Ultrasound Image Deconvolution in Symmetrical Mirror Wavelet Bases

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ABSTRACT

Observed medical ultrasound images are degraded representations of true tissue images. The degradation is a combination of blurring due to the finite resolution of the imaging system and the observation noise. This paper presents a new wavelet based deconvolution method for medical ultrasound imaging. We design a new orthogonal wavelet basis known as the symmetrical mirror wavelet basis that can provide more desirable frequency resolution. Our proposed ultrasound image restoration with wavelets consists of an inversion of the observed ultrasound image using the estimated two-dimensional (2-D) point spread function (PSF) followed by denoising in the designed wavelet basis. The tissue image restoration is then accomplished by modelling the tissue structures with the generalized Gaussian density (GGD) function using the Bayesian estimation. Both subjective and objective measures show that the deconvolved images are more appealing in the visualization and resolution gain.

Keywords: Deconvolution, mirror wavelet, denoising, generalized Gaussian distribution, Bayes estimator

1. INTRODUCTION

The observed radio frequency (RF) ultrasound images are known to be degraded representations of tissue structures due to the 2-D convolution of the biological tissues with the spatially varying ultrasonic 2-D pulse-echo wavelet or PSF. And resultant images are further contaminated by additive noise.

In image processing, wavelets have been well known for its excellent performance in image denoising and compression. Its property of sparse signal representation enables noise reduction and image compression to be achieved with a simple threshold operation. Our proposed ultrasound image restoration with wavelets consist of an inversion of the observed ultrasound images with the estimated 2-D PSF and followed by denoising in our designed wavelet basis.

Since the ultrasonic 2-D PSF is typically lowpass and bandpass responses in the lateral and axial directions respectively, there will be almost null at high frequencies in the lateral direction and at both the low and high frequencies in the axial direction. Inverting the observed ultrasound images with the estimated 2-D PSF in presence of the additive noise amplifies the noise and causes it to become colored in the neighborhood of these frequencies. Under this condition, the noise covariance will not be diagonal and the existing dyadic wavelet decomposition cannot concentrate the noise energy. This degrades the effectiveness of the thresholding operator in removing the noise.

In view of this, we design a new orthogonal symmetrical mirror wavelet basis that can provide more desirable frequency resolution. Denoising is then accomplished by modelling the tissue structures with GGD function and under Bayesian philosophy.

This paper is organized as follows. Section 2 discusses the problem in direct inversion of the ultrasound image using the 2-D PSF. Section 3 presents the mirror wavelet basis, known as the symmetrical mirror wavelet basis, designated for ultrasound deconvolution. In Section 4, the wavelet coefficients are denoised and the methodology further author information: (Send correspondence to Cishen Zhang)
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is described. The mirror wavelet coefficients of the tissue is modelled as GGD function whereas the scale-dependent estimator in the mirror wavelet basis is employed for its estimation. In Section 5, we present the experimental results, and address performance of the proposed algorithm under different configurations. Finally, Section 6 concludes the paper.

2. ULTRASOUND DECONVOLUTION

In ultrasound image formation, the images are blurred in both the axial and lateral directions due to the finite resolution of the imaging system. Under the assumptions of linear propagation and weak scattering, an expression for the received pulse-echo pressure field is derived using the first-order Born approximation presented in a convolution model in the sampled domain:\(^6\):

\[
y(m, n) = v(m, n) * s(m, n) + w(m, n) \\
s(m, n) = f(m, n) + d(m, n),
\]

where \(y(m, n)\) is the ultrasound RF echo, \(v(m, n)\) is the 2-D PSF, \(s(m, n)\) is the noisy tissue image, both \(w(m, n)\) and \(d(m, n)\) are additive noise, and \(*\) denotes the discrete 2-D linear convolution operator with respect to the variables \(m\) and \(n\) representing the horizontal and vertical discrete sample coordinates. In other words, the distortion introduced by the ultrasound system is modelled as a convolution of the desired signal with the ultrasonic system impulse response. Inverting the effects of the distortions corresponds to a deconvolution procedure. A direct inversion is not feasible because the observed ultrasound RF echo is also contaminated by the environmental noise which makes the deconvolution problem more difficult.

