The noise properties of 42 millisecond pulsars from the European Pulsar Timing Array and their impact on gravitational-wave searches


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ABSTRACT

The sensitivity of Pulsar Timing Arrays to gravitational waves (GWs) depends on the noise present in the individual pulsar timing data. Noise may be either intrinsic or extrinsic to the pulsar. Intrinsic sources of noise will include rotational instabilities, for example. Extrinsic sources of noise include contributions from physical processes which are not sufficiently well modelled, for example, dispersion and scattering effects, analysis errors and instrumental instabilities. We present the results from a noise analysis for 42 millisecond pulsars (MSPs) observed with the European Pulsar Timing Array. For characterizing the low-frequency, stochastic and achromatic noise component, or ‘timing noise’, we employ two methods, based on Bayesian and frequentist statistics. For 25 MSPs, we achieve statistically significant measurements of their timing noise parameters and find that the two methods give consistent results. For the remaining 17 MSPs, we place upper limits on the timing noise amplitude at the 95 per cent confidence level. We additionally place an upper limit on the contribution to the pulsar noise budget from errors in the reference terrestrial time standards (below 1 per cent), and we find evidence for a noise component which is present only in the data of one of the four used telescopes. Finally, we estimate that the timing noise of individual pulsars reduces the sensitivity of this data set to an isotropic, stochastic GW background by a factor of >9.1 and by a factor of >2.3 for continuous GWs from resolvable, inspiralling supermassive black hole binaries with circular orbits.

Key words: gravitational waves – methods: data analysis – pulsars: general.

1 INTRODUCTION

Over the past decades, pulsar astronomy has been instrumental in the experimental tests of general relativity (GR) and alternative theories of gravity. Some of the most notable highlights from this research field include the first evidence of the existence of gravitational waves (GW; Taylor & Weisberg 1989), the most precise tests of GR (Kramer et al. 2006b), as well as tests of alternative theories of gravity, such as tensor–scalar gravity, in the quasi-stationary, strong-field regime (see e.g. Freire et al. 2012; Shao et al. 2013). These results rely on the pulsar timing technique (e.g. Lorimer & Kramer 2005), which fits the precisely recorded times-of-arrival (TOAs) of the pulses with a model of the pulsar’s rotational, astrometric and orbital parameters, as well as signal propagation delays induced by the ionized interstellar medium between the pulsar and Earth. The differences between the observed TOAs and those predicted by the model are called the timing residuals and contain the effects of any unmodelled physical or instrumental processes.

One of the applications of pulsar timing is the possibility of direct detection of GWs via the precise timing of an ensemble of pulsars, commonly referred to as a Pulsar Timing Array (PTA; Foster &
Backer 1990). The expected effects of GW propagation on the TOAs were first examined by Sazhin (1978). Later, the idea of using a PTA for unambiguous direct detection of low-frequency (nHz regime) GWs based on the predicted cross-correlation of the residuals of pulsars in various sky positions was proposed by Hellings & Downs (1983). Subsequent work has identified the potential of modern timing data for detecting nHz GWs and formulated the detection methodologies (e.g. Jenet et al. 2004, 2005; Sanidas, Battye & Stappers 2012).

PTAs are sensitive to the stochastic GW background (GWB) resulting from the incoherent superposition of the GW signals from the cosmos population of unresolved inspiralling supermassive black hole binaries (SMBHBs; e.g. Rajagopal & Romani 1995), continuous GWs (CGWs) from individual, resolvable SMBHB systems (e.g. Estabrook & Wahlquist 1975), the GWB created from the decaying loops of a cosmic string network that may have formed in the early Universe (e.g. Kibble 1976), a cosmological relic GW from the Universe’s inflationary era (e.g. Grishchuk 2005) and the memory term (long-term change in the GW’s amplitude) from GW bursts from SMBHB mergers (e.g. Favata 2009). Prior to the detection, upper limits on the GW amplitudes can impose limits on the properties of the cosmic SMBHB population (e.g. Shannon et al. 2015), and rule out the presence of nearby SMBHBs proposed by independent observations (Jenet et al. 2004). In the era of GW astronomy, PTAs using future, hypersensitive telescopes will also be able to test theories of gravity in the radiative regime. The GW polarization modes predicted by GR or alternative theories result in different cross-correlations of the pulsar timing residuals (e.g. Chamberlin & Siemens 2012). These cross-correlations can be further modified if the graviton is not massless as predicted by GR (e.g. Lee 2013).

The pursuit of GW detection using pulsar timing is coordinated by three consortia: the European Pulsar Timing Array (EPTA; Kramer & Champion 2013) in Europe, the North-American Nanohertz Observatory for Gravitational Waves (NANOGrav; McLaughlin 2013) in North America and the Parkes Pulsar Timing Array (PTA; Hobbs 2013) in Australia. The PTAs employ in total eight large single-dish radio telescopes. The EPTA uses five telescopes, namely the Effelsberg Radio Telescope (EFF), the Nançay Radio Telescope (NRT), the Lovell Telescope (JBO), the Westerbork Synthesis Radio Telescope (WSRT) and the Sardinia Radio Telescope. NANOGrav uses two telescopes, the Green Bank Telescope and the Arecibo Radio Telescope, while the PPTA uses the Parkes Radio Telescope. The three consortia cooperate under the International Pulsar Timing Array (IPTA) consortium, maximizing the observing efficiency and data set sensitivity.

The sensitivity of a given PTA is mainly limited by the uncertainties of the TOA measurements, the number of observations and the data time-span, the number of pulsars, their sky distribution and the presence of low-frequency noise in the data (see e.g. Lee et al. 2012; Siemens et al. 2013). While improvements in the instrumentation, increase of the allocated telescope time to PTAs and discoveries of new pulsars can address the first three factors, low-frequency noise needs to be characterized and understood on a pulsar-by-pulsar basis.

A number of methods have been developed to mitigate the dominant sources of noise in pulsar timing. Temporal variations in the dispersion measure (DM; integrated free electron density of the interstellar medium) along the line of sight to the pulsar are a primary source of low-frequency stochastic noise. DM time delays, however, depend on the observing frequency, ν, as ΔνDM ∝ DMν−2, and therefore DM variations can be, to a large degree, corrected using multifrequency data (e.g. Keith et al. 2013; Lee et al. 2014). Improper calibration of the gain of the two receiver feeds or cross-coupling between the two feeds can potentially lead to distortions of the total intensity profiles. These instrumental artefacts will introduce additional non-stationary noise components in the timing residuals (van Straten & Bailes 2003; van Straten 2006). By performing standard calibration observations during every observing run, we can minimize the presence of such noise in the data (e.g. Britton 2000). By comparing the noise properties of the same pulsars using overlapping data from different telescopes, uncorrected noise from instrumental instabilities can potentially be identified (Lentati et al. submitted).

Unfortunately, pulsar timing data also exhibit some levels of ‘timing noise’ (TN), low-frequency, stochastic, achromatic noise, the physical origin of which is unknown and, as such, cannot be mitigated. TN is primarily thought to be caused by pulsar rotational instabilities from various mechanisms. One approach is to consider simultaneous random walks and discrete jumps (caused, e.g. by microglitches) in the pulsar’s spin frequency and the spin-down rate (e.g. Cordes & Downs 1985; D’Alessandro et al. 1995; Shannon & Cordes 2010). Based on observational evidence, it is also suggested that TN can result from accumulated periodic and quasi-periodic changes in the spin-down rates due to magnetospheric state switching (Kramer et al. 2006a; Lyne et al. 2010). In addition, intrinsic noise also has been proposed to be the result of undetected (and therefore unmodelled) bodies in orbit, such as asteroid belts (Shannon et al. 2013) or planetary-mass objects in long, decadal orbits (Thorsett et al. 1999). Clearly, the measured TN in pulsar timing data can be a superposition of noise intrinsic to the pulsar, and any of the above non-intrinsic noise which is not properly mitigated, e.g. noise by DM variations not properly corrected due to the lack of sufficient multifrequency data.

While young pulsars show large amounts of low-frequency noise, millisecond pulsars (MSPs), typically show very low levels of such noise (Verbiest et al. 2009). It is theorized that MSPs have spun-up to the observed ms-order rotational periods via mass transfer from their companions during the system’s evolution (e.g. Alpar et al. 1982). Their highly stable rotations, short periods and absence of significant temporal changes in their pulse profile shapes (see e.g. Shao et al. 2013) make them excellent celestial clocks which can be timed to sub-100 ns precision over decades. MSPs are therefore the observed sources for GW-detection experiments, and indeed for all high-precision pulsar timing applications.

Despite their demonstrated rotational stability, some MSPs show significant amounts of TN. While their TN is considerably weaker than that of non-recycled pulsars, it can be significant enough to hinder GW detection. PSR B1937+21 (J1939+2134), the first ever discovered MSP, is a notable example of an MSP with strong TN (Kaspi, Taylor & Ryba 1994; Shannon et al. 2013). Other MSPs show more moderate noise levels, comparable to the predicted strength of the targeted GWs signals (e.g. PSR J1713+0747; see Zhu et al. 2015). The characterization of TN is therefore of central importance in high-precision pulsar timing applications.

The measured TN will also contain signals from spatially correlated low-frequency noise (e.g. Tiburzi et al. 2016). Primary examples are the long sought-after stochastic GWB, the signal caused by errors in the reference terrestrial time standards (see e.g. Hobbs et al. 2012) and errors in the Solar system ephemeris (see Champion et al. 2010). These signals can be distinguished by the spatial cross-correlations they induce on the timing residuals. The GWB induces a quadrupole signature (see Section 7.1). Errors in the
terrestrial time standards produce a fully correlated signal in all
pulsars (see Section 6), while errors in the Solar system ephemeris
can potentially produce a superposition of dipolar correlations be-
tween pulsars, each produced by the error in the predicted location
of a Solar system body. PTAs allow such correlated signals to be
recovered or put upper limits on their power.

