Front evolution and dynamics of elliptical gravity currents on a uniform slope

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Abstract. Gravity currents are essentially flows driven by the density difference between the fluids. They are relevant in many geophysical, environmental and engineering problems, such as dust storms, snow avalanches and marine oil spillages. Much work has been carried out on planar or circular gravity currents along a horizontal boundary. However, the effects of initial shape of the gravity current and topography can play an important role in many situations. In the present investigation, we report data from direct numerical simulations of elliptical, finite release, Boussinesq gravity currents propagating down a uniform slope. The study comprises a series of simulations of elliptical gravity currents on a range of slope angles (5° ≤ θ ≤ 20°) at a Reynolds number of 5000. The shape parameters are varied to study the effects of cross-sectional aspect ratio on the dynamics of the gravity current. It is found that the long-time development of the current is influenced by its initial shape at low slope angles (θ = 5° and 10°) whereas the long-time dynamics of the gravity currents is relatively insensitive to its initial shape but is dependent on the slope angle. The physical mechanisms governing the pertinent dynamics of the gravity current will be presented.

Introduction

Gravity currents, also known as density currents, are buoyancy driven flows caused by a density difference between a light fluid and a heavy fluid. The density difference may be due to variations of temperature, dissolved materials, or inhomogeneous distribution of suspended particles. Gravity currents can be found in natural environments and industrial applications, such as thunderstorms, pyroclastic flows of hot volcanic tephra, snow avalanches and hazardous chemical spillages. These examples show the variety of gravity currents and the importance of studying such flows. A wide range of applications and laboratory studies of gravity currents can be found in [1].

Gravity currents have received a substantial amount of attention and extensive research has been performed using theoretical, experimental and numerical methods. The work of [2] is some of the earliest theoretical attempts to describe the spreading rate of gravity currents using Bernoulli’s theorem. Benjamin [3] arrived at the same conclusions by applying the theorem of hydraulic jumps. Much of the existing work has concentrated on the problem of gravity currents on a horizontal boundary through one of two canonical configurations, namely planar or axisymmetric release. Many experiments have been performed to study the various aspects of these types of flows, including spreading pattern, rate of spreading and flow structures [4,5,6]. Huppert and Simpson [7] described the spreading of a gravity current in three phases. There is a first slumping phase, during which the current develops and moves at a constant velocity. This may be followed by a self-similar inertial phase, wherein the buoyancy force is balanced by the inertial force. Finally, a self-similar viscous phase in which the viscous force becomes significant compared to the buoyancy force. Various analytical models have been developed to predict the bulk motion of the current, including the integral model [7] and shallow water model [8]. The study of the dynamics of gravity currents using high-resolution numerical calculations has gained considerable momentum with the recent
advancement in computational fluid dynamics (CFD) methodologies. Direct numerical simulation (DNS) resolves the entire range of spatial and temporal scales of the fluid motion, which provides new insights into the structure and dynamics of gravity currents. It is demonstrated that DNS is capable of not only reproducing the global flow properties observed in the experiments [9,10] but also capturing the detailed flow structure and dynamics of the currents [11,12].

In comparison, investigations of an instantaneous release on an unconfined slope have been relatively limited. A current released on a slope with no lateral boundaries loses its symmetry or axisymmetry, which makes the flow fully three-dimensional. Such a flow configuration has many practical relevance such as snow avalanches and pyroclastic flows spreading downhill. Webber et al. [13] presented a model of the motion of a circular heavy gas cloud down a uniform slope without entrainment using the solutions of shallow water equations. Tickle [14] expanded the circular wedge integral model from [13] to include the effect of entrainment. Ross et al. [15] performed Boussinesq saline experiments and extend the original model to a triangular wedge integral model based on the experimental observations. Recently, Zgheib et al. [16] reported data on the dynamics of circular finite release Boussinesq gravity currents on a uniform slope using three-dimensional direct numerical simulations. Their data showed that the gravity current evolves to a shape that is similar to a triangular wedge shape as shown in [15]. Interestingly, the front velocity of the gravity current is found to go through two acceleration phases, which is not observed in any of the previous studies [13,15].

Studies of gravity currents beyond the classical canonical configurations are very scarce [17] despite the fact that the majority of currents in real situations have an arbitrary, non-circular or non-axisymmetric, initial source. In the present investigation, we present results from direct numerical simulations of three-dimensional elliptical, finite release, Boussinesq gravity currents propagating down an unconfined uniform slope. The study comprises a series of simulations of elliptical gravity currents on a range of slope angles with various cross-sectional aspect ratios. The physical mechanisms governing the pertinent dynamics of the gravity current will be presented.

