LETTER

Interpreting the macroscopic pointer by analysing the elements of reality of a Schrödinger cat

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Interpreting the macroscopic pointer by analysing the elements of reality of a Schrödinger cat

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Abstract
We examine Einstein–Podolsky–Rosen’s (EPR) steering nonlocality for two realisable Schrödinger cat-type states where a meso/macroscopic system (called the ‘cat’-system) is entangled with a microscopic spin-1/2 system. We follow EPR’s argument and derive the predictions for ‘elements of reality’ that would exist to describe the cat-system, under the assumption of EPR’s local realism. By showing that those predictions cannot be replicated by any local quantum state description of the cat-system, we demonstrate the EPR-steering of the cat-system. For large cat-systems, we find that a local hidden state model is near-satisfied, meaning that a local quantum state description exists (for the cat) whose predictions differ from those of the elements of reality by a vanishingly small amount. For such a local hidden state model, the EPR-steering of the cat vanishes, and the cat-system can be regarded as being in a mixture of ‘dead’ and ‘alive’ states despite it being entangled with the spin system. We therefore propose that a rigorous signature of the Schrödinger cat-type paradox is the EPR-steering of the cat-system and provide two experimental signatures. This leads to a hybrid quantum/classical interpretation of the macroscopic pointer of a measurement device and suggests that many Schrödinger cat-type paradoxes may be explained by microscopic nonlocality.

Keywords: Schrödinger-cat, Einstein–Podolsky–Rosen paradox, macroscopic superposition state, measurement problem, entanglement, steering

Supplementary material for this article is available online

(Some figures may appear in colour only in the online journal)
The original arguments of Einstein–Podolsky–Rosen (EPR) and Bell dealt with small symmetrical systems: two particles or two spins [1, 2]. The arguments are based on EPR’s notion of local realism (LR)—put simply, that there can be no nonlocal effect (‘spooky action-at-a-distance’) [3] on one system as a result of measurements made on the other. In revealing inconsistencies between the predictions of quantum mechanics and the premise of LR, these arguments have had profound implications for physics [4]. Schrödinger recognised that the consequences of such paradoxes would be significant for larger systems [5]. He analysed a gedanken experiment whereby a macroscopic system C (likened to a cat and that we refer to as the ‘cat-system’) becomes entangled with a microscopic spin 1/2 system S, the final state being the superposition

$$|\psi\rangle = \frac{1}{\sqrt{2}} \left( |A\rangle_C |\uparrow\rangle_S + |D\rangle_C |\downarrow\rangle_S \right).$$  

(1)

Here, $|A\rangle_C$ and $|D\rangle_C$ represent macroscopically distinguishable states in which the ‘cat’ is ‘dead’ (if given by $|D\rangle_C$) or ‘alive’ (if given by $|A\rangle_C$). The $|\uparrow\rangle, |\downarrow\rangle$ are the eigenstates of the Pauli spin $\sigma_Z$. The spin and ‘cat’ systems can in principle become spatially separated.

While Schrödinger pointed out the natural interpretation of this state—that the ‘cat’ cannot be viewed as either ‘dead’ or ‘alive’ until measured—he did not construct an EPR-type experiment that would demonstrate such failure of reality for a practical realisation of (1). While such signatures have since been developed (for example [6–14]), experimental work has mainly focused on providing evidence (such as a fidelity or entanglement measure) for the state (1) within quantum theory [15–20] and most signatures do not directly examine the reality of the cat-system C itself, as distinct from that of the spin-system S. Understanding the precise nature of the failure of classical realism for the state (1) is of topical interest: many proposals have been put forward so that the paradoxical situation in which a ‘cat’ is apparently both alive and dead can be better understood [2, 21–26].

The objective of this letter is to probe the nature of the entanglement of the asymmetrical ‘micro-macro’ superposition state (1) by way of an EPR paradox. In this more general situation that extends beyond EPR’s original gedanken experiment, we follow recent treatments and refer to an EPR paradox as an ‘EPR-steering paradox’ [27–29]. Where EPR’s version of local realism premise is shown to fail, so there is some sort of ‘spooky action’ effect on a system due to the measurements made elsewhere, we say the system is ‘steerable’ and that there is an EPR-steering of that system. The important feature of our analysis is then to recognise that EPR’s local realism is defined asymmetrically with respect to two systems, so one may consider either ‘spooky’ action on the cat-system C by measurements on the spin-system S, or vice versa [1]. This opens up the possibility that EPR nonlocality can manifest asymmetrically between the two systems [30, 31]. In fact, ‘one-way’ EPR-steering (where there is EPR-steering for one system by another, but not vice versa) has recently been confirmed experimentally between two microscopic qubit systems [32].

