Optical parametric amplification in dispersion-flattened highly nonlinear photonic crystal fibers

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ABSTRACT

Photonic Crystal Fibers (PCFs) are optical fibers with unique guiding characteristics as well as unusual nonlinear and dispersion properties. Since PCFs offer the possibility to engineer the zero-dispersion wavelength, the dispersion curve and the nonlinear coefficient value, they are very interesting for optical parametric amplification. In the present paper the phase-matching condition has been deeply analyzed in different triangular PCFs configurations. In particular, highly nonlinear PCFs have been designed to achieve flattened dispersion curves around the zero-dispersion wavelength in the C band. Very flat parametric gain, around 16 dB, on a bandwidth up to 35 nm can be obtained with short PCF and low pump power level.

Keywords: Photonic crystal fiber, parametric amplification, nonlinear effect, dispersion properties

1. INTRODUCTION

Parametric amplification offer a new possibility to amplify signals in optical transmission systems besides erbium-doped or Raman fiber amplifiers. The parametric gain is based on highly efficient four wave mixing (FWM) relying on the relative phase between four interacting photons.1,2 By pumping the fiber with an intense wave, a wide and flat gain can be obtained over two wavelength bands surrounding the pump. Modern high power sources have increased the interest in optical parametric amplifiers (OPAs), whose gain band can be tailored to operate at any wavelength, providing amplification outside the conventional erbium-doped one. Besides broadband amplification at arbitrary wavelength, parametric process offers a variety of applications such as, for example, conversion wavelength, pulse reshaping and soliton-soliton interaction.3 Multiple pump schemes can further enhance the OPA efficiency, both in terms of maximum gain and bandwidth.4 Fiber nonlinearity and dispersion are fundamental aspects for a successful OPA design. In fact, to achieve high and broadband gain in OPAs, the phase matching requirement demands a low dispersion slope, while the efficiency of the nonlinear process requires a small fiber effective area, in order to have an high nonlinearity.5–8 In the last few years, highly nonlinear optical fibers with nonlinear parameters five to ten times higher than conventional fibers have been introduced and OPA gains up to 50 dB have been experimentally demonstrated.9 Photonic Crystal Fibers (PCFs) have also recently emerged as fibers with unique guiding, nonlinear and dispersion characteristics. Highly nonlinear PCFs,10 as well as fibers with very flat dispersion curves around the zero-dispersion wavelength have been successfully designed.11 Also PCFs with both these properties have been proposed.12 Since PCFs offer the possibility to engineer the zero-dispersion wavelength, the dispersion curve and the nonlinear coefficient value, they seem to be very attractive for OPA applications.13

The present work provides a study of all-silica triangular lattice PCFs designed with a flattened dispersion curve, the zero-dispersion wavelength around 1550 nm and a high nonlinear coefficient. They have silica cores with different shapes, obtained by removing or by changing the holes surrounding the single defect core. In this way, the main advantage offered by PCFs, that is the possibility to control their guiding characteristics by properly changing the refractive index profile, has been exploited. In order to show that triangular lattice PCFs have interesting properties for parametric amplification, the phase-matching condition has been analyzed by varying the parameters which define the fiber cross-section geometry. In particular, three triangular PCFs with high nonlinear coefficient values and flattened dispersion have been considered and the resulting parametric gain evaluated. As an example, 16 dB OPA gain on bandwidths up to 35 nm can be obtained with short PCFs and low pump power levels.

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Figure 1. Cross-section of the considered triangular lattice PCFs. Λ is the hole-to-hole pitch and d is the hole diameter; (a) structure designed with three reduced holes having diameter \( d_f \), (b) structure designed by changing the diameter \( d_1 \), \( d_2 \) and \( d_3 \) of the holes belonging to the first three rings.

2. THE FINITE ELEMENT METHOD

The present analysis has been performed by using the Finite Element Method (FEM) which has been already successfully used to study PCF dispersion\textsuperscript{12,14} and nonlinear properties.\textsuperscript{15} The method has been exploited to evaluate the mode field distribution, necessary to compute the nonlinear coefficient, and the mode propagation constant as a function of the wavelength which, in turn, provides the fiber dispersion curve. The FEM allows the PCF cross-section in the transverse \( x - y \) plane to be divided into a patchwork of triangular elements which can be of different sizes, shapes and refractive indices. In this way any kind of geometry, including PCF holes, as well as medium characteristic, can be accurately described.

