)

where \( x, y \in [0,1] \).

### 2.2.3. Fuzzy Relations and Compositions

**Cartesian product:** Let \( A_1, A_2, ..., A_n \) are fuzzy sets in \( U_1, U_2, ..., U_n \), respectively. The Cartesian product of \( A_1, A_2, ..., A_n \) is a fuzzy set in the product space \( U_1, U_2, ..., U_n \) with the membership function as:

\[
\mu_{A_1 \times A_2 \times \ldots \times A_n}(x_1, x_2, ..., x_n) = \min\{\mu_{A_1}(x_1), \mu_{A_2}(x_2), ..., \mu_{A_n}(x_n)\} \tag{2.2.7}
\]

so, the Cartesian product of \( A_1, A_2, ..., A_n \) are donated by \( A_1 \times A_2 \times \ldots \times A_n \).

**Fuzzy relations:** Fuzzy relations are mapping elements of universe \( X \) to another universe \( Y \), through the Cartesian product of two universe and is expressed as

\[
R(X, Y) = \{(x, y) \mid \mu_R(x, y) \} \quad (x, y) \in (X \times Y) \tag{2.2.8}
\]
where the fuzzy relation $R$ has a membership function

$$\mu_R(x, y) = \mu_{A \times B}(x, y) = \min\{\mu_A(x), \mu_B(y)\} \quad (2.2.9)$$

**Compositions**: Composition of fuzzy relations used to combine fuzzy relations on different product spaces. As fuzzy relations are fuzzy sets in the product space, algebraic operations can be defined using operators for fuzzy intersection, union, and complement. Let $R$ and $S$ be fuzzy relations in $U \times V$. The intersection and union of $R$ and $S$, which are compositions of the two relations, are defined as

$$\mu_{R \cap S}(x, y) = \mu_R(x, y) \ast \mu_S(x, y) \quad (2.2.10)$$

and

$$\mu_{R \cup S}(x, y) = \mu_R(x, y) \oplus \mu_S(x, y) \quad (2.2.11)$$

respectively, where $\ast$ is t-norm operator and $\oplus$ is t-conorm operator.

2.2.4. The Extension Principle

The extension principle provides a general procedure for extending crisp domains of mathematical expressions to fuzzy domains [31], [32]. This procedure generalizes an ordinary mapping of a function $f$ to a mapping between fuzzy sets. It has been extensively used in fuzzy literature.

**Definition 2.2.1**: Suppose $f$ is a function from $X$ to $Y$ and $A$ is a fuzzy set on $X$ defined as:

$$A = \{(x_1, \mu_A(x_1)), (x_2, \mu_A(x_2)), \ldots, (x_n, \mu_A(x_n))\} \quad (2.2.12)$$

where $\mu_A(\cdot)$ is the membership function of $A$

then the extension principle states that the image of fuzzy set $A$ under the mapping $f$ can be expressed as a fuzzy set $B \subseteq Y$. 

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\[ B = f(A) = \{(y, \mu_B(y))\} \]  
(2.2.13)

where

\[ \mu_B(y) = \max_{x=f^{-1}(y)} \{\mu_A(x)\} \]  
(2.2.14)

**Definition 2.2.2**: Let \( X \) be a Cartesian product of a universal set \( X = X_1 \times X_2 \times \ldots \times X_n \) and \( A_1 \times A_2 \times \ldots \times A_n \) be \( n \) fuzzy sets in the universal set. The Cartesian product of fuzzy sets \( A_1, A_2, \ldots, A_n \) are donated by \( A_1 \times A_2 \times \ldots \times A_n \) as in (2.2.7).

Suppose \( f \) is a function from \( X \) to \( Y \),

\[ f(x_1, x_2, \ldots, x_n): X \to Y \]  
(2.2.15)

then fuzzy set \( B \) in \( Y \) can be obtained by function \( f \) and fuzzy sets \( A_1, A_2, \ldots, A_n \) as follows:

\[ \mu_B(y) = \max_{y=f^{-1}(x_1, x_2, \ldots, x_n)} \{\min\{\mu_{A_1}(x_1), \mu_{A_2}(x_2), \ldots, \mu_{A_n}(x_n)\}\} \]  
(2.2.16)

we assume that \( y = f^{-1}(x_1, x_2, \ldots, x_n) \) is not empty. If \( y = f^{-1}(x_1, x_2, \ldots, x_n) \) is empty, define \( \mu_B(y) = 0 \).

2.2.5. Fuzzy If-Then Rules

Fuzzy If-Then rules are also known as fuzzy implications. They are used to formulate the conditional statements that comprise fuzzy logic, and are often employed to capture the imprecise modes of reasoning in an environment of uncertainty.

A single fuzzy if-then rule assumes the form

IF \( x \) is \( A \) Then \( y \) is \( B \)
where $A$ and $B$ are linguistic values defined by fuzzy sets on the ranges (universes of discourse) $X$ and $Y$, respectively. The if-part of the rule "$x$ is $A$" is called the *antecedent* or premise, while the then-part of the rule "$y$ is $B$" is called the *consequent* or conclusion. An example of such a rule might be

$$\text{IF temperature is cold Then heater is high}$$

where *temperature* and *heater* are linguistic variables, *cold* and *high* are linguistic values that are characterized by membership functions. The concept of *cold* is represented as a number between 0 and 1. Conversely, *high* is represented as a fuzzy set, and so the consequent is an assignment that assigns the entire fuzzy set $B$ to the output variable $y$.

In 1985, Takagi and Sugeno proposed another form of fuzzy If-Then, the rule consequents are a linear model [4]. An example of T-S fuzzy If-Then rule, which describes the resistant force on the car braking system, is shown as follows:

$$\text{IF pressure is high Then force} = a \cdot \text{pressure} + b$$

where $a$ and $b$ are constant parameters, the consequent part is described with a linear equation of the input variable, pressure.

Compared with most of the fuzzy If-Then rules, the T-S fuzzy If-Then rule gives a more precise way to model control actions using numerical data.

### 2.2.6. Fuzzy Inference Systems

Fuzzy inference systems (FISs) have been widely used in many applications such as automatic control, data classification, decision analysis, optimization, and image processing. FISs are also known as fuzzy rule-based systems, fuzzy expert systems, fuzzy models, fuzzy associative memory, fuzzy logic controller and simply fuzzy systems [8], [14], [47]-[51]. The basic structure of an FIS consists of four conceptual
components: fuzzification interface, knowledge base, inference engine, and defuzzification interface. Figure 2.3 shows the block diagram of an FIS.

![Fuzzy inference system](image)

Figure 2.3  Fuzzy inference system

The fuzzification interface transforms the crisp input values into linguistic values, by computing their membership to all linguistic terms defined in the corresponding input domain. The knowledge base consists of a rule base and a data base. The rule base is the procedural part of the knowledge base which contains a number of fuzzy if-then rules. The data base is the declarative part of the knowledge base which defines the membership functions of the fuzzy set used in the fuzzy rules. The inference engine is a reasoning mechanism which performs the fuzzy inference process, by computing the activation degree and the output of each rule. Finally, the defuzzification interface transforms the fuzzy results of the inference into a crisp output.

FIS perform a fuzzy inference process in four steps. The first step is to take the inputs and determine the linguistic values to which they belong to each of the appropriate fuzzy sets via the membership functions. After the inputs are fuzzified, the linguistic values are combined to get the firing strength to each rule. Third, depending on the firing strength, the qualified subsequent of each rule is generated. Finally, all the qualified consequents are aggregated to produce a crisp output.

In general, an FIS is designed based on the past known behavior of a target system. The fuzzy system is then expected to be able to reproduce the behavior of the target system [47].
2.3. Fuzzy Systems

The concept of fuzzy set theory can be employed in the modelling of complex systems. There are two major types of fuzzy systems: the Mamdani fuzzy systems and the T-S fuzzy systems. The main difference is: the rule consequents of Mamdani fuzzy systems are fuzzy sets, while the rule consequents of T-S fuzzy systems are linear functions. Therefore in this section, the basic concepts of both fuzzy systems are summarized. The advantages of T-S fuzzy systems against Mamdani fuzzy systems are highlighted at the end of this section.

2.3.1. Mamdani Fuzzy Systems

The Mamdani fuzzy modelling of a complex system can be performed in the following steps: first, the whole state-space of the complex system is decomposed into a few subspaces. Second, within each subspace, the complex system is approximated using a fuzzy set. Finally, the global Mamdani fuzzy model of the complex system is constructed by aggregating all the fuzzy sets of the subsystems using the weight average fuzzy inference, and then obtaining the crisp outputs by defuzzifying the fuzzy output as shown in Figure 2.4.

A general form of Mamdani fuzzy if-then rules (also called the linguistic fuzzy if-then rules) is described as follows:
\[ R^i : \quad \text{IF} \quad z_1 \text{ is } F^i_1 \AND \ldots \AND z_n \text{ is } F^i_n \quad \text{THEN} \]
\[ y \text{ is } B^i \quad \text{for } i = 1, 2, \ldots, m \]

where \( R^i \) denotes the \( i^{th} \) fuzzy inference rule, \( m \) the number of inference rules, \( F^i_j (j = 1, 2, \ldots, n) \) the fuzzy sets, \( n \) the number of fuzzy sets, \( y \) is the output linguistic variable, \( B^i \) is the fuzzy set of the consequent part for the \( i^{th} \) fuzzy inference rule. \( z(t) := [z_1, z_2, \ldots, z_n] \) are some measurable system variables.

Denote \( \mu_B(y) \) as the aggregated output membership function, and \( S \) as the support of \( \mu_B(y) \). By using Centroid of Area (COE) defuzzification [47][52], the crisp output \( y^* \) of the Mamdani system could be represented as

\[ y^* = \frac{\int_S y \cdot \mu_B(y) dy}{\int_S \mu_B(y) dy} \quad (2.3.2) \]

2.3.2. T-S Fuzzy Systems

Due to the lack of an explicit model of the controlled system, the Mamdani approach was limited in its further extension to the topics of control theory-stability analysis, optimal control and robustness [53], [54]. Therefore, Takagi and Sugeno [4] developed the method of modelling complex systems by their fuzzy decomposition into fuzzy linear subsystems.

The following steps are often used to model a complex system using T-S fuzzy modelling technique: first, the whole state-space of the complex system is decomposed into a few subspaces. Second, within each subspace, the complex system is approximated using a linear time-invariant (LTI) model. Finally, the global T-S fuzzy model of the complex system is constructed using weight average fuzzy inference to aggregate all the subsystem matrices as shown in Figure 2.5.
A general form of T-S fuzzy if-then rules is described as follows:

\[ R^i : \text{IF} \quad z_1 \text{ is } F_1^i \text{ AND } ... \text{ z}_n \text{ is } F_n^i \]

THEN

\[
\begin{align*}
\dot{x}(t) &= A_i x(t) + B_i u(t) \\
y(t) &= C_i x(t)
\end{align*}
\] (2.3.3)

for \( i = 1, 2, ..., m \)

where \( R^i \) denotes the \( i^{th} \) fuzzy inference rule, \( m \) the number of inference rules, \( F_j^i (j = 1, 2, ..., n) \) the fuzzy sets, \( n \) the number of fuzzy sets, \( x(t) \in \mathbb{R}^n \) the system state vector, \( u(t) \in \mathbb{R}^1 \) the system input variable, and \( y(t) \in \mathbb{R}^1 \) the system output. \( z(t) := [z_1, z_2, ..., z_n] \) are some measurable system variables and the matrices \( A_i, B_i \) and \( C_i \) are \( n \times n, n \times 1 \) and \( 1 \times n \) parameter matrices of the \( i^{th} \) subsystem.

Denote \( \mu_i (z(t)) \) as the normalized fuzzy membership function

\[
\mu_i (z(t)) = \frac{w_i(t)}{\sum_{i=1}^{m} w_i(t)}
\] (2.3.4)

where
Using the T-S fuzzy inferences approach [15], [16], we obtain the global fuzzy model of the system as follows:

\[ \dot{x}(t) = A(t)x(t) + B(t)u(t) \]
\[ y(t) = C(t)x(t) \]  

where

\[ A(t) = \sum_{i=1}^{m} \mu_i(t)A_i \]  
\[ B(t) = \sum_{i=1}^{m} \mu_i(t)B_i \]  
\[ C(t) = \sum_{i=1}^{m} \mu_i(t)C_i \]  

It is important to note that the T-S fuzzy system in (2.3.8) can be treated as a time-varying system where \( A(t), B(t) \) and \( C(t) \) are \( n \times n, n \times 1 \) and \( 1 \times n \) parameter matrices and their entries are functions of time.

2.3.3. Comparison between Mamdani Fuzzy Systems and T-S Fuzzy Systems

The dynamic behaviour of the original systems can be precisely described using T-S fuzzy modelling technique. Unlike the Mamdani fuzzy model, the rule consequences of the T-S fuzzy models are described with linear functions. Therefore, the dynamic behaviour of the original system could be easily preserved. The system output of Mamdani fuzzy systems is calculated by aggregating all the fuzzy sets, and then
performing a numerical integration of the entire fuzzy set surface as in (2.3.2). However, the aggregation step is not required in T-S fuzzy system.

The T-S fuzzy systems are more compact and computationally efficient representations than Mamdani fuzzy systems. In practical implementation, the T-S fuzzy systems have the following three major advantages over Mamdani fuzzy systems:

- It works well with linear control techniques
- It is well suited to mathematical analysis.
- It has guaranteed continuity of the output surface.

Therefore, the T-S fuzzy systems have been more widely applied to fuzzy modelling and control of engineering systems than Mamdani fuzzy systems.

2.4. Lyapunov Stability Theory

The Lyapunov stability theory plays an important role in engineering system design and analysis. Lyapunov stability is named after Aleksandr Lyapunov, a Russian mathematician and mechanician who laid the foundation of the theory in 1892 [54], [55]. As the nonlinearities and possible time-varying parameters exist in nonlinear systems, linear stability criteria e.g Routh’s stability criterion or Nyquist stability criterion cannot be generalized and carried over into the systems for stability analysis. The Lyapunov stability theory introduced in this section is the most general approach to determine the stability of the linear or nonlinear systems.

In this section, the basic concepts of Lyapunov stability theory are summarized which are necessary in understanding the stability analysis of the proposed control scheme for complex systems via T-S fuzzy models.

2.4.1. Basic Definitions

Consider a dynamical system which satisfies
\[ \dot{x} = f(x, t) \]  

(2.4.1)

where \( x \in \mathbb{R}^{n \times 1} \) is the state variable vector, and \( n \) is the order of the system. \( f(x, t) \in \mathbb{R}^{n \times 1} \) is a set of functions of \( x(t) \).

**Definition 2.4.1**: The state \( x_e \in \mathbb{R}^{n \times 1} \) is an equilibrium point of system (2.4.1) if \( x_e \) satisfies the following equations:

\[
f(x_e, t) = 0 \quad \text{for all time } t
\]

(2.4.2)

**Definition 2.4.2**: The equilibrium point at the origin of (2.4.1) is said to be stable in the sense of Lyapunov if for any real number \( \epsilon > 0 \) there exists a \( \delta > 0 \) such that

\[
\|x(t_0)\| < \delta \Rightarrow \|x(t)\| < \epsilon, \quad \forall t \geq t_0
\]

(2.4.3)

**Definition 2.4.3**: The equilibrium point at the origin of (2.4.1) is said to be asymptotically stable in the sense of Lyapunov if it is stable and there exists a \( \delta > 0 \) such that

\[
\|x(t_0)\| < \delta \Rightarrow \lim_{t \to \infty} x(t) = 0
\]

(2.4.4)

It is important to note that the definitions 2.4.2 and 2.4.3 are local definitions; they only describe the behaviour of a system near an equilibrium point which is not very useful in practice. In order to archive global stability, the Lyapunov direct method is represented in the following to handle this drawback.

2.4.2. Lyapunov’s Direct Method

Lyapunov’s direct method (also called the second method of Lyapunov) allows us to determine the stability of a system without explicitly integrating the differential equation in (2.4.1). The method is a generalization of the idea that if there is some “measure of energy” in a system, then we can study the rate of change of the energy of the system to determine the stability [30], [37], [56-58]. If the rate of change is a
negative value, the total energy of a mechanical system continuously dissipates. Therefore, the system must eventually settle down to an equilibrium point. Specifically, we need to construct a scalar “energy-like” function for a given dynamic system. By examining the scalar function, we can perform stability analysis of the dynamic system.

**Definition 2.4.4**: A scalar function \( \mathcal{E}(\mathbf{x}) \) is said to be positive definite in a region \( S \) including the system origin if \( \mathcal{E}(\mathbf{x}) > 0 \) for all nonzero states in the region \( S \) and \( \mathcal{E}(\mathbf{x}) = 0 \) when \( \mathbf{x} \) is at the origin.

In the definition 2.4.4, the scalar function \( \mathcal{E}(\mathbf{x}) \) is simply the function of the state variable vector \( \mathbf{x} \), and it does not depend on time \( t \). Therefore, \( \mathcal{E}(\mathbf{x}) \) is commonly constructed for a time-invariance system.

**Theorem 2.4.1**: Let \( D \subset \mathbb{R}^{n \times 1} \) be a domain containing the system origin and that \( V(x): D \rightarrow \mathbb{R} \) is a continuously differentiable and satisfies \( V(x) = 0 \) when \( x = 0 \) such that

\[
V(x) > 0, \forall x \neq 0 \tag{2.4.5}
\]

\[
\|x\| \rightarrow \infty \Rightarrow V(x) \rightarrow \infty \tag{2.4.6}
\]

and

\[
\dot{V}(x) \leq 0 \tag{2.4.7}
\]

then the equilibrium at the origin is globally stable. In addition, the equilibrium at the origin is globally asymptotically stable if

\[
\dot{V}(x) < 0, \forall x \neq 0 \tag{2.4.8}
\]

where the function \( V(x) \) is called a Lyapunov function.

For a time-varying system, the Lyapunov function is in the form of \( V(x, t) \), and the positive definite \( V(x, t) \) is redefined as follows:
**Definition 2.4.5**: The $V(x, t)$ is said to be a positive definite in a region $S$ including the system origin if there exists a positive function $V(x) > 0$ such that $V(x, t) > V(x)$ for all $t \geq t_0$ and $V(x, t) = 0$ when $x$ is at the origin.

**Theorem 2.4.2**: Let $D \subset \mathbb{R}^{n\times1}$ be a domain containing the system origin and that $V(x) : D \rightarrow \mathbb{R}$ is a continuously differentiable and satisfies $V(x, t) = 0$ when $x = 0$ such that

$$V(x, t) > 0, \forall x \neq 0 \quad (2.4.9)$$

$$\|x\| \rightarrow \infty \Rightarrow V(x, t) \rightarrow \infty \quad (2.4.10)$$

and

$$\dot{V}(x, t) \leq 0 \quad (2.4.11)$$

then the equilibrium at the origin is globally stable. In addition, the equilibrium at the origin is globally asymptotically stable if

$$\dot{V}(x, t) < 0, \forall x \neq 0 \quad (2.4.12)$$

**2.5. Basic Concepts of SMC**

The SMC technique was first proposed by Russian scientists, Emel’yanov and Barbashin, in the early 1960s [59], [60]. The related works were published in English by Itkis and Utkin in the 1970’s [61]-[63]. The central feature of SMC is its sliding motion. In the sliding mode, the dynamic motion of the system is effectively constrained to lie within a certain subspace of the full state space. The sliding motion is then achieved by altering the system dynamics along sliding mode surfaces in the states space. On the sliding mode surface, the system is equivalent to an unforced system of lower order, which is insensitive to both system uncertainties and disturbances.

The basic idea of SMC is described as follows:
(i) The desired system dynamics is first defined on a sliding surface in the state space.

(ii) A controller is then designed, using the output measurements and system uncertainties bounds, to drive the closed-loop system to reach the sliding surface.

(iii) The desired dynamics of the closed-loop system is then obtained on the sliding surface.

In this thesis, the concepts of SMC have been used to design the proposed sliding mode learning control scheme. Therefore, the proposed sliding mode learning controller can ensure the asymptotic convergence of the closed-loop system. This section presents the basic concepts, mathematic and design aspects of SMC. The conventional methods of suppressing chattering and eliminating the uncertain system dynamics are summarized at the end of this section.

2.5.1. Single-input LTI System

Consider the following general single-input LTI system:

\[ \dot{x}(t) = Ax(t) + Bu(t) \]  

(2.5.1)

where \( x(t) \in \mathbb{R}^n \) is the system state vector, \( u(t) \in \mathbb{R}^1 \) is the control input and \( A \) and \( B \) are \( n \times n \) and \( n \times 1 \) constant parameter matrices. It is assumed that the pair \((A,B)\) is completely controllable and the controllability matrix \([B \ AB \ A^2B \ ... \ A^{n-1}B]\) has full rank.

The sliding variable \( s(t) \) is defined as

\[ s(t) = Cx(t) = c_0x + c_1\dot{x} + \ldots + c_{n-2}x^{(n-2)} + x^{(n-1)} \]  

(2.5.2)
where $C = [c_0, c_1, \ldots, c_{n-1}, 1]$ is the sliding parameter vector. The parameter $c_i (i = 0, 1, 2, \ldots n - 2)$ should be chosen such that the solution of the following differential equation is asymptotically stable:

$$C x(t) = c_0 x + c_1 \dot{x} + \ldots + c_{n-2} x^{(n-2)} + x^{(n-1)} = 0$$ (2.5.3)

It is worth noting that the behavior of the system in the sliding mode depends on the parameter $c_i (i = 0, 1, 2, \ldots n - 2)$. This invariance is the most essential feature of SMC when controlling a time-varying system or treating disturbance rejection problems.

Lyapunov’s direct method can be used to design a sliding mode controller for the system in (2.5.1) with the prescribed desired system dynamics in (2.5.2). Generally, the following Lyapunov function is often used in the sliding mode controller design:

$$V(t) = \frac{1}{2} s^2(t)$$ (2.5.4)

Differentiating $V(t)$ with respect to time $t$

$$\dot{V}(t) = s(t) \dot{s}(t)$$ (2.5.5)

The reachability condition for the sliding variable to reach the sliding surfaces can be expressed as follows:

$$\dot{V}(t) = s(t) \dot{s}(t) < 0$$ (2.5.6)

It has been noted that the design of most sliding mode controllers is based on the reachability condition in (2.5.6) to ensure the sliding mode controller can drive the sliding variable $s(t)$ to asymptotically converge to zero.

The equivalent control based SMC has the following form:

$$u(t) = u_{eq}(t) + u_s(t)$$ (2.5.7)
\[ u_{eq}(t) = -(CB)^{-1}CAx(t) \]  \hspace{1cm} (2.5.8)

\[ u_s(t) = -\eta(CB)^{-1}\text{sign}(s(t)) \]  \hspace{1cm} (2.5.9)

where \( u_{eq}(t) \) is an equivalent control and \( u_s(t) \) is a discontinuous or switched component, \( \eta(>0) \) is a constant control gain, and

\[
\text{sign}(s(t)) = \begin{cases} 
1, & s(t) > 0 \\
0, & s(t) = 0 \\
-1, & s(t) < 0 
\end{cases}  \hspace{1cm} (2.5.10)
\]

Substituting (2.5.1) and (2.5.7) into (2.5.5)

\[
\dot{V}(t) = s(t)CAx(t) + s(t)CBu(t) \\
= -\eta |s(t)| < 0 \hspace{1cm} (2.5.11)
\]

From (2.5.11), we can conclude that the ideal sliding mode is guaranteed to be reached in finite time.

2.5.2. Robust Control for LTI Systems

Robustness is an important feature of the SMC system. The system uncertainties and disturbances are always factored in the SMC controller design. With consideration of system uncertainties and disturbances, the general LTI system in (2.5.1) is described by

\[
\dot{x}(t) = (A + \Delta A)x(t) + (B + \Delta B)u(t) + w(t) \hspace{1cm} (2.5.12)
\]

which can be rewritten in the following form:

\[
\dot{x}(t) = Ax(t) + Bu(t) + f(t) \hspace{1cm} (2.5.13)
\]

where \( x(t) \in R^n \) is the system state vector, \( u(t) \in R^1 \) is the control input, \( f(t) = \Delta Ax(t) + \Delta Bu(t) + w(t) \), \( \Delta A \) and \( \Delta B \) are the system uncertainties, as well as the
external disturbances $w(t)$. It is assumed that the pair $(A,B)$ is completely controllable and the controllability matrix $[B\ AB\ A^2B\ ...\ A^{n-1}B]$ has full rank. $f(t)$ is piecewise continuous and square-integrable fulfilling the matching conditions, i.e., there exists $g(t) \in R^1$ such that $f(t) = Bg(t)$ [25].