In this letter, we utilise the asymmetry of the EPR paradox to gain an understanding of the discrepancy between quantum and classical descriptions for each of the sub-systems (the ‘cat’ C and the spin S). This information is not given by the observation of entanglement alone. (We refer to this discrepancy as ‘quantumness’, noting that the discrepancy is zero for a sub-system that is consistent with a classical interpretation, and nonzero otherwise.) It might be expected that this discrepancy can be different for the two sub-systems. We find this is indeed the case, and are able to show that for some minutely-perturbed systems where the cat-system is large, the discrepancy of the cat-system can vanish to zero, to the point where surprisingly, the cat-system can be described as ‘dead’ or ‘alive’, despite the fact that the two sub-systems remain
entangled. In this limit, the cat-system acts as a classical measuring device for the microscopic system, which maintains its nonzero ‘quantumness’. In this paper, the discrepancy will be evaluated by considering the ‘elements of reality’ defined in EPR’s original paradox, and by considering compatibility with the local hidden quantum states defined in the separable models of [27–29].

This motivates the question of how to precisely determine when the cat-system itself is paradoxical, along the lines suggested by Schrödinger. Such a bound is not set at the certification of entanglement, but (we show) is set by the certification of an EPR-steering of the cat-system. This type of EPR-steering manifests as a falsification of certain hidden states for the cat-system, that are implied by the premise of LR. These hidden states (or ‘elements of reality’, as EPR called them) predetermine results for measurements on the ‘cat’. In this paper, we calculate details of such elements of reality for realisations of (1) involving coherent states and Greenberger–Horne–Zeilinger (GHZ) spin states. By revealing contradictions, we arrive at measurable signatures for the EPR-steering of the cat-system for two experimentally accessible mesoscopic superposition states. We confirm that as the cat-system becomes larger, the predictions of the hidden states become effectively indistinguishable from those of classical states (in which the cat-system is ‘alive’ or ‘dead’). With minute perturbation to the cat-system, a ‘one-way’ steering regime can be obtained. Here, the EPR-steering of the cat-system vanishes, but we verify that EPR-steering of the spin remains possible, so that measurements on the cat-system can certify the ‘quantumness’ of the spin.

The regime where the cat-system is large is particularly interesting, because in this limit the cat-system models the pointer of a measurement device for the spin \( \sigma_Z \). For the quantum state of type (1), paradoxes arise, because the interpretation of the ‘cat’ being simultaneously ‘alive and dead’ is then that the pointer is simultaneously at two macroscopically separated positions of the measurement dial. While decoherence mechanisms preclude such a result, it is a fundamental question to understand whether and how the collapse of the pointer into a state of one position or the other occurs without decoherence. Our results indicate a regime for mixed states where the pointer is entangled, but with ‘elements of reality’ consistent with the classical reality of being in one place or the other. In this regime, the pointer is said to be ‘non-steerable’. In fact, a regime of one-way EPR-steering exists, where localised measurements on a non-steerable pointer can lead to the nonlocal ‘steering’ effect of the microscopic system. We will show that for the pure state (1), steering is possible both ways (even when the cat-system is large). However, we cannot infer that any nonlocal effect associated with the steering of the pointer has more than a microscopic effect on the pointer system. Finally, we discuss how this analysis supports a hybrid quantum-classical picture of the pointer, that immediately after interaction with the microscopic system, the pointer is located at (near) one of the two macroscopically distinct positions, but with an indeterminacy potentially related to nonlocal effects bounded in size by the uncertainty relation.