Different methods have been proposed and employed to char-
acterize the statistical properties of TN in pulsar data and to per-
form pulsar timing analysis in the presence of correlated noise.
These cover techniques based on frequentist (Matsakis, Taylor &
Eubanks 1997; Coles et al. 2011) and Bayesian statistics (e.g. van
Haasteren et al. 2009; Lentati et al. 2014), both in the time- and
frequency-domain. As part of the efforts to detect GWs, an increas-
ing number of algorithms are being used by the various PTAs to
determine the TN properties of MSPs, motivating work to examine
the possible biases inherent to different methods. In this con-
text, we perform characterization of the TN using two established
methods based on different statistical analyses, Bayesian and fre-
quenstist, and make a comparison of their performance and results.

We subsequently use the measured TN properties to search for
the presence of TN unique to specific observing systems, place an
upper limit on the contribution of clock errors to the measured noise
and investigate the impact of the TN on the data set’s sensitivity to
GWs.

This paper is organized as follows. In Section 2, we describe
the data we use. In Section 3, we present the methods used to
calculate the noise parameters. The results from both methods are
presented in Section 4. In Section 5, we check for TN present only in individual
data subsets and continue to investigate systematics by making a
search for a correlated clock error signal in Section 6. In Section
7, we evaluate the effects of the TN present in our data on their
sensitivity to GWBs and CGWs and finally discuss our conclusions
in Section 8.

2 THE EPTA DATA RELEASE 1.0

We use the EPTA Data Release 1.0 that is presented in Desvignes
et al. (submitted; henceforth D15). The data set is composed of data
recorded with four EPTA radio telescopes: The Effelsberg in Germany,
the NRT in France, the WSRT in the Netherlands and the JBO
in the United Kingdom. The data-recording systems (backends)
used are the Effelsberg–Berkeley Pulsar Processor (EBPP), the
Berkeley–Orléans–Nançay (BON), the Pulsar Machine I (PuMaI)
and the Digital Filterbank (DFB), respectively. A more detailed
description of the instruments and data reduction techniques can
be found in D15, where the timing solutions of the pulsars are also
presented.

The data set includes TOAs from 42 MSPs. Their key proper-
ties are summarized in Table 1. We identify observing systems as
unique combinations of telescope, backend and central observing
frequency (receiver). In total, the data set has 18 distinct systems.
The EBPP L-band\(^1\) data have the longest time-span, with a maxi-
mum of 18 yr, starting from 1996 October, divided into two observing
systems, due to a change in the receiver in 2009. For most of
the sources with EBPP data, all other instruments started recording
from 2007 onwards, dividing our longest pulsar data sets into two
subsets: the first, with single-telescope, single-frequency data and
the second, with multitelescope, multifrequency data. The lack of
multifrequency data in the first half of the data set makes direct

\(^{1}\) 1–2 GHz range in centre frequency.
3 METHODS FOR ESTIMATING NOISE PROPERTIES

For the estimation of the noise properties, we use two different methods. The first method follows a Bayesian approach, in the time-frequency domain and is described in Lentati et al. (2014). The second method uses frequentist statistics based on power-spectral estimation of the residuals and using algorithms described in Section 3.3, which are an extension of those introduced in Coles et al. (2011). We first discuss the noise model components, which we use for both approaches, and then present the details of each method used.

3.1 Noise modelling

We form the timing residuals using the pulsar timing analysis package TEMPO2 (Hobbs, Edwards & Manchester 2006), which iteratively performs a weighted least-squares (WLS) fit of the model to the TOAs until the reduced chi-squared of the residuals is minimized. Timing models are gradually improved over many years by incorporating more data. These solutions will often result in timing residuals scattered beyond what would be expected based on their formal uncertainties, due to the absence, at this point, of the stochastic signals in the model. These signals are in general divided into the time-correlated and uncorrelated components.

The uncorrelated (white-noise) components correct the uncertainties of the timing residuals. The formal uncertainties of the TOAs are derived by the cross-correlation of the recorded integrated pulse profile with a reference template, which is constructed using the best available observations. These uncertainties are correct if the recorded profiles are characterized solely by (white) radiometer noise and the profile template precisely represents the intrinsic shape of the integrated profile. However, possible presence of un-excised radio frequency interference (RFI), temporal variations in the pulse profile, artefacts in the profiles from instrumental instabilities or imperfect profile templates can lead to errors in the uncertainty estimations (e.g. Liu et al. 2011). It is therefore common practice to include a multiplicative correction factor called error factor (EFAC). We also add a correction term quadratically to the formal uncertainty to account for additional scatter in the TOAs caused by statistically independent physical processes, such as pulse phase jitter noise (e.g. Shannon et al. 2014). This term is commonly referred to as error added in quadrature (EQUAD). We do not investigate the physical origin of the noise included in the EQUADs. This requires a more detailed analysis of the white noise; for example, jitter noise is dependent on the integration time of the observation and this needs to be properly taken into consideration if one wants the EQUAD number to describe an underlying physical process.

We include one EFAC and one EQUAD term per observing system to mathematically model the uncorrelated noise from all possible processes. The white-noise correction factors should be such that the data satisfy the central assumption of pulsar timing, that they are drawn from a random Gaussian process. In other words, when subtracting the waveforms (induced residuals) of all calculated stochastic signals from the residuals, their uncertainties should be such that the residuals are white and the timing solution has a reduced chi-squared of unity. The original TOA uncertainty, $\sigma$, EFAC ($f$), EQUAD ($q$) and corrected uncertainty, $\delta$, are related as

\[ \delta^2 = (\sigma^2 + f^2) + q^2. \]

We include two stationary time-correlated noise components, namely the chromatic low-frequency noise from DM variations and the achromatic TN. Previous studies (e.g. Shannon & Cordes 2010; Coles et al. 2011) have shown that the low-frequency power spectra of pulsar timing residuals can be adequately modelled with single power laws for the majority of MSPs. This does not mean that the TN is necessarily a pure power law, but rather that this functional form is sufficient to describe the data, given the measurement precision. We examined whether deviations from the single power-law model are supported by the data using the Bayesian analysis method. In particular, we performed the noise analysis with two additional models for the TN spectrum: (i) a model that allows the power of individual frequency bins to vary independently from the power-law model and (ii) a model that includes the power law and an additional sinusoid signal of varying frequency, amplitude and phase. We evaluated the results using the Bayes factor, i.e. the ratio of the Bayesian evidence of two competing models (see also Section 3.2). A common interpretation of the Bayes factor is given by Kass & Raftery (1995), based on which we required a value equal or greater than 3 to justify the addition of any extra model parameter. This was not the case for any of the models we compared to the simple single power-law model.

In this work, we have followed the single power-law formalism for both analysis methods in order to facilitate their comparison and the comparison of the measured TN parameters with those usually used as GW stochastic parameters in the PTA literature. For isotropic GW signals (see Section 7) one of the most important properties is the characteristic strain spectrum, $h_c$, of the GWB on the one-sided power spectrum of the induced timing residuals. For most models of interest, this can be written as a power-law function of the GW frequency (e.g. Jenet et al. 2005), $f$ as

\[ h_c(f) = A \left( \frac{f}{f_c} \right)^\alpha, \]

where $A$ is the (dimensionless) amplitude of the wave, $\alpha$ is the spectral index and $f_c$ is the reference frequency, typically set to $1 \text{ yr}^{-1}$. The one-sided power-spectral density of the signal is then given by

\[ S(f) = \frac{A^2}{12\pi^2} \left( \frac{f}{f_c} \right)^{-\gamma}, \]

where the power spectrum and strain spectral indices are related as $\gamma = 3 - 2\alpha$. This is the functional form we use to model the TN. We set a cut-off at frequency $1/T$, where $T$ is the time-span of the data. The cut-off arises naturally because the fitted pulsar’s spin and spin-down absorb the power from any achromatic low-frequency signal below the cut-off frequency. It has been shown (van Haasteren et al. 2009; Lee et al. 2012) that if the spectral index is $\gamma \lesssim 7$ (which is the case for all MSPs in this paper), the cut-off at frequency $1/T$ is sufficient.

The DM variations have been mitigated using first- and second-order DM derivatives in the timing model (which are first- and second-order polynomials) and additionally a power law equivalent to equation (3). The DM derivatives absorb any power from the stochastic DM component below the cut-off frequency, in the same way the spin and spin-down do for the achromatic TN (Lee et al. 2014). The observing frequency dependence of the DM variations signal is measured in the time-domain via the (multifrequency) timing residuals, as we show in Section 3.2. The choice of a power-law
3.2 Noise parameter estimation using Bayesian inference

The first Bayesian investigation of the GWB detectability with PTAs was performed by van Haasteren et al. (2009). The algorithms were later applied on EPTA data to derive the EPTA GWB upper limit (van Haasteren et al. 2011). In that analysis, the TN parameters of the MSPs were simultaneously estimated with the GWB parameters. Further work on Bayesian analysis methods for pulsar timing provided more algorithms, both in time- and time-frequency-domains, to characterize the properties of TN and DM variations and to perform robust pulsar timing analysis in the presence of correlated noise (e.g. van Haasteren & Levin 2013; Lentati et al. 2013; Lee et al. 2014).

Bayes’ theorem, which is the central equation for these analysis methods, states that

\[ P(\theta) = \frac{L(\theta)\pi(\theta)}{Z} , \]

where \( \theta \) is the model’s parameters, \( P(\theta) \) is the posterior probability distribution (PPD) of the parameters (probability distribution of the parameters given the model and the data), \( \pi(\theta) \) is the prior probability distribution of the parameters for a given model, \( L(\theta) \) is the likelihood function (which gives the probability that the data are described by a given model) and \( Z \) is the Bayesian evidence. Following Feroz & Hobson (2008), \( Z \) is only a normalizing factor independent of \( \theta \) and can therefore be ignored when one is interested only in parameter estimation, such that \( P(\theta) \propto L(\theta)\pi(\theta) \). On the other hand, when one is interested in model selection, the ratio of the evidence between two different models, \( R \), known as the Bayes factor, is used. The probability, \( P, \) of a model compared to another, can be expressed (Kass & Raftery 1995) as

\[ P = \frac{R}{1 + R} . \]

The various Bayesian analysis algorithms are distinguished by the mathematical description of the model parameters and the computational methods used to sample the unnormalized PPD.

