

## Further Evidence for Cosmological Evolution of the Fine Structure Constant

J. K. Webb,<sup>1</sup> M. T. Murphy,<sup>1</sup> V. V. Flambaum,<sup>1</sup> V. A. Dzuba,<sup>1</sup> J. D. Barrow,<sup>2</sup> C. W. Churchill,<sup>3</sup>  
J. X. Prochaska,<sup>4</sup> and A. M. Wolfe<sup>5</sup>

<sup>1</sup>*School of Physics, University of New South Wales, Sydney, NSW 2052, Australia*

<sup>2</sup>*DAMTP, Centre for Mathematical Sciences, Wilberforce Road, Cambridge University, Cambridge CB3 0WA, United Kingdom*

<sup>3</sup>*Department of Astronomy & Astrophysics, Pennsylvania State University, University Park, Pennsylvania 16802*

<sup>4</sup>*Carnegie Observatories, 813 Santa Barbara Street, Pasadena, California 91101*

<sup>5</sup>*Department of Physics and Center for Astrophysics and Space Sciences, University of California, San Diego, C-0424, La Jolla, California 920923*

(Received 29 December 2000; published 9 August 2001)

We describe the results of a search for time variability of the fine structure constant  $\alpha$  using absorption systems in the spectra of distant quasars. Three large optical data sets and two 21 cm and mm absorption systems provide four *independent* samples, spanning  $\sim 23\%$  to  $87\%$  of the age of the universe. Each sample yields a smaller  $\alpha$  in the past and the optical sample shows a  $4\sigma$  deviation:  $\Delta\alpha/\alpha = -0.72 \pm 0.18 \times 10^{-5}$  over the redshift range  $0.5 < z < 3.5$ . We find no systematic effects which can explain our results. The only potentially significant systematic effects push  $\Delta\alpha/\alpha$  towards *positive* values; i.e., our results would become more significant were we to correct for them.

DOI: 10.1103/PhysRevLett.87.091301

PACS numbers: 98.80.Es, 06.20.Jr, 95.30.Dr, 95.30.Sf

A common property of unified theories, applied to cosmology, is that they allow space and time dependence of the coupling constants [1]. Spectroscopy of gas clouds which intersect the sight lines to distant quasars provide stringent constraints on variation of the fine structure constant  $\alpha \equiv e^2/\hbar c$ . Observing quasars at a range of redshifts provides the substantial advantage of being able to probe  $\alpha$  over most of the history of the universe.

*The many-multiplet method.*—Variations in  $\alpha$  would cause detectable shifts in the rest wavelengths of redshifted UV resonance transitions seen in quasar absorption systems. For the relativistic fine structure splitting in alkali-type doublets, the separation between lines is proportional to  $\alpha^2$ , so small variations in the relative separation are proportional to  $\alpha$  [2]. The “alkali-doublet” (AD) method offers the advantage of being simple, but fails to exploit the available precision since it compares transitions with respect to the *same* ground state. In recent papers [3,4] we introduced a new technique, the “many-multiplet” (MM) method, which is far more sensitive than the AD method and which offers other important advantages. The MM method allows the simultaneous use of any combination of transitions from many multiplets, comparing transitions relative to *different* ground states. Simultaneously using species with widely differing atomic masses enhances the sensitivity because the difference between ground-state relativistic corrections can be large and even of opposite sign. The AD method also fails to fully exploit the available data since only a single doublet is analyzed at a time. Using several different species at the same time improves the statistics and, importantly, provides an invaluable means of minimizing systematic effects.