The analysis assumes the periodic boundary condition, which means that all convolutions are circular. The noise \(w(m, n)\) is circular stationary and its covariance is therefore diagonalized by the discrete Fourier basis. We suppose that the 2-D PSF is obtained using contemporary PSF estimation techniques such as the complex cepstrum method\(^4,5\) and the noise is known through calibration.

Let \(V(\omega_m, \omega_n)\) denote the Fourier transform (FT) of \(v(m, n)\), i.e., \(V(\omega_m, \omega_n) = \mathcal{F}[v(m, n)]\). Direct inversion of (1) with \(V(\omega_m, \omega_n)\) necessarily amplifies noise components at frequencies where \(V(\omega_m, \omega_n)\) is small. We employ a conditional inversion to moderate the noise amplification. It is the pseudo-inverse filter \(v^{-1}(m, n)\) whose FT \(V^{-1}(\omega_m, \omega_n)\) is:

\[
V^{-1}(\omega_m, \omega_n) = \begin{cases} 
\frac{1}{|V(\omega_m, \omega_n)|}, & |V(\omega_m, \omega_n)| > \tau \\
0, & |V(\omega_m, \omega_n)| \leq \tau
\end{cases}
\]

where \(\tau\) is a threshold controlling the tradeoff between the amount of noise suppression and the amount of signal distortion. The range of \(\tau\) varies from 0 to the maximum value of \(|V(\omega_m, \omega_n)|\). Setting \(\tau = 0\) gives an unbiased but noisy estimate whereas setting it to the maximum value completely suppresses the noise but also completely distort the signal. A small \(\tau\) is desired as it can lead to huge reduction in the degree of noise amplification only at the expenses of slight increase in signal distortion. In our studies, \(\tau\) is set to 1 % the magnitude of \(|V(\omega_m, \omega_n)|\) which is equivalent to having the effect of setting the frequency segments above critical frequencies to small values\(^a\).

The use of the pseudo-inverse filter avoids amplification of the deconvolved noise in frequencies where there is no observed signal. Applying \(v^{-1}(m, n)\) to \(y(m, n)\) rewrite the task of restoration of tissue signals into a standard denoising problem:

\[
x(m, n) = v^{-1}(m, n) * y(m, n) \\
= c(m, n) * f(m, n) + z(m, n),
\]

where we have assigned \(z(m, n) = c(m, n) * d(m, n) + v^{-1}(m, n) * w(m, n)\) to account for all the undesirable signals. Estimating \(f(m, n)\) from \(y(m, n)\) is equivalent to estimate it from \(x(m, n)\) where \(c(m, n) = v^{-1}(m, n) * v(n)\)

\(^a\)In practice, the value of \(\tau\) is obtained empirically with the smaller the value the better. Exactly how small it can be depends on the quality of the estimated 2-D PSF and the amount of observation noise existing within the measurement.
projects \(s(m,n)\) over frequencies where \(V(\omega_m, \omega_n)\) does not vanish. Hence, \(C(\omega_m, \omega_n) = F[c(m,n)]\) has the following spectrum:

\[
C(\omega_m, \omega_n) = \begin{cases} 
1 & |V(\omega_m, \omega_n)| > \tau \\
0 & |V(\omega_m, \omega_n)| \leq \tau 
\end{cases}
\]

(5)

Since both \(d(m,n)\) and \(w(m,n)\) are circular stationary, the deconvolved noise \(z(m,n)\) is also circular stationary with its power spectrum \(P_z(\omega_m, \omega_n)\) relating to that of \(d(m,n)\) and \(w(m,n)\) as follows:

\[
P_z(\omega_m, \omega_n) = C(\omega_m, \omega_n) P_D(\omega_m, \omega_n) + P_W(\omega_m, \omega_n) |V^{-1}(\omega_n)|^2,
\]

(6)

where \(P_D(\omega_m, \omega_n)\) and \(P_W(\omega_m, \omega_n)\) are the power spectrum of \(d(m,n)\) and \(w(m,n)\) respectively. Since \(V(\omega_m, \omega_n)\) has a band-limited frequency response, the frequency response \(V^{-1}(\omega_m, \omega_n)\) becomes huge, in the neighborhood of frequencies where \(V(\omega_m, \omega_n)\) vanishes. Thus the deconvolved noise spectrum \(P_W(\omega_n) |V^{-1}(\omega_n)|^2\) is amplified considerably.

