Stringent Null Constraint on Cosmological Evolution of the Proton-to-Electron Mass Ratio

Julian A. King and John K. Webb
School of Physics, University of New South Wales, Sydney, NSW, 2052, Australia

Michael T. Murphy
Centre for Astrophysics and Supercomputing, Swinburne University of Technology, Victoria, 3122, Australia

Robert F. Carswell
Institute of Astronomy, University of Cambridge, Cambridge CB3 0HA, United Kingdom

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We present a strong constraint on variation of the proton-to-electron mass ratio $\mu$ over cosmological time scales using molecular hydrogen transitions in optical quasar spectra. Using high quality spectra of quasars Q0405 − 443, Q0347 − 383, and Q0528 – 250, variation in $\mu$ relative to the present day value is limited to $\Delta \mu/\mu = (2.6 \pm 3.0) \times 10^{-6}$. We reduce systematic errors compared to previous works by substantially improving the spectral wavelength calibration method and by fitting absorption profiles to the forest of hydrogen Lyman $\alpha$ transitions surrounding each $\mathrm{H}_2$ transition. Our results are consistent with no variation, and inconsistent with a previous $\approx 4\sigma$ detection of $\Delta \mu$ variation involving Q0405 − 443 and Q0347 − 383. If the results of this work and those suggesting that $\alpha$ may be varying are both correct, then this would tend to disfavor certain grand unification models.

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Searches have been undertaken in recent years for cosmological variations in fundamental, dimensionless constants. These searches are motivated by predictions of Kaluza-Klein theory, string theory, and other grand unification theories that the so-called “fundamental constants” may evolve over cosmological time scales. Although much of the focus has been on $\alpha$, the fine structure constant, others have examined the proton-to-electron mass ratio, $\mu = m_p/m_e$. The quantum chromodynamical scale, to which $\mu$ is sensitive, may vary faster than the quantum electrodynamical scale; hence, $\mu$ may vary more than $\alpha$ [1]. The wavelengths of the Lyman and Werner transitions of the $\mathrm{H}_2$ molecule are sensitive to $\mu$, and examination of $\mathrm{H}_2$ absorption systems in quasar spectra allows one to search for any such variation, as was first noted by [2].

Attempts from 1995 to 2004 to detect a variation in $\mu$ yielded results statistically consistent with no change [3–11]. These searches (with the exception of [11]) were impaired by insufficiently accurate laboratory measurements of the $\mathrm{H}_2$ wavelengths, as well as lower quality quasar spectra. Recent laboratory extreme ultraviolet (XUV) laser measurements [12,13] have yielded substantial improvements in $\mathrm{H}_2$ wavelength accuracy.

Using these newly available wavelengths, Reinhold et al. [14] reanalyzed the observed $\mathrm{H}_2$ wavelengths derived by Ivanchik et al. [11] from Very Large Telescope (VLT) spectra of absorbers associated with Q0405 − 443 (at redshift $z = 2.595$) and Q0347 − 383 (at $z = 3.025$), finding a change in $\mu$ of $\Delta \mu/\mu = (2.4 \pm 0.6) \times 10^{-5}$, where $\Delta \mu/\mu \equiv (\mu_z - \mu_0)/\mu_0$, $\mu_z$ is the measured value of $\mu$ at redshift $z$, and $\mu_0$ is the present day laboratory value. However, it has since been demonstrated [15] that the techniques used to calibrate the wavelength scale of the Ultraviolet and Visual Echelle Spectrograph (UVES, on the VLT) produce both long- and short-range calibration errors [15]. These calibration errors directly impact the calculation of $\Delta \mu/\mu$. It is therefore important to reanalyze these spectra using the improved wavelength calibration techniques of [15], and we do so here. We also analyze an absorber towards Q0528 − 250 (at $z = 2.811$), which provides a new, strong constraint on $\Delta \mu/\mu$. We use the Voigt profile fitting program VPFIT to analyze our spectra.