The dependence of the observed wave number,  $\omega_z$ , on  $\alpha$  is conveniently expressed as  $\omega_z = \omega_0 + q_1x + q_2y$  where  $x = [(\frac{\alpha_z}{\alpha_0})^2 - 1]$ ,  $y = [(\frac{\alpha_z}{\alpha_0})^4 - 1]$ ,  $\alpha_0$  is the

present day value, and  $\alpha_z$  is the value at the absorption redshift,  $z$ .  $q_1, q_2$  are coefficients which quantify the relativistic correction for a particular atomic mass and electron configuration. These coefficients have been calculated in [3,5,6] using accurate many-body theory methods. The accuracy of the laboratory wave numbers,  $\omega_0$ , dictates the precision of  $\omega_z$  and hence the constraints on  $\Delta\alpha/\alpha$ . New high precision laboratory measurements of many species have been carried out using Fourier transform spectrographs specifically for the purpose of searching for varying  $\alpha$  [7].

The first application of the MM method [4] used FeII, MgI, and MgII transitions in 30 absorption systems towards 17 quasars and yielded an order of magnitude gain over previous AD method constraints. The results suggest  $\alpha$  may have been smaller in the past:  $\Delta\alpha/\alpha = -1.09 \pm 0.36 \times 10^{-5}$  for  $0.5 < z < 1.6$ , where  $\Delta\alpha/\alpha = (\alpha_z - \alpha_0)/\alpha_0$ .

*The data.*—In the present work, we have reanalyzed our initial sample [4,8]. Small changes in the definitions of the spectral fitting regions and in the selection of systems mean we now have 28 Mg/FeII systems covering redshifts  $0.5 < z_{\text{abs}} < 1.8$ . The Mg  $q$  coefficients are small compared to those for FeII, so Mg can be thought to act as an “anchor” against which shifts in the FeII lines can be measured. This large difference between the  $q$  coefficients enabled the dramatic sensitivity increase compared to the AD method.

We include new data [9], also obtained using the HIRES echelle spectrograph on the Keck I telescope. The spectral resolution is  $\sim 7$  km/s for the entire data set and the signal-to-noise ratio per pixel is  $\sim 30$  for most of the spectra. This sample is dominated by 18 damped Lyman- $\alpha$  absorption systems covering redshifts  $1.8 < z_{\text{abs}} < 3.5$  towards 13 quasars but also includes three new Mg/FeII

absorption systems. Two further Keck/HIRES absorption systems are included [10,11]. The redshift range is on average higher than the data from [4,8], so different transitions are used to constrain  $\Delta\alpha/\alpha$ . The transitions used primarily involve multiplets of NiII, CrII, and ZnII. However, other transitions (MgI, MgII, AlII, AlIII, FeII) are also included. Al and Si play an analogous anchor role to Mg in the lower redshift sample.

There is an important contrast between the previous Mg/FeII measurements and these new ones: the NiII, CrII, and ZnII  $q$  coefficients vary not only in magnitude but also in sign. Some wavelengths thus shift in *opposite* directions for a given change in  $\alpha$ . This, and the greater difference between the  $q$  coefficients (compared to Mg/FeII), provides a further sensitivity gain. It also dilutes any possible systematic effects, especially any associated with wavelength calibration of the data (although careful tests already eliminate this as a source of significant error [12]). A summary of all  $q$  coefficients and all the wave numbers used in our analysis, which are related to the same reference calibration scale, is given in Tables 1 of [13,14].

A third large new optical data set is also included in the present analysis. This comprises 21 SiIV absorption doublets towards 13 quasar spectra [9].

HI 21 cm absorption lines can be compared with molecular transitions detected at mm wavelengths to constrain  $g_p\alpha^2$  ( $g_p$  is the proton  $g$  factor). We have re-analyzed the data from [15], including additional molecular absorption lines. This provides two new  $\Delta\alpha/\alpha$  estimates at  $z = 0.25$  and  $0.68$  (see [16]).

