

# An Overview of Recent Advances in Event-Triggered Consensus of Multiagent Systems

Lei Ding, Qing-Long Han<sup>✉</sup>, *Senior Member, IEEE*, Xiaohua Ge, *Member, IEEE*,  
and Xian-Ming Zhang, *Member, IEEE*

**Abstract**—Event-triggered consensus of multiagent systems (MASs) has attracted tremendous attention from both theoretical and practical perspectives due to the fact that it enables all agents eventually to reach an agreement upon a common quantity of interest while significantly alleviating utilization of communication and computation resources. This paper aims to provide an overview of recent advances in event-triggered consensus of MASs. First, a basic framework of multiagent event-triggered operational mechanisms is established. Second, representative results and methodologies reported in the literature are reviewed and some in-depth analysis is made on several event-triggered schemes, including event-based sampling schemes, model-based event-triggered schemes, sampled-data-based event-triggered schemes, and self-triggered sampling schemes. Third, two examples are outlined to show applicability of event-triggered consensus in power sharing of microgrids and formation control of multirobot systems, respectively. Finally, some challenging issues on event-triggered consensus are proposed for future research.

**Index Terms**—Consensus, even-based sampling, model-based event-triggered scheme, multiagent systems (MASs), sampled-data-based event-triggered scheme, self-triggered sampling scheme.

## I. INTRODUCTION

**M**ULTIAGENT systems (MASs), in which distributed sensing, communication, computing, and control are integrated, are usually employed to achieve coordinated tasks by letting a group of agents work cooperatively with each other [1], [2]. As a fundamental problem of cooperative control of MASs, consensus has attracted an interest of researchers due to their widespread applications in various areas, such as attitude alignment of satellites [3], formation of multiple robots [4]–[6], estimation over sensor networks [7]–[11], power management in power networks [12]–[14], distributed optimization [15]–[17], and so on. An essential issue on consensus of MASs is how to design a suitable control scheme such that the states of all agents can reach a common quantity

Manuscript received July 30, 2017; revised October 12, 2017; accepted November 2, 2017. This work was supported by the Australian Research Council Discovery Project under Grant DP160103567. This paper was recommended by Associate Editor Z. Zeng. (*Corresponding author: Qing-Long Han.*)

The authors are with the School of Software and Electrical Engineering, Swinburne University of Technology, Melbourne, VIC 3122, Australia (e-mail: qhan@swin.edu.au).

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Digital Object Identifier 10.1109/TCYB.2017.2771560

of interest. Large-scale participation of agents makes it costly or even impractical to control and manage MASs in a centralized manner. To solve this problem as well as to improve reliability and scalability of MASs, it is preferable to carry out distributed control by utilizing local information exchanges among neighbors via shared communication networks. As a result, numerous research on distributed consensus control for MASs has been conducted in recent years, see [1], [18]–[27]. Some related research topics on consensus control problems are surveyed in [28].

In conventional consensus control settings, it is assumed that MASs can access to continuous measurements and/or control signals. Such an assumption mandates support from sufficient computation resources and an ideal communication environment for MASs. Obviously, it is unrealistic in some practical applications, especially when agents themselves or their internal devices are powered by batteries; or communication bandwidth and channels are limited. Thus, an important criterion on designing a suitable distributed control scheme for MASs should not only guarantee the desirable control performance but also be capable of saving limited communication and computation resources. One possible approach is to use sampled-data control in an MAS [29]–[33], where sampling is triggered after the elapse of a fixed time interval. However, it is generally acknowledged that such a time-triggered sampling scheme may lead to excessive consumption of both communication and computation resources of MASs, especially when the system states nearly approach their equilibriums and there are no disturbances imposed on the systems [34], [35]. On the other hand, notwithstanding beneficial control performance in the sense that fast sampling can efficiently capture useful states of systems, time-triggered sampling results in a high frequency of data updates along with detrimental consequences, such as rising costs and traffic congestion, thereby imposing restrictions on other critical system monitoring and protection functions. It is well recognized that communication congestion may cause long latency, increased packet loss and reduced throughput, inevitably degrading system stability, performance and reliability [36]–[38]. Therefore, one important issue to be addressed is how to design suitable control schemes which can sustain the satisfactory control performance of MASs while significantly reducing over-consumption of communication and computation resources.

The introduction of event-triggered consensus control provides a positive solution to the above issue. Compared with

a traditional time-triggered control scheme with a fixed sampling period, a remarkable feature of an event-triggered control scheme is that the time instants when sampling actions and control updates should be performed are determined by a predefined event triggering condition (ETC) related closely to system measurements (e.g., system states or outputs) or performance levels [34]. If the ETC is violated, which means that the current system measurement exceeds a certain threshold, the current system measurement should be sampled and transmitted. This adequately establishes a link between sampling and control actions and system measurement. In this sense, an event-triggered scheme is advantageous over a time-triggered scheme in reducing unnecessary utilization of limited communication and computation resources [39], [40]. Thanks to this advantage, event-triggered control has been widely investigated in the past decade and some theoretical methodologies have been developed to address several control issues [34], [36], [41]–[52]. From these results, it is verified that the frequency of sampling and control executions can be significantly decreased while maintaining the prescribed control performance. Deep analysis and investigations on event-triggered control and filtering in networked control systems are referred to some recent survey papers [38], [53].

In the context of MASs, it is found that designing a suitable event-triggered consensus control scheme is more complicated and challenging compared with single-agent-based systems. The difficulties primarily come from the following three aspects: 1) strong information coupling in distributed control protocols involving information from both individual agents and their neighbors specified by a fixed or time-varying interaction/network topology; 2) high complexity in designing distributed ETCs; and 3) analysis on excluding the Zeno behavior in the sense that infinite events happen over a finite time interval. Inspired by an idea in [34], both centralized and distributed event-triggered control schemes are presented in [54] such that average consensus of single-integrator networks can be achieved by employing Lyapunov stability theory. It is shown in [54] that the proposed event-triggered scheme requires fewer control updates for agents while eliminating Zeno behavior. Subsequently, a great number of research results on distributed event-triggered consensus of MASs have been reported from various perspectives [55]–[61], which have been briefly reviewed in [62]–[64].

Different from [28] and [62]–[64], this paper aims to provide a state-of-the-art overview on event-triggered consensus control of MASs. Detailed analysis is made and insightful understanding is given with respect to recent results on event-triggered multiagent consensus control reported in the literature. The remainder of this paper is organized as follows. A general framework for event-triggered consensus control in an MAS is described in Section II, where an operational mechanism of an event-triggered scheme is concisely explained. Section III focuses on reviewing the latest theoretical results and their respective advantages and disadvantages. Practicability of distributed event-triggered consensus schemes is shown in Section IV through two practical examples, such as reactive power sharing of microgrids and formation control of multirobot systems. Section VI presents some challenging issues.

## II. EVENT-TRIGGERED CONSENSUS CONTROL FRAMEWORK

In this section, we first introduce a consensus control protocol for a linear MAS. Then, we present a general event-triggered consensus control framework for MASs.

### A. System Dynamics and Consensus Protocol

In order to conveniently summarize some exiting results in a unified way, we consider a general linear MAS, where single-integrator and double-integrator can be viewed as its special cases. The MAS consists of  $N$  agents whose dynamics is modeled by [25], [65]

$$\dot{x}_i(t) = Ax_i(t) + Bu_i(t), \quad i = 1, 2, \dots, N \quad (1)$$

where  $A \in \mathbb{R}^{n \times n}$  and  $B \in \mathbb{R}^{n \times m}$  are constant matrices;  $x_i \in \mathbb{R}^n$  and  $u_i \in \mathbb{R}^m$  are the state and the control input of agent  $i$ , respectively. It is assumed that  $(A, B)$  is stabilizable. The initial condition of (1) is given by  $x_i(0) = x_i^0$ . The communication topology is modeled as a weighted directed or undirected graph  $G = \{\mathcal{V}, E, \mathcal{W}\}$ , where  $\mathcal{V} = \{v_1, v_2, \dots, v_N\}$  and  $E \subseteq \mathcal{V} \times \mathcal{V}$ , which stand for the set of nodes and the set of edges, respectively; and  $\mathcal{W} = [w_{ij}] \in \mathbb{R}^{N \times N}$  is a weighted adjacency matrix with  $w_{ii} = 0$  for any  $i$ . If node  $v_i$  can receive information from node  $v_j$ , then node  $v_j$  is considered as a neighbor of node  $v_i$ , and node  $v_i$  is in turn called an out-neighbor of node  $v_j$ . Denote  $N_i$  the set of neighbors of node  $v_i$ . It is assumed that  $w_{ij} > 0$  if  $j \in N_i$ , otherwise,  $w_{ij} = 0$ . In a directed graph, if there exists a directed path from one node to any other nodes, then the graph  $G$  is said to have a spanning tree. In an undirected graph, the graph  $G$  is said to be connected if there exists a path between any nodes.