During the sliding motion, the state vector of the system satisfied the following equations:

$$ Cx(t) = 0 $$  \hspace{2cm} (2.5.14)  

$$ C(Ax(t) + Bu(t) + f(t)) = 0 $$  \hspace{2cm} (2.5.15)  

From expression (2.5.15), the equivalent control can be achieved as

$$ u(t) = u_{eq}(t) = -(CB)^{-1}C(Ax(t) + f(t)) $$  \hspace{2cm} (2.5.16)  

and the equivalent system equation is then given by

$$ \dot{x}(t) = [I - B(CB)^{-1}C](Ax(t) + f(t)) $$  \hspace{2cm} (2.5.17)  

In [25], the SMC system in (2.5.14) is insensitive to system uncertainties and external disturbances if only if $f(t)$ satisfies the matching condition, that is, $f(t) = Bg(t)$. This invariance property makes SMC an efficient tool for controlling uncertain systems and provides a strong motivation for continuing research interest in the control area. However, equivalent control action is dependent on an unknown exogenous signal and therefore cannot be realized in practice.

### 2.5.3. Robust Control for Nonlinear Systems

In Section 2.5.1, we have briefly reviewed the basic SMC theory for LTI systems. Most of these ideas can be extended to SMC for nonlinear systems. However, the complexity of both the analysis and the controller designs may be increased due to nonlinearity and time-variance in the nonlinear system. In practice, the system dynamics of a nonlinear
system is different from its nominal system model due to the parameter variations. From the control engineering point of view, the following nonlinear system with uncertain parameters is often considered [38]:

\[
\dot{x}(t) = [f(t, x(t)) + \Delta f(t, x(t), r(t))] + [B(t, x(t)) + \Delta B(t, x(t), r(t))]u(t) \tag{2.5.18}
\]

where the state variable vector \(x(t) \in \mathbb{R}^n\), the control input vector \(u(t) \in \mathbb{R}^1\), time-varying function \(f(t, x) \in \mathbb{R}^n\), and \(B(t, x) \in \mathbb{R}^{n \times m}\). Each entry in \(f(t, x)\) and \(B(t, x)\) is assumed to be continuous with a continuous bounded derivative with respect to \(x\). \(r(t)\) is a vector function of uncertain parameters. Following the matching condition proposed in [26], [27], [38], [64] and [65], the parameter variations \(\Delta f(t, x, r)\) and \(\Delta B(t, x, r)\) are required to lie in the image of \(B(t, x)\) for all variables \(t\) and \(x\). The nonlinear system in (2.5.1) can be rewritten in the following form:

\[
\dot{x}(t) = f(t, x) + B(t, x)u(t) + B(t, x)e(t, x, r, u) \tag{2.5.19}
\]

where \(e(t, x, r, u)\) represent system uncertainties.

Following the sliding variable design for a general LTI system in (2.5.2), the method of SMC is used to design the control signal \(u(t)\) to ensure the system motion is restricted to the sliding mode surface \(s(t) = 0\).

The \(u(t)\) is characterized by the SMC structure defined by

\[
u(t) = \begin{cases} u(t)^+, & \text{for } s(t) > 0 \\ u(t)^-, & \text{for } s(t) < 0 \end{cases} \tag{2.5.20}
\]

If \(e(t, x, r, u)\) is bounded by positive function \(\rho(t, x)\)

\[
\|e(t, x, r, u)\|_2 \leq \rho(t, x) \tag{2.5.21}
\]

The control input \(u(t)\) in the SMC usually has the following form [38]:

\[
\|e(t, x, r, u)\|_2 \leq \rho(t, x) \tag{2.5.21}
\]
\[ u(t) = u_{eq}(t) + u_n(t) \]  

(2.5.22)

where

\[ u_{eq}(t) = - \left[ \frac{\partial s}{\partial x} B(t,x) \right]^{-1} \left[ \frac{\partial s}{\partial t} + \frac{\partial s}{\partial x} f(t,x) \right] \]  

(2.5.23)

\[ u_n(t) = - \frac{B^T(t,x) \nabla_x V(t,x)}{\|B^T(t,x) \nabla_x V(t,x)\|_2} \hat{\rho}(t,x) \]  

(2.5.24)

\[ \hat{\rho}(t,x) = \alpha + \rho(t,x), \alpha > 0 \]  

(2.5.25)

\[ \nabla_x V(t,x) = \left[ \frac{\partial s}{\partial x}(t,x) \right]^T s(t,x) \]  

(2.5.26)

then the system state trajectory can reach the sliding surfaces \( s(t) = 0 \), and the desired system dynamics can be obtained by the suitable choice of the sliding parameters.

The motivation for exploring uncertain systems in Sections 2.5.2 and 2.5.3 is because the model identification of real-world systems introduces modelling errors. A whole body of literature over 20 years is concerned with the deterministic stabilization of systems having system uncertainties lying within known bounds [66-70]. Such control strategies are usually based on Lyapunov’s direct method.

The advantage of SMC in Sections 2.5.2 and 2.5.3 is its insensitivity to matched uncertainties in the sliding mode. However, if the matching condition is not satisfied, the motion of the sliding mode is dependent on the system uncertainties because the condition for the robustness of SMC does not hold.

For SMC, the idea of rejecting the system uncertainties and disturbances is different from H-infinity control approaches. Compared with H-infinity control which attempts to minimise the sensitivity of the closed-loop system in the sense that the effect of uncertainties and disturbance can be attenuated, the SMC can completely reject uncertain system dynamics which satisfy the matching condition. However, the
chattering problem in the SMC system is the one of the key challenges to overcome. In the next sub-section, the disadvantages of the chattering phenomenon and common chattering elimination approaches will be discussed in detail.

2.5.4. The Chattering Phenomenon

An idea sliding mode does not exist in practice since it would imply that the control commutes at an infinite frequency. Due to imperfections in switching devices, SMC suffers from chattering, the discontinuity in the feedback control produces a particular dynamic behaviour in the vicinity of the sliding surface as shown in Figure 2.6 [25], [26], [30], [71], [72].

![Figure 2.6 The chattering phenomenon](image)

In Figure 2.6, the system trajectory in the region $s(t) > 0$ heading towards the sliding surface $s(t) = 0$. It first hits the surface at point A. In an ideal SMC the trajectory should start sliding on the surface from point A. However, due to a delay between the time the sign of $s(t)$ changes and the time the control switches, the trajectory reverses its direction and heads again towards the surface. The repetition of this process creates the “zig-zag motion” which oscillates around the predefined sliding surface.
The chattering results in low control accuracy, high heat losses in electric power circuits and high wear of moving mechanical parts. It may excite unmodeled high-frequency dynamics, which degrades the performance of the system and may even lead to instability. Various techniques have been proposed to eliminate the chattering [30], [73-77]. The boundary layer technique is one of the common approaches to eliminate the chattering.

The discontinuous or switched component of the SMC controller in (2.5.7) is of the form:

\[ u_s(t) = -\eta(CB)^{-1} \text{sign}(s(t)) \]  \hspace{1cm} (2.5.27)

The boundary layer technique can be used to eliminate the chattering by replacing the sign function in (2.5.27) with a saturation function shown in Figure 2.7 as follows:

\[ u_s(t) = -\eta(CB)^{-1} \text{sat}(s(t)) \]  \hspace{1cm} (2.5.28)

where \( \text{sat}(s(t)) \) is the saturation function defined by

\[ \text{sat}(s(t)) = \begin{cases} \frac{s(t)}{\varepsilon}, & \text{if } |s(t)| \leq \varepsilon \\ \text{sign}(s(t)), & \text{if } |s(t)| > \varepsilon \end{cases} \]  \hspace{1cm} (2.5.29)

and a positive constant \( \varepsilon (> 0) \) should be chosen in simulation or experiment to guarantee that the chattering can be eliminated and a reasonable control performance can be obtained.
This smoothing technique has often been employed in order to prevent chattering. However, although the chattering can be removed, the robustness of the sliding mode is also compromised. Such an approach might lead to a loss of asymptotic stability. Therefore, the boundary layer technique is not a perfect solution to eliminate chattering.

Another solution to cope with chattering is based on continuous approximation method (also called the pseudo-sliding mode method in the literature) in which the sign function in (2.5.24) is replaced by a continuous approximation as follows [78]-[82]:

$$u_s(t) = -\eta(CB)^{-1} \left( \frac{s(t)}{|s(t)| + \epsilon} \right)$$  \hspace{1cm} (2.5.30)

where $\epsilon(> 0)$ is a small positive number. However, this approach gives rise to a high-gain control when the states are in the close neighbourhood of the sliding surface.

2.6. Basic Concepts of ILC
ILC has been an active topic of research since the inception of the idea in 1967 [83]. Inspired by the interest on the research of the LC, a series of related articles were published in 1984 [84]-[86]. Since then, the ILC has been receiving a great deal of attention in many engineering disciplines. As shown in [87]-[93], the ILC has been successfully applied to industrial robots, semibatch chemical reactors, wafer processes, freeway traffic control, glycemic control, and atomic force microscope imaging. The ILC is well-recognized as an effective method for improving the transient response of systems and robust performance against uncertain dynamics of systems that operate repetitively over a fixed time interval.

Although various conventional control methods provide numerous solutions for improving the performance of a dynamic system, it is not possible to achieve a desired level of control performance. This is mainly due to a lack of exact information on unmodeled dynamics, parametric uncertainties or disturbances in the controller design. ILC is a design tool that can help overcome the shortcomings of traditional controllers, making it possible to achieve the desired level of control performance when there are system uncertainties and disturbances in the dynamic system. The major advantage of ILC is that an ILC controller can be designed without an accurate model of the system.

In this section, the basic concepts of ILC are summarized which are necessary in understanding the learning capability of the sliding mode learning controller scheme.

2.6.1. Simple SISO LTI System

Consider the following SISO LTI system, in continuous time:

\[ \dot{x}_i(t) = Ax_i(t) + Bu_i(t) \]
\[ y_i(t) = Cx_i(t) \]  \hspace{1cm} (2.6.1)

where \( x_i(t) \in R^n \) is the system state vector, \( u_i(t) \in R^1 \) is the control input, and \( y_i(t) \in R^1 \) the system output. The matrices \( A, B \) and \( C \) are \( n \times n, n \times 1 \) and \( 1 \times n \) parameter matrices of the system. The control objective is to drive the system output \( y_i(t) \) to track the reference output vector \( y_r(t) \) on a fixed time interval \( t \in [0,T] \) as the
iteration $i$ increases. In classical ILC, the following basic assumptions are required [94]-[96]:

(i) Every iteration ends in a fixed time interval.
(ii) The initial state $x_i(0)$ can be set to the same point at the beginning of each iteration.
(iii) The system dynamics are deterministic.
(iv) Invariance of the system dynamics is ensured throughout repetition.

Under these assumptions, the D-type ILC scheme usually has the following form [84], [97]:

$$u_{i+1}(t) = u_i(t) + K \dot{e}_i(t) \tag{2.6.2}$$

where $e_i(t) = y_r(t) - y_i(t)$, and $K$ is a diagonal learning gain matrix, and ensures that

$$\lim_{i \to \infty} y_i(t) \to y_r(t), \forall t \in [0, T] \tag{2.6.3}$$

if

$$\|I - CBK\|_q < 1 \tag{2.6.4}$$

where $\|\cdot\|_q$ is an operator norm and $q \in \{1, 2, \ldots, \infty\}$.

It is important to note that the major advantage of ILC is the selection of learning gain $K$ in (2.6.2). As the selection criterion in (2.6.4) does not require the information about the system parameter matrix $A$, the ILC requires less a prior knowledge of the system while still achieving perfect tracking under rather weak conditions.

2.6.2. ILC for Nonlinear Systems
In practice, most plants are characterized by nonlinear dynamics. In this sub-section, the applicability of linear ILC schemes for nonlinear system is investigated. Consider the following nonlinear system:

\[
\begin{align*}
\dot{x}_i(t) &= f(x_i(t), u_i(t)), \quad x_i(0) = x_0 \\
y_i(t) &= g(x_i(t), u_i(t))
\end{align*}
\]  

(2.6.5)

where \(f(\cdot)\) is a smooth vector-valued function and \(g(\cdot)\) is a smooth function.

In [98], [99], the simple linear ILC scheme has the following form:

\[
u_{i+1}(t) = u_i(t) + Ke_i(t)
\]  

(2.6.6)

The convergence condition is determined by the relation

\[
|u_{i+1}|_\lambda \leq |1 - Kg_u||u_i|_\lambda \\
|1 - Kg_u| = \alpha < 1
\]  

(2.6.7)

where \(g_u = \frac{\partial g}{\partial u}\) and \(\cdot|_\lambda\) is the time-weight norm defined as \(\max_{t \in [0,T]} e^{-\lambda t} \cdot\). The learnability condition is that the system gain \(g_u \in [\beta_1, \beta_2]\), either \(\beta_1 > 0\) or \(\beta_2 < 0\). By choosing a proper learning gain \(K\), the condition (2.6.7) can be fulfilled if the interval \([\beta_1, \beta_2]\) is known a priori.

Xu et al. [100] observed that, the traditional ILC schemes in [101]-[112] are required that the target trajectory must be invariant in all iterations. If there is a change in the target trajectory due to the task specifications, the control system will have to start the learning process from the beginning and the previously learned control input profiles can no longer be used. In order to improve the learnability of ILC, a number of novel ILC algorithms derived from a contraction mapping approach and energy function approach, have been proposed in [113]-[115]. However, certain conditions are required to learn an invariant set in the iteration domain.

2.6.3. Robust ILC for Norm-bounded Uncertainties
Consider the following nonlinear uncertain system:

\[
\dot{x} = f(t, x) + u(t), \quad x_i(0) = x_0
\]  

(2.6.8)

where \(f(t, x)\) is the unknown function representing a lumped uncertainty which is assumed to be continuous with a continuous bounded derivative with respect to \(x\).

If \(f(t, x)\) is bounded by a positive function \(\rho(t, x)\)

\[
\lVert f(t, x) \rVert \leq \rho(t, x)
\]  

(2.6.9)

In [98], [116], the robust ILC has the following form:

\[
u_i = \text{proj}(u_{i-1}) + (\bar{\rho}_i \eta_i + 1)e_i
\]  

(2.6.10)

\[
\bar{\rho}_i = \sqrt{\dot{r}^2 + u_m} + \rho_i
\]  

(2.6.11)

\[
\eta_i = \frac{\sqrt{e_i^2 + 3u_m^2} + 8u_m}{\left(\sqrt{e_i^2 + 3u_m^2} + u_m\right)^2}
\]  

(2.6.12)

\[
\text{proj}(a) \triangleq \begin{cases} 
    a & |a| \leq u_m \\
    u_m \text{sign}(a) & |a| > u_m
\end{cases}
\]  

(2.6.13)

where \(u_m > 0\) is a projection bound satisfying \(u_m > \sup_{t \in [0,T]} |u_r(t)|\), and \(u_r(t)\) is the desired control profile that, though unknown, can be described by the system inversion \(u_r(t) = f(r) - \dot{r}\). In practice, \(u_m\) is either a system physical constraint or virtual saturation bound that can be arbitrarily large but finite.

To derive the learning convergence, the following time weighted composite energy function (CEF) is used:
\[ E_i(t) = e^{-\lambda t}e_i^2 + \int_0^t e^{-\lambda \tau} \delta u_i^2 \, d\tau \]  
(2.6.14)

where \( \delta u_i = u_r - u_i \). The convergence analysis of the robust ILC scheme (2.6.10) is shown in [116]. The authors conclude that \( u_i(t) \) converges to \( u_r(t) \) almost everywhere and \( e_i(t) \) converges to zero uniformly in \([0, T]\) as \( i \to \infty \).

Recently the robust ILC has been actively explored using Q-filter, H-infinity or LMI approaches. In the past few years dozens of papers have been dedicated to robust ILC, but unanimously for linear systems. Robust ILC for nonlinear systems subject to various norm-bounded uncertainties is still an open and challenging topic.

### 2.7. Concluding Remarks

The dynamics of complex systems to be controlled are difficult to model. It is possible to linearize them around some given operating points such that a linear control theory can be applied in the local region. However, obtaining a global model to represent complex systems by aggregating a set of local linear models is not an easy task. Fuzzy modelling offers an effective method to aggregate a set of local linear models into a global model by using fuzzy membership functions.

Sections 2.2 and 2.3 provide an overview of fuzzy logic and fuzzy systems. The fundamentals of fuzzy set theory have been briefly outlined. The basic concepts of Mamdani fuzzy systems and T-S fuzzy systems have been discussed. It is seen that T-S fuzzy modelling is more computationally efficient and suitable to mathematical analysis and controller implementations than Mamdani fuzzy systems. However, the stability properties of T-S fuzzy systems have not been fully explored.

The concepts of Lyapunov stability theory are summarized in Section 2.4. Lyapunov’s direct method is the most suitable approach to determine the stability and design of the controller for T-S fuzzy systems. The theory of SMC systems has been briefly surveyed in Section 2.5. As SMC theory has many advantages, it has been widely used for controlling linear and nonlinear systems. Although robustness can be
achieved, the information about the uncertain system dynamics is required in the SMC controller design. Excessive control input and severe control chattering may excite undesired system behaviour in the SMC system, and thus, degrade the performance of the closed-loop system. The key challenges of SMC are how to design an effective SMC without knowing the dynamics of the entire systems and how to eliminate the chattering efficiently.

In order to achieve a desired control performance for complex systems with T-S fuzzy models, LC control theory has been used to enhance the proposed sliding mode learning control algorithms. Therefore, in Section 2.6, the basic concepts of ILC have been briefly discussed. The ILC can help overcome the shortcomings of conventional controllers such as a lack of exact information on uncertain system dynamics and hard to achieve desired level of control performance.
In this chapter, a sliding mode-like learning control scheme is developed for a class of SISO complex systems. First, the T-S fuzzy modelling technique is employed to model the complex dynamical systems. Second, a sliding mode-like learning control is designed to drive the sliding variable to converge to the sliding surface, and the system states can then asymptotically converge to zero on the sliding surface. The advantages of this scheme are that:

- The information about the uncertain system dynamics and the system model structure is not required for the design of the learning controller.
- The closed-loop system behaves with a strong robustness with respect to uncertainties and disturbances.
- The control input is chattering-free.

The sufficient conditions for the sliding mode-like learning control to stabilize the global fuzzy model are discussed in detail. A simulation example for the control of an inverted pendulum cart is presented to demonstrate the effectiveness of the proposed control scheme.

3.1. Introduction

Fuzzy modellings of complex systems, based on the fuzzy set theory of Zadeh, have been extensively studied and applied in many engineering disciplines [14], [16], [31],
Generally speaking, there are two major types of fuzzy modelling techniques, that is, Mamdani fuzzy modelling and T-S fuzzy modelling. Mamdani fuzzy modelling realizes a nonlinear mapping in terms of fuzzifying the crisp system inputs, computing the outputs of all fuzzy rules based on their fuzzy antecedents, aggregating all fuzzy outputs, and then obtaining the crisp outputs by defuzzifying the fuzzy outputs.

However, for the T-S fuzzy modelling of a complex system, the local dynamics of the complex system are represented by a group of simple linear models. The global T-S model of the system can then be derived by using the aggregation of all the local linear models [15]. It is worth noting that the global T-S fuzzy model not only provides a universal approximation of a complex system, as the Mamdani model does, but also reveals the internal dynamics of the system [9], [118]. Thus, T-S modelling has become an effective approach to model complex systems with uncertain dynamics [127-129].

In order to design effective controllers for the T-S fuzzy model-based complex systems, some advanced control methodologies are often required to achieve the closed-loop stability and strong robustness with respect to nonlinearities and uncertain dynamics. Among many modern control techniques, such as feedback linearization, SMC, adaptive control and PDC [4], [11], [15-17], [117], [118], [130-132], [136-138], SMC is one of a few effective control schemes. In references [17], [117], [118], and [130-132], the conventional linear SMC techniques are used to stabilize the closed-loop T-S systems and improve the robustness with respect to uncertain dynamics and bounded external disturbances.

However, the chattering which occurs in the control inputs may excite undesired system behaviours in conventional SMC, and thus, degrades the closed-loop system performance [134]. In addition, the conventional SMC also requires a prior knowledge of both the upper and the lower bounds of uncertainties and internal parameters, in order to design sliding mode controllers. The above limitations of the conventional SMC have greatly restricted the applications of SMC to the control of complex systems with T-S fuzzy models.
In this chapter, we propose a new sliding mode-like learning control, based on [133], [135], [139-141], for a class of SISO complex systems with their T-S fuzzy models. It will be shown that, like the recursive learning control algorithms [142-144], in each subspace, a local sliding mode-like learning controller is designed with its most recent control signal and a correction term. The correction term plays the role of searching the sliding surface and correcting the motion direction of the closed-loop system based on the most recent stability status of the closed-loop system. The aggregated global controller can then drive the sliding variable to converge to the sliding surface asymptotically and the system states can then asymptotically converge to zero.

It should be noted that the proposed sliding mode-like learning control in this chapter is based on the concept of the Lipschitz-like condition proposed in [133], which states that the difference between the current value of the gradient of the sliding variable and its most recent value is very small as the sampling period is sufficiently small. The merit of using the Lipschitz-like condition is that the system uncertainties and nonlinearities are all embedded in this condition. Therefore, no prior information about the uncertain system dynamics and the system model structure is required in the controller design.

The rest of the chapter is organized as follows: In Section 3.2, the T-S fuzzy modelling for SISO complex systems and the novel sliding mode-like learning control are formulated. In Section 3.3, the convergence analysis of the closed-loop system with the proposed sliding mode-like learning control scheme is discussed in detail. In Section 3.4, the control of an inverted pendulum cart is simulated in support of the developed new scheme. Section 3.5 provides the conclusions and some further work.

3.2. Problem Formulation

Consider a class of SISO complex systems represented by the following local dynamical fuzzy models [14], [16], [117], [118]:

\[ R^i : \text{IF } z_1 \text{ is } F^i_1 \text{ AND } \ldots \text{ z}_n \text{ is } F^i_n \]
THEN

\[ \dot{x}(t) = A_i x(t) + B_i u(t) + D_i d(t) \quad (3.2.1) \]

for \( i = 1, 2, ..., m \)

where \( R^i \) denotes the \( i^{th} \) fuzzy inference rule, \( m \) the number of inference rules, \( F^i_j (j = 1, 2, ..., n) \) the fuzzy sets, \( n \) the number of fuzzy sets, \( x(t) \in R^n \) the system state vector, \( u(t) \in R^1 \) the system input variable, \( d(t) \) the external disturbance. \( z(t) := [z_1, z_2, ..., z_n] \) contains some measurable system variables, and \( A_i, B_i \) and \( D_i \) are \( n \times n, n \times 1 \) and \( 1 \times n \) parameter matrices, respectively.

Denote \( \mu_i(z(t)) \) as the normalized fuzzy membership function:

\[ \mu_i(z(t)) = \frac{w_i(t)}{\sum_{i=1}^{m} w_i(t)} \quad (3.2.2) \]

where

\[ w_i(t) = \bigcap_{j=1}^{n} F^i_j (z(t)) \quad (3.2.3) \]

\[ \mu_i(t) > 0 \quad (3.2.4) \]

and

\[ \sum_{i=1}^{m} \mu_i(t) = 1 \quad (3.2.5) \]

Using the T-S fuzzy inferences \([14], [17], [16], [117], [118], [130]\), we obtain the global fuzzy model of the system as follows:

\[ \dot{x}(t) = A(t)x(t) + B(t)u(t) + D(t)d(t) \quad (3.2.6) \]

where

\[ A(t) = \sum_{i=1}^{m} \mu_i(t) A_i \quad B(t) = \sum_{i=1}^{m} \mu_i(t) B_i \]

\[ [z_1, z_2, ..., z_n] \] contains some measurable system variables, and \( A_i, B_i \) and \( D_i \) are \( n \times n, n \times 1 \) and \( 1 \times n \) parameter matrices, respectively.
\[ D(t) = \sum_{i=1}^{m} \mu_i(t) D_i \]  

(3.2.7)

For future consideration, we have the following assumptions [17] and [118]:

**Assumption 3.2.1:** Each local subsystem in (3.2.1) is controllable, that is, the controllability matrix

\[ M_i = [B_i, A_i B_i, ..., A_i^{n-1} B_i], \text{ for } i = 1, ..., m, \]

has a full rank of \( n \).