*Analysis details.*—The analysis methods used in the present work are as described in [4] apart from the following improvements.  $\Delta\alpha/\alpha$  is now explicitly included as a free parameter in a multiparameter fit. Previously we had varied  $\Delta\alpha/\alpha$  externally. The velocity width ( $b$  parameter) of an absorption line is related to the FWHM of the gaseous atomic velocity distribution by  $b = \text{FWHM}/1.66$ , and  $b^2 = \frac{2kT}{M} + b_{\text{turb}}^2$  for an ionic species with mass  $M$ . The first term describes the thermal component of the line broadening at kinetic temperature,  $T$ , and the second describes a possible turbulent motion.  $T$  and  $b_{\text{turb}}$  are also now included as free parameters, and are not degenerate when there are  $\geq 2$  species in a fit. Note that  $\Delta\alpha/\alpha$  and  $z$  are also not degenerate when there are  $\geq 2$  species in a fit. We have reanalyzed the MgII and FeII data reported in [4] using the modified method, and the two sets of results are statistically indistinguishable.

As in [4], to achieve optimal precision from the data, all physically related parameters ( $z$ 's and  $b$ 's) are tied in the  $\chi^2$  minimization. A single  $z$  parameter is used for different corresponding species. Parameter errors were estimated using the diagonal terms of the inverse of the Hessian matrix (i.e., the covariance matrix) at the best fit solution. Monte Carlo simulations verified the reliability of the errors derived in this way.

Rigorous consistency checks are imposed before a fit is statistically acceptable. The reduced  $\chi^2$  for each fit

must be  $\sim 1$ . Each fit is carried out in three different ways, first assuming thermal broadening (so  $b_{\text{turb}} = 0$ ), second assuming turbulent broadening (so  $\frac{2kT}{M} = 0$ ), and third treating  $b_{\text{turb}}$  and  $T$  as free parameters. Variations in  $\Delta\alpha/\alpha$  over the three fits must not exceed  $1\sigma$ . Only two fits out of the optical data set failed this test, which provides a simple robustness check on the derived velocity structure for each absorption complex. The final adopted value was that with the smallest reduced  $\chi^2$  (which was, as expected, in all but three cases, the third type of fit above).

*Results.*—We now have 72 individual estimates of  $\Delta\alpha/\alpha$  spanning a large redshift range, providing the most comprehensive constraints so far obtained. The seven solid circles (annotated “many-multiplet”) in Fig. 1 show the binned results for the reanalyzed absorption systems presented in [4] and the new points based on the higher redshift Ni/Cr/Zn data, a total of 49 points [13].

The hollow triangle (annotated “alkali-doublet”), illustrates the average result for the 21 SiIV alkali doublets [14]. Table I presents a summary of the results for each sample. The overall deviation from  $\Delta\alpha/\alpha = 0$  for the whole optical sample is significant at the  $4.1\sigma$  level.

The new results for the HI 21 cm and mm data are  $\Delta\alpha/\alpha = (-0.10 \pm 0.22) \times 10^{-5}$  at  $z = 0.25$  and  $\Delta\alpha/\alpha = (-0.08 \pm 0.27) \times 10^{-5}$  at  $z = 0.68$ , assuming, without justification, constant  $g_p$ . The error for each