For the MAS (1), it is said to achieve consensus if there exists a suitable control protocol  $u_i(t)$  such that all the agents' states  $x_i(t)$  reach a common value or vector, i.e.,  $\lim_{t \rightarrow +\infty} \|x_i(t) - x_j(t)\| \leq \varepsilon$ , where  $\varepsilon \geq 0$  is a very small constant [1]. If  $\varepsilon = 0$ , it is referred to as accurate consensus [1], [18]; otherwise, bounded consensus or practical consensus [23], [32]. In order to solve the consensus problem, the following distributed control protocol is commonly used [65]:

$$u_i(t) = K \sum_{j \in N_i} w_{ij} (x_j(t) - x_i(t)) \quad (2)$$

where  $K$  is a control gain matrix.

In practical implementation of (2), it is unrealistic to have access to continuous signals from agents and their neighbors. To overcome this problem, a sampled-data control method is used. Generally, there exist two sampling ways: 1) time-triggered sampling and 2) even-triggered sampling. Since the even-triggered sampling has attractive advantages in reducing utilization of communication and computation resources, much attention has been paid to event-triggered control of MASs. In what follows, we show its basic framework.

### B. Event-Triggered Mechanism

A general event-triggered control configuration for each agent can be shown in Fig. 1. Different from traditional MASs, an event detector is introduced and installed between the

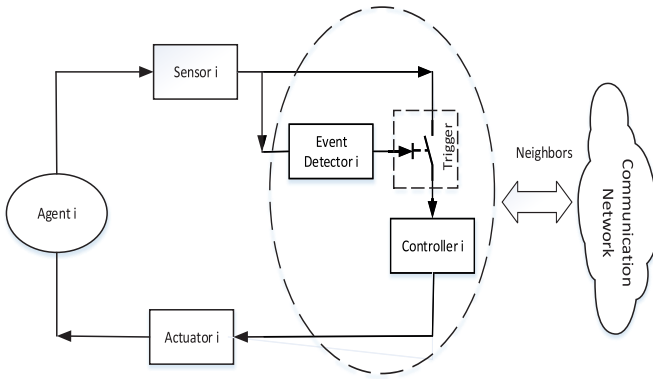


Fig. 1. Basic event-triggered control configuration for agent  $i$ .

sensor and the controller of agent  $i$ . The event detector, as a core component in the event-triggered control, is responsible for determining when agent  $i$ 's measurement should be triggered for controller updates of itself and its neighbors. The principle of making such a decision is based on an ETC preset or embedded in the event detector of agent  $i$  [54]. Suppose that the measurement of agent  $i$  can be constantly acquired by sensor  $i$  and sent to event detector  $i$ . Then, the ETC can be checked at all times. Once the ETC is not satisfied, the current measurement of agent  $i$  will be "requested" for the MAS and it should be broadcast. According to the above mechanism, the tasks of event detector  $i$  include: 1) collecting all the measurements (from itself or its neighbors) required by its ETC; 2) making a decision on when the measurement should be sampled and broadcasted by using the received information; and 3) generating an execution signal if the ETC is violated. After the trigger of agent  $i$  receives the triggering signal, agent  $i$ 's measurement will be authorized to be broadcast and then be used to update the control inputs of its own and its neighbors.

In order to ensure the effectiveness of the above mechanism, some important issues need to be carefully addressed.

- 1) *ETC*: In an event-triggered control scheme, the ETC is a key component for determining event time instants, which is closely related to reducing the number of control updates and communication among neighbors. In principle, the selection of ETCs should be physically explained. Besides, ETCs should be easy to implement from a practical perspective. These two points raise difficulties in designing a suitable ETC for MASs. Note that ETCs used in the literature are mainly: a) *centralized* [54]: using all the agents' measurement information; b) *decentralized* [55], [66], [67]: using its own information; and c) *distributed* [56], [57], [60]: using the information from itself and its neighbors. Since it is not practical to use information of all agents' states to design ETCs, the decentralized and distributed event-triggered schemes are attractive for MASs.
- 2) *Control Protocol*: When an event-triggered scheme is applied to an MAS, the measurements of agents and their neighbors can be available only at event time instants. Hence, an important issue on consensus protocol design

is how to efficiently make use of such information available for control updates under the designed ETCs. In fact, different ETCs may lead to a great variety of consensus protocols (see Section III in detail). Furthermore, it is really a challenging issue to design both the controller gain  $K$  in (2) and threshold parameters of event-triggered schemes in a unified framework [60].

- 3) *Interevent Time*: When the interevent time is zero, Zeno behavior occurs. In this case, the event-triggered scheme fails to be used [34]. Thus, it is compulsive to guarantee that the lower bound of the interevent-time is strictly greater than zero. However, it is in fact not an easy task, especially when agents are affected by external disturbances and/or the thresholds in ETCs heavily depend on the measurements of agents and their neighbors [54].

The above three issues should be taken into account in designing an appropriate event-triggered control scheme for MASs. Up to date, a plenty of notable results on event-triggered control schemes have been derived in the literature, which are briefly reviewed in the next section.

### III. EVENT-TRIGGERED CONSENSUS CONTROL SCHEMES

As mentioned in Section II, the key points of event-triggered schemes lie in the design of control protocols and ETCs. Recalling some existing results on MASs, there are mainly four types of event-triggered schemes: 1) event-based sampling scheme [54]; 2) model-based event-triggered scheme [66]; 3) sampled-data-based event-triggered scheme [60]; and 4) self-triggered sampling scheme [68]. In what follows, in-depth analysis on them is made.

#### A. Event-Based Sampling Schemes

A key feature of an event-based sampling scheme is that whether or not the current measurement is sampled is determined by a predefined ETC. This implies that event-based sampling can reduce unnecessary samplings significantly, thus having more potential in saving the lifespan of energy-based devices. The event-based sampling scheme is initially tailored in a centralized way for single-integrator MASs [54], where the ETC is dependent on the global state measurement errors between the last event and current instants as well as network topology. Such a centralized event-based sampling scheme is difficult to implement, particularly when the number of agents is huge and communication resources are limited. Thus, the centralized event-based sampling scheme should give way to a distributed event-based sampling scheme, which is introduced in [54] and [69] for consensus control of MASs. In such a scheme, the state measurement error for agent  $i$  is defined as  $e_i(t) = x_i(t_k^i) - x_i(t)$ ,  $t \in [t_k^i, t_{k+1}^i)$ , where  $t_k^i$  ( $k = 1, 2, \dots$ ) denotes the event instants for agent  $i$ . Then, a distributed event-based consensus control scheme for agent  $i$  is given by [54], [69]

$$\Pi_1 : \begin{cases} \text{Protocol: } u_i(t) = -K \sum_{j \in N_i} w_{ij} (x_i(t_k^i) - x_j(t_{k'}^j)) \\ \text{ETC: } f_i(e_i(t)) \leq \Delta_i(z_i(t)) \end{cases} \quad (3)$$

where  $t_{k'}^j \triangleq \arg \min_{p \in \mathbb{N}} \{t - t_p^j | t \geq t_p^j\}$ ;  $K$  is the controller gain matrix;  $f_i(\cdot)$  and  $\Delta_i(\cdot)$  are the error function and



the threshold function of agent  $i$ , respectively, and  $z_i(t) = \sum_{j \in N_i} w_{ij}(x_i(t) - x_j(t))$ . If the threshold function  $\Delta_i$  is related to the states of agents, it is called a *state-dependent threshold*, otherwise, a *state-independent threshold*. In the distributed setting, agent  $i$ 's event time instants  $t_k^i$  are determined once its triggering condition in (3) is violated. A reasonable explanation is that the state change between the consecutive sampling instants is not allowed to go beyond a threshold. Since the distributed event-based consensus control scheme in (3) is only dependent on its own and neighbors' information, it is suitable for MASs. Moreover, in [54] and [69], it is shown that the proposed distributed event-based sampling scheme (3) can exclude Zeno behavior efficiently and decrease the frequency of agents' control actuation significantly.