**Assumption 3.2.2:** The global fuzzy model in (3.2.6) is controllable in the state space, that is, the controllability matrix

\[ M = [B, AB, A^2 B, ..., A^{n-1} B] \]

has a full rank of \( n \).

In this chapter, a sliding variable \( s(t) \) is defined as:

\[ s(t) = Cx(t) \]  

(3.2.8)

where \( C \in R^{nx1} \) is the sliding mode parameter matrix, selected such that the dynamics of \( s(t) = 0 \) is Hurwitz.

For the fuzzy system described in (3.2.1)-(3.2.7), we propose the following local sliding mode-like learning control for the \( i^{th} \) subsystem:

\[
C^i : \quad \begin{cases} 
\text{IF } z_1 \text{ is } F_1^i \text{ AND } \cdots \text{ AND } \text{z}_n \text{ is } F_n^i \\
\text{THEN} \\
u_i(t) = u_i(t - \tau) - \Delta u_i(t)
\end{cases} 
\]  

(3.2.9)

for \( i = 1, 2, ..., m \)
with the correction term

\[
\Delta u_i(t) = \begin{cases} 
\alpha_i s(t) + \beta_i \hat{s}(t - \tau) & \text{for } s(t) \neq 0 \\
0 & \text{for } s(t) = 0
\end{cases}
\]  
(3.2.10)

where \( u_i(t) \) in (3.2.9) is the \( i^{th} \) local control input, \( \alpha_i (> 0) \) and \( \beta_i (> 0) \) in (3.2.10) are the control parameters to be determined later, \( \tau \) is the time delay and \( \hat{s}(t - \tau) \) is the estimate of \( \dot{s}(t - \tau) \), defined as:

\[
\hat{s}(t - \tau) = \frac{s(t) - s(t - \tau)}{\tau}
\]  
(3.2.11)

The global control \( u(t) \) can then be defined as:

\[
u(t) = \sum_{i=1}^{m} \mu_i(t) u_i(t)
\]

\[
= \sum_{i=1}^{m} \mu_i(t) \left( u_i(t - \tau) - \Delta u_i(t) \right)
\]  
(3.2.12)

**Remark 3.2.1:** The term \( \alpha_i s(t) \) in (3.2.10) behaves like an inertia term and, by properly choosing the value of \( \alpha_i \), the convergence of the closed-loop system can be improved, as seen in the next section. In addition, the term \( \beta_i \hat{s}(t - \tau) \) in the correction term is used to check the most recent stability status of the closed-loop system and update the control signal to ensure that the system states can asymptotically converge to zero. Most importantly, if the most recent information shows that the system is unstable, the term \( \beta_i \hat{s}(t - \tau) \) is capable of modifying the control signal in the sense that the closed-loop system can be driven from an unstable domain to the stable domain. The merits of this point can be seen from the convergence analysis in the next section.

**Remark 3.2.2:** Please note that the minimum value of time delay \( \tau \) is actually equal to the sampling period in practice. As the sampling frequency is very high, the time delay
\(\tau\) is sufficiently small. Thus, \(\hat{s}(t - \tau)\) can be reasonably assumed to have the same sign as \(\dot{s}(t - \tau)\), that is,

\[
\text{sign}(\hat{s}(t - \tau)) = \text{sign}(\dot{s}(t - \tau)) \tag{3.2.13}
\]

for \(\hat{s}(t - \tau) \neq 0\) and \(\dot{s}(t - \tau) \neq 0\),

where

\[
\dot{s}(t - \tau) = C \left[ A(t - \tau)x(t - \tau) + B(t - \tau) \sum_{i=1}^{m} \mu_i(t - \tau) u_i(t - \tau) \\
+ D(t - \tau)d(t - \tau) \right] \tag{3.2.14}
\]

**Remark 3.2.3:** Considering the controllability of both subsystem models and the global model, as in Assumption 3.2.1 and Assumption 3.2.2, we assume the following inequalities,

\[
CB_i > 0 \text{ and } CB(t) > 0 \tag{3.2.15}
\]

**Remark 3.2.4:** It is seen that, no sign function of the sliding variable \(s(t)\) is involved in the control signals and therefore both the local control inputs and the global control input enjoy chattering-free characteristics.

**Remark 3.2.5:** In practice, the subsystems of some complex systems may satisfy the controllability condition \(CB_i < 0\) and \(CB(t) < 0\). In such a case, the local control \(u_i(t)\) in (3.2.9) can be slightly modified as follows:

\[
u_i(t) = u_i(t - \tau) + \Delta u_i(t) \tag{3.2.16}
\]

Without loss of generality, in the next few sections, we consider the case of \(CB_i > 0\) and \(CB(t) > 0\) only.
3.3. Convergence Analysis

Theorem 3.3.1: Consider the global T-S fuzzy model in (3.2.6). If the sliding variable is chosen as in (3.2.8), the local control input $u_i(t)$ and global control input $u(t)$ are designed as in (3.2.9) and (3.2.12), respectively, then the system states $x(t)$ can asymptotically converge to zero.

Proof: The Lyapunov candidate for the closed-loop global system is chosen as:

$$V(t) = 0.5s(t)^2 \quad (3.3.1)$$

Differentiating $V(t)$ with respect to time $t$ leads

$$\dot{V}(t) = s(t)\dot{s}(t) = s(t)C\dot{x}(t)$$

$$= s(t)C \left[ A(t)x(t) + B(t) \sum_{i=1}^{m} \mu_i(t)u_i(t - \tau) - B(t) \sum_{i=1}^{m} \mu_i(t)\alpha_i s(t) \right. \left. - B(t) \sum_{i=1}^{m} \mu_i(t)\beta_i \hat{s}(t - \tau) + D(t)d(t) \right]$$

$$= s(t) \left[ CA(t)x(t) + CB(t) \sum_{i=1}^{m} \mu_i(t)(u_i(t - \tau)) - \alpha(t)CB(t)s(t) \right. \left. - \beta(t)CB(t)\hat{s}(t - \tau) + CD(t)d(t) \right] \quad (3.3.2)$$

where

$$\alpha(t) = \sum_{i=1}^{m} \mu_i(t)\alpha_i > 0 \quad \beta(t) = \sum_{i=1}^{m} \mu_i(t)\beta_i > 0 \quad (3.3.3)$$

From (3.3.2), we can express $\dot{s}(t)$ as follows:

$$\dot{s}(t) = \dot{s}(t, t - \tau) - \beta(t)CB(t)\hat{s}(t - \tau) - \alpha(t)CB(t)s(t) \quad (3.3.4)$$
where

$$\dot{s}(t, t - \tau) = C \left[ A(t)x(t) + B(t) \sum_{i=1}^{m} \mu_i(t)(u_i(t - \tau)) + D(t)d(t) \right] \quad (3.3.5)$$

Adding the term $\dot{s}(t - \tau) - \dot{s}(t - \tau)$ to (3.3.4), we have

$$\dot{s}(t) = \dot{s}(t, t - \tau) + \dot{s}(t - \tau) - \dot{s}(t - \tau) - \beta(t)CB(t)\dot{s}(t - \tau) - \alpha(t)CB(t)s(t)$$

$$\leq |\dot{s}(t, t - \tau) - \dot{s}(t - \tau)| + \dot{s}(t - \tau) - \beta(t)CB(t)\dot{s}(t - \tau)$$

$$- \alpha(t)CB(t)s(t) \quad (3.3.6)$$

**Remark 3.3.1:** Considering the continuity of both $\dot{s}(t)$ and $\dot{s}(t, t - \tau)$, we see that, as the time delay $\tau$ is sufficiently small, we can always find a constant $M >> 1$ such that the following inequality is held:

$$|\dot{s}(t, t - \tau) - \dot{s}(t - \tau)| < \frac{1}{M} |\ddot{s}(t - \tau)| \quad (3.3.7)$$

for $\dot{s}(t, t - \tau) \neq 0$, $\dot{s}(t - \tau) \neq 0$ and $\ddot{s}(t - \tau) \neq 0$.

Using (3.3.7) in (3.3.6), we obtain

$$\dot{s}(t) < \frac{1}{M} |\ddot{s}(t - \tau)| + \dot{s}(t - \tau) - \beta(t)CB(t)\dot{s}(t - \tau) - \alpha(t)CB(t)s(t) \quad (3.3.8)$$

Further, the approximation error between $\dot{s}(t - \tau)$ and $\ddot{s}(t - \tau)$ is defined as:

$$\delta(\dot{s}(t - \tau)) = \dot{s}(t - \tau) - \ddot{s}(t - \tau) \quad (3.3.9)$$

For $\ddot{s}(t - \tau) \neq 0$ and $\dot{s}(t - \tau) \neq 0$, there exists a positive number $\gamma$, such that the following inequality is held:
\[ |\hat{\delta}(s(t-\tau))| = |\hat{s}(t-\tau) - \hat{s}(t-\tau)| < \gamma |\hat{s}(t-\tau)| \]  \hspace{1cm} (3.3.10)

with \( 0 < \gamma \ll 1 \).

Then, using (3.3.9) and (3.3.10) in (3.3.8) leads
\[
\begin{align*}
\dot{s}(t) &\leq \frac{1}{M} |\hat{s}(t-\tau)| + \hat{s}(t-\tau) + \gamma |\hat{s}(t-\tau)| - \beta(t)CB(t)\hat{s}(t-\tau) \\
&\quad - \alpha(t)CB(t)s(t)
\end{align*}
\]
\hspace{1cm} (3.3.11)

For the case that \( s(t) > 0 \):

- If \( \hat{s}(t-\tau) > 0 \),

\[
\dot{s}(t) \leq \dot{s}(t-\tau) + \frac{1}{M} |\hat{s}(t-\tau)| - \beta(t)CB(t)|\hat{s}(t-\tau)| \\
- \alpha(t)CB(t)s(t)
\]
\hspace{1cm} (3.3.12)

If the control parameter \( \beta(t) \) is chosen to satisfy the following condition:

\[
\left(1 - \frac{1}{M} - \gamma \right) > \beta(t)CB(t) > \left(\frac{1}{M} + \gamma \right)
\]
\hspace{1cm} (3.3.13)

then

\[
\dot{s}(t) \leq \dot{s}(t-\tau) - \left[\beta(t)CB(t) - \frac{1}{M}\right]|\hat{s}(t-\tau)| - \alpha(t)CB(t)s(t)
\]
\hspace{1cm} < \dot{s}(t-\tau)
\]
\hspace{1cm} (3.3.14)

(3.3.14) shows that the sliding mode-like learning controller in (3.2.12) always makes the \( \dot{s}(t) \) smaller than \( \dot{s}(t-\tau) \) as \( \dot{s}(t-\tau) > 0 \).

Suppose that, at time \( t = t_1 \), \( \dot{s}(t_1) = 0 \). Then, at the time \( t = t_1 + \tau \), we can express (3.3.4) as:
\[ \dot{s}(t_1 + \tau) = \dot{s}(t_1 + \tau, t_1) - \beta(t_1 + \tau)CB(t_1 + \tau)\dot{s}(t_1) \]
\[ -\alpha(t_1 + \tau)CB(t_1 + \tau)s(t_1 + \tau) \quad (3.3.15) \]

Considering the fact that
\[ -\alpha(t_1 + \tau)CB(t_1 + \tau)s(t_1 + \tau) < 0 \quad (3.3.16) \]
if the control parameter \( \alpha(t_1 + \tau) \) is chosen to satisfy the following condition:
\[ \dot{s}(t_1 + \tau, t_1) - \beta(t_1 + \tau)CB(t_1 + \tau)\dot{s}(t_1) < \alpha(t_1 + \tau)CB(t_1 + \tau)s(t_1 + \tau) \quad (3.3.17) \]
then
\[ \dot{s}(t_1 + \tau) = \dot{s}(t_1 + \tau, t_1) - \beta(t_1 + \tau)CB(t_1 + \tau)\dot{s}(t_1) \]
\[ -\alpha(t_1 + \tau)CB(t_1 + \tau)s(t_1 + \tau) < 0 \quad (3.3.18) \]

(3.3.12)-(3.3.18) indicate that the sliding mode-like learning controller in (3.2.13) is capable of driving the closed-loop system dynamics to move from the unstable domain to the stable domain, and thus, ensure that
\[ s(t)\dot{s}(t) < 0 \]

- If \( \dot{s}(t - \tau) < 0 \), (3.3.11) can be expressed as:
\[ \dot{s}(t) \leq \frac{1}{M}|\hat{s}(t - \tau)| + \hat{s}(t - \tau) + \gamma|\hat{s}(t - \tau)| - \beta(t)CB(t)\hat{s}(t - \tau) \]
\[ -\alpha(t)CB(t)s(t) \]
\[ = -|\hat{s}(t - \tau)| + \frac{1}{M}|\hat{s}(t - \tau)| + \gamma|\hat{s}(t - \tau)| + \beta(t)CB(t)|\hat{s}(t - \tau)| \]
\[ -\alpha(t)CB(t)s(t) \]
\[ = -\left[ (1 - \frac{1}{M} - \gamma) - \beta(t)CB(t) \right]|\hat{s}(t - \tau)| - \alpha(t)CB(t)s(t) \]
\[ < -\alpha(t)CB(t)s(t) < 0 \] \hspace{1cm} (3.3.19)

Thus,

\[ s(t)\dot{s}(t) < 0 \]

For the case that \( s(t) < 0 \):

- If \( \dot{s}(t - \tau) > 0 \), we can express (3.3.6) as:

\[
\dot{s}(t) \geq -|\dot{s}(t, t - \tau) - \dot{s}(t - \tau)| + \dot{s}(t - \tau) - \beta(t)CB(t)\dot{s}(t - \tau) - \alpha(t)CB(t)s(t)
\]

\[
\geq -\frac{1}{M}|\dot{s}(t - \tau)| + \dot{s}(t - \tau) - \beta(t)CB(t)|\dot{s}(t - \tau)| - \alpha(t)CB(t)s(t)
\]

\[
= \left[ (1 - \frac{1}{M} + \gamma) - \beta(t)CB(t) \right]|\dot{s}(t - \tau)| + \alpha(t)CB(t)|s(t)| \hspace{1cm} (3.3.20)
\]

Thus,

\[ s(t)\dot{s}(t) < 0 \]

- If \( \dot{s}(t - \tau) < 0 \),

\[
\dot{s}(t) \geq -|\dot{s}(t, t - \tau) - \dot{s}(t - \tau)| + \dot{s}(t - \tau) - \beta(t)CB(t)\dot{s}(t - \tau) - \alpha(t)CB(t)s(t)
\]

\[
\geq -\frac{1}{M}|\dot{s}(t - \tau)| + \dot{s}(t - \tau) + \beta(t)CB(t)|\dot{s}(t - \tau)| - \alpha(t)CB(t)s(t)
\]

\[
= \dot{s}(t - \tau) + \left[ \beta(t)CB(t) - \frac{1}{M} \right]|\dot{s}(t - \tau)| + \alpha(t)CB(t)|s(t)|
\]

\[ > \dot{s}(t - \tau) \hspace{1cm} (3.3.21)\]
Suppose that, at time $t = t_1$, $\dot{s}(t_1) = 0$

At the time $t = t_1 + \tau$, we can express (3.3.4) as:

$$\dot{s}(t_1 + \tau) = \dot{s}(t_1 + \tau, t_1) - \beta(t_1 + \tau)CB(t_1 + \tau)\dot{s}(t_1) - \alpha(t_1 + \tau)CB(t_1 + \tau)s(t_1 + \tau)$$  \hspace{1cm} (3.3.22)

Considering the fact that

$$-\alpha(t_1 + \tau)CB(t_1 + \tau)s(t_1 + \tau) > 0$$  \hspace{1cm} (3.3.23)

we can also choose a proper $\alpha(t_1 + \tau)$ in the sense that the inequality in (3.3.17) is satisfied. Therefore,

$$\dot{s}(t_1 + \tau) = \dot{s}(t_1 + \tau, t_1) - \beta(t_1 + \tau)CB(t_1 + \tau)\dot{s}(t_1) - \alpha(t_1 + \tau)CB(t_1 + \tau)s(t_1 + \tau) > 0$$  \hspace{1cm} (3.3.24)

(3.3.21) to (3.3.24) means that

$$s(t)\dot{s}(t) < 0$$

In summary, we can conclude that the global controller $u(t)$ in (3.2.12) can ensure

$$\dot{V}(t) = s(t)\dot{s}(t) < 0$$  \hspace{1cm} (3.3.25)

Both the sliding variable $s(t)$ and the state variable vector $x(t)$ can then asymptotically converge to zero.

**Remark 3.3.2:** The inequality in (3.3.7) is called the *Lipschitz-like condition* [133] that is simply another version of the continuity of the derivative of the Lyapunov function $V(t)$ in the state space. However, the embedded advantage of using the Lipschitz-like condition is that no information on the upper and the lower bounds of uncertain parameters is required for the design of the learning control in (3.2.12). Thus, the
control system design has been greatly simplified, compared with the conventional SMC design in [14], [16], [17], [117], [118], and [130-132],

**Remark 3.3.3:** It is seen from Theorem 1 and the related mathematical discussions in this section that the sliding mode-like learning control in (3.2.12) is capable of continuously adjusting the direction of the convergence of the sliding variable, based on the recent stability history of the closed-loop system, and driving the closed-loop system to move towards to the sliding surface. Although the sliding variable may not reach the sliding surface in a finite time, it can asymptotically converge to the sliding surface, and the system states can also asymptotically converge to zero.

**Remark 3.3.4:** The claims in (3.3.16) and (3.3.23) are valid. Recall that, at time $t = t_1 + \tau$,

\[ \mu_i(t_1 + \tau) > 0 \quad (3.3.26) \]

\[ B(t_1 + \tau) = \sum_{i=1}^{m} \mu_i(t_1 + \tau)B_i \quad (3.3.27) \]

and

\[ \text{sign}(B(t_1 + \tau)) = \text{sign}(B(t)) \quad (3.3.28) \]

The control parameter $\alpha(t_1 + \tau)$ can then be defined as:

\[ \alpha(t_1 + \tau) = \sum_{i=1}^{m} \mu_i(t_1 + \tau)\alpha_i > 0 \quad (3.3.29) \]

Thus, the following inequality is held:

\[ \alpha(t_1 + \tau)CB(t_1 + \tau) = \alpha(t_1 + \tau) \sum_{i=1}^{m} \mu_i(t_1 + \tau)CB_i > 0 \quad (3.3.30) \]

and together with the discussions in (3.3.15)-(3.3.18) and (3.3.22)-(3.3.24), we ensure
Remark 3.3.5: The robustness of the proposed scheme is another important feature. It is seen from (3.2.12) that no information on the system parameters and model structure is required for controller design, although the complex system may have uncertainties. The sliding mode-like learning control signal in (3.2.12) is simply computed based on the estimation of the most recent stability status of the closed-loop system and the current measurements of the state variables.

3.4. An illustrative Example

Consider the following inverted pendulum on a cart with its dynamics as follows [15], [17]:

\[
\begin{align*}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= \frac{g \sin(x_1) - \frac{am^2x_2^2 \sin(2x_1)}{2} - \cos(x_1)u}{\frac{4l}{3} - amlcos^2(x_1)}
\end{align*}
\]  

(3.4.1)

where \(x_1\) is the angle of the pendulum from the vertical, \(x_2\) is the angular velocity, \(g = 9.81\text{m/s}\) is the gravity constant, \(m\) is the mass of the pendulum rod, \(M\) is the mass of the cart, \(2l\) is the length of the pendulum, \(u\) is the control force applied to the cart, parameter \(a = 1/(m + M)\) and the initial values of the state variables are \(x_1(t_0) = \pi/10\) and \(x_2(t_0) = 0\), respectively.

In this simulation, the system parameters are as follows:

\[m = 2.0\text{kg, } M = 8.0\text{kg, } 2l = 1.0\text{m}\]

with the T-S modelling technique described in Section 3.2, we represent the system in (3.4.1) in state space as follows:

\[R^1 : \text{IF } x_1 \text{ is about } 0, \text{ } x_2 \text{ is about } 0\]
\[
\begin{align*}
\text{THEN} \\
\dot{x}(t) &= A_1 x(t) + B_1 u(t) \\
R^2 : &\quad \text{IF } x_1 \text{ is about 0, } x_2 \text{ is about } \pm 4 \\
&\quad \text{THEN} \\
\dot{x}(t) &= A_2 x(t) + B_2 u(t) \\
R^3 : &\quad \text{IF } x_1 \text{ is about } \pm \pi/3, x_2 \text{ is about 0} \\
&\quad \text{THEN} \\
\dot{x}(t) &= A_3 x(t) + B_3 u(t) \\
R^4 : &\quad \text{IF } x_1 \text{ is about } +\pi/3, x_2 \text{ is about } +4 \\
&\quad \text{or } x_1 \text{ is about } -\pi/3, x_2 \text{ is about } -4 \\
&\quad \text{THEN} \\
\dot{x}(t) &= A_4 x(t) + B_4 u(t) \\
R^5 : &\quad \text{IF } x_1 \text{ is about } +\pi/3, x_2 \text{ is about } -4 \\
&\quad \text{or } x_1 \text{ is about } -\pi/3, x_2 \text{ is about } +4 \\
&\quad \text{THEN} \\
\dot{x}(t) &= A_5 x(t) + B_5 u(t) \quad (3.4.2) \\
\text{where} \\
A_1 &= \begin{bmatrix} 0 \\ 17.2941 \\ 0 \end{bmatrix}, B_1 = \begin{bmatrix} 0 \\ -0.1765 \end{bmatrix} \\
A_2 &= \begin{bmatrix} 0 \\ 14.4706 \\ 0 \end{bmatrix}, B_2 = \begin{bmatrix} 0 \\ -0.1765 \end{bmatrix} \\
A_3 &= \begin{bmatrix} 0 \\ 5.8512 \\ 0 \end{bmatrix}, B_3 = \begin{bmatrix} 0 \\ -0.0779 \end{bmatrix} \\
A_4 &= \begin{bmatrix} 0 \\ 7.2437 \\ 0.5399 \end{bmatrix}, B_4 = \begin{bmatrix} 0 \\ -0.0779 \end{bmatrix} \\
A_5 &= \begin{bmatrix} 0 \\ 7.2437 \\ 0.5399 \end{bmatrix}, B_5 = \begin{bmatrix} 0 \\ -0.0779 \end{bmatrix} \quad (3.4.3)
\end{align*}
\]
The fuzzy membership functions for state variables $x_1(t)$ and $x_2(t)$ are chosen as in Figures 3.1(a) and 3.1(b), respectively.

Based on (3.2.9)-(3.2.12), we have designed the global sliding mode-like learning controller as follows:
\[ u(t) = \sum_{i=1}^{5} \mu_i(t)(u_i(t - \tau) + \alpha_i s(t) + \beta_i \dot{s}(t - \tau)) \]  

Figures 3.2(a), 3.2(b) and 3.2(c) show the sliding variable \( s(t) \), system state \( x_1(t) \) as well as its derivative \( x_2(t) \), and the global control input \( u(t) \), respectively, where the sliding mode parameter matrix is chosen as \( C = [5 \ 1] \), the sampling period is \( \Delta T = 0.005 \text{sec} \), the time delay \( \tau = \Delta T \), and the control parameters in (3.4.4) are set to \( \alpha_1 = \alpha_2 = \cdots = \alpha_5 = 20 \) & \( \beta_1 = \beta_2 = \cdots = \beta_5 = 1 \).

It is seen that the sliding variable \( s(t) \) converges to zero in 2 seconds, the system states \( x_1(t) \) and \( x_2(t) \) then exponentially converges to zero. In addition, the global control signal \( u(t) \) is completely chattering-free.

