High-harmonic-generation (HHG) lasers are becoming an increasingly important source of coherent soft x rays [1,2]. By their nature, HHG sources produce a light spectrum that consists of many harmonics. Prior knowledge of the spectrum will play a crucial role in many experiments.

It is generally convenient to have an in situ capacity to monitor the HHG spectrum being produced. Recovery of the HHG spectrum from a fringe pattern created by a Young’s double-slit experiment has been reported earlier [3]. This approach used a Fourier transform (FT) to extract the spectral information. In this Letter we present a more flexible approach to this problem involving an application of the maximum-entropy method (MEM).

Consider the experimental arrangement shown in Fig. 1(a). In the case of a quasi-monochromatic illumination with frequency \( \nu \) and a complex degree of coherence \( \gamma_{12} \), the measured intensity distribution can be written in the form

\[
I(\tau|\nu) = 2[1 + |\gamma_{12}|\cos(2\pi \nu \tau)]\text{sinc}^2\left(\frac{W}{D} \nu \tau\right),
\]

where \( \tau = \Delta r/c = XD/cZ \) is the time delay introduced by the path-length difference \( \Delta r \); \( x \) is the spatial coordinate on the detector plane; \( c \) is the speed of light; \( \text{sinc}(a) = \sin(a)/a \); and \( Z, W, \) and \( D \) are the experimental parameters defined in Fig. 1(a). We assume that the incident light has a spectrum, \( F(\nu) \), such that the optical energy in frequency interval \( \Delta \nu \) is given by \( F(\nu) \Delta \nu \). The total observed intensity distribution \( I_T(\tau) \) is the superposition of the fringe patterns from each part of the spectrum, so that in a discrete form

\[
I_T(\tau) = \sum_{k=1}^{N} F(\nu_k)I(\tau|\nu_k),
\]

where the spectrum is considered over the interval \( \nu_0 - \Delta \nu N/2 \leq \nu_k \leq \nu_0 + \Delta \nu N/2, \) \( N \) is the total number of points in the HHG spectrum to be reconstructed, and \( \nu_0 \) is the central frequency of the light field. The problem we address here is the recovery of \( F(\nu_k) \) from a measurement of \( I_T(\tau_j) \) given that we know \( I(\tau_j|\nu_k) \).

The classic formulation of the maximum entropy problem is that of determining a distribution, \( x \), consistent with a given set of average values, \( b \) (see, for example [4], and references therein) from expression of the form \( \sum x_k A_{kj} = b_j \), where the \( A_{kj} \) are known. This is precisely the form of Eq. (2).

The MEM formalism has been presented in detail elsewhere [5]. The general idea of a MEM-based reconstruction is that the most probable HHG spectrum distribution is defined to be the distribution with the maximum value of the entropy functional,

\[
S = -\sum_{k=1}^{N} p_k \cdot \log(p_k/p_k^{(0)}),
\]

Fig. 1. (a) Schematic representation of the Young’s double-slits experimental setup. (b) The idealized model of the HHG spectrum.
which satisfies the following condition:

\[
C = \frac{1}{M} \sum_{j=1}^{M} \frac{|I_T^{MEM}(\tau_j) - I_T(\tau_j)|^2}{\sigma^2(I_T(\tau_j))} = 1,
\]

where \(M\) is the number of experimental points; \(p_k^{(n)} = F(\nu_k)/\sum_{k=1}^{N} F(\nu_k)\) is the normalized intensity of the HHG spectrum at the frequency \(\nu_k\); \(p_k^{(0)}\) is the normalized intensity of the HHG spectrum at \(\nu_k\) derived from prior information; \(I_T(\tau_j)\) is the experimental intensity distribution of the interference pattern; \(I_T^{MEM}(\tau_j)\) is the MEM estimate of \(I_T(\tau_j)\); and \(\sigma(I_T(\tau_j))\) is the estimated uncertainty in the measurement \(I_T(\tau_j)\). The degree of coherence, \(\gamma_2\), can be obtained from the data [6] and, in the experimental results reported here, was found to have the value \(\gamma_2 = 1\) to within experimental error, which corresponds to the fully spatially coherent case. The MEM-based HHG spectrum distribution is reached through iterations \(n = 0, 1, 2, \ldots\) defined by

\[
p_k^{(n+1)} = p_k^{(n)} \exp(-\alpha \mathcal{L}/\partial p_k^{(n)})/T,
\]

where \(T = \sum_{k=1}^{N} p_k^{(n)} \exp(-\alpha \mathcal{L}/\partial p_k^{(n)})\) and \(\alpha\) is the Lagrange multiplier. The reliability of reconstruction is measured using

\[
R_F(\%) = 100 \sum_{j=1}^{M} \frac{|I_T^{MEM}(\tau_j) - I_T(\tau_j)|}{\sum_{j=1}^{M} I_T(\tau_j)}.
\]