For a given $\mathrm{H}_2$ transition observed in an absorbing cloud at redshift $z_{\text{abs}}$, the first-order shift in the wavelength $\lambda_i$ compared to the laboratory wavelength $\lambda_0$ is given by

$$
\lambda_i = \lambda_0 (1 + z_{\text{abs}}) (1 + K_i \Delta \mu/\mu),
$$

where $K_i$ is the sensitivity coefficient associated with each transition, given by $K_i = (d \ln \lambda_i)/(d \ln \mu)$. $z_{\text{abs}}$ is the redshift of the transitions measured provided that $\Delta \mu/\mu = 0$. If $\Delta \mu/\mu \neq 0$, $z_{\text{abs}}$ corresponds to the redshift, determined from the ensemble of available transitions, of a transition with $K_i = 0$. Previous works have calculated $K_i$ within a semiempirical framework [14,16]; Reinhold et al. [14] recently produced $K_i$ coefficients of improved accuracy by including effects beyond the Born-Oppenheimer approximation. We use the $K_i$ coefficients calculated by Reinhold et al. [14] and Ubachs et al. [16].

For a series of $\mathrm{H}_2$ transitions, the best-fit value of $\Delta \mu/\mu$ may be determined in one of two ways. In the first, the “reduced redshift method” (RRM), one calculates an ob-
served redshift $z_i$ for each transition, and then defines a reduced redshift

$$\xi_i = \frac{z_i - z_{\text{abs}}}{1 + z_{\text{abs}}} = \frac{\Delta \mu}{\mu} K_i.$$  \hspace{1cm} (2)

\(\Delta \mu/\mu\) can then be determined by a linear fit to the observed $\xi_i$ vs $K_i$ distribution.

The second method (VPFIT method) involves fitting all available transitions simultaneously and solving for a single redshift for each identifiable absorbing H$_2$ component in the system. We refer to each fitted redshift as a velocity component because of the close proximity of these components in velocity space. $\Delta \mu/\mu$ is estimated by perturbing the laboratory H$_2$ wavelengths as $\lambda_0 \rightarrow \lambda_0[1 + K_i(\Delta \mu/\mu)]$ and minimizing $\chi^2$ for the spectral data fitted. The value of $\Delta \mu/\mu$ at the minimum $\chi^2$ is the best-fit value.

Although the VPFIT method has previously been used to construct $\chi^2$ vs $\Delta \mu/\mu$ curves, from which the best value of $\Delta \mu/\mu$ can be estimated, we instead include $\Delta \mu/\mu$ as a free parameter in the fit (within VPFIT), to be solved for concomitantly with the other line parameters; this yields a substantial improvement in computational speed and robustness.

The RRM has been used in most previous measurements and was the method used by [14]. This method is appealing because the required numerical methods are relatively simple. However, the VPFIT method is preferable in that fewer parameters are required to fit the data. In particular, the VPFIT method has $n_v(n_v - 1)$ fewer free parameters, where $n_v$ is the number of velocity components and $n_i$ is the number of transitions used. For Q0405 – 443 the VPFIT method yields 51 fewer parameters, for Q0347 – 373 it yields 67 fewer parameters, and for Q0528 – 250 it yields 252 fewer parameters.

It should be noted that the VPFIT method also improves the stability of the fitting process. In systems with multiple velocity components, particular transitions may have very poorly constrained line parameters, despite the fact that the best-fit line parameters may be well constrained over many transitions. In the RRM, this can cause the fitting algorithm to reject certain velocity components in some transitions, rendering those transitions unsuitable for inclusion in the fit. Using the VPFIT method, the reduction in the number of free parameters as a result of requiring each velocity component to occur at a single redshift helps to stabilize the fit, allowing for the inclusion of a greater number of transitions.

Each of the molecular hydrogen transitions involved falls within the Lyman $\alpha$ forest, a dense series of absorption lines blueward of the hydrogen Lyman $\alpha$ emission line of the quasar. These transitions substantially complicate the analysis; the narrow molecular hydrogen absorption lines are often situated deep within much broader, and usually complex, Lyman $\alpha$ lines. These contaminating atomic Lyman $\alpha$ transitions are insensitive to a change in $\mu$. In contrast to previous works, we fit absorption profiles to all of the Lyman $\alpha$ transitions in the vicinity of each H$_2$ transition. This allows the inclusion of a greater number of H$_2$ transitions which would otherwise be excluded from the fit.