3. MIRROR WAVELET BASIS FOR ULTRASOUND DECONVOLUTION

In the following context, we propose a wavelet basis which is a modification of the orthonormal mirror wavelet basis proposed by Mallat et al. 6, 7

3.1. Choice of Optimal Basis

For simplicity, the presentation in the following is based on the one-dimensional model in the axial direction. Unless otherwise specified, it is also valid for the model in the lateral direction.

Wavelet bases are well-known for their efficiency in concentrate the signal energy and have been widely used to enhance images corrupted by noise. A \(N\)-periodic discrete orthonormal dyadic scale wavelet basis is written as:

\[
\psi_{j,q}(n) = \frac{1}{\sqrt{2^j}} \psi \left( \frac{n - N 2^j q}{2^j} \right) = \psi_j(n - N 2^j q) = \psi_j(n - 2^j - L q),
\]

(7)

where \(\int_{-\infty}^{\infty} \psi(t) dt = 0, 0 \leq q < 2^{-j}\), and \(\Psi_j(\omega_n) = F[\psi_j(n)]\) has an energy essentially concentrated in the frequency band \(|\omega_n| \in [2^{-j-1}, 2^{-j}]\). The wavelet family \(\{\psi_{j,q}\}_{L+1}^{\infty} 1 \leq j \leq L, 0 < q < 2^{-j}\) is an orthonormal basis of \(C^N\), with \(L = -\log_2 N\) and the largest scale wavelet is constant, i.e., \(\psi_{1,0}(n) = 1\). It has been proven by Kalifa and Mallat 7 that the noise energy concentrates in the wavelet basis if there exists a constant \(\lambda_j\) for the wavelet basis as follows:

\[
\lambda_j \min_{\omega_n \in [2^{-j-1}, 2^{-j}]} |V^{-1}(\omega_n)| \geq \max_{\omega_n \in [2^{-j-1}, 2^{-j}]} |V^{-1}(\omega_n)|.
\]

(8)

To simply notations, we use \(\alpha(\omega_n) \sim \beta(\omega_n)\) to represent the following relationship for two constant \(A\) and \(B\) of the order of 1 such that:

\[
A \alpha(\omega_n) \leq \beta(\omega_n) \leq B \alpha(\omega_n).
\]

(9)

For \(|V^{-1}(\omega_n)|\) having a rational growth, it can be modelled as \(|V^{-1}(\omega_n)| \sim |\omega_n|^\gamma\) whereby \(\gamma\) is a real exponential. In this case, it can be shown that (8) is satisfied for \(\lambda_j \sim 2^{|h|}\).

3.2. Modification of Optimal Basis for Ultrasound Deconvolution

In ultrasound imaging, the 2-D PSF has a lowpass response in the lateral direction and a bandpass response in the axial direction. Therefore, the 2-D PSF can be modelled by transfer function separable in the lateral and axial directions with zeros of order \(p_1, p_2, p_3 \geq 1\) written as follows:

\[
|V(\omega_m)| \sim \left| \frac{2\omega_m}{N} - 1 \right|^{p_2},
\]

(10)

\[
|V(\omega_n)| \sim |\omega_n|^{|p_1|} \left| \frac{2\omega_n}{N} - 1 \right|^{p_3}.
\]

(11)
Therefore, the deconvolved noise has the following spectrum in the lateral and axial directions respectively:

\[
P_Z(\omega_m) = C(\omega_m) P_D(\omega_m) + P_W(\omega_m) \left| V^{-1}(\omega_m) \right|^2 \sim \sigma^2 \left| \frac{2\omega_m}{N} - 1 \right|^{-2p_2},
\]

\[
P_Z(\omega_n) = C(\omega_n) P_D(\omega_n) + P_W(\omega_n) \left| V^{-1}(\omega_n) \right|^2 \sim \sigma^2 \left| \frac{2\omega_n}{N} - 1 \right|^{-2p_3}.
\]