Lentati et al. (2014) introduced TEMPO2, a Bayesian software package for the analysis of pulsar timing data, available to use as a TEMPO2 plug-in. The timing solution and the additional stochastic parameters such as EFACs, EQUADs, DM variations and the TN (referred to as ‘excess red noise’) can be determined simultaneously. TEMPO2 uses the Bayesian inference tool MULTINEST (Feroz & Hobson 2008) to explore this joint parameter space, whilst using TEMPO2 as an established means of evaluating the timing model at each point in that space. For the PPD sampling, TEMPO2 uses the nested sampling Monte Carlo method (Skilling 2004).

We perform a joint analysis for the timing model and the stochastic parameters. Both the TN and the DM variations are modelled as Gaussian stochastic signals with power-law spectra as described by equation (3). TEMPO2 employs the time-frequency analysis described in Lentati et al. (2013). The TN waveform is expressed as (here, and henceforth we use boldface characters in equations to denote matrices) \( \mathbf{t}_{\text{TN}} = \mathbf{F}_{\text{TN}} \mathbf{a} \), where \( \mathbf{F}_{\text{TN}} \) is the Fourier transform with elements \( \mathbf{F} = \sin(2\pi f) + \cos(2\pi f) \) and corresponding coefficients, \( \mathbf{a} \), which are free parameters. The Fourier frequencies take values \( f = n/T \), with \( n \) integers ranging from 1 up to the value necessary to sample frequencies as high as 1/14 d\(^{-1}\). The covariance matrix of the TN is then described by the following equation (see Lentati et al. 2015):

\[
\mathbf{C}_{\text{TN}} = \mathbf{C}_a^{-1} - \mathbf{C}_a^{-1} \mathbf{F}_{\text{TN}} \left[ (\mathbf{F}_{\text{TN}})\mathbf{T} \mathbf{C}_{\text{TN}} \mathbf{T} \mathbf{F}_{\text{TN}} + (\Psi)^{-1} \right]^{-1} (\mathbf{F}_{\text{TN}})^\mathbf{T} \mathbf{C}_a^{-1} .
\]

(6)

Here, \( \Psi = (\mathbf{a}, \mathbf{a}) \), is the covariance matrix of the Fourier coefficients and \( \mathbf{C}_a \) is the covariance matrix of the white-noise component, a diagonal matrix with the main diagonal populated by the residual uncertainties squared, \( \sigma^2 \) (as in equation 1). The superscript \( \mathbf{T} \) denotes the transpose of the matrix.

The covariance matrix for the DM variations, \( \mathbf{C}_{\text{DM}} \), is equivalent to equation (6), but including an observing frequency dependence. This is achieved by replacing the \( \mathbf{F} \) elements with \( \mathbf{F}_{\text{DM}} = \mathbf{F}_i \mathbf{D}_i \), where the \( i \) indices denote the residual numbers, \( \mathbf{D}_i = 1/(\nu_i, \nu_i^2) \), \( \nu_i \) is the observing frequency of the TOA, typically set as the central frequency of the observing band, and \( \nu = 2.41 \times 10^{-16} \text{ Hz}^-2 \text{ cm}^-3 \text{ pc}s^-1 \), is the dispersion constant.

The likelihood function is the probability that the data (TOAs), noted as \( \mathbf{t} \), are fully described by the timing model signal, \( \mathbf{r}(\mathbf{\epsilon}) \), with parameters \( \mathbf{\epsilon} \) and the stochastic noise. The latter is encoded in the residuals’ total covariance matrix, \( \mathbf{C} = \mathbf{C}_a + \mathbf{C}_{\text{DM}} + \mathbf{C}_{\text{TN}} \).

Following van Haasteren et al. (2009), and noting that the difference \( \mathbf{t} - \mathbf{r}(\mathbf{\epsilon}) \) is a vector of residuals, we can write the likelihood function as:

\[
L = \frac{1}{\sqrt{(2\pi)^n |\mathbf{C}|}} e^{-\frac{1}{2} (\mathbf{t} - \mathbf{r}(\mathbf{\epsilon}))^\mathbf{T} \mathbf{C}^{-1} (\mathbf{t} - \mathbf{r}(\mathbf{\epsilon}))} .
\]

(8)

After the noise properties are estimated, we produce the TN waveforms, which can be estimated from the data using the maximum likelihood (ML) value of its statistical model parameters, \( \mathbf{A} \) and \( \mathbf{\gamma} \). As shown in Lee et al. (2014), the ML waveform, \( \mathbf{t}_{\text{TN}} \), and its uncertainties, \( \sigma_{\text{TN}} \), are optimally estimated as

\[
\mathbf{t}_{\text{TN}} = \mathbf{C}_{\text{TN}} \mathbf{C}_a^{-1} \mathbf{t} ,
\]

(9)

with uncertainties estimated as

\[
\sigma_{\text{TN}} = \mathbf{C}_{\text{TN}} - \mathbf{C}_{\text{TN}} \mathbf{C}_a^{-1} \mathbf{C}_{\text{TN}}^\mathbf{T} .
\]

(10)

The uncertainties are estimated as the standard deviation of the estimator. However, as noted in Lee et al. (2014), since the data points of TN waveforms are correlated, their interpretation in terms of uncertainties is meaningless, since this is only valid under the assumption that the noise is uncorrelated. The uncertainties can therefore only be used as an indication of the variance of each point.
for the parameter value while for uniform pPDs, we assign the same probability for all values. The uninformative log-uniform pPDs will result in PPDs for the parameters that are the least affected by the pPD and therefore are what we consider as the parameter measurement. If no significant noise can be detected in the data, the PPDs are unconstrained and the distribution’s upper limit is dependent on the lower limit of the pPD. Therefore, a separate analysis is required using uniform pPDs in order to obtain robust upper limits. If the signal is strong and the result from a log-uniform pPD analysis is a well-constrained PPD, then the change of the pPD should not affect the result significantly and the PPDs should be almost identical. As a result, we performed the analysis with the following combinations of pPDs.

(a) Uniform EQUAD pPDs and log-uniform pPDs for TN and DM variation amplitudes. This set of pPDs results in upper limits for EQUADs. As such, the solutions have the highest possible timing residuals uncertainties, resulting in weaker TN and DM variations detections. The TN and the DM variations are treated in the same way, giving no prior information that can favour the one over the other when multifrequency data are not sufficient to de-couple them. In the absence of multifrequency data one can therefore expect that their PPDs will not be well constrained.

(b) Uniform TN amplitude and log-uniform pPDs for EQUADs and DM variation amplitudes: The total white-noise levels of these solutions are lower, since EQUAD PPDs can be flat if the data do not support them to be measurable. The use of uniform pPDs for the TN amplitude and log-uniform for the DM variations results in solutions in favour of the TN against the DM variations in the absence of multifrequency data. This set of pPDs will provide the strictest upper limits on the TN amplitudes. We used the PPDs from this analysis to calculate the amplitude upper limits at the 95 per cent confidence level (CL).

3.3 Noise parameter estimation using power-spectral analysis

Power-spectral analysis of pulsar timing data using standard discrete Fourier transforms is complicated by highly variable error bars, irregular sampling, data gaps (due to difficulties in being granted telescope time at exact regular intervals but also due to loss of data from technical difficulties, weather conditions, telescope maintenance or from weak pulses on particular days due to unfavourable interstellar scintillation) and the presence of TN which has a steeped spectrum. Fourier transforms require equispaced data points. Interpolation of data points on regular grids introduces time-correlations in data points and the presence of strong TN introduces spectral leakage. In order to bypass such problems, Coles et al. (2011) introduced an algorithm for pulsar timing analysis in the presence of correlated noise which employs the use of generalized least-squares (GLS) analysis of the timing data using the covariance matrix of the residuals (as described in Section 3.2). In brief, the covariance matrix of the residuals is used to perform a linear transformation that whitens both the residuals and the timing model. The transformation is based on the Cholesky decomposition of the covariance matrix.

For this algorithm, initial estimates of the residuals covariance matrix are necessary, and are obtained using the Lomb-Scargle periodogram (LSP), which can calculate the power spectrum of irregularly sampled data. Spectral leakage in the presence of strong TN with steep power-law spectra is mitigated with pre-whitening using the difference filter. The difference pre-whitening filter of any order, $k$, can be described by $y_{w,k} = y_{w,k-1}(t_i) - y_{w,k-1}(t_{i-1})$, where $t_i$ is the $i$th sampling time and $y_{w,k}$ is the whitened residual of difference order $k$ ($k = 0$ corresponds to the original residuals). It was suggested to use the lowest order necessary to whiten the data enough to mitigate spectral leakage. Effectively, this filter is equivalent to multiplying the power spectrum by a filter (e.g. for first-order difference, the filter is the square of the transfer function). After the spectrum is estimated using the pre-whitened data, one corrects the power spectrum by dividing it with the same filter, a process known as post-darkening. The low-frequency spectrum can be fitted with a power-law model leading to the first estimation of the covariance matrix. Through an iterative process, new estimates of the spectrum can be achieved by using LSP after whitening the data using the Cholesky decomposition of the covariance matrix.

Coles et al. (2011) have demonstrated that the implementation of this method allows better timing solutions with more robust timing parameters and uncertainty calculations. In particular, the measured spin and spin-down of the pulsar show the largest improvements, since they have low-frequency signatures in the Fourier domain and correlate with TN. However, this method is not optimized to accurately estimate the TN properties through detailed fitting of a noise model to the power spectrum. The algorithm described in Coles et al. (2011) focuses on obtaining a linear, unbiased estimator of the timing parameters. For this purpose, they demonstrate that using the GLS timing solutions using the covariance matrices of any TN models which whiten the data sufficiently to remove spectral leakage, are statistically consistent. In this work, we extend the algorithms of Coles et al. (2011), focusing on the precise evaluation of the power spectra and the power-law model parameters. To this end, we have developed an independent power-spectral analysis and model fitting code.

A fully frequentist analysis should include a white-noise and DM-correction analysis. However, in order to focus on comparing the methods with regards to the estimation of the TN properties, we use the ML EFAC and EQUAD values and subtract the ML DM-variations waveforms derived from the Bayesian analysis.