Although, the distributed event-triggered control scheme (3) offers some advantages in reducing the number of control updates, there are several limitations to its practical implementation, which are shown as follows.

- L1) *High Frequency of Control Updates*: Notice that from the consensus control protocol in (3), the control input updates of agent  $i$  are triggered at its own event instants  $t_k^i$  and its neighbors' event instants  $t_{k'(t)}^j$  as well. As a result, if the agent has a larger number of neighbors, the minimum time-interval between any two consecutive control updates may become smaller and smaller, eventually leading to Zeno behavior [57], [70]. In this case, efficiency and applicability of the event-based sampling scheme will be definitely degraded.
- L2) *Requirement on Continuous Communication*: From (3), one can see clearly that the threshold function  $\Delta_i$  of agent  $i$  relies on the real-time state information from itself and its neighbors, which means that continuous communication among neighbors is necessary. Hence, more communication resources are required to implement the event-based sampling scheme (3), which substantially augments the operational costs and contradicts the intention of event-triggered strategies.
- L3) *Limitations of System Dynamics*: In [54] and [69], it is shown that, the distributed event-based consensus control scheme (3) is effective for an MAS whose agents' states eventually converge to an equilibrium point. However, it fails to work when all the agents' states are synchronized to a time-varying trajectory. More specifically, when the system dynamics of the MAS (1) reach consensus under the ETC of (3), the threshold  $\Delta_i(z_i(t))$  approaches to zero as  $z_i(t) \rightarrow 0$ . But the state error  $e_i(t)$  does not converge to zero since the final consensus state trajectory is time-varying. As a result, the ETC in (3) will be violated at every time, unavoidably leading to Zeno behavior.

In order to overcome the above limitations, significant effort has been made to improve the distributed event-based consensus control scheme (3) for MASs. Regarding the limitation L1), the following distributed even-based consensus scheme is proposed [57], [70], [71]:

$$\Pi_2 : \begin{cases} \text{Protocol: } u_i(t) = -K \sum_{j \in N_i} w_{ij}(x_i(t_k^i) - x_j(t_k^i)) \\ \text{ETC: } f_i(e_i(t), e_{ij}(t)) \leq \Delta_i(z_i(t), t) + \delta_i \end{cases} \quad (4)$$

where  $e_{ij}(t) = e_j(t_k^i) - e_j(t)$  and  $\delta_i \geq 0$  is a constant. Intuitively, the event-based consensus scheme (4) can reduce the number of control updates compared with (3) due to the fact that the control updates of agent  $i$  occur only at its own event instants  $t_k^i$ . In the case of  $\delta_i = 0$ , the event-triggered consensus problem is investigated for MASs with single-integrators [72], double integrators [70], linear dynamics [73], and nonlinear dynamics [71], [74]. It should be pointed out that, as analyzed in L3), the event-based consensus scheme (4) is no longer effective for double-integrator networks since the Zeno behavior may occur after consensus is reached. To cope with the problem, a small constant  $\delta_i \neq 0$  is introduced in [57] to the threshold function  $\Delta_i$ . This leads to the event-based scheme (4) to a relaxed constraint that the system matrix  $A$  does not need to be stable. It should be emphasized that when the threshold function  $\Delta_i$  includes a positive constant  $\delta_i$ , the event-triggered scheme (4) can achieve only bounded consensus rather than complete consensus [57].

It should be mentioned that a combinational measurement approach to designing the event-triggered scheme is developed in [56] and [75] to overcome the limitation in L1), where the combinational measurement error is defined by  $\tilde{z}_i(t) = z_i(t) - z_i(t_k^i)$ , and the ETC in (4) is modified as

$$f_i(\tilde{z}_i(t)) \leq \Delta_i(z_i(t), t) + \delta_i. \quad (5)$$

Based on the ETC (5), some research work is conducted for leader-following consensus [76], consensus with input time delay [78], and output consensus [79]. Besides the reduction of control updates, the other advantages of the ETC (5) lie in that it can avoid the effects of system dynamics as stated in L3) on the event-triggered scheme because the combinational measurement error will converge to zero as the consensus is completed. Note that the relative measurement errors among neighbors in the ETC (5) are available to each agent. It is commonplace to access to these combinational states via a communication layer. Therefore, although ETCs (4) and (5) can decrease the control updates of each agent, they still depend on continuous communication. In some special cases, the relative measurement can be either taken directly by sensor devices, or generated indirectly by complicated computation [56], [76]. For example, in a vehicle platoon the measurement sensors are embedded on each vehicle to obtain the relative positions and velocities between vehicles. It is clear that such the direct and undirected measurement requires support from extra hardwares, thus, it will cause additional cost when implementing ETC (5).

In a state-dependent event-triggered scheme, one solution to relax constraints on continuous communication is that it uses only triggered state information from neighbors. Then, a modified version of the event-based consensus scheme (4) is proposed in [57], where each agent uses only the latest control information updated from its neighbors. However, control information of agents sometimes is private or unavailable to others due to malicious attacks. For example, the typical DoS attacks jam the shared network medium to prevent agents from communicating with others [77]. Thus, this modified event-triggered scheme may not be practical from the perspective of information privacy or security. Moreover, from the protocol

in (4), it is known that the sampled states of neighbors at the event time  $t_k^i$  are requested for the control updates. In reality, such a mechanism requires additional processing devices, leading to extra running cost of the whole MAS.

In order to circumvent continuous communication among agents, the following distributed event-triggered scheme is presented [58]:

$$\Pi_3 : \begin{cases} \text{Protocol: } u_i(t) = -K \sum_{j \in N_i} w_{ij} (x_i(t_k^i) - x_j(t_{k'}^j)) \\ \text{ETC: } f_i(e_i(t)) \leq \Delta_i(\hat{z}_i(t)) \end{cases} \quad (6)$$

where  $\hat{z}_i(t) = x_i(t_k^i) - x_j(t_{k'}^j)$ . It is clear that the threshold of ETC in (6) depends on both the discrete signals  $x(t_k^i)$  and  $x_j(t_{k'}^j)$  from agent  $i$  and its neighbors, respectively, which implies that an individual agent is not required to have continuous access to its neighbors' information. Based on the scheme (6), practical asymptotic tracking of the dynamic average of the time-varying agents' reference inputs over networks with time-varying connected undirected graphs is achieved in [80]. The scheme (6) is extended in [81] to address the event-based consensus problem for linear MASs with time delay and a directed graph. In [82], output synchronization of networked passive systems without or with network induced delays is investigated. Different from [80]–[82], a fully distributed event-triggered scheme is proposed to achieve average consensus of single-integrator networks in [58] by employing the Lyapunov method. As such a fully distributed event-triggered scheme does not need any knowledge of global network topology for each agent, it is very attractive to practical implementation, especially in the presence of large-scale networks. However, only single-integrator MASs are considered in [58] and the analysis method therein may be difficult to accommodate general linear MASs. For this reason, it would be appealing to further investigate a distributed event-triggered scheme in more complicated cases of agent dynamics.