We have reduced the total number of free parameters in the fit by tying together physically linked parameters. In particular, we tie the Doppler (linewidth) parameters together for H$_2$ transitions with the same rotational quantum number $J$ of the initial state. For the transitions we have analyzed, $J \in [0, 4]$. Each error estimate is multiplied by $\sqrt{\chi^2}$ (where $\chi^2_\nu$ is the $\chi^2$ per degree of freedom for the whole fit), to account for a nonideal fit.

Our inclusion of the Lyman $\alpha$ forest within the fitting process increases the number of free parameters in the fit substantially (to over 1000 in each quasar spectrum). With such a large parameter space, convergence of the optimization algorithm must be checked. References [17,18] demonstrated that the results of [19–22] (in relation to $\alpha$) were flawed in this respect. Our algorithm demonstrates the proper convergence, in that $\Delta \mu/\mu$ vs $\chi^2$ curves possess the correct parabolic shape, with derived 1$\sigma$ error bounds on $\Delta \mu/\mu$ that agree with those produced by VPFIT.

The system towards Q0347 – 373 contains a single H$_2$ velocity component. The system towards Q0405 – 443 has a second velocity component, separated by $\approx 13 \text{ km s}^{-1}$ in velocity space [14]. However, many of its transitions are weak or are heavily blended, and so we have not utilized the second component here.

The H$_2$ system towards Q0528 – 250 is more complicated. Previous attempts to examine this system have yielded varied and comparatively poor results [6,10] because the spectra used had substantially lower signal-to-noise ratio than those currently available from VLT. Ledoux et al. [23] report the detection of multiple velocity components in the Q0528 – 250 system (that is, multiple systems separated in velocity space), and Srianand et al. [24] model the absorber with two components.

We have tried modeling the absorber towards Q0528 – 250 with 2, 3, and 4 velocity components. Two velocity components are plainly obvious as a substantial asymmetry with 2, 3, and 4 velocity components. Two velocity components are particularly clear, being due to chance is $p = 4 \times 10^{-8}$. That is, a three component model is very strongly preferred to a two component model.

Comparing a model with four velocity components to a model with three gives $p = 1.8 \times 10^{-8}$. That is, the four component model is preferred over both the two and three component models. The use of a five component model produces a fit that is highly unstable numerically, and so we use the four component model as the fiducial model.

In modeling the multiple velocity components, we require that the ratio of the column densities between each velocity component is the same for transitions with the same quantum number $J$. Certain line parameters for the
the line parameters for each of the velocity components
are similar to each other and to the result for the four
component model. This demonstrates the insensi-
tivity of the result to having used the statistically preferred
velocity structure.

Combining the three measurements of $\Delta \mu / \mu$ obtained
within VPFIT using a weighted mean yields the value
$\Delta \mu / \mu = (2.6 \pm 3.0) \times 10^{-6}$. This is null at a 1$\sigma$
confidence level. This is our main result from a combined
analysis of all three quasar absorbers.

For comparison with Reinhold et al. [14], a reduced
redshift plot (Fig. 1) including only Q0405−443 and
Q0347−373 produces the result $\Delta \mu / \mu = (8.5 \pm 5.7) \times
10^{-6}$ (weighted fit) and $\Delta \mu / \mu = (7.9 \pm 8.1) \times 10^{-6}$(un-
weighted fit). We also attempt to compare with [14] by
including in the fit only those transitions used in that Letter.
For Q0405−443, this removes 16 transitions and adds 3,
the latter of which appear to be contaminated and were
excluded from our main analysis. This yields, from the
RRM, a Q0405−443 result of $\Delta \mu / \mu = (10.2 \pm 8.9) \times
10^{-6}$. This is offset from the result of $(27.8 \pm 8.8) \times 10^{-6}$
in [14]. For Q0347−373, we remove 35 transitions that are
not used in [14], and include 4 which appear to be
contaminated, to give a result of $\Delta \mu / \mu = (12.0 \pm
14.0) \times 10^{-6}$, compared with $(20.6 \pm 7.9) \times 10^{-6}$ from
[14]. The weighted mean of our results in this circumstance
is $\Delta \mu / \mu = (10.7 \pm 7.5) \times 10^{-6}$. It is difficult to make a
direct statistical comparison, due to the fact that the spectra
analyzed are not independent; however, in both cases we
see a shift of $\Delta \mu / \mu$ towards 0. Although the inclusion of
Q0528−250 clearly shifts the combined Q0405−443 +
Q0347−373 result towards zero, our combined Q0405−
443 + Q0347−373 result is null under all the circum-
stances considered.