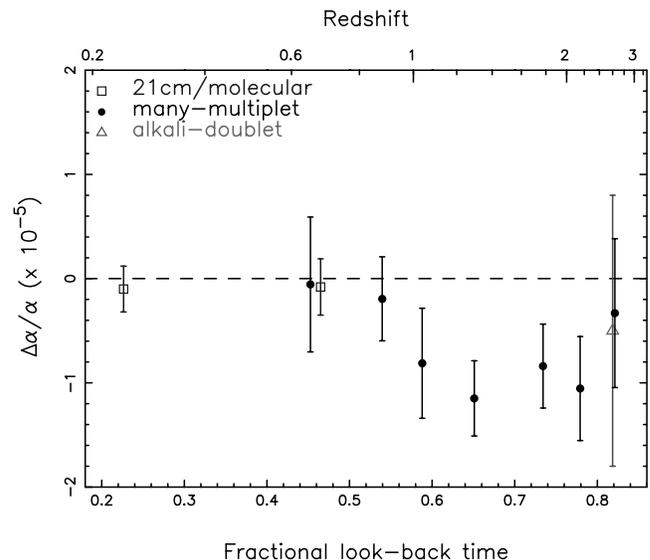


FIG. 1.  $\Delta\alpha/\alpha$  vs fractional look-back time to the big bang. The conversion between redshift and look-back time assumes  $H_0 = 68 \text{ km s}^{-1} \text{ Mpc}^{-1}$ ,  $(\Omega_M, \Omega_\Lambda) = (0.3, 0.7)$ , so that the age of the universe is 13.9 Gyr. A total of 72 quasar absorption systems contribute to this binned-data plot. The hollow squares correspond to two HI 21 cm and molecular absorption systems [16]. Those points assume no change in  $g_p$ , so should be interpreted with caution. The seven solid circles are binned results for 49 quasar absorption systems. The lower redshift points (below  $z \approx 1.6$ ) are based on (MgII/FeII) and the higher redshift points on (ZnII, CrII, NiII, AlIII, AlII, SiII) [13]. Of these 49 systems, 28 correspond to the sample used in [4]. The hollow triangle represents the average over 21 quasar SiIV absorption doublets using the alkali-doublet method [14].

TABLE I. Summary of results for four independent samples. Values of  $\Delta\alpha/\alpha$  are weighted means in units of  $10^{-5}$ . MM and AD indicate “many multiplet” and “alkali doublet.”  $N_{\text{abs}}$  is the number of absorption systems in each sample.

Sample	Method	$N_{\text{abs}}$	Redshift	$\Delta\alpha/\alpha$
FeII/MgII	MM	28	$0.5 < z < 1.8$	$-0.70 \pm 0.23$
NiII/CrII/ZnII	MM	21	$1.8 < z < 3.5$	$-0.76 \pm 0.28$
SiIV	AD	21	$2.0 < z < 3.0$	$-0.5 \pm 1.3$
21 cm and mm	radio	2	0.25, 0.68	$-0.10 \pm 0.17$

point includes a component of  $0.2 \times 10^{-5}$  to allow for possible spatial and velocity segregation of the HI and mm absorption. This could be due to slightly different lines of sight to the background quasar continuum (at such different wavelengths), or differences along the same line of sight, or both. A recent analysis [17] of the same two absorption systems adds a systematic error of  $1.7 \times 10^{-5}$ . Our value is derived empirically using measurements of the galactic interstellar medium (see Fig. 2 of [15]).

*Potential systematic errors.*—We have carried out a comprehensive search for any systematic effects [12] which could potentially cause the result we report. These include laboratory wavelengths errors, heliocentric velocity variation during a quasar integration, isotopic saturation and abundance variation, hyperfine structure, magnetic fields, kinematic effects, wavelength calibration and air-vacuum wavelength conversion errors, temperature variations during the observations, line blending, atmospheric dispersion effects, and variations in the intrinsic instrumental profile. None of these are able to explain our result. For example, kinematic effects (due to velocity segregation for different species) could introduce a scatter in  $\Delta\alpha/\alpha$  greater than the statistical error bars, which is not seen. Only *two* potentially significant systematic effects were identified: atmospheric dispersion and isotopic abundance evolution. If the spectrograph slit is not parallel to the atmospheric dispersion direction (i.e., is not perpendicular to the horizon), differential dispersion will place the quasar light at different slit positions, depending on wavelength. In fact, this effect turns out to push  $\Delta\alpha/\alpha$  to more *positive* values for each of the three optical samples. If we apply a *maximum* correction, on a case-by-case basis for the actual spectrograph slit angle, the result for the MM sample as a whole would become  $\Delta\alpha/\alpha = (-1.19 \pm 0.17) \times 10^{-5}$ .