Another effective approach to avoiding continuous communication is to use a state-independent threshold for the relative error  $e_i(t)$ . In contrast to the state-dependent thresholds in (3)–(6), a state-independent threshold has a simpler form. Then, the distributed event-triggered scheme is given by [55], [83]

$$\Pi_4 : \begin{cases} \text{Protocol: } u_i(t) = -K \sum_{j \in N_i} w_{ij} (x_i(t_k^i) - x_j(t_{k'}^j)) \\ \text{ETC: } f_i(e_i(t)) \leq \Delta_i(t) = c_0^i + c_1^i e^{-\alpha^i t} \end{cases} \quad (7)$$

where  $c_0^i$ ,  $c_1^i$ , and  $\alpha^i$  are constants satisfying  $c_0^i \geq 0$ ,  $c_1^i \geq 0$ ,  $c_0^i + c_1^i > 0$ , and  $\alpha^i > 0$ . Clearly, the threshold function  $\Delta_i(t)$  is independent of agents' states. For MASs with single-integrators and double-integrators, the scheme (7) is used in [55] to achieve consensus with convergence to a neighbor ball of equilibrium if  $c_0^i \neq 0$ . For linear MASs, unbounded consensus is considered in [83] by using the scheme (7). The merits of such a state-independent threshold include: 1) when each agent broadcasts its state to neighbors depends only on the change of its own state, but not on that of its neighbors. As a result, continuous communication among neighbors is no longer required; 2) the lower bound of the interevent time can be easily computed so that Zeno behavior can be excluded,

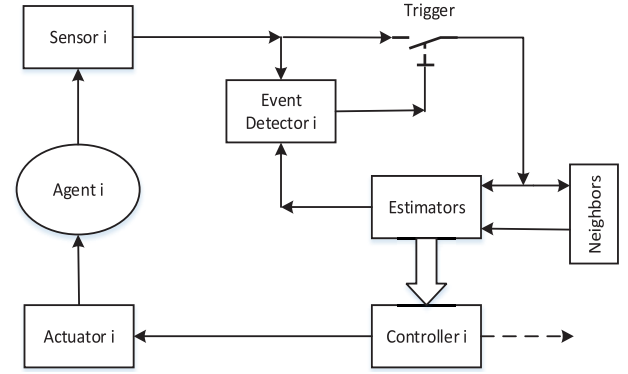


Fig. 2. Model-based event-triggered scheme for agent  $i$  with  $d_i+1$  estimators.

especially for  $c_0^i \neq 0$ ; and 3) there is no need to consider the limitation of system dynamics aforementioned. Nevertheless, compared with the state dependent thresholds in (3)–(6), state-independent thresholds also have some disadvantages. On the one hand, the sampling action cannot reflect the nature of system dynamics explicitly due to the fact that the thresholds are totally independent of system states. On the other hand, when  $c_0^i \neq 0$ , the distributed event-triggered scheme (7) can only guarantee bounded consensus.

### B. Model-Based Event-Triggered Schemes

Apart from event-based sampling schemes, a model-based event-triggered control strategy is proposed [84]. Based on the periodic event-trigger scheme, a model-based event-triggered predictive control is constructed first in [85] for the networked control systems considering the effect of data dropout. With the model-based event-triggered scheme, an estimate of system states on update intervals is made. Based on the estimate errors, an ETC is defined to determine when the actual system states should be transmitted. Such a model-based event-triggered control scheme tailored for an MAS is shown in Fig. 2, see detail in [59]. Recalling existing results, there are two approaches to dealing with model-based event-triggered consensus control problems: an open-loop estimation approach and a closed-loop estimation approach.

1) *Open-Loop Estimation Approach*: In this fashion, each agent needs to equip  $d_i + 1$  estimators for agent  $i$  to complete the estimates of its own and neighbors' states based on their state information at the latest triggered times, where  $d_i$  is the number of agent  $i$ 's neighbors. Denote by  $\hat{x}_j(t)$  the estimates of the agent  $j$ 's states. Then the dynamics of estimators in agent  $i$  can be described by

$$\begin{cases} \dot{\hat{x}}_j(t) = A\hat{x}_j(t), & t \in [t_k^j, t_{k+1}^j) \\ \hat{x}_j(t_k^j) = x_j(t_k^j), & j \in N_i. \end{cases} \quad (8)$$

Since all the agents have an identical system matrix  $A$ , the estimates of agent  $i$ 's states made by agent  $i$  and its neighbor agents  $j \in N_i$  are identical. Let  $\tilde{x}_i(t) = \hat{x}_i(t) - x_i(t)$  denote the error between the actual state of agent  $i$  and its estimate. Then a distributed model-based event-triggered consensus scheme is presented by

$$\Pi_5 : \begin{cases} \text{Protocol: } u_i(t) = -K \sum_{j \in N_i} w_{ij} (\hat{x}_i(t) - \hat{x}_j(t)) \\ \text{ETC: } f_i(\tilde{x}_i(t)) \leq \Delta_i(\hat{z}_i(t), t) + \delta_i \end{cases} \quad (9)$$

where  $\hat{z}_i(t) = \sum_{j \in N_i} w_{ij}(\hat{x}_i(t) - \hat{x}_j(t))$ . From (9), the control input of agent  $i$  is updated continuously by using all the estimates available from (8). Note that the ETC of agent  $i$  only uses local information from its  $d_i + 1$  estimators. Thus, the event-triggered consensus scheme (9) is decentralized [59]. Since agent  $i$  can predict its neighbors' states based on (8), it is reasonable to use the estimate  $\hat{x}_j(t)$  in the combinational states  $\hat{z}(t)$  even though there is no continuous communication. When the triggering condition of agent  $i$  in (9) is violated at times  $t_k^i$ , its state  $x_i(t_k^i)$  will be transmitted to its out-neighbors. Once its neighbors  $j$  receive the new state information from agent  $i$ , they will update the corresponding estimate of agent  $i$  in terms of (8).

There are lots of results on the event-triggered consensus scheme (9) reported in [59], [67], and [86]–[93]. For MASs with an undirected and connected topology graph, several sufficient conditions on achieving consensus are presented in [59], [89], and [91], where the ETCs are state-dependent. It is shown in [59] that, if  $\delta_i = 0$ , accurate consensus can be completed while Zeno behavior may happen; otherwise, only bounded consensus can be achieved with Zeno behavior excluded. The scheme is then extended to a leader-following framework with fixed and switching network topologies [91]. Considering a directed graph with a spanning tree, some conditions on event-triggered consensus are obtained in [67] and [90], where a state-independent ETC is introduced. From (8), one has

$$\hat{x}_i(t) = e^{A(t-t_k^i)} x_i(t_k^i), \quad t \in [t_k^i, t_{k+1}^i). \quad (10)$$

The estimate error is rewritten as

$$\tilde{x}_i(t) = e^{A(t-t_k^i)} x_i(t_k^i) - x_i(t). \quad (11)$$

Based on the event-triggered scheme (9) with (10) and (11), the consensus problem for MAS with general linear dynamics under a general directed graph is investigated in [67], where the threshold function of ETC is time-dependent. Different from [67], event-triggered consensus of linear MASs with undirected topology is completed in [86] by an observer-based output feedback control protocol. By using the event-triggered (9) with (10) and (11), the leader-following consensus problems of MASs under fixed and switching directed topology are dealt with in [92]. Then, the event-triggered control scheme is extended in [93] to accommodate the case of quantization communication for leader-following consensus of MASs.

Following the line of (8), some edge state estimators are introduced in [94] to estimate the relative state errors between neighbors. For a connected edge  $(i, j)$ , the point-to-point communication link between agents  $i$  and  $j$  is available only when an edge event is triggered. Let  $t_k^{ij}$  be the event times when the edge triggering condition of  $(i, j)$  is not satisfied. If the graph of network topology is undirected, one has  $t_k^{ij} = t_k^{ji}$ . Define a connected edge  $(i, j)$  as  $z_{ij}(t) = x_i(t) - x_j(t)$ . Similar to (8), the estimate  $\hat{z}_{ij}(t)$  of the edge  $z_{ij}(t)$  satisfies

$$\begin{cases} \hat{z}_{ij}(t) = A\hat{z}_{ij}(t), & t \in [t_k^{ij}, t_{k+1}^{ij}) \\ \hat{z}_{ij}(t_k^{ij}) = z_{ij}(t_k^{ij}), & j \in N_i. \end{cases} \quad (12)$$

Then, based on the edge estimate (12), the following event-triggered consensus scheme is proposed [94]:

$$\Pi_6 : \begin{cases} \text{Protocol: } u_i(t) = -K \sum_{j \in N_i} w_{ij} \hat{z}_{ij}(t) \\ \text{ETC: } \begin{cases} f_{ij}(h_{ij}(t), \Delta_i(t, \delta_i)) \leq 0 \\ f_{ji}(h_{ji}(t), \Delta_i(t, \delta_i)) \leq 0 \end{cases} \end{cases} \quad (13)$$

where  $h_{ij}(t) = \left\| \int_{t_k^i}^t e^{A(t-\tau)} B u_i(\tau) d\tau \right\|$ . Due to the fact that  $h_{ij}(t)$  is a function of control input  $u_i(t)$ , the ETC in (13) is called an input-based triggering condition [94]. Note that the scheme (9) has similar properties to (13). Both of them have common advantages and disadvantages.