![Figure 3.2(a) Sliding variable \( s(t) \) (Sliding mode-like learning control)](image)
Figure 3.2(b) States $x_1(t)$ and $x_2(t)$ responses (Sliding mode-like learning control)

Figure 3.2(c) The control input $u(t)$ (Sliding mode-like learning control)

For comparison purpose, Figures 3.3(a), 3.3(b) and 3.3(c) show the simulation results of the closed-loop system with the conventional SMC [118], where the initial values of the state variables and the sampling period are the same as the ones in Figures 3.2(a), 3.2(b) and 3.2(c). It has been seen that, although the convergence speeds of both the proposed learning control and the conventional SMC are nearly the same, the learning control signal in Figure 3.2(c) is smooth and chattering-free. Such an excellent property makes the proposed learning control more practical than the conventional one.
Figure 3.3(a) Sliding variable $s(t)$ (Conventional SMC)

Figure 3.3(b) States $x_1(t)$ and $x_2(t)$ responses (Conventional SMC)
3.5. Conclusion

In this chapter, a sliding mode-like learning control for a class of SISO complex systems with the T-S fuzzy models has been developed. It has been shown that the Lipschitz-like condition has been used to replace the upper and the lower bounds’ information for the controller design. Such a revolutionary idea has made the controller design much easier than the one of the conventional sliding mode controllers. In addition, the learning technique, proposed in this research, is another light point in the sense that it can make the closed-loop system move from the unstable domain to the stable domain, and thus, guarantees the stability of the closed-loop system. The further work is to combine the sliding mode-like learning control technique with the dominant control principle in [16], [17], [118] to enhance the function of the sliding mode-like learning control with various types of applications and its robust control.
Chapter 4

Sliding Mode Learning Control of Fuzzy Uncertain Continuous-time SISO Systems

In this chapter, a sliding mode learning control scheme is developed for a class of continuous-time SISO complex systems. It is seen that a global T-S fuzzy model is first built up to describe the complex system dynamics in the state-space, and a local sliding mode learning controller is designed based on each local linear model. The concept of dominant control principle can then be used to determine the global control signal to ensure that the closed-loop system is globally asymptotically stable. As a learning control, the control signal is updated by a correction term designed with the information of the most recent stability history of the closed-loop system, and such a learning strategy can always ensure that the asymptotic convergence of the closed-loop system is achieved. The major advantage of the proposed control scheme is that the information about the system uncertainties is no longer required for the controller design. A simulation example is presented to demonstrate the effectiveness of the proposed SMC control through comparison with the conventional SMC and H-infinity control.

4.1. Introduction

The fuzzy modelling technique offers an effective method to model the practical systems that are highly complex and nonlinear [14], [128], [145-147]. Although a neural network based modelling technique can also perform well in many cases, the fuzzy modelling technique is capable of utilizing both the qualitative and quantitative information about a complex system to construct its mathematical model with the help of IF-THEN rules. The T-S fuzzy modelling technique is commonly used to develop a systematic approach to approximate a class of complex systems by generating fuzzy
rules from a given input-output data set [4], [128], [129], [148]. The performance of this technique depends on the number of fuzzy rules, the types of membership functions, the consequent regressors and the measurable system variables. A great number of theoretical results on system approximation have shown that, the T-S fuzzy modelling technique can approximate any complex system to any desired degree of accuracy [4], [9], [14-17], [117], [118], [128], [129], [148], [149].

Over the past two decades, the development of control schemes for better robustness, performance and efficiency of T-S model-based fuzzy control systems has attracted a lot of attention from researchers. There are plenty of control schemes related to this research topic e.g. H-infinity control, SMC and guaranteed cost control [11], [14-19], [117], [128], [137], [150-152]. The key technical problems such as conservative stability condition, chattering and requiring information about the uncertain system dynamics associated with the controller designs remain challenging research questions in control engineering. In [15], the H-infinity control scheme is developed based on norm bounded uncertainties. However, the drawback of this approach is that information about system uncertainties needs to be known in the controller design. In [16], [117], [132] and [157], the conventional SMC have been used to stabilize T-S model-based fuzzy control systems and improve the robustness with respect to system uncertainties and disturbances.

Nevertheless, the undesired chattering in the control signal is the main obstacle for practical sliding mode controller implementation [2]. The chattering phenomena may excite unmodeled high frequency dynamics, which degrades the performance of the system and may lead to instability. Moreover, it results in low control accuracy, high heat losses in electric power circuits and high wear of moving mechanical parts. The boundary layer control technique is the common approach for eliminating the chattering by replacing the sign function in the sliding mode controller designs with a saturation function. However, the robustness of SMC is also compromised. Hence, the quest for developing a new control technique for eliminating chattering while preserving the robustness of the sliding mode appears to be a major challenge in the development of SMC.
The traditional control structure for a T-S model based fuzzy control system is PDC in which the global fuzzy control law can be obtained by aggregating all the local controllers using a fuzzy inference law [4], [153]. In [4], [11] and [148], a fuzzy feedback control law is designed based on some stability criteria. However, the stability conditions require that for all the local linear models a common positive-definite matrix $P$ must be found [154], [155]. Finding the common positive matrix from a Riccati equation might lead to a feasibility problem in which there is no solution for an LMI expression and the process of finding the solution is complex.

In [156], a sliding mode-like learning control scheme is developed to drive the sliding variable move towards to the sliding surface. Although the sliding variable may not reach the sliding surface in a finite time, it can asymptotically converge to the sliding surface, and the system states can then asymptotically converge to zero. Similar to the fuzzy PDC control structure, the aggregated control parameters are required to satisfy the design criteria based on some stability conditions. Therefore, this constraint has greatly limited applications of the learning control scheme for the T-S model based fuzzy system. It is hard to stabilize the global fuzzy system by means of amalgamating the control signal from each subspace using a fuzzy inference law.

In this chapter, the global control signal is determined by the control signal of the dominant linear model which dominates the entire dynamics of the global fuzzy system as in [16], [17], [118]. In order to design an effective controller for better performance, efficiency and robustness, a sliding mode learning control scheme has been developed. The control signal is updated by a correction term designed with the information of the most recent stability history of the closed-loop system. The effect of the system uncertainties on the closed-loop system has been completely eliminated by designing the sliding mode learning controller based on the Lipschitz-like condition proposed in [133]. It is worth noting that the uncertain system dynamics can be embedded into the Lipschitz-like condition as shown in [133], [135], [139-141], [156]. The controller designs with the aid of the Lipschitz-like condition can ensure the closed-loop system behaves with a strong robustness with respect to unknown uncertainties and disturbances. The major advantage of the proposed control scheme is that the information about the system uncertainties is no longer required in the controller design.
Meanwhile, the chattering phenomenon that frequently appears in conventional SMC systems is also eliminated without deteriorating the robustness of the system.

The remainder of the chapter is organised as follows: In Section 4.2, the T-S fuzzy modelling of a class of continuous-time SISO complex systems is formulated. In Section 4.3, the sliding mode learning control scheme is proposed. In Section 4.4, the convergence analysis of the closed-loop system with the proposed control scheme is discussed in detail. In Section 4.5, simulation results are provided to show the effectiveness of the proposed control scheme. Section 4.6 provides conclusions and some further work.

4.2. Problem Formulation

Consider the following T-S fuzzy models to represent a class of complex SISO systems with both fuzzy inference rules and local uncertain linear dynamic models [15]-[17], [117], [118]:

\[ R^i : \text{IF } z_1 \text{ is } F^i_1 \text{ AND ... } z_n \text{ is } F^i_n \]

\[ \text{THEN} \]

\[ \dot{x}(t) = (A_i + \Delta A_i)x(t) + (B_i + \Delta B_i)u(t) \] \hspace{1cm} (4.2.1)

for \( i = 1, 2, ..., m \)

where \( R^i \) denotes the \( i^{th} \) fuzzy inference rule, \( m \) the number of inference rules, \( F^i_j \) \((j = 1, 2, ..., n)\) the fuzzy sets, \( n \) the number of fuzzy sets, \( z(t) := [z_1, z_2, ..., z_n] \) contains some measurable system variables, \( A_i \) and \( B_i \) are \( n \times n \) and \( n \times 1 \) parameter matrices, \( x(t) \in \mathbb{R}^n \) the system state vector, \( u(t) \in \mathbb{R}^1 \) is the control input, \( \Delta A_i \) and \( \Delta B_i \) are \( n \times n \) and \( n \times 1 \) unknown system uncertainties, respectively.

Denoting \( \mu_i(z(t)) \) as the normalized fuzzy membership function:

\[ \mu_i(z(t)) = \frac{w_i(t)}{\sum_{i=1}^{m} w_i(t)} \] \hspace{1cm} (4.2.2)
where

\[ w_i(t) = \bigcap_{j=1}^{n} F_{i}^{j}(z(t)) \]  

(4.2.3)

\[ \mu_i(t) > 0 \]  

(4.2.4)

and

\[ \sum_{i=1}^{m} \mu_i(t) = 1 \]  

(4.2.5)

Using the T-S fuzzy inferences [16], [117], [132], we obtain the global fuzzy model of the complex systems as follows:

\[ \dot{x}(t) = A(t)x(t) + B(t)u(t) + f(t) \]  

(4.2.6)

where

\[ A(t) = \sum_{i=1}^{m} \mu_i(t)A_i \quad B(t) = \sum_{i=1}^{m} \mu_i(t)B_i \]  

(4.2.7)

\[ f(t) = \sum_{i=1}^{m} \mu_i(t)[\Delta A_i x(t) + \Delta B_i u(t)] \]  

(4.2.8)

**Definition 4.2.1:** The fuzzy set \( F_i^{j}(z(t)) \) is a binary set \( F_i^{j}(z(t)) = [\mu_i(z(t)), \bar{z}(t)] \) where \( \bar{z}(t) \) is the point at \( \mu_i(\bar{z}(t)) = 1 \) and \( \bar{z}(t) \) is called the crisp point of \( F_i^{j}(z(t)) \) [157].

**Definition 4.2.2:** If the controllability matrix \( M_i = [B_i, A_iB_i, \ldots, A_i^{n-1}B_i] \), for \( i = 1,2,\ldots, m \) has a full rank of \( n \), the local subsystems in (4.2.1) are locally controllable.

**Definition 4.2.3:** If the controllability matrix \( M = [B, AB, A^2B, \ldots, A^{n-1}B] \) has a full rank of \( n \). The global fuzzy model in (4.2.6) is globally controllable in the state space.
To proceed, we will assume that the (4.2.1) and (4.2.6) are both locally and globally controllable. By using the sliding mode learning control technique [133], [156] and dominant control principle [16], [17], [118], a global sliding mode learning controller can be designed such that the closed-loop system is asymptotically stable.

### 4.3. Sliding Mode Learning Control

In this chapter, a sliding variable $s(t)$ is defined as:

$$s(t) = Cx(t)$$  \hspace{1cm} (4.3.1)

where $C \in R^{1 \times n}$ is the sliding parameter matrix, which is selected to make the dynamics of $s(t) = 0$ Hurwitz.

For the fuzzy system described in (4.2.1)-(4.2.8), we propose the following local sliding mode learning controller for the $i^{th}$ subsystem:

$$C^i : IF z_1 is F^i_1 AND ... z_n is F^i_n \hspace{1cm} \text{THEN}$$

$$u_i(t) = u_i(t - \tau) - \Delta u_i(t)$$  \hspace{1cm} (4.3.2)

for $i = 1,2, ..., m.$

with the correction term

$$\Delta u_i(t) = \begin{cases} \frac{(CB_i)^{-1}}{s(t)} \left[ \alpha_i V(t) + \beta_i V(t - \tau) + \eta_i \hat{V}(t - \tau) \right] & \text{for } s(t) \neq 0 \\ 0 & \text{for } s(t) = 0 \end{cases}$$  \hspace{1cm} (4.3.3)

where $\alpha_i(>0), \beta_i(>0)$ and $\eta_i(>0)$ in (4.3.3) are the control parameters to be determined later, $\hat{V}(t - \tau)$ is the estimate of $V(t - \tau), \tau$ is the time delay. The minimum value of $\tau$ is actually equal to the sampling period in practice.
The estimation of $\dot{V}(t - \tau)$ can be achieved by using a numerical differentiator as follows:

$$
\dot{V}(t - \tau) = \frac{V(t) - V(t - \tau)}{\tau}
$$

(4.3.4)

**Remark 4.3.1**: The terms $\alpha_i V(t)$ and $\beta_i V(t - \tau)$ in (4.3.3) behave like inertia terms, and by properly choosing the value of $\alpha_i$ and $\beta_i$, the asymptotic convergence of the closed-loop system can be achieved. In addition, the correction term $\eta_i \dot{V}(t - \tau)$ is used to check the most recent stability status of the closed-loop system and update the control signal. If the most recent information shows that the system is unstable, the term $\eta_i \dot{V}(t - \tau)$ is capable of adjusting the control signal in the sense that the closed-loop system can be driven from the unstable domain to the stable domain. The merits of these points can be seen from the convergence analysis in the next section.

**Remark 4.3.2**: It is important to note that the local linear models in (4.2.1) represent the local dynamics of a class of continuous-time SISO complex systems in the subspaces of the state space which are smoothly connected by fuzzy membership functions. However, although each linear model with its local sliding mode controller in each subspace is asymptotically stable, global asymptotic stability may not be guaranteed in the overlapping regions of fuzzy sets where several subsystems are activated at the same time to certain degrees [17].

Considering the fact that the local linear model in (4.2.1) with the maximum membership function dominates the entire system dynamics, we can design the global control $u(t)$ [17], [157]. Firstly, the global control is inferred as:

$$
u(t) = \sum_{i=1}^{m} \mu_i(t) u_i(t)
$$

(4.3.5)

Secondly, according to the dominant control principle [16], [17], [118], we can use $u_k(t)$, the local control signal for the $k$th subsystem model with the maximum membership function, to replace $u_i(t)$ in (4.3.5). The global control can be expressed as
\[ u(t) = \sum_{i=1}^{m} \mu_i(t) [u_k(t - \tau) - \Delta u_k(t)] \quad (4.3.6) \]

\[ = u_k(t - \tau) - \Delta u_k(t) \]

where

\[ k = j: \mu_j(t) = \max[\mu_i(t), \ldots, \mu_m(t)] \quad (4.3.7) \]

**Remark 4.3.3:** It is seen that, no sign function of the sliding variable \( s(t) \) is involved in the control signals and hence both the local control inputs and the global control input enjoy chattering-free characteristics.

**Remark 4.3.4:** As shown in [16], [17], [118], the proposition of the dominant control principle can greatly simplify the T-S model based fuzzy control system designs for complex systems. Compared with the PDC approach [4], [153], the dominant control approach can avoid the problem of finding a common positive solution to satisfy the design criteria based on some stability conditions. The merits of the proposed control with the aid of a dominant control principle can be seen from a section of the convergence analysis and simulation results.

### 4.4. Convergence Analysis

**Theorem 4.4.1:** Consider the global fuzzy system in (4.2.6). If the sliding variable is chosen as in (4.3.1), the local control input \( u_i(t) \) and global control input \( u(t) \) are designed as in (4.3.2) and (4.3.6), respectively, then the system trajectory can asymptotically converge to zero.

**Proof:** The candidate of Lyapunov function for the global closed-loop system is chosen as:

\[ V(t) = 0.5s(t)^2 \quad (4.4.1) \]

Differentiating \( V(t) \) with respect to time \( t \) leads to
\[ \dot{V}(t) = s(t)\dot{s}(t) = s(t)C\dot{x}(t) \]

\[ = s(t)C \sum_{i=1}^{m} \mu_i(t) [A_i x(t) + B_i u(t) + \Delta B_i u(t) + \Delta A_i x(t)] \]

\[ = s(t)CA(t)x(t) + s(t) \sum_{i=1}^{m} \mu_i(t) [CB_i u(t)] + s(t)Cy(t) \]

\[ = s(t)CA(t)x(t) + s(t)Cf(t) \]

\[ + s(t) \sum_{i=1}^{m} \mu_i(t) \left[ CB_i u_k(t - \tau) \right. \]

\[ - \frac{CB_i(CB_k)^{-1}}{s(t)} \left( \alpha_k V(t) + \beta_k V(t - \tau) + \eta_k \hat{V}(t - \tau) \right) \]

\[ = s(t)CA(t)x(t) + s(t)CB(t)u_k(t - \tau) + s(t)Cf(t) \]

\[ - \sum_{i=1}^{m} \mu_i(t) \left[ CB_i(CB_k)^{-1} \left( \alpha_k V(t) + \beta_k V(t - \tau) + \eta_k \hat{V}(t - \tau) \right) \right] \]

\[ = \dot{V}(t, t - \tau) \]

\[ - \sum_{i=1}^{m} \mu_i(t) \left[ CB_i(CB_k)^{-1} \left( \alpha_k V(t) + \beta_k V(t - \tau) + \eta_k \hat{V}(t - \tau) \right) \right] \text{(4.4.2)} \]

where

\[ \dot{V}(t, t - \tau) = s(t)CA(t)x(t) + s(t)CB(t)u_k(t - \tau) + s(t)Cf(t) \quad \text{(4.4.3)} \]

Adding the term \( \dot{V}(t - \tau) - \dot{V}(t - \tau) \) to (4.4.2), we have

\[ \dot{V}(t) = \dot{V}(t, t - \tau) + \dot{V}(t - \tau) - \dot{V}(t - \tau) \]

\[ - \sum_{i=1}^{m} \mu_i(t) [CB_i(CB_k)^{-1}] \left( \alpha_k V(t) + \beta_k V(t - \tau) + \eta_k \hat{V}(t - \tau) \right) \]
\[
\begin{align*}
\leq & |\dot{V}(t, t - \tau) - \dot{V}(t - \tau)| + \dot{V}(t - \tau) \\
& - \sum_{i=1}^{m} \mu_i(t) [CB_i(CB_k)^{-1}] \left( \alpha_k V(t) + \beta_k V(t - \tau) + \eta_k \hat{V}(t - \tau) \right) \tag{4.4.4}
\end{align*}
\]

where

\[
\dot{V}(t - \tau) = s(t - \tau)CA(t - \tau)x(t - \tau) + s(t - \tau)CB(t - \tau)u_k(t - \tau) \\
+ s(t - \tau)f(t - \tau) \tag{4.4.5}
\]

**Remark 4.4.1:** Considering the continuity of both \(\dot{V}(t - \tau)\) and \(\dot{V}(t, t - \tau)\), with the time-delay \(\tau\) chosen to be sufficiently small, we may always find a constant \(M \gg 1\) such that the following inequality is held \[133\], \[156\]:

\[
|\dot{V}(t, t - \tau) - \dot{V}(t - \tau)| < \frac{1}{M} |\dot{V}(t - \tau)| \tag{4.4.6}
\]

for \(\dot{V}(t, t - \tau) \neq 0\), \(\dot{V}(t - \tau) \neq 0\) and \(\dot{V}(t) \neq 0\).

It is important to note that the inequality (4.4.6) is called the Lipschitz-like condition \[133\], \[156\]. The \(\dot{V}(t, t - \tau)\) is the derivative of the present Lyapunov function with the recent control input, \(\dot{V}(t - \tau)\) is the derivative of the most recent Lyapunov function. The Lipschitz-like condition which describes the difference between these two derivatives is very small as the sampling time is sufficiently small. The uncertain system dynamics are all embedded into the left-hand side of (4.4.6). Therefore, the information on the upper and the lower bounds of the system uncertainties is no longer required in the controller design with the aid of the Lipschitz-like condition. In addition, the Lipschitz-like condition provides a learning control strategy for us to design the controller. The control signal is recursively updated based on the most recent stability history of the closed-loop system. The merits of this control structure can be seen from the following stability analysis.

Using (4.4.4) in (4.4.6), we obtain
\[
\dot{V}(t) < \frac{1}{M} \left| \dot{V}(t - \tau) \right| + \dot{V}(t - \tau) - G(t) \left( \alpha_k V(t) + \beta_k V(t - \tau) + \eta_k \hat{V}(t - \tau) \right)
\]  
(4.4.7)

where
\[
G(t) = \sum_{i=1}^{m} \mu_i(t) [CB_i(CB_k)^{-1}]
\]  
(4.4.8)

**Remark 4.4.2:** As the time delay \(\tau\) is sufficiently small, \(\hat{V}(t - \tau)\) can be reasonably assumed to have the same sign as \(\dot{V}(t - \tau)\), that is,
\[
\text{sign} \left( \hat{V}(t - \tau) \right) = \text{sign} \left( \dot{V}(t - \tau) \right)
\]  
(4.4.9)

for \(\hat{V}(t - \tau) \neq 0\) and \(\dot{V}(t - \tau) \neq 0\).

The approximation error between \(\dot{V}(t - \tau)\) and \(\hat{V}(t - \tau)\) is defined as:
\[
\delta \left( \dot{V}(t - \tau) \right) = \dot{V}(t - \tau) - \hat{V}(t - \tau)
\]  
(4.4.10)

For \(\hat{V}(t - \tau) \neq 0\) and \(\dot{V}(t - \tau) \neq 0\), there exists a positive number \(\gamma\), such that the following inequality is held:
\[
\left| \delta \left( \dot{V}(t - \tau) \right) \right| = \left| \dot{V}(t - \tau) - \hat{V}(t - \tau) \right| < \gamma \left| \hat{V}(t - \tau) \right|
\]  
(4.4.11)

with \(0 < \gamma \ll 1\),

then using (4.4.10) and (4.4.11) in (4.4.7) leads to
\[
\dot{V}(t) < \frac{1}{M} \left| \hat{V}(t - \tau) \right| + \dot{V}(t - \tau) + \gamma \left| \hat{V}(t - \tau) \right|
\]
\[-G(t) \left( \alpha_k V(t) + \beta_k V(t - \tau) + \eta_k \hat{V}(t - \tau) \right) \quad (4.4.12)\]

- If $\hat{V}(t - \tau) > 0$

From (4.4.7), we can express $\hat{V}(t)$ as follows:

\[
\hat{V}(t) < \hat{V}(t - \tau) - \left( \eta_k G(t) - \frac{1}{M} \right) |\hat{V}(t - \tau)|
- G(t)(\alpha_k V(t) + \beta_k V(t - \tau))
< \hat{V}(t - \tau) - \left( \eta_k G(t) - \frac{1}{M} \right) |\hat{V}(t - \tau)|
\quad (4.4.13)
\]

**Remark 4.4.3:** It is important to note that the sliding mode parameter $C$ for each linearized subsystem is designed to ensure the closed-loop system is globally asymptotically stable, and also it satisfies the following conditions:

\[1 \geq K_{\min} = \lambda_{\min}(C B_i (C B_k)^{-1}) > 0 \quad (4.4.14)\]

\[\left(1 - \frac{1}{M} - \gamma\right) (K_{\min})^2 > \eta_k K_{\min} > \frac{1}{M} + \gamma \quad (4.4.15)\]

From (4.4.15), we can determine the control parameter $\eta_k$. Then, using (4.4.15) in (4.4.13) leads to

\[
\hat{V}(t) < \hat{V}(t - \tau) - \left( \eta_k G(t) - \frac{1}{M} \right) |\hat{V}(t - \tau)| - \alpha_k CB(t)V(t) - \beta_k CB(t)V(t - \tau)
\leq \hat{V}(t - \tau) - \left( \eta_k K_{min} - \frac{1}{M} \right) |\hat{V}(t - \tau)| - \alpha_k CB(t)V(t) - \beta_k CB(t)V(t - \tau)
< \hat{V}(t - \tau)
\quad (4.4.16)\]
The inequality (4.4.16) shows that the global sliding mode learning controller in (4.3.6) always makes the $\dot{V}(t)$ smaller than $\dot{V}(t - \tau)$ as $\dot{V}(t - \tau) > 0$.

Suppose that, at time $t = t_1$, $\dot{V}(t_1) = 0$

$$\dot{V}(t_1) = \dot{V}(t_1, t_1 - \tau) - G(t_1) \left[ \eta_k \dot{V}(t_1 - \tau) + \alpha_k V(t_1) + \beta_k V(t_1 - \tau) \right]$$  \hspace{1cm} (4.4.17)

From (4.4.17), we can express the terms as

$$\dot{V}(t_1, t_1 - \tau) = G(t_1) \left[ \eta_k \dot{V}(t_1 - \tau) + \alpha_k V(t_1) + \beta_k V(t_1 - \tau) \right] > 0$$  \hspace{1cm} (4.4.18)

For $\dot{V}(t_1 - \tau) \neq 0$, (4.4.18) means that

$$G(t_1) \left[ \eta_k \dot{V}(t_1 - \tau) + \alpha_k V(t_1) + \beta_k V(t_1 - \tau) \right] > \dot{V}(t_1) = 0$$  \hspace{1cm} (4.4.19)

It is seen from (4.4.18) that, the term $\dot{V}(t_1, t_1 - \tau)$ neutralises the negative force from the global sliding mode learning controller in (4.3.6). However, the inequality in (4.4.19) shows that the controller still continuously tries to drive the system energy towards the direction of $\dot{V}(t) < 0$. Moreover, the controller is designed to have inertial terms $\alpha_k V(t)$ and $\beta_k V(t - \tau)$ to force the $\dot{V}(t) < 0$.