**Numerical Formulation**

We consider the case of a slanted elliptical cylinder containing heavy fluid surrounded by an infinite extent of light fluid on a sloping boundary. The density difference between the two fluids is assumed to be small enough so that the Boussinesq approximation is valid. With this approximation, density variations are only retained in the buoyancy term. The dimensionless system of equations governing the motion of the flow reads

\[ \nabla \cdot \mathbf{u} = 0, \]  
\[ \frac{\partial \mathbf{u}}{\partial t} = \rho \mathbf{e}_g - \nabla p + \frac{1}{Re} \nabla^2 \mathbf{u}, \]  
\[ \frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = \frac{1}{Sc Re} \nabla^2 \rho, \]  

Here, \( \mathbf{u} \), \( p \) and \( \rho \) are the divergence-free dimensionless velocity vector, pressure and density in the flow, respectively, and \( \mathbf{e}_g \) is a unit vector pointing in the direction of gravity. The dimensionless density and pressure are given by

\[ \rho = \frac{\rho^* - \rho_0^*}{\rho_0^*}, \quad p = \frac{p^*}{\rho_0^*(U^*)^2}. \]  

Any variable with an asterisk is to be understood as dimensional. The variables \( \rho^* \), \( \rho_0^* \) and \( \rho_1^* \) represent the local, ambient and initial heavy fluid densities, respectively. Hence, the value of \( \rho \) is bounded between 0 and 1. The variables \( p^* \) and \( U^* \) denote the local pressure and velocity scale. The two dimensionless numbers in Eq. 3 are the Reynolds number \( (Re = U^* L^*/v^*) \) and Schmidt number \( (Sc = v^*/\kappa^*) \), where \( v^* \) is the kinematic viscosity and \( \kappa^* \) is the molecular diffusivity of the fluid. The length scale \( L^* \), the velocity scale \( U^* \) and the time scale \( T^* \), following the definition from [16], are given by

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where $V'_0$ is the initial volume of heavy fluid in the slanted elliptical cylinder and $g^*$ denotes the gravitational acceleration.

\[ L^* = (V'_0)^{1/3}, \quad U^* = \frac{g^* \rho_{1} - \rho_{0}}{\rho_{0}} L^*, \quad T^* = L^*/U^*, \] (5)

All the simulations considered in the current study are solved in a rectangular box of size $L_x \times L_y \times L_z$, as shown in Fig. 1a. The governing equations are solved using a de-aliased pseudospectral code [18,19]. Periodic boundary conditions are employed along the streamwise ($x$) and spanwise ($y$) directions. This implies that a periodic array of gravity currents is being simulated. In the wall normal direction ($z$), no slip boundary conditions is used for the velocity field at the bottom wall and a free slip boundary condition is used at the top wall. Owing to the periodic boundary conditions, a large domain size of $L_x = 18$, $L_y = 15$ and $L_z = 2.5$ was chosen to allow unhindered development of the current on a sloping boundary. Simulations are conducted at $Re = 5000$ with a grid resolution of $700 \times 600 \times 201$ ($N_x \times N_y \times N_z$). The grid resolution was selected to be consistent with the requirement of a grid size of the order of $O(ReSc)^{-1/2}$ [20]. The heavy fluid is initially confined inside a truncated elliptical segment with a mean height $h_0 = 1$ and an initial volume $V_0 = 1$ (see Fig. 1b), whose cross section is an ellipse with an aspect ratio $AR = a/b$, where $a$ is the axis along the streamwise direction and $b$ is the axis along the spanwise direction. The lock radius $r_0$ along the streamwise direction can be calculated as $a/\cos(\theta)$, where $\theta$ is the slope of the bottom boundary. Combinations of four slope angles ($5^\circ, 10^\circ, 15^\circ$ and $20^\circ$) and seven cross-sectional aspect ratios ($0.1$, $0.2$, $0.5$, $1$, $2$, $5$ and $10$) are considered. The flow was started from rest with a small random disturbance superposed on the density field to accelerate the three-dimensional development [10]. The Schmidt number of unity was employed for all the simulations as it has been shown that the dynamics of gravity currents is weakly dependent on $Sc$, provided that Reynolds number of the flow is large [21]. The time step was chosen to ensure a Courant number less than 0.5.

Results

**Mass Distribution and Front Evolution.** In order to better understand the spatial development of the gravity current, we investigate the equivalent height of the current,

\[ h(x,y,t) = \int_{0}^{t} \rho(x,y,z,t)dz, \] (6)

which is the wall normal integrated height of the current. $h$ is a good indicator of the spatial mass distribution in the streamwise and spanwise directions and the shape of the current. The evolution of $h$ for $AR = 1$ and $\theta = 15^\circ$ is shown in Fig. 2. High value of $h$ is shown in red whereas low value of $h$ is coloured in blue. It can be observed that the influence of the slope becomes obvious after some finite time after the release. The current initially spreads in an axisymmetric manner, forming a
nearly circular shape at \( t = 1 \) and 2. The majority of the heavy fluid spreads out in the radial direction and starts rearranging and redistributing itself to the front of the current down the slope. At \( t = 6 \), the effect of the inclined boundary is apparent. The current has developed into a crescent-like structure in which the majority of the heavy fluid accumulates near the front, which is known as the head of the gravity current. A very thin layer of fluid resides in the rear. As the current propagate further downstream (\( t = 10, 20 \) and 30), the mass continues moving towards the head along the circumference of the current, forming a triangular wedge shape as observed in [15]. In the meantime, the flow becomes fully three-dimensional and turbulent.

