Development of an Intelligent Perception System for an Automotive Brake-by-Wire System

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A thesis submitted to the Faculty of Engineering and Industrial Sciences, Swinburne University of Technology, in fulfilment of the requirements for the degree of Doctor of Philosophy, 2009.
I declare that:

This thesis contains no material which has been accepted for the award of any other degree or diploma, except where due reference is made in the text of the thesis. To the best of my knowledge, this thesis contains no material previously published or written by another person except where due reference is made in the text of the thesis.

Name:..................................................

Signed:..........................................

Date:.............................................
Dedicated to my loving family and friends
Abstract

This thesis considers the problem of estimating clamp force in an electro-mechanical brake (EMB) for an automotive brake-by-wire (BBW) system. A clamp force sensor is typically used in EMB designs and the elimination of this component is strongly demanded due to implementation difficulties and cost issues. The motivation behind this thesis is to make developmental inroads into the deficiencies provided by previous attempts to estimate clamp force. Previous attempts have deficiencies for high speed braking applications as well as handling thermally dependent parameter variations.

A dynamic stiffness model to estimate clamp force is developed that relies on the actuator resolver sensor and two additional temperature sensors. Previous attempts to estimate clamp force have been stiffness based and have not been capable of successfully modelling parameter variations in response to heating. This thesis introduces new developments on how to model stiffness parameter variations under the influence of heating. Two temperature sensors are required to be employed in this new approach. These additional sensors will not have a considerable impact towards the cost savings created by omitting a clamp force sensor. A torque balance model to estimate clamp force is also developed that relies on the actuator resolver sensor and actuator motor current sensors. A training strategy is used for the dynamic stiffness and torque balance models to estimate clamp force so that wear dependent parameters can be adapted.

The two independent models to estimate clamp force are fused using various sensor fusion algorithms to give improved estimates of clamp force. A maximum-likelihood estimator is used to optimize the root-mean-square error (RMSE) of estimation. This is followed by implementing a Kalman filter to estimate clamp force in an EMB. The dynamic stiffness model is used as the state space equation in the Kalman filter, whilst the torque balance model is used to give measurement updates. Experimental verification showed that the Kalman filter was more accurate than the maximum-likelihood estimator as expected, however the Kalman filter required more computational burden. A RMSE of 0.5 kN was attained for the Kalman filter and 0.56 kN for the maximum-likelihood estimator.
Acknowledgments

Many people provided valuable support in helping to produce this thesis. I would particularly like to thank Associate Professor Alireza Bab-Hadiashar for the privilege of receiving his invaluable supervisory guidance. Many helpful insights were received from him and his active involvement shaped the outcome of this thesis.

I would like to thank Dr. Reza Hoseinnezhad, my second supervisor, who kindly took the time to help me become competent with relevant software and programming. He also proof-read many of my papers that emerged from this thesis.

The technical support given by Dr. Johannes van der Walt with regards to the thermal aspects of this thesis ensured that steady progress was maintained. I am thankful for his generous assistance.

During my PhD studies, I received an Australian Postgraduate Research Industry (APAI) scholarship from the Australian Research Council (ARC) under Linkage Grant LP0349130. I am thankful for being given this opportunity. The industry partner was Pacifica Group Technologies (PGT). The Research Centre for Advanced By-Wire Technologies (RABiT) provided the medium under which this collaborative work could be undertaken between Swinburne University of Technology (SUT) and PGT.

I would like to thank all my friends and colleagues at SUT and PGT. I am grateful for all the technical support provided by the engineers at PGT, in particular with regards to setting up test rigs.

Finally, the love and support I received from my family and friends was a constant motivating factor and I am forever grateful for this.
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1
Introduction

1.1 Brake-by-Wire System

One of the primary intentions for the introduction of drive-by-wire (DBW) technologies into the automotive industry has been to ultimately develop intelligent vehicle control systems that improve performance by benefiting from the integration of electronic systems [15, 37]. Throttle-by-wire (TBW) is a DBW technology currently in use. Other DBW technologies such as BBW and steer-by-wire (SBW) are still in the development phases. DBW technologies are discussed further in [11, 20, 23, 59]. DBW is also intended to improve actuation response times by replacing mechanically actuated systems used in conventional vehicles [31]. Design and implementation of BBW systems has been focused upon by researchers and industry experts [5, 16, 18, 24-26, 32-33, 49-51, 56, 61-62, 65]. Figure 1-1 shows a schematic diagram of a typical BBW system. The human-machine interface in a BBW system is provided by a pedal.

*Figure 1-1. BBW system.*
feel emulator. Such a pedal is equipped with sensors that indicate the level of brake demand required by a driver. The output signals from these sensors are processed by an electronic control unit that appropriately controls the actuators. A high level of safety is employed in BBW systems to ensure fault tolerant operations [1, 19, 22, 27, 69].

There are 2 actuation designs that are preferred by BBW system developers. The first involves the use of electro-hydraulic components that retain many of the hydraulic mechanisms adopted in conventional vehicles. Here an electric motor driven pump, in conjunction with proportioning valves, provides the method of brake control to each wheel. Due to the convenience of using existing parts, this concept is the first proposed approach for implementation of a BBW system in the automotive industry [28]. The second BBW approach, which is of sole interest in this thesis from this point forth, reduces weight and is more environmentally friendly (due to brake fluid omission) than electro-hydraulic technologies. This scheme uses an electric motor drive coupled to a reduction gear set-up to provide brake control to each wheel. The motor is typically a 3 phase permanent magnet brushless DC type for the purpose of compactness and improved commutation efficiency. The reduction gearing generally consists of a planetary gear-train connected to a ball-screw that can generate clamp forces of up to 50 kN. In practice an automotive brake is expected to operate within an approximately 0-40 kN range; around 5 kN for normal vehicle braking, and up to 40 kN for panic braking [17]. Figure 1-2 displays a sectioned view of a floating EMB being developed at PGT.

![Figure 1-2. EMB from Pacifica Group Technologies (PGT).](image)

1. stator field winding
2. brake pads
3. ball-screw
4. planetary gear-train
5. thrust bearing
6. clamp force sensor location
7. resolver sensor location
8. permanent magnet rotor location
9. load distribution plate
10. nut
11. aluminium caliper bridge
An EMB uses a clamp force sensor to close a loop for the purpose of controlling dynamic performance. The control of an EMB with an internal clamp force sensor can be achieved using a standard motion control architecture adopted for servomotors (cascaded position, velocity and current control loops) that is slightly modified to suit the application at hand. Line et al. [37] replace the position control loop with a force control loop to control an EMB. This architecture is shown in Figure 1-3. A resolver sensor is typically used to sense velocity. A resolver sensor is an absolute position sensing device which allows for efficient and smooth control schemes to be implemented for EMB designs [34]. An alternative control architecture that is less expensive to implement uses Hall sensors which replace the resolver sensor. The drawbacks with this scheme are that the resolution is lower which leads to inefficient commutation and torque ripples [34]. Control architectures that utilize a resolver sensor in an EMB tend to be the most prevalent. It is this control architecture which is of interest in this thesis from this point forth. Previous control developments in EMB systems can be found in [39].

![Figure 1-3. An EMB control architecture.](image)

1.2 Problem Statement

A clamp force sensor is a relatively expensive component in an EMB (approximately $ 7-10 AUD). The cost is derived from its high unit value from a supplier, as well as significant production expenses due to its inclusion. The latter arises from the complex assembly procedures dealing with small tolerances, as well as online calibration for performance variability amongst clamp force sensors. The successful use of a clamp force sensor in an EMB poses a challenging engineering task. If a clamp force sensor is placed close to a brake pad, it will then be subject to severe temperature conditions reaching up to 800 °C that will challenge its mechanical integrity. This situation can be avoided by embedding a clamp force sensor deep within the EMB, i.e. at the near end of the ball-screw, view Figure 1-2. It has been shown that embedding this sensor leads to hysteresis that is influenced by friction between the clamp force
sensor and the pad and disc interface [54]. This hysteresis significantly influences the accuracy of clamp force measurement.

Due to the cost issues and engineering challenges involved with including the clamp force sensor, it is highly desirable to eliminate this component from an EMB. A potential opportunity to achieve this presents itself in the development of a virtual sensor. That is, to accurately estimate the clamp force based on alternative EMB system sensory measurements leading to the omission of a clamp force sensor. Figure 1-3 shows that motor current sensors as well as a velocity sensor (typically in the form of a resolver sensor) are part of commonly used control architectures for an EMB. Such sensors have been previously used to help estimate clamp force in an EMB [54]. Previous efforts however have limitations for certain scenarios. That is, highly dynamic braking causes significant inaccuracies in clamp force estimation. This is due to dynamics not being considered in model structures which will be detailed in Chapter 3. Previous efforts also require real time parameter adaptation techniques to be employed during braking so that the estimated clamp force continually tracks the true clamp force. These adaptation techniques cannot be applied for highly dynamic braking. Considering that the adaptation techniques can only be applied during braking, they also fail to sense the parameter variations that occur during non-braking cooling periods.

The motivation for this thesis has emerged from the short comings of existing methods to estimate clamp force for an EMB actuator in automotive BBW systems. Therefore, further developments on this matter would be considered a significant contribution to state of the art knowledge. Also, any new developments on an EMB actuator could provide useful material for other research topics on this emerging technology.

### 1.3 Aim

The focus of this thesis is to further develop a virtual clamp force sensor for EMB actuators in automotive BBW systems. This involves extending previous methodologies and developing new methodologies.

### 1.4 Scope

In this thesis multiple independent models are proposed that estimate clamp force with the help of remaining system sensory information (motor current sensors and resolver sensor). These models are then fused using various fusion algorithms to optimize the RMSE of estimation for each algorithm. The proposed clamp force estimator is intended to make developmental inroads into the deficiencies provided by previous efforts to estimate clamp force.
1.5 Methodology

New approaches to estimate clamp force are presented which incorporate elements from earlier efforts to estimate clamp force by Schwarz et al. [54]. Previous efforts to estimate clamp force were based on stiffness modelling. The resolver sensor used in an EMB measures the motor angle displacement. This sensor has been used to estimate clamp force in a spring system approach. This method to estimate clamp force is inaccurate for highly dynamic braking. This is because a dynamic system is present for motor angle input and clamp force output that is undetected by a stiffness model. A frequency response [42] is conducted to better understand the nature of this dynamic so that a more comprehensive model to estimate clamp force using a resolver sensor is developed. Any stiffness components to estimate clamp force are subject to thermally dependant parameter variations due to the wide ranging temperature conditions experienced in an EMB and disc assembly. Temperature prediction schemes are set-up to help model the change in stiffness parameters to estimate clamp force.

An alternative approach to estimate clamp force is also developed. Motor current sensors are involved in all EMB designs. The torque produced by a permanent magnet DC motor is linearly proportional to its field current. A torque balance model to estimate clamp force in an EMB is constructed considering motor torque is definable amongst other things. Inertial torques necessary for the torque balance are provided with the aid of the resolver sensor. Also, friction modelling of the EMB reduction gearing is devised for use in the torque balance.

The developed independent models to estimate clamp force are fused using a maximum-likelihood estimator to optimize the RMSE of estimation. This form of fusion algorithm is subsequently extended to the Kalman filter to investigate its usefulness.

1.6 Contributions

The main contributions of this thesis are as follows:

- A new dynamic stiffness model to estimate clamp force which is different to the existing purely stiffness based approach. It is shown that the new dynamic stiffness model outperforms the existing stiffness model as it is sensitive to rate dependencies within the system.

- Developmental inroads are made to handle thermally dependent stiffness parameters so that these parameter variations do not go unnoticed as in previous efforts to estimate clamp force using stiffness. The major outcomes achieved towards this end are as follows:
  - One dimensional temperature model developed to predict pad temperatures.
Pad friction thermal boundary condition defined for non-braking cooling scenarios.

Stiffness parameters modelled as functions of thermal conditions.

A new method to estimate clamp force based on a torque balance approach that relies on inputs from EMB motor current sensors and resolver sensor is developed.

A Kalman filter is implemented to improve clamp force estimation by fusing the multiple models to estimate clamp force. The dynamic stiffness model is used as the state space system equation and the torque balance model is used as the source for measurement updates.

Material presented could provide useful support to future EMB researchers for this emerging technology in the automotive industry.

1.7 Thesis Overview

To achieve the aim described above, this thesis is organized as follows:

In Chapter 2 relevant background detail is given before an existing method to estimate clamp force is described.

In Chapter 3 the concept of dynamic is introduced to previous stiffness based models to estimate clamp force.

In Chapter 4 a torque balance model to estimate clamp force is presented.

In Chapter 5 a literature review is given on relevant sensor fusion techniques. A maximum-likelihood estimation scheme is described as well as the Kalman filter.

In Chapter 6 the dynamic stiffness and torque balance models are fused to give improved estimates of clamp force. A maximum-likelihood estimation scheme is used as well as a Kalman filter.

In Chapter 7 it is shown that the stiffness parameters from the dynamic stiffness model have thermal sensitivities. Background information is given to help develop a method to handle such parameter variations for in-service applications. The idea of using a pad thermal model is proposed to help handle thermally dependent stiffness parameter variations.

In Chapter 8 developmental inroads are made on how to handle the thermally dependent parameter variations in the dynamic stiffness model during in-service applications. The cooling boundary condition for the pad thermal model during non-braking scenarios is developed. An idea is proposed on how the heating boundary condition for the pad thermal model during braking scenarios could be handled.
In Chapter 9 prior developments in the thesis are merged together and validated against empirical data.

Finally, Chapter 10 summarises this thesis and discusses future research.

Figure 1-4 summarizes all the chapters within this thesis.

1.8 Publications


- Saric S., Bab-Hadiashar A. and Hoseinnezhad R., A Sensor Fusion Approach to Estimate...

2

Literature Review: Stiffness Modelling

2.1 Characteristic Curve Definition

The pseudo-static relationship that exists between motor angle and induced clamp force in an EMB is hysteretic in nature. Fitting a line of best fit to this hysteretic relationship leads to a non-linear curve otherwise known as the characteristic curve. Figure 2-1 shows the characteristic curve for an EMB. Schwarz et al. [54] propose to use an EMB characteristic curve solely to estimate clamp force for feedback control in Figure 1-3.

![Figure 2-1. Characteristic curve for an EMB.](image)
2.2 Characteristic Curve Dependence on Pad Thickness

Figure 2-2 shows the characteristic curve for various pad thicknesses. The figure shows that the characteristic curve is dependent on pad thickness. Schwarz et al. [54] developed a method to adapt the characteristic curve for pad wear amongst other things. This method is described immediately after the next section which is necessary to give relevant background detail before Schwarz et al. [54] method can be explained.

![Figure 2-2. Characteristic curves for different pad thicknesses.](image)

2.3 Brushless Motor Fundamentals

In the last couple of decades, brushless motors have been implemented as the choice of servomotor replacing their brush counterparts. The advantages of brushless designs are compactness, improved commutation efficiency, less maintenance and better reliability. Brushless motors rely on electronics to perform commutation rather than a mechanical self-commutating arrangement that brush motors use. Brushless motors are typically 3 phase permanent magnet DC in nature. Figure 2-3 shows the mechanical construction of such a motor in its simplest form. The 3 field coils are connected according to various configurations, the most commonly used is the wye connection scheme [6]. In this scheme the 3 field coils from Figure 2-3 are connected at a junction whilst the remaining 3 terminals have voltages applied to them.