### Coherent cat-states

Consider the following prototype for the superposition state (1) [15]:

\[
|\psi_{\text{coh}}\rangle = \frac{1}{\sqrt{2}} \left( |\alpha\rangle |\uparrow\rangle_Z + | - \alpha\rangle |\downarrow\rangle_Z \right).
\]  

(2)

Here \( |\alpha\rangle \) is a coherent state for a quantum harmonic oscillator system that we refer to as the cat-system \( C \). The \( |\uparrow\rangle_Z, |\downarrow\rangle_Z \) are eigenstates of \( \sigma_Z \) for the spin system \( S \). We take \( \alpha \) as real. Observers Alice and Bob can make measurements on the spin and cat-systems respectively. We
consider the systems to be spatially separated after the interaction that created the entanglement. If Alice measures $\sigma_2$ and the result is 1, then the state of system $C$ is given by $|\alpha\rangle$. Similarly, if the result is $-1$, the state is $|-\alpha\rangle$. Thus, Alice can predict the statistics for Bob’s measurements, conditional on her outcome. Suppose Bob makes a measurement of either the ‘position’ $X$ or ‘momentum’ $P$ quadrature defined by $X = \frac{1}{\sqrt{2}} (a^\dagger + a)$ and $P = \frac{1}{\sqrt{2}} (a - a^\dagger)$ ($a^\dagger, a$ are standard boson operators for system $C$). If Alice’s outcome is $\pm 1$, then the conditional probability distribution $P(x)$ for the outcome $x$ of Bob’s measurement $X$ is the Gaussian hill

$$P_\pm(x) = \frac{1}{\sqrt{\pi}} \exp \left\{ -(x \mp \sqrt{2}\alpha)^2 \right\}$$

(3)

centred at $\pm\sqrt{2}\alpha$, respectively, and with a variance $(\Delta x)^2 = 1/2$ (as for a coherent state). As $\alpha \to \infty$, the $\pm$ hills are macroscopically distinguishable, and are labelled ‘alive’ and ‘dead’.

EPR postulated that the measurement by Alice makes no difference to the system of the other observer. Bell’s expression of EPR’s LR is that the joint probability $P_{CS}(x, y)$ for outcomes $x$ and $y$ of measurements made at $C$ and $S$ respectively can be described by a local hidden variable (LHV) model [2, 4]

$$P_{CS}(x, y) = \int_\lambda d\lambda \rho(\lambda) P_C(x|\theta, \lambda) P_S(y|\phi, \lambda).$$

(4)

Here $\lambda$ symbolises a set of hidden variables with distribution $\rho(\lambda)$; $\phi$ and $\theta$ are the measurement choices for $S$ and $C$ respectively. The locality assumption is that $P_C(x|\theta, \lambda)$ is independent of Alice’s measurement choice $\phi$ and the outcome $y$ at location $S$; similarly $P_S(y|\phi, \lambda)$ is independent of Bob’s choice $\theta$ and the outcome $x$ at $C$ [33]. We note there is an asymmetry in the locality assumption for the EPR experiment, because the measurements $x$ and $\theta$ by Bob are spacelike separated from those of Alice but are in the future. We call this premise LR, and the model becomes the local hidden state (LHS) model [27, 28]

$$P_{CS}(x, y) = \int_\lambda d\lambda \rho(\lambda) P_C(x|\theta, \lambda) P_S(y|\phi, \lambda).$$

(5)

the falsification of which is certification of EPR-steering of the cat-system $C$.

A strong statistical correlation between systems $S$ and $C$ places restrictions on the hidden variables $\lambda$ given in (4). This is the case for the state (2), where Alice can predict whether Bob will find the ‘cat’ dead or alive, without apparently interacting with his system. LR thus implies the local cat-system to be consistent with being in a mixture of hidden states that predetermine the cat-system to be either ‘dead’ or ‘alive’ [1]. EPR used the term ‘elements of reality’ to describe such a predetermination. This is expressed as the following result (refer to the online supplementary material, stacks.iop.org/JPhysA/50/41LT01/mmedia).

Result (1a)

Assuming a LHV model (4) to be valid, the local cat-system $C$ is consistent with being either in a hidden-variable state with distribution for $X$ given by $P_+(x)$ (‘alive’) or in a state with distribution given by $P_-(x)$ (‘dead’). This implies that the hidden variable set $\{\lambda\}$ includes
a variable $\lambda_Z$, which defines whether the ‘cat’ is ‘dead’ or ‘alive’, prior to measurement. The two predetermined states are denoted by $\lambda_Z = +1$ or $-1$ respectively. The hidden variable is ‘macroscopic’, in the sense that the two values describe macroscopically distinguishable outcomes.