Our spectral analysis code calculates a generalized LSP; i.e. it performs a wLS fit of sine and cosine pairs at each frequency. We follow an iterative procedure as follows: (1) We first use TEMPO2 to obtain the wLS post-fit residuals, while subtracting the ML DM variations signal estimated with the Bayesian methods described in Section 3.2. (2) We calculate the spectrum of these residuals using a chi-squared minimization fit on all frequency points. (3) TEMPO2 is re-run using the covariance matrix of the initial noise model to perform a GLS fit. (4) Finally, we re-run the spectral analysis code on the residuals from the GLS timing solution to update the TN model and repeat steps 3 and 4 until the solution converges. Typically, this required no more than one iteration.

Our code implements a generalized LSP to account for the timing residual uncertainties. Denoting each pair of time and residual as $(t_i, y_i)$, the LSP is formed by fitting sine–cosine pairs of the form $\hat{y}(\omega, t_i) = a_i \cos(\omega t_i) + b_i \sin(\omega t_i)$ at all angular frequencies, $\omega_k = 2\pi f_k$, with $f_k$ the frequency. The solution is obtained by minimizing the chi-squared for each $\omega_k$, weighted by the summed uncertainties of the timing residuals as

$$x_k^2 = \sum_i \left( \frac{y_i - a \sin(\omega t_i) - b \cos(\omega t_i)}{\sigma_i} \right)^2.$$  \hspace{1cm} (11)

Once the LSP is calculated, noting the number of timing residuals as $N$, the spectral density is finally computed as

$$S(f) = \frac{2|y|^2 T}{N^2}.$$  \hspace{1cm} (12)
We examine whether spectral leakage is present following the same routine as in Coles et al. (2011). Visual inspection of the original spectrum allows us to approximately define the frequency where the red component of the spectrum intersects the flat, white component. We apply a low-pass filter in time-domain to separate the high-frequency from the low-frequency residuals and calculate their individual spectra. The high-frequency spectrum should be consistent with the high-frequency part of the spectrum of the original data. If that is not the case, and instead the high-frequency spectrum is significantly weaker, then leakage is important and we need to apply the pre-whitening filter. The code allows for any order of difference whitening. For this data set, we required only up to second order. We then proceed with calculating the LSP as before and finally post-darken the spectrum before calculating the final spectral density.

We fit the power spectrum with the following function:

\[ S(f) = S_0 \left( \frac{f}{f_0} \right)^{-\gamma} + S_W, \quad S_0 = \frac{A^2}{12 \pi^2}. \]  

(13)

Here, \( S_W \) is the spectral density of the high-frequency (white) component. The power-law description of the low-frequency component is equivalent to equation (3), with \( S_i \) the spectral density at reference frequency \( f_i \), which is set to \( 1 \text{yr}^{-1} \). A fit of only the low-frequency component is proven difficult; due to the steepness of the spectrum at low frequencies and moderate power of the TN in many MSPs, only about five frequency points would be included in a pure power-law fit of only the red part of the spectrum. This leads to unstable fits without meaningful error estimations.

The fit minimizes the chi-squared, \( \chi^2_f \). Chi-squared minimization assumes that the spectrum is normally distributed. In principle, the power spectrum is a chi-squared distribution. However, in logarithmic space, the distribution is approximately Gaussian with variance of order unity. Therefore this is a good approximation if we fit the power-law model to the spectrum in logarithmic space. By doing so, we minimize the chi-squared defined as

\[ \chi^2_f = \sum_{i=1}^{N} \left[ \log S_i - \log \left( S_0 \left( \frac{f_i}{f_0} \right)^{-\gamma} + S_W \right) \right]^2, \]  

(14)

where \( S_i \) and \( f_i \) define the points of the spectrum for each frequency bin, \( i \), while simultaneously fitting for \( S_0, \gamma \) and \( S_W \). We first fit the spectrum while setting the uncertainties of the LSP points to one and then scale the uncertainties to achieve a reduced chi-squared of order unity. Therefore this is a good approximation if we fit the power-law model to the spectrum in logarithmic space. By doing so, we minimize the chi-squared defined as

\[ \chi^2_f = \sum_{i=1}^{N} \left[ \log S_i - \log \left( S_0 \left( \frac{f_i}{f_0} \right)^{-\gamma} + S_W \right) \right]^2, \]  

(14)

where \( S_i \) and \( f_i \) define the points of the spectrum for each frequency bin, \( i \), while simultaneously fitting for \( S_0, \gamma \) and \( S_W \). We first fit the spectrum while setting the uncertainties of the LSP points to one and then scale the uncertainties to achieve a reduced chi-squared of unity.

Once we obtain the values for the noise parameters, we construct the covariance matrix of the TN, \( C_{\text{TN}} \). The Fourier transform of the TN power-law model gives the covariance function, \( c_{\text{TN}}(\tau) = (t_{\text{TN},i}, t_{\text{TN},j}) \). The \( i \) and \( j \) indices refer to the time epoch of the observation and \( \tau = t_i - t_j \). The TN covariance matrix is then formed by the elements \( C_{\text{TN},ij} = c(\tau_{ij}), \) where \( \tau_{ij} = |t_i - t_j| \). Using the total covariance matrix (equation 7), we then perform a TEMPO2 GLS fit on the TOAs, repeat the power-spectrum analysis and power-law fit to update the model parameter values and iterate these steps until we converge to a stable solution.

For the cases where the spectra are white-noise dominated and no measurement of the TN parameters can be achieved on a \( \sigma \) level, we derived upper limits for the TN amplitude. The limits are at the 95 per cent CL and are calculated as the \( 2\sigma \) upper limit of the white-noise level \( (S_W \text{ in equation 13 and Table 3}) \).

4 http://www. epta. eu/ aom/ DR1noise. html
Table 2. Timing-noise characteristics of EPTA MSPs based on Bayesian inference for a single power-law model as described by equation (3). The results are divided based on the quality of the PPDs as described in Section 4.1. We tabulate the ML and median (med) values of the dimensionless amplitude, A, at reference frequency of 1 yr\(^{-1}\) and the spectral index, γ. For A, we also tabulate the 95 per cent confidence upper limits. The 1σ uncertainties are calculated such that the 68 per cent of the area under the one-dimensional marginalized PPD of the parameter is symmetrically distributed around the median. As described in Section 4.1, for unconstrained PPDs we only consider the upper limits analysis results.

<table>
<thead>
<tr>
<th>J-Name</th>
<th>A_{ML}</th>
<th>A_{med}</th>
<th>A_{95}^{\gamma}</th>
<th>\gamma_{ML}</th>
<th>\gamma_{med}</th>
</tr>
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<tbody>
<tr>
<td>J0030+0451</td>
<td>−14.9</td>
<td>−14.9</td>
<td>12.0</td>
<td>6.3</td>
<td>5.8</td>
</tr>
<tr>
<td>J0218+4232</td>
<td>−13.1</td>
<td>−14.1</td>
<td>12.3</td>
<td>2.7</td>
<td>3.9</td>
</tr>
<tr>
<td>J0610–2100</td>
<td>−18.7</td>
<td>−16.0</td>
<td>12.4</td>
<td>1.4</td>
<td>2.7</td>
</tr>
<tr>
<td>J0613–0200</td>
<td>−13.7</td>
<td>−14.4</td>
<td>12.4</td>
<td>2.8</td>
<td>4.1</td>
</tr>
<tr>
<td>J0751+1807</td>
<td>−18.8</td>
<td>−15.9</td>
<td>12.4</td>
<td>6.5</td>
<td>3.0</td>
</tr>
<tr>
<td>J1024–0719</td>
<td>−14.0</td>
<td>−16.3</td>
<td>13.1</td>
<td>5.3</td>
<td>3.0</td>
</tr>
<tr>
<td>J1455–3330</td>
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<td>−14.2</td>
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<td>0.8</td>
<td>3.6</td>
</tr>
<tr>
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<td>−13.1</td>
<td>12.8</td>
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<tr>
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<td>−17.7</td>
<td>−13.3</td>
<td>12.8</td>
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<td>−11.7</td>
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<td>11.5</td>
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<td>1.9</td>
</tr>
<tr>
<td>J1730–2304</td>
<td>−12.8</td>
<td>−14.7</td>
<td>12.6</td>
<td>1.8</td>
<td>2.0</td>
</tr>
<tr>
<td>J1801–1417</td>
<td>−14.4</td>
<td>−15.1</td>
<td>12.2</td>
<td>6.3</td>
<td>3.3</td>
</tr>
<tr>
<td>J1802–2124</td>
<td>−17.0</td>
<td>−15.6</td>
<td>12.2</td>
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<tr>
<td>J1843–1113</td>
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<td>−12.9</td>
<td>12.5</td>
<td>0.6</td>
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</tr>
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<td>−14.1</td>
<td>13.8</td>
<td>2.4</td>
<td>2.4</td>
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<tr>
<td>J1918–0642</td>
<td>−16.9</td>
<td>−14.5</td>
<td>12.6</td>
<td>1.7</td>
<td>5.4</td>
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<td>J2145–0750</td>
<td>−14.4</td>
<td>−14.0</td>
<td>12.9</td>
<td>5.2</td>
<td>4.1</td>
</tr>
</tbody>
</table>

The comparison of the results derived with these two methods should also consider the effects of the Bayesian ML DM-variations waveform subtraction from the residuals before performing the power-spectral analysis. In the case of semi-constrained PPDs, the
amplitude parameters for the two TN and DM variations are naturally highly correlated. When this is the case, the ML parameter estimates are not as reliable, as the particular ML solution might correspond to either significant DM variations and no TN, or significant TN and no DM variations. This can lead to over- or underestimations of the DM variations which will lead to either part of the TN being subtracted as well or part of the DM signal leaking into the TN.

As an example, we show in the left-hand panel of Fig. 2 the two- and one-dimensional marginalized PPDs for the amplitudes of the TN and DM variations for PSR J0751+1807 (semi-constrained PPDs case). One can see the strong covariance between the two parameters. The data support that the TN amplitude is more likely to be very low (the TN tail has more probability than the DM tail), however, there is still a non-zero probability that the DM variations signal is weaker than the ML model suggests. For well-constrained PPDs, DM variations and TN are de-coupled, as seen in the right-hand panel of Fig. 2 for the case of J1012+5307, and the DM ML waveform subtraction is more reliable. If the statistical assumptions

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### Table 3. Timing-noise characteristics of EPTA MSPs based on power-spectral analysis for a single power-law model as described by equation (3).