It can be seen in Figure 2-3 that current passing through the fixed field coils induce magnetic fields. These magnetic fields interact with the magnetic field from the permanent magnet rotor and can thus cause a torque to be produced on the rotor. The appropriate regulation of current magnitudes and current directions through the field coils can control the motion of the rotor.
The objective is to ensure that mainly tangential forces are applied to the rotor. That is, radial forces on the rotor should be minimized as they do not contribute to anything useful and therefore cause inefficiency. Radial forces on the rotor mainly lead to heat energy dissipation at the bearings. Minimizing the radial forces on the rotor from a brushless motor is achieved as follows. The magnetic flux density of a field coil is proportional to the level and direction of current passing through it. Motor current sensors are used in brushless motors so the current passing through the field coils is known. Current space vectors can be set-up as shown by the arrows next to the field coils in Figure 2-3. These vectors can be summed to find the resultant current space vector as viewed in Figure 2-4. Furthermore, the resultant current space vector can be broken down into components along the tangential and radial axis. These axes are shown in Figure 2-4 and rotate with the rotor. The aim is to ensure the resultant current space vector is on or as close to the tangential axis as possible. In order to achieve this, knowledge of the position of the rotor is required. For this reason brushless motors have position sensing capabilities. The torque produced by a permanent magnet brushless motor is linearly proportional to the tangential component of the resultant current space vector such that:

\[ T_m = K_m I_m, \]

where \( T_m, K_m \) and \( I_m \) are the motor torque, the motor torque constant and the tangential component of the resultant current space vector respectively. The motor torque constant is determined from empirical data.

Depending on the control scheme implemented, the resultant current space vector tracks the
tangential axis to various degrees. The trapezoidal commutation control scheme is implemented when Hall sensors are used to sense rotor position [34]. This scheme sees the resultant current space vector track the tangential axis in a rough manner because of the low resolution of rotor position measurements. Accompanying this inefficiency are undesirable torque ripples inherent with trapezoidal commutation. The use of an absolute position sensor such as a resolver to sense rotor position allows for more efficient control schemes to be implemented, for example, sinusoidal commutation [34]. The sinusoidal commutation control scheme tries to make the resultant current space vector exactly track the tangential axis. This scheme can produce continuous smooth torque. Figure 2-5 helps display how this is performed. Three current sinusoids of equal amplitude are passed through the 3 field coils. Each current sinusoid is phase shifted 120 degrees from the other 2. Regulating these current sinusoids in such a manner relative to the position of the rotor can ensure that the resultant current space vector exactly tracks the tangential axis in a constant circular manner as shown in Figure 2-5. Hence, continuous smooth torque is generated. The phase shift of 120 degrees arises from the fact that the 3 field coils in Figure 2-5 are physically located 120 degrees apart from each other.

2.4 Parameter Adaptation

The motor in an EMB provides a torque input which in turn induces a clamp force at the brake pads. Therefore, there must be a degree of correlation between motor torque and induced clamp force. Equation 2-1 gives a definition of the torque induced by a permanent magnet brushless motor. It is evident in Eq. 2-1 that motor torque is dependent on the current passing through the
field coils. As has been mentioned previously, motor current sensors are involved in all EMB designs, therefore motor currents are known. To calculate an induced clamp force in an EMB using motor currents amongst other things, a torque balance can be used. The torque balance equation states that the torque supplied by the motor $T_m$ equals the sum of the torques required to generate the clamp force $T_a$ - also called application torque, to overcome inertial resistance $T_i$ and to overcome frictional resistance $T_f$. Application torque is linearly proportional to the clamp force $F_{cl}$ with reduction gearing gain $\gamma_{tot}$ which is determined by using specifications from a gear-train and a ball-screw. Inertial torque is linearly proportional to the motor angular acceleration $\frac{d^2\theta_m}{dt^2}$ with a lumped inertia gain $J_{tot}$ that involves both rotational and translational motions. The torque balance equation to estimate clamp force is as follows:

$$T_m = T_a + T_i + T_f$$  \hspace{1cm} Eq. 2-2

$$I_m K_m = \gamma_{tot} F_{cl} + J_{tot} \frac{d^2\theta_m}{dt^2} + T_f$$  \hspace{1cm} Eq. 2-3

$$F_{cl} = \frac{I_m K_m - J_{tot} \frac{d^2\theta_m}{dt^2} - T_f}{\gamma_{tot}}.$$  \hspace{1cm} Eq. 2-4

Note that the motor angle term $\theta_m$ is provided by an absolute position sensing device in the form of a resolver sensor which is generally part of all EMB designs. The lumped inertia gain is
normally determined using empirical data where an energy balance, over a period of motor acceleration, is set-up to find this parameter.

The friction torque is undefined in Eq. 2-4, this is because as Olsson et al. [43] describe, deriving an accurate friction model from first principles is simply not possible due to the random nature of friction. Physical phenomena that cause friction are described in [8-9, 46, 60]. Schwarz et al. [54] use a form of the torque balance without the need for a friction model to adapt the characteristic curve for pad wear and thermally dependant stiffness changes. They achieve this by superimposing a high frequency low amplitude sinusoid on the otherwise normal motion from the motor. This serves to force the motor to pass the same location in a short period of time between a clamping and releasing action. Applying the torque balance to both instants yields:

\[ T_{m,cl} = \gamma_{tot} F_{cl} + J_{tot} \frac{d^2 \theta_{m,cl}}{dt^2} + T_f \] \hspace{1cm} Eq. 2-5

\[ T_{m,rl} = \gamma_{tot} F_{cl} + J_{tot} \frac{d^2 \theta_{m,rl}}{dt^2} - T_f \] \hspace{1cm} Eq. 2-6

where the subscripts \( cl \) and \( rl \) mean clamping and releasing respectively. The friction term changes its sign as the direction of motion changes. Schwarz et al. [54] state that the viscous contribution to friction is very small for an EMB and therefore negligible. Thus, the modulus of the friction torques in Eq’s 2-5 and 2-6 are approximately equal. Therefore, adding Eq’s 2-5 and 2-6 followed by some algebraic manipulation leads to the following equation to estimate clamp force \( F_{cl}^* \) where the friction torques have been taken to cancel each other out:

\[ F_{cl}^* = \frac{T_{m,cl} + T_{m,rl} - J_{tot} \frac{d^2 (\theta_{m,cl} + \theta_{m,rl})}{dt^2}}{2\gamma_{tot}} \] \hspace{1cm} Eq. 2-7

The major issue with this method for clamp force estimation in general is its limitations for high speed applications. This is because on geometrical grounds, capturing a clamping and releasing action at the same motor angle is very difficult at high speed. Also, whether an EMB has the dynamic control ability to reverse direction at high speeds in a short interval of time is contentious. Schwarz et al. [54] propose to use the characteristic curve solely to estimate clamp force for feedback control in Figure 1-3. In the instants where Eq. 2-7 can be applied, it is used to adapt the parameter variations in the characteristic curve associated with pad wear and thermally dependant stiffness changes.
2.5 Clearance Management

The control architecture used by Line et al. [37] for an EMB during clamping is shown in Figure 1-3. To handle the initial clearance existing between pads and disc, they revert to position control where the outer force control loop in Figure 1-3 is replaced by a position control loop. The action of the clamp force sensor indicates which control strategy is implemented at which time. It should be mentioned that efforts are made by EMB designers to keep the clearance length constant irrespective of pad wear so that braking response times are kept consistent.

The omission of a clamp force sensor from an EMB leads to a situation where initial contact between the brake pads and disc cannot be sensed. The use of a virtual clamp force sensor will not be practical unless knowledge of when initial contact has occurred is known. Schwarz et al. [54] overcome this problem by developing a clearance management scheme. They use the following equation to achieve this:

\[ \lambda = \frac{dT_m}{d\theta_m} \]

*Eq. 2.8*

Once the \( \lambda \) value exceeds a certain threshold, engagement between pads and disc is signalled.
3

Stiffness Modelling: Dynamic System Considerations

3.1 Static Experimental Environment

The test rig described in this section is used to attain all data from this point forth unless otherwise stated. Also, it was used to attain previous data presented in Chapter 2. The static nature of the test rig pertains to the fact that a rotating disc is not implemented.

An external servomotor is used to provide actuation to a prototype EMB by coupling it to the internal reduction gearing from the EMB as shown in Figure 3-1. The external motor is of the permanent magnet brushless type, with ratings of 55.5 N.m and 5000 rpm and ensures that maximum clamp forces can be achieved. To interface with the drive of this motor, the RS232 protocol is utilized. The Simulink package and xPC block-set from MATLAB provides a real time operating system that is implemented to control the external motor angle. The external motor is controlled by proportional-integrative-derivative (PID) controllers within a standard motion control architecture; cascaded position, velocity and current control loops. Sensory information is logged by uploading the signal data to the host PC from the target PC, marked 1 and 2 respectively in Figure 3-1. The logged data is stamped at 100 µs time step intervals. Both the host and target PC’s have Pentium 4 processors operating at 2.4 GHz. To measure the EMB motor angle, an encoder output is taken from the 1:1 coupled external servomotor. The resolution of this encoder output provides 8192 counts per revolution. An external torque sensor
Figure 3-1. Test rig using a brushless permanent magnet external servomotor.

1. host PC  
2. target PC  
3. brushless permanent magnet external servomotor  
4. external torque sensor  
5. EMB  
6. external clamp force sensor  
7. break out boxes  
8. low-pass filter and amplifier for external clamp force sensor  
9. DC power supply  
10. ethernet hub

is used to sense torque input to the EMB. An external clamp force sensor is placed in-between the brake pads to measure the true force induced by the brake pads.

For the reasons of clarity and unless otherwise mentioned, the external motor angle and torque data from this test rig are considered to be received from an EMB itself since current sensors and a resolver sensor are available in an EMB.

3.2 Dynamic Considerations

Figure 3-2 shows clamp force versus motor angle where the latter is varied in a uniform random manner every 100 ms. Uniform random means all numbers within a prescribed range have an equal chance of occurring. It is apparent from Figure 3-2 that there is significant dynamic in the system and that the use of a characteristic curve, as shown in Figure 2-1, to estimate clamp force
Chapter 3 - Dynamic Stiffness Modelling

Figure 3-2. Clamp force versus motor angle for highly dynamic case.

has its limitations for highly dynamic cases. The cause of this dynamic will be described and modelled ahead in this chapter.

It was ensured that the data presented in Figure 3-2 captured the dynamic from the EMB and not that from the test rig used in Figure 3-1. Manufacturers specifications indicated that the external clamp force sensor, external encoder output and the filtering applied to the clamp force signal had no significant dynamic response behaviour over the expected operating range. In addition, amplification of the clamp force signal was indicated to have insignificant gain and offset errors. The clamp force signal was filtered using a 1000 Hz low-pass filter. This cut-off frequency is well beyond the operating demands of an EMB. Thus, the EMB dynamic has been captured in Figure 3-2 and not that of the test rig used to attain the data.

3.3 Dynamic Stiffness Model Development

Figure 3-3 shows a flow chart describing the process used to develop a dynamic stiffness model to estimate clamp force that relies solely on the measured motor angle. A frequency response analysis [42] is performed to extract a linear model with motor angle as input and clamp force as output. The influence of significant non-linearity is compensated for until estimation error is acceptable. It should be noted that within this section a constant pad thickness is used, more discussion will be given to the significance of this in Subsection 3.3.2.

3.3.1 Frequency Response

As was mentioned previously, motor angle will be taken as an input variable while the induced clamp force will be taken as an output variable for use in a frequency response analysis. A range
Figure 3-3. Dynamic stiffness model development flow chart.

of separate sinusoidal inputs with frequencies from 0-5 Hz are applied to the test rig shown in Figure 3-1. Motor angle and induced clamp force are logged for each case. A fast Fourier transform (FFT) [44] is applied to the logged signals for each frequency. The peak absolute value and accompanying phase angle are then extracted from each signal at each frequency. A decibel value is taken for the ratio of absolute values at each frequency to generate the continuous trace in the gain plot of Figure 3-4a. The difference in phase angles for each frequency is taken to generate the continuous trace in the phase shift plot of Figure 3-4b.

Since the roll-off from Figure 3-4a is close to −20 dB per decade and the phase shift range from Figure 3-4b is near −90 degrees, Figure 3-4 is indicative of a first order system [55]. Based on this observation, the relationship between motor angle and induced clamp force is modelled as a first order dynamic system. This first order dynamic is attributed to viscoelasticity exhibited
mainly by the caliper bridge [48] and to some degree the brake pads [4]. Figure 3-5 shows dynamic data of clamp force vs motor angle at various pad thicknesses. The motor angle is varied in a uniform random manner every 100 ms to attain this data. Figure 3-5 shows that the main influence to the dynamic is the caliper bridge as no noticeable attenuation in dynamic occurs as a result of the pad thickness reducing.

Figure 3-4. Frequency domain plots for motor angle input and clamp force output.
Chapter 3 - Dynamic Stiffness Modelling

To determine the parameters of the first order dynamic, a least-squares regression (LSR) is used as follows. For the sake of simplicity, continuous time notations will be used. Consider a first order transfer function expressed by the following equation:

$$
G(s) = \frac{F_{cl}^*(s)}{\Theta_m(s)} = \frac{K}{\tau s + 1},
$$

Eq. 3-1

where $\Theta_m$ and $F_{cl}^*$ denote the motor angle and estimated clamp force in $s$-domain respectively. The gain $K$ and time constant $\tau$ are the parameters to be determined. In frequency domain, Eq. 3-1 can be rewritten as shown ahead:

$$
\frac{1}{|G(j2\pi f)|^2} = \frac{4\pi^2 \tau^2}{K^2} f^2 + \frac{1}{K^2},
$$

Eq. 3-2

where $f$ is the frequency of the input signal in Hz. Eq. 3-2 is in a form $y = a_1x + a_2$, where the unknowns $a_1$ and $a_2$ contain the gain and time constant parameters. Applying a LSR to solve for $a_1$ and $a_2$ using the empirically determined frequency response data allows the first order gain and time constant to be subsequently evaluated. With a linear model defined, a frequency response is estimated as shown by the dashed traces within Figures 3-4a and 3-4b to be compared with the empirically determined frequency response as shown by the continuous traces. Figure 3-4b shows that the influence of non-linearity is clear since there are trace deviations [55]. To illustrate the significance of this non-linearity a clamp force response and its linear system estimate are shown in Figure 3-6 for a sinusoid case. In both instances the input sinusoids were identical with frequencies of 1.18 Hz. The error is obviously significant with a
difference of over 10 kN shown in areas. Therefore, the introduction of compensation is required to improve the accuracy of the fundamental describing function determined.

3.3.2 Non-Linear Compensation

Converting Eq. 3-1 into a time domain form yields:

$$F_{cl}^* = K\theta_m^0 - \tau \frac{dF_{cl}^*}{dt}.$$  \hspace{1cm} \text{Eq. 3-3}

In a low speed case $dF_{cl}^*/dt \approx 0$, therefore the estimated clamp force will nearly be linearly proportional to the motor angle. However, as it is shown in Figure 2-1, a characteristic curve for an EMB is non-linear and can be accurately described by a third order polynomial. This non-linearity can be at the very least attributed to a variation in stiffness exhibited by the brake pads [10, 63] and the caliper bridge [53]. Based on this, a more accurate variation of Eq. 3-3 is as follows:

$$F_{cl}^* = A_2\theta_m^1 + A_1\theta_m^2 + A_0\theta_m^3 - \tau \frac{dF_{cl}^*}{dt},$$  \hspace{1cm} \text{Eq. 3-4}

where $A_2$, $A_1$, and $A_0$ are stiffness parameters. The discrete form of Eq. 3-4 is more practical for use in a digital processing system and is expressed in a simplified form as follows:

$$F_{cl}^*(i) = \alpha_3\theta_m^1(i) + \alpha_2\theta_m^2(i) + \alpha_1\theta_m(i) + \alpha_0 F_{cl}^*(i-1).$$  \hspace{1cm} \text{Eq. 3-5}
To determine the values of the coefficients in Eq. 3-5 a LSR is used on dynamic data. The motor angle is varied in a uniform random manner every 100 ms to attain this data. Uniform random data is selected since numerous frequency components are involved that promote the attainment of more robust coefficients. Applying the defined form of Eq. 3-5 to new uniform random data yields the validation results shown in Figure 3-7. The RMSE was found to be 0.29 kN for this high speed case. Since high speed data was used for parameter tuning in Eq. 3-5, such scenarios are weighted more heavily with regards to accuracy. This is a desired outcome as performance for high deceleration safety-critical braking situations can be optimized.