![Figure 2](image2.png)

**Fig. 2** Evolution of \( \tilde{h} \) at \( Re = 5000 \), aspect ratio = 1 and \( \theta = 15^\circ \) at \( t = 1, 2, 6, 10, 20 \) and 30 from top left to the bottom right.

![Figure 3](image3.png)

**Fig. 3** Front location evolution over time. Front visualized by contours of \( \tilde{h} = 0.01 \) for aspect ratios of 0.1, 1, 10 (left to right) and slope angles of 5°, 20° (top to bottom). The red contour presents the original boundary of the current. The time separation between contours is \( \Delta t = 0.4 \). The details show continuous lobe splitting and merge.

Fig. 3 shows the evolution of the front location identified by the contour of \( \tilde{h} = 0.01 \) at the bottom boundary. The front location at several equally spaced time interval of \( \Delta t = 0.4 \) are superimposed for \( AR = 0.1, 1, 10 \) and \( \theta = 5^\circ, 20^\circ \). The red contour presents the initial boundary of the current. The composite pictures provide a clear view of the formation of lobe-and-cleft structures and evolution of the current shape. At \( \theta = 5^\circ \) and \( AR = 0.1 \), the current travels the farthest in the streamwise direction with relatively less displacement in the spanwise direction. Here the initial minor axis is in the streamwise direction. At \( \theta = 5^\circ \) and \( AR = 1 \), the current propagates almost radially outwards in all directions. At \( \theta = 5^\circ \) and \( AR = 10 \), rapid expansion of the current...
front in the spanwise direction with a dual front location is identified while the initial minor axis is in the spanwise direction. It is found that the initial shape of the heavy fluid has significant influence on the long-time development of the current at low slope angles ($\theta = 5^\circ$ and $10^\circ$), which agrees well with the ‘switching axis’ experimental observation in [17]. However, such influence diminishes at higher slope angles ($\theta = 15^\circ$ and $20^\circ$).

**Front Location and Velocity.** The temporal evolution of the front velocity $u_F$ is shown in Fig. 4. Firstly, the front location of the current is identified as the maximum downstream location of a small threshold value $\epsilon = 0.01$ of $\bar{h}$. Subsequently, the front velocity data, represented by the symbols, are obtained using a central finite difference scheme on the front location data. Each solid line is a smoothing spline for guiding the trend of the front velocity. The velocity curves reveal that there is a second acceleration phase immediately following the first acceleration phase as reported in [16]. The second acceleration can be explained by the rearrangement and redistribution of the heavy fluid at the end of the first acceleration, which increases the buoyancy at the downstream end of the current (see Fig. 2). It is observed that the maximum velocity at the end of the first acceleration generally increases as the cross-sectional aspect ratio decreases whereas the peak velocity at the end of the second acceleration phase increases together with the cross-sectional aspect ratio. Another interesting aspect of the current study is the final velocity of the gravity current appears to only depend on the slope angle but not on the initial shape of the current.

![Fig. 4 Temporal evolution of the front velocity $u_F$ for the four slope angles ($5^\circ$, $10^\circ$, $15^\circ$ and $20^\circ$) with the seven cross-sectional aspect ratios (0.1, 0.2, 0.5, 1, 2, 5 and 10).](image)

**Summary**

We present data from highly resolved numerical simulations to investigate the dynamics of elliptical gravity currents on a uniform slope. Simulations were performed at $Re = 5000$ with four slope angles ($5^\circ$, $10^\circ$, $15^\circ$ and $20^\circ$) and seven cross-sectional aspect ratios (0.1, 0.2, 0.5, 1, 2, 5 and 10). The ‘switching axis’ was evident for the current with a non-circular shape ($AR \neq 1$). A second acceleration phase is observed immediately following the first acceleration phase due to the convergence of mass towards the current head at the downstream. Temporal evolution of the front location reveals that the initial shape of the heavy fluid is found to have significant influence on the long-time development of the current at low slope angles ($\theta = 5^\circ$ and $10^\circ$) while the influence diminishes as the slope angle increases. It is found that the maximum velocity at the end of the first acceleration generally increases as $AR$ decreases whereas the peak velocity at the end of the second
acceleration phase increases with larger ARs. The final velocity of the gravity current appears to only depend on the slope angle.

References