We tabulate the dimensionless amplitude, $A$, at reference frequency of $1 \text{yr}^{-1}$, the spectral index, $\gamma$, and the white-noise power level, $S_W$, and their respective 1σ uncertainties. We also tabulate the pre-whitening level used ($\text{level}_{pw}$). For the pulsars where the measurement of TN was not possible, we quote the 95 per cent confidence upper limits for the amplitude. The table is divided as Table 2 for easier comparison.

<table>
<thead>
<tr>
<th>PSR J-Name</th>
<th>$\log(A)$</th>
<th>$\gamma$</th>
<th>$\log(S_W(\text{yr}^3))$</th>
<th>$\text{level}_{pw}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>J0621+1002</td>
<td>$-12.3 \pm 0.1$</td>
<td>$2.8 \pm 0.6$</td>
<td>$-26.94 \pm 0.04$</td>
<td>1</td>
</tr>
<tr>
<td>J1012+5307</td>
<td>$-13.01 \pm 0.07$</td>
<td>$1.7 \pm 0.3$</td>
<td>$-28.60 \pm 0.02$</td>
<td>1</td>
</tr>
<tr>
<td>J1022+1001</td>
<td>$-13.2 \pm 0.2$</td>
<td>$2.0 \pm 0.6$</td>
<td>$-27.97 \pm 0.03$</td>
<td>0</td>
</tr>
<tr>
<td>J1600–3053</td>
<td>$-13.6 \pm 0.1$</td>
<td>$1.3 \pm 0.5$</td>
<td>$-29.36 \pm 0.05$</td>
<td>0</td>
</tr>
<tr>
<td>J1713+0747</td>
<td>$-14.2 \pm 0.2$</td>
<td>$4.9 \pm 0.6$</td>
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<td>2</td>
</tr>
<tr>
<td>J1744–1134</td>
<td>$-13.6 \pm 0.2$</td>
<td>$3.0 \pm 0.6$</td>
<td>$-28.90 \pm 0.03$</td>
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</tr>
<tr>
<td>J1857+0943</td>
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<td>$2.3 \pm 0.7$</td>
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</tr>
<tr>
<td>J1939+2134</td>
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<td>$6.7 \pm 0.5$</td>
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</tr>
<tr>
<td>J0300+0451</td>
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<tr>
<td>J0218+4232</td>
<td>$-12.6 \pm 0.2$</td>
<td>$2.3 \pm 0.6$</td>
<td>$-26.69 \pm 0.03$</td>
<td>0</td>
</tr>
<tr>
<td>J0610–2100</td>
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<td>$2.1 \pm 0.6$</td>
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<tr>
<td>J0613–0200</td>
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<tr>
<td>J0751+1807</td>
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<tr>
<td>J1024–0719</td>
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<td>2</td>
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<td>$-28.25 \pm 0.03$</td>
<td>0</td>
</tr>
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<td>J1721–2457</td>
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<td>$2.7 \pm 0.8$</td>
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<td>0</td>
</tr>
<tr>
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<td>$1.7 \pm 0.5$</td>
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</tr>
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<tr>
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<td>J1843–1113</td>
<td>$-12.8 \pm 0.1$</td>
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<tr>
<td>J2145–0750</td>
<td>$-13.7 \pm 0.3$</td>
<td>$3.5 \pm 0.7$</td>
<td>$-28.36 \pm 0.03$</td>
<td>0</td>
</tr>
</tbody>
</table>

### Upper limits

<table>
<thead>
<tr>
<th>PSR J-Name</th>
<th>$\log(A^\text{UL})$</th>
<th>$\log(S_W(\text{yr}^3))$</th>
<th>$\text{level}_{pw}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>J0034–0534</td>
<td>$-12.4$</td>
<td>$-27.02 \pm 0.05$</td>
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<tr>
<td>J0900–3144</td>
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<tr>
<td>J1738+3333</td>
<td>$-12.6$</td>
<td>$-27.36 \pm 0.04$</td>
<td>0</td>
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<tr>
<td>J1751–2857</td>
<td>$-12.1$</td>
<td>$-27.3 \pm 0.6$</td>
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<tr>
<td>J1804–2717</td>
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<td>$-26.57 \pm 0.09$</td>
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<td>J1853+1303</td>
<td>$-12.7$</td>
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<td>J1910+1256</td>
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<td>$-27.38 \pm 0.06$</td>
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<td>$-27.88 \pm 0.1$</td>
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<td>J1955+2908</td>
<td>$-12.1$</td>
<td>$-26.46 \pm 0.06$</td>
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<td>$-27.95 \pm 0.04$</td>
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<td>J2322+2057</td>
<td>$-12.3$</td>
<td>$-26.78 \pm 0.05$</td>
<td>0</td>
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</tbody>
</table>
Figure 1. Two- and one-dimensional marginalized PPDs for the timing-noise parameters of three pulsars: J1012+5307, J0751+1807 and J2229+2643. In the two-dimensional distributions, the solid, dashed and dotted contours represent the 68, 95 and 99.7 per cent ($1\sigma$, $2\sigma$ and $3\sigma$) confidence intervals and the red star marks the ML solution. The one-dimensional distributions have the median and $1\sigma$ uncertainties marked as dashed and solid lines, respectively. For J2229+2643, the right-hand figure shows the distribution of the noise parameters from the upper limits analysis. Note the different ranges on the amplitude axes. See text in Section 4.1 for discussion.

of the Bayesian and frequentist analysis are valid, the results for the TN of pulsars with well-constrained PPDs should be consistent between the two methods.

4.2 Timing-noise parameters

Out of the 42 sources, the Bayesian analysis resulted in well-constrained PPDs for both the amplitude and the spectral index of the TN power-law model for eight sources. For these, the Bayesian ML and median values are always consistent at the $1\sigma$ level. The two methods are always consistent at the $1\sigma$ level for the spectral index, while for the amplitude, three sources show deviations, though consistency remains at the $2\sigma$ level (Fig. 3, top row).

For 17 MSPs, the PPDs of at least one of the TN parameters is semi-constrained. The Bayesian ML and median values show inconsistencies at the $1\sigma$ level in four pulsars (Fig. 3, middle row). The power-spectral analysis results are in agreement with the Bayesian median values. All Bayesian upper limits are in agreement with the rest of the results. We note that for PSR J1909–3744, we did not achieve a $3\sigma$ measurement for the spectral index with the power-spectral analysis.

The rest of the sources, 17 in total, show flat, unconstrained PPDs. The bottom row of Fig. 3 shows the 95 per cent CL upper limits from the two methods. Given the low significance of the TN measurement in these cases, inconsistencies in the amplitudes do not have statistically significant effects on the timing solutions when using the total covariance matrix to perform GLS timing analysis.

The agreement between the two methods for the sources with statistically significant TN measurements, supports the confidence in the methods and the results. When covariances between noise properties cannot be decoupled by the data, the interpretation of the results requires more attention. For this reason, we propose that
4.3 White-noise parameters

Radiometer noise estimation is typically robust when the pulse has a medium to high signal-to-noise ratio (S/N) (Taylor 1992), so EFACs are expected to be close to unity for most observing systems. The EQUADs results indicate for which observing systems there may be additional scatter in the residuals from physical processes related to the pulsars (e.g. pulse phase jitter) or RFI.

Fig. 4 shows the distribution of the ML EFAC values. As expected, the distribution strongly peaks around unity. A few systems show EFAC values up to $\sim 5$. These are typically high-frequency observations with very weakly detected pulses. The cases where EFACs take values significantly lower than one are either due to strong overestimation of the uncertainties or when a system’s EFAC and EQUAD are highly correlated.

We examine in a similar way the distribution of EQUAD values. Fig. 5 shows the distribution of the measured ML EQUAD values from the analysis using log-uniform EQUAD pPDs, and the distribution of their upper limits. As expected, in the vast majority of cases, the EQUADs are much below the TOA precision, which typically ranges from 0.5 to 10 $\mu$s (D15).

We have examined the EQUAD PPDs from the analysis with log-uniform pPDs to determine the cases where EQUADs have well-constrained PPDs and therefore show measurable EQUADs. For some of these cases, this could reflect signs of jitter noise present in the data. We list these pulsars and observing systems in Table 4. We note that there are 49 cases where the EQUAD PPDs are semi-constrained and significantly covariant with EFACs, and therefore cannot be considered as significant EQUAD measurements. From Table 4, we can see that the vast majority of EQUADs come from L-band systems, which typically have the most sensitive data. For each pulsar there are usually only one or two systems with clear EQUAD measurement with the exception of PSR J1022+1001. This source is known to require a high level of polarimetric calibration (van Straten 2013) and to show phase jitter noise (Liu et al. 2015). Only part of the NRT data were fully calibrated and this may explain the high levels of EQUADs in this source. We stress once again that more detail investigation is required to comment on the origin of the EQUAD measurements. It is likely that EQUADs could reflect additional scatter in the residuals from instrumental instabilities or analysis systematics, which could explain the EQUAD measurements in systems where the TOA precision is too low to expect any measurements of pulse jitter noise (as in the case e.g. of PSR J2033+1734, see Table 4.)

5 TIMING-NOISE FROM INDIVIDUAL OBSERVING SYSTEMS

For MSPs which have large enough data span with overlapping data from various observing systems, we examine whether part of the measured TN is present only in specific observing systems. We perform the noise analysis on selected pulsars with data from one telescope removed at a time. For the Effelsberg data, this is more complicated for many MSPs where it is the only telescope with data in the first half of the data set, so removing its data automatically means a loss of about half the data span. We note that this test may not be feasible in some cases with this data set, e.g. when a significant fraction of the residuals sensitivity to the TN is lost when removing a set of dominant, very precise data points. When the TN was absent after removing data from one telescope, we confirmed that the rest of the data would be sufficient to detect the noise by simulating realizations of the new data and performing the noise analysis after injecting TN with the measured properties.