Figure 3-8 shows the performance of the non-linear dynamic clamp force estimator against the empirical data used in Figure 3-6. Figure 3-6 shows validation results for the uncompensated linear estimator. When comparing Figures 3-6 and 3-8 it is apparent the improvement non-linear compensation has made.

Figure 2-2 shows that pad thickness has a significant influence on the characteristic curve. The modelling in this section has not considered pad thickness variation. The next section introduces methods to handle pad thickness variation for the modelling developed within this section.

### 3.4 Pad Thickness Variation Considerations

The terms $A_2$, $A_1$ and $A_0$ from Eq. 3-4 are characteristic curve coefficients. The discrete form of Eq. 3-4 is as follows:

$$F_{cl}^+(i) = A_2 \theta_m^3(i) + A_1 \theta_m^2(i) + A_0 \theta_m(i) + \tau \frac{F_{cl}^+(i) - F_{cl}^+(i-1)}{\Delta t}.$$  \hspace{1cm} \text{Eq. 3-6}

![Figure 3-7. Uniform random data estimator validation, 100 ms motor angle variation.](image)
where $\Delta t$ is the sample time. Rearranging to make the most recent clamp force estimate the subject yields:

$$F_{cl}^*(i) = \frac{1}{1 + \frac{\tau}{\Delta t}} \left( A_2 \theta_m^3(i) + A_1 \theta_m^2(i) + A_0 \theta_m(i) \right) + \frac{\tau}{\Delta t} F_{cl}^*(i-1).$$  \hspace{1cm} \text{Eq. 3-7}$$

The time constant is dependent on the damping in the system as well as the stiffness [55]. Stiffness and to a lesser degree damping are dependent on the pad thickness. Therefore, the time constant is dependent on pad thickness. In an approximation a constant linear time constant is proposed for use in Eq. 3-7. To determine the varying characteristic curve coefficients for in-service application, Schwarz et al. [54] parameter adaptation technique is to be used. Low-pass filtering is applied for torque sensing so that high frequency noise is smoothed out. Throughout this thesis, unless otherwise stated, sensed motor torque is filtered. Eq. 3-7 is simplified to give:

$$F_{cl}^*(i) = \beta_1 \left( A_2 \theta_m^3(i) + A_1 \theta_m^2(i) + A_0 \theta_m(i) \right) + \beta_0 F_{cl}^*(i-1),$$  \hspace{1cm} \text{Eq. 3-8}$$

where,
\[ \beta_0 = \frac{\tau}{\Delta t}, \]

Eq. 3-9

and

\[ \beta_1 = \left( \frac{1}{1 + \frac{\tau}{\Delta t}} \right). \]

Eq. 3-10

The \( \beta \) coefficients are proposed to be constant irrespective of pad wear for a set sample time. Six pad pairs were machined to set pad thicknesses; 0.5 mm, 3.5 mm, 5 mm, 6.5 mm, 9.5 mm and 11.5 mm. For each pad thickness, except for the 5 mm case, a characteristic curve is determined via a LSR using sensed motor angle and sensed clamp force measurements. Dynamic data of sensed motor angle and sensed clamp force is obtained for each pad thickness. The motor angle is varied in a uniform random manner every 100 ms to attain these data sets. All the uniform random data, except for the 5 mm case, is used to tune the best \( \beta \) coefficients from Eq. 3-8 where the empirically based characteristic curve data is used. A LSR approach is applied.

Using Schwarz et al. [54] parameter adaptation technique, the characteristic curve for the 5 mm pad thickness is determined. Figure 3-9 shows the nature of the input used for parameter adaptation where a sinusoid of high frequency and low amplitude is superimposed on another sinusoid of lower frequency and higher amplitude. Two points in Figure 3-9, clamp and release, are marked at the same motor angle. Eq. 2-7 is used to give an estimate of the clamp force for

Figure 3-9. EMB training signal.
both points in time. This is repeated throughout the signal so that a series of clamp force estimates are obtained. Keeping a log of these clamp force estimates, along with the associated motor angles (using a resolver sensor), characteristic curve coefficients via a LSR are determined. Schwarz et al. [54] parameter adaptation technique can be used in-service during instances such as when the park brake is locked. Using the adapted characteristic curve as well as the determined $\beta$ coefficients, validation is performed on uniform random data obtained previously for the 5 mm pad thickness. Figure 3-10 shows the results where a RMSE of 0.35 kN is obtained. This is an increase with regards to the RMSE of 0.29 kN found in the previous section where pad thickness was not considered. The reason for this increase is attributed to greater sensory noise influences resulting from Schwarz et al. [54] parameter adaptation technique as well as greater model structure approximations in Eq. 3-8.

![Figure 3-10. Uniform random data estimator validation, 100 ms motor angle variation.](image)
4

Torque Balance Modelling

4.1 Electro-Mechanical Brake Frictional Phenomena

Eq. 2-4 estimates clamp force via a torque balance approach. The friction torque $T_f$ is undefined in Eq. 2-4. This is because as Olsson et al. [43] describe, deriving an accurate friction model from first principles is simply not possible. To improve accuracy, general friction models should be used in accordance with compensations for friction phenomena that occur in a particular system. Such frictional phenomena are identified from empirical data. Generally, the use of a friction model of any sort to estimate clamp force for an EMB application tends to be avoided due to the difficulty in developing an accurate model that is robust to wear. Figure 4-1 shows clamp force plotted versus motor torque for 2 cases: a relatively new EMB, and an EMB that is well run-in. In both cases the input signals (motor angle) were identical and each EMB was of the same mechanical construction. It can be seen that for the same motor torque, a difference in clamp force of up to 5 kN is observed. This is attributed to frictional variation in the reduction gearing because of wear. Therefore, the use of a torque balance approach to estimate clamp force in an EMB, as given by Eq. 2-4, will fail if the friction model is not tolerant to wear. In this chapter an adaptive approach is proposed to handle friction parameter variations in an EMB.

The frictional variation occurring in an EMB is not apparent after a limited number of cycles. Figure 4-2 shows the motor torque versus time for a high speed cyclic case where motor angle is taken as the input signal. The clamp force is approximately between 0-20 kN. It is observed that


Figure 4-1. Clamp force versus motor torque for variously aged EMB reduction gearing.

Figure 4-2. Motor torque versus time.

the motor torque has a nearly identical trace at the beginning and the end of the test. This shows that the frictional parameters remain constant during short time intervals. Hence, the updating of any friction model used to estimate clamp force is not required after every braking action, rather after a numerous number of braking actions.

It is shown that the viscous contribution to friction in the reduction gearing from an EMB is small compared to the clamp force dependent Coulomb friction component [54]. Therefore, viscous friction can be neglected with regards to friction modelling. Figure 4-3 shows the frictional motor torque component in an EMB versus clamp force where the former has not been filtered. To attain this graph a torque balance was performed on data where the motor angle was varied in a slow and continuous manner so that inertial torque did not have to be considered. It
can be seen in Figure 4-3 that the friction torque is approximately linearly proportional to clamp force.

Based on the frictional characteristics discussed in this section for an EMB, the next section develops modelling to estimate clamp force using a torque balance approach. Validation results are provided.

4.2 Torque Balance Model Development

A simplified friction model [3, 43] is included in Eq. 2-2 as follows:

\[
T_m = T_a + T_i + \left( \mu_k F_{cl} + \eta_k \right) \text{sgn}\left( \frac{d\theta_m}{dt} \right).
\]

*Eq. 4-1*

where \( \mu_k \) and \( \eta_k \) are the EMB reduction gearing coefficient of kinetic Coulomb friction and kinetic friction offset respectively. The kinetic friction offset is required to take into account frictional resistance prior to inducing a clamp. The sign function \( \text{sgn}(.) \) (1 for positive and \(-1\) for negative arguments) is included to model the friction sign change that occurs between clamping and releasing. It should be noted that stiction is not included in the friction model within Eq. 4-1. This is because this thesis is concerned only with estimating clamp force for dynamic braking scenarios. As Line *et al.* [37] discuss, stiction is significant during clamped scenarios and must be considered from a controls point of view. Stiction can be included in the friction modelling from Eq. 4-1 for any future work as an extension to the framework provided here.
Figure 4-1 shows the level of frictional variation occurring in the reduction gearing from an EMB over time. To estimate clamp force using a torque balance approach, it is necessary that the friction model parameters be updated at timely intervals. A discrete form of Eq. 4-1 for practical use in clamp force estimation is as follows:

\[ T_m(i) = \gamma_{tot} F^*_c(i) + \frac{J_{tot}}{\Delta t^2} (\theta_m(i) - 2\theta_m(i-1) + \theta_m(i-2)) + (\mu_g F^*_c(i) + \eta_g) \text{sgn}(\theta_m(i) - \theta_m(i-1)). \]

**Eq. 4-2**

The reduction gearing gain \( \gamma_{tot} \) can be determined using reduction gearing specifications. The lumped inertia gain \( J_{tot} \) is normally determined using empirical data. Using Schwarz *et al.* [54] parameter adaptation technique as shown in the previous chapter, a series of clamp force estimates is obtained. Using these clamp force estimates along with the associated motor torques and motor angles, the friction model parameters are determined via a LSR using Eq. 4-2. Adapting the friction model parameters in Eq. 4-2 can be performed at timely intervals throughout the service life of an EMB. After some algebraic manipulation of Eq. 4-2 a parametric expression to estimate clamp force in real time is attained as follows:

\[ F^*_c(i) = \frac{T_m(i) - \frac{J_{tot}}{\Delta t^2} (\theta_m(i) - 2\theta_m(i-1) + \theta_m(i-2)) - \eta_g \text{sgn}(\theta_m(i) - \theta_m(i-1))}{\gamma_{tot} + \mu_g \text{sgn}(\theta_m(i) - \theta_m(i-1))}. \]

**Eq. 4-3**

The linearity in the Coulomb friction model has enabled the estimated clamp force to easily be arranged to become the subject in Eq. 4-3. Validating Eq. 4-3 on dynamic data yields Figure 4-4 where an RMSE of 0.61 kN is attained. The motor angle is varied in a uniform random manner.

![Figure 4-4. Uniform random data estimator validation, 100 ms motor angle variation.](image)
every 100 ms to attain this data.
Chapter 5 - Literature Review: Sensor Fusion

5

Literature Review: Sensor Fusion

5.1 Introduction

Sensor fusion, also known as multi-sensor data fusion, is the integration of data obtained from sensory sources such that the resulting information is an improvement in some way on any information prior [12, 21, 29, 40, 70]. Sensor fusion can be categorized into 3 sub-groups; direct fusion, indirect fusion and the combination of the two. Direct fusion relies on data from sources such as homogenous or heterogeneous sensors, soft sensors and sensor history data. Soft sensors are virtual in nature and rely on signals within the system to be processed to give sensed information of the variable of interest, i.e the clamp force estimators proposed thus far in this thesis. Indirect fusion relies on data from sources such as human input and a priori knowledge of the environment.

The essence of sensor fusion is that any data sources used in the fusion process can only serve to improve the resulting information. The improvement can be viewed as more accurate, more comprehensive, or more reliable amongst other things.

5.2 Maximum-Likelihood Estimation

Maximum-likelihood estimation is an optimal estimation scheme that uses weighting to determine the relative quality of the Gaussian noise corrupted data sources at hand. An example is given as follows where an optimal estimate $\hat{x}$ is needed given knowledge of two measurements $x_1$ and $x_2$ whose statistics are required to be Gaussian and known. It is necessary
that the noises of \( x_1 \) and \( x_2 \) are independent of each other. The first thing required to proceed is to define a criterion of optimality. Due to the random nature of \( x_1 \) and \( x_2 \), optimality must be defined in a statistical sense. The estimator giving \( \hat{x} \) will be optimal if it minimizes a loss function on average. A loss function is described as follows:

\[
L = \left( (x - \hat{x})^2 \mid x_1, x_2 \right),
\]

\textit{Eq. 5-1}

where \( L \) is a measure of loss. Eq. 5-1 states that the loss is determined from the squared error given two measurements \( x_1 \) and \( x_2 \). Since the statistics of \( x_1 \) and \( x_2 \) are known, then the expectation or mean of the loss function can be established. The expectation for minimizing Eq. 5-1 is written as:

\[
\mathbb{E}[L] = \mathbb{E}\left[ \left( (x - \hat{x})^2 \mid x_1, x_2 \right) \right] \quad \text{is minimized.}
\]

\textit{Eq. 5-2}

The minimum expectation of the loss can be defined by finding where the minima occurs. This can be achieved by taking the partial derivative of Eq. 5-2 with respect to \( \hat{x} \), setting the resulting equation equal to 0 and then solving for \( \hat{x} \). The partial derivative of Eq. 5-2 with respect to \( \hat{x} \) is given as follows:

\[
\frac{\partial \mathbb{E}[L]}{\partial \hat{x}} = 2\mathbb{E}\left[ (x - \hat{x}) \mid x_1, x_2 \right].
\]

\textit{Eq. 5-3}

Setting Eq. 5-3 equal to 0 as well as applying some algebraic manipulation yields:

\[
0 = 2\mathbb{E}[x \mid x_1, x_2] - 2\mathbb{E}[\hat{x} \mid x_1, x_2].
\]

\textit{Eq. 5-4}

The \( \mathbb{E}[\hat{x} \mid x_1, x_2] \) can be equal to anything selected and it is set appropriately to \( \hat{x} \). Therefore, Eq. 5-4 reduces to:

\[
\hat{x} = \mathbb{E}[x \mid x_1, x_2].
\]

\textit{Eq. 5-5}

Eq. 5-5 states that the optimal estimator that minimizes the loss on average from Eq. 5-1 is equal to \( \mathbb{E}[x \mid x_1, x_2] \).