The algorithm was tested on several simulated interference patterns. First, a model of the HHG spectrum was created [Fig. 1(b)]. Three sets of interference patterns, \(I_T(\tau_j)\), were simulated for different values of \(D, W,\) and \(Z\): model 1 (\(D=575\ \mu m, W =20.0\ \mu m, Z=2.85\ m\)), model 2 \((D=50.0\ \mu m, W =10.0\ \mu m, Z=1.0\ m)\), and model 3 \((D=20.0\ \mu m, W =5.0\ \mu m, Z=0.22\ m)\). The MEM reconstruction was started from a uniformly distributed HHG spectrum. Figure 2 shows results of the MEM-based reconstruction of the HHG spectrum from three different models of \(I_T(\tau_j)\). It can be seen from Fig. 2 that HHG spectra reconstructed using the MEM-based approach are in good agreement with the simulated spectrum [Fig. 1(b)]. The values of \(R_F\) for three sets of interference patterns were found to be 2.8\%, 3.3\%, and 4.8\%, respectively. We have tested a MEM-based reconstruction algorithm with various models of the HHG spectrum as well as with various values of \(D, W,\) and \(Z\) and always obtained comparable quality results \(R_F < 5\%\). We also calculated \(FT[I_T(\tau_j)]\) for three models (dashed curves in Fig. 2) to compare results of MEM-based reconstruction with the FT method presented in [3]. It should be noted that MEM yields an estimate of the \(F(\nu)\) distribution; however, the FT approach yields the convolution of HHG spectrum with the FT of the diffraction pattern from the slit, \(T(\nu) \circ F(\nu)\) [3], where \(\circ\) is the convolution operator and, in our case, \(T(\nu) = FT[\text{sinc}^2(\pi W \nu/D)]\). This implies that the resolution of the reconstruction is limited by the \(W/D\) ratio of the Young’s slits. Both methods show almost identical results for model 1, but the FT approach did not resolve the spectral peaks for models 2 and 3.

The experimental system has been described elsewhere [7]. Data were acquired using a multiple-pass Ti:sapphire amplifier system producing 29\ fs laser pulses with a 1-kHz repetition rate and operating at a central wavelength of 810 nm. The laser pulses are focused by a 500 mm focal length lens into a 10 mm long argon gas cell with 150 \(\mu\)m pinholes at the entrance and exit, which are also used to isolate the vacuum chamber from the gas-filled cell. The cell is operated at a pressure of 20 Torr, the fundamental beam is truncated to a diameter of 8.25 mm, and the focal point was placed at a distance of approximately 15 mm from the exit plane of the gas cell. The effective peak intensity at the focus is approximately \(5 \times 10^{14}-10^{15}\ W/cm^2\). The harmonic beam is separated from the fundamental beam by 0.2 \(\mu\)m thick Al foil and spatially filtered by means of a precision 100 \(\mu\)m diameter pinhole. The silicon wafer with two different pairs of Young double slits [S1: \(D = (20.0 \pm 0.1)\ \mu m, W = (5.0 \pm 0.5)\ \mu m\), S2: \(D = (50.0 \pm 0.1)\ \mu m, W = (10.0 \pm 0.5)\ \mu m\)] were placed downstream at \(\sim 700\ mm\) from the spatial filtering pinhole. The interference pattern was detected using a CCD (Princeton Instruments) with an array of 1300 \(\times\) 1340 pixels of size 20 \(\times\) 20 \(\mu m\) placed at \(Z = (235.0 \pm 0.5)\ mm\) from the double slits.

Figure 3 illustrates a fit between the experimental and calculated intensity distributions of the interference patterns. As we can see from the figure, the reconstructed interference patterns are in good agreement with the experimental data \(R_F = 4.8\%\) for S1 and \(R_F = 5.9\%\) for S2). The reconstructed spectra are shown in Fig. 4. The HHG spectrum was also measured by a grazing incidence monochromator–spectrometer (GIMS 4, Setpoint). To avoid the need to calibrate the effect of detector efficiency, the same CCD camera that has been described previously was used; the results are shown in Fig. 4. The configuration of our experiment was such that this data was
acquired on a different day and the error bars on the measurement indicate the extent to which the spectrum will vary as a result of the shot-to-shot variation and the degree to which the precise experimental conditions could be replicated. It can be seen from Fig. 4 that relative intensities and positions of harmonics of the reconstructed HHG spectra for two experiments are in good agreement with that obtained directly using a grazing-incidence spectrometer.

Uncertainty in the experimental geometry leads to systematic errors in the MEM analysis. As was mentioned above, the intensity distribution of the interference pattern can be written in a simple form

\[ \Sigma_{\delta x \delta \tau} A_{ij} = b_j. \]

The accuracy of the HHG spectrum reconstruction, therefore, depends on the accuracy of calculation of the two-dimensional matrix \( A_{ij} \) which, in turn, depends on the accuracy of the time delay calibration, \( \Delta \tau / \tau \), and the accuracy of the estimation of the \( W/D \) ratio. The error of the time delay calibration can be written as \( \Delta \tau / \tau \approx \Delta P / P + \Delta x / x \), where \( P = D / Z \) and \( \Delta x \) is the pixel size of the CCD. The overall accuracy of harmonic positions in the reconstruction results are \( \Delta \lambda \approx 0.16 \text{ nm} \) for S1 and \( \approx 0.25 \text{ nm} \) for S2. A value of \( W/D \) must be estimated before the start of the MEM analysis. The FT approach, presented in [3], can be used for this purpose. According to [3], the \( W/D \) ratio can be estimated with very good accuracy from the width of the central peak in the FT of the intensity pattern. The inverse FT of this peak largely characterizes the size and quality of the slit system and this information can be used in the analysis.

In summary, the maximum-entropy method was used to reconstruct the HHG spectrum from the Young’s double slits interference pattern. Results presented in this Letter clearly show the ability of this method to reconstruct the HHG spectrum from simulated and experimentally collected data. The proposed method does not require any prior information about the number of harmonics, as well as their positions or shapes and has yielded reliable spectra in conditions where simpler Fourier-based methods [3] cannot.

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