Our analysis shows evidence for TN specific to the NRT data. Fig. 6 shows the PPDs for the TN parameters when using the full data set and when excluding the NRT data, and the respective ML TN waveforms. For PSR J1022+1001, the PPDs become significantly broader when excluding the NRT data. The mean value of the amplitude reduces by two orders of magnitude and the TN waveform becomes smoother, although the waveform has almost unchanged peak-to-peak variations. The TN parameters PPDs of
Figure 3. Comparison of the timing-noise parameters estimated with the Bayesian (blue, filled diamonds for the ML values, black, filled circles for the median values with 1σ error bars and blue, filled triangle for upper limits) and frequentist method (red, filled squares and red, open triangles for upper limits). Top row: results for the cases where the Bayesian analysis resulted in well-constrained PPDs for both parameters. Middle row: results for the cases where the Bayesian analysis resulted in semi-constrained PPDs for at least one of the parameters. Bottom row: results for the cases where the Bayesian code resulted in unconstrained PPDs for at least one of the parameters.

PSR J2145−0750 show a bimodality, which is not present when removing the NRT data. The two TN waveforms are almost identical, apart from the fact that the waveform of the full data set shows a bump around MJD 56000, which is not present when removing the NRT data. These effects are most likely caused either by additional noise in the NRT data from instrumental instabilities or by some additional non-instrumental noise component that only the NRT data are sensitive to, having indeed the highest precision TOAs. We stress that since we have assumed the TN to be stationary, the properties of instrumental noise during a specific time-interval can...
Figure 4. Distribution of EFAC values for all MSPs and observing systems. The black, solid line refers to the results of the Bayesian analysis for which the EQUAD priors are set to uniform to get their upper limit values, while the blue, dashed line is for the analysis were EQUAD priors are uninformative log-uniform.

Figure 5. Distribution of EQUAD values for all MSPs and observing systems. The solid line refers to the results of the Bayesian analysis for which the EQUAD priors are set to uniform to get their upper limit values, while the dashed line is for the analysis were EQUAD priors are uninformative log-uniform.

Table 4. List of the pulsars and observing systems for which we have well-constrained PPDs for the EQUADs. The last column shows the EQUADs ML values from a Bayesian analysis with log-uniform EQUAD prior distribution. The telescope and backend acronyms are as introduced in Section 2.

<table>
<thead>
<tr>
<th>PSR J-Name</th>
<th>Telesc.</th>
<th>Backend</th>
<th>Freq. (MHz)</th>
<th>EQUAD ML (µs)</th>
</tr>
</thead>
<tbody>
<tr>
<td>J0751+1807</td>
<td>EFF</td>
<td>EBPP</td>
<td>1360</td>
<td>5.0</td>
</tr>
<tr>
<td>J1012+5307</td>
<td>EFF</td>
<td>EBPP</td>
<td>1360</td>
<td>3.4</td>
</tr>
<tr>
<td>J1022+1001</td>
<td>JBO</td>
<td>DFB</td>
<td>1520</td>
<td>1.4</td>
</tr>
<tr>
<td>J1643−1224</td>
<td>JBO</td>
<td>DFB</td>
<td>1520</td>
<td>1.3</td>
</tr>
<tr>
<td>J1744−1134</td>
<td>JBO</td>
<td>DFB</td>
<td>1520</td>
<td>3.9</td>
</tr>
<tr>
<td>J1857+0943</td>
<td>NRT</td>
<td>BON</td>
<td>1400</td>
<td>2.5</td>
</tr>
<tr>
<td>J1939+2134</td>
<td>NRT</td>
<td>DDS</td>
<td>1400</td>
<td>1.0</td>
</tr>
<tr>
<td>J2033+1734</td>
<td>NRT</td>
<td>BON</td>
<td>1400</td>
<td>0.9</td>
</tr>
<tr>
<td>J2145−0750</td>
<td>NRT</td>
<td>BON</td>
<td>2000</td>
<td>0.3</td>
</tr>
<tr>
<td>J0751+1807</td>
<td>EFF</td>
<td>EBPP</td>
<td>1410</td>
<td>0.3</td>
</tr>
<tr>
<td>J0751+1807</td>
<td>EFF</td>
<td>EBPP</td>
<td>1410</td>
<td>0.3</td>
</tr>
</tbody>
</table>

During pulsar timing observations, the TOAs are referenced against the local atomic clock (e.g. hydrogen maser clock) or a Global Positioning System (GPS) clock. These clocks are stable on time-scales of weeks, allowing good phase keeping (1-pulse-per-second signal) during observations. These clocks, however, show instabilities on time-scales of months to years and the TOAs recorded using them, are therefore not suitable for high-precision pulsar timing projects. This problem can be mitigated through the application of a series of corrections based on monitoring the offsets between pairs of clocks (see e.g. Hobbs et al. 2006, 2012).

Cross-correlating the pulse profiles with the template profile references all arrival times to the same (arbitrary) phase, forming the topocentric TOAs. Unless the time-stamping was performed using a GPS clock, the TOAs are then converted to GPS-based Universal Coordinated Time (UTC) time, using clock correction files, created by calculating the difference between the local atomic clock and the GPS times. This is then converted to UTC and subsequently to the International Atomic Time (TAI) standard. TAI is formed by the weighted average of the time-scales of several hundred atomic clocks around the world and subsequent frequency adjustments using primary frequency standards. These adjustments are made over time-scales of years, a process known as ‘steering’. As a result, TAI can have errors during the steering periods which are never retroactively corrected. For these reasons, for pulsar timing we use the corrections on TAI provided by the Bureau International des Poids et Mesures (BIPM). These corrections are made through measuring offsets between various clock pairs to achieve the best possible precision and are regularly updated.

Any possible remaining errors in the BIMP terrestrial time standard or error propagated to the TOAs by systematics when referencing the TOAs to the various time standards, will lead to a ‘clock error’ signal, a monopolar correlated signal in the PTA sources, i.e. a signal with the same waveform in all pulsars and observing systems. As discussed in Tiburzi et al. (2016), the mitigation of the clock error signal is of central importance in PTA efforts for GW detection. In this section, we search for a terrestrial clock error in the data set to determine how much of the measured noise can be attributed to clock error noise. Previously, Hobbs et al. (2012) presented their measurement of the clock error using data from

6 TIMING-NOISE FROM ERRORS IN TERRESTRIAL TIME STANDARDS

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5 This is the case for the NRT data.

6 http://www.bipm.org/
6.1 Methodology and results

We use an ML estimator to infer the clock error signal. The clock error noise is modelled as a red-noise process power law with power-spectral density described by equation (3), with amplitude $A_{\text{clk}}$ and spectral index $\gamma_{\text{clk}}$. Using the results on these parameters, we subsequently construct the ML signal waveform.

For this analysis, we set the TN parameters of the MSPs to the ML values from the Bayesian analysis with uniform pPDs on the TN amplitude (as described in Section 3.2). In this way, we derive the ML solution for the clock error noise with the higher possible amplitude, given our TN results. We use the residuals after subtracting the ML DM variations signal as in Section 3.3, to focus on the TN only. The likelihood function is similar to equation (8) but with the extension to multiple pulsars to investigate the clock signal, which is identical in all pulsars, therefore its covariance matrix elements can be expressed as $C_{\text{clk},I,J} = C_{\text{clk}}(t_I - t_J)$ with $C_{\text{clk},I,J} = 1$ for all $I,J$ pairs.

The likelihood function shows that for the estimation of the clock noise parameters, we consider both the clock error signal on the residuals of each pulsar (autocorrelation effect) and the cross-correlation of the residuals between pulsar pairs.

We make the linear approximation of the timing model as described in van Haasteren et al. (2009), i.e. considering linear deviations of the true timing parameter values, $\epsilon$, from the least-squares fit timing model values, $\epsilon_0$, via the linear relation $\delta(\epsilon) = \epsilon - \epsilon_0$. We therefore substitute the expression for the residuals in equation (15), $t - \tau(\epsilon)$, with $\delta t = \delta t_{\text{post}} - M\delta(\epsilon)$; $\delta t_{\text{post}}$ are the post-fit timing residuals and $M$ is the design matrix of the timing parameters. We marginalize analytically over all timing parameters and get the reduced likelihood function:

$$L \propto \frac{1}{\sqrt{|C|}} e^{-\frac{1}{2} \sum_{i,j,I,J} \delta t_{I,i} C_{I,J}^{-1} \delta t_{J,j}},$$

where the index $I, J$ are for pulsars, and index $i, j$ are for the time epoch. The total covariance matrix now includes the covariance matrix of the clock error signal, $C_{\text{clk}}$, while not including the matrix of the DM variations such that, $C = C_w + C_{\text{TN}} + C_{\text{clk}}$. The intrinsic noise of pulsars is not correlated between pulsar pairs, so $C_{w,I,J} = 0$ and $C_{\text{TN},I,J} = 0$ for $I \neq J$. The clock error waveform is identical in all pulsars, therefore its covariance matrix elements can be expressed as $C_{\text{clk},I,J} = C_{\text{clk}}(t_I - t_J)$ with $C_{\text{clk},I,J} = 1$ for all $I,J$ pairs.

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with \( C = C^{-1} - C^{-1}M(M^T C^{-1} M)^{-1} M^T C^{-1} \). Going one step further, we split the deterministic signal between that of parameters for which we want to marginalize over (usually the timing model parameters), \( \delta t' \) and the signal of parameters we assume unknowns of the likelihood function (see e.g. Section 7.2). We note the latter parameters with the vector lambda, and assume their waveforms to be described by the \( S(\lambda) \). The likelihood function is then re-written as

\[
L \propto \frac{1}{\sqrt{|C|}} e^{-\frac{1}{2} \sum_{i,j,I,J}(t_{i,I} - S(\lambda_{I,J}))C_{i,j,I,J}^{-1}(t_{J,J} - S(\lambda_{I,J}))},
\]

We sample \( A_{\text{clk}} \) and \( \gamma_{\text{clk}} \) over a uniform grid of values and search for the model that maximizes the likelihood. The amplitude is sampled with values of \( \log(A_{\text{clk}}) \) ranging from \(-17.0 \) to \(-14.0 \) with a step of \(0.1\), while the spectral index values range from \(0.5 \) to \(5 \) with a step of \(0.1\). Due to the large condition number of the clock error’s covariance matrix, the individual likelihood computations are unstable. As such, the direct search for the ML solution with uniform grids produces non-desirable artefact (non-physical likelihood maxima). To avoid these effects, we performed a large number of trials by dithering noise parameters with randomized offset values within each search grid. The likelihood value of the grid is taken to be the maximum of all trials.

