Three Body Recombination of Ultracold Dipoles to Weakly Bound Dimers

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We use universality in two-body dipolar physics to study three-body recombination. We present results for the universal structure of weakly bound two-dipole states that depend only on the $s$-wave scattering length ($a$). We study threshold three-body recombination rates into weakly bound dimer states as a function of the scattering length. A Fermi golden rule analysis is used to estimate rates for different events mediated by the dipole-dipole interaction and a phenomenological contact interaction. The three-body recombination rate in the limit where $a \gg D$ contains terms which scale as $a^4$, $a^2 D^2$, and $D^4$, where $D$ is the dipolar length. When $a \ll D$, the three-body recombination rate scales as $D^4$.

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Universalities have become increasingly important and applicable in ultracold physics. Two remarkable examples are strongly interacting fermions [1–3] and few-body systems with large scattering lengths. Three-body recombination, $A + A + A \rightarrow A_2 + A + KE$, is of special interest in an ultracold gas. It usually is the limiting process determining the lifetime of the gas in the absence of two-body losses. Three-body recombination also has known to be an important probe in understanding the basic quantum mechanical nature of universal few-body systems [4]. The experimental observation of universal three- and four-body loss features [5–7], associated with a class of the three-body-bound state predicted by Efimov [8], is one of the major success stories in this field. These few-body complexes have found great utility because of the control over the two-body scattering length with a magnetic Fano-Feshbach resonance [9]. In a magnetic field, the last bound state can be tuned through the scattering threshold giving complete control over the scattering length. When $a$ is much greater than the range of the short-range interaction, many properties become independent of the details of the two-body interaction. For example, the three-body recombination takes on universal scaling behavior of $a^4$ in this regime [10].

The powerful idea of universality can be exploited to study complex systems. Now, ultracold polar molecules have emerged [11,12], and these systems will have a new type of collisional control [13]. This control emerges in the form of the dipole-dipole interaction, $V_{dd} = d^2 [1 - 3(\hat{\omega} \cdot \hat{r})^2] / r^3$, where $r$ is the interparticle separation vector and $d$ is the induced dipole moment along an external field axis ($\hat{\omega}$). The collisional control can be achieved through the induced dipole moment $d$ which is determined by the strength of the electric field. As the electric field is increased, there is a series of $s$-wave dominated resonances [14–16]. These resonances occur when the system gains a weakly bound long-range state.

The short-range bound states structure of the interaction is largely independent of the external fields strength. These threshold resonances could be used to control scattering length, but the importance of these resonances will go further. The interaction is long-range and anisotropic which will mediate fundamentally novel quantum behavior. What is striking about this interaction is that the length scale describing it can be incredibly large, many orders of magnitude larger than the short-range interaction. The dipolar length scale is $D = \mu_d d^2 / \hbar^2$ and $\mu_d$ is the reduced mass of a two-body system. This large length scale can be used to create a highly correlated system.

The two-dipole scattering cross section has been demonstrated to be universal [15,17,18]. In this work we extend the concept of universality in two-body dipolar physics and apply it to three-body recombination. We present results showing the universal structure of weakly bound two-dipole states that depend only on $a$. Using this, we then study threshold three-body recombination rates into weakly bound dimer states as a function of $a$.

To start, we study the universality of the dipolar threshold resonances. These universal dipolar collisions depend only on the $s$-wave scattering length. To tune the scattering length we increase $D$ while enforcing a node at a finite length $r_0$ below which the interaction becomes system dependent. In a real system increasing $D$ mimics increasing the electric field and the molecule’s polarization. This creates a series of wide $s$-wave dominated resonances at the scattering threshold. Changes in the short-range boundary condition can shift the position of these resonances, but not the behavior we are concerned with. There are other resonances which are not $s$-wave dominated, but these will be narrow and rare [15] and we are not interested in them.

Examples of universal two-body scattering are well known, for instance when the scattering length is large and positive, large dimers form with binding energies $E_b = \hbar^2 / (2 \mu_d a^2)$. Weakly bound two-body dipolar states...
near an s-wave resonance ($a \gg D$) also exhibit this universal property. The binding energy for these states is shown in Fig. 1(a) in units of the dipolar energy $E_D = \hbar^2 / \mu_{2b} D^2$ as a function of $a/D$. We have plotted the binding energies for three different resonances, the first (black circles), second (blue diamonds), and eighth (red squares) resonance as $D$ is increased. As expected, the three resonances agree very well in the large scattering length limit, but surprisingly the agreement is very good even when $a < D$.