At the time $t = t_1 + \tau$, we can express (4.4.17) as:

$$\dot{V}(t_1 + \tau) = \dot{V}(t_1 + \tau, t_1) - G(t_1 + \tau)\dot{V}(t_1)$$

$$- G(t_1 + \tau) \left( \alpha_k V(t_1 + \tau) + \beta_k V(t_1) \right)$$

$$\leq \dot{V}(t_1 + \tau, t_1) - G(t_1 + \tau)\dot{V}(t_1)$$

$$- K_{\text{min}} \left( \alpha_k V(t_1 + \tau) + \beta_k V(t_1) \right)$$  \hspace{1cm} (4.4.20)

if the control parameters $\alpha_k$ and $\beta_k$ are chosen to satisfy the following condition
\[
|\dot{V}(t_1 + \tau, t_1)| + |G(t_1 + \tau)\dot{V}(t_1)| < K_{\text{min}}(\alpha_k V(t_1 + \tau) + \beta_k V(t_1)) \tag{4.4.21}
\]
then using (4.4.21) in (4.4.20),

\[
\dot{V}(t_1 + \tau) \leq \dot{V}(t_1 + \tau, t_1) - \eta_k G(t_1 + \tau)\dot{V}(t_1) - K_1(\alpha_k V(t_1 + \tau) + \beta_k V(t_1)) < 0 \tag{4.4.22}
\]

- If \( \dot{V}(t - \tau) < 0 \)

Using (4.4.15) in (4.4.12) leads to

\[
\dot{V}(t) \leq -\left(1 - \frac{1}{M} - \gamma\right) - \eta_k G(t) - \frac{\eta_k}{K_{\text{min}}} |\dot{V}(t - \tau)| - \alpha_k G(t)V(t - \tau) - \beta_k G(t)V(t - \tau) < -\alpha_k C(t)V(t) - \beta_k C(t)V(t - \tau) \text{ for } s(t) \neq 0 \tag{4.4.23}
\]

(4.4.13)-(4.4.23) indicate that the global sliding mode learning controller in (4.3.6) is capable of driving the closed-loop system dynamics to move from the unstable domain to the stable domain, and thus, ensures that \( \dot{V}(t) < 0 \). The sliding mode learning control law in (4.3.6) ensures that both the sliding variable \( s(t) \) and the system states asymptotically converge to zero. The merits of both the asymptotic convergence of the closed-loop system and the chattering-free characteristics of the proposed sliding mode learning controller can be further verified from the simulation results in the next section.

**Remark 4.4.4:** As the sliding variable \( s(t) \) asymptotically converges to zero, the relationship between \( s(t) \) and \( s(t - \tau) \) as time \( t \to \infty \) is definite as:

\[
\lim_{s \to \infty} \frac{s(t)}{s(t - \tau)} = \rho \tag{4.4.24}
\]

where \( 0 < |\rho| \leq 1 \).
then the correction term $\Delta u_i(t)$ in the sliding mode learning controller in (4.3.2) satisfies:

$$
\lim_{s \to 0} (\Delta u_i(t)) = \lim_{s \to 0} \left( \alpha_i (CB_i)^{-1} V(t) s(t) + \beta_i (CB_i)^{-1} \frac{V(t - \tau)}{s(t)} + \eta_i (CB_i)^{-1} \frac{\ddot{V}(t - \tau)}{s(t)} \right)
$$

$$
= \lim_{s \to 0} \left( \frac{(CB_i)^{-1}}{2s(t)} \left( \alpha_k s^2(t) + \beta_k s^2(t - \tau) + \frac{\eta_k}{\tau} (s^2(t) - s^2(t - \tau)) \right) \right)
$$

$$
= \lim_{s \to 0} \left( \frac{(CB_i)^{-1}}{2} \left( \alpha_k s(t) + \frac{\beta_k}{\rho} s(t) + \frac{\eta_k}{2\tau} \left( 1 - \frac{1}{\rho} \right) s(t) \right) \right)
$$

$$
= 0 \quad (4.4.25)
$$

Therefore, there is no singularity in the control signal $u_i(t)$. This point will be clearly seen from a section of the simulation results.

**Remark 4.4.5**: The design of sliding surface is a major problem in this chapter. The pole placement approach [158] and the hyperplane design approach [159] are commonly used to design sliding parameter $C$ of sliding surface $s(t)$. All the methods can be used for the design of $C$ for the fuzzy control system. However, the design of $C$ needs to satisfy the design criteria specified in **Remark 4.4.3**

**Remark 4.4.6**: It is well known that, in conventional SMC, the upper and lower bounds of system uncertainties are often required for designing a sliding mode controller. However, as seen in both the local sliding mode learning control in (4.3.2) and the global fuzzy control law in (4.3.6) in this chapter, no information about unknown system parameter is needed for the design of both the local sliding mode and the global sliding mode controller. Therefore, the proposed control scheme is robust and is the most suitable for control of large-scale systems with large uncertain dynamics and unknown dynamic interactions.
4.5. Simulation Results

Consider an inverted pendulum on a cart with its uncertain system dynamics as follows [11], [132], [133]:

\[
\begin{align*}
\dot{x}_1 &= \Delta a_1 x_1 + x_2 \\
\dot{x}_2 &= \frac{gsin(x_1) - \frac{amlx_2^2 \sin(2x_1)}{2} - (acos(x_1) + \Delta b_1 \cos(x_1))u}{4l/3 - amlcos^2(x_1)},
\end{align*}
\]

\[
\Delta a_1 = 4 \sin(2\pi t), \quad |\Delta a_1| \leq 4
\]

\[
\Delta b_1 = 0.5 \tag{4.5.1}
\]

where \(x_1\) is the angle of the pendulum from the vertical, \(x_2\) is the angular velocity, \(g = 9.81\text{m/s}\) is the gravity constant, \(m\) is the mass of the pendulum rod, \(M\) is the mass of the cart, \(u\) is the control force applied to the cart, \(\Delta a_1\) and \(\Delta b_1\) are the system uncertainties, parameter \(a = 1/(m + M)\), and \(2l\) is the length of the pendulum respectively. The initial values of the state variables are chosen as \(x_1(t_0) = \pi/10\text{ rad}\) and \(x_2(t_0) = 0\text{ rad/s for } t_0 = 0\). In this simulation, the system parameters are chosen as \(m = 2.0\text{kg}, M = 8.0\text{kg}, \) and \(2l = 1.0\text{m}\). The following fuzzy model can be obtained by linearizing the nonlinear dynamic equation (4.5.1) over a number of operating points \((x_1, x_2)\) in state space as in [16] for approximating range \(x_1 \in (-\pi/3, \pi/3)\) and \(x_2 \in (-4,4)\).

\[
R^1 : \quad \text{IF } x_1 \text{ is about 0, } x_2 \text{ is about 0} \quad \text{THEN} \quad \dot{x}(t) = (A_1 + \Delta A_1)x(t) + (B_1 + \Delta B_1)u
\]

\[
R^2 : \quad \text{IF } x_1 \text{ is about 0, } x_2 \text{ is about } \pm 4 \quad \text{THEN} \quad \dot{x}(t) = (A_2 + \Delta A_2)x(t) + (B_2 + \Delta B_2)u
\]
\[ R^3 : \text{ IF } x_1 \text{ is about } \pm \pi/3, x_2 \text{ is about 0} \]
\[ \text{THEN} \]
\[ \dot{x}(t) = (A_3 + \Delta A_3)x(t) + (B_3 + \Delta B_3)u \]

\[ R^4 : \text{ IF } x_1 \text{ is about } +\pi/3, x_2 \text{ is about } +4 \]
\[ \text{or } x_1 \text{ is about } -\pi/3, x_2 \text{ is about } -4 \]
\[ \text{THEN} \]
\[ \dot{x}(t) = (A_4 + \Delta A_4)x(t) + (B_4 + \Delta B_4)u \]

\[ R^5 : \text{ IF } x_1 \text{ is about } +\pi/3, x_2 \text{ is about } -4 \]
\[ \text{or } x_1 \text{ is about } -\pi/3, x_2 \text{ is about } +4 \]
\[ \text{THEN} \]
\[ \dot{x}(t) = (A_5 + \Delta A_5)x(t) + (B_5 + \Delta B_5)u \]  \hspace{1cm} (4.5.2)

where

\[ A_1 = \begin{bmatrix} 0 & 1 \\ 17.2941 & 0 \end{bmatrix} \quad B_1 = \begin{bmatrix} 0 \\ -0.1765 \end{bmatrix} \]
\[ A_2 = \begin{bmatrix} 0 & 1 \\ 14.4706 & 0 \end{bmatrix} \quad B_2 = \begin{bmatrix} 0 \\ -0.1765 \end{bmatrix} \]
\[ A_3 = \begin{bmatrix} 0 & 1 \\ 5.8512 & 0 \end{bmatrix} \quad B_3 = \begin{bmatrix} 0 \\ -0.0779 \end{bmatrix} \]
\[ A_4 = \begin{bmatrix} 0 & 1 \\ 7.2437 & 0.5399 \end{bmatrix} \quad B_4 = \begin{bmatrix} 0 \\ -0.0779 \end{bmatrix} \]
\[ A_5 = \begin{bmatrix} 0 & 1 \\ 7.2437 & 0.5399 \end{bmatrix} \quad B_5 = \begin{bmatrix} 0 \\ -0.0779 \end{bmatrix} \]
\[ \Delta A_1 = \Delta A_2 \ldots \Delta A_5 = \begin{bmatrix} \Delta a_1 & 0 \\ 0 & 0 \end{bmatrix} \]
\[ \Delta B_1 = 0.5B_1 \quad \Delta B_2 = 0.5B_2 \quad \Delta B_3 = 0.5B_3 \]
\[ \Delta B_4 = 0.5B_4 \quad \Delta B_5 = 0.5B_5 \]  \hspace{1cm} (4.5.3)

we use the following fuzzy membership functions for state variables \( x_1(t) \) and \( x_2(t) \):

\[ \mu_{A_{-\pi/3}}(x_1) = \frac{1}{1 + e^{(3\pi(x_1+\pi/6))}} \quad \mu_{B_{-4}}(x_2) = \frac{1}{1 + e^{(3\pi(x_2+\pi/3))}} \]
\[ \mu_{A_{\pi/3}}(x_1) = \frac{1}{1 + e^{(-3\pi(x_1-\pi/6))}} \quad \mu_{B_4}(x_2) = \frac{1}{1 + e^{(-3\pi(x_2-\pi/3))}} \]
\[ \mu_{A_0}(x_1) = 1 - \mu_{-\pi/3}(x_1) - \mu_{\pi/3}(x_1) \quad \mu_{B_0}(x_2) = 1 - \mu_{-4}(x_2) - \mu_4(x_2) \]  \hspace{1cm} (4.5.4)
These membership functions are shown in Figure 4.1.

![Membership Function of fuzzy set x1 (rad)](image1)

![Membership Function of fuzzy set x2 (rad/s)](image2)

Figure 4.1  The membership functions of fuzzy sets $x_1(t)$ and $x_2(t)$

The membership functions for 1$^{st}$-5$^{th}$ rules of the fuzzy model are selected based on the extension principle.

$$
R^1: \mu_1 = \mu_{A0}(x_1) \ast \mu_{B0}(x_2) \\
R^2: \mu_2 = \max(\mu_{A0}(x_1) \ast \mu_{B4}(x_2), \mu_{A0}(x_1) \ast \mu_{B-4}(x_2)) \\
R^3: \mu_3 = \max \left( \mu_{A_{\pi/3}}(x_1) \ast \mu_{B0}(x_2), \mu_{A-\pi/3}(x_1) \ast \mu_{B-0}(x_2) \right) \\
R^4: \mu_4 = \max \left( \mu_{A_{\pi/3}}(x_1) \ast \mu_{B0}(x_2), \mu_{A-\pi/3}(x_1) \ast \mu_{B-0}(x_2) \right) \\
R^5: \mu_5 = 1 - \mu_1 - \mu_2 - \mu_3 - \mu_4 \\
(4.5.5)
$$

Based on (4.3.2)-(4.3.7), we select the control signal of the dominant subsystem as the global sliding mode learning controller:

$$u_k(t) = u_k(t - \tau) + \Delta u_k(t) \quad (4.5.6)$$

with the correction term
\[
\Delta u_k(t) = \begin{cases} 
\frac{(CB_k)^{-1}}{s(t)} \left[ \alpha_k V(t) + \beta_k V(t - \tau) + \eta_k \hat{V}(t - \tau) \right] & \text{for } s(t) \neq 0 \\
0 & \text{for } s(t) = 0 
\end{cases}
\tag{4.5.7}
\]

where

\[
k = j: \mu_j(t) = \max[\mu_1(t), ..., \mu_2(t)]
\]

The sliding mode parameter matrix is chosen as \( C = \begin{bmatrix} 5 & 1 \end{bmatrix} \), the sampling period is \( \Delta T = 0.001 \text{sec} \), the time delay \( \tau = \Delta T \).

It is easy to check that

\[
\lambda_{\min}(CB_i(CB_k)^{-1}) = 0.4414 > 0 \tag{4.5.8}
\]

is satisfied for all \( i \) and \( k \). Also, in this simulation, we assume \( M=20 \) and \( \gamma = 0.001 \)

\[
\frac{1}{M} + \gamma = 0.051 \tag{4.5.9}
\]

The control parameter \( \eta_k \) is designed to satisfy the following condition:

\[
0.1907 > \eta_k \cdot 0.4414 > 0.051 \tag{4.5.10}
\]

Therefore, the control parameters in (4.5.7) are set to \( \alpha_k = 6 \), \( \beta_k = 0.2 \) and \( \eta_k = 0.4 \). Figures 4.2(a) and 4.2(b) show the sliding variable \( s(t) \), the global control input \( u_k(t) \) and system states \( x_1 \) and \( x_2 \). It is clearly seen that in Figures 4.2(a) and 4.2(b) the sliding variable \( s(t) \) converges to zero in 0.5 seconds, the states \( x_1 \) and \( x_2 \) can then asymptotically converge to zero. The concept of the Lipschitz-like condition is verified in Figure 4.2(c), the inequality in (4.4.6) is always satisfied. In addition, the control signal \( u_k(t) \) is completely chattering-free.
Figure 4.2(a) Sliding variable $s(t)$ and control input $u_k(t)$ (Sliding mode learning control)

Figure 4.2(b) States $x_1(t)$ and $x_2(t)$ responses (Sliding mode learning control)
For comparison purposes, the conventional sliding mode and H-infinity controller are developed based on T-S fuzzy models in (4.5.2). Firstly, Figures 4.3(a) and 4.3(b) show the simulation results of the closed-loop system using the conventional SMC [118], where the control gains $K_1 = K_2 \ldots K_5 = 15$, the initial values of the state variables and the sampling period are the same as the previous ones. It has been seen that, although the convergence speeds of the proposed sliding mode learning control and the conventional SMC are nearly the same, the sliding mode learning control signal in Figure 4.2(a) is smooth and chattering-free. Such an excellent property obviously makes the proposed sliding mode learning control more practical.
Next, the H-infinity control scheme is used to compare the control performance of different controllers. The following control gains can be determined by the H-infinity attenuation technique in [132]:

\[
K_1 = [222.48 \ 50.38], \quad K_2 = [188.25 \ 46.34]
\]
\[ K_3 = [211.65 \ 78.21], \quad K_4 = [228.10 \ 85.38] \]
\[ K_5 = [235.94 \ 72.35] \quad \text{(4.5.11)} \]

![Figure 4.4(a) The control input \( u(t) \) (H-infinity control)](image1)

![Figure 4.4(b) States \( x_1(t) \) and \( x_2(t) \) responses (H-infinity control)](image2)

The control performance of the H-infinity in Figures 4.4(a) and 4.4(b) are not as good as the proposed sliding mode learning control method. The settling time is much longer than conventional SMC and proposed sliding mode controller.

For a further comparison, we amplify the system uncertainty \( \Delta a_2 \) as follows:
\[ \Delta a_1 = 10 \sin(2\pi t) \]  

(4.5.12)

Based on the uncertainty in (4.5.12), we redesign the control gains for the H-infinity controller

\[
K_1 = [1143.2 \ 258.9], \quad K_2 = [967.3 \ 238.1] \\
K_3 = [1279.6 \ 477.1], \quad K_4 = [1329.6 \ 497.7] \\
K_5 = [1375.3 \ 421.7] 
\]

(4.5.13)

The attenuation performance of the H-infinity control with the control gains as set in (4.5.13) are shown in Figures 4.5(a) and 4.5(b). In order to maintain the control performance, a set of large gains are required for H-infinity control to compensate for the effects of the uncertainties. However, large control gains will generate a large overshoot in the control signal which is not suitable in real-time implementation.

Fig. 4.5(a): The control input \( u(t) \) (H-infinity control)
Figure 4.5(b) States $x_1(t)$ and $x_2(t)$ responses (H-infinity control)

The simulation results of the proposed sliding mode learning controller are shown in Figures 4.6(a) and 4.6(b). The sliding mode parameter matrix is set as $C = [15 1]$, the control parameters of the proposed controller remain the same as the previous ones. The overshoot of the proposed control input signal in Figure 4.6(a) is much smaller than the H-infinity control method in Figure 4.5(a). Meanwhile the proposed sliding mode learning controller can ensure that the closed-loop system is globally asymptotically stable without the need of adjusting the control gains $\alpha_k$, $\beta_k$ and $\eta_k$.

Figure 4.6(a) Sliding variable $s(t)$ and control input $u_k(t)$
The merit of using the sliding mode learning control technique is that the controller can guarantee the system states smoothly converge to zero. The system uncertainties have been embedded in the Lipchitz-like condition as in (4.4.6). Hence, without relying on the upper and the lower bounds of system uncertainties, the controller tunes the current stability status of the system as it moves towards the stable region. The proposed learning control method is less conservative than the conventional SMC method and H-infinity control method. Therefore, it can be used for more complex control systems.

4.6. Conclusion

In this chapter, the sliding mode learning control has been developed for fuzzy uncertain continuous-time SISO systems. It has been seen that the proposed sliding mode learning control is capable of driving both the sliding variable and systems states to converge to zero asymptotically based on the most recent stability status of the closed-loop system. The concept of dominant control principle is used to facilitate the sliding mode learning control scheme and enhance the function of the controller in different types of control applications. The design criteria for sliding parameter $C$ have been proposed to ensure
the stability of the closed-loop system. It has been further shown in the simulation section that the proposed sliding mode learning control achieves much better results in terms of robustness and performance as compared to the conventional SMC and H-infinity control. The further work to apply the sliding mode learning control with the adaptive estimation of control parameters for the large-scale systems is placed under investigation.
Chapter 5
Robust Sliding Mode Learning Model Reference Tracking Control for a Class of Large-Scale Systems with Adaptive Estimation

This chapter studies the problem of a sliding mode learning model reference tracking control for a class of large-scale systems. The considered large-scale system contains a number of nonlinear interconnected subsystems. The interconnection between these subsystems may be nonlinear and unknown. The local dynamics of subsystems is first described by a local linear model in the state-space. It is seen that a global T-S fuzzy model can then be achieved by smoothly connecting the local linear model in each subspace of the state space together via membership functions. A set of local chattering-free sliding mode learning controllers is designed to ensure the stability of its corresponding local linear model. According to the principle that the dominant subsystem governs the global fuzzy model dynamics, a global control signal can be determined to override the effects of nonlinear interconnections such that good tracking performance is achieved. The advantage of the control structure is that a prior knowledge of the upper bounds of interconnection among subsystems is not required in the proposed learning controller design. A simulation example is given to show the excellent tracking performance and effectiveness of the proposed sliding mode learning control scheme.

5.1. Introduction

In recent years, there has been rapidly growing interest in the control of a class of large-scale systems. There are plenty of successful applications related to this research topic such as multirobot systems, electric power systems, aerospace systems, process control
systems and transportation networks. A large-scale system consists of a number of subsystems that serve particular functions and exchange information between the subsystems in the sense that the subsystems have become increasingly large in complexity in their structure. That is the main reason why control of complex behaviours in large-scale systems has received a great deal of attention in recent studies [160]-[164]. From a system theoretic point of view, a large-scale system can be treated as a set of interconnected complex nonlinear systems.

In the past few years, many modelling methodologies have been developed to model large-scale systems for investigating the stability property of the systems and designing effective control strategies [165]-[170]. In [167], a large-scale system is composed of linearized interconnected subsystems, and the authors have defined a switching function for each subsystem. Jaint et al. considered a class of interconnected systems which can be transformed to the decentralized strict feedback form in [168]. However, the major drawbacks of conventional modelling approaches are attributed to (i) the lack of a systematic method for its stability analysis, (ii) various assumptions required to model the interconnected subsystems, (iii) the linearization technique and linear robust control are only applicable to some specific large-scale systems, (iv) difficult to obtain the system dynamics in practice [165]. Recently, T-S fuzzy modelling technique is used to develop a systematic approach to model the large-scale systems [166], [171], [172]. It has the capability of approximate the systems to arbitrary degrees of accuracy.

In order to provide a simple and robust control scheme for a class of large-scale systems, the researchers have noted many possible control solutions of applying the T-S fuzzy modelling technique. Wang et al. developed the decentralized PDC for large-scale T-S Fuzzy systems [166]. Some sufficient conditions are based on Lyapunov criterion and Riccati-inequality. In [171], a robust control problem is studied for a class of large-scale networked control systems. Tseng et al. developed the $H$-infinity decentralized fuzzy control scheme to deal with the model reference tracking control problem [172]. The major defect of the decentralized PDC approaches is that the upper bounds of nonlinear interconnections need to be known in controller designs. For large-scale T-S Fuzzy systems, if the number of sub-models is large, finding the common positive
matrix by using LMI might cause a feasibility problem in which there is no solution for LMI expression. In addition, the decentralized control scheme attempts to avoid difficulties in complexity of design and storage requirements. However, there is still no efficient way to cope with the decentralized control problem of large-scale nonlinear interconnected systems i.e., to design a decentralized controller so that the states of the system asymptotically track the given model reference inputs independent of any nonlinear interconnections among the systems. As a result, many theoretical and practical problems remain unsolved in this field.

The objective of this chapter is to address the problem of model reference tracking control for a class of large-scale systems by applying sliding mode learning control theory based on [133], [135], [139]-[141], [156]. Our interest in this problem is motivated by the recent relative research works in [165]-[166] and [172] on T-S fuzzy modelling and control of large-scale systems. In a large-scale system, global T-S fuzzy models are first developed to model dynamics of nonlinear interconnected subsystems. A set of local sliding mode learning controllers can then be designed to ensure the stability of its corresponding local linear models which are smoothly connected by fuzzy membership functions. For each subsystem, global fuzzy control can be determined by the control signal of the dominant local linear model which dominates the dynamics of the global T-S fuzzy model.

As a learning control, the control signal is updated by a correction term designed with the information of the most recent stability history of the closed-loop system to drive both the sliding variable and tracking error between system state and model reference to asymptotically converge to zero. The prior information about the nonlinear interconnections among subsystems is no longer required in the controller design with the aid of the Lipschitz-like condition. The proposed sliding mode learning control in this chapter is designed based on the concept of adaptive control laws in [17]. Unlike the control strategy in [156], in which the sliding mode learning control parameters are determined based on some stability criteria, in this chapter we use adaptive estimation laws such that the control parameters are obtained as well for the proposed controller design. The adaptive estimation laws are designed based on the most recent stability
status. It can be seen that, the estimation stops after the sliding variable reach the sliding surface in which the behavior of the system depends on the design of the sliding surface.

The rest of this chapter is organised as follows: in Section 5.2, the T-S fuzzy modelling of a class of large-scale systems and the sliding mode learning control with adaptive estimation laws are formulated. In Section 5.3, the convergence analysis of the closed-loop system with the proposed sliding mode learning control is discussed in detail. In Section 5.4, a simulation example is given to illustrate the effectiveness of the proposed control scheme. Section 5.5 provides the conclusions and some further works.

### 5.2. Problem Formulation

Suppose there is a large-scale system composed of \( q \) nonlinear interconnected subsystems. The dynamics of each subsystem \( H_i \) can be represented by T-S fuzzy models in subspaces of the state-space. The \( i \)th rule of the T-S fuzzy model for \( i \)th subsystem is of the following form \([166],[173]\):

\[
H_i^l: \begin{cases}
\text{IF } z_{i1} \text{ is } F_{i1}^l \text{ AND } ... \text{ AND } z_{iy} \text{ is } F_{iy}^l \\
\text{THEN } \dot{x}_i(t) = A_{il}x_i(t) + B_{il}u_i(t) + B_{il} \sum_{j=1, j \neq i}^{N} f_{ijl}(x_j(t))
\end{cases} \tag{5.2.1}
\]

for \( i = 1,2, ..., q; l = 1,2, ..., p \)

where \( q \) the number of subsystems, \( p \) the number of inference rules, \( F_{io}^l \) \((o = 1,2, ..., y)\) the fuzzy sets, \( y \) the number of fuzzy sets, \( x_i(t) \in \mathbb{R}^n \) the state vector, \( u_i(t) \in \mathbb{R}^1 \) the input variable. \( z_i(t) := [z_{i1}, z_{i2}, ..., z_{iy}] \) contains measurements of system variables, \( f_{ijl}(x_j(t)) \) represents the interconnection between subsystem \( H_i \) and subsystem \( H_j \), \( A_{il} \) and \( B_{il} \) are \( n \times n \) and \( n \times 1 \) parameter matrices, respectively.