To reduce the computational cost of the analysis we use the ‘restricted data set’ proposed in Babak et al. (2016). This consists of six MSPs from the full data set, which give 90 per cent of the sensitivity to CGWs. This ‘restricted data set’ has also been used in the derivation of upper limits to the amplitude of GWs with the EPTA Data Release 1.0 (Lentati et al. 2015; Taylor et al. 2015; Babak et al. 2016). The ‘restricted data set’ contains the pulsars PSRs J0613–0200, J1012+5307, J1600–3053, J1713+0747, J1744–1134 and J1909–3744.

We find an ML solution at \( A_{\text{clk}} = -15.2 \) and \( \gamma_{\text{clk}} = 4.8 \). We use these values to calculate the ML waveform of the signal, shown in Fig. 7. The clock signal waveform can be estimated in the same way we did for TN:

\[
t_{\text{clk}} = C_{\text{clk}} C^{-1} t.
\]
presence of TN are in fact comparable\(^7\) to the amplitude limits derived in Lentati et al. (2015) and Babak et al. (2016) using more rigorous algorithms.

In its general form, the CRLB is formulated as follows. Given a likelihood function, \(f(\lambda, x)\), where \(x\) is the data and \(\lambda\) are the model parameters, the CRLB is

\[
\text{Cov}(\lambda) = \langle \sigma_{\lambda_{i}} \sigma_{\lambda_{j}} \rangle \geq I_{ij}^{-1},
\]

(20)

where the indices \(i\) and \(j\) denote the different parameters and \(I_{ij}\) is

\[
I_{ij} = \left\langle \frac{\partial \ln f(x, \lambda)}{\partial \lambda_{i}} \frac{\partial \ln f(x, \lambda)}{\partial \lambda_{j}} \right\rangle = -\left\langle \frac{\partial^{2} \ln f(x, \lambda)}{\partial \lambda_{i} \partial \lambda_{j}} \right\rangle.
\]

(21)

It is well known that I can be analytically calculated for Gaussian likelihood functions (as is equation 8), and results in the so-called Slepian–Bangs formula (Slepian 1954; Bangs 1971):

\[
I_{ij} = \frac{1}{2} \left\langle tr \left[ C^{-1} \frac{\partial C}{\partial \beta_{i}} C^{-1} \frac{\partial C}{\partial \beta_{j}} \right] + \frac{\partial S(\lambda)^{T}}{\partial \lambda_{i}} C^{-1} \frac{\partial S(\lambda)}{\partial \lambda_{j}} \right\rangle.
\]

(22)

Here, \(\beta_{i}\) are the model parameters describing the covariance matrix, \(\lambda_{i}\), are the parameters describing the unknown waveform \(S\) and \(tr\) is the matrix trace.

We make use of the same ML estimator as in Section 6 (equation 17), but we replace the stochastic clock error signal with that of a stochastic and isotropic GWB and set \(S(\lambda)\) to be the CGW signal from a single SMBHB, as detailed in Section 7.2. The likelihood function (equation 17) uses a total covariance matrix which includes the covariance matrix of the GWB, such that \(C = C_{\text{TN}} + C_{\text{gwb}}\). The GWB’s covariance matrix, is dictated by the expected correlation coefficient in the residuals of every pulsar pair, described by the overlap reduction function (Finn, Larson & Romano 2009), \(\Gamma(\zeta)\), defined as

\[
\Gamma(\zeta) = \frac{3}{8} \left[ 1 + \frac{\cos \zeta_{IJ}}{3} + 4(1 - \cos \zeta_{IJ}) \ln \left( \frac{\sin \zeta_{IJ}}{2} \right) \right] (1 + \delta_{IJ}).
\]

(23)

Here, \(\zeta_{IJ}\) is the angular separation between the \(I\)-th and the \(J\)-th pulsar, and \(\delta_{IJ}\) is the Kronecker delta. In principle, both the Earth and a pulsar term contribute to the correlation and \(\delta_{IJ}\) accounts for the latter. In the short-wavelength approximation, i.e. when the pulsars are separated away from the Earth and from each other by many GW wavelengths, the overlap reduction function is also known as the Hellings–Downs curve (Hellings & Downs 1983). The elements of the covariance matrix of the GWB are then expressed as \(C_{\text{gwb},(i,j)} = C_{\text{gwb}}(t_{i} - t_{j})\Gamma(\zeta_{IJ})\). As in the case of the clock error covariance matrix (Section 6.1), the form of the covariance matrix allows the calculation of the CRLB to include both the autocorrelation and cross-correlation effects of the GW.

For this analysis, we use the same six MSPs that we used to estimate the clock error noise parameters in Section 6.1 and we set the TN properties to their ML values as estimated with the Bayesian pulsar noise analysis described in Section 3.2 and presented in Table 2. As discussed in Section 6.1, the estimation that the sensitivity loss to GWs when using this data set is below 10 per cent was made for the case of CGWs. For low-frequency stochastic signals such as the GWB or the clock error signal, the sensitivity loss should be less. For CGWs, adding a pulsar with precise data only in part of

\[
\text{its data span can increase the } S/N/\text{of a detection significantly if the SMBHB orbit is fully sampled. In the case of the GWB, however, the targeted correlated signal must be found in cross-correlations of TOAs across a long time-span of order equal to the inverse of the GW frequency, with sufficient precision. We have verified this by calculating the CRLB for the GWB using 40 MSPs and noting an improvement in the amplitude limit of the order of 2 per cent. The scaling of the sensitivity to GWs with the number of MSPs, the } S/N/\text{regime of the targeted signal and other factors have been studied elsewhere (e.g. Babak & Sesana 2012; Siemens et al. 2013) and is outside the scope of this work.}
\]

In order to focus on the impact of TN only, we mitigate the DM variations before subtracting the ML DM variations waveforms from the residuals. For detailed derivations and astrophysical interpretations on GW limits using the EPTA Data Release 1.0, we refer the reader to Lentati et al. (2015), Taylor et al. (2015) and Babak et al. (2016) for the cases of a stochastic and isotropic GWB, the anisotropy in the GWB and the CGW from individual SMBHBs, respectively.

### 7.1 Stochastic gravitational wave background

When estimating the CRLB for the GWB amplitude, the terms with partial derivatives of \(S\) are zero and equation (22) reduces to

\[
I_{ij} = \frac{1}{2} tr \left[ C^{-1} \frac{\partial C}{\partial \beta_{i}} C^{-1} \frac{\partial C}{\partial \beta_{j}} \right].
\]

(24)

We calculate the CRLB for the GWB amplitude, keeping each time the GWB spectral index fixed. We do so for a range of spectral indices, from \(-2\) to \(1\), which covers GWB signals often discussed in PTAs literature, e.g. from SMBHBs, cosmic strings and the relic GWB from the inflationary era.

This simplified approach intends to provide an understanding of the difficulties the TN imposes on the detection of the various GWBs probed by PTAs. It is not exhaustive, since each of these GWBs can in general have a range of possible spectral index values. In the case of SMBHBs, this depends on the orbital eccentricities and whether the SMBHBs are coupled to their stellar and gaseous environment or they are driven by GW emission only (Sesana 2013). The often used power-law index of \(-2/3\) refers to circular, GW-driven SMBHBs (Rajagopal & Romani 1995; Jaffe & Backer 2003). Strong environment coupling and high orbital eccentricities can cause a turnover of the spectrum at low frequencies (e.g. fig. 2 in Sesana 2013). The value \(-7/6\) we have used for the spectral index of the cosmic string GWB has been analytically derived using a simplified approximation of the loop number density and assuming cusp emission (e.g. Damour & Vilenkin 2005). However, especially in the frequencies probed by PTAs, a wide range of spectral indices is possible, depending on some characteristic parameters used to describe the evolution of the network and the details of the dominant GW emission mechanism, and one typically sets limits on the amplitude for a range of these parameters (Sanidas et al. 2012). For the cosmological relic GWB, a spectral index of \(-1\) is often cited (Grishchuk 2005). For more details on the sources of the various GWBs and details on the derivation of amplitude limits as function of the spectral index and other physical parameters, we refer the reader to Lentati et al. (2015) and Arzoumanian et al. (2015).

The CRLBs are calculated using the TN parameters from the two Bayesian analyses, using different types of pPDs on the TN noise amplitude. For each set of TN results, we calculate the CRLB for two cases, namely assuming the presence of the measured white and TN, or assuming only the measured white-noise levels, and

\(\text{Note that the CRLB refers to the equivalent of a 68 per cent CL. Typically, the 95 per cent CL is reported in the PTA literature for the amplitude of GWs.}\)
upper limits) on the timing-noise amplitude. For both Bayesian analyses, with uniform (ML TN) and log-uniform priors (TN upper limits) on the timing-noise amplitude.

These results demonstrate how strongly TN can reduce the data’s sensitivity to stochastic GWBs. To stress this even further, we note that the upper limits on the GWB amplitude by SMBHBs (spectral indices representative of GWBs from SMBHBs, cosmic strings and relic GWs are presented in Table 5. The results for the CRLB on a GWB for the expected signals from SMBHBs ($\alpha = -2/3$), cosmic strings ($\alpha = -7/6$) and cosmological relic GWs ($\alpha = -1$) (see Section 7.1 for a discussion on the noted spectral indices). We tabulate the CRLB when considering both the white- and timing-noise levels ($A_{\text{GWBr}}$ and $A_{\text{GWBw}}$) and when assuming no timing noise present in the data ($A_{\text{GWBr}}$). Results were derived using the noise parameters from the Bayesian analysis with log-uniform priors on the timing-noise amplitude.