More details regarding our results can be found in [26].
In particular, the reader may locate individual copies of our
$\zeta_i$ vs $K_i$ plots there, as well as some example spectra fits.
Reference [26] does not contain any information important
to our result that is not included here.

Our final result of $\Delta \mu / \mu = (2.6 \pm 3.0) \times 10^{-6}$ repres-
ts a significant increase in precision over previous
works (a factor of ≈ 2). This result is entirely consistent
with $\Delta \mu / \mu = 0$ over cosmological time scales. It is also
consistent with the recently published work of Murphy
et al. [27], who find that $\Delta \mu / \mu = (0.74 \pm 0.47_{stat} \pm
0.76_{syst}) \times 10^{-6}$ using the inversion transitions of amm-
onia. Note, however, that the ammonia constraint is at $z =
0.685$ while all our constraints are at $z > 2.5$; they may not
be directly compared without a theory of cosmologically
evolving $\mu$.

<table>
<thead>
<tr>
<th>Quasar spectrum</th>
<th>$\Delta \mu / \mu$ (VPFIT)</th>
<th>$\chi^2_\nu$</th>
<th>$z$</th>
<th>$n$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Q0405−443</td>
<td>$(10.1 \pm 6.2) \times 10^{-6}$</td>
<td>1.42</td>
<td>2.595</td>
<td>52</td>
</tr>
<tr>
<td>Q0347−373</td>
<td>$(8.2 \pm 7.4) \times 10^{-6}$</td>
<td>1.28</td>
<td>3.025</td>
<td>68</td>
</tr>
<tr>
<td>Q0528−250</td>
<td>$(-1.4 \pm 3.9) \times 10^{-6}$</td>
<td>1.22</td>
<td>2.811</td>
<td>64</td>
</tr>
<tr>
<td>Weighted mean</td>
<td>$(2.6 \pm 3.0) \times 10^{-6}$</td>
<td>$n/a$</td>
<td>2.81</td>
<td>$n/a$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Quasar spectrum</th>
<th>$\Delta \mu / \mu$ (RRM)</th>
<th>$\chi^2_\nu$</th>
<th>$z$</th>
<th>$n$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Q0405−443</td>
<td>$(10.9 \pm 7.1) \times 10^{-6}$</td>
<td>1.01</td>
<td>2.595</td>
<td>52</td>
</tr>
<tr>
<td>Q0347−373</td>
<td>$(6.4 \pm 10.3) \times 10^{-6}$</td>
<td>1.13</td>
<td>3.025</td>
<td>68</td>
</tr>
<tr>
<td>Q0405 + Q0347</td>
<td>$(8.5 \pm 5.7) \times 10^{-6}$</td>
<td>1.06</td>
<td>2.810</td>
<td>120</td>
</tr>
<tr>
<td>Q0528−250</td>
<td>$\ldots$</td>
<td>$\ldots$</td>
<td>$\ldots$</td>
<td>$\ldots$</td>
</tr>
</tbody>
</table>
The unification of all interactions clearly requires that any cosmological variations in the various fundamental constants will be linked to each other. Grand unified theories typically predict $\Delta \mu/\mu = R\Delta \alpha/\alpha$ [28–30], where both the sign and magnitude of $R$ are strongly model dependent. $|R| \approx 30–40$ emerges from many grand unified theory models [28–30]. Generally speaking, $|R| \gg 1$. The most reliable constraint on $\alpha$ variation at present is $\Delta \alpha/\alpha = (-5.7 \pm 1.0) \times 10^{-6}$ [31]; the works of [19–22] and others have been demonstrated to be unreliable [17,18]. Taking both this and our new null result at face value, any variation in $\mu$ is almost 2 orders of magnitude below that expected on the basis of the $\alpha$-variation results. If both these results are correct, those grand unified models which predict $|R| \gg 1$ are disfavored.

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