Properties of Galaxy Groups in the SDSS:
II.– AGN Feedback and Star Formation Truncation

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ABSTRACT
Successfully reproducing the galaxy luminosity function and the bimodality in the galaxy distribution requires a mechanism that can truncate star formation in massive haloes. Current models of galaxy formation consider two such truncation mechanisms: strangulation, which acts on satellite galaxies, and AGN feedback, which predominantly affects central galaxies. The efficiencies of these processes set the blue fraction of galaxies, \( f_{\text{blue}}(L, M) \), as function of galaxy luminosity, \( L \), and halo mass, \( M \). In this paper we use a galaxy group catalogue extracted from the Sloan Digital Sky Survey (SDSS) to determine \( f_{\text{blue}}(L, M) \). To demonstrate the potential power of these data as a benchmark for galaxy formation models, we compare the results to the semi-analytical model for galaxy formation of Croton et al. (2006). Although this model accurately fits the global statistics of the galaxy population, as well as the shape of the conditional luminosity function, there are significant discrepancies when the blue fraction of galaxies as a function of mass and luminosity is compared between the observations and the model. In particular, the model predicts (i) too many faint satellite galaxies in massive haloes, (ii) a blue fraction of satellites that is much too low, and (iii) a blue fraction of centrals that is too high and with an inverted luminosity dependence. In the same order, we argue that these discrepancies owe to (i) the neglect of tidal stripping in the semi-analytical model, (ii) the oversimplified treatment of strangulation, and (iii) improper modeling of dust extinction and/or AGN feedback. The data presented here will prove useful to test and calibrate future models of galaxy formation and in particular to discriminate between various models for AGN feedback and other star formation truncation mechanisms.

Key words: galaxies: formation – galaxies: evolution – galaxies: general – galaxies: statistics – methods: statistical

1 INTRODUCTION
One of the most challenging outstanding problems in galaxy formation is to explain the detailed shape of the galaxy luminosity function (hereafter LF). In particular, the relatively shallow faint-end slope and the exponential cut-off at the bright end of the LF have proven difficult to explain (e.g., White & Frenk 1991; Benson et al. 2003). In the traditional scenario for galaxy formation, it is envisioned that lower cooling efficiencies in massive galaxies would explain the exponential tail of the LF (Rees & Ostriker 1977; Silk 1977; White & Rees 1978), while supernova feedback is typically invoked to reduce the star formation efficiency in low mass haloes (Larson 1974; White & Rees 1978; Dekel & Silk 1986). Although the latter can indeed be tuned to reproduce the faint-end slope of the galaxy LF, it worsens the problems at the bright end. As nicely demonstrated by Benson et al. (2003), supernova feedback causes a drastic increase of the amount of diffuse hot gas that remains in larger halos. This gas is able to cool onto the central galaxies in these haloes, producing too many bright galaxies. In addition, this causes the bright model galaxies to have relatively young stellar populations, in disagreement with observations (e.g., Kauffmann & Charlot 1998; Heavens et al. 2004; Thomas

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et al. 2005). What is needed is a mechanism that can truncate the star formation in these massive, central galaxies at relatively late times.

Star formation truncation is also the main mechanism that is thought to underlie the bimodality of galaxy properties. The local population of galaxies consists roughly of two types: red galaxies, which reveal an early type morphology and which have very little or no ongoing star formation, and blue galaxies with active star formation and a late-type morphology (e.g., Strateva et al. 2001; Blanton et al. 2003a; Kauffmann et al. 2003, 2004; Baldry et al. 2004; Brinchmann et al. 2004; Balogh et al. 2004a,b). Although a non-negligible fraction of red galaxies are clearly edge-on disk galaxies that owe their red colour to an enhanced extinction, the most pronounced distinction between ‘red sequence’ and ‘blue sequence’ galaxies is their current star formation rate. Since only relatively small amounts of ongoing star formation tend to make a galaxy appear ‘blue’, the colour bimodality basically reflects star formation truncation: red sequence galaxies have their star formation truncated, while blue sequence galaxies are still forming stars today.

Semi-analytical models for galaxy formation consider a number of mechanisms that can prevent, delay or truncate star formation. In low mass haloes one typically invokes reionization and supernova feedback in order to suppress or truncate star formation. In haloes with \( M \gtrsim 10^{12} h^{-1} M_\odot \), on which we will focus in this paper, two additional truncation effects are considered. The first one, called strangulation, only affects satellite galaxies. As soon as a dark matter halo is accreted by a larger halo, its central galaxy becomes a satellite galaxy. It is often assumed that this accretion process causes the satellite galaxy to be stripped of its hot gas reservoir. Consequently, after a delay time in which the galaxy consumes (part of) its cold gas, star formation is truncated, and the satellite galaxy becomes red (Larson, Tinsley & Caldwell 1980; Balogh, Navarro & Morris 2000). Ram-pressure stripping (Gunn & Gott 1972) may shorten the time-delay, by also stripping the satellite of its cold gas reservoir, but since the time scale for strangulation is already relatively short, the addition of ram-pressure stripping does not have a large impact. Virtually all semi-analytical models of galaxy formation, starting with Kauffmann, White & Guiderdoni (1993), have taken this strangulation mechanism into account. In fact, it is the main mechanism that causes red-sequence model galaxies to preferentially reside in overdense regions such as groups and clusters of galaxies, in good agreement with observations (e.g. Oemler 1974; Dressler 1980; Hogg et al. 2004; Balogh et al. 2004b; Weinmann et al. 2006).