Quasar absorption system abundances are generally below solar values [8,9], so isotopic abundance ratios may differ from terrestrial values. Therefore, the centroid wavelengths for each rest-frame transition (from laboratory measurements) may not be quite correct. Observations [18] and theoretical estimates [19] allow us to estimate the importance of this [12]. To do this we remove all weaker isotopes in all relevant species and refit the entire sample, deriving a new set of  $\Delta\alpha/\alpha$ . Again, we find that this effect would push  $\Delta\alpha/\alpha$  to more *positive* values for each of the three optical samples. If

we were to apply a correction, we would obtain  $\Delta\alpha/\alpha = (-0.96 \pm 0.17) \times 10^{-5}$  for the whole MM sample.

To summarize the above: (i) a thorough investigation reveals no systematic effect which can produce our results, and (ii) applying either of the two significant corrections would *enhance* the significance of our results. The results we quote in Table I are not corrected for these systematic effects.

*Other constraints.*—Constraints on  $\alpha$  variation come from a variety of independent sources. Laboratory measurements made over a 140 day period [20] yield  $|\dot{\alpha}/\alpha| \leq 3.7 \times 10^{-14} \text{ yr}^{-1}$ . Another terrestrial constraint comes from the Oklo natural uranium fission reactor [21], active  $\sim 1.8 \times 10^9$  years ago (corresponding to a “redshift” of  $z \approx 0.1$ ). Recent analyses [22,23] suggest  $\Delta\alpha/\alpha = (-0.4 \pm 1.4) \times 10^{-8}$  (although a second, significantly nonzero solution is also permitted adopting a different Sm resonance level shift). The limit above (favored by [23]) is well below our detection. The discrepancy is easily removed for a nonlinear time evolution in  $\Delta\alpha/\alpha$  since the quasar data probe an earlier epoch. Note that Fig. 1 shows that our data are consistent with no variation for  $z \lesssim 1$ . One may also interpret the combination of the Oklo and quasar results as the absence of temporal variation and the existence of spatial variation of  $\alpha$ . Also, unlike the optical quasar data, the Oklo data do not constrain  $\Delta\alpha/\alpha$  directly, but constrain  $e^2/r_0 \sim \alpha m_\pi c^2$  ( $r_0$  is the nucleon-nucleon separation, and  $m_\pi$  is the  $\pi$ -meson mass). Even then, this relies on the unjustified assumption that the strong interaction and nucleon kinetic energies are constant. The Oklo result is thus not as “clean” as the quasar results and a reliable interpretation of the apparent discrepancy requires further work.

Interesting limits can be obtained by comparing the hyperfine 21 cm HI transition with optical atomic transitions in the same gas cloud. Defining  $X = \alpha^2 g_p m_e / m_p$  ( $m_e/m_p$  is the ratio of electron and proton masses), a  $z_{\text{abs}} = 1.8$  gas cloud provides a limit of  $\Delta X/X = 0.7 \pm 1.1 \times 10^{-5}$  (95% confidence limit) [24]. Comparison with our result constrains any variation of  $W = g_p m_e / m_p$  and would give a new result of  $\Delta W/W = 2.1 \pm 0.7 \times 10^{-5}$  (68% limits). However, the error quoted in [24] on  $\Delta X/X$  does not include any component associated with spatial and velocity segregation, which is very likely to be present when comparing transitions at widely different frequencies, and will be important for a single

measurement. The true error on  $\Delta W/W$  is therefore probably significantly larger than this.

The cosmic microwave background (CMB) probes  $z \sim 1000$ , within  $\sim 10^6$  years of the big bang. Future experiments [25] may reach  $\Delta\alpha/\alpha \sim 10^{-2}-10^{-3}$  [26], although degeneracy with any electron mass change may reduce this [27]. The light element abundances constrain the scale lengths of additional dimensions at the time of primordial nucleosynthesis ( $z \sim 10^8-10^9$ , a few seconds after the big bang). The  ${}^4\text{He}$  yield is sensitive to the (uncertain) electromagnetic contribution to the neutron-proton mass difference [28]. This problem is avoided for heavier elements, and a recent analysis [29] yields  $|\Delta\alpha/\alpha| < 2 \times 10^{-2}$ .