The considered formulation is based on the curl-curl equation. For a medium described by the relative dielectric permittivity \( \varepsilon_r \) and the magnetic permeability \( \mu_r \) it reads:

\[
\nabla \times \left( p \nabla \times \mathbf{V} \right) - k_0^2 q \mathbf{V} = 0 \tag{1}
\]

where \( k_0 \) is the wavenumber in the vacuum while \( p \) and \( q \) represent \( \varepsilon_r^{-1} \) and \( \mu_r \) when \( \mathbf{V} \) is the magnetic field \( \mathbf{H} \) and \( \mu_r^{-1} \) and \( \varepsilon_r \) when \( \mathbf{V} \) is the electric field \( \mathbf{E} \). By applying the variational finite element procedure, the full vector equation (1) yields the algebraic problem\textsuperscript{16}:

\[
\left( \begin{bmatrix} A \end{bmatrix} - (n_{eff})^2 \begin{bmatrix} B \end{bmatrix} \right) \{v\} = 0 \tag{2}
\]

where \( \{v\} \) is the unknown eigenvector representing the magnetic or the electric field while the eigenvalue directly provides the complex effective mode index \( n_{eff} \). The matrices \( [A] \) and \( [B] \) are sparse and symmetric thus allowing an efficient resolution of Eq. 2 by means of high performance algebraic solvers for both real and complex problems. Complex formulations can be very useful, for instance, to evaluate PCF leakage losses due to the finite number of hole rings in the cladding lattice.\textsuperscript{17}

3. PHOTONIC CRYSTAL FIBER DESIGN

3.1. Structure

All the PCFs here studied are simply made of silica, with refractive index 1.45, and have a triangular lattice of air-holes in the cross-section, which is characterized by the pitch \( \Lambda \) and the air-hole diameter \( d \). Two different kinds of PCFs have been considered.
In the first type, three of the holes belonging to the first ring have a different diameter \(d_f<d\), as it is shown in Fig. 1a. In this case the PCF core has a shape which is almost triangular and it sustains a field whose magnetic components are reported in Fig. 2 for the particular case of \(\lambda=1550\) nm, \(\Lambda=1.7\) \(\mu m\), \(d=0.54\) \(\mu m\) and \(d_f=0.2\) \(\mu m\). The distribution of the fundamental component has a triangular symmetry with a quasi-Gaussian shape in the center of the core thus allowing high coupling values with standard fibers. Notice that by choosing \(d_f=0\) the three holes around the core are eliminated.

Since all the studied PCFs have small \(\Delta d/\Lambda\) values, between 0.3 and 0.4, twelve air-hole rings have been considered to obtain negligible leakage losses. The second type of fiber has been designed by changing the diameter \(d_1\), \(d_2\) and \(d_3\) of all the holes belonging to the first three rings around the core, as it is shown in Fig. 1b. This fiber has been selected because it has been already demonstrated that a proper choice of \(\Lambda\), \(d\), \(d_1\), \(d_2\) and \(d_3\) can give very high nonlinear coefficients and very flat dispersion curves. In particular, the following geometric parameters have been selected, \(\Lambda=0.9\) \(\mu m\), \(d=0.81\) \(\mu m\), \(d_1=0.42\Lambda\), \(d_2=0.87\Lambda\) and \(d_3=0.86\Lambda\).

### 3.2. Dispersion

In order to achieve a flattened dispersion curve around 1550 nm, the pitch \(\Lambda\) has been modified in the range \(1.4\div1.7\) \(\mu m\) and the air-hole diameter \(d\) has been properly chosen between 0.5 \(\mu m\) and 0.7 \(\mu m\). In addition, the zero-dispersion wavelength position in the C band has been optimized by changing \(d_f\). In Fig. 3a and 3b the dispersion curves of the PCFs of the first type, with \(d=0.65\) \(\mu m\) and \(\Lambda=1.6\) \(\mu m\) and 1.7 \(\mu m\), are reported.

### Figure 2

**H_x, H_y and H_z** components of the magnetic field at \(\lambda=1550\) nm for the PCF with \(\Lambda=1.7\) \(\mu m\), \(d=0.54\) \(\mu m\) and \(d_f=0.2\) \(\mu m\).