- 1) *Advantages:* a) Communication among agents only occurs at event times, which implies that continuous communication is no longer necessary and b) the system matrix  $A$  is allowed to be unstable due to the use of model-based estimates.
- 2) *Disadvantages:* a) The control updates are continuously performed. Thus, agents' on-board energy for computation cannot be saved, which is unfavorable for MASs with battery-powered nodes; b) to implement the event-triggered scheme (9) and (13), it is required to deploy extra  $d_i + 1$  estimators for agent  $i$ . The number of such estimators will become large as agent  $i$ 's neighbors grow, thus leading to much cost and high complexity of implementation; and c) the necessary assumption that the system matrix  $A$  should be identical and known *a priori* is required in (9), which limits the application scope of the event-triggered scheme.

2) *Closed-Loop Estimation Approach:* In [95] and [96], a closed-loop estimate approach is developed. It is assumed that each agent can access to the control update values of its neighbors. Then, agent  $i$  can take state estimate for its neighbor agent  $j$  by

$$\hat{x}_j(t) = A\hat{x}_j(t) + B u_j(t_k^{ij}), \quad j \in N_i. \quad (14)$$

Define the auxiliary state of edge  $(i, j)$  by  $\hat{z}_{ij}(t) = \hat{x}_i(t) - \hat{x}_j(t)$  and the estimate mismatch between its current and last triggered states by  $\tilde{z}_{ij}(t) = \hat{z}_{ij}(t_k^{ij}) - \hat{z}_{ij}(t)$ ,  $t \in [t_k^{ij}, t_{k+1}^{ij})$ . Then, the following event-triggered consensus scheme is given as:

$$\Pi_7 : \begin{cases} \text{Protocol: } u_i(t) = -K \sum_{j \in N_i} w_{ij} (\hat{x}_i(t_k^{ij}) - \hat{x}_j(t_k^{ij})) \\ \text{ETC: } f_i(\tilde{z}_{ij}(t)) \leq \Delta_i(\hat{z}_{ij}(t), t) + \delta_i. \end{cases} \quad (15)$$

When the ETC in (15) is violated at times  $t_k^{ij}$ , agent  $i$  will compute its control input  $u_i(t_k^{ij})$  and then send it to its respective neighbors for estimating according to (14). Based on this scheme, both complete and bounded synchronization for MASs under an undirected and fixed network topology are investigated in [96] by setting the state-independent threshold functions with  $\delta_i = 0$  or  $\delta_i \neq 0$ , respectively. For switching networks, an event-triggered pinning control problem under (15) is studied in [95].

Analogous to (9) and (13), the implementation of (15) can ensure that information between neighbors is only exchanged at each event time, which is beneficial to communication resources saving. In addition, different from (9) with continuous control signals, the scheme (15) only uses



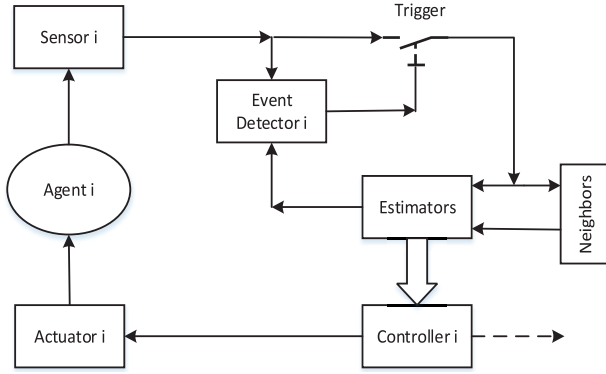


Fig. 3. Sampled-data-based event-triggered mechanism for agent  $i$  [60].

piecewise-constant signals for control updates, further reducing consumption of computation resources. From (14), one can see clearly that each agent requires its neighbors' control input values at event times to predict the states of its neighbors. In general, information of controller updates for each agent is regarded as being private and may be inaccessible to other agents. This will make it impossible to carry out the event-triggered scheme (15) in some practical applications.

### C. Sampled-Data-Based Event-Triggered Schemes

Note that the event-triggered schemes mentioned in Sections III-A and III-B have one common feature that the triggering conditions are needed to be constantly checked and computed. On the one hand, it is because of such real-time detection and computation that these event-triggered schemes in essence cannot accomplish the purpose of reducing the computational resources in entire systems, even though the control updates are only triggered at event times. On the other hand, undertaking the continuous event detection requires support from extra hardwares, which definitely increases costs of system design, operation, and maintenance. In order to overcome this drawback, motivated by [45], a distributed sampled-data-based event-triggered consensus protocol is proposed in [60], whose framework is shown in Fig. 3. The core idea of the sampled-data-based event-triggered strategy is that the event detection is only carried out at sampling times rather than at continuous times. Besides, the minimum of interevent times is inherently lower bounded by one sampling period, implying that Zeno behavior is absolutely excluded [97].

To be more specific, it is assumed that the state of each agent  $i$  is sampled by a sampler at a constant sampling period  $h > 0$ . The measurement error of sampled-data at the  $k$ th sampling time is defined as  $e_i(kh) \triangleq x_i(kh) - x_i(t_m^i h)$ ,  $t_m^i \leq k < t_{m+1}^i$ . Then, a sampled-data-based event-triggered consensus scheme is given by [60]

$$\Pi_8 : \begin{cases} \text{Protocol:} \\ u_i(t) = -K \sum_{j \in N_i} w_{ij} \left( x_i(t_m^i h) - x_j(t_{m_j'}^j h) \right) \\ \text{ETC: } f_i(e_i(kh)) \leq \Delta_i(\hat{z}_i(kh)) \end{cases} \quad (16)$$

where  $\hat{z}_i(kh) = \sum_{j=1}^N w_{ij} [x_i(t_m^i h) - x_j(t_{m_j'}^j h)]$  with  $m_j' \triangleq \arg \min_p \{t_m^i + l_i - t_p^j + l_j \geq t_p^j, p \in \mathbb{N}\}$ . Whether or not

sampled-data of agent  $i$  should be broadcast or used at the sampling instant  $kh$  ( $k \in \mathbb{N}$ ) depends on when its ETC is violated. It is clear to know from (16) that, at the  $k$ th sampling instant, the event-triggered condition for agent  $i$  is closely related to the sampled-data error  $e_i(kh)$  and the sampled-data  $\hat{z}_i(kh)$  including the latest transmitted sampled-data  $x_i(t_m^i h)$  of agent  $i$  and the latest transmitted sampled-data  $x_j(t_{m_j'}^j h)$  of its neighbors. If the ETC in (16) is satisfied, the sampled-data of agent  $i$  is not needed to be transmitted to its neighbor agents. It is worth mentioning that, when the ETC in (16) is always violated at each sampling time, it means that the sampled-data of agent  $i$  at each sampling time is required to be broadcasted. In this case, the event-triggered control scheme (16) is reduced to the standard periodic sampled-data one. In this sense, it is clear that the sampled-data-based event-triggered scheme (16) has remarkable advantages in reducing frequency of both sampled-data transmission and control input updates compared with a traditional periodic sampling scheme.

Under the event-triggered scheme (16), sufficient conditions guaranteeing average consensus of multiple single integrators over fixed or switching undirected and connected communication topology [58], [98] are derived by using the Lyapunov theory. For linear MASs with a directed graph containing a spanning tree, the sampled-data-based event-triggered consensus problem is dealt with in [60], where the ETC is set as  $e_i^T(kh) \Phi e_i(kh) \leq \sigma_i \hat{z}_i^T(kh) \Phi \hat{z}_i(kh)$  with  $\sigma_i > 0$  being a threshold parameter and  $\Phi > 0$  being a weighting matrix. By dividing the event intervals, a closed-loop error system with time delays is obtained. Then, by using a Lyapunov–Krasovskii method, the MAS (1) achieves asymptotic consensus. Along the method used in [60], an adaptive event-triggered scheme is proposed in [99], where the threshold parameters  $\sigma_i$  are no longer static but dynamically varying with the relative sampled-data errors.