Interestingly, independent results are now emerging which support the trend in  $\Delta\alpha/\alpha$  we find. The most recent CMB data are consistent with  $\alpha$  being smaller in the past by a few percent [30]. Also, varying speed of light models [31] are appealing because they may explain the supernovae results for a nonzero cosmological constant and solve other cosmological problems (e.g., the horizon, flatness, monopole problems) [32]. These also require a smaller  $\alpha$  in the past. We anticipate that further independent quasar data will provide a definitive check on our results.

We thank Juliet Pickering and Anne Thorne for crucial experimental work, and Ulf Griesmann, Sverner Johansson, Rainer Kling, Ulf Litzén for similar measurements. J. K. W. and M. T. M. thank the IoA for hospitality, where some of this work was done. J. D. B. acknowledges support from PPARC, the Royal Society, and the UNSW Gordon Godfrey fund. A. M. W. acknowledges partial support from NSF Grant No. AST0071257. We thank the John Templeton Foundation for support.

---

[1] P. Forgács and Z. Horváth, *Gen. Relativ. Gravit.* **11**, 205 (1979); W. Marciano, *Phys. Rev. Lett.* **52**, 489 (1984); J. D. Barrow, *Phys. Rev. D* **35**, 1805 (1987); T. Damour and A. M. Polyakov, *Nucl. Phys.* **B423**, 532 (1994); Y. Fujii, M. Omote, and T. Nishioka, *Prog. Theor. Phys.* **92**, 521 (1994); L.-X. Li and J. R. Gott III, *Phys. Rev. D* **58**, 103513 (1998).

[2] J. N. Bahcall, W. L. W. Sargent, and M. Schmidt, *Astrophys. J.* **149**, L11 (1967); D. A. Varshalovich, V. E. Panchuk, and A. V. Ivanchik, *Astron. Lett.* **22**, 6 (1996).

[3] V. A. Dzuba, V. V. Flambaum, and J. K. Webb, *Phys. Rev. Lett.* **82**, 888 (1999).

[4] J. K. Webb, V. V. Flambaum, C. W. Churchill, M. J. Drinkwater, and J. D. Barrow, *Phys. Rev. Lett.* **82**, 884 (1999).

[5] V. A. Dzuba, V. V. Flambaum, and J. K. Webb, *Phys. Rev. A* **59**, 230 (1999).

[6] V. A. Dzuba, V. V. Flambaum, M. T. Murphy, and J. K. Webb, *Phys. Rev. A* **63**, 042509 (2001).

[7] J. C. Pickering, A. P. Thorne, and J. K. Webb, *Mon. Not. R. Astron. Soc.* **300**, 131 (1998); J. C. Pickering, A. P. Thorne, J. E. Murray, U. Litzén, S. Johansson, V. Zilio, and J. K. Webb, *Mon. Not. R. Astron. Soc.* **319**, 163 (2000);

U. Griesmann and R. Kling, *Astrophys. J.* **536**, L113 (2000).

[8] C. W. Churchill, Lick Technical Report No. 74, 1995; Ph.D. thesis, UC Santa Cruz, 1997; C. W. Churchill, J. R. Rigby, J. C. Charlton, and S. S. Vogt, *Astrophys. J. Suppl.* **120**, 51 (1999).

[9] J. X. Prochaska and A. M. Wolfe, *Astrophys. J.* **470**, 403 (1996); **474**, 140 (1997); *Astrophys. J. Suppl.* **121**, 369 (1999); *Astrophys. J.* **533**, L5 (2000).

[10] P. J. Outram, F. H. Chaffee, and R. F. Carswell, *Mon. Not. R. Astron. Soc.* **310**, 289 (1999).