To show EPR-steering, we suppose Alice measures $\sigma_X$ [11]. The state (2) can be written in terms of the eigenstates $|\uparrow>_{X}$, $|\downarrow>_{X}$ of $\sigma_X$, as

$$|\psi_{\text{coh}}> = \frac{1}{2N_+}|\psi_+>\uparrow + \frac{1}{2N_-}|\psi_->\downarrow$$

(6)

where $|\psi_{\pm}> = N_{\pm}(\alpha)\frac{1}{\sqrt{2}}(|\uparrow> - |\downarrow>)$ and $N_{\pm}$ are normalisation constants. Measurement of $\sigma_X$ by Alice projects Bob’s system into one of the even or odd coherent superpositions $|\psi_{\pm}>$ according to whether the result of the measurement is 1 or $-1$. Alice is able to predict the probability distribution $P_{\pm}(p)$ for Bob’s measurement $P$ on system $C$, conditional on her outcome $\pm 1$.

The conditional distribution is

$$P_{\pm}(p) = \frac{1}{\sqrt{\pi}}\exp(-p^2)(1 \pm \sin(2\sqrt{2}p\alpha)).$$

(7)

The distribution has a variance $(\Delta p)^2 = \frac{1}{4} - 2\alpha^2 e^{-4\alpha^2}$, below that of the coherent state, for which $(\Delta p)^2 = \frac{1}{2}$. Result (1a) leads to the conclusion the cat-system $C$ is in one or other of two states, that correspond to the distributions $P_{+}(p)$ and $P_{-}(p)$ respectively. We denote these hidden states by the variable $\lambda_X$, which assumes the value $+1$ or $-1$ in each case. For consistency with the LHV model (4), we show (result (1b)) in the supplemental material) that the local cat-system $C$ would simultaneously be described by both variables: $\lambda_Z$ and $\lambda_X$. We represent such an element of reality state by the ordered pair $(\lambda_Z, \lambda_X)$.

Now we note the inconsistency that gives an EPR-steering paradox. There are four element of reality states of the cat-system, as depicted in figure 1: each $(\lambda_Z, \lambda_X)$ has predictions for $X$ and $P$ given by $P_{\lambda_Z}(x)$ and $P_{\lambda_X}(p)$ respectively. We see that for each of these states, $\Delta X\Delta P = \frac{1}{4}(1 - 4\alpha^2 e^{-4\alpha^2})^{1/2} < \frac{1}{2}$, which contradicts the Heisenberg uncertainty relation $\Delta X\Delta P \geq \frac{1}{2}$. Thus, the element of reality states cannot be quantum states: the inequality $\Delta X\Delta P < 1/2$ is violation of the EPR-steering inequality

$$\Delta_{\inf}X\Delta_{\inf}P \geq 1/2$$

(8)

where $(\Delta_{\inf}X)^2 = \sum_{\Sigma_2} P(\sigma_2)(\Delta(X|\sigma_2))^2$ and $(\Delta_{\inf}P)^2 = \sum_{\Sigma_2} P(\sigma_2)(\Delta(P|\sigma_2))^2$ are the average inference variances for $X$ and $P$. Here, $P(\sigma_2)$ is the probability of outcome $\sigma_2$ for $\lambda_X$ and $(\Delta(X|\sigma_2))^2$ is the variance of the conditional distribution $P(X|\sigma_2)$. The violation of the EPR-steering inequality signifies the failure of all LHS models (5) and hence an EPR-steering of the cat-system [11, 27, 34].

In other words, the violation of the inequality (8) negates that the local cat-system $C$ is in any mixture of any ‘dead’ or ‘alive’ quantum states as consistent with the LHV model (4). This is proved for all $\alpha$ (as is consistent with the results of [35] that suggest pure entangled states to violate the LHV model). However, as $\alpha \to \infty$, the falsification of the LHS model (5) (evident by the fringe pattern) becomes unverifiable. This is shown in figure 2, where for $\alpha \sim 100$, the LHS model (5) cannot be falsified visually given the finite resolution of the graphics.