Choosing the \( N \) periodic discrete orthonormal dyadic scale wavelet basis presented in (7), it can be shown that we can obtain (8) under the following conditions:

\[
\lambda_j \omega_m \in \left[ 2^{-j-1}, 2^{-j} \right] \quad \left| \frac{2\omega_m}{N} - 1 \right|^{-p_2} \geq \max_{\omega_m \in \left[ 2^{-j-1}, 2^{-j} \right]} \left| \frac{2\omega_m}{N} - 1 \right|^{-p_2},
\]

\[
\lambda_j \omega_n \in \left[ 2^{-j-1}, 2^{-j} \right] \quad \left| \frac{2\omega_n}{N} - 1 \right|^{-p_3} \geq \max_{\omega_n \in \left[ 2^{-j-1}, 2^{-j} \right]} \left| \frac{2\omega_n}{N} - 1 \right|^{-p_3}.
\]

For the lowpass response \( V(\omega_m) \) of (10), (14) is valid (i.e., \( \lambda_j \) should remain of the order of 1) for \( j > L+1 \). However, at the finest scale \( j = L + 1 \), \( V^{-1}(\omega_m) \) varies by a huge factor within the interval \( [N/4, N/2 - 1] \) with \( \lambda_j \sim (N/4)^{p_3} \). This is an indication that the finest wavelet does not have enough frequency resolution to concentrate the noise energy. As for the bandpass response \( V(\omega_n) \) of (11), there is huge amplification of noise in both the low and the high frequencies. Due to the dyadic scale of wavelet basis which results having a higher frequency resolution in the larger scale of \( j > L+1 \), (15) is then satisfied for \( j > L+1 \) with \( \lambda_j \sim 2^{p_3} \). Likewise, at the finest scale \( j = L+1 \), (15) is not valid with \( \lambda_j \sim (N/4)^{p_3} \).

We will now modify the wavelet basis so that it can concentrate both signal and noise energies. We know that the wavelet basis of scale \( j > L+1 \) is able to adapt well to the amplified noise with \( \lambda_j \sim 2^{p_3} \) while the signal is still well approximated by relative small proportion of nonzero wavelet coefficients. For this reason, we are of interest in constructing a new wavelet basis such that it is a mirror reflection of the original wavelet basis about the frequency \( N/4 \). Hence, the newly built wavelet basis is known as the mirror wavelet basis. The frequency support of the mirror wavelet \( \psi_{j,q}(n) \) is given by \( [N/2 - 2^{-j}, N/2 - 2^{j-1}] \). One does not want to further reduce the size of these frequency intervals because it would increase the spatial support of the basis vectors which reduce the efficiency of signal representation.

The designated composite wavelet basis composes of the original wavelet basis and the mirror wavelet basis, both are used with scales \( j > L+1 \). This is the composite wavelet basis we propose for ultrasound image deconvolution. It has satisfied \( \lambda_j \sim 2^{p_3} \) in the wavelet basis and \( \lambda_j \sim 2^{p_3} \) in the mirror wavelet basis while still well adapts to approximate signals efficiently.

The mirror wavelet basis is realized by combining two wavelet bases. A discrete and periodic orthogonal wavelet basis is given as:

\[
\mathcal{B} = \{ \psi_{j,q}(n) \}_{L+1 \leq j \leq 1; 0 \leq q < 2^{-j}},
\]

where \( \psi_{j,q}(n) = 2^{-j/2} \psi(2^{-j}n - Nq) = \psi_j(n - N2^jq) = \psi_j(n - 2^{j-L}q) \). The finest wavelet \( \psi_{L+1,q}(n) \) is replaced by wavelets whose discrete Fourier support decrease exponentially at high frequencies known as the mirror wavelet:

\[
\left\{ \tilde{\psi}_{j,q}(n) = (-1)^{n-1} \psi_{j,q}(1-n) \right\}_{L+2 \leq j \leq 1; 0 \leq q < 2^{-j}},
\]

whereby \( \Psi_j(\omega_n) = F[\psi_{j,q}(n)], \tilde{\Psi}_j(\omega_n) = F[\tilde{\psi}_{j,q}(n)] \) and \( |\tilde{\Psi}_j(\omega_n)| = |\Psi_j(N/2 - \omega_n)| \). The result of combining the two wavelet bases forms an orthonormal basis of \( C^N \), called the mirror wavelet basis.