[11] L. Lu, W. L. W. Sargent, D. S. Womble, and M. Takada-Hidai, *Astrophys. J.* **472**, 509 (1996).

[12] M. T. Murphy, J. K. Webb, V. V. Flambaum, C. W. Churchill, and J. X. Prochaska, astro-ph/0012420, 2000.

[13] M. T. Murphy, J. K. Webb, V. V. Flambaum, C. W. Churchill, J. X. Prochaska, and A. M. Wolfe, astro-ph/0012419.

[14] M. T. Murphy, J. K. Webb, V. V. Flambaum, J. X. Prochaska, and A. M. Wolfe, astro-ph/0012421 [*Mon. Not. R. Astron. Soc.* (to be published)].

[15] M. J. Drinkwater, J. K. Webb, J. D. Barrow, and V. V. Flambaum, *Mon. Not. R. Astron. Soc.* **295**, 457 (1998).

[16] M. T. Murphy, J. K. Webb, V. V. Flambaum, M. J. Drinkwater, F. Combes, and T. Wiklind, astro-ph/0101519.

[17] C. L. Carilli, K. M. Menten, J. T. Stocke, E. Perlman, R. Vermeulen, F. Briggs, A. G. de Bruyn, J. Conway, and C. P. Moore, *Phys. Rev. Lett.* **85**, 5511 (2000).

[18] P. L. Gay and D. L. Lambert, *Astrophys. J.* **533**, 260 (2000).

[19] F. X. Timmes and D. D. Clayton, *Astrophys. J.* **472**, 723 (1996); F. X. Timmes, S. E. Woosley, and T. A. Weaver, *Astrophys. J. Suppl.* **98**, 617 (1995).

[20] J. D. Prestage, R. L. Tjoelker, and L. Maleki, *Phys. Rev. Lett.* **74**, 3511 (1995).

[21] A. I. Shylakhter, *Nature (London)* **264**, 340 (1976); ATOMKI Report No. A/I, 1983.

[22] T. Damour and F. J. Dyson, *Nucl. Phys.* **B480**, 37 (1996).

[23] Y. Fujii *et al.*, *Nucl. Phys.* **B573**, 377 (2000).

[24] L. L. Cowie and A. Songalia, *Astrophys. J.* **453**, 596 (1995).

[25] P. de Bernardis *et al.*, *Nature (London)* **404**, 955 (2000); S. Hanany *et al.*, astro-ph/0005123, 2000.

[26] M. Kaplinghat, R. J. Scherrer, and M. S. Turner, *Phys. Rev. D* **60**, 023516 (1999); S. Hannestad, *Phys. Rev. D* **60**, 023515 (1999).

[27] J. Kujat and R. J. Scherrer, *Phys. Rev. D* **62**, 023510 (2000).

[28] E. W. Kolb, M. J. Perry, and T. P. Walker, *Phys. Rev. D* **33**, 869 (1986).

[29] L. Bergström, S. Iguri, and H. Rubinstein, *Phys. Rev. D* **60**, 045005 (1999).

[30] P. P. Avelino, C. J. A. P. Martins, G. Rocha, and P. Viana, *Phys. Rev. D* **62**, 123508 (2000); R. A. Battye, R. Crittenden, and J. Weller, *Phys. Rev. D* **63**, 043505 (2001).

[31] J. W. Moffat, *Int. J. Mod. Phys. D* **2**, 351-365 (1993); J. D. Barrow and J. Magueijo, *Classical Quantum Gravity* **16**, 1435 (1999).

[32] J. D. Barrow and J. Magueijo, *Astrophys. J.* **532**, L87 (1999); A. Albrecht and J. Magueijo, *Phys. Rev. D* **59**, 043516 (1999); J. D. Barrow and J. Magueijo, *Phys. Lett. B* **443**, 104 (1998).