To further illustrate universal features, we look at the partial wave populations as a function of $a$. We have numerically obtained the molecular wave function: $\psi_{d,l}(\mathbf{r}_{12}) = \sum_{N} Y_{l}(\mathbf{r}_{12}) f_{l}^{d}(r_{12})$, where $l$ is the partial wave. We require that $\sum_{l} N_{l} = 1$, where $N_{l} = \langle f^{d}_{l} | f^{d}_{l} \rangle$ is the partial wave population. Figure 1(b) shows the s- (solid), d- (dashed), and g-wave (dotted) populations for the first (black) and second (blue) resonances as a function of $a/D$. This figure shows that the partial wave populations are universal, and that these molecules are mostly $s$ and $d$ wave. For large $a$, the $s$-wave contribution dominates and is near unity for large $a$ and $\psi_{d,l}$ takes on the universal form $\psi_{d,l} \propto \exp(-r/a)$. As $a$ is decreased the $d$-wave contribution becomes significant and reaches $\sim 40\%$. Even for very small $a$, the g-wave, i.e., $l = 4$, contribution is at most 2%. The dipolar interaction conserves parity and this selection rule prevents an s-wave channel coupling to any odd partial wave. This implies that the present theory applies to only bosonic or distinguishable dipoles.

There is of course a caveat to universality. The dipole size limit, but surprisingly the agreement is very good even when $a < D$.

FIG. 1 (color online). Universal properties of two-body dipolar scattering. (a) The binding energy (symbols with curves) of the dipolar dimer as a function of the s-wave scattering length for the first (black circles), second (blue diamonds), and eighth (red square) resonance. (b) The partial wave molecular populations are shown for the s-wave (solid), d-wave (dashed), and g-wave (dotted) populations for the first (black) and second (blue) resonance.
matrix element that connects the outgoing dipole-dimer state to the incoming plane-wave state.

Expanding the plane waves and dimer-wave function in Eq. (3) in spherical harmonics and integrating over the angular degrees of freedom yields the total distinguishable momenta greater than zero in both channels made up of two sets of partial wave, one for the appropriate for a gas phase experiment.

The results of the Fermi gold rule calculations are shown in Fig. 2 as a function of scattering length for the first resonance in Fig. 1. The total rates have the expected $a^4$ scaling at large $a$ due to the contact interaction, but as $a/D$ is decreased they eventually flatten out, and when $D > a$ the rate is entirely controlled by the dipolar interaction, scaling as $D^4$: $K_3^{ident} \sim 495hD^4/\mu_{2D}$. We have compared the recombination rates for several two-body resonances and have found nearly perfect agreement, to within $\sim 1\%$, indicating that the three-body recombination behavior found here is universal.

The dipolar interaction directly couples the incident channel to final states with $L' = 0, 2, 4, \ldots$ and a molecular state with $l' = 0, 2, \ldots$. The largest term in the $a > D$ regime, $l'l' = sS$ (dashed red curve), comes from the contact interaction and scales as $a^4$. The next largest contribution from the $dS$ channel (dotted red curve) scales as $D^2a^2$. Additional contributions from higher order $l'l'$ channels are shown (dot-dashed red), but contribute little in the large $a$ regime. However in the $a < D$ regime, the dominant contribution comes from the $l'l' = dS$ and $dG$ terms, the first of which is only weakly dependent on the
scattering length. As one might expect there are two universal regimes that appear. The first regime appears where the long-range dipole interactions are dominant and the scattering length is small, i.e., \(a < D\), where \(K_3\) is independent of \(a\) and scales as \(D^3\). The second regime appears when \(a\) is the dominant length scale, \(a > D\), and the \(a^4\) \(s\)-wave contribution is dominant over all others. The analytic results of Eq. (6) are seen to be in near perfect agreement with the numerical calculation in Fig. 2 in the \(a > D\) limit. The inset of Fig. 2 shows that the total recombination rate is in surprisingly good agreement with Eq. (6) for \(a > D\).

The Fermi’s golden rule based approach is only capable of describing the scaling behavior of three-body recombination, and does not incorporate more complex three-body correlations. The recombination rate can have a variety of interesting resonant behaviors on top of the envelope behavior presented here. For instance, we expect that in the \(a > D\) regime, a geometrically spaced set of Efimov minima are likely to appear in the \(|L|L' = ss\) outgoing channel [4].

We have shown the universal structure of weakly bound two-dipole states that depend only on the scattering length of the system, shown in Fig. 1. This has allowed us to obtain threshold three-body recombination rates into weakly bound dimer states as a function of \(a\). We used a Fermi golden rule analysis to estimate the contributions to the rate from different events mediated by the dipole-dipole interaction and a contact interaction. These rates are shown in Fig. 2. The individual terms have many different scaling behaviors. When \(a > D\), we find the dominant term scales as \(a^4\). When \(D > a\) the recombination rate scales as \(D^4\). The case of negative scattering length is not addressed in this work, but it is not unreasonable to expect that the envelope behavior will be similar to that presented here. More detailed three-body calculations which include extra resonance structures are the subject of future work. The \(|a| > D\) regime might also hold a variety of interesting phenomena such as universal three-dipole Efimov states.

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