Since there is a tradeoff between control performance and resource utilization, how to design the suitable controller gain and the threshold parameters in event-triggered control scheme (16) becomes one important issue. Another contribution of [60] is to address this issue by introducing an average transmission rate  $J$  on a finite interval  $[0, Th)$  as

$$J \triangleq \frac{1}{T \sum_{i=1}^N \mathcal{N}_i} \sum_{k=0}^{T-1} \sum_{i=1}^N \mathcal{N}_i \rho_k^i \quad (17)$$

where  $T$  is the sampling number,  $\mathcal{N}_i$  is the total sampled-data number of the agent  $i$  broadcasted to other agents and

$$\rho_k^i = \begin{cases} 1, & \text{if } x_i(kh) \text{ is transmitted} \\ 0, & \text{otherwise} \end{cases} \quad (18)$$

stands for the sampled-data transmission signal of agent  $i$  at each sampling time  $kh$ . Consequently, the co-design issue comes to simultaneously designing the parameters  $\sigma_i$  and  $\Phi$  and the controller gain  $K$  by minimizing  $\varrho = |J - J^*|$ , where  $J^*$  is the expected average transmission rate. It should be emphasized that there are few related results considering the co-design of consensus controller and the threshold parameters of ETCs except [60].

Recently, sampled-data-based event detections are also introduced to edge-event triggered control of

TABLE I  
ADVANTAGES AND DISADVANTAGES OF EVENT-TRIGGERED CONTROL SCHEMES

Schemes	Related literature	Advantages & Disadvantages				
		L1)	L2)	L3)	Event monitoring	Analysis of Zeno behavior
Event-Based Sampling Schemes	$\Pi_1$ : [54], [69]	×	×	×	Continuous	Complicated
	$\Pi_2$ : [56], [57], [70]–[76], [78], [79]	✓	×	×	Continuous	Complicated
	$\Pi_3$ : [58], [66], [80]–[82], [99]	×	✓	×	Continuous	Complicated
	$\Pi_4$ : [55], [83]	×	✓	✓	Continuous	Simple
Model-Based Event-Triggered Schemes	$\Pi_5$ : [59], [67], [86]–[93]	×	✓	✓	Continuous	Complicated
	$\Pi_7$ : [95], [96]	×	✓	✓	Continuous	Complicated
Sampled-Data-Based Event-Triggered Schemes	$\Pi_8$ : [60], [98], [100]–[106]	×	✓	×	Discrete	Not needed
Self-Triggered Sampling Schemes	$\Pi_9$ : [54], [68], [107], [108]	×	✓	✓	Discrete	Not needed

If the scheme can overcome the limitation, it is marked by ✓, otherwise, by ×.

MASs [100]–[106]. Under such a periodic edge-event triggered scheme, average consensus of single-integrator networks is investigated in [100], which is extended to some more complicated cases with measurement errors, quantized data, and time delays in [102]. For double-integrator networks, edge event hybrid-driven rules to ensure state consensus are, respectively, proposed in a leader-following framework [103] and in a leaderless framework [109]. Furthermore, consensus of double-integrator networks with communication delays is achieved in [104] and [105] by using periodic edge-event triggered schemes. It should be pointed out, the existing results on edge-event triggered control schemes are mainly concerned with undirected single/double-integrator networks and it remains an open issue to develop a suitable periodic edge-event triggered scheme for general linear or nonlinear MASs with directed network topologies.

While the sampled-data-based event-triggered scheme shows its appealing advantages in preventing continuous event monitoring and precluding Zeno behavior, its implementation may suffer from some restrictions listed below.

- 1) In some existing results [60], [98], [100]–[106], all the agents are assumed to be synchronously sampled at a constant sampling period. In practice, such sampling clock synchronization is very difficult to implement for large-scale networks. Thus, it is more practical to use sampled-data-based event-triggered schemes in which each agent has its own sampling period.
- 2) Since events are only detected at sampling times, the sampled-data-based event-triggered schemes will inherit the properties of sampled-data systems, especially including their shortcomings. For instance, some useful states may be ignored in the process of sampling when they are fluctuating between a large range.

#### D. Self-Triggered Sampling Schemes

Besides sampled-data-based event-triggered schemes, another effective policy avoiding continuous event monitoring is self-triggered sampling, where the next sampling time is predicted at control update instants based on the last triggered data and knowledge of plant dynamics [39], [110]–[114]. Following this idea, a distributed self-triggered control

scheme is presented in [54] to achieve asymptotic consensus of single-integrator MASs and the next time  $t_{k+1}^i$  is computed by

$$t_{k+1}^i = t_k^i + h_i(x_i(t_k^i), x_j(t_{k_j(t)}^j)) \quad (19)$$

where  $h(\cdot)$  is the function depending on agent's and its neighbors' states at their last event times. In [54], it is demonstrated that the self-triggered sampling scheme results in more controller updates than the event-triggered scheme, but it seems more robust. This is mainly because the self-triggered sampling scheme includes the over-approximation by individual agent on the state of environment and the network [108]. It is also pointed out in [54] that zero interexecution times, i.e.,  $h_i = 0$ , are allowed when the local average value of an agent approach to zero in finite time. In order to solve this problem, a self-triggered algorithm with Zeno free triggers is given in [68]. Moreover, compared with [54], the self-triggered scheme in [68] can provide a simpler and more efficient way to compute and determine the events. Self-triggered practical consensus with ternary controllers is achieved in [107] and it is verified that such a self-triggered scheme has high robustness against various uncertainties and disturbances, such as inaccuracy of clock, delays, and restrictions in data rates. Taking advantage of both the strengths of event- and self-triggered control, a so-called team-triggered coordination scheme is presented in [108] for networked cyber-physical systems.

#### E. Summary

In this section, we mainly review four types of the event-triggered schemes proposed in the literature, namely, event-based sampling schemes, model-based event-triggered schemes, sampled-data-based event-triggered schemes, and self-triggered sampling schemes. Based on the review above, the key characteristics of these schemes are listed in Table I, which can explicitly shows the advantages and disadvantages of the event-triggered schemes. From Table I it is easy to know that, although various event-triggered schemes for MASs have been presented in the literature to reduce consumption of resources, there are still many critical issues to be further addressed in the future work.



#### IV. PRACTICAL EXAMPLES OF EVENT-TRIGGERED CONSENSUS STRATEGIES

In this section, we provide a brief discussion on how an event-triggered consensus algorithm is applied to deal with some seemingly different problems but closely to consensus in various areas which have gained increasing interest from researchers in recent years.

##### A. Distributed Event-Triggered Power Sharing in Microgrids

A microgrid, as a new-style power grid with integration of distributed generation (DG), smart infrastructure, advanced communication, and management technology, has gained much attention due to its advantages of high reliability, efficiency sustainability. Since a microgrid usually includes a large number of DGs, achieving power control and management of microgrids in a centralized way is usually costly in communication and computation. To reduce the cost and to enhance expandability, distributed control strategies have been introduced into microgrids. For example, in [115] an MAS-based hierarchical hybrid control is initially proposed to solve optimal control of smart microgrids, which is more flexible and more effective than most existing methods. Moreover, the above method is further extended in [116] and [117] to solve security control of hybrid energy generation systems by using a security-evaluation-based event-triggered scheme, which is verified to have remarkable efficiency in controlling complex power systems, especially with a wider access of new energy, such as PV and wind power. In a microgrid, a fundamental concern is how to efficiently dispatch power supply of DGs to distributed loads, which refers to power sharing issues including economic dispatch [118]–[120] and proportional power sharing [12], [13]. Considering limited bandwidth of communication network, distributed event-triggered communication schemes are introduced in [14], [121], and [122] to achieve power sharing in microgrids. In what follows, we briefly present a recent case study on distributed event-triggered reactive power sharing in microgrids.

Consider a Kron-reduced microgrid consisting of  $N$  DGs interfaced via an ac inverter, where each DG can be viewed as a node of undirected and connected graph  $\mathcal{G}$ . Nodes  $i$  and  $m$  are connected by a physical power line with a complex admittance  $Y_{im}$ . It is assumed that all the power lines are lossless, i.e., admittances of all lines are purely inductive. Then, we denote the admittance between node  $i$  and  $m$  by  $Y_{im} = jB_{im}$ , where  $B_{im} < 0$  is the inductive susceptance. If there is no connection between  $i$  and  $m$ , then  $Y_{im} = 0$ . At node  $i$ , the apparent power flow is represented by  $S_i = P_i + jQ_i$ , where  $P_i$  and  $Q_i$  are the active and reactive power, respectively. It is well known that the reactive power distribution can be achieved by voltage control. By means of model simplification, the  $Q$ – $V$  dynamics for node  $i$  are given by

$$\dot{V}_i = u_i \quad (20)$$

$$Q_i = |B_{ii}|V_i^2 - \sum_{m \in \mathcal{N}_i} |B_{im}|V_i V_m \quad (21)$$

where  $V_i$  is the voltage amplitude of node  $i$ ;  $u_i$  is the voltage control input;  $B_{ii}$  is the shunt susceptance at node  $i$  and

$\mathcal{N}_i$  is the neighbor set of node  $i$  in the electrical network  $\mathcal{G}$ . The simplified form (21) of reactive power flow  $Q_i$  is obtained by using a standard decoupling assumption [13]. The microgrid is said to achieve proportional reactive power sharing if  $(Q_i/\chi_i) = (Q_m/\chi_m)$ , where  $Q_i^s$  and  $Q_m^s$  are the reactive power flows at the steady states for node  $i$  and  $m$ , respectively, as well as  $\chi_i$  and  $\chi_m$  are the proportional coefficients.