The main point of this paper is that where the LHS model (5) is not falsifiable, there can be no demonstration of the loss of classical reality of the cat-system itself. This is because the expression (5) describes the cat-system in a classical mixture of the local hidden quantum states $\rho_{C,\lambda}$ consistent with the hidden variable $\lambda_Z$ and therefore being in quantum states either
It is known that the LHS model (5) can hold, despite that the two systems are entangled [27, 36]. Entanglement is certified by negating the quantum separable model

\[ P_{CS}(x, y) = \int_\lambda d\lambda \rho(\lambda) P_C(x|\theta, \lambda, \lambda) q P_S(y|\phi, \lambda) q \]

(9)

where the predictions \( P_S(y|\phi, \lambda) \) are also constrained to be consistent with a quantum density operator. The negation of model (5) certainly implies entanglement because the set of local hidden variable states includes all local quantum states [27, 36]. Similarly, entanglement is confirmed by the negation of the LHS model

\[ P_{CS}(x, y) = \int_\lambda d\lambda \rho(\lambda) P_C(x|\theta, \lambda, \lambda) q P_S(y|\phi, \lambda) q. \]

(10)

If this LHS model (10) is negated, we demonstrate an EPR-steering of the spin-system \( S \). In this case the hidden states ("elements of reality") of the spin system that would be consistent with the separable LHS model (5) cannot be described as a local quantum state. Because it leads to EPR’s paradoxical situation where quantum mechanics cannot describe the local states [1], we call such a system ‘nonclassical’. In short, entanglement can be confirmed based
on the nonclassicality of the spin-system $S$, regardless of the classicality of the cat-system, and is a less rigorous signature of the cat-paradox.

Next, we give an explicit method for demonstrating a regime of entanglement, in the absence of EPR-steering of the cat-system. Specifically, we derive the ‘elements of reality’ for the spin system, and show how these demonstrate EPR-steering of the spin system (and hence entanglement). From (2) we see that as $\alpha \to \infty$, measurement of $X$ is also a perfect measurement of spin $\sigma_Z$. Hence in this regime, the error in inferring the value of $\sigma_Z$ is zero and $\Delta_{\text{inf}}\sigma_Z \to 0$. We see from (6), since the states $|\psi_{\pm}\rangle$ have even and odd numbers of photons, that a number measurement of cat-system $C$ by Bob can distinguish the two states $|\psi_{\pm}\rangle$ and hence measure the spin $\sigma_X$ with perfect accuracy. Hence $\Delta_{\text{inf}}\sigma_X = 0$. This is true for all $\alpha$ but requires perfect resolution of number. These results verify the EPR steering of the spin-system, since the EPR steering inequality

$$\left(\Delta_{\text{inf}}\sigma_Z\right)^2 + \left(\Delta_{\text{inf}}\sigma_X\right)^2 \geq 1$$

(11)
is violated. The steering inequality is based on the two-component uncertainty relation $(\Delta \sigma_Z)^2 + (\Delta \sigma_X)^2 \geq 1$ that holds for spin 1/2 systems [37, 38], and is derived straightforwardly following the techniques of [27, 29, 34, 39]. In the limit of large $\alpha$, the elements of reality for the spin-system are given as the precise values $+1$ and $-1$, for each of the variables $\sigma_X$ and $\sigma_Z$. Unlike those for the cat-system, they remain unchanged for large $\alpha$.

Thus, one can achieve a regime of EPR-steering of the spin provided the measurements of number (and spin) have perfect resolution. For number, this generally becomes increasingly difficult in practice for larger $\alpha$, because of detection inefficiencies. On the other hand, the condition for the steering of the cat-system based on the measurements of $X$ and $P$ requires an increasingly higher resolution of the phase measurement $P$ as $\alpha$ increases. Thus it is clear that operational regimes of one-way steering can be achieved for large $\alpha$. This corresponds to EPR-steering of the spin (and hence entanglement), but no EPR-steering of the cat-system. A remaining open question is to demonstrate a regime of genuine one-way steering, which requires to demonstrate the existence of an actual LHS model (5) in which the cat-system is non-steerable [32]. The existence of such states has been proven for two qubit systems [27, 28, 36].