Denoting \( \mu_i(z(t)) \) as the normalized fuzzy membership function:

\[
\mu_i(z(t)) = \frac{w_i(t)}{\sum_{l=1}^{p} w_l(t)} \tag{5.2.2}
\]
where

\[ w_i(t) = \bigcap_{\alpha=1}^{n} F^i_\alpha(z(t)) \]  \quad (5.2.3)

\[ \mu_i(t) > 0 \]  \quad (5.2.4)

and

\[ \sum_{i=1}^{p} \mu_i(t) = 1 \]  \quad (5.2.5)

and using the T-S fuzzy inferences \([117],[156],[174]\), we obtain the global fuzzy model of \(i\)th subsystem as follows:

\[ \dot{x}_i(t) = A_i(t)x_i(t) + B_i(t)u_i(t) + v_{ij}(x_j(t)) \]  \quad (5.2.6)

where

\[ A_i(t) = \sum_{i=1}^{p} \mu_i(t)A_{il} \quad B_i(t) = \sum_{i=1}^{p} \mu_i(t)B_{il} \]

\[ v_{ij}(x_j(t)) = \sum_{i=1}^{p} \mu_i(t)B_{il} \sum_{j=1, j\neq i}^{N} f_{ij}(x_j(t)) \]  \quad (5.2.7)

For future consideration, we have the following assumptions \([15],[133]\):

**Assumption 5.2.1:** Each local subsystem in (5.2.1) is controllable, that is, the controllability matrix

\[ M_{il} = [B_{il}, A_{il}B_{il}, \ldots, A_{il}^{n-1}B_{il}], \text{ for } i = 1,2,\ldots,N \]

has a full rank of \(n\).
**Assumption 5.2.2:** The global fuzzy model in (5.2.6) is controllable in the state space, that is, the controllability matrix

\[ M_i = [B_i, A_i B_i, A_i^2 B_i, \ldots, A_i^{n-1} B_i] \]

has a full rank of \( n \).

Consider a reference model for \( i \)th subsystem [117], [132], [166]:

\[ \dot{x}_{ri}(t) = A_{ri} x_{ri}(t) + r_i(t) \]  \hspace{1cm} (5.2.8)

where \( x_{ri}(t) \in \mathbb{R}^n \) the reference state vector, \( A_{ri} \in \mathbb{R}^{n \times n} \) the specific asymptotically stable matrix and \( r_i(t) \in \mathbb{R}^n \) is the bounded reference input.

The tracking error \( e_i(t) \) is defined as:

\[ e_i(t) = x_i(t) - x_{ri}(t) = [\varepsilon_i(t) \ \dot{\varepsilon}_i(t) \ \varepsilon_i^{(n-1)}(t)]^T \]  \hspace{1cm} (5.2.9)

where \( \varepsilon_i^m(t) = x_i^m(t) - x_{ri}^m(t) \) for \( m = 0, 1, \ldots, n - 1 \).

The error dynamics of \( i \)th closed-loop system can be obtained by using (5.2.6), (5.2.8) and (5.2.9).

\[ \dot{e}_i(t) = A_i(t) e_i(t) + (A_i(t) - A_{ri}) x_{ri}(t) + B_i(t) u_i(t) - r_i(t) \]
\[ + v_{ij} \left(x_j(t)\right) \]  \hspace{1cm} (5.2.10)

**Remark 5.2.1:** The objective of this chapter is to determine a control signal \( u_i(t) \) such that the error dynamics of the \( i \)th closed-loop system asymptotically converges to zero. It will be shown that prior knowledge of the upper bounds of the nonlinear interconnections among subsystems is not required in the proposed controller design. Such an advantage makes the proposed control scheme exhibit strong robustness against unknown nonlinear interconnections.
For \(i\)th closed-loop system, a sliding variable \(s_i(t)\) is defined as:

\[
s_i(t) = C_i e_i(t)
\]  
(5.2.11)

where \(C_i \in \mathbb{R}^{1 \times n}\) is the sliding mode parameter matrix, which is selected to make the characteristic matrix of \(s_i(t) = 0\) Hurwitz.

In this chapter, the fuzzy sliding mode learning controller for its corresponding \(i\)th rule of the T-S fuzzy model for \(i\)th subsystem is proposed as follows:

\[
s_i(t) = \mu_i(t) \sum_{l=1}^{p} \mu_i(t) u_{il}(t)
\]  
(5.2.15)

Remark 5.2.2: One natural candidate of the control for the global fuzzy model in (5.2.6) may be
The global control (5.2.15) requires the aggregated sliding mode learning control parameters to satisfy the design criteria based on some stability conditions [156]. This constraint has greatly limited applications of the learning control scheme for the control of large-scale systems via T-S fuzzy models. Therefore, the concept of dominant control principle is used to determine the global control signal and the robust control scheme can then be developed to ensure that the \( i \)th closed-loop system is globally asymptotically stable.

For \( i \)th closed-loop system, the global control \( u_i(t) \) is defined as:

\[
    u_i(t) = \sum_{l=1}^{p} \mu_l(t) u_{ik}(t) \\
    = \sum_{l=1}^{p} \mu_l(t) \left( u_{ik}(t - \tau) - \Delta u_{ik}(t) \right) \tag{5.2.16}
\]

where

\[
    k = j : \mu_j(t) = \max[\mu_1(t), \ldots, \mu_p(t)] \tag{5.2.17}
\]

**Theorem 5.2.1**: For the global fuzzy model of \( i \)th subsystem (5.2.6), if

\[
    1 \geq K_i = \lambda_{\min}(C_iB_{il}(C_iB_{ik})^{-1}) > 0 \tag{5.2.18}
\]

and the control input (5.2.16) with the adaptation laws

\[
    \dot{\alpha}_{ik} = \omega_{1i} K_i V_i(t) \tag{5.2.19} \\
    \dot{\beta}_{ik} = \omega_{2i} K_i \left| \hat{V}_i(t - \tau) \right| \tag{5.2.20} \\
    \dot{\eta}_{ik} = \omega_{3i} K_i \left| \hat{V}_i(t - \tau) \right| \tag{5.2.21}
\]

then there exists a moment \( t_0 \) such that for \( t > t_0 \), the system is asymptotically stable [17].
Remark 5.2.3: It can be seen from the next section that the correction term $\Delta u_{ik}(t)$ is continuous all the time. Therefore, the local control inputs $u_i(t)$ and global control input $u_i(t)$ in (5.2.12) and (5.2.16) are continuous as well. The merits of this point can be seen in the stability analysis and simulation sections.

5.3. Convergence Analysis

Theorem 5.3.1: Consider the global T-S fuzzy model in (5.2.5). If the sliding mode variable is chosen as in (5.2.11), the local control input $u_i(t)$ and global control input $u_i(t)$ are designed as in (5.2.16) with adaptation laws in (5.2.19), (5.2.20) and (5.2.21), then the overall closed-loop system of the large-scale system, consisting of $q$ subsystems, is asymptotically stable. Thus, the tracking error of each subsystem can asymptotically converge to zero.

Proof: The Lyapunov candidate for the overall closed-loop system of the large-scale system is chosen as:

$$V(t) = \sum_{i=1}^{q} V_i(t)$$

$$= \sum_{i=1}^{q} 0.5s_i^2(t) + 0.5\omega_{1i}^{-1}(\alpha_{ik} - \hat{\alpha}_{ik})^2$$

$$+ 0.5\omega_{2i}^{-1}(\beta_{ik} - \hat{\beta}_{ik})^2$$

$$+ 0.5\omega_{3i}^{-1}(\eta_{ik} - \hat{\eta}_{ik})^2$$

(5.3.1)

for $i = 1, 2, \ldots N$

where $(\alpha_{ik} - \hat{\alpha}_{ik})$, $(\beta_{ik} - \hat{\beta}_{ik})$ and $(\eta_{ik} - \hat{\eta}_{ik})$ are adaptation errors.

The time derivative of $V(t)$ is
\[
\dot{V}(t) = \sum_{i=1}^{q} s_i(t) C_i \hat{e}_i(t) + \omega_{1i}^{-1} \hat{\alpha}_{ik}(\alpha_{ik} - \hat{\alpha}_{ik}) + \omega_{2i}^{-1} \hat{\beta}_{ik}(\beta_{ik} - \hat{\beta}_{ik}) \\
+ \omega_{3i}^{-1} \hat{\eta}_{ik}(\eta_{ik} - \hat{\eta}_{ik}) 
\] (5.3.2)

Substituting (5.2.10) and (5.2.15) into (5.3.2) yields

\[
\dot{V}(t) = \sum_{i=1}^{q} s_i(t) \left[ C_i A_i(t) e_i(t) + C_i (A_i(t) - A_{ri}) x_{ri}(t) + C_i \sum_{l=1}^{p} \mu_l(t) (B_{li} u_i(t)) \\
- C_i r_i(t) + c_i v_{ij} (x_j(t)) \right] + \omega_{1i}^{-1} \hat{\alpha}_{ik}(\alpha_{ik} - \hat{\alpha}_{ik}) + \omega_{2i}^{-1} \hat{\beta}_{ik}(\beta_{ik} - \hat{\beta}_{ik}) \\
- \hat{\beta}_{ik} + \omega_{3i}^{-1} \hat{\eta}_{ik}(\eta_{ik} - \hat{\eta}_{ik}) 
\] (5.3.3)

From (5.3.3), we can express \( \dot{V}_i(t) \) as follows:

\[
\dot{V}_i(t) = s_i(t) \left[ C_i A_i(t) e_i(t) + C_i (A_i(t) - A_{ri}) x_{ri}(t) \\
+ C_i \sum_{l=1}^{p} \mu_l(t) (B_{li} u_i(t - \tau) - B_{li} \Delta u_{ik}(t)) - C_i r_i(t) + C_i v_{ij} (x_j(t)) \right] \\
+ \omega_{1i}^{-1} \hat{\alpha}_{ik}(\alpha_{ik} - \hat{\alpha}_{ik}) + \omega_{2i}^{-1} \hat{\beta}_{ik}(\beta_{ik} - \hat{\beta}_{ik}) + \omega_{3i}^{-1} \hat{\eta}_{ik}(\eta_{ik} - \hat{\eta}_{ik}) \\
= \dot{V}_i(t, t - \tau) - \sum_{l=1}^{p} \mu_l(t) (C_i B_{li} \Delta u_{ik}(t)) + \omega_{1i}^{-1} \hat{\alpha}_{ik}(\alpha_{ik} - \hat{\alpha}_{ik}) \\
+ \omega_{2i}^{-1} \hat{\beta}_{ik}(\beta_{ik} - \hat{\beta}_{ik}) + \omega_{3i}^{-1} \hat{\eta}_{ik}(\eta_{ik} - \hat{\eta}_{ik}) \\
= \dot{V}_i(t, t - \tau) - \sum_{l=1}^{p} \mu_l(t) (C_i B_{li} (C_i B_{ik})^{-1}) (\alpha_{ik} V_i(t) + \beta_{ik} \dot{\hat{V}}_i(t - \tau)) \\
+ \eta_{ik} \dot{V}_i(t - \tau) + \omega_{1i}^{-1} \hat{\alpha}_{ik}(\alpha_{ik} - \hat{\alpha}_{ik}) + \omega_{2i}^{-1} \hat{\beta}_{ik}(\beta_{ik} - \hat{\beta}_{ik}) \\
+ \omega_{3i}^{-1} \hat{\eta}_{ik}(\eta_{ik} - \hat{\eta}_{ik})
\]
\[ \dot{V}_i(t, t - \tau) - \alpha_{ik} G_i(t) V_i(t) - \beta_{ik} G_i(t) \left| \dot{V}_i(t - \tau) \right| - \eta_{ik} G_i(t) \dot{V}_i(t - \tau) + \omega_{2i}^{-1} \dot{\alpha}_{ik} (\alpha_{ik} - \dot{\alpha}_{ik}) + \omega_{3i}^{-1} \dot{\beta}_{ik} (\beta_{ik} - \dot{\beta}_{ik}) \]

where

\[ \dot{V}_i(t, t - \tau) = s_i(t) \left[ C_i A_i(t) e_i(t) + C_i (A_i(t) - A_{ri}) x_{ri}(t) + C_i B_i(t) u_{ik}(t - \tau) - C_i r_i(t) + C_i v_{ij} (x_j(t)) \right] \]

and

\[ G_i(t) = \sum_{l=1}^{p} \mu_l(t) (C_l B_{kl} (C_l B_{lk})^{-1}) \]

Adding the term \( \dot{V}_i(t - \tau) - \dot{V}_i(t - \tau) \) to (5.3.4), we have

\[ \dot{V}_i(t) = \dot{V}_i(t, t - \tau) + \dot{V}_i(t - \tau) - \dot{V}_i(t - \tau) - \alpha_{ik} G_i(t) V_i(t) - \beta_{ik} G_i(t) \left| \dot{V}_i(t - \tau) \right| - \eta_{ik} G_i(t) \dot{V}_i(t - \tau) + \omega_{2i}^{-1} \dot{\alpha}_{ik} (\alpha_{ik} - \dot{\alpha}_{ik}) + \omega_{3i}^{-1} \dot{\beta}_{ik} (\beta_{ik} - \dot{\beta}_{ik}) + \omega_{3i}^{-1} \dot{\eta}_{ik} (\eta_{ik} - \dot{\eta}_{ik}) \]

\[ \leq \left| \dot{V}_i(t, t - \tau) - \dot{V}_i(t - \tau) \right| + \dot{V}_i(t - \tau) - \alpha_{ik} G_i(t) V_i(t) - \beta_{ik} G_i(t) \left| \dot{V}_i(t - \tau) \right| - \eta_{ik} G_i(t) \dot{V}_i(t - \tau) + \omega_{2i}^{-1} \dot{\alpha}_{ik} (\alpha_{ik} - \dot{\alpha}_{ik}) + \omega_{3i}^{-1} \dot{\beta}_{ik} (\beta_{ik} - \dot{\beta}_{ik}) + \omega_{3i}^{-1} \dot{\eta}_{ik} (\eta_{ik} - \dot{\eta}_{ik}) \]

where

\[ \dot{V}_i(t - \tau) = s_i(t - \tau) \left[ C_i A_i(t - \tau) e_i(t - \tau) + C_i (A_i(t - \tau) - A_{ri}) x_{ri}(t - \tau) + C_i B_i(t) u_{ik}(t - \tau) - C_i r_i(t - \tau) + C_i v_{ij} (x_j(t - \tau)) \right] \]

**Remark 5.3.1:** Considering the continuity of both \( \dot{V}_i(t - \tau) \) and \( \dot{V}_i(t, t - \tau) \), with the time-delay \( \tau \) chosen to be sufficiently small, we may always find a constant \( M_i >> 1 \) such that the following Lipschitz-like condition inequality is held [156]:

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\[ |\dot{V}_i(t, t - \tau) - \hat{\dot{V}}_i(t - \tau)| < \frac{1}{M_i} |\hat{\dot{V}}_i(t - \tau)| \quad (5.3.9) \]

for \(\dot{V}_i(t, t - \tau) \neq 0, \hat{\dot{V}}_i(t - \tau) \neq 0\) and \(\hat{\dot{V}}_i(t - \tau) \neq 0\).

The inequality (5.3.9) is called the Lipshitz-like condition [133]. It describes that the difference between the present value of the gradient of the Lyapunov function and its most recent value is very small as the time interval \(\tau\) is sufficiently small. The uncertain system dynamics are all embedded into left-hand side of (5.3.9). For the controller designs with the aid of the Lipshitz-like condition, the knowledge of the upper bounds of the interconnection between subsystem \(H_i\) and subsystem \(H_j\) is no longer required. It is expected that, in the control of large-scale systems with unknown dynamic interactions, the Lipshitz-like condition will play a very essential role to relax constraints on the robust controller design.

Using (5.3.9) in (5.3.7), we obtain

\[
\dot{V}_i(t) < \frac{1}{M_i} |\hat{\dot{V}}_i(t - \tau)| + \dot{V}_i(t - \tau) - \alpha_{ik} G_i(t) V_i(t) - \beta_{ik} G_i(t) |\hat{\dot{V}}_i(t - \tau)|
- \eta_{ik} G_i(t) \hat{\dot{V}}_i(t - \tau) + \omega_{2i}^{-1} \hat{\alpha}_{ik} (\alpha_{ik} - \hat{\alpha}_{ik}) + \omega_{2i}^{-1} \hat{\beta}_{ik} (\beta_{ik} - \hat{\beta}_{ik})
+ \omega_{3i}^{-1} \eta_{ik} (\eta_{ik} - \hat{\eta}_{ik})
\quad (5.3.10)\
\]

**Remark 5.3.2:** If the time delay \(\tau\) is sufficiently small, it is reasonable to assume that

\[ \text{sign} \left( \dot{V}_i(t - \tau) \right) = \text{sign} \left( \dot{V}_i(t - \tau) \right) \quad (5.3.11) \]

\[ \delta \left( \dot{V}_i(t - \tau) \right) = \dot{V}_i(t - \tau) - \hat{\dot{V}}_i(t - \tau) \quad (5.3.12) \]

and

\[ |\delta \left( \dot{V}_i(t - \tau) \right)| < \gamma_i |\hat{\dot{V}}_i(t - \tau)| \quad (5.3.13) \]

for \(\hat{\dot{V}}_i(t - \tau) \neq 0, \dot{V}_i(t - \tau) \neq 0\), and \(0 < \gamma \ll 1\).
The inequality (5.3.13) implies that the approximation error between \( \hat{V}_i(t - \tau) \) and \( \hat{V}_i(t - \tau) \) is very small as the time interval \( \tau \) is sufficiently small.

Using (5.3.12) and (5.3.13) in (5.3.10) leads to

\[
\hat{V}_i(t) < \frac{1}{M_i} \left| \hat{V}_i(t - \tau) \right| + \hat{V}_i(t - \tau) + \gamma_i \left| \hat{V}_i(t - \tau) \right| - \alpha_{ik} G_i(t) V_i(t) \\
- \beta_{ik} G_i(t) \left| \hat{V}_i(t - \tau) \right| - \eta_{ik} G_i(t) \hat{V}_i(t - \tau) + \omega_{2i}^{-1} \tilde{\alpha}_{ik}(\alpha_{ik} - \hat{\alpha}_{ik}) \\
+ \omega_{2i}^{-1} \hat{\alpha}_{ik}(\beta_{ik} - \beta_{ik}) + \omega_{3i}^{-1} \hat{\eta}_{ik}(\eta_{ik} - \hat{\eta}_{ik})
\]

(5.3.14)

- If \( \hat{V}_i(t - \tau) > 0 \)

Substituting the condition (5.2.19), (5.2.20) and (5.2.21) into (5.3.10) yields

\[
\hat{V}_i(t) < \hat{V}_i(t - \tau) + \frac{1}{M_i} \left| \hat{V}_i(t - \tau) \right| - \alpha_{ik} G_i(t) V_i(t) - \beta_{ik} G_i(t) \left| \hat{V}_i(t - \tau) \right| \\
- \eta_{ik} G_i(t) \hat{V}_i(t - \tau) + K_i V_i(t) (\alpha_{ik} - \hat{\alpha}_{ik}) + K_i \left| \hat{V}_i(t - \tau) \right| (\beta_{ik} - \hat{\beta}_{ik}) \\
+ K_i \left| \hat{V}_i(t - \tau) \right| (\eta_{ik} - \hat{\eta}_{ik})
\]

\[
\leq \hat{V}_i(t - \tau) + \frac{1}{M_i} \left| \hat{V}_i(t - \tau) \right| - \alpha_{ik} K_i V_i(t) - \beta_{ik} K_i \left| \hat{V}_i(t - \tau) \right| \\
- \eta_{ik} K_i \hat{V}_i(t - \tau) + K_i V_i(t) (\alpha_{ik} - \hat{\alpha}_{ik}) + K_i \left| \hat{V}_i(t - \tau) \right| (\beta_{ik} - \hat{\beta}_{ik}) \\
+ K_i \left| \hat{V}_i(t - \tau) \right| (\eta_{ik} - \hat{\eta}_{ik})
\]

\[
= \hat{V}_i(t - \tau) + \frac{1}{M_i} \left| \hat{V}_i(t - \tau) \right| - \alpha_{ik} K_i V_i(t) - \beta_{ik} K_i \left| \hat{V}_i(t - \tau) \right| \\
- \eta_{ik} K_i \left| \hat{V}_i(t - \tau) \right|
\]

(5.3.15)

The targeted \( \hat{\beta}_{ik} \) is defined as

\[
\hat{\beta}_{ik} > \frac{1}{M_i K_i}
\]

(5.3.16)
Then, using (5.3.16) in (5.3.15) leads to

\[
\dot{V}_i(t) < \dot{V}_i(t - \tau) - \left( \hat{\beta}_{ik} K_i - \frac{1}{M_i} \right) \left| \dot{V}_i(t - \tau) \right| - \hat{\alpha}_{ik} K_i |V_i(t)| - \dot{\eta}_{ik} K_i |\dot{V}_i(t - \tau)|
\]

\[
< \dot{V}_i(t - \tau)
\]

(5.3.17)

The inequality (5.3.17) indicates that the global sliding mode learning controller in (5.2.16) always makes the value of \( \dot{V}_i(t) \) decrease when \( \dot{V}_i(t - \tau) > 0 \).

**Remark 5.3.3**: There exists a moment \( t_0 \) such that for \( t > t_0 \), \( \alpha_{ik} > \hat{\alpha}_{ik}, \beta_{ik} > \hat{\beta}_{ik} \) and \( \eta_{ik} > \hat{\eta}_{ik} \). As long as the sliding surface is not reached, \( \alpha_{ik}, \beta_{ik} \) and \( \eta_{ik} \) will increase by \( \omega_{1i}, \omega_{2i} \) and \( \omega_{3i} \) until \( s_i(t) = 0 \). Therefore, the requirement of a prior knowledge of upper bounds of the \( \hat{\alpha}_{ik}, \hat{\beta}_{ik} \) and \( \hat{\eta}_{ik} \) is removed.