Let the graph  $G$  be a communication topology. The communication network topology  $G$  can be identical but not necessarily to the electrical counterpart  $\mathcal{G}$ . For (20) and (21), a distributed event-triggered voltage control law for proportional reactive power sharing is given in [14] by

$$u_i(t) = -\frac{\kappa}{\chi_i} V_i(t) \sum_{m \in \mathcal{N}_i} w_{ij} \left( \frac{Q_i(t_{k_i}^i)}{\chi_i} - \frac{Q_m(t_{k_m}^m)}{\chi_m} \right) \quad (22)$$

where  $k_i(t) \triangleq \arg \max_k \{t_{k_i}^i \leq t\}$  and  $\mathcal{N}_i$  is the neighbor set of node  $i$  in the communication network  $G$ . The ETC determining the event times  $t_{k_i}^i$  of node  $i$ , is designed as [14]

$$|e_i(t)r| \leq \eta \chi_i \left| \sum_{m \in \mathcal{N}_i} w_{ij} \left( \frac{Q_i(t_{k_i}^i)}{\chi_i} - \frac{Q_m(t_{k_m}^m)}{\chi_m} \right) \right| \quad (23)$$

where  $e_i(t) = Q_i(t_k^i) - Q_i(t)$  and  $0 < \eta < \lambda_N$  with  $\lambda_N$  being the maximum eigenvalues of Laplacian matrix of the communication topology  $G$ . Clearly, the voltage controller updates (22) and the ETC (23) are formally similar to (6). By the simulation of [14], it is verified that the event-triggered scheme (22) and (23) is able to efficiently decrease the total number of communication in microgrids while maintaining the nearly identical precision for reactive power sharing as that in a periodic sampling scheme. However, it is noteworthy that the voltage controller input (22) of node  $i$  is continuously updated since it needs the real-time voltage measurement  $V_i(t)$ , which shows the limitation of the event-triggered scheme in [14].

##### B. Distributed Event-Triggered Formation Control of Multirobot Systems

Recently, formation control of multirobot systems, aiming at driving a group of robots to realize and preserve a desired geometric structure, has been well explored in both robotics and control communities [4]. Up till now, several approaches have been developed to tackle the problem of multiagent formation control in the literature, such as the behavior-based approach [123], the virtual structure approach [124], [125], the leader-following approach [126], [127], and the consensus approach [4]–[6]. One of the major challenges for multirobot formation lies in how inter-robot communication should be performed such that information among the robots can be shared and exchanged efficiently in terms of time and energy. In order to overcome this challenge, event-triggered control scheme is employed to regulate the frequency of inter-robot information exchange. In what follows, from a control perspective, we briefly describe an application of event-triggered consensus strategies to deal with the formation control problem of a multirobot system.

Consider a team of nonholonomic mobile robots of unicycle type. The  $i$ th robot's kinematic model is represented by [6], [126], [128]

$$\dot{\bar{x}}_i(t) = v_i(t) \cos(\phi_i(t)), \quad \dot{\bar{y}}_i(t) = v_i(t) \sin(\phi_i(t)), \quad \dot{\phi}_i(t) = \omega_i(t)$$

for any  $i \in \mathcal{V} = \{1, 2, \dots, N\}$ , where  $[\bar{x}_i(t), \bar{y}_i(t)]^T \in \mathbb{R}^2$  is the Cartesian coordinates of the center of mass;  $v_i(t) \in \mathbb{R}$  is the linear velocity;  $\phi_i(t) \in \mathbb{R}$  is the heading angle in the inertial frame; and  $\omega_i(t) \in \mathbb{R}$  is the angular velocity. Denote  $x_i(t) = [\bar{x}_i(t), \bar{v}_i^x(t), \bar{y}_i(t), \bar{v}_i^y(t)]^T$  and  $u_i(t) = [\bar{u}_i^x(t), \bar{u}_i^y(t)]^T$  along the  $x$  and  $y$  axes. The above multirobot model can be represented by (1) using a dynamic feedback linearization method [126], where the system parameter matrices in (1) are given by  $A = I_2 \otimes \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$  and  $B = I_2 \otimes [0, 1]^T$ . Denote an augmented column vector by  $F = [f_1^T, f_2^T, \dots, f_N^T]^T \in \mathbb{R}^{4N}$ , where  $f_i \in \mathbb{R}^4$ ,  $i \in \mathcal{V}$ , is a formation vector of robot  $i$ . The multirobot system is said to be formable specified by  $F$  if  $\lim_{t \rightarrow \infty} \{x_i(t) - x_j(t)\} = (f_i - f_j)$ ,  $i \in \mathcal{V}$ , for any given bounded initial condition [4]. Define  $z_i(t) = x_i(t) - f_i$ . It follows that the multirobot system is formable if  $\lim_{t \rightarrow \infty} \{z_i(t) - z_j(t)\} = 0$ ,  $i \in \mathcal{V}$ , for any given bounded initial. Thus, the distributed formation control problem for the above multirobot system reduces to a consensus problem with regard to  $z_i(t)$  for all  $i \in \mathcal{V}$ .

In [4], the following formation control protocol is studied:

$$u_i(t) = K \sum_{j \in N_i} w_{ij} \left( z_i(t_k^i h) - z_j(t_{k_j}^j h) \right) + H f_i$$

for any  $i \in \mathcal{V}$  and  $t \in [t_k^i h, t_{k+1}^i h)$ , where  $x_j(t_{k_j}^j h)$  denotes the recently transmitted local measurement received from robot  $j$ 's neighbor  $k_j$  with  $k_j \triangleq \arg \min_k \{\vartheta - t_k^j \mid \vartheta > t_k^j, k \in \mathbb{N}\}$  and  $h > 0$  representing the sampling period. The consensus term  $z_i - z_j$  stands for the desired offset committed by robot  $i$  and its neighbors, enabling one to adopt the desired formation information (e.g., relative position) between robots. To schedule inter-robot communication, the following dynamic event-triggered communication mechanism is proposed for robot  $i$  in [4] so as to determine whether robot  $i$ 's current measurement  $x_i(\vartheta h)$  should be transmitted to its neighboring robots

$$t_{k+1}^i h = \inf \{ \vartheta h > t_k^i h \mid f_i(\vartheta h) > 0, \vartheta = t_k^i, \dots, t_{k+1}^i - 1 \}$$

where  $f_i(\vartheta h) = e_i^T(\vartheta h) \Psi e_i(\vartheta h) - \xi(\vartheta h) x_i^T(t_k^i h) \Phi x_i(t_k^i h)$  with  $e_i(\vartheta h) = x_i(t_k^i h) - x_i(\vartheta h)$ ;  $\Psi$  and  $\Phi$  are positive symmetric weighting matrices to be designed; and  $\xi(\vartheta h)$  is a dynamic threshold parameter whose value is computed according to the following dynamic rule:

$$\xi(kh) = \xi(kh - h) - \theta \xi(kh) \xi(kh - h) e_s(kh - h)$$

with  $e_s(kh - h) = \sum_{i=1}^N e_i^T(kh - h) \Psi e_i(kh - h)$ ;  $\theta$  denoting a given positive scalar; and  $\sigma(0) \in [0, 1)$  representing the initial condition. In [4], it is proved that the above dynamic threshold parameter is monotonically nonincreasing and satisfies  $0 \leq \xi(kh) \leq \xi(0) < 1$ . Then, applying a model transformation, the event-triggered formation control problem is converted into an asymptotic stability problem of a reduced-order system. It is also shown in [4] that, by using the dynamic event-triggered

communication mechanism, one may find a better tradeoff between preserving formation and/or consensus performance and reducing inter-robot communication frequency than using some existing static counterparts.