At low frequencies \( (\omega_m, \omega_n) \in [0, N/4]^2 \), a 2-D separable wavelet basis is constructed with separable products of discrete periodic scaling functions \( \phi_j(n) \) and the corresponding periodic wavelet \( \psi_j(n) \). If we let \( \psi_{1,q}(m) = \phi_{0,q}(m) = \phi_{0,q}(n) \), we have:

\[
\mathcal{E}_c = \{ \psi_{1,j}(m) \psi_{2,q}(n) \}_{L+2 \leq j_1, j_2 \leq 1; 0 \leq p < 2^{-j_1}, 0 \leq q < 2^{-j_2}}.
\]
The remaining frequency annulus is covered by the 2-D symmetrical mirror wavelet basis that is formed by the separable products of mirror wavelets and the periodic orthogonal wavelets:

\[ B_1 = \{ \psi_{j_1,p}(m)\tilde{\psi}_{j_2,q}(n), \tilde{\psi}_{j_1,p}(m)\psi_{j_2,q}(n), \psi_{j_1,p}(m)\tilde{\psi}_{j_2,q}(n) \} \quad 0 \leq p < 2^{-j_1}, 0 \leq q < 2^{-j_2} \]  \hspace{1cm} (19)

The union of these two bases \( B = B_0 \cup B_1 \) defines an orthonormal symmetrical mirror wavelets basis of image of \( N^2 \) pixels. The segmentation of the 2-D frequency plane by the composite wavelet basis, for a 3-level implementation, is shown in Figure 1.

**Figure 1.** Proposed symmetrical mirror wavelet basis. The grey squares correspond to critical frequencies beyond which the estimator sets all coefficients to zeros. Setting the threshold \( \tau \) to 1% the magnitude of the PSF automatically determined these critical frequencies and prevents the wavelet coefficients from over amplification.
4. NOISE REDUCTION BY THRESHOLDING MIRROR WAVELET COEFFICIENTS

Now, we recall that the estimated 2-D PSF is used to invert the received ultrasound RF image \( y(m,n) \) to yield \( x(m,n) \), which is then decomposed under the proposed symmetrical mirror wavelet basis for denoising.

This basis is designed for ultrasound RF images to facilitate denoising since it is capable of representing signal structures with few large wavelet coefficients and providing the necessary frequency resolution in the frequency interval \([N/4, N/2 - 1]\) to decorrelate the colored noise. This section describes the employment of the scale-dependent estimator in the mirror wavelet basis. Following the general definition of an estimator, the deconvolved signal \( x(n) = v^{-1}(n) * y(n) \) is decomposed in the proposed composite wavelet basis and a decision operator \( \rho_m \) is used to estimate the noiseless tissue wavelet coefficients.

\[
\tilde{x}(n) = D \left[ x(n) \right] = \sum_{j=L+2}^{j=L+1} \sum_{q=0}^{2^j-1} \rho_m \left( \langle x(n), \psi_{j,q}(n) \rangle \right) \tilde{x}_{j,m} 
\]

4.1. Modelling of Noiseless Tissue Wavelet Coefficients

It has been observed that sparse signal representation of the wavelet transform gives rise to wavelet coefficients of non-Gaussian statistics. An intuitive interpretation of this is that images typically contain smooth regions which lead to small filter responses which generate sharp peaks at zero. Whereas, localized features of images produce large-amplitude responses that generate the extensive tails in probably density function (PDF). These densities are accurately modelled with a two-parameter PDF known as the generalized Gaussian distribution or GGD of the form:

\[
p_{f_B} (f_B(n)) = \frac{\beta \eta(\beta)}{2 \Gamma \left( \frac{1}{\beta} \right) \sigma_{f_B}^{\beta}} e^{-\beta (f_B(n) - \eta(\beta))}^{\frac{1}{\beta}}, \tag{21}
\]

where \( \Gamma(t) = \int_0^\infty e^{-ut} u^{t-1} du \), \( f_B(n) \) is the wavelet coefficient, \( \sigma_{f_B} \) be the standard deviation, \( \beta \) is the shape parameter, and \( \eta(\beta) / \sigma_{f_B} = 1 / \alpha \). The distribution is zero-mean and symmetric.