**Figure 3.** Dispersion parameter \(D\) as a function of \(d_f\) for the PCFs with \(d=0.65\) \(\mu m\), (a) \(\Lambda=1.6\) \(\mu m\) and (b) \(\Lambda=1.7\) \(\mu m\).
for different \( d_f \) values. Notice that for both the considered \( \Lambda \) values the \( D \) parameter decreases as \( d_f \) varies from 0 to 0.3 \( \mu m \). A further increase of this diameter up to 0.4 \( \mu m \) causes a significant but undesired change in the dispersion curve slope. By properly fixing \( d_f = 0.29 \mu m \), when \( \Lambda = 1.6 \mu m \), a triangular-core PCF with a zero-dispersion wavelength \( \lambda_0 = 1550.5 \text{ nm} \) and a dispersion slope \( S_0 \) of about \(-1.8 \cdot 10^{-2} \text{ ps/km} \cdot \text{nm}^2\) at \( \lambda_0 \) can be obtained. Simulation results have shown that the best \( d_f \) is 0.32 \( \mu m \), when the higher pitch is assumed, being in this case \( \lambda_0 \approx 1563.3 \text{ nm} \) and the dispersion slope \(-1.3 \cdot 10^{-2} \text{ ps/km} \cdot \text{nm}^2\).

The pitch \( \Lambda = 1.7 \mu m \) and the diameter \( d_f = 0.2 \mu m \) have then been fixed, and \( d \) has been changed between 0.53 \( \mu m \) and 0.65 \( \mu m \), in order to show the influence of the air-hole dimension on the PCF dispersion properties. As shown in Fig. 4, \( D \) values decrease in all the wavelength range considered as the air-holes become smaller, while the slope of the dispersion curve is only slightly modified. The dispersion parameter is always negative when \( d = 0.54 \mu m \), reaching a maximum of about \(-0.14 \text{ ps/km} \cdot \text{nm}\) at 1525 \( \text{nm} \), with a very low \( S_0 \), that is

![Figure 4](https://example.com/figure4.png)

**Figure 4.** Dispersion parameter \( D \) as a function of \( d \) for the PCFs with \( d_f = 0.2 \mu m \) and \( \Lambda = 1.7 \mu m \).

![Figure 5](https://example.com/figure5.png)

**Figure 5.** Dispersion parameter \( D \) versus the wavelength in the range 1480 \( \text{nm} \) ÷ 1600 \( \text{nm} \) for the triangular-core PCFs with the best set of \( \Lambda \), \( d \) and \( d_f \) values.
about $-1.7 \cdot 10^{-3}$ ps/km·nm$^2$ at 1550 nm.

The same analysis has been performed also for different configurations of the PCF cross-section. Fig. 5 shows the best dispersion curves obtained considering new $\Lambda$ values with proper hole diameters $d$ and $d_f$. It is important to underline that the PCF with $\Lambda = 1.4 \, \mu m$ has $d_f = 0 \, \mu m$, that is the three air-holes with diameter $d_f$ have been completely removed. Also in this case it is possible to achieve a good dispersion slope of about $-3.8 \cdot 10^{-2}$ ps/km·nm$^2$. Tab. 1 summarizes the zero dispersion wavelengths and the dispersion slopes of the designed PCFs. The values for the fiber of the second type are also reported in the last row. In this case the zero wavelength occurs at a slightly lower value, that is 1510.5 nm, and the dispersion slope is very low as well.

### 3.3. Nonlinearity

The nonlinear coefficient can be evaluated through

$$\gamma = \frac{2\pi}{\Lambda} \int \int n_2(x,y)i^2(x,y)dxdy,$$

where $n_2(x,y)$ is $3 \cdot 10^{-20}$ m$^2$/W in the silica bulk and zero in the air-holes. The normalized intensity $i(x,y)$ is evaluated according to the definition of the Poynting vector. The distribution of the field components have been obtained through the FEM solver.\(^{19}\) It has been observed that, as the diameter $d$ of all the air-holes decreases, the field is less confined and this, in turn, limits the value of $\gamma$. For example the nonlinear coefficient of the first two fibers reported in Tab. 1 at a wavelength equal to 1550 nm are, respectively, 9.7 (W·km)$^{-1}$ and 6.8 (W·km)$^{-1}$. The opposite effect can be achieved by reducing the pitch $\Lambda$, that is the core dimension. This is confirmed by the value of $\gamma = 10.97$ (W·km)$^{-1}$ and $\gamma = 10.92$ (W·km)$^{-1}$ of the third and fourth fiber in Tab.1.