Since the statistics of \( x_1 \) and \( x_2 \) are Gaussian, the probability density function (PDF) for each is:

\[
p(x_1 \mid x) = \frac{1}{\sqrt{2\pi\sigma_{x_1}^2}} e^{-\frac{(x_1 - x)^2}{2\sigma_{x_1}^2}}
\]

\textit{Eq. 5-6}
where \( \sigma_{x_1} \) and \( \sigma_{x_2} \) are the standard deviations of \( x_1 \) and \( x_2 \) respectively. Equation 5-5 requires knowledge of \( \mathbb{E}[x \mid x_1, x_2] \) so that the optimal estimator \( \hat{x} \) can be defined. If the PDF \( p(x \mid x_1, x_2) \) can be determined, then \( \mathbb{E}[x \mid x_1, x_2] \) can be found by averaging out this PDF. According to Bayes theorem [67] the PDF \( p(x \mid x_1, x_2) \) can be expressed as follows:

\[
p(x \mid x_1, x_2) = \frac{p(x_1, x_2 \mid x)p(x)}{\int_{-\infty}^{\infty} p(x_1, x_2 \mid x)p(x)dx}.
\]

Eq. 5-8

It is assumed that the PDF \( p(x) \) is uniform in nature which means that all possible values for \( x \) are equally likely to occur. Such a PDF is written as:

\[
p(x) = \begin{cases} 
\frac{1}{x_{\text{max}} - x_{\text{min}}} & \text{for } x_{\text{min}} \leq x \leq x_{\text{min}}, \\
0 & \text{for } x < x_{\text{min}} \text{ or } x > x_{\text{max}},
\end{cases}
\]

Eq. 5-9

where \( x_{\text{min}} \) and \( x_{\text{max}} \) are the minimum and maximum possible values for \( x \) respectively. Since the PDF \( p(x) \) is a constant it cancels out in Eq. 5-8. which reduces to:

\[
p(x \mid x_1, x_2) = \frac{\int_{-\infty}^{\infty} p(x_1, x_2 \mid x)p(x)dx}{\int_{-\infty}^{\infty} p(x_1, x_2 \mid x)p(x)dx}.
\]

Eq. 5-10

The PDF \( p(x_1, x_2 \mid x) \) can be attained by multiplying Eq’s 5-6 and 5-7 to get:

\[
p(x_1, x_2 \mid x) = \frac{1}{2\pi\sigma_{x_1}\sigma_{x_2}} e^{-\frac{(x_1-x_2)^2}{2\sigma_{x_1}^2} - \frac{(x_2-x)^2}{2\sigma_{x_2}^2}}.
\]

Eq. 5-11

The integral of the PDF \( p(x_1, x_2 \mid x) \) with respect to \( x \) is required in Eq. 5-10. This is given as follows:

\[
\int_{-\infty}^{\infty} p(x_1, x_2 \mid x)dx = \sqrt{\frac{2\pi}{\sigma_{x_1}^2 + \sigma_{x_2}^2}} e^{-\frac{(x_1-x_2)^2}{2(\sigma_{x_1}^2 + \sigma_{x_2}^2)}}.
\]

Eq. 5-12
Since all the PDF’s on the right-hand side of Eq. 5-10 have been determined, the PDF \( p(x_1, x_2 \mid x) \) can be defined as follows:

\[
p(x \mid x_1, x_2) = \frac{1}{2\pi} \sqrt{\frac{1}{\sigma_{x_1}^2} + \frac{1}{\sigma_{x_2}^2}} \left( -\frac{(x_1-x)^2}{2\sigma_{x_1}^2} \right) e^{\frac{-(x_2-x)^2}{2\sigma_{x_2}^2}} e^{\frac{-(x_1-x)^2}{2(\sigma_{x_1}^2 + \sigma_{x_2}^2)}}. \quad \text{Eq. 5-13}
\]

The exponent from Eq. 5-13 can be rearranged to give:

\[
\frac{- (x_1 - x)^2}{2\sigma_{x_1}^2} + \frac{-(x_2 - x)^2}{2\sigma_{x_2}^2} + \frac{(x_1 - x_2)^2}{2(\sigma_{x_1}^2 + \sigma_{x_2}^2)} = \frac{1}{\sigma_{x_1}^2} + \frac{1}{\sigma_{x_2}^2}
\]

\[
\left( \frac{1}{\sigma_{x_1}^2} + \frac{1}{\sigma_{x_2}^2} \right) x^2 - \left( \frac{2x_1}{\sigma_{x_1}^2} + \frac{2x_2}{\sigma_{x_2}^2} \right) x + \frac{x_1^2}{\sigma_{x_1}^2} - \frac{(x_1 - x_2)^2}{\sigma_{x_1}^2 + \sigma_{x_2}^2} + \frac{x_2^2}{\sigma_{x_2}^2}.
\]

The right-hand side of Eq. 5-14 is a perfect square and can be simplified to yield:

\[
\left( \frac{1}{\sigma_{x_1}^2} + \frac{1}{\sigma_{x_2}^2} \right) x^2 - \left( \frac{2x_1}{\sigma_{x_1}^2} + \frac{2x_2}{\sigma_{x_2}^2} \right) x + \frac{x_1^2}{\sigma_{x_1}^2} - \frac{(x_1 - x_2)^2}{\sigma_{x_1}^2 + \sigma_{x_2}^2} + \frac{x_2^2}{\sigma_{x_2}^2} = \frac{-1}{2} \left( \frac{1}{\sigma_{x_1}^2} + \frac{1}{\sigma_{x_2}^2} \right) \left( x - \frac{\sigma_{x_1}^2 x_1 + \sigma_{x_2}^2 x_2}{\sigma_{x_1}^2 + \sigma_{x_2}^2} \right)^2.
\]

Now the fraction within the square on the right-hand side of Eq. 5-15 can be expressed as:

\[
\frac{\sigma_{x_1}^2 x_1 + \sigma_{x_2}^2 x_2}{\sigma_{x_1}^2 + \sigma_{x_2}^2} = x_1 - \frac{\sigma_{x_1}^2}{\sigma_{x_1}^2 + \sigma_{x_2}^2} (x_1 - x_2). \quad \text{Eq. 5-16}
\]

Based on Eq’s 5-14, 5-15 and 5-16, Eq. 5-13 can be rewritten as follows:

\[
p(x \mid x_1, x_2) = \frac{1}{2\pi} \sqrt{\frac{1}{\sigma_{x_1}^2} + \frac{1}{\sigma_{x_2}^2}} e^{-\frac{1}{2} \left( \frac{1}{\sigma_{x_1}^2} + \frac{1}{\sigma_{x_2}^2} \right) \left( x - \frac{\sigma_{x_1}^2 x_1 + \sigma_{x_2}^2 x_2}{\sigma_{x_1}^2 + \sigma_{x_2}^2} \right)^2}.
\]

Now it is evident that Eq. 5-17 is in a Gaussian form. The average of this PDF is equal to the \( \text{E}[x \mid x_1, x_2] \) which is the optimal estimator \( \hat{x} \). Due to the Gaussian nature of Eq. 5-17, this occurs at the peak value. Therefore, the optimal estimator \( \hat{x} \) can be defined as follows:
\[
\hat{x} = x_1 - \frac{\sigma^2_{x_1}}{\sigma^2_{x_1} + \sigma^2_{x_2}} (x_1 - x_2).
\]

\text{Eq. 5-18}

Eq. 5-18 is the maximum-likelihood estimator \( \hat{x} \) given all possible values of \( x \) are equally likely to occur as well as the measurement noises \( x_1 \) and \( x_2 \) having Gaussian statistics. It should be noted that the noises from the measurements are required to be statistically uncorrelated from each other.

The concept of weighting is evident in Eq. 5-18 and is described as follows. Firstly the weighting factor from Eq. 5-18 is identified as:

\[
m = \frac{\sigma^2_{x_1}}{\sigma^2_{x_1} + \sigma^2_{x_2}}.
\]

\text{Eq. 5-19}

It can be seen that the weighting factor approaches 1 as the standard deviation of \( x_1 \) becomes much larger than the standard deviation of \( x_2 \). Alternatively, if the standard deviation of \( x_1 \) becomes much smaller than the standard deviation of \( x_2 \), then the weighting factor approaches 0.

The weighting factor serves to give more weight to the more accurate measurement source. For example, if \( x_1 \) has a larger standard deviation than \( x_2 \), the difference between \( x_1 \) and \( x_2 \) from Eq. 5-18 is given more weight in the overall equation due to multiplication with the resulting weighting factor. Therefore, \( x_1 \) has a quantity subtracted from it that has an influence in causing the overall estimate to tend to \( x_2 \). Alternatively, if \( x_1 \) has a smaller standard deviation than \( x_2 \), the difference between \( x_1 \) and \( x_2 \) from Eq. 5-18 is given less weight in the overall equation due to multiplication with the resulting weighting factor. Therefore, \( x_1 \) becomes the dominant measurement source to provide the estimate.

The maximum-likelihood estimator given by Eq. 5-18 is optimal in the sense that it provides the best possible estimates on average. This means at times the estimates may not be very accurate but on average the RMSE is minimized with regards to any other possible model structure not represented by Eq. 5-18. The RMSE of the estimator given by Eq. 5-18 is described as:

\[
\sigma_{\hat{x}} = \sqrt{\mathbb{E}[(x - \hat{x})^2]}.
\]

\text{Eq. 5-20}

Substituting Eq. 5-18 for \( \hat{x} \) into Eq. 5-20 allows for the RMSE to be reduced to:

\[
\sigma_{\hat{x}} = \sqrt{\frac{\sigma^2_{x_1} \sigma^2_{x_2}}{\sigma^2_{x_1} + \sigma^2_{x_2}}}.
\]

\text{Eq. 5-21}
Chapter 5 - Literature Review: Sensor Fusion

The RMSE for the estimator given by Eq. 5-18 is such that it is less than or equal to the minimum standard deviation from either $x_1$ or $x_2$.

The maximum-likelihood estimator presented in this section is used in the next chapter to estimate clamp force in an EMB. The clamp force estimation models presented in Chapters 3 and 4 are applied within the maximum-likelihood estimator.

5.3 Kalman Filtering

The maximum-likelihood estimator shown in the previous section does not have a recursive nature, therefore previous knowledge in time is not used in the estimation process. This is not the case with a Kalman filter which would further improve estimation accuracy [58]. A Kalman filter is a maximum-likelihood estimator that has a recursive aspect. It is a linear discrete time estimation algorithm that minimizes the RMSE of estimation. A Kalman filter is implemented widely in control systems to give improved system state estimates. Figure 5-1 shows a block diagram representation of a Kalman filter for a control system. A Kalman filter uses system dynamics as well as other measurement sources to estimate states. Typically the later is attained from direct sensory measurements. The noises which affect both kinds of estimates the Kalman filter receives, view Figure 5-1, are required to be uncorrelated and Gaussian.

The derivation of the Kalman filtering algorithm is given as follows. Firstly the system equation for a multidimensional case is shown in state space form as:

$$\mathbf{x}(i) = \Phi \mathbf{x}(i-1) + \Psi \mathbf{u}(i) + \mathbf{w}(i-1).$$

**Eq. 5-22**

where $\Phi$, $\Psi$, $\mathbf{u}(i)$ and $\mathbf{w}(i)$ are the transition matrix, control input matrix, control input vector and system noise vector respectively. It is assumed that the transition matrix and control input

![Figure 5-1. Typical Kalman filter application.](image)
matrix are not time varying. The bold notations in Eq. 5-22 indicate either matrix or vector which is also the case throughout this section. The covariance matrix for the system noise vector is given as:

\[ Q(i) = E[w(i)w(i)^T], \quad Eq. \ 5-23 \]

where the superscript \( Tr \) indicates transposition. The measurement equation for a multidimensional case is expressed as follows:

\[ z(i) = Hx(i) + v(i), \quad Eq. \ 5-24 \]

where \( H \) and \( v(i) \) are the measurement matrix and measurement noise vector respectively. It is assumed that the measurement matrix is not time varying. The measurement matrix serves to denote what part of the state vector \( x(i) \) is measured. The covariance matrix for the measurement noise vector is defined by:

\[ R(i) = E[v(i)v(i)^T], \quad Eq. \ 5-25 \]

In a similar manner to the optimal estimator defined by Eq. 5-5 from the previous section, the optimal estimator for the Kalman filter is defined as follows:

\[ \hat{x}(i \mid i) = E[x(i) \mid z(i), u(i)], \quad Eq. \ 5-26 \]

where the discrete time notation \( i \mid i \) indicates that the information at \( i \) is determined given knowledge at \( i \). A linear estimator is the required end outcome and can be shown to be:

\[ \hat{x}(i \mid i) = K_1(i)\hat{x}(i \mid i-1) + K_0(i)z(i), \quad Eq. \ 5-27 \]

where \( K_1(i) \) and \( K_0(i) \) are matrices that are required to be determined. The discrete time notation \( i \mid i-1 \) in Eq. 5-27 indicates that the information at \( i \) is determined given knowledge at \( i-1 \), hence it is a priori in nature. Such notation in Eq. 5-27 displays the recursive aspect of the Kalman filter. It is evident in Eq. 5-27 that the notation \( i \mid i \) signifies a posteriori in nature.

The state-error vector can be described as follows:

\[ \tilde{x}(i \mid i) = x(i) - \hat{x}(i \mid i). \quad Eq. \ 5-28 \]

The following expression can be written where the principle of orthogonality is used:

\[ E[\tilde{x}(i \mid i)z(i)^T] = 0. \quad Eq. \ 5-29 \]
Substituting Eq. 5-24 into Eq. 5-27 for $z(i)$, followed by substituting this resulting equation into Eq. 5-28 for $\hat{x}(i|\hat{d})$, followed by substituting this resulting equation into Eq. 5-29 for $\tilde{x}(i|\hat{d})$, leads to the expression ahead:

$$
E[(x(i) - K_1(i)\hat{x}(i| i-1) - K_0(i)Hx(i) - K_0v(i)z(i)^T)z(i)^T] = 0.
$$

Eq. 5-30

It can be found that:

$$
E[v(i)z(i)^T] = 0.
$$

Eq. 5-31

Including $(K_1(i)-K_0(i))x(i)$ within the major rounded brackets in Eq. 5-30, and with knowledge of Eq. 5-31, the following equation can be determined:

$$
E[(I - K_0(i)H - K_1(i))x(i)z(i)^T + K_1(i)x(i) - \hat{x}(i| i-1)z(i)^T] = 0,
$$

Eq. 5-32

where $I$ is the identity matrix. It is apparent based on the principle of orthogonality that:

$$
E[(x(i) - \hat{x}(i| i-1))z(i)^T] = 0.
$$

Eq. 5-33

Using Eq. 5-33, Eq. 5-32 can be reduced to:

$$(I - K_0(i)H - K_1(i))E[x(i)z(i)^T] = 0.
$$

Eq. 5-34

It is evident that Eq. 5-34 can only be satisfied if:

$$
I - K_0(i)H - K_1(i) = 0.
$$

Eq. 5-35

Substituting Eq. 5-35 into Eq. 5-27 for $K_1(i)$ allows for the equation ahead to be determined:

$$
\hat{x}(i| i) = \hat{x}(i| i-1) + K_0(i)z(i) - H\hat{x}(i| i-1)).
$$

Eq. 5-36

The matrix $K_0(i)$ is required to be defined in Eq. 5-36 and is known as the Kalman gain. The similarities between Eq. 5-36 and the maximum-likelihood estimator given by Eq. 5-18 are apparent with a difference being the recursive aspect of the former. It can be shown using the principle of orthogonality that:

$$
E[(x(i) - \hat{x}(i| i))\tilde{z}(i)^T] = 0,
$$

Eq. 5-37

where $\tilde{z}(i)$ is the measurement-error vector and is defined as follows:

$$
\tilde{z}(i) = z(i) - H\hat{x}(i| i-1).
$$

Eq. 5-38
Substituting Eq. 5-24 into Eq. 5-38 for \( z(i) \) permits the equation ahead to be attained:

\[
\tilde{z}(i) = v(i) + H\tilde{x}(i \mid i-1).
\]  

Eq. 5-39

Substituting Eq. 5-24 into Eq. 5-36 for \( z(i) \) allows the following expression for the state-error vector to be determined:

\[
x(i) - \hat{x}(i \mid i) = (I - K_0(i)H)\tilde{x}(i \mid i-1) - K_0(i)v(i).
\]  

Eq. 5-40

Substituting Eq. 5-39 and Eq. 5-40 into Eq. 5-37 for \( \tilde{z}(i) \) and \( x(i) - \hat{x}(i \mid i) \) respectively, leads to the expression ahead to be obtained:

\[
(I - K_0(i)H)E[\tilde{x}(i \mid i-1)\tilde{x}(i \mid i-1)^T]H^T - K_0(i)E[v(i)v(i)^T] = 0.
\]  

Eq. 5-41

The a priori covariance matrix for the state-error vector is given as follows:

\[
P(i \mid i-1) = E[\tilde{x}(i \mid i-1)\tilde{x}(i \mid i-1)^T].
\]  

Eq. 5-42

Rewriting Eq. 5-41 using Eq’s 5-25 and 5-42, followed by making \( K_0(i) \) the subject yields:

\[
K_0(i) = P(i \mid i-1)H^T[H^TP(i \mid i-1)H^T + R(i)]^{-1}.
\]  

Eq. 5-43

Equation 5-43 formulates the Kalman gain.