Finally, calculate their ratios. Fig. 8 shows the results for both cases. The results for the spectral indices representative of GWBs from SMBHBs, cosmic strings and relic GWs are presented in Table 5. The improvement factor on the lower bound when assuming no TN in the data is about an order of magnitude, ranging from 9.1 to 11.4. The results for the CRLB on a GWB for the expected signals from SMBHBs ($\alpha = -2/3$), cosmic strings ($\alpha = -7/6$) and cosmological relic GWs ($\alpha = -1$) (see Section 7.1 for a discussion on the noted spectral indices). We tabulate the CRLB when considering both the white- and timing-noise levels ($A_{\text{GWBr}}$ and $A_{\text{GWBw}}$) and when assuming no timing noise present in the data ($A_{\text{GWBr}}$). Results were derived using the noise parameters from both Bayesian analyses, with uniform (ML TN) and log-uniform priors (TN upper limits) on the timing-noise amplitude.

**Table 5.** Results for the CRLB on a GWB for the expected signals from SMBHBs ($\alpha = -2/3$), cosmic strings ($\alpha = -7/6$) and cosmological relic GWs ($\alpha = -1$) (see Section 7.1 for a discussion on the noted spectral indices). We tabulate the CRLB when considering both the white- and timing-noise levels ($A_{\text{GWBr}}$ and $A_{\text{GWBw}}$) and when assuming no timing noise present in the data ($A_{\text{GWBr}}$). Results were derived using the noise parameters from both Bayesian analyses, with uniform (ML TN) and log-uniform priors (TN upper limits) on the timing-noise amplitude.

<table>
<thead>
<tr>
<th>$\alpha_{\text{GWB}}$</th>
<th>$A_{\text{GWBr}}$</th>
<th>$A_{\text{GWBw}}$</th>
<th>$A_{\text{GWBr}} / A_{\text{GWBw}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-2/3$</td>
<td>$8.3 \times 10^{-16}$</td>
<td>$9.1 \times 10^{-17}$</td>
<td>9.1</td>
</tr>
<tr>
<td>$-7/6$</td>
<td>$4.6 \times 10^{-17}$</td>
<td>$4.4 \times 10^{-18}$</td>
<td>10.3</td>
</tr>
<tr>
<td>$-1$</td>
<td>$3.5 \times 10^{-16}$</td>
<td>$3.5 \times 10^{-18}$</td>
<td>10.0</td>
</tr>
<tr>
<td>$-2/3$</td>
<td>$9.2 \times 10^{-16}$</td>
<td>$8.5 \times 10^{-17}$</td>
<td>10.7</td>
</tr>
<tr>
<td>$-7/6$</td>
<td>$4.7 \times 10^{-17}$</td>
<td>$4.1 \times 10^{-18}$</td>
<td>11.4</td>
</tr>
<tr>
<td>$-1$</td>
<td>$3.7 \times 10^{-16}$</td>
<td>$3.3 \times 10^{-17}$</td>
<td>11.1</td>
</tr>
</tbody>
</table>

Finally, calculate their ratios. Fig. 8 shows the results for both cases. The results for the spectral indices representative of GWBs from SMBHBs, cosmic strings and relic GWs are presented in Table 5. The improvement factor on the lower bound when assuming no TN in the data is about an order of magnitude, ranging from 9.1 to 11.4. These results demonstrate how strongly TN can reduce the data’s sensitivity to stochastic GWBs. To stress this even further, we note that the upper limits on the GWB amplitude by SMBHBs (spectral index $-2/3$) by PTAs have improved by a factor of 10 over the past 10 yr.

7.2 Gravitational waves from single SMBHBs

Here we focus on CGWs from resolvable, GW-driven SMBHBs with circular orbits and without measurable frequency evolution of the signal due to energy loss from the binary by GW emission over the observing interval (an effect known as frequency chirping, see e.g. Hughes 2009). The waveform ($S$) of CGWs has been calculated by many independent studies (e.g. Wahlquist 1987; Blanchet 2006; Hughes 2009). For each SMBHB, the waveform is characterized by seven parameters, namely the GW amplitude, frequency and phase, the SMBHB’s sky co-ordinates (right ascension and declination), orbital inclination and orientation. It has been shown (Lee et al. 2011) that the precision estimation of the GW source position using CRLB would be poor, due to the lack of a unique unbiased estimator for the single source problem. The statistics of the amplitude estimator, on the other hand, can be well described by the CRLB, which determines the sensitivity of a PTA as function of frequency. The sensitivity depends on the GW source position. We estimate the CRLB for three scenarios: placing the SMBHB at the sky position where the PTA has the minimum and maximum sensitivity as well as the average of all positions on the sky. Our results are given in Fig. 9. The low-frequency sensitivity extends to values lower than $10^{-17}$ (3.17×10$^{-8}$ Hz) is caused by the pulsar sky position fitting.

The improvement in the PTA sensitivity at low frequencies is obvious from Fig. 9. One can clearly notice how the presence of TN flattens the sensitivity below $\sim 10$ nHz, which, in contrast, keeps improving in the case of timing data free of TN. In the absence of TN, the sensitivity at low GW frequencies is only limited by the PTA’s frequency resolution. Table 6 summarizes the CRLBs

![Figure 8.](http://example.com/figure8.png)
In order to focus the comparison between the methods correlated TN, DM variations and uncorrelated noise (white-noise) analysis package TEMPONEST to simultaneously determine the time-on power-spectral analysis. We used the Bayesian pulsar timing two established methods, one based on Bayesian and the other one in the left-hand panel (red, solid line) for better comparison.

In this paper, we have characterized the noise properties for 42 MSPs, using the EPTA Data Release 1.0. While the central focus is on the TN properties, we have also characterized the white-noise properties of the data. The long time-spans of the pulsar data sets is on the TN properties, we have also characterized the white-noise and for the white and the TN of the data. Right-hand panel: sensitivity curves when only accounting the white noise of the data. The additional blue, thick double dot–dashed line is the case for mean PTA sensitivity when including the TN as in the left-hand panel (red, solid line) for better comparison.

Table 6. Results for the CRLB on the strain amplitude of CGWs from resolvable SMBHBs with circular orbits and without measurable frequency chirping. We quote the limits for the cases when the SMBHB is at the sky location where the PTA has the maximum (max) and minimum (min) sensitivity, and the average of all sky positions (avg) at GW frequencies of 5 and 7 nHz. For each case, we quote the limits when accounting for the white and the TN of the data, $A_{\text{CGW}_{\text{w}}}$ and for the white noise only, $A_{\text{CGW}_{\text{w}}}$.

<table>
<thead>
<tr>
<th>GW freq. (nHz)</th>
<th>$A_{\text{CGW}_{\text{w}}}$</th>
<th>$A_{\text{CGW}_{\text{w}}}$</th>
<th>$A_{\text{CGW}<em>{\text{w}}}/A</em>{\text{CGW}_{\text{w}}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>$1.2 \times 10^{-14}$</td>
<td>$2.1 \times 10^{-15}$</td>
<td>5.6</td>
</tr>
<tr>
<td>7</td>
<td>$9.1 \times 10^{-15}$</td>
<td>$3.8 \times 10^{-15}$</td>
<td>2.4</td>
</tr>
<tr>
<td>5</td>
<td>$4.0 \times 10^{-15}$</td>
<td>$8.1 \times 10^{-16}$</td>
<td>5.0</td>
</tr>
<tr>
<td>7</td>
<td>$2.7 \times 10^{-15}$</td>
<td>$1.1 \times 10^{-15}$</td>
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<td>$2.4 \times 10^{-16}$</td>
<td>5.3</td>
</tr>
<tr>
<td>7</td>
<td>$1.0 \times 10^{-15}$</td>
<td>$4.4 \times 10^{-16}$</td>
<td>2.3</td>
</tr>
</tbody>
</table>

for the CGWs amplitude at frequencies of 5 and 7 nHz and the improvement factors to the sensitivity when the data do not have TN, which range from 2.3 to 5.6.

8 CONCLUSIONS

In this paper, we have characterized the noise properties for 42 MSPs, using the EPTA Data Release 1.0. While the central focus is on the TN properties, we have also characterized the white-noise properties of the data. The long time-spans of the pulsar data sets (the shortest being 6.9 yr and the longest 24.1 yr long) of high-quality timing data, are especially valuable for determining the TN. In order to increase our confidence in the results, we have employed two established methods, one based on Bayesian and the other one on power-spectral analysis. We used the Bayesian pulsar timing analysis package TEMPONEST to simultaneously determine the time-correlated TN, DM variations and uncorrelated noise (white-noise) properties. In order to focus the comparison between the methods on the TN characterization, we used the ML TEMPONEST results on DM variations and white-noise parameters as a priori known information when performing the frequentist analysis, based on a developed power-spectral analysis code described in this paper. For pulsars with statistically significant TN measurements, the two methods give statistically consistent results.

The lack of sufficient multifrequency data in 17 pulsars where TN is detected leads to strong covariances between the TN and DM variations, causing the posterior distributions of the noise parameters derived from the Bayesian analysis to have probability tails extending to ±∞. These reflect the small probabilities of the noise amplitude to be zero, causing some deviations between the ML and mean values of the parameters. The values of the ML and mean parameters as well as the parameter values estimated with the power-spectral analysis, are still however statistically consistent. Upper limit analysis is performed in these cases to set robust upper limits on the TN amplitude.

Our analysis shows evidence of TN specific to the NRT data, which are likely linked to improper polarization calibration in a roughly six-month-long epoch. We have also placed an upper limit on clock error TN and find that it contributes at most 1 per cent to the total noise in the MSPs under examination. Finally, we assessed the role of TN in the efforts for GW detection using PTAs. We did so by estimating the CRLB on the strain amplitude of a stochastic G WB and CGWs from resolvable SMBHBs, accounting only for the measured white noise first and then adding the measured TN properties. We find that, for GWBs, the TN in this data set reduces the sensitivity of this data set by a factor of 9.1–11.4, depending on the GWB spectral index. For CGWs, the sensitivity reduces by a factor of 2.3–5.6, depending on the GW frequency and the sky position of the SMBHB with respect to the sky position where the PTA is most sensitive.

The results of this paper stress in a clear way the imperative need of PTAs to improve the noise characterization and mitigation techniques and the development of good observing and data reduction practices to avoid introducing TN due to systematics. It also demonstrates the demand for new discoveries of MSPs that are not only bright, but also exhibit stable rotation. The rotational stability of pulsars can only be evaluated via TN characterization on data...
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