### Table 2. Nonlinear coefficient $\gamma$ for the PCFs of the first (first two rows) and the second (last row) type.

<table>
<thead>
<tr>
<th>$\Lambda$ ($\mu m$)</th>
<th>$d$ ($\mu m$)</th>
<th>$d_f$ ($\mu m$)</th>
<th>$\gamma$ (W·km)$^{-1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.6</td>
<td>0.65</td>
<td>0.29</td>
<td>10.97</td>
</tr>
<tr>
<td>1.4</td>
<td>0.55</td>
<td>0.0</td>
<td>10.92</td>
</tr>
<tr>
<td>0.9</td>
<td>0.81</td>
<td>—</td>
<td>40.73</td>
</tr>
</tbody>
</table>
Notice that the PCFs with the lowest slopes have also the lowest $\gamma$ values, as previously predicted. This suggests that for this kind of fibers, a proper trade-off between the dispersion slope and the nonlinear coefficient values must be found. For this reason, considering the PCFs of the first type, the attention will be focused on the third and fourth fiber of Tab. 1.

As already observed, the fiber of the second type has been selected for its high $\gamma$ value, that is 40.73 (W km)$^{-1}$, which is due to its small pitch that strongly focuses the field within the core, thus increasing the local intensity. Tab. 2 summarizes the nonlinear coefficient for the three fibers which will be considered in the following analysis.

### 4. Phase Matching Condition

In order to show how the triangular lattice PCFs here proposed can be successfully used for optical parametric amplification, the phase-matching condition has been analyzed. Under the assumption of undepleted pump, this condition reads:

$$\Delta \beta + 2\gamma P_p = 0 ,$$  \hfill (4)

where $\Delta \beta$ is the linear wave-vector mismatch and $P_p$ is the pump power. When this condition is satisfied, the maximum gain can be obtained through the parametric amplification, since the power flow from the pump at $\lambda_p$ to the signal at $\lambda_s$, which are involved in the FWM process, is highly efficient. The phase-matching is obtained when the nonlinear phase shift $2\gamma P_p$ is compensated by a negative $\Delta \beta$. The linear component of the phase-mismatch parameter can be calculated by expanding in Taylor series the propagation constant $\beta(\omega)$ around the zero-dispersion frequency $\omega_0 = 2\pi c/\lambda_0$, that is

$$\Delta \beta = \left\{ \beta_3(\omega_p - \omega_0) + \frac{\beta_4}{2} (\omega_p - \omega_0)^2 + \frac{1}{6} (\omega_p - \omega_s)^2 \right\} (\omega_p - \omega_s)^2 ,$$  \hfill (5)

where $\beta_3$ and $\beta_4$ are, respectively, the third and fourth derivative of $\beta(\omega)$ calculated at $\omega_0$; $\omega_p$ is the pump frequency and $\omega_s$ the signal one. In the present analysis the contribution from $\beta_4$ has been considered, as shown in Eq. 5. In fact, when taking into account PCFs, the waveguide contribution to the dispersion curve is significant, thus higher order derivative of $\beta(\omega)$ are usually larger than in conventional fibers and can not be neglected.

The values of $\beta_3$ and $\beta_4$ for the three fibers are reported in Tab. 3 and have been evaluated by deriving the 8-th order polynomial fitted to the dispersion curve. Notice that their accuracy has been checked following a second approach, besides the expression given by Eq. 5. In particular the linear wave-vector mismatch has been calculated also through the relation

$$\Delta \beta = \beta(\omega_s) + \beta(\omega_i) - 2\beta(\omega_p),$$

being $\beta(\omega_s)$, $\beta(\omega_i)$ and $\beta(\omega_p)$ the propagation constant, respectively, of signal, idler and pump, which have been obtained by the FEM solver. The agreement between the two approaches is very good.

The linear wave-vector mismatch versus the wavelength difference between the signal and the pump, $|\lambda_s - \lambda_p|$, has been calculated in the range 0~60 nm for the considered PCFs. For example, Fig. 6 and 7 report two sets