Equation 5-36 uses a priori estimate of the state vector to determine a posteriori estimate of the state vector. The a priori estimate is determined using Eq. 5-22 where the a posteriori estimate of Eq. 5-36 is used from the previous time step. It should be noted that the system noise vector is not used in Eq. 5-22 here. One can therefore see the recursive aspect of the Kalman filter.

Equation 5-22 is used in a prediction sense, whilst Eq. 5-36 is used in a filtering sense. The Kalman gain from Eq. 5-43 is dependent on the a priori covariance matrix for the state-error vector which is defined as follows. The a priori state-error vector can be expressed as:

\[
\tilde{x}(i \mid i-1) = x(i) - \hat{x}(i \mid i-1).
\]  

Eq. 5-44

The terms on the right-hand side of Eq. 5-44 are substituted using Eq. 5-22 to give:

\[
\tilde{x}(i \mid i-1) = \Phi x(i-1) + \Psi u(i) + w(i-1) - \Phi x(i-1 \mid i-1) - \Psi u(i).
\]  

Eq. 5-45

It should be noted that the system noise vector is not used to describe \( \hat{x}(i \mid i-1) \) from Eq. 5-44 in Eq. 5-45. Equation 5-45 can be reduced to:
\[
\hat{x}(i \mid i-1) = \Phi \hat{x}(i-1) + w(i-1).
\]

Substituting Eq. 5-46 into Eq. 5-42 for \( \hat{x}(i \mid i-1) \) and using the principle of orthogonality allows the a priori covariance matrix for the state-error vector to be expressed as follows:

\[
P(i \mid i-1) = \Phi P(i-1 \mid i-1) \Phi^T + Q(i-1).
\]

It can be seen in Eq. 5-47 that the a posteriori covariance matrix for the state-error vector from the previous time step is required. The a posteriori covariance matrix for the state-error vector can be written as:

\[
P(i \mid i) = E[\hat{x}(i \mid i)\hat{x}(i \mid i)^T].
\]

Substituting Eq. 5-40 into Eq. 5-48 for \( \hat{x}(i \mid i) \) and using the principle of orthogonality allows the a posteriori covariance matrix for the state-error vector to be expressed as follows:

\[
P(i \mid i) = (I - K_\theta(i)H)P(i \mid i-1)(I - K_\theta(i)H)^T + K_\theta(i)R(i)K_\theta(i)^T.
\]

Making use of Eq. 5-43, Eq. 5-49 can be reorganized to make the a posteriori covariance matrix for the state-error vector dependent on the a priori covariance matrix for the state-error vector, such an equation is shown ahead:

\[
P(i \mid i) = (I - K_\theta(i)H)P(i \mid i-1).
\]

It can be seen that Eq’s 5-47 and 5-50 are together used to recursively propagate the covariance matrix for the state-error vector.

A Kalman filter involves the recursive application of prediction and filtering cycles as seen in the block diagram of Figure 5-2.

A Kalman filter is used in the next chapter to estimate clamp force in an EMB. The clamp force estimation models presented in Chapters 3 and 4 are applied within this filter.

Figure 5-2. Block diagram representation of a Kalman filter.
6 Clamp Force Estimation: Sensor Fusion

6.1 Clamp Force Estimation: Maximum-Likelihood Estimation

The dynamic stiffness and torque balance models to estimate clamp force, as developed previously in Chapters 3 and 4 respectively, are shown as follows:

\[ F_{cl}^*(i) = \beta_1 \left( A_2 \theta_m^*(i) + A_1 \theta_m^2(i) + A_0 \theta_m(i) \right) + \beta_0 F_{cl}^*(i-1) \]  \hspace{1cm} \text{Eq. 6-1}

\[ T_m(i) = \frac{J_{tot}}{\Delta t^2} \left( \theta_m(i) - 2 \theta_m(i-1) + \theta_m(i-2) \right) - \eta_s \text{sgn} \left( \theta_m(i) - \theta_m(i-1) \right) \]

\[ F_{cl}^*(i) = \frac{\gamma_{tot} + \mu_s \text{sgn} \left( \theta_m(i) - \theta_m(i-1) \right)}{\sigma_{tot}^2 + \sigma_{tot}^2} (\dot{F}_{cl,ds}^*(i) - F_{cl,ds}^*(i-1)) \]  \hspace{1cm} \text{Eq. 6-2}

The maximum-likelihood estimator given by Eq. 5-18 from the previous chapter can be used to fuse the models above to optimize the RMSE of estimation. Equation 5-18 is written in a discrete form in terms relevant to this chapter as:

\[ \hat{F}_{cl}(i) = F_{cl,ds}^*(i) - \frac{\sigma_{ds}^2 \left( F_{cl,ds}^*(i) - F_{cl,ds}^*(i-1) \right)}{\sigma_{ds}^2 + \sigma_{tb}^2} \]

\[ \text{Eq. 6-3} \]

where the subscripts \( ds \) and \( tb \) denote dynamic stiffness and torque balance respectively. It is assumed that for Eq. 6-3 to be useful, the noises associated with the dynamic stiffness and torque balance models have Gaussian statistics and are uncorrelated from each other. In Chapters 3 and 4 it was shown that the RMSE’s for the dynamic stiffness and torque
balance models was 0.35 kN and 0.61 kN respectively. With the statistical parameters of Eq. 6-3 defined, Figure 6-1 shows the performance of Eq. 6-3 in tracking high speed clamp force measurements where motor angle is varied in a uniform random manner every 100 ms. A new RMSE of 0.32 kN results which is an approximately 10 % improvement on the RMSE from the dynamic stiffness model given by Eq. 6-1. Equation 5-21 is written in terms relevant to this chapter as follows:

\[ \sigma_{\hat{F}_{cl}} = \sqrt{\frac{\sigma_{ds}^2 \sigma_{lb}^2}{\sigma_{ds}^2 + \sigma_{lb}^2}}. \]

Equation 6-4 gives the theoretically improved RMSE as a result of fusion. Substituting the RMSE from the dynamic stiffness and torque balance models into Eq. 6-4 leads to a value of 0.3 kN to be calculated. It is apparent that the empirically determined RMSE of 0.32 kN for Eq. 6-3 does not fully realize the theoretical improvement indicated by Eq. 6-4. This is attributed to the noises associated with the dynamic stiffness and torque balance models not having perfectly Gaussian distributions.

### 6.2 Clamp Force Estimation: Kalman Filtering

Figure 5-2 from the previous chapter shows the Kalman filtering algorithm. This figure will be cross referenced within this section to describe the components necessary to estimate clamp force in an EMB based on the Kalman filtering approach.

The dynamic stiffness model given by Eq. 6-1 is used as the state space equation for a Kalman

![Figure 6-1. Uniform random data estimator validation, 100 ms motor angle variation.](image)
filter to estimate clamp force in an EMB. The constant $\beta_0$ from Eq. 6-1 is taken to be equal to $\Phi$ from the Kalman filtering algorithm shown in Figure 5-2. Note that typical matrix notations are not required due to the unit state space dimension of Eq. 6-1. The clamp force in Eq. 6-1 is non-linearly proportional to the motor angle input. This non-linearity does not require the use of an extended Kalman filter (EKF) [13] or other Kalman filter variant [30] because it is not state dependant. If there was non-linearity which was state dependent, an EKF could be used which performs a linearization procedure. To integrate the non-linearity from Eq. 6-1 into Figure 5-2, the following equality is applied:

$$\Psi_\theta(i) = \beta_1 \left( A_2 \theta_3(t) + A_1 \theta_2(t) + A_0 \theta_1(t) \right).$$  \hspace{1cm} \textit{Eq. 6-5}

From Figure 5-2, $\hat{x}$ is taken to be $\hat{F}_{cl}$ for the purposes within this chapter. The a priori estimate of the state $\hat{F}_{cl} \mid i-1$ is taken to be directly equal to $\hat{z}(i-1)$ from Figure 5-2. This means that $H$ takes on a unit value from Figure 5-2. The torque balance model given by Eq. 6-2 is used as the source for measurement updates which is equivalent to $z(i)$ from Figure 5-2. The RMSE for the dynamic stiffness and torque balance models is 0.35 kN and 0.61 kN respectively as found from the previous couple of chapters. These values squared are used as assumed constant $Q$ and $R$ from Section 5.3 and are necessary to mechanize a Kalman filter.

The Kalman filter set-up described for clamp force estimation purposes in an EMB is applied to uniform random data. The uniform random data involves varying the motor angle in a uniform random manner with a sample time of 100 ms. To initialize the clamp force estimate mean-square error (MSE) at time equal to zero, $Q$ is used. Figure 6-2 shows the performance of the method presented in this section to estimate clamp force in an EMB based on the Kalman

![Figure 6-2. Uniform random data estimator validation, 100 ms motor angle variation.](image)
filtering approach. An RMSE of 0.29 kN results which is an approximately 20 % improvement on the RMSE from the dynamic stiffness model given by Eq. 6-1. It was found that the use of a maximum-likelihood estimator in the previous section to estimate clamp force in an EMB gave an RMSE of 0.32 kN. Therefore it has been demonstrated that the use of a Kalman filter, which has a recursive aspect, improves the RMSE of clamp force estimation by approximately 10 % with regards to the methods of the previous section which do not have a recursive aspect. An advantage of the maximum-likelihood estimator from the previous section to the Kalman filter of this section is that the former is more computationally efficient. Therefore, the methods in the previous section are more suitable for memory-critical applications.
Chapter 7 - Thermal Considerations

7

Thermal Considerations

7.1 Introduction

In the previous chapters a static test rig was used to obtain data for analysis and validation purposes. This test rig, as shown in Figure 3-1 in Chapter 3, is static in the sense that a rotating disc is not used and therefore no braking heat is being generated. In-service pad temperatures can reach up to 800 °C. The dynamic stiffness model to estimate clamp force developed previously in Chapter 3 is reliant on the characteristic curve. Considering brake pads have polymeric constituencies [47, 57] and that the temperatures were essentially kept constant in the development of the dynamic stiffness model, the influences of temperature on the characteristic curve should be investigated under practical circumstances. For this purpose a test rig with a rotating disc is built and is described in greater detail in the following section.

Figure 7-1 shows the characteristic curve using 10 mm thick pads at ambient and heated conditions. This data was obtained using the test rig described in the next section where thermocouples were embedded in the pads. It is apparent from Figure 7-1 that the characteristic curve varies with increased temperatures as a result of braking. Schwarz et al. [54] algorithm for estimating clamp force as described in Chapter 2 has limitations for certain scenarios. One scenario is after vehicle braking has taken place followed by a period of cooling. This is because the cooling will result in an alteration in the characteristic curve which will go undetected according to Schwarz et al. [54] method. Upon braking at least the first application will be
operating from an incorrect characteristic curve until adaptation has taken place. As discussed in Chapter 2 Schwarz et al. [54] algorithm also has limitations for high speed actuation. Therefore, a series of quick brake applications will result in no update of the characteristic curve. In this and the following chapter foundations are proposed for a new approach that attempts to track the characteristic curve parameter variations for high speed applications as well as during cooling instances.

7.2 Dynamic Experimental Environment

The test rig described in this section is used to attain all data from this point forth. The dynamic nature of the test rig pertains to the fact that a rotating disc is implemented.

A prototype EMB is used on the test rig shown in Figures 7-2a and 7-2b. The brake actuator is driven by an external motor via coupling to its reduction gearing. This motor is capable of a peak torque of 90 N·m (3 times continuous stall torque) and a maximum speed of 2000 rpm. These specifications ensure that comparable EMB performance can be achieved as that expected from in-service application. The angle of this motor is proportionally controlled.

Clamp force is sensed using the EMB internal clamp force sensor. Calibration of this sensor is initially undertaken in an environmental chamber for temperature drifts, span shifts and frictional hysteresis against a calibrated external clamp force sensor of known temperature sensitivities. The calibration set-up is shown in Figure 7-3. A thermistor is used alongside the internal clamp force sensor from the EMB so that local temperature is known at all times. The EMB motor angle is measured by using the encoder output from the EMB external motor drive. The resolution of this encoder output is 4096 counts per revolution. The torque input to the
Figure 7-2. Test rig: (a) Control tower. (b) Mechanical set-up.

1. host PC
2. target PC
3. signal conditioning unit
4. disc motor drive
5. disc motor
6. temperature data logger and signal conditioner
7. EMB
8. external torque sensor
9. EMB external motor
10. disc brake
11. brake torque sensor
12. EMB external motor drive

Figure 7-3. EMB internal force sensor calibration, environmental chamber set-up.

1. EMB
2. external clamp force sensor
3. environmental chamber
4. internal clamp force sensor location
EMB is sensed by an external torque sensor. For the purpose of clarity the sensory information from this test rig, where applicable, will be written as being received from an EMB as would be the case for in-service application.

The brake disc is driven by a motor which is capable of a maximum speed of 1500 rpm and is connected to a 4:1 planetary gear-train. The disc angle is measured by using the encoder output from the disc motor drive. The resolution of this encoder output is 16384 counts per revolution. The analog voltage ports in the disc motor drive are used to control the speed of the disc. Brake torque is sensed by a brake torque sensor. The maximum permissible brake torque which can be generated by the test rig is 660 N·m which is set by the limits of the brake torque sensor used. This constraint limits the level of clamp force that can be applied. Clamp forces for normal vehicle braking can be applied (approximately 5 kN), however clamp forces for panic braking will exceed the brake torque limits and therefore cannot be applied [17].

MATLAB’s Simulink package along with the xPC block-set provides the software interface which is used to control the perturbation signals of the test rig. Clamp force, brake torque, disc angle, EMB motor angle, EMB motor torque and time are logged by uploading data from the target PC to the host PC. The host and target PC’s have Pentium 4 processors which operate at 2.4 GHz. K-type thermocouples are used to sense pad, caliper bridge and disc temperatures. This type of temperature sensor has an approximate maximum working temperature of 1000 ºC at the sensed location which is generally considered adequate for vehicle braking applications. All temperature measurements are logged using a temperature data logger. A trigger signal is used to synchronise the commencement of data logging between the target PC and the temperature data logger. The target PC and temperature data logger log data at 100 µs and 10 ms time step intervals respectively.

7.3 EMB and Disc Assembly Compliance Analysis

The overall compliance (inverse of stiffness) of an EMB and disc assembly can be broken down into sub-components as described by the following equation:

\[ c_a = c_c + c_g + c_d + 2c_p, \]

where \( c_a \), \( c_c \), \( c_g \), \( c_d \) and \( c_p \) are the compliances from the EMB and disc assembly, the caliper bridge, the reduction gearing, the disc and a pad respectively. The contribution to the overall compliance by the reduction gearing and disc are considered to be insignificant for both hot and cold conditions [53]. The modulus of elasticity of an aluminium caliper bridge is expected to change negligibly for the temperatures expected in-service of up to 200 ºC. Therefore, the pads are the only components in Eq. 7-1 that have a significant influence over the characteristic curve.
during heated conditions, view Figure 7-1. To successfully estimate clamp force using the characteristic curve it is required that thermally dependent compliance changes of the pads be adequately managed. One way to achieve this would be to develop a method where pad temperatures can be attained in-service and used to indicate the level of compliance changes undergone. This is the approach taken in this thesis.

7.4 Pad Isothermal Planes Assumption

The temperature in a brake pad can be inferred as varying approximately in 1 spatial dimension only, that is, along its thickness. The explanation is as follows. A pad tends to wear uniformly and its frontal surface area is large relative to its thickness. Uniform wear causes the same amount of heat to be applied across the entire contact surface area of a pad, and since the thickness direction offers a path of reduced thermal resistance, isothermal planes would tend to be formed that vary along the thickness direction of a pad. Empirical verification of this inference was performed. Fifteen thermocouples were embedded across the frontal surface area of a pad, each at depths of 7 mm from the friction surface as depicted in Figure 7-4. Holes were drilled in through the backing plate and thermocouples subsequently glued in place. This thermocoupled pad was heated using the test rig shown in Figures 7-2a and 7-2b. It then cooled once frictional heating had ceased. Small clamp forces were used for the initial heating phase. Figure 7-5 shows the responses from the 15 thermocouples. It can be seen that the temperatures are similar and therefore the assumptions described previously are supported empirically.