GHZ states

An EPR-steering cat-signature can also be obtained for the GHZ realisation of the state (1). The GHZ state $|\psi_{\text{GHZ}}\rangle = \frac{1}{\sqrt{2}} \left( |\uparrow\rangle \otimes |\uparrow\rangle - |\downarrow\rangle \otimes |\downarrow\rangle \right)$ is formed from $N$ spin-1/2 particles [18, 19, 40]: here we define the $N$-spin eigenstate by $|\uparrow\rangle \otimes |\uparrow\rangle = \prod_{k=1}^{N} |\uparrow\rangle^{(k)}$ where $|\uparrow\rangle^{(k)}$ is the eigenstate of $\sigma^{(k)}_Z$, the $\sigma_Z$ observable for the $k$th particle. If we separate the $N$-th spin from the remaining $N-1$ spins, the GHZ state is a microscopic spin $S$ entangled with a larger system $C$:

$$|\psi_{\text{GHZ}}\rangle = \frac{1}{\sqrt{2}} \left( |\uparrow\rangle^{C}_{\otimes N-1} |\uparrow\rangle^{(N)} - |\downarrow\rangle^{C}_{\otimes N-1} |\downarrow\rangle^{(N)} \right).$$

(12)

Alice makes a measurement on the single spin, while Bob measures the cat-system $C$ of $N-1$ particles. We define the collective spin for the system $C$ as $\sigma_C = \sum_{k=1}^{N-1} \sigma^{(k)}_Z$. Measurement of $\sigma_C$ by Alice reduces the cat-system to the ‘alive’ state $|\uparrow\rangle^{\otimes N-1}$ if her result is $+1$, or to the ‘dead’ state $|\downarrow\rangle^{\otimes N-1}$ if her result is $-1$. Assuming LR and applying Result 1, the cat-system is deduced to be ‘alive’ or ‘dead’ i.e. always in one or the other of two hidden states...
that correspond to outcomes \( \pm(N - 1)/2 \) for \( \sigma^B_Y \) respectively. We denote these states by a hidden variable \( \lambda_Z \) with values \( \pm 1 \). To demonstrate EPR steering, we suppose Alice measures \( \sigma^{(N)}_X \). We choose \( N = 3, 7, \ldots \). Measurement of \( \sigma^{(N)}_X \) predicts precise outcomes for Bob’s \( \Pr^B_Y = \prod_{k=1}^{N-1} \sigma^{(k)}_Y \). Assuming LR, Result 1 implies system \( C \) to be specified by a hidden variable \( \lambda_X \), where the value \( \lambda_X = \pm 1 \) predetermines outcomes \( \pm 1 \) for \( \Pr^B_Y \). Suppose Alice measures \( \sigma^{(k)}_Y \). Assuming LR, the system \( C \) is specified by a third hidden variable \( \lambda_Y \) where the value \( \lambda_Y = \pm 1 \) corresponds to outcomes for all products \( \Pr^B_J = \sigma^{(J)}_X \prod_{k=1, \neq J}^{N-1} \sigma^{(k)}_Y \), \( J = 1, \ldots, N - 1 \) being \( \pm 1 \). Result (1b) implies the cat-system \( C \) to be in one of the element of reality states \( (\lambda_Z, \lambda_X, \lambda_Y) \) in which the outcomes for \( \sigma^B_X, \Pr^B_Y \), \( \Pr^B_J \) are simultaneously each predetermined with no uncertainty. Yet, observables \( \sigma^B_X, \Pr^B_Y \), \( \sum_{J=1}^{N-1} \Pr^B_J \) satisfy the Heisenberg uncertainty relation \( \Delta(\sigma^B_X)\Delta(\Pr^B_Y) \geq |\langle \sum_{J=1}^{N-1} \Pr^B_J \rangle|/2 \). The hidden states \( (\lambda_Z, \lambda_X, \lambda_Y) \) (which specify predetermined nonzero values for each of these observables) contradict this relation. As with inequality (8), this leads to a violation of the EPR-steering inequality (derived according to [27, 29, 34, 39])

\[
\Delta_{\text{inf}}(\sigma^B_X)\Delta_{\text{inf}}(\Pr^B_J) \geq |\sum_{J=1}^{N-1} \Pr^B_J|/2.
\]

This violation signifies an EPR-steering of the cat-system and negates any mixture of quantum ‘dead’ and ‘alive’ hidden states consistent with LR.