## V. FUTURE RESEARCH PROSPECT

An overview of multiagent event-triggered consensus control has been provided. Although some event-triggered control issues of MASs have been well addressed in the literature, there are still limitations from strict assumptions and special requirements, which potentially bring some space for improvement over the existing results and methodologies. In what follows, some important and yet challenging research topics worthy of further investigations are suggested.

- 1) *Dynamic Event Triggering Mechanism*: It has been shown in [129]–[131] that the proposed dynamic triggering mechanism, wherein the threshold involves an internal dynamic variable, can allow for the larger minimum interevent times than a static counterpart. However, it becomes much more difficult when dynamical event triggering mechanisms are applied to deal with consensus control for MASs due to the fact that ETCs generally involve coupled information among the agents while requiring to be implemented in a fully distributed fashion. Up to date, it remains an open issue on distributed dynamic event-triggered schemes for MASs.
- 2) *Finite-Time Event-Triggered Consensus*: It is noteworthy that most existing event-triggered schemes guarantee asymptotic consensus of MASs. In practice, as convergence rate is a significant performance metric of the proposed consensus protocols, it is much expected that practical MASs can achieve event-triggered consensus in a finite time, referred to as finite-time event-triggered consensus [132]. Nevertheless, an event-triggered consensus scheme aims to reduce the sampling actions and/or control updates, which in turn may decrease the convergence rate. Hence, it would be a promising topic to design a suitable finite-time event-triggered consensus protocol for MASs, which ensures a fast convergence rate while decreasing utilization of computation and communication resources. Furthermore, the settling time of the finite-time consensus cannot be preset since it closely depends on both the initial conditions and some design parameters. Therefore, it is more desirable to develop a *fixed-time event-triggered consensus protocol* such that MASs can reach consensus within a preset settling time while reducing resource usage.
- 3) *Event-Triggered Consensus in the Presence of Stochasticity*: In practical MASs, the stochasticity may exist in many different forms, such as stochastic process noise, stochastic measurement noise, stochastic communication noise, stochastic communication topologies, and so forth. These stochastic phenomena pose significant challenges for the event-triggered consensus control of MASs. This is because that most studies of event-triggered consensus are concerned with deterministic agent dynamics and/or deterministic

communication channels, thus the consensus protocols therein are no longer applicable due to the existence of stochasticity. To the best of our knowledge, the problem of event-triggered consensus under stochastic phenomena has been not been adequately investigated.

- 4) *Event-Triggered Consensus and Distributed Optimization of Constrained MASs*: Recently, distributed optimization in MASs, which aims at finding an optimal control strategy subject to some given constraints concerning consensus convergence speed or some specific cost functions, has sparked considerable interest due to its wide applications in areas, such as power control, sensor networks, and source localization [133]–[135]. However, most of the existing results along this line of research have not considered issues on resource utilization of communication and computation. Essentially, it would be challenging to explicitly reveal the relationship between constrained objective functions and utilization of resources in multiagent optimization problems, which deserves deep investigation.

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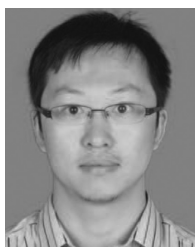
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**Lei Ding** received the B.Sc. degree in automation, and the M.Sc. and Ph.D. degrees in control theory and control engineering from Dalian Maritime University, Dalian, China, in 2007, 2009, and 2014, respectively.

From 2010 to 2012, he was a visiting Ph.D. student sponsored by China Scholarship Council, Centre for Intelligent and Networked Systems, Central Queensland University, Rockhampton, QLD, Australia. From 2015 to 2016, he was a Visiting Research Fellow with Western Sydney University, Sydney, NSW, Australia. From 2016 to 2017, he was a Post-Doctoral Fellow with Wayne State University, Detroit, MI, USA. Since 2017, He has been a Visiting Research Fellow with the Swinburne University of Technology, Melbourne, VIC, Australia. His current research interests include event-triggered control and distributed cooperative control of multiagent systems and their applications to smart grids.



**Qing-Long Han** (M'09–SM'13) received the B.Sc. degree in mathematics from Shandong Normal University, Jinan, China, in 1983, and the M.Sc. and Ph.D. degrees in control engineering and electrical engineering from the East China University of Science and Technology, Shanghai, China, in 1992 and 1997, respectively.

From 1997 to 1998, he was a Post-Doctoral Researcher Fellow with the Laboratoire d'Automatique et d'Informatique Industrielle (currently, Laboratoire d'Informatique et d'Automatique pour les Systèmes), École Supérieure d'Ingénieurs de Poitiers (currently, École Nationale Supérieure d'Ingénieurs de Poitiers), Université de Poitiers, Poitiers, France. From 1999 to 2001, he was a Research Assistant Professor with the Department of Mechanical and Industrial Engineering, Southern Illinois University, Edwardsville, IL, USA. From 2001 to 2014, he was a Laureate Professor, an Associate Dean (Research and Innovation) with Higher Education Division, and the Founding Director of the Centre for Intelligent and Networked Systems, Central Queensland University, Rockhampton, QLD, Australia. From 2014 to 2016, he was the Deputy Dean (Research) with the Griffith Sciences, and a Professor with the Griffith School of Engineering, Griffith University, Gold Coast, QLD, Australia. In 2016, he joined the Swinburne University of Technology, Melbourne, VIC, Australia, where he is currently a Pro Vice-Chancellor (Research Quality) and a Distinguished Professor. In 2010, he was appointed as a Chang Jiang (Yangtze River) Scholar Chair Professor by the Ministry of Education China, Beijing, China. His current research interests include networked control systems, neural networks, time-delay systems, multiagent systems, and complex dynamical systems.

Prof. Han was a recipient of the Highly Cited Researcher Award in Engineering by Thomson Reuters. He is one of The World's Most Influential Scientific Minds: 2014–2016. He is an Associate Editor of a number of international journals including the IEEE TRANSACTIONS ON INDUSTRIAL ELECTRONICS, the IEEE TRANSACTIONS ON INDUSTRIAL INFORMATICS, the IEEE TRANSACTIONS ON CYBERNETICS, and *Information Sciences*.



**Xiaohua Ge** (M'18) received the B.Eng. degree in electronic and information engineering from Nanchang Hangkong University, Nanchang, China, in 2008, the M.Eng. degree in control theory and control engineering from Hangzhou Dianzi University, Hangzhou, China, in 2011, and the Ph.D. degree in computer engineering from Central Queensland University, Rockhampton, QLD, Australia, in 2014.

He was a Research Assistant with the Centre for Intelligent and Networked Systems, Central Queensland University, from 2011 to 2013, where he was a Research Fellow with the Centre for Intelligent and Networked Systems, in 2014. From 2015 to 2016, he was a Research Fellow with the Griffith School of Engineering, Griffith University, Gold Coast, QLD, Australia. In 2017, he joined the Swinburne University of Technology, Melbourne, VIC, Australia, where he is currently a Lecturer with the School of Software and Electrical Engineering. His current research interests include distributed networked control systems, multiagent systems, and sensor networks.



**Xian-Ming Zhang** (M'16) received the M.Sc. degree in applied mathematics and the Ph.D. degree in control theory and engineering from Central South University, Changsha, China, in 1992 and 2006, respectively.

In 1992, he joined Central South University, where he was an Associate Professor with the School of Mathematics and Statistics. From 2007 to 2014, he was a Post-Doctoral Research Fellow and a Lecturer with the School of Engineering and Technology, Central Queensland University, Rockhampton, QLD, Australia. From 2014 to 2016, he was a Lecturer with the Griffith School of Engineering, Griffith University, Gold Coast, QLD, Australia. In 2016, he joined the Swinburne University of Technology, Melbourne, VIC, Australia, where he is currently a Senior Lecturer with the School of Software and Electrical Engineering. His current research interests include H-infinity filtering, event-triggered control systems, networked control systems, neural networks, distributed systems, and time-delay systems.

Dr. Zhang was a recipient of the National Natural Science Award (Level 2) in China in 2013, and the Hunan Provincial Natural Science Award (Level 1) in Hunan Province in China in 2011, both jointly with Prof. M. Wu and Prof. Y. He, and the IET Premium Award in 2016, jointly with Prof. Q.-L. Han. He serves as an Associate Editor for the *Journal of the Franklin Institute* and an Editorial Board Member of *Neural Computing and Applications*.