It should be noted that the pad and disc interface during braking is restricted to small areas. That is, only asperity regions are in contact [17] and not the whole area of the pad. Given that the heat liberated at each such contact point is very small, irregularities in the pad temperature will only be detectable to a very short depth [64]. Thus, thermal equilibrium takes place very near to the friction surface of the pad that promotes the formation of approximate isothermal planes throughout the subsequent depth of the pad.

![Figure 7-4. Thermocouple positions as viewed from pad backing plate. Each hole to depth of 7 mm from friction surface.](image)
Chapter 7 - Thermal Considerations

7.5 Pad Thermal Boundary Conditions

If the thermal boundary conditions on a brake pad could be defined, as well as having knowledge of pad thermal properties and initial temperature, then time dependent pad temperature profiles can be determined. Friction heating occurs at the pad and disc interface during braking. However, after the brake pads have disengaged cooling takes place that is dependent on the speed of the disc. This is demonstrated by the following set of tests. A brake pad with a thermocouple embedded 2 mm from its friction surface was heated on two separate occasions using the test rig shown in Figures 7-2a and 7-2b. The heating phases were similar in both instances however, after braking ceased the disc continued to rotate at different rates for each case (1 and 5 turns per second). The thermocouple data from these tests can be viewed in Figure 7-6. It can be seen that after disengaging the pads, as indicated roughly by the 200 °C point, the 5 turn per second case causes significantly faster cooling than the 1 turn per second case. Therefore, it can be seen that the disc speed has an influence on how fast the pads cool after braking. In the next chapter methods are proposed to predict the thermal boundary condition at the pad friction surface during cooling for in-service applications. Also, in the next chapter ideas are presented on tackling the thermal boundary condition during braking at the pad friction surface for in-service applications.

Heat conduction occurs at the pad and pad backing plate interface. It is proposed that a temperature sensor is to be used at this location for each pad during in-service application so that these thermal boundary conditions are defined. That is, based on the isothermal planes assumption described earlier, these sensed temperatures represent their entire respective surface
areas. The addition of two temperature sensors in the form of thermocouples will not have a significant influence over the cost savings which would be acquired by omitting a clamp force sensor from an EMB system.

7.6 Proposed Temperature Prediction Scheme

Section 7-4 showed that the temperature distribution in a pad tends to be 1 dimensional for both braking and cooling instances. Therefore, the formulation of the 1 dimensional planar thermal problem to predict temperatures for a pad can be defined by the following set of equations:

\[
\rho_p C_p(T_p) \frac{\partial T_p(x_p,t)}{\partial t} = \frac{\partial}{\partial x_p} \left( k_p(T_p) \frac{\partial T_p(x_p,t)}{\partial x_p} \right) \quad \text{in} \quad 0 < x_p < L, \quad t > 0 \quad \text{Eq. 7-2a}
\]

\[
-k_p(T_p) \frac{\partial T_p(x_p,t)}{\partial x_p} = f(t) \quad \text{at} \quad x_p = 0, \quad t > 0 \quad \text{Eq. 7-2b}
\]

\[
T_p(x_p,t) = f(t) \quad \text{at} \quad x_p = L, \quad t > 0 \quad \text{Eq. 7-2c}
\]

\[
T_p(x_p,t) = T_{p,0} \quad \text{for} \quad t = 0, \quad \text{Eq. 7-2d}
\]

where \( T_p, x_p \) and \( t \) are the pad temperature, space and time variables respectively. Pad properties are represented by \( \rho_p, C_p, k_p \) and \( L \) which are the density, specific heat, thermal conductivity and pad thickness respectively. The initial pad temperature is given as \( T_{p,0} \). The derivation of Eq. 7-2a is shown in Appendix A. For now it is convenient to write the thermal boundary condition
at the friction surface of a pad as a heat flux that accounts for both heating and cooling as given by Eq. 7-2b. The analytical solution of Eq. 7-2a, given Eq’s 7-2b, 7-2c and 7-2d, is problematical because of non-linearity and time dependent boundary conditions. The main non-linearity is the dependence of pad thermal conductivity on temperature. An appropriate solution strategy of Eq. 7-2a is to take a numerical approach, such as finite differences. The explicit finite difference scheme is chosen here to solve Eq. 7-2a because it is relatively simple and straightforward for programming [14]. The explicit finite difference solution of Eq. 7-2a is given as follows:

\[
T_p(i+1, n) = \tau_p(i, n) \left[ T_p(i, n-1) + T_p(i, n+1) \right] + \left[ 1 - 2\tau_p(i, n) \right] T_p(i, n) \tag{7-3a}
\]

\[
T_p(i+1, 0) = 2\tau_p(i, 0) T_p(i, 1) + \left[ 1 - 2\tau_p(i, 0) \right] T_p(i, 0) + \frac{2\tau_p(i, 0) \Delta x_p \dot{q}(i)}{k_p(i, 0)} \tag{7-3b}
\]

where \(\Delta x_p\) is the space step. The \(i\) and \(n\) scripts denote the sample number and the particular nodal point along the thickness of a pad respectively. The heat flux and pad mesh Fourier number are represented by \(\dot{q}(i)\) and \(\tau_p(i, n)\) respectively. The pad mesh Fourier number is defined as follows:

\[
\tau_p(i, n) = \frac{k_p(i, n) \Delta t}{\rho_p C_p(i, n) \Delta x_p^2} \tag{7-4}
\]

Equation 7-3a is applied to interior nodes whilst Eq. 7-3b is applied to the pad friction surface boundary. The explicit finite difference scheme is conditionally stable. As described in [14], the stability criterion given Eq’s 7-3a and 7-3b is:

\[
\tau_p(i, n) \leq \frac{1}{2} \tag{7-5}
\]

Using typical values for the pad thermal properties as provided by Limpert [36], along with a sample time and space step of 4 ms and 2 mm respectively, a pad mesh Fourier number of 0.00032 is attained. The sample time and space step used are representative of what would be intended to be used in-service. A 4 ms sample time is a practicable in-service rate. The pad mesh Fourier number is well within the limits imposed by Eq. 7-5 by a factor of approximately 1500. Due to this indicated solid stability, it is assumed that variations in pad thermal properties with temperature will not be so extreme as to cause instability.
A proportion of the heating power generated during braking is transferred into the pads with the rest going into the disc which acts as a heat sink. The heating power going into both pads, for the same brake assembly, is not the same since the thermal resistance networks associated with each is different. Therefore, the use of Eq’s 7-3a and 7-3b for pad temperature prediction must be applied independently for each pad within the same brake assembly given that the heat distribution characteristics will be different.

7.7 Discussion

In this chapter it was shown that the characteristic curve has significant dependency on the thermal condition of the pads. Equation 3-8 uses a characteristic curve to estimate clamp force in an EMB based on a dynamic stiffness approach. This equation cannot handle thermally dependent characteristic curve variation. In this chapter the idea was put forward of using a pad thermal model to appropriately adjust the characteristic curve to varying pad temperatures. Relevant background work was conducted in this chapter for the development of a pad temperature model. The next chapter carries on from here by putting into practice the developments from this chapter. Methods to handle the pad friction surface boundary condition during non-braking scenarios are shown in the next chapter. Also, ideas to handle the pad friction surface boundary condition during braking scenarios are presented. Modelling of the characteristic curve dependence on pad thermal conditions is performed and empirically verified in the next chapter.
Chapter 8 - Thermally Sensitive Characteristic Curve Modelling

8

Thermally Sensitive Characteristic Curve Modelling

8.1 Pad Thermal Properties and Heat Distribution Terms

The proposed temperature prediction scheme in Section 7.6 from the previous chapter requires knowledge of pad thermal properties. It also requires knowledge of what proportion of the heating power goes into a pad during braking. To help obtain this information specific to the pads and hardware used, two identically thermocoupled pads were firstly fabricated. Holes were drilled in through the backing plate to various depths and thermocouples glued in place as shown in Figure 8-1. The thicknesses of the pads used were 10 mm. The pads were heated using the test rig shown in Figures 7-2a and 7-2b. Using logged data from this test rig, programming in MATLAB was performed to manually tune pad thermal properties as well as heat distribution terms. The programming involved the use of Eq’s 7-3a and 7-3b, the logged thermocouple data and knowledge of the overall heating power generated. A space step of 2 mm was applied with regards to Eq’s 7-3a and 7-3b. The overall heating power generated was determined from the sensed brake torque and sensed disc speed by multiplying these two variables. Equation 7-3b requires knowledge of the heat flux into a pad during braking. This heat flux is determined by taking a portion of the overall heating power generated and dividing by the frontal surface area of the pad. Constant heat distribution factors were assumed specific to each pad within the same brake assembly. A linear pad thermal conductivity model was employed which was dependent on temperature. Constants were assumed for the specific heat and density of the pad.
The nature of the inputs to the test rig to attain data for tuning was as follows. Initial clamp load was set at approximately 3 kN and the disc speed varied in a sinusoid manner from 0-6 turns per second with a frequency of 0.0167 Hz. Initial values for the pad thermal properties and heat distribution factors were taken from Limpert [36]. Manual adjustments were made in order to improve fits between model predicted temperatures and sensed temperatures. Successful validation results are provided in Subsection 8-3 ahead on new data using the pad thermal properties and heat distribution factors attained here.

Equation 8-1 shows a condition which was necessary in the pad thermal model to accommodate for short periods of small heat flux input after a temperature gradient existed along the thickness direction of the pad:

\[
T_p(i+1,0) = \begin{cases} 
T_p(i,0) & \text{if } T_p(i+1,0) - T_p(i,0) < 0 \\
T_p(i+1,0) & \text{otherwise.}
\end{cases}
\]

Eq. 8-1

This was necessary because the temperature of the pad friction surface tends to remain constant under these circumstances. If the pad initially had a temperature gradient along its thickness, then without the use of Eq. 8-1 during small heat flux inputs, the model would drop the pad friction surface temperature immediately due to thermal equilibrium taking place within the pad. Contact with the disc for the circumstances mentioned tends to prevent a temperature decline at the friction surface of the pad, at least for short term, because the disc is a relatively large thermal mass which tends to be at uniform temperature close to that of the pad friction surface at all times. More will be discussed on this in the following section.

8.2 Modelling Pad Friction Surface Thermal Boundary Condition: Cooling

As has been described previously and shown in Figure 7-6, the rate at which the pads cool after being heated from braking is influenced by the subsequent disc speed. The temperatures of the
pad friction surface and disc friction surface tend to be similar at all times, including during cooling. This is shown in Figure 8-2 where temperatures after a prolonged braking event are given for a pad and disc whilst rotation of the disc is maintained. The disc temperature was measured using a thermocouple designed for use on a moving surface as shown in Figure 8-3.

![Figure 8-2. Disc and pad temperatures during cooling after braking.](image)

![Figure 8-3. Thermocouple set-up for moving surface.](image)
The pad friction surface temperature was estimated using Eq. 7-3a and internal thermocouples embedded within a pad (the pad from Figure 8-1 was used). Equation 7-3a was applied to the first interior node from the pad friction surface and then the equation rearranged to make the surface temperature the subject. Pad thermal properties (as determined from the previous section) were used to allow the pad friction surface temperature to be calculated. It can be seen in Figure 8-2 that the disc temperature approaches that of the pad friction surface the longer cooling is maintained. The small initial temperature difference is attributed to thermal contact resistance that exists between the pad and disc interface during braking.

The applicability of convective lumped thermal modelling is determined by firstly calculating the Biot number [14]. The Biot number is defined as follows:

\[ Bi = \frac{hL_c}{k} \]

where \( h \), \( L_c \) and \( k \) are the convection heat transfer coefficient, characteristic length and thermal conductivity of the solid body respectively. The characteristic length is the ratio of the volume of the solid body to its surface area. According to [14], it is generally accepted that lumped thermal modelling is applicable if:

\[ Bi \leq 0.1. \]

When this criterion is satisfied, the differential temperature between the solid body and fluid is accurate to within 5% [14]. The application of this criterion is now performed for a ventilated disc. Limpert [36] uses a convection heat transfer coefficient of 142 W/m\(^2\)·°C for a ventilated disc during vehicle speeds of about 30 km/h. A typical value for the thermal conductivity of a disc is 50 W/m·°C. A typical figure for the characteristic length of a ventilated disc is 0.019 m (solid disc geometry used). Applying these numbers to Eq. 8-2 results in a Biot number of 0.026 which satisfies Eq. 8-3. This shows that the ventilated disc can be tentatively assumed to be uniform in temperature during periods of heat transfer.

It was shown in Figure 8-2 that after a braking event the disc temperature remains very close to the pad friction surface temperature during cooling. An explanation for this is as follows. After disengaging the pads from the electro-mechanical actuator design used, the pads continue to remain in loose contact with the disc. The disc then proceeds to cool and since the pads are in contact with the disc (which is a relatively large thermal mass) the pad friction surface temperatures track that of the disc. To be able to predict internal pad temperatures during pad disengagement, a thermal boundary condition has to be defined at the pad friction surface. It is
proposed to model the disc as a lumped thermal system after braking. This in turn defines a
temperature boundary condition at the pad friction surface since the pad and disc temperatures
remain close to each other at all times. It is assumed that after braking the only heat transfer
associated with the disc is outwards. That is, the influence of any heat retained within the pads
after braking is considered insignificant since cooling of the disc is much more substantial.

The analytical solution for a lumped system undergoing convective cooling is as follows:

\[
T(t) = (T_0 - T_\infty) e^{-bt} + T_\infty
\]

where \( T_0 \), \( T_\infty \) and \( b \) are the initial temperature of the solid body prior to the onset of convective
cooling, the temperature of the fluid and a constant respectively. The derivation of Eq. 8-4 is
provided in Appendix B. The constant in Eq. 8-4 is defined as follows:

\[
b = \frac{hA}{\rho VC},
\]

where \( A \), \( \rho \), \( V \) and \( C \) are the surface area, density, volume and specific heat of the solid body
respectively. Limpert [35] uses approximated convection heat transfer coefficient formulas for a
disc that are dependent on disc speed amongst other things. However, the convection heat
transfer coefficient in Eq. 8-5 is constant. It is intended to model \( b \) from Eq. 8-5 as a function of
disc speed in the manner described now. Firstly Eq. 8-4 is rearranged as follows and includes
some additional notation for convenience:

\[
-\ln\left(\frac{T_d(t) - T_\infty}{T_{d,0} - T_\infty}\right) = b_d t,
\]

where the subscript \( d \) indicates with respect to disc. The heating of a disc is performed through
friction braking using the test rig shown in Figures 7-2a and 7-2b. A thermocouple is placed on
the disc which is designed for use on a moving surface as shown in Figure 8-3. Upon pad
disengagement the disc continues to rotate at a constant speed until near ambient temperatures
are reached. Considering that all the terms in Eq. 8-6 are available except for \( b_d \), a LSR is
applied to determine an optimal value of this quantity for the set disc speed. This process is
repeated for a number of disc speeds. Figure 8-4 shows the results and it is apparent that there is
a linear relationship. A line of best fit is fitted to the data which is used to model the \( b_d \) term as a
function of disc speed. It should be noted here that inherent with this approach is the fact that it
is assumed that the rest of the parameters in Eq. 8-5, apart from the convection heat transfer
coefficient, remain constant.
Radiation cooling becomes a significant factor in the transfer of heat from a disc at elevated temperatures [36, 66]. Radiation has been indirectly considered in the disc cooling model proposed in the form of a combined heat transfer coefficient approach. That is, empirical data was used to tune the $b_d$ terms for each set disc speed. Theoretically $b_d$ was defined according to Eq. 8-5 where convection cooling was considered and radiation cooling was not. However, since tuning was performed using empirical data, the convection heat transfer coefficient in Eq. 8-5 would have taken a combined heat transfer coefficient role. In this way radiation has been considered in an approximate sense.