So far, there is no falsification that the cat-system can be described as a mixture of ‘dead’ or ‘alive’ hidden variable states. Such a falsification is achieved if the full LHV model (4) can be violated for the cat-state. We note that this has been shown possible for the GHZ state, which for arbitrary \( N \) shows a violation of Svetlichny-type Bell inequalities [6, 19]. The inequalities involve products of the spin of each of the \( N \) particles, and the violations require accurate measurement of each spin, which is increasingly difficult, for fixed detection inefficiencies, as \( N \to \infty \).

**Discussion**

The cat-signatures derived in this paper give a falsification of the LHS model (5). Put another way, the cat-signatures of this paper falsify that the cat-system can be described as either in a ‘dead’ or ‘alive’ state—but where it is required that those ‘dead’ or ‘alive’ descriptions be consistent with there being no nonlocal effects between the cat and spin systems. This is so, because the LHS model assumes full locality (separability) between the cat and spin systems. The locality assumption is evident by the factorisation that occurs in the integrands of (5) (and (4)). If nonlocal effects or interactions are present, then this assumption is not valid.

The criticism could be made that nonlocality (defined as a failure of Bell’s LHV model (4)) has been verified experimentally [2, 4]. This raises the question of the validity of signifying a cat-state using the LHS and LHV models, which are based on the full locality assumption. This leads to two responses: first, we note that, to date, any nonlocal effect observed in experiments is microscopic only, in the sense of corresponding to predictions of \( \sim \) one spin unit, or as a detection of a single photon. By contrast, the validity of the macroscopic hidden variable \( \lambda_Y \) (that in the LHV or LHS model predetermines that the cat is either dead or alive) does not depend on the assumption of microscopic locality. Rather, the macroscopic hidden variable can be deduced (using the logic of the original EPR argument, summarised below) based on there being no macroscopic nonlocal effect, that would cause the measurement on the spin
to instantly change the cat-system from being dead to alive, or vice versa. (The original EPR reasoning applied to this case is as follows: in view of the correlation between the spin and the cat-subsystems, Alice can predict with certainty whether the ‘cat’ will be found by Bob to be dead or alive. Assuming Alice’s spin measurement does not induce a macroscopic change to the cat-system, then the conclusion is that the result of Bob’s measurement was predetermined, thus justifying the macroscopic hidden variable $\lambda_Z$.) Thus, the argument used to justify the macroscopic predetermination, as given by the variable $\lambda_Z$, is not negated by the current experiments.

The second response is that the cat-signatures of this paper do depend on the full assumption of locality (microscopic and macroscopic), because they require measurements (like $P$) other than those (like $X$) that determine whether the cat is dead or alive, and these measurements require a microscopic resolution. In fact, the cat-signatures of this paper negate particular realisations of the ‘dead’ and ‘alive’ states of the cat, that involve the predictions of other measurements conjugate to $X$. This negation is not addressed by the Bell experiments and is of fundamental interest, being the main point of this paper.

A relevant question then becomes whether microscopic nonlocal effects could ‘explain’ the proposed cat-paradox? In such an interpretation, small nonlocal interactions could perturb the state of the cat-system, thus making a predetermination of the microscopic results of the measurements (e.g. $P$), in the context of models like (4) or (5), not possible. The signatures then indicate a failure to be able to describe the dead and alive states of the cat, in a way that is consistent with of microscopic LR. Our results suggest this cannot be ruled out: the deviation (as evident in figure 2) between the ‘elements of reality’ for the quantum superposition state and the classical elements of reality (in which the cat is dead or alive) becomes vanishingly small for large cat size. Hence, if we allow that there could be microscopic nonlocal effects, we can no longer deduce a paradox.

To explain, we introduce a quantification of EPR’s local realism (LR) premise: For $\delta$-scopic LR $S \rightarrow C$, it is assumed that Alice’s measurement of the spin does not affect the (value of measurement on the) cat-system by an amount more than $\pm \delta$. This modified premise thus allows for a small amount (up to $\pm \delta$) of nonlocal effect. Following the logic of the original EPR argument [1], hidden variables for the cat-system can then be defined but with an indeterminacy up to $\pm \delta$ in the prediction for local measurements (due to an amount of allowed nonlocal change $\pm \delta$).