Suppose that, at the time \( t = t_1 + \tau \) and \( \dot{V}_i(t_1) = 0 \), we can express (5.3.4) as:

\[
\dot{V}_i(t_1 + \tau) = \dot{V}_i(t_1 + \tau, t_1) - G_i(t_1 + \tau) \left[ \alpha_{ik} V_i(t_1 + \tau) + \beta_{ik} \left| \dot{V}_i(t_1) \right| + \eta_{ik} \dot{V}_i(t_1) \right] \\
+ K_i V_i(t_1 + \tau)(\alpha_{ik} - \hat{\alpha}_{ik}) + K_i \left| \dot{V}_i(t_1) \right| (\beta_{ik} - \hat{\beta}_{ik}) \\
+ K_i \left| \ddot{V}_i(t_1) \right| (\eta_{ik} - \hat{\eta}_{ik})
\]

\[
\leq \dot{V}_i(t_1 + \tau, t_1) - K_i \left[ \alpha_{ik} V_i(t_1 + \tau) + \beta_{ik} \left| \dot{V}_i(t_1) \right| + \eta_{ik} \dot{V}_i(t_1) \right] \\
+ K_i V_i(t_1 + \tau)(\alpha_{ik} - \hat{\alpha}_{ik}) + K_i \left| \dot{V}_i(t_1) \right| (\beta_{ik} - \hat{\beta}_{ik}) \\
+ K_i \left| \ddot{V}_i(t_1) \right| (\eta_{ik} - \hat{\eta}_{ik})
\]

\[
= \dot{V}_i(t_1 + \tau, t_1) - \eta_{ik} K_i \dot{V}_i(t_1) - \hat{\alpha}_{ik} K_i V_i(t_1 + \tau) - \hat{\beta}_{ik} K_i \left| \dot{V}_i(t_1) \right| + K_i \left| \ddot{V}_i(t_1) \right| (\eta_{ik} - \hat{\eta}_{ik})
\]

(5.3.18)

Consider the fact that
\[
\hat{V}_i(t_1) \equiv 0 \quad (5.3.19)
\]
and
\[
\hat{a}_{ik} K_i V_i(t_1 + \tau) > 0 \quad (5.3.20)
\]

The targeted \( \hat{a}_{ik} \) is defined as
\[
\hat{a}_{ik} > \frac{\dot{V}_i(t_1 + \tau, t_1)}{K_i V_i(t_1 + \tau)} 
\]  
(5.3.21)

then using (5.3.21) in (5.3.18),
\[
\dot{V}_i(t_1 + \tau) \equiv \dot{V}_i(t_1 + \tau, t_1) - \hat{a}_{ik} K_i V_i(t_1 + \tau) < 0 \quad (5.3.22)
\]

• If \( \hat{V}_i(t - \tau) < 0 \)

From (5.3.14), we can express \( \ddot{V}(t) \) as follows:
\[
\ddot{V}(t) < -\left( 1 - \frac{1}{M_i} - \gamma_i \right) |\dot{V}_i(t - \tau)| - a_{ik} G_i(t) V_i(t) - \beta_{ik} G_i(t) |\dot{V}_i(t - \tau)| + \eta_{ik} G_i(t) \dot{V}_i(t - \tau) + K_i V_i(t) (a_{ik} - \hat{a}_{ik}) + K_i |\dot{V}_i(t - \tau)| (\beta_{ik} - \hat{\beta}_{ik})
+ K_i |\hat{V}_i(t - \tau)| (\eta_{ik} - \hat{\eta}_{ik})
\]
\[
\leq -\left( 1 - \frac{1}{M_i} - \gamma_i \right) |\dot{V}_i(t - \tau)| - a_{ik} K_i V_i(t) - \beta_{ik} K_i |\dot{V}_i(t - \tau)| + \frac{\eta_{ik}}{K_i} |\dot{V}_i(t - \tau)| + K_i V_i(t) (a_{ik} - \hat{a}_{ik}) + K_i |\dot{V}_i(t - \tau)| (\beta_{ik} - \hat{\beta}_{ik})
+ K_i |\hat{V}_i(t - \tau)| (\eta_{ik} - \hat{\eta}_{ik})
\]
\[
\leq -\left( 1 - \frac{1}{M_i} - \gamma_i \right) |\dot{V}_i(t - \tau)| + \left( \eta_{ik} K_i + \frac{\eta_{ik}}{K_i} \right) |\dot{V}_i(t - \tau)| - \hat{\eta}_{ik} K_i |\dot{V}_i(t - \tau)| - \hat{a}_{ik} K_i V_i(t) - \beta_{ik} K_i |\dot{V}_i(t - \tau)| \quad (5.3.23)
\]

If the estimated control parameter \( \eta_{ik} \) is designed to satisfy the following condition:
\[
(1 - \frac{1}{M_i} - \gamma_i) \left(\frac{K_i}{K_i^2 + 1}\right) > \eta_{lk} > 0 \tag{5.3.24}
\]

**Remark 5.3.4:** It is worth noting that condition (5.3.24) highly depends on the design of sliding parameter \(C_i\) for each subsystem in the large-scale system. The design of \(C_i\) needs to ensure the closed-loop system is globally asymptotically stable, and also it satisfies the condition specified in (5.3.24) and (5.2.18).

Using (5.3.24) in (5.3.23) leads to

\[
\dot{V}_i(t) < - \left(1 - \frac{1}{M_i} - \gamma_i\right) |\hat{V}_i(t - \tau)| + \left(\eta_{ik} K_i + \eta_{ik} K_i - \hat{\eta}_{ik} K_i\right) |\hat{V}_i(t - \tau)|
- \alpha_{ik} K_i V_i(t) - \beta_{ik} K_i |\hat{V}_i(t - \tau)| - \hat{\eta}_{ik} K_i |\hat{V}_i(t - \tau)|
\]

\[
< 0 \tag{5.3.25}
\]

Specifically, (5.3.25) means that

\[
\hat{V}(t) = \sum_{i=1}^{N} \dot{V}_i(t)
\]

\[
< \sum_{i=1}^{N} -\alpha_{ik} K_i V_i(t) - \beta_{ik} K_i |\hat{V}_i(t - \tau)| - \hat{\eta}_{ik} K_i |\hat{V}_i(t - \tau)| < 0 \tag{5.3.26}
\]

The proof indicates that the global sliding mode learning control in (5.2.16) is capable of driving the \(i\)th closed-loop system dynamics to move from the unstable domain to the stable domain. Thus, the overall closed-loop system of the large-scale system is asymptotically stable and the sliding mode learning control law in (5.2.12) ensures that both the sliding variable \(s_i(t)\) and tracking error \(e_i(t)\) also asymptotically converge to zero.
Remark 5.3.5: As the sliding variable \( s_i(t) \) asymptotically converges to zero, the relationship between \( s_i(t) \) and \( s_i(t - \tau) \) as time \( t \to \infty \) is definite as:

\[
\lim_{s_i \to 0} \frac{s_i(t)}{s_i(t - \tau)} = \rho
\]

(5.3.24)

where \( 0 < |\rho| \leq 1 \),

then the correction term \( \Delta u_{il}(t) \) in the sliding mode learning controller in (5.2.13) satisfies:

\[
\lim_{s \to 0}(\Delta u_{il}(t)) = \lim_{s_i \to 0} \left( \frac{(C_i B_{ii})^{-1}}{s_i(t)} \left[ \alpha_{il} V_i(t) + \beta_{ii} \left| \hat{V}_i(t - \tau) \right| + \eta_{il} \hat{V}_i(t - \tau) \right] \right)
\]

\[
= \lim_{s_i \to 0} \left( \frac{(C_i B_{ii})^{-1}}{s_i(t)} \left( \alpha_{il} s_i^2(t) + \beta_{ii} \left| s_i^2(t) - s_i^2(t - \tau) \right| \right.ight.
\]

\[
+ \left. \frac{\eta_{il}(s_i^2(t) - s_i^2(t - \tau))}{2\tau} \right) \right)
\]

\[
= \lim_{s_i \to 0} \left( \frac{(C_i B_{ii})^{-1}}{s_i(t)} \left( \alpha_{il} s_i^2(t) + \beta_{ii} \left| s_i^2(t) - \frac{(1 - \frac{1}{\rho})}{2\tau} \right| \right.ight.
\]

\[
+ \left. \frac{\eta_{il}s_i^2(t)(1 - \frac{1}{\rho})}{2\tau} \right) \right)
\]

\[
= \lim_{s_i \to 0} \left( \frac{(C_i B_{ii})^{-1}}{s_i(t)} \left( \alpha_{il} s_i(t) + \beta_{ii} \frac{s_i(t)}{2\tau} \left| 1 - \frac{1}{\rho} \right| \right.ight.
\]

\[
+ \left. \frac{\eta_{il}s_i(t)(1 - \frac{1}{\rho})}{2\tau} \right) \right)
\]

\[
= 0
\]
**Remark 5.3.5:** It is important to note that, in conventional decentralized feedback control approaches, the upper bound of interconnections is often required for designing a feedback controller. In addition, solving the LMI matrix does not guarantee that the positive definite $P$ is always available in practice, especially for large-scale systems. However, in the proposed control scheme, no information about the unknown interconnections is needed and no LMI matrix is required to be solved for the proposed control scheme. Therefore, the proposed control scheme is suitable for large-scale systems with unknown dynamic interactions.

The merits of both the asymptotic error convergence and the chattering-free characteristics of the proposed sliding mode learning controller can be further verified from the simulation example in the next section.

### 5.4. Simulation Results

Consider a large-scale system $S$ that is composed of three subsystems $S_1$, $S_2$ and $S_3$. The rules of the T-S fuzzy model for each subsystem are shown as follows:

**$S_1$:**

$R^1$ : IF $x_{11}(t)$ is about 0, $x_{12}(t)$ is about 0
THEN

$$\dot{x}_1(t) = A_{11}x_1(t) + B_{11}u_1 + B_{11} \sum_{j=1, j \neq 1}^{3} f^1_{1j}x_j(t)$$

$R^2$ : IF $x_{11}(t)$ is about 0, $x_{12}(t)$ is about ± 1
THEN

$$\dot{x}_1(t) = A_{12}x_1(t) + B_{12}u_1 + B_{12} \sum_{j=1, j \neq 1}^{3} f^2_{1j}x_j(t)$$

$R^3$ : IF $x_{11}(t)$ is about ± 1, $x_{12}(t)$ is about 0
THEN

$$\dot{x}_1(t) = A_{13}x_1(t) + B_{13}u_1 + B_{13} \sum_{j=1, j \neq 1}^{3} f^3_{1j}x_j(t)$$

**$S_2$:**

$R^1$ : IF $x_{21}(t)$ is about 0, $x_{22}(t)$ is about 0
THEN

$$\dot{x}_2(t) = A_{21}x_2(t) + B_{21}u_2 + B_{21} \sum_{j=1, j \neq 2}^{3} f^1_{2j}x_j(t)$$

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\[ R^2: \text{IF } x_{21}(t) \text{ is about } 0, \ x_{22}(t) \text{ is about } \pm 1 \]
\[ \text{THEN} \]
\[ \dot{x}_2(t) = A_{22}x_2(t) + B_{22}u_2 + B_{22} \sum_{j=1,j \neq 2}^{3} f_{2j}^2x_j(t) \]

\[ S_3:\]
\[ R^1: \text{IF } x_{31}(t) \text{ is about } 0, \ x_{32}(t) \text{ is about } 0 \]
\[ \text{THEN} \]
\[ \dot{x}_3(t) = A_{31}x_3(t) + B_{31}u_3 + B_{31} \sum_{j=1}^{3} f_{3j}^1x_j(t) \]

\[ R^2: \text{IF } x_{31}(t) \text{ is about } 0, \ x_{32}(t) \text{ is about } \pm 2 \]
\[ \text{THEN} \]
\[ \dot{x}_3(t) = A_{32}x_3(t) + B_{32}u_3 + B_{32} \sum_{j=1,j \neq 2}^{3} f_{3j}^2x_j(t) \]  
(5.4.1)

where

\[
A_{11} = \begin{bmatrix} 0 & 1 \\ -2 & 3 \end{bmatrix} \quad A_{12} = \begin{bmatrix} 0 & 1 \\ -2.4 & 3.3 \end{bmatrix} \quad A_{13} = \begin{bmatrix} 0 & 1 \\ -3 & 4 \end{bmatrix} \\
A_{21} = \begin{bmatrix} 0 & 1 \\ -3 & 3 \end{bmatrix} \quad A_{22} = \begin{bmatrix} 0 & 1 \\ -3.4 & 3.1 \end{bmatrix} \\
A_{31} = \begin{bmatrix} 0 & 1 \\ -6 & 5.5 \end{bmatrix} \quad A_{32} = \begin{bmatrix} 0 & 1 \\ -6.2 & 5.8 \end{bmatrix} \\
B_{11} = B_{21} = B_{31} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad (5.4.2)
\]

The fuzzy membership functions for state variables \( x_{11}(t), \ x_{12}(t), \ x_{21}(t), \ x_{22}(t), \ x_{31}(t), \ x_{32}(t) \) are chosen as in Figures 5.1 (a), 5.1(b) and 5.1(c).
Figure 5.1(a) Membership functions of $x_{11}(t)$ and $x_{12}(t)$

Figure 5.1(b) Membership functions of $x_{21}(t)$ and $x_{22}(t)$
The interconnections among the three systems are given as follows:

\[
\begin{align*}
    f^1_{12} &= f^1_{12} = f^3_{12} = 0.5 \sin(x_2) \\
    f^1_{13} &= f^2_{13} = f^3_{13} = 0.5 \sin(x_3) \\
    f^1_{21} &= f^2_{21} = 0.6 \sin(x_1) \\
    f^1_{23} &= f^2_{23} = 0.6 \sin(x_3) \\
    f^3_{31} &= f^3_{31} = 0.5 \sin(x_1) \\
    f^3_{32} &= f^3_{32} = 0.5 \sin(x_2)
\end{align*}
\] (5.4.3)

For tracking purpose, the each subsystem reference model is designed as follows:

\[
\begin{align*}
    A_{r1} &= A_{r2} = A_{r3} = \begin{bmatrix} 0 & 1 \\ -10 & -10 \end{bmatrix} \\
    r_1(t) &= r_3(t) = \begin{bmatrix} 0 \\ 10 \cos(0.5t) \end{bmatrix} \\
    r_2(t) &= \begin{bmatrix} 0 \\ 10 \sin(0.5t) \end{bmatrix}
\end{align*}
\] (5.4.4)

Based on (5.2.12)-(5.2.17), we select the control signal of the local system model with the maximum membership function as the global sliding mode tracking control for each subsystem:
\[ u_i(t) = u_{ik}(t - \tau) - \Delta u_{ik}(t) \quad (5.4.5) \]

for \( i = 1, 2 \) and \( 3 \)

with the correction term

\[ \Delta u_{ik}(t) = \begin{cases} \left( (C_i B_{ik})^{-1} \right) \left[ \alpha_{ik} \hat{V}_i(t) + \beta_{ik} \left| \hat{V}_i(t - \tau) \right| + \eta_{ik} \hat{V}_i(t - \tau) \right] & \text{for } s_i(t) \neq 0 \\ 0 & \text{for } s_i(t) = 0 \end{cases} \quad (5.4.6) \]

and

\[ k = j; \mu_j(t) = \max[\mu_l(t), \ldots, \mu_p(t)] \quad (5.4.7) \]

The sliding mode parameter matrices are chosen as \( C_1 = C_2 = C_3 = [10 \ 1] \), the sampling period is \( \Delta T = 0.001 \) sec, the parameters \( \omega_{1i}, \omega_{2i} \) and \( \omega_{3i} \) in (5.2.19)-(5.2.21) are set to be \( \omega_{1i} = 20, \omega_{2i} = 0.01 \) and \( \omega_{3i} = 0.1 \) for \( i = 1, 2 \) and \( 3 \), the time delay \( \tau = \Delta T \) and initial condition for each subsystem is \( x_1(0) = [0.2; -0.5]^T, x_2(0) = [-0.2; -0.1]^T \) and \( x_3(0) = [0.2; -0.1]^T \).

It can be seen from (5.4.1) that

\[ \lambda_{\min}(C_1 B_{11}(C_1 B_{1k})^{-1}) = \lambda_{\min}(C_2 B_{21}(C_2 B_{2k})^{-1}) = \lambda_{\min}(C_3 B_{31}(C_3 B_{3k})^{-1}) = 1 \quad (5.4.8) \]

Also, in this simulation, we assume \( M=50 \) and \( \gamma_i = 0.001 \)

\[ \frac{1}{M} + \gamma_i = 0.021 \quad (5.4.9) \]

The performance of the sliding mode learning controllers is shown in Figures 2(a), 2(b), 2(c) and 2(d).

Figures 2(a), 2(b), 2(c) and 2(d) show the state variables \( x_1(t) = [x_{11}(t), x_{12}(t)], x_2(t) = [x_{21}(t), x_{22}(t)] \) and \( x_3(t) = [x_{31}(t), x_{32}(t)] \), and global control input \( u_1(t) \), \( u_2(t) \) and \( u_3(t) \), respectively. It is seen that, within 1 second, the tracking
error converges to zero. Also, the global sliding mode learning control signal is continuous and chattering-free.

Figure 5.2(a) The states $x_{11}(t)$ and $x_{12}(t)$ responses

Figure 5.2(b) The states $x_{21}(t)$ and $x_{22}(t)$ responses
It is clearly seen that in Figures 5.3, 5.4 and 5.5, the control parameters $\alpha_{ik}$, $\beta_{ik}$ and $\eta_{ik}$ accumulated until $s_i(t) = 0$. Therefore, the asymptotical stability of the global fuzzy system is guaranteed.

It is easy to check that

$$0.4895 > \eta_{ik} > 0$$

is satisfy for all $i$
Figure 5.3(a) The adaptation parameters $\alpha_{1k}$, $\beta_{1k}$ and $\eta_{1k}$ for $S_1$

Figure 5.3(b) Sliding variable $s_1(t)$
Figure 5.4(a) The adaptation parameters $\alpha_{2k}$, $\beta_{2k}$ and $\eta_{2k}$ for $S_2$

Figure 5.4(b) Sliding variable $s_2(t)$
5.5. Conclusion

This chapter has developed an adaptive sliding mode learning control scheme for the trajectory tracking control of a class of large-scale systems with T-S fuzzy models. It has been shown that the proposed controller is capable of efficiently adjusting the closed-loop response based on the most recent history of the closed-loop stability and ensuring a robust tracking performance. Meanwhile, the principle of the dominant
subsystem helps simplify the global controller design. The adaptive estimation laws accumulate the nonzero distance to the sliding surface for the estimation of the control parameters. The simulation results have demonstrated the excellent tracking performance of the proposed scheme. The further work to extension the proposed scheme to the control of MIMO nonlinear systems, sampled data systems, and the design of the sliding mode learning control based observers are under the investigation.
Chapter 6
Conclusions

In this chapter, the major contributions of the current work are highlighted, and some of the relevant future research directions are given.

6.1. Summary of Contributions

This thesis has developed a number of robust sliding mode learning control algorithms for complex systems with dynamic fuzzy models. The focus of this thesis is on the study and improvement of sliding mode learning control design, in order to achieve strong robustness against parameter variations, external disturbances, and nonlinear interconnections. Different from conventional SMC, the information about the unknown system dynamics is not required for the design of the learning controller and the control input is chattering-free. The key contributions of this thesis are summarized as follows:

Chapter 2 of this thesis has provided the fundamental fuzzy set theory for fuzzy logic and fuzzy systems which are being applied in this thesis. The fuzzy sets, fuzzy set operations, fuzzy relations and compositions, extension principle and fuzzy inference systems have been outlined. The basic concepts of Mamdani fuzzy systems and T-S fuzzy systems have been discussed. By comparing these two systems, the advantages of T-S fuzzy systems against Mamdani fuzzy systems in the control area are highlighted. Lyapunov stability theory is provided in order to analyse the stability properties of T-S fuzzy systems. A brief survey of the basic SMC and ILC theories has been provided. The limitations of these control techniques have also been highlighted.
In Chapter 3, a sliding mode-like learning control scheme has been proposed for a class of complex systems with T-S fuzzy models. The sufficient conditions for the sliding mode-like learning control to stabilize the global fuzzy model are discussed in detail. The control of an inverted pendulum cart is presented to demonstrate the effectiveness of the proposed control scheme.

In Chapter 4, the concepts of dominate control principle are employed to facilitate the design of the sliding mode learning control. It has been seen that the information of the uncertain system dynamics is no longer required for the controller design. The robustness of the proposed sliding mode learning control has been verified in the simulation section through comparison with the conventional SMC and H-infinity control.

In Chapter 5, the adaptive sliding mode learning control algorithm has been developed for the large-scale system. A large-scale system consists of a number of subsystems with nonlinear interconnection among subsystems. It has been shown that the controller can override the effect of nonlinear interconnection among subsystems and ensure good tracking performance is achieved. The upper bound information of the interconnection terms is not required in the proposed controller design.

6.2. Future Research

Some areas for future research are as follows:

6.2.1. Multi-input Multi-output (MIMO) Systems

All of the theories developed in this thesis relate to the SISO complex system only. However, the control of complex multivariable systems is very challenging in the areas of system and control. Motivated by the work in [153], it is expected that, the sliding mode learning control schemes developed in Chapter 3, Chapter 4 and Chapter 5 can be extended to control a class of MIMO systems. Using multiple sliding mode surfaces and appropriate stability criteria, the sliding mode learning can be implemented for MIMO
systems via T-S fuzzy models. This research work will further expand the applications of proposed sliding mode learning algorithms.

6.2.2. Sampled Data Systems

All the proposed control algorithms provided in this thesis are for continuous time systems. However, many practical controllers nowadays are commonly implemented in digital electronics due to the increasingly affordable microprocessor hardware. Therefore, it is necessary to carry out research investigations for the sliding mode learning controller for the discrete-time case.

6.2.3. Observer Design for State Estimation

In this thesis, the estimate of the first order derivative of the Lyapunov function is always required in the controller design. In practice, this is often not possible, since the system states are not available or are too expensive to measure [175]. The implementation of a system state observer is needed to overcome this drawback. The design of a observer based sliding mode learning control is under the authors’ investigation.

6.2.4. Time Delays Systems

Time delays often occur in many dynamic systems especially for the network control systems. The existence of the time delays in the system states and control input often become the sources of deterioration of the performance of the control systems and cause instability in the network systems. In order to ensure the closed-loop systems have strong robustness, [176] and [177] proposed sliding mode predictive controller based on discrete-time system. However, the proposed control scheme cannot reduce the chattering of the sliding mode. In this way, the proposed sliding mode learning control is the best candidate for networked control systems with variable time delay via T-S fuzzy models.

6.2.5. Real-world Applications
Lastly, it is also important to expand the real-world applications of the sliding mode learning control, so that more practical problems can be solved in order to demonstrate good performance of the proposed learning control schemes. In [178], the sliding mode learning control scheme has been successfully applied to steer-by-wire systems. Therefore, it is expected that more practical work in the development of sliding mode learning control will be conducted in the future for better robustness and performance of the control system in real-world applications.
Bibliography


Appendix

Matlab Codes

1.1 Matlab code for Chapter 3

clear
clc

%Sampling time
dt=0.005;

%System parameter matrices
A1=[0 1;17.2941 0];
B1=[0;-0.1765];

A2=[0 1;14.4706 0];
B2=[0;-0.1765];

A3=[0 1;5.8512 0];
B3=[0;-0.0709];

A4=[0 1;7.2437 0.5399];
B4=[0;-0.0779];

A5=[0 1;7.2437 -0.5399];
B5=[0;-0.0779];

%Initial value
x=[pi/10;0];
u1=0.1;
u2=0.1;
u3=0.1;
u4=0.1;
u5=0.1;
u=0;

%Control paramter
kk1=20;
gg1=1;

%Sliding parameter
lamda=5;
for i=1:2000

% x1 membership
un3pi=1/(1+exp(-3*pi*(x(1)-pi/3)));  
up3pi=1/(1+exp(3*pi*(x(1)+pi/3)));  
ud0=1-up3pi-un3pi;

up3g(i)=up3pi;
un3g(i)=un3pi;
ud0g(i)=ud0;

% x2 membership
un4=1/(1+exp(-3*1*(x(2)-4)));  
up4=1/(1+exp(3*1*(x(2)+4)));  
u0=1-up4-un4;

up4g(i)=up4;
un4g(i)=un4;
u0g(i)=u0;

% 5 fuzzy rules
r1=(u0*ud0);
r2=max(ud0*up4,ud0*un4);
r3=max(up3pi*u0,un3pi*u0);
r4=max(up3pi*up4,un3pi*un4);
r5=1-r1-r2-r3-r4;

s=x(2)+lamda*x(1);
s1(i)=s;
c=[lamda/1 1/1];

if i==1
   du1=0;
   du2=0;
   du3=0;
   du4=0;
   du5=0;
else
   sd1=(s1(i)-s1(i-1))/dt;
sd2=(s1(i)-s1(i-1))/dt;
sd3=(s1(i)-s1(i-1))/dt;
sd4=(s1(i)-s1(i-1))/dt;
sd5=(s1(i)-s1(i-1))/dt;

   du1=-(kk1*(s)+(gg1*(sd1)));
   du2=-(kk1*(s)+(gg1*(sd2)));
   du3=-(kk1*(s)+(gg1*(sd3)));
   du4=-(kk1*(s)+(gg1*(sd4)));
   du5=-(kk1*(s)+(gg1*(sd5)));
end

u1=u1-du1;
u2=u2-du2;
u3=u3-du3;
u4=u4-du4;
u5=u5-du5;
% global matrix
A = (r1*A1+r2*A2+r3*A3+r4*A4+r5*A5);
B = (r1*B1+r2*B2+r3*B3+r4*B4+r5*B5);
r=[r1 r2 r3 r4 r5];

u= (r1*u1+r2*u2+r3*u3+r4*u4+r5*u5);

x=x+dt*(A*x+B*u);

uu(i)=u;
x1(i)=x(1);
x2(i)=x(2);

end

t=0.005:0.005:10;

figure(1)
plot(t,s1);
xlabel('Time t (sec)')
ylabel('The sliding mode variable s(t)')
figure(2)
plot(t,x1,'-',t,x2,'--')
xlabel('Time t (sec)')
ylabel('The system states x1(rad) and x2(rad/s)')
LEGEND('x1','x2')
figure(3)
plot(t,uu);
xlabel('Time t (sec)')
ylabel('The control input signal (N)')

1.2 Matlab code for Chapter 4

clear
clc
dt=0.001;
Tdt=0.001;