A discrete form of Eq. 8-4 for practical use in the disc cooling model is:

$$T_d(t+1) = (T_a(t) - T_{\infty})e^{-b_d(\omega)\Delta t} + T_{\infty},$$  \hspace{1cm} Eq. 8-7

where $\omega$ is the disc speed. To initialize Eq. 8-7 after pad disengagement, the pad friction surface temperature is used since it is close to that of the disc. The pad friction surface temperature required for initialization will most likely vary for both pads within the same brake assembly. In an approximation Eq. 8-7 is initialized separately for both pads. Equation 8-7 requires knowledge of the ambient air temperature which is a signal that is available from modern day vehicles. Validation results are provided in the next section for the cooling model presented in this section. Validation results are also provided in the next section for the thermal properties and heat distribution factors obtained in the previous section.

### 8.3 Pad Temperature Model Validation

Figures 8-5a and 8-5b show empirically determined and predicted pad temperatures, the latter
Figure 8-5. Pad thermal property and cooling model validation for 10 mm thick pads at 2 mm, 4 mm, 6 mm and 8 mm locations from friction surface: (a) Inner pad, EMB drive-train side. (b) Outer pad.

is determined according to the findings of the previous 2 sections in this chapter. The test rig shown in Figures 7-2a and 7-2b and thermocoupled pads of the type shown in Figure 8-1 were used to attain the empirical data. The predicted temperatures were determined essentially using Eq’s 7-3a and 7-3b, Eq. 8-1 and the pad thermal properties and heat distribution factors determined previously in Section 8.1. The pad thermal boundary conditions used were as follows. The overall heating power produced during braking was determined from the sensed brake torque and sensed disc speed. The heat distribution factors were then used to help
determine the heat flux into each pad during braking. For non-braking scenarios, the friction surface boundary condition was switched to the cooling model described in the previous section. At the pad and pad backing plate interface sensed temperature was used for the boundary condition.

The nature of the inputs used on the test rig to generate the empirical data shown in Figures 8-5a and 8-5b were as follows. The disc was ramped up from 0-6 turns per second over a 60 second period followed by 30 seconds of rotation at constant speed. This cycle was repeated 18 times. The constant speed rotation varied between cycles from 1-6 turns per second in unit increments starting at 1 turn per second and resetting after 6 turns per second. Braking occurred during the ramping phase of the disc speed and stopped thereafter. The overall RMSE between sensed and estimated temperatures from Figures 8-5a and 8-5b was found to be 3.7 °C. It should be noted that the estimated results in Figures 8-5a and 8-5b were attained with knowledge of the overall heating power generated during braking. In practice the sensed brake torque signal at a wheel is not available in vehicles. Therefore, a method must be adopted where the heat flux into a pad during braking can be attained for in-service application. In Section 8.5 ahead ideas are presented on tackling this thermal boundary condition for in-service applications.

It should be mentioned that results attained using the test rig shown in Figures 7-2a and 7-2b are assumed to be indicative of in-service applications for a vehicle.

8.4 Modelling Characteristic Curve Variation with Thermal Conditions

As has been shown previously in Figure 7-1, the characteristic curve has dependency on the thermal conditions of the pads. In order to develop a model that predicts the characteristic curve variation with pad thermal conditions, it is firstly necessary to attain data for analysis from the test rig shown in Figures 7-2a and 7-2b as follows. Thermocoupled pads of the type shown in Figure 8-1 were initially heated via braking to a maximum temperature of approximately 300 °C at the friction surface. This was then followed by the disc ceasing rotation and the motor angle of the EMB varying in a sinusoid manner (frequency 0.2 Hz) whilst the pads cooled to near ambient temperatures (cooling took approximately 4 hours). Figure 8-6 shows the characteristic curves during the cooling phase as well as prior to any braking. The EMB internal clamp force sensor was used to determine when pad and disc contact was initiated between sinusoidal cycles. The average temperatures shown in Figure 8-6 are for the pads and are determined by using the thermocouple data as well as predicted pad friction surface temperatures. The predicted pad friction surface temperatures were determined using Eq. 7-3a in accordance with thermocouple data and the pad thermal properties attained previously in Section 8.1. The
average temperatures were taken near the beginning of a sinusoid cycle. The variation in
temperature at any point in a pad with respect to time was considered to be negligible
throughout a sinusoid cycle.

It can be seen in Fig. 8-6 that the characteristic curve with average pad temperatures of 223 °C,
taken at the commencement of cooling, is more compliant than the characteristic curve taken
prior to braking at ambient temperatures which is as expected. However, as the pads continue to
cool it can be seen that the characteristic curve tends to become more compliant as indicated by
the 98 °C average pad temperature curve. This might be somewhat unexpected as it may
initially be thought that the pads would tend to stiffen as they cooled. After continued cooling
the pads eventually stiffen up and the characteristic curve prior to braking is approached as
indicated by the 37 °C average pad temperature curve. What is indicated by Figure 8-6 is the
fact that the level of compliance the pads undergo is not only dependent on temperature but also
the length of time the pads remain at elevated temperatures.

It is proposed to model the characteristic curve variation with thermal conditions based on the
following formulation given in discrete time form:

\[ F_{cl}(i) = \alpha(i)\left( A_2\left(\theta_m(i)\right)^3 + A_1\left(\theta_m(i)\right)^2 + A_0\theta_m(i)\right) \]

Eq. 8-8

where \( A_2, A_1 \) and \( A_0 \) are characteristic curve coefficients for ambient conditions. The \( \alpha(i) \)
coefficient is a stiffness correction factor which has dependency on pad thermal conditions. This
term acts to appropriately adjust the third order polynomial characteristic curve. The stiffness
correction factor is defined as follows:
\[ a(i) = -B_1 \zeta(i) - B_0 \Delta T_{p,avg}(i) + 1, \quad \text{Eq. 8-9} \]

where \( B_1 \) and \( B_0 \) are empirically determined coefficients which are attained in a manner to be described later in this section. The \( \zeta(i) \) term is expressed as follows:

\[ \zeta(i) = \Delta t \sum_{l}^{i} (T_{p,avg}(l) - T_{p,c}), \quad \text{Eq. 8-10} \]

where \( T_{p,avg}(l), T_{p,c} \) and \( k \) are the average pad temperature, the ambient temperature at which the characteristic curve coefficients of Eq. 8-8 are determined at and a dummy variable representing sample the number respectively. The \( l \) term is defined according to:

\[ l = \begin{cases} 1 & \text{if } i \leq N \\ i - N & \text{otherwise,} \end{cases} \quad \text{Eq. 8-11} \]

where the nature of \( N \) will be explained in greater detail later in this section. The \( T_{p,c} \) term can be defined since modern day vehicles have ambient temperature signals. For in-service applications it is intended to use nodal data from the numerical scheme described by Eq’s 7-3a and 7-3b to determine average pad temperatures. A space step of 2 mm is to be used for the greater part considering a pad thickness is not always an exact multiple of 2 mm. Schwarz et al. [54] adaptation technique, which was described previously in Chapter 2, would be used in-service to determine the characteristic curve coefficients from Eq. 8-8. This technique is necessary because it does not require use of a clamp force sensor.

The \( \zeta(i) \) term is essentially an area measure of the average pad temperature with time. It is intended to model the time dependent relaxation that occurs the longer the pads remain at elevated temperatures. The \( \Delta T_{p,avg}(i) \) term from Eq. 8-9 is equal to \( T_{p,avg}(i) - T_{p,c} \). This term is intended to model the pad stiffness temperature dependence. It can be seen that the stiffness correction factor not only has an instantaneous dependence on pad temperatures, but also on the temperature history of the pads. The \( B_1 \) and \( B_0 \) coefficients are required to be determined using empirical data. These coefficients are dependent on pad thickness, however in this thesis values for just 10 mm thick pads will be determined. The purpose within this section is to just show a working method to handle pad thermal issues which can be extended thereafter as required.

The \( \zeta(i) \) term requires a scheme whereby it does not infinitely increase as a result of the pads remaining at elevated temperatures for prolonged periods of time. That is, it must be sensitive to the fact that there are practical limitations as to the amount of relaxation the pads undergo when heated. In order to achieve this, a moving window approach is proposed. That is, the amount of sampling data in Eq. 8-10 is limited to a set number of most recent samples for a certain time.
step, hence the \( N \) term in Eq. 8-11. In order to determine the value of this limiting amount of samples for a certain time step, as well as the \( B_1 \) and \( B_0 \) coefficients, previous data which was applied in the development of Figure 8-6 is used. A series of characteristic curves are firstly determined that cover the entire span of cooling to near ambient conditions. A fixed number of samples is arbitrarily selected which defines the size of the moving window. A LSR is then performed on the empirically determined characteristic curve data, along with the accompanying thermal data from the pads, to determine the best \( B_1 \) and \( B_0 \) coefficients for the specified window width. This process is repeated for a number of window widths until the best overall \( B_1 \) and \( B_0 \) coefficients are determined along with the accompanying window width. Using the proposed method to handle characteristic curve variations with thermal conditions, a RMSE of 1 kN was found for 13 empirically determined characteristic curves as taken from the data required to develop Figure 8-6. These empirically determined characteristic curves covered the entire cooling range. Figure 8-7 shows the results for 3 such cases.

8.5 Discussion

The thermal boundary condition at the pad friction surface during braking has been defined in this chapter with the help of sensory information which is not otherwise available in modern day vehicles. Therefore, such an approach cannot be used for in-service applications. Brake torque and disc speed were sensed in the test rig shown in Figures 7-2a and 7-2b. By multiplying these two quantities, the overall heating power generated during braking can be determined. The use of heat distribution factors then allowed for the determination of what proportion of heat went into a particular pad. The sensed brake torque is not available in modern day vehicles whilst the wheel speed is. Therefore, a method must be devised to handle the pad friction surface boundary

![Figure 8-7. Validation modelling of characteristic curve variation with thermal conditions.](image)
condition for in-service braking applications.

In a simplified model, the brake torque induced for a rotating wheel can be defined as follows:

\[ T_b = 2r_{eff} F_i \mu_b, \]

Eq. 8-12

where \( T_b, r_{eff} \) and \( \mu_b \) are the brake torque, effective radius and friction coefficient respectively. It has been found that the friction coefficient is dependent on clamp force, wheel speed and the pad and the disc interface temperature [2, 38, 68]. Therefore, Eq. 8-12 is somewhat more complex. The idea is now proposed on how the pad friction surface boundary condition could be defined during braking for in-service applications. The overall heating power generated during braking at a wheel is dependent on clamp force, wheel speed and the pad and disc interface temperature. This is apparent from Eq. 8-12 as it has dependencies on the variables mentioned, and after multiplying the brake torque by the disc speed one gets the overall heating power generated. Since it is intended to estimate clamp force and the pad friction surface temperature, as well as the fact that wheel speed is sensed in modern day vehicles, the idea is proposed of using an empirically determined look up table that relies on these variables to approximately give the heat flux into a pad during braking for in-service applications. A look up table is opted for because modelling is considered to be too complex due to the difficulties in mainly handling the friction coefficient which has non-linear dependencies [45].

An experimental environment such as that shown in Figures 7-2a and 7-2b provides sensed disc speed, clamp force and the overall heating power generated during braking. With the ability to measure pad surface temperature as well as knowledge of the heat distribution factor, it is suggested that a number of heat fluxes into a pad during braking can be determined for a series of set clamp force, disc speed and pad surface temperature combinations. Two such look up tables would have to be developed for each pad within the brake assembly since the thermal resistance networks associated with each is different. Numerically speaking, knowledge of the heat flux into a pad during braking would enable clamp force and pad temperature to be estimated, which in turn would allow for updated heat flux’s to be determined and so on for a braking situation. Multivariate interpolation techniques can be used on the data from the look up tables in the instances where the heat flux cannot be immediately cross-referenced [7, 41].

Figure 8-8 displays an interesting feature from the data required to develop Figure 8-6. Figure 8-8 shows clamp force given for several sinusoidal cycles over the entire cooling period. It can be seen that initially the clamp force decreases. This is somewhat unexpected since it would be assumed that as the pads become cooler they become stiffer and therefore the clamp force should always be increasing as cooling continues. The reason the clamp force initially
Figure 8-8. Several sinusoidal clamps over entire cooling phase of pads.

decreases during cooling in Figure 8-8 is mainly due to thermal contractions in volume of the disc during cooling. This causes greater EMB motor angle travel before a clamp force is induced, and due to the sinusoidal nature of the EMB motor angle input, a reduction initially in clamp force during cooling results as shown in Figure 8-8. This is counteracted as cooling continues where the stiffening of the pads have a greater influence and cause the clamp force to increase. In practice thermal expansion and contraction of the disc volume should for the most part not hinder clamp force estimation significantly. This is because disengagement offers the opportunity for the system to adjust for such events. That is, Schwarz et al. [54] clearance management schemes enable engagement to be determined without the use of a physical clamp force sensor for automotive EMB systems. Such clearance management schemes rely on remaining EMB system sensory information. The thermal expansion and contraction of the disc will therefore have no influence on Schwarz et al. [54] clearance management schemes. The only instance when thermal expansion of the disc will be an issue is for prolonged braking events such as downhill braking. Therefore, methods must be adopted to handle such scenarios.
9

Integrating Developments

9.1 Introduction

Previous chapters within this thesis have served as component parts towards the practical development of a clamp force estimator for an EMB. In this chapter these component parts are integrated together and validated using empirical data. The next subsection provides some brief material necessary before any validation can be performed. The focus in the next subsection is to determine what significance thermal conditions have on damping from the dynamic stiffness model developed previously in Chapter 3. After this subsection all developments within this thesis are merged and validated against empirical data.

9.2 Parameter Thermal Dependence Investigation

The dynamic stiffness model developed in Chapter 3 to estimate clamp force in an EMB is given as follows:

\[ F_{ci}^+ (i) = \beta_1 \left( A_2 \theta_m^3 (i) + A_1 \theta_m^2 (i) + A_0 \theta_m (i) \right) + \beta_0 F_{ci}^+ (i - 1), \]

Eq. 9-1

where,

\[ \beta_0 = \frac{\tau}{\Delta t}, \]

Eq. 9-2

and
\[ \beta_i = \left( \frac{1}{1 + \frac{\tau}{\Delta t}} \right). \quad Eq. 9.3 \]

The developments of Chapter 3 were obtained with the help of the static test rig shown in Figure 3-1. This test rig did not produce any significant heat during operation and so the temperature of the EMB was more or less at ambient. The \( \beta \) coefficients from Eq. 9-1 have not been examined directly to see what influence temperature may have on the damping aspect. This is done now as follows.