**Result (2a)**

The EPR-steering signatures are a negation of a fully separable LHS model (5). We determine the value $\delta$ such that if we allow nonlocality by an amount greater than $\delta$, then the cat-system becomes indistinguishable from the classical mixture. We find $\delta$ (which is a measure of the perturbation required so that the elements of reality become indistinguishable from a classical realism description) is classifiable as microscopic.

To illustrate, the EPR-steering manifests in the cat-state (2) at large $\alpha$ through very fine fringes in distributions for $P$. One needs only relax the full locality condition by a small amount (to assume $\delta$-scopic LR which allows for a very small change $\delta$ in $P$) to nullify the steering effect. For the GHZ state, the EPR steering of the cat-system is lost when $\delta$ is a single spin unit. This ultra-sensitivity is consistent with proven fundamental requirements for signifying macroscopic quantum superpositions [22, 41]. Moreover, we see from figure 2 that as $\alpha \rightarrow \infty$ the value $\delta$ becomes smaller. This explains the fragility to decoherence as the size $\alpha$ of the cat-state increases—the cat-like behaviour is more difficult to observe because the
elements of reality (which give a predetermination of the results of measurement) are closer to classically consistent values.

Continuing with the discussion, it is the hidden states for the cat-system that are negated by the EPR-steering cat-signatures. These states have microscopic predictions. By contrast to the hidden states, the macroscopic hidden variable \( \lambda_Z \) (that predetermines the outcome for the measurement \( X \) distinguishing the cat-system as either ‘alive’ or ‘dead’) can be defined with a macroscopic indeterminacy \( \Delta \). Because the ‘dead’ and ‘alive’ outcomes are macroscopically separated, \( \lambda_Z \) predetermines the cat-system to be ‘dead’ or ‘alive’, without full specification of the microscopic details of the prediction. In the EPR argument summarised above, the variable \( \lambda_Z \) requires the assumption of \( \Delta \)-scopic LR for its justification, but this is a weaker assumption than \( \delta \)-scopic LR (\( \delta < \Delta \)). Result (2a) therefore indicates that for typical cat-type scenarios (where microscopic perturbation nullifies the cat-signature), the macroscopic hidden variable \( \lambda_Z \) cannot be directly negated. We quantify with the following\(^1\).

**Result (2b)**

Suppose the uncertainty relation for \( X \) and \( P \) is \( (\Delta X)(\Delta P) \geq k \) where \( k \) is a constant. Suppose we assume \( \Delta_\gamma \)-scopic LR and \( \delta_\gamma \)-scopic LR to deduce the hidden variables for measurement \( X \) and \( P \) respectively. If \( \Delta_\gamma \delta_\gamma \geq k \), we cannot signify the cat-paradox by negation of the LHV model based on \( X, P \) measurements.

**Conclusion**

For typical models of the entangled state (1), we cannot falsify the macroscopic element of reality \( \lambda_Z \) for the cat-system. The interpretation of the cat-system being ‘both dead and alive’ becomes debatable in this context. This is clear because \( \lambda_Z \) is precisely the variable that predetermines the outcome of the measurement distinguishing whether the cat-system is ‘dead’ or ‘alive’. We can however falsify (by EPR-steering inequalities) that the cat-system is predetermined to be in a ‘dead’ or ‘alive’ local hidden state, where that hidden state has microscopic predictions independent of measurements made on the spin system (as in the LHS model (5)). If the cat-system is a measuring-device pointer, then an Ockham’s Razor interpretation is illustrated by figure 2(c). The pointer is positioned at one place on the dial or the other (as determined by the macroscopic element of reality \( \lambda_Z \) that cannot be negated) but with its position/ momentum microscopically indeterminate due to microscopic nonlocality (illustrated by the fringes that reveal the falsification of the LHS model). On the other hand, we note from Result (2b) that the quantification of the relevant nonlocal effect is given by the uncertainty bound \( k \) which more generally need not be considered microscopic [42]. We note similarities with other interpretations, including [23] which describes quantum mechanics using interacting classical systems, and [26] which considers the correlation between the pointer and the spin to justify a collapse.

The cat-states we describe are potentially realizable for photonic or trapped ion GHZ states, for a mechanical oscillator coupled to a two-level atom or optical system, or for microwave fields coupled to Rydberg atoms or in superconducting cavities [12, 15, 16, 18, 19, 31, 43]. The steering signatures are thus likely testable by experiment.

\(^1\) See online supplementary material.
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References

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