%ith subsystem parameter matrices
A1=[0 1;17.2941 0];
B1=[0;-0.1765];
A2=[0 1;14.4706 0];
B2=[0;-0.1765];
A3=[0 1;5.8512 0];
B3=[0;-0.0709];
A4=[0 1;7.2437 0.5399];
B4=[0;-0.0779];
A5=[0 1;7.2437 -0.5399];
B5=[0;-0.0779];

%Initial values of the state variables x1 and x2
x=[pi/10;0];
% Sliding mode control parameters for

% alpha
alpha1=4;
alpha2=5;
alpha3=4;
alpha4=5;
alpha5=4;
% beta
beta1=0.2;
beta2=0.2;
beta3=0.2;
beta4=0.2;
beta5=0.2;
% Eta
Eta1=0.4;
Eta2=0.4;
Eta3=0.4;
Eta4=0.4;
Eta5=0.4;

% Initial values of the control signal u
u1=0.1;
u2=0.1;
u3=0.1;
u4=0.1;
u5=0.1;
u=0;

% Sliding mode parameter
% C=[lamda, 1]
  lamda=5;
% lamda=15;

for i=1:5000

% System uncertainties
  db=4*sin(2*pi*i*Tdt);
  db=10*sin(2*pi*i*Tdt);
  dA1=[db 0;0 0];
dA2=dA1;
dA3=dA1;
dA4=dA1;
dA5=dA1;
  dA1r(i)=db;
  dA2r(i)=db;
  dA3r(i)=db;
  dA4r(i)=db;
  dA5r(i)=db;
  dB1=0.5*B1;
  dB2=0.7*B2;
  dB3=0.2*B3;
  dB4=0.3*B4;
  dB5=0.5*B5;
dB11r(i)=dB1(1);
dB12r(i)=dB1(2);
dB21r(i)=dB2(1);
dB22r(i)=dB2(2);
dB31r(i)=dB3(1);
dB32r(i)=dB3(2);
dB41r(i)=dB4(1);
dB42r(i)=dB4(2);
dB51r(i)=dB5(1);
dB52r(i)=dB5(2);

% x1 membership
up3pi=1/(1+exp(-3*pi*(x(1)-pi/6)));
un3pi=1/(1+exp(3*pi*(x(1)+pi/6)));
ud0=1-up3pi-un3pi;

up3g(i)=up3pi;
un3g(i)=un3pi;
ud0g(i)=ud0;

% x2 membership
up4=1/(1+exp(-3*pi*(x(2)-pi/3)));
un4=1/(1+exp(3*pi*(x(2)+pi/3)));
u0=1-up4-un4;

up4g(i)=up4;
un4g(i)=un4;
u0g(i)=u0;

% 5 fuzzy rules
r1=(u0*ud0);
r2=max(ud0*up4,ud0*un4);
r3=max(up3pi*u0,un3pi*u0);
r4=max(up3pi*up4,un3pi*un4);
r5=1-r1-r2-r3-r4;

% Sliding variable
s=x(2)+lamda*x(1);
s1(i)=s;
c=[lamda 1];

% Lyapunov function
v1=0.5*s*s;
v(i)=0.5*s*s;

% Correction term
if i==1
    du1=0;
    du2=0;
    du3=0;
    du4=0;
    du5=0;
    sd1=0;
    sd2=0;
    sd3=0;
    sd4=0;
sd5=0;
vd1=0;
v2=0;
v3=0;
v4=0;
v5=0;

elseif i==2
du1=0;
du2=0;
du3=0;
du4=0;
du5=0;
sd1=0;
sd2=0;
sd3=0;
sd4=0;
sd5=0;

vd1=0;
v2=0;
v3=0;
v4=0;
v5=0;

else
% estimated vd
%     vd1=(3/2*v(i-2)*v(i-1)+0.5*v(i-2))/dt;
vd1=(v(i)-v(i-1))/dt;
    du1=inv(c*B1)/s*(alpha1*(v(i))+beta1*(v(i-1))+Eta1*(vd1));
    du2=inv(c*B2)/s*(alpha1*(v(i))+beta1*(v(i-1))+Eta1*(vd1));
    du3=inv(c*B3)/s*(alpha1*(v(i))+beta1*(v(i-1))+Eta1*(vd1));
    du4=inv(c*B4)/s*(alpha1*(v(i))+beta1*(v(i-1))+Eta1*(vd1));
    du5=inv(c*B5)/s*(alpha1*(v(i))+beta1*(v(i-1))+Eta1*(vd1));
end

% local control signal
    u1=u1-du1;
    u2=u2-du2;
    u3=u3-du3;
    u4=u4-du4;
    u5=u5-du5;

% global matrix
B = (r1*(B1+dB1)+r2*(B2+dB2)+r3*(B3+dB3)+r4*(B4+dB4)+r5*(B5+dB5));
    r=[r1 r2 r3 r4 r5];

% Membership function
    r1r(i)=r1;
    r2r(i)=r2;
    r3r(i)=r3;
    r4r(i)=r4;
    r5r(i)=r5;

% Dominant control principle
for j=1:4
    if r(j)>=r(j+1)
        rmax=r(j);
        r(j)=r(j+1);
        r(j+1)=rmax;
    else
        rmax=r(j+1);
    end
end

if rmax==r1
    u=u1;
elseif rmax==r2
    u=u2;
elseif rmax==r3
    u=u3;
elseif rmax==r4
    u=u4;
else
    u=u5;
end
uu(i)=u;

% Lipschitz condition

% Vd(t-tao)
if i==1
% Vd(t-tao)
    vdL1=0;
    vdL2=0;
    vdL3=0;
    vdL4=0;
    vdL5=0;
end

% Vd(t-tao)
    vdK1=0;
    vdK2=0;
    vdK3=0;
    vdK4=0;
    vdK5=0;

Vleft(i)=0;
Vright(i)=0;

else
% Vd(t-tao)
    vdL1=s1(i-1)*c*((A1+[dA1r(i-1) 0;0 0])*[x1(i-1) x2(i-1)]'+(B1+[dB11r(i-1);dB12r(i-1)])*uu(i-1));
    vdL2=s1(i-1)*c*((A2+[dA2r(i-1) 0;0 0])*[x1(i-1) x2(i-1)]'+(B2+[dB21r(i-1);dB22r(i-1)])*uu(i-1));
    vdL3=s1(i-1)*c*((A3+[dA3r(i-1) 0;0 0])*[x1(i-1) x2(i-1)]'+(B3+[dB31r(i-1);dB32r(i-1)])*uu(i-1));
    vdL4=s1(i-1)*c*((A4+[dA4r(i-1) 0;0 0])*[x1(i-1) x2(i-1)]'+(B4+[dB41r(i-1);dB42r(i-1)])*uu(i-1));
    vdL5=s1(i-1)*c*((A5+[dA5r(i-1) 0;0 0])*[x1(i-1) x2(i-1)]'+(B5+[dB51r(i-1);dB52r(i-1)])*uu(i-1));
vdLT=r1r(i-1)*vdL1+r2r(i-1)*vdL2+r3r(i-1)*vdL3+r4r(i-1)*vdL4+r5r(i-1)*vdL5;

%Vd(t,t-tao)
vdk1=s1(i)*c*{(A1+dA1)*x+(B1+dB1)*uu(i-1)};
vdk2=s1(i)*c*{(A2+dA2)*x+(B2+dB2)*uu(i-1)};
vdk3=s1(i)*c*{(A3+dA3)*x+(B3+dB3)*uu(i-1)};
vdk4=s1(i)*c*{(A4+dA4)*x+(B4+dB4)*uu(i-1)};
vdk5=s1(i)*c*{(A5+dA5)*x+(B5+dB5)*uu(i-1)};
vdkT=r1r(i)*vdK1+r2r(i)*vdK2+r3r(i)*vdK3+r4r(i)*vdK4+r5r(i)*vdK5;

|M=50

Vleft(i)=abs(vdKT-vdLT);
Vright(i)=(1/20)*abs(vd1);

%estimation error
err(i)=vdLT-vd1;

e=+dt*(A*x+B*u);

t=0.001:0.001:5;

figure(1)
subplot (2,1,1),plot(t,s1);
xlabel('Time t (sec) sampling period:0.001s')
ylabel('The sliding mode variable s(t)')

subplot (2,1,2),plot(t,uu);
xlabel('Time t (sec) sampling period:0.001s')
ylabel('The control input signal(N)')

figure(2)
subplot (2,1,1),plot(t,x1,'-');
xlabel('Time t (sec) sampling period:0.001s')
ylabel('The system states x1(rad)')

subplot (2,1,2),plot(t,x2,'-');
xlabel('Time t (sec) sampling period:0.001s')
ylabel('The system states x2(rad/s)')

figure (3)
plot(t,uu);
xlabel('Time t (sec) sampling period:0.001s')
ylabel('The control input signal(N)')

figure (4)
plot(t,Vleft,'--',t,Vright,'-');
xlabel('Time t (sec) sampling period:0.001s')
ylabel('Lipschitz-like condition (4.4.6)')
legend('Left hand side of (4.4.6)','Right hand side of (4.4.6)');
1.3 Matlab code for Chapter 5

clear
c1c
close all

%Sampling time
dt=0.001;
Dt1=0.001;
Dt2=0.001;
Dt3=0.001;

%system A, system B and system C
%system parameters
%system A
A11=[0 1;-2 3];
A12=[0 1;-2.4 3.3];
A13=[0 1;-3 4];
B1=[0;1];
D1=[1;0];

%system B
A21=[0 1;-3 3];
A22=[0 1;-3.4 3.1];
B2=[0;1];
D2=D1;

%system C
A31=[0 1;-6 5.5];
A32=[0 1;-6.2 5.8];
B3=[0;1];
D3=D1;

%Initial state value
x11=0.2;

x12=-0.5;

x21=-0.2;

x22=-0.1;

x31=0.2;

x32=-0.1;

x1=[x11 x12]';
x2=[x21 x22]';
x3=[x31 x32]';

%Initial control input
%For global fuzzy model
u1=0.1;
u2=0.1;
u3=0.1;

%For local linear model
%system A
u11=0.1;
u12=0.1;
u13=0.1;
%system B
u21=0.1;
u22=0.1;

%system C
u31=0.1;
u32=0.1;

%Reference Signal
xr1=[0;0];
xr2=[0;0];
xr3=[0;0];

%Reference signal
ArX1=[0 1;-10 -10];
ArX2=ArX1;
ArX3=ArX1;
Br=[0;1]
 xr=[0;0];

%control gain
alpha11=0;
alpha12=alpha11;
alpha13=alpha11;

beta11=0;
bet a12=beta11;
bet a13=beta11;

et a11=0;
et a12=eta11;
et a13=eta11;

alpha21=0;
alpha22=alpha21;

beta21=0;
beta22=beta21;

eta21=0;
et a22=eta21;

alpha31=0;
alpha32=alpha31;

beta31=0;
beta32=beta31;

eta31=0;
et a32=eta31;

%sliding mode variable
lamda=10;

for i=1:20000

%Reference signal
R1=[0; 10*cos(0.5*i*D t1)];
R2=[0; 10*sin(0.5*i*Dt2)];
R3=[0; 10*cos(0.5*i*Dt3)];

xr1=xr1+Dt1*(ArX1*xr1+R1);
xr2=xr2+Dt2*(ArX2*xr2+R2);
xr3=xr3+Dt3*(ArX3*xr3+R3);

%system A, x1 and x2
xr11(i)=xr1(1);
xr12(i)=xr1(2);
R1r(i)=10*cos(0.5*i*Dt1);

%system B, x1 and x2
xr21(i)=xr2(1);
xr22(i)=xr2(2);
R2r(i)=10*sin(0.5*i*Dt2);

%system C, x1 and x2
xr31(i)=xr3(1);
xr32(i)=xr3(2);
R3r(i)=10*cos(0.5*i*Dt3);

qr1=xr1(1);
qrd1=xr1(2);
qrx11(i)=qr1;
qrdx12(i)=qrd1;

qr2=xr2(1);
qrd2=xr2(2);
qrx21(i)=qr2;
qrdx22(i)=qrd2;

qr3=xr3(1);
qrd3=xr3(2);
qrx31(i)=qr3;
qrdx32(i)=qrd3;

%system A, e and ed
e1=x1(1)-xr1(1);
ed1=x1(2)-xr1(2);
err1=[e1;ed1];
er1(i)=e1;

%system B, e and ed
e2=x2(1)-xr2(1);
ed2=x2(2)-xr2(2);
err2=[e2;ed2];
er2(i)=e2;

%system C, e and ed
e3=x3(1)-xr3(1);
ed3=x3(2)-xr3(2);
err3=[e3;ed3];
er3(i)=e3;

% system A
% x1 membership
unx11=1/(1+exp(-4*pi*(x1(1)-0.5)));
upx11=1/(1+exp(4*pi*(x1(1)+0.5)));
udx11=1-upx11-unx11;
% x2 membership
ux12=1/(1+exp(-4*pi*(x1(2)-0.5)));
upx12=1/(1+exp(4*pi*(x1(2)+0.5)));
udx12=1-upx12-ux12;

%system B
% x1 membership
ux21=1/(1+exp(-4*pi*(x2(1)-0.5)));
upx21=1/(1+exp(4*pi*(x2(1)+0.5)));
udx21=1-upx21-ux21;

% x2 membership
ux22=1/(1+exp(-4*pi*(x2(2)-0.5)));
upx22=1/(1+exp(4*pi*(x2(2)+0.5)));
udx22=1-upx22-ux22;

% system C
% x1 membership
ux31=1/(1+exp(-2*pi*(x3(1)-pi/3)));
upx31=1/(1+exp(2*pi*(x3(1)+pi/3)));
udx31=1-upx31-ux31;

% x2 membership
ux32=1/(1+exp(-2*pi*(x3(2)-pi/3)));
upx32=1/(1+exp(2*pi*(x3(2)+pi/3)));
udx32=1-upx32-ux32;

% 7 fuzzy rules
% system A
% rule 1
r11=(udx11*udx12);
% rule 2
r12=max(udx11*ux12,udx11*upx12);
% rule 3
r13=1-r11-r12;

% system B
% rule 1
r21=(ux21*ux22);
% rule 2
r22=1-r21;

% system C
% rule 1
r31=(ux31*ux32);
% rule 2
r32=1-r31;

%sliding variable s1,s2,s3
%s1
s1=ed1+lamba*e1;
s1r(i)=s1;
c1=[lamba/1 1/1];
v1(i)=0.5*s1*s1;

%s2
s2=ed2+lamba*e2;
s2r(i)=s2;
\%c2=[15 1];
c2=[\text{lamda}/1 1/1];
\%v2(i)=0.5*s2*s2;

\%s3
s3=ed3+\text{lamda}*e3;
s3r(i)=s3;
c3=[\text{lamda}/1 1/1];
\%v3(i)=0.5*s3*s3;

\% local sliding mode learning controller
\%Correction term
if i==1

\%system A
du11=0;
du12=0;
du13=0;
\%system B
du21=0;
du22=0;
\%system C
du31=0;
du32=0;

v1(i)=0.5*s1*s1;
v2(i)=0.5*s2*s2;
v3(i)=0.5*s3*s3;
else
\%sd
v1(i)=0.5*s1*s1;
v2(i)=0.5*s2*s2;
v3(i)=0.5*s3*s3;

\%system A
vd11=(v1(i)-v1(i-1))/dt;
v12=(v1(i)-v1(i-1))/dt;
v13=(v1(i)-v1(i-1))/dt;
\%system B
vd21=(v2(i)-v2(i-1))/dt;
v22=(v2(i)-v2(i-1))/dt;
\%system C
vd31=(v3(i)-v3(i-1))/dt;
v32=(v3(i)-v3(i-1))/dt;

\%adaptation laws
alpha11=alpha11+dt*(20*abs(v1(i)));
beta11=beta11+dt*(0.01*abs(vd11));
etta11=etta11+dt*(0.1*abs(vd11));

alpha21=alpha21+dt*(20*abs(v2(i)));
beta21=beta21+dt*(0.01*abs(vd11));
etta21=etta21+dt*(0.1*abs(vd21));

alpha31=alpha31+dt*(20*abs(v3(i)));
beta31=beta31+dt*(0.01*abs(vd11));
etta31=etta31+dt*(0.1*abs(vd31));
alpha12 = alpha11;
alpha13 = alpha11;
beta12 = beta11;
beta13 = beta11;
etal12 = eta11;
etal13 = eta11;
alpha22 = alpha21;
beta22 = beta21;
etal22 = eta21;
alpha32 = alpha31;
beta32 = beta31;
etal32 = eta32;

% system A

du11 = inv(c1*B1)/s1*(alpha11*v1(i)+beta11*abs(vd11)+eta11*vd11);
du12 = inv(c1*B1)/s1*(alpha12*v1(i)+beta12*abs(vd12)+eta12*vd12);
du13 = inv(c1*B1)/s1*(alpha13*v1(i)+beta13*abs(vd13)+eta13*vd13);

% system B

du21 = inv(c2*B2)/s2*(alpha21*v2(i)+beta21*abs(vd21)+eta21*vd21);
du22 = inv(c2*B2)/s2*(alpha22*v2(i)+beta22*abs(vd22)+eta22*vd22);

% system C

du31 = inv(c3*B3)/s3*(alpha31*v3(i)+beta31*abs(vd31)+eta31*vd31);
du32 = inv(c3*B3)/s3*(alpha32*v3(i)+beta32*abs(vd32)+eta32*vd32);

end

% local control input
u11 = u11 - du11;
u12 = u12 - du12;
u13 = u13 - du13;
u21 = u21 - du21;
u22 = u22 - du22;
u31 = u31 - du31;
u32 = u32 - du32;

% global matrix
A1 = (r11*(A11)+r12*(A12)+r13*(A13));
A2 = (r21*(A21)+r22*(A22));
A3 = (r31*(A31)+r32*(A32));

% internal connection term
\[ f_{112} = 0.5 \sin(2\pi x_2 \cdot dt); \]
\[ f_{212} = f_{112}; \]
\[ f_{312} = f_{112}; \]

\[ f_{113} = 0.5 \sin(2\pi x_3 \cdot dt); \]
\[ f_{213} = f_{113}; \]
\[ f_{313} = f_{113}; \]

\[ f_{121} = 0.6 \sin(2\pi x_1 \cdot dt); \]
\[ f_{221} = f_{121}; \]
\[ f_{123} = 0.6 \sin(2\pi x_3 \cdot dt); \]
\[ f_{223} = f_{123}; \]

\[ f_{131} = 0.5 \sin(2\pi x_1 \cdot dt); \]
\[ f_{231} = f_{131}; \]
\[ f_{132} = 0.5 \sin(2\pi x_2 \cdot dt); \]
\[ f_{232} = f_{132}; \]

\[ A_{in1} = r_{11} (f_{112} + f_{113}) + r_{12} (f_{212} + f_{213}) + r_{13} (f_{312} + f_{313}); \]
\[ A_{in2} = r_{21} (f_{121} + f_{123}) + r_{22} (f_{221} + f_{223}); \]
\[ A_{in3} = r_{31} (f_{131} + f_{132}) + r_{32} (f_{231} + f_{232}); \]

\[ r_{x1} = [r_{11} \ r_{12} \ r_{13}]; \]
\[ r_{x2} = [r_{21} \ r_{22}]; \]
\[ r_{x3} = [r_{31} \ r_{32}]; \]

% dominant control principle
for j=1:2
    if \( r_{x1}(j) >= r_{x1}(j+1) \)
        \( r_{maxx1} = r_{x1}(j); \)
        \( r_{x1}(j) = r_{x1}(j+1); \)
        \( r_{x1}(j+1) = r_{maxx1}; \)
    else
        \( r_{maxx1} = r_{x1}(j+1); \)
    end
end

% global control input
if \( r_{maxx1} == r_{11} \)
    \( u_{1} = u_{11}; \)
elseif \( r_{maxx1} == r_{12} \)
    \( u_{1} = u_{12}; \)
else
    \( u_{1} = u_{13}; \)
end

if \( r_{x2}(1) >= r_{x2}(2) \)
    \( u_{2} = u_{21}; \)
else
    \( u_{2} = u_{22}; \)
end

if \( r_{x3}(1) >= r_{x3}(2) \)
    \( u_{3} = u_{31}; \)
else
    u3=u32;
end

u1 = (r11*(u11)+r12*(u12)+r13*(u13));
u2 = (r21*(u21)+r22*(u22));
u3 = (r31*(u31)+r32*(u32));

uu1(i)=u1;
uu2(i)=u2;
uu3(i)=u3;

x1=x1+dt*(A1*x1+Ain1+(B1)*u1);
x2=x2+dt*(A2*x2+Ain2+(B2)*u2);
x3=x3+dt*(A3*x3+Ain3+(B3)*u3);

x11r(i)=x1(1);
x12r(i)=x1(2);
x21r(i)=x2(1);
x22r(i)=x2(2);
x31r(i)=x3(1);
x32r(i)=x3(2);

alpha1kr(i)=alpha11;
beta1kr(i)=beta11;
etakr(i)=eta11;

alpha2kr(i)=alpha21;
beta2kr(i)=beta21;
etakr(i)=eta21;

alpha3kr(i)=alpha31;
beta3kr(i)=beta31;
etakr(i)=eta31;
end

t=0.001:0.001:20;

figure(1)
s subplot (2,1,1),plot(t,x11r,'-',t,xr11,'--');
xlabel('Time t (sec) sampling period:0.001s')
title('The system state x11(rad) and reference state xr11(rad)')
legend('x11','x11 reference signal');

subplot (2,1,2),plot(t,x12r,'-',t,xr12,'--');
xlabel('Time t (sec) sampling period:0.001s')
title('The system state x12(rad/s) and reference state xr12(rad/s)')
legend('x12','x12 reference signal');

figure(2)
s subplot (2,1,1),plot(t,x21r,'-',t,xr21,'--');
xlabel('Time t (sec) sampling period:0.001s')
title('The system state $x_{21}(\text{rad})$ and reference state $x_{21}(\text{rad})$')
legend('x_{21}', 'x_{21}$ reference signal');

subplot (2,1,2), plot(t,x22r,'-',t,xr22,'--');
xlabel('Time $t$ (sec) sampling period: 0.001s')
title('The system state $x_{22}(\text{rad/s})$ and reference state $x_{22}(\text{rad/s})$')
legend('x_{22}', 'x_{22}$ reference signal');

figure(3)
subplot (2,1,1), plot(t,x31r,'-',t,xr31,'--');
xlabel('Time $t$ (sec) sampling period: 0.001s')
title('The system state $x_{31}(\text{rad})$ and reference state $x_{31}(\text{rad})$')
legend('x_{31}', 'x_{31}$ reference signal');

subplot (2,1,2), plot(t,x32r,'-',t,xr32,'--');
xlabel('Time $t$ (sec) sampling period: 0.001s')
title('The system state $x_{32}(\text{rad/s})$ and reference state $x_{32}(\text{rad/s})$')
legend('x_{32}', 'x_{32}$ reference signal');

figure(4)
plot(t,uu1,'-',t,uu2,'--',t,uu3,'-.');
xlabel('Time $t$ (sec) sampling period: 0.001s')
title('The global fuzzy control input $u_1$, $u_2$ and $u_3$')

figure(5)
plot(t,alpha1kr,'-',t,beta1kr,'--',t,eta1kr,'-.');
xlabel('Time $t$ (sec) sampling period: 0.001s')
title('The adaptation parameters for S1')
legend('alpha_1k', 'beta_1k', 'eta_1k');
AXIS([0 3 0 1.5])

figure(6)
plot(t,alpha2kr,'-',t,beta2kr,'--',t,eta2kr,'-.');
xlabel('Time $t$ (sec) sampling period: 0.001s')
title('The adaptation parameters for S2')
legend('alpha_2k', 'beta_2k', 'eta_2k');
AXIS([0 3 0 3.5])

figure(7)
plot(t,alpha3kr,'-',t,beta3kr,'--',t,eta3kr,'-.');
xlabel('Time $t$ (sec) sampling period: 0.001s')
title('The adaptation parameters for S3')
legend('alpha_3k', 'beta_3k', 'eta_3k');
AXIS([0 3 0 2.5])

figure(8)
plot(t,s1r,'-');
xlabel('Time $t$ (sec) sampling period: 0.001s')
title('The sliding variable for S1')
AXIS([0 3 -0.5 1.6])

figure(9)
plot(t,s2r,'-');
xlabel('Time $t$ (sec) sampling period: 0.001s')
title('The sliding variable for S2')
AXIS([0 3 -2.5 0.5])

figure(10)
plot(t,s3r,'-');
xlabel('Time t (sec) sampling period:0.001s')
title('The sliding variable for S3')
AXIS([0 3 -0.5 2])
List of Publications