Let \( \kappa \) be the kurtosis which is the fourth moment divided by squared of the variance, the parameters \( \alpha \) and \( \beta \) of GGD are directly related to the second moment and fourth moments of GGD as follows:

\[
\alpha = \sqrt{\frac{\Gamma \left( \frac{3}{\beta} \right)}{\sigma_{f_B}^{2} \Gamma \left( \frac{1}{\beta} \right)}} \quad \text{and} \quad \kappa = \frac{\Gamma \left( \frac{1}{\beta} \right) \Gamma \left( \frac{3}{\beta} \right)}{\Gamma \left( \frac{5}{\beta} \right)^2} \cdot \tag{22}
\]

The distribution is Laplacian with \( \beta \) of 1.0 and it becomes Gaussian when \( \beta \) is 2.0. This implies that random variables following GGD with small \( \beta \) are highly impulsive which is an evidence of the presence of resolvable structures in the underlying data. Figure 2 shows the density function of GGD with various \( \beta \). It shows that smaller \( \beta \) gives a sharper peak at zero with broad tails indicating a larger kurtosis which is typically well above that of the Gaussian distributed data.
4.2. The MMSE Estimator

In the previous section, modification of the wavelet basis to the symmetrical mirror wavelet basis is proposed so that the resultant subband wavelet coefficient is the combination of the signal coefficient which is of GGD distribution and observation noise coefficient that is nearly white. In the following, we use a formulation of Bayesian estimator known as the minimum mean square error (MMSE) estimator for the estimation of the noiseless coefficients in the wavelet domain. The MMSE estimate is obtained as the coefficient that minimizes the mean square error cost function defined as:

$$ r(D, f_B(n)) = E \left\{ \| f_B(n) - D(x_B(n)) \|^2 \right\}. \quad (23) $$

The mean of the posterior distribution provides an optimal estimate of the noiseless coefficient, $f_B(n)$, given a measurement of $x_B(n)$:

$$ f_B(n) = \arg \min_{D[x_B(n)]} r(D, f_B(n)) $$

$$ = \int f_B(n) p_{f_B/x_B}(f_B(n)/x_B(n)) \, df_B $$

$$ = \int f_B(n) p_{x_B}(x_B(n) - f_B(n)) p_{f_B}(f_B(n)) \, df_B $$

$$ = \int p_{x_B}(x_B(n) - f_B(n)) p_{f_B}(f_B(n)) \, df_B $$

where for a given set of observation data, the PDF $p_{x_B}(x_B(n))$ is a constant and has only a normalizing effect. From (24), the posterior PDF is proportional to the product of the likelihood $p_{x_B/f_B}(x_B(n)/f_B(n))$ that the observation $x_B(n)$ is contributed by signal coefficient, $f_B(n)$, and the prior PDF $p_{f_B}(f_B(n))$.

Figure 3 shows the set of numerically computed MMSE estimators with three different values of the shape parameters. Smaller values of $\beta$ produce a nonlinear shrinkage operator. For $\beta = 0.5$, the estimator preserves large coefficients and suppress the small coefficients. This is intuitively sensible because small value of $\beta$ corresponds to sparse wavelet representation. As the value of $\beta$ increases toward 2.0, this indicates that the statistic of the underlying data tends toward more Gaussian. Therefore, the MMSE estimator eventually attained the linear form when $\beta = 2.0$.