As described in Chapter 3, the \( \beta \) coefficients are used to approximate dynamic that exists between motor angle and induced clamp force. It was found in Chapter 3 that the main component causing such dynamic was the caliper bridge. To view what influence temperature may have on the damping aspect from the \( \beta \) coefficients, dynamic data of motor angle input and clamp force output is gathered at various caliper bridge temperatures using the test rig shown in Figures 7-2a and 7-2b. Via friction braking, this test rig generates heat and so increases the temperature of the EMB. To measure caliper bridge temperatures a thermocouple is glued on the caliper bridge as shown in Figure 9-1. Figures 9-2a and 9-2b show dynamic data of clamp force versus motor angle for various caliper bridge temperatures. To attain this data the

\[ \text{Figure 9-1. Thermocouple glued on caliper bridge.} \]
motor angle was varied similarly each time in a dynamic uniform random manner. The temperatures of the caliper bridge were taken at the beginning of each uniform random cycle. Considering the relatively short duration of these cycles, any variation of caliper bridge temperatures during each cycle was regarded as negligible. It should be noted that the characteristic curves did not vary significantly for the data shown in Figures 9-2a and 9-2b. It can be seen then in Figures 9-2a and 9-2b that the level of damping experienced at the different caliper bridge temperatures is similar. Based on Figures 9-2a and 9-2b, the damping aspect of the $\beta$ coefficients from Eq. 9-1 is approximated to have no temperature dependence at least for the measured temperature range. It should be mentioned that the caliper bridge is not expected to exceed temperatures of more than 200 °C in-service [53]. Reaching such a temperature using the test rig shown in Figures 7-2a and 7-2b was impractical due to the static atmospheric environment which could lead to fire. Throughout the rest of this thesis the $\beta$ coefficients are assumed to have no temperature dependence.
Chapter 9 - Integrating Developments

The characteristic curve varies with pad wear and pad thermal conditions. Considering that the $\beta$ coefficients have stiffness dependencies, an approximation is assumed by using constant $\beta$ coefficients.

Given that the $\beta$ coefficients were attained using a different test rig as to that shown in Figures 7-2a and 7-2b, they are nonetheless still relevant to the test rig shown in Figures 7-2a and 7-2b.

9.3 Empirical Validation of Developments

In order to collectively validate the developments within this thesis to estimate the clamp force in an EMB, relevant data firstly needs to be attained using the test rig shown in Figures 7-2a and 7-2b. Figures 9-3a and 9-3b show part of such data. It can be seen that the brake torque and disc speed are varied over a lengthy period of time. During periods where the disc was stationary, the EMB motor angle was varied in a dynamic uniform random manner. Also, at the beginning the EMB motor angle was varied in the manner shown in Figure 3-9 while the disc was stationary.

This was necessary so that characteristic curve coefficients could be determined using Schwarz et al. [54] parameter adaptation technique as described in Section 2.4. These characteristic curve coefficients were required for use in the dynamic stiffness model to estimate clamp force described in Chapter 3. Also, the friction parameters in the torque balance model to estimate clamp force described in Chapter 4 were determined using Schwarz et al. [54] parameter adaptation technique on the initial training data. To sense when contact between pads and disc was made, Schwarz et al. [54] clearance management scheme was used as described in Section 2.5.

The previous chapter focused on modelling how the characteristic curve varied with thermal conditions. These developments can be integrated into the dynamic stiffness model given by eq. 9-1 to estimate clamp force since characteristic curve coefficients are contained within eq. 9-1.

Such an extended form of Eq. 9-1 is given as follows:

$$ F_{cl,i}(i) = \beta_0 \alpha(i) (A_{2,0} \theta_m^3(i) + A_0 \theta_m(i) + A_{0,0} \theta_e(i)) + \beta_1 F_{cl,0}(i-1), $$  

*Eq. 9-4*

where the $\alpha(i)$ coefficient is described in Section 8.4. Using dynamic uniform random data associated with Figures 9-3a and 9-3b, eq. 9-4 is applied to predict clamp force where an RMSE of 1.09 kN resulted. Using the maximum-likelihood estimator described in Section 5.2, Equation 9-4 and the torque balance model to estimate clamp force described in Chapter 4 are fused to estimate clamp force using the dynamic uniform random data associated with Figures 9-3a and 9-3b. Figure 9-4a shows the results of such fusion where an RMSE of 0.56 kN was attained. In the same manner, except that a Kalman filter is used as described in Section 5.3, the
outputs from the two clamp force estimation models are fused. Figure 9-4b shows the results using a Kalman filter where an RMSE of 0.5 kN is determined. The significance of the results obtained in this section will be briefly discussed in the next chapter.

It should finally be mentioned that clamp force estimator validations were performed with no rotating disc. In practice the disc has varying dimensional tolerances. The formation of heat spots on the disc during braking will also cause varying dimensional tolerances. During rotation of the disc, varying dimensional tolerances will influence clamp force estimation where stiffness
Figure 9-4. Clamp force estimator validations on dynamic uniform random data: (a) Maximum-likelihood estimator. (b) Kalman filter.

is used. Therefore, the significance of varying dimensional tolerances of the disc on clamp force estimation where stiffness is used should be investigated for future studies.
Chapter 10 - Conclusions and Future Research

10 Conclusions and Future Research

10.1 Summary and Conclusions

The aim in this thesis was to further develop a virtual clamp force sensor that would lead to the omission of a physical clamp force sensor from an EMB in an automotive BBW system, thus reducing costs. Previous attempts to estimate clamp force in an EMB have been reliant on stiffness based models. That is, by utilizing the actuator resolver sensor a spring system approach was taken to estimate the clamp force based on motor angle displacements. The curve representing such a relationship was termed the characteristic curve. The extreme heat generated by the actuator causes stiffness parameter variations in the characteristic curve. An adaptation technique has been proposed previously in an attempt to accommodate such parameter variations. Prior efforts to estimate clamp force have failed to handle certain scenarios.

In this thesis the problems associated with existing clamp force estimation efforts were considered. Also, new methodologies to estimate clamp force incorporating previous techniques have been presented. In this thesis it has been shown that a dynamic system existed between motor angle input and clamp force output. Such a dynamic system could not be handled appropriately by a purely stiffness based model to estimate clamp force. To solve this problem, a dynamic stiffness model to estimate clamp force was developed using empirical data. Developmental inroads were made for a new approach to handle thermally dependent stiffness parameter variations. The existing method attempting to track such parameter variations fail during high speed braking applications as well as during cooling instances after disengagement.
of the brake pads has occurred. The proposed solution to this problem in this thesis focused on developing a pad thermal model that could be used to predict thermally dependent stiffness parameter variations. This course of action identified the 1 dimensional nature of brake pad temperature propagation based on empirical data. Also, relaxation phenomena of the brake pad was observed from empirical data. It has been found in this thesis, using empirical data, that the pad friction surface temperature remained very close to that of the disc friction surface temperature at all times for the actuator design used. This enabled the pad friction surface thermal boundary condition during cooling to be greatly simplified by developing a disc cooling model that served to indicate what the pad friction surface temperature was during cooling for in-service applications. Ideas were given as to how the pad friction surface thermal boundary condition could be handled during braking for in-service applications. For practical purposes within this thesis, such a boundary condition was defined with the help of sensory information not available for in-service applications. The use of a temperature sensor, in the form of a thermocouple, at the pad and pad backing plate interface was proposed for in-service application so that this thermal boundary condition for the pad was defined. The addition of 2 such temperature sensors will still result in cost savings being achieved as a clamp force sensor for an automotive EMB is significantly more expensive.

A torque balance model to estimate clamp force was developed in this thesis that relied on the actuator resolver sensor and the actuator motor current sensors. This model as well as the dynamic stiffness model to estimate clamp force with new thermal sensitivities were fused in this thesis using various sensor fusion algorithms to give improved estimates of clamp force. A maximum-likelihood estimator and a Kalman filter were used. Empirical data was applied to show the accuracy of the 2 fusion algorithms in tracking dynamic data. The Kalman filter was more accurate as expected however it required more computational burden then the maximum-likelihood estimator. It is difficult to ascertain what level of clamp force accuracy is required for in-service applications. This is because such empirically supported knowledge is scarce given the emerging nature of this technology. Regardless of this, this thesis has persisted with trying to improve the estimation of clamp force. Considerable amounts of empirical data have been presented in this thesis which can serve to provide material support for continued future research in this emerging technology.

10.2 Future Research

In order to improve the methods presented in this thesis to estimate clamp force in an EMB for an automotive BBW system, further research is required. The proposals in this thesis have focused on deficiencies in existing efforts to estimate clamp force as well as introduce new
methodologies to further improve clamp force estimation. However, as described previously, there are limitations. Thermal expansion of the disc volume was not modelled. Thermal expansion of the disc volume during prolonged periods of braking, such as going downhill, will cause considerable error in any clamp force estimator that is reliant on stiffness. This is because the motor angle position that initiates clamp force has changed significantly during such a scenario. Considering this thesis showed that a ventilated disc can be tentatively assumed to be uniform in temperature during periods of heat transfer, and that such temperatures are intended to be approximately known in-service based on pad friction surface temperatures, an empirically based model of disc thermal expansion with temperature could be developed in future work to help solve this problem.

Clamp force estimator validations in this thesis were not performed on a rotating disc. Considering a disc has varying dimensional tolerances in practice, a clamp force estimator where stiffness is used will be influenced during disc rotation. The significance of this should be investigated in future research.

The thermal boundary condition at the pad friction surface during braking was not modelled in this thesis. This is a necessary requirement for the pad temperature prediction model to have practical significance. An idea of how this boundary condition could be handled was proposed for future research. It involved using look up tables to approximately give the heat flux into a pad during braking based on clamp force, pad friction surface temperature and disc speed measurements which are all intended to be available in-service.

The RMSE of clamp force estimation provided throughout this thesis require a benchmark to determine what is acceptable. The most appropriate benchmark would be that determined based on empirical data. Such knowledge is scarce at the moment and future research in this area is necessary.


Reference List


Appendix A

Heat Conduction Equation

Heat conduction occurs in a solid medium when a temperature difference exists in it. Heat flows from higher temperatures to lower temperatures until thermal equilibrium is reached. The geometry of the solid medium as well as the boundary conditions dictates whether the heat flow is 1-dimensional or multidimensional. For the case of a prism geometry where the cross sectional area is large relative to its thickness, a heat flux at one side of the prism will result in mainly a 1-dimensional heat flow across the thickness direction. An example of this is the metal plate from an iron. To obtain a temperature profile with respect to the space and time dimensions for such a case, a differential equation must firstly be determined. This is shown as follows. Figure A-1 shows an arbitrary prism which is part of a larger wall. The thermal properties of the prism are assumed to be constant. A volume element of finite thickness $\Delta x$
is highlighted in Figure A-1. An energy balance over a finite time interval on the volume element from Figure A-1 can be expressed as follows:

\[
\left( \text{Rate of heat flow at } x \right) - \left( \text{Rate of heat flow at } x + \Delta x \right) = \left( \text{Rate of change of energy content within volume element} \right).
\]

\[A-1\]

or in algebraic terms as:

\[\dot{Q}_x - \dot{Q}_{x+\Delta x} = \frac{\Delta E_c}{\Delta t}. \]

\[A-2\]

The change in energy content of the volume element over the finite time step can be defined as follows:

\[\Delta E_c = \rhoCA\Delta x(T_{t+\Delta t} - T_t), \]

\[A-3\]

where \(A\), \(T_t\) and \(T_{t+\Delta t}\) are the surface area of the plane perpendicular to the path of heat transfer, the temperature at time \(t\) of the volume element and the temperature at time \(t+\Delta t\) of the volume element.

Substituting Eq. A-3 for \(\Delta E_c\) into Eq. A-2 gives:

\[\dot{Q}_x - \dot{Q}_{x+\Delta x} = \frac{\rho CA\Delta x(T_{t+\Delta t} - T_t)}{\Delta t}. \]

\[A-4\]

Dividing Eq. A-4 by \(A\Delta x\) allows the following expression to be attained:

\[- \frac{1}{A} \frac{\dot{Q}_{x+\Delta x} - \dot{Q}_x}{\Delta x} = \rho C \frac{T_{t+\Delta t} - T_t}{\Delta t}. \]

\[A-5\]

Setting the limits \(\Delta x \to 0\) and \(\Delta t \to 0\) in Eq. A-5 leads to:

\[- \frac{1}{A} \frac{\partial \dot{Q}}{\partial x} = \rho C \frac{\partial T}{\partial t}. \]

\[A-6\]

Fourier’s law of heat conduction is given as follows:

\[\dot{Q} = -kA \frac{dT}{dx}. \]

\[A-7\]

Substituting Eq. A-7 for \(\dot{Q}\) into Eq. A-6 and taking temperature to be dependant on both space and time variables yields:

\[- \frac{1}{A} \frac{\partial}{\partial x} \left( kA \frac{\partial T}{\partial x} \right) = \rho C \frac{\partial T}{\partial t}. \]

\[A-8\]

The thermal conductivity and area in Eq. A-8 are taken to be constants and therefore can be taken outside the partial derivative to give:
Equation A-9 is the differential equation that must be solved to give temperature as a function of space and time for 1-dimensional heat flow through a prism as described earlier in this appendix.

$$k \frac{\partial^2 T}{\partial x^2} = \rho C \frac{\partial T}{\partial t}. \quad A-9$$
Appendix B

Convective Lumped Thermal Modelling

For certain convective heat transfer situations, the solid body undergoing heat transfer can be assumed to behave as a time varying isothermal volume. That is, no significant temperature variations with respect to the space dimensions of the solid body exist. An example would be a small silver coin taken out of a fridge whereupon it is heated by the atmosphere to ambient temperature. The mechanisms and directions of heat flow for such a case are shown in Figure B-1. Since silver has a relatively high thermal conductivity, thermal equilibrium takes place rapidly within the coin ensuring that uniform temperature is approximately maintained throughout the heat transfer process.

The derivation of a model to predict temperature of a solid body during convective heat transfer processes where uniform temperature of the solid body can be assumed is given as follows. Such modelling is also known as convective lumped thermal modelling. A solid body of arbitrary shape is considered as shown in Figure B-2. The solid body is initially at a uniform temperature $T_0$ and is placed in an environment where the fluid is of say higher constant temperature $T_\infty$. Heat transfer takes place via convection between the solid body and its surroundings such that heat flows into the solid body. Newton’s law of cooling for convection is given as follows:

$$\dot{Q} = hA(T_s - T_\infty)$$  \hspace{1cm} \textit{B-1}$$

where $h$, $A$ and $T_s$ are the convection heat transfer coefficient, surface area and surface temperature respectively. An energy balance on the solid body from Figure B-2 over a differential interval of time is

![Figure B-1. Atmospheric heating of small silver coin that is initially cold. Mechanisms and directions of heat flow shown.](image-url)
Incorporating Newton’s law of cooling for convection into Eq. B-2, and using algebraic terms throughout to describe Eq. B-2 yields:

\[ hA(T_\infty - T)dt = mc dT, \quad B-3 \]

where \( T \) is the temperature of the solid body. Equation B-3 can be rearranged and written as follows:

\[ \frac{d(T - T_\infty)}{T - T_\infty} = -\frac{hA}{\rho VC} dt, \quad B-4 \]

where \( d(T - T_\infty) = dT \). Integrating Eq. B-4 with respect to the time interval \([0, t]\) which has an associated temperature interval \([T_0, T(t)]\) gives:

\[ \ln \frac{T(t) - T_\infty}{T_0 - T_\infty} = -\frac{hA}{\rho VC} t. \quad B-5 \]

Rearranging Eq. B-5 after taking the exponential on both sides allows the following expression to be obtained which gives the temperature of the solid body over time:

\[ T(t) = (T_0 - T_\infty)e^{-bt} + T_\infty, \quad B-6 \]

where,

\[ b = \frac{hA}{\rho VC}. \quad B-7 \]

A criterion must be defined on the applicability of convective lumped thermal modelling since not all situations can be satisfactorily approximated in this manner. The Biot number is used to define this criterion and is expressed as:
\[ Bi = \frac{hL_c}{k}, \]  \hspace{1cm} B-8

where,

\[ L_c = \frac{V}{A}, \]  \hspace{1cm} B-9

If the Biot number is less than or equal to 0.1, the differential temperature between the solid body and fluid is accurate to within 5%. Convective lumped thermal modelling is generally considered applicable when the Biot number is less than or equal to 0.1.