<table>
<thead>
<tr>
<th>$\Lambda$ ($\mu m$)</th>
<th>d ($\mu m$)</th>
<th>$d_f$ ($\mu m$)</th>
<th>$\beta_3$ (ps$^3$/km)</th>
<th>$\beta_4$ (ps$^4$/km)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.4</td>
<td>0.55</td>
<td>0.0</td>
<td>-6.31·10$^{-2}$</td>
<td>5.57·10$^{-4}$</td>
</tr>
<tr>
<td>1.6</td>
<td>0.65</td>
<td>0.29</td>
<td>-2.84·10$^{-2}$</td>
<td>2.99·10$^{-4}$</td>
</tr>
<tr>
<td>0.9</td>
<td>0.81</td>
<td>—</td>
<td>-1.38·10$^{-2}$</td>
<td>1.40·10$^{-4}$</td>
</tr>
</tbody>
</table>
of $\Delta \beta$ curves obtained by choosing different $\lambda_p$ in order to get similar values of the $\Delta \beta$ minimum. Notice that, being $\beta_3$ negative for all the PCFs, $\omega_p$ must be greater than $\omega_0$, that is $\lambda_p < \lambda_0$, in order to obtain a negative $\Delta \beta$, as shown by Eq. 5. Moreover, the presence of the symmetrical minima in all the $\Delta \beta$ curves is due to the different sign of $\beta_3$ and $\beta_4$. In fact, according to Eq. 5, as $|\lambda_s - \lambda_p|$ increases, the positive contribution of $\beta_4$ on the linear wave-vector mismatch becomes higher, until it dominates the negative one provided by $\beta_3$. As a consequence, $\Delta \beta$ decreases initially when $\lambda_s \simeq \lambda_p$, it reaches a negative minimum value and then it increases, becoming positive and no longer useful to satisfy the phase-matching condition. The value and the position of the minimum is strictly related to the choice of the pump wavelength. In particular, it becomes more negative and farther from $\lambda_p$ when $|\lambda_p - \lambda_0|$ increases, as can be easily observed by comparing Fig. 6 with Fig. 7. Moreover, it is possible to understand the influence of $\beta_3$ and $\beta_4$. For example, considering the first two fibers, the minima of $\Delta \beta$ and the condition $\Delta \beta = 0$ can be obtained for greater values of $|\lambda_s - \lambda_p|$ if the PCF with the lowest $\beta_3$
Figure 8. Signal power gain $G$ versus $\lambda_s - \lambda_p$. The pump power level used for each PCF is indicated in the figure label. All the fibers are 1.0 km long.

and $\beta_4$ is considered, that is the one with $\Lambda = 1.6 \, \mu m$.

Finally, it is important to underline that for $2\gamma P_p$ values lower than the absolute value of the $\Delta \beta$ minimum, the phase-matching condition is satisfied for two different signal wavelengths. This is shown in the figures by the curve intersections with the horizontal lines which represent an arbitrary value of the nonlinear phase shift. Of course, $P_p$ must be chosen in order to maximize the gain value and bandwidth, according to the selected kind of fiber. For example, by increasing $P_p$ the two intersection points go far away one from the other. This increases the bandwidth but can cause the gain curve to be affected by strong ripples. On the contrary, two close intersection points results in a flattened gain curve with a reduced bandwidth.

5. PARAMETRIC GAIN

The signal power gain can be expressed as

\[
G = 10 \log 10 \left( 1 + \frac{\gamma P_p}{g} \sinh(gL) \right),
\]

where $L$ is the fiber length and $g = \sqrt{(\gamma P_p)^2 - (\kappa/2)^2}$ is the parametric gain coefficient.\(^3\) The signal gain of the considered PCFs has been calculated versus $|\lambda_s - \lambda_p|$ for two different lengths, 1 km and 0.5 km, by varying $P_p$ so that the product between the pump power, the nonlinear coefficient and the fiber length is constant and, consequently, the maximum $G$ is kept almost fix.\(^3\) As shown in Fig. 8 and Fig. 9, a very flat gain can be obtained over a wide signal wavelength range. For example, considering the fiber of the second type, a 3 dB bandwidth of 30 nm and 35 nm has been reached by satisfying the phase matching condition, respectively, for a nonlinear phase shift of 5 km\(^{-1}\), corresponding to $P_p = 0.062 \, W$ and a fiber length equal 1 km and a nonlinear phase shift of 10 km\(^{-1}\), corresponding to $P_p = 0.124 \, W$ and a fiber length equal 0.5 km.

6. CONCLUSION

Triangular lattice PCFs can be designed to engineer the dispersion curve and the nonlinear coefficient value and consequently they can be successfully exploited to satisfy the FWM phase-matching condition to enhance the parametric amplification process. The dispersion and the nonlinear properties of all-silica triangular lattice PCFs have been analyzed by a full-vector modal solver based on the FEM. Simulation results have demonstrated that it is possible to design PCFs with flattened dispersion curves in the $C$ band and high nonlinear coefficients to increase the gain values and enlarge the amplifier bandwidth. These results have been achieved by properly
choosing the air-hole distance and diameter in the triangular lattice and the dimension of the air-holes surrounding the single defect core of the PCF.

REFERENCES


Figure 9. Signal power gain $G$ versus $\lambda_s - \lambda_p$. The pump power level used for each PCF is indicated in the figure label. All the fibers are 0.5 km long.