5. EXPERIMENTAL RESULTS

A set of eight different unfiltered RF data is recorded from the abdomen of adult volunteers with a VIVID3 (GE, Medical ultrasound, Inc.) commercial ultrasound scanner, equipped with a special data-transfer board. These
Figure 3. MMSE estimators for the model given in equation (24), with three different shape parameters. The noise is additive and Gaussian, with variance one third that of the signal. Dash line indicates the identity function.

data are obtained by a linear array probe, with a central frequency of about 3.5 MHz. The images are simple in visualizations and do not require a scan conversion stage. The data are acquired with a single transmission focal point, localized approximately at the center of the field of view. All the data are composed, on average, of 200 RF-line, each of 1024 sample points in length. The sampling rate and resolution are 20 MHz and 16-bit, respectively.

In the experiments, we record our results using maximally asymmetric wavelet of Daubechies with 4 vanishing moments and 3 levels of wavelet decomposition. We compare the results of our algorithm under different configurations. To get an appealing image, the estimated noise standard deviations are multiplied by a factor of 5. Increasing the estimated noise standard deviation has the effect of increasing the thresholding by a multiplication factor. This is a treatment quite commonly used in wavelet denoising, and the multiplication factor is called the threshold multiplier.\textsuperscript{13}

Furthermore, as the reduction in speckle size suggests a gain in resolution, we adopt the resolution gain (RG) which is defined as the ratio of the correlation lengths of the RF data before and after deconvolution. It is computed for both the -5 dB and -10 dB correlation lengths of the RF data corresponding to the images before and after deconvolution. Defining the resolution at two different dB levels allow us to have a better idea of the shape of the 2-D autocovariance function (ACF). These computations are performed on both the axial slice, $A_{00}$, and lateral slice, $L_{00}$, where they are extracted through the peak of the 2-D ACF. This criteria is used in a number of works\textsuperscript{2-5} in evaluation of ultrasound imaging in the RF signal domain. Besides, the reduction of speckle improves the discrimination of fine details in the ultrasound image during diagnostic examination. While there is no metric to best quantify this improvement, visual inspection is also important for the evaluation.

Table 1. Resolution gains obtained due to deconvolution using the proposed mirror wavelet solution computed in the RF domain with the different configurations: proposed standard method (Proposed), and Hard Threshold (HardThres). These values are tabulated in terms of mean ± one standard deviation.

<table>
<thead>
<tr>
<th>Description</th>
<th>Axial slice, $A_{00}$</th>
<th>Lateral slice, $L_{00}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>RG\textsubscript{5dB}</td>
<td>RG\textsubscript{10dB}</td>
</tr>
<tr>
<td>Proposed</td>
<td>1.9014 ± 0.2651</td>
<td>2.0685 ± 0.5304</td>
</tr>
<tr>
<td>Proposed +</td>
<td>2.2110 ± 0.3047</td>
<td>2.3380 ± 0.6507</td>
</tr>
<tr>
<td>HardThres</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 1 tabulates the resolution gains of the proposed algorithm under different configurations such as that with and without an additional hard threshold decision operator, and also studies the situation if cycle-spinning technique\textsuperscript{7,14} is employed. As shown in the tabulated results, if the deconvolved image is further denoised by the hard thresholding operator will improve the result. This is also observed by Kalifa et al.\textsuperscript{7} Visual examination of
the deconvolved image in Figure 4 confirms that the size of the speckle is reduced and small details of the tissue structures are now sharper. Moreover, the blurring in the far-field of Figure 4(a) is corrected in Figure 4(b).

![Figure 4](image)

**Figure 4.** Deconvolution of the real ultrasound image using blind mirror wavelet transform. (a) Original, and (b) the proposed algorithm with an additional hard threshold.

6. CONCLUSION

In conclusion, we have proposed an ultrasound image deconvolution algorithm where the RF images are deblurred by inverse filtering with the estimated 2-D PSF and the noise removal is accomplished through the use of a thresholding estimator in a newly designed symmetrical mirror wavelet basis dedicated for ultrasound imaging. Our experiments have shown that the proposed method can improve the ultrasound image quality in terms of the resolution gain. It has also shown visually that the size of the speckle is reduced and the visibility of the ultrasound image has improved in the deconvolved image.

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