The second star formation truncation mechanism that operates in massive haloes is feedback from active galactic nuclei (AGN), which predominantly affects central galaxies. Although the potential importance of AGN feedback has long been recognized (e.g., Tabor & Binney 1993; Ciotti & Ostriker 1997), it only recently has been given serious consideration in galaxy formation models. This has largely been motivated by X-ray observations which reveal that AGN can indeed impact the hot IGM of galaxy clusters (e.g., Fabian et al. 2003; McNamara et al. 2005). Numerous recent studies have demonstrated that the inclusion of AGN feedback in galaxy formation models can help to explain the bright end exponential cut-off of the galaxy luminosity function (e.g. Granato et al. 2004; Croton et al. 2006; Cattaneo et al. 2006a; Sijacki & Springel 2005; Bower et al. 2005) and the fact that the most massive galaxies contain the oldest stars (Croton et al. 2006; Scannapieco, Silk & Bouwens 2005; Bower et al. 2005).

Despite these successes, we are still far from a proper understanding of how AGN feedback may establish an equilibrium state where it can efficiently suppress star formation in the centers of massive haloes. In the studies mentioned above, AGN feedback is typically modeled using oversimplified, heuristic scaling relations, often based on very different views of how AGN feedback might operate. In particular, it is still unclear which mode of AGN activity is most important for the star formation truncation discussed above; the merger induced ‘quasar mode’ which leads to an initial starburst followed by a quenching of star formation, or the ‘radio mode’, which is caused by continual and quiescent accretion of hot gas onto the central supermassive black hole. Hopkins et al. (2006) have suggested that merger-induced AGN activity (the ‘quasar mode’) is responsible for the transition from blue, star forming to red, passive galaxies. Springel, Di Matteo & Hernquist (2005a), Menci et al. (2005) and Kang, Jing & Silk (2005) have shown that this ‘quasar mode’ feedback can indeed terminate star formation and expel the gas from the center of the galaxy, once the supermassive black holes become sufficiently massive. On the other hand, Croton et al. (2006), Bower et al. (2005), Cattaneo et al. (2006a), Nusser et al. (2006) and Sijacki & Springel (2006) have argued that the ‘radio mode’ of AGN activity (or AGN feedback operating in quasi-hydrodynamically cool haloes) is the main mechanism to truncate star formation in massive galaxies. How exactly this radio mode feedback operates, however, is still unclear, as is evident from the fact that the aforementioned studies all use very different formulations.

All these different AGN feedback models mainly differ in the way in which the feedback efficiency scales with halo mass and with galaxy properties such as black hole mass and gas mass fraction. Since AGN feedback causes star formation truncation, one way to discriminate between these various models is therefore to investigate the relative fractions of blue and red galaxies as function of halo mass and galaxy properties. Such a study will also help to improve our understanding of strangulation, the star formation truncation mechanism for satellites. Although strangulation is typically modeled as being independent of halo mass, one might argue that the ability of a host halo to strip a subhalo of its hot gas reservoir depends on the presence and density of the hot corona of the host halo, which in turn may well be mass dependent. Again, knowledge of the fractions of blue and red (satellite) galaxies as function of halo mass should allow us to discriminate between these different possibilities.

Nowadays, with large galaxy surveys, such as the Two Degree Field Galaxy Redshift Survey (Colless et al. 2001) and the Sloan Digital Sky Survey (York et al. 2000), the number of galaxies is sufficiently large that, in principle, one could accurately measure the red and blue fractions as a function of various variables. In this paper, we use our SDSS group catalogue, presented in Weinmann et al. (2006, hereafter paper I), to compute the fractions of blue and red galaxies as function of both halo mass and galaxy luminosity. To emphasize the potential constraining power of these data we compare our results to the semi-analytical
galaxy formation model of Croton et al. (2006), which includes both strangulation and ‘radio-mode’ AGN feedback. We show that although this model accurately fits the galaxy luminosity function, the color-bimodality, and many other global statistics of the galaxy population, it fails dramatically when it comes down to the blue fraction of galaxies as a function of halo mass and luminosity. We argue that this has its origin in the way that strangulation and AGN feedback have been incorporated and we briefly discuss possible modifications. The aim of this paper, however, is not to present a new, improved model for star formation truncation, but merely to present observational constraints that will hopefully prove useful in discriminating between the various models.

This paper is organized as follows. Section 2 describes our SDSS group catalogue, which we compare to a similar group catalogue extracted from the semi-analytical model of Croton et al. (2006) described in Section 3. The actual comparison is presented in Section 4, while Section 5 discusses the possible implications for galaxy formation models. We summarize our results in Section 6.

2 THE SDSS GROUP CATALOGUE

2.1 Data

The Sloan Digital Sky Survey (SDSS; York et al. 2000) is a joint, five passband (u, g, r, i and z) imaging and medium-resolution (R ~ 1800) spectroscopic survey. In this paper, we focus on the subset of galaxies that are in the New York University Value-Added Galaxy Catalogue (NYU-VAGC) based on the SDSS Data Release 2 (Blanton et al. 2005). This NYU-VAGC is based on an independent, significantly improved data reduction. From this catalogue we select all galaxies with an extinction corrected apparent magnitude in the r-band, k-corrected to z = 0.19, which corresponds to the redshift for which a galaxy with M_r = -19.5 + 5 log h has an apparent magnitude that is equal to the luminosity of all group members brighter than L_{group}, where only the brightest members are observed, to an empirically normalized luminosity scale. As common luminosity scale we use L_{19.5}, defined as the luminosity of all group members brighter than 0.1 M_r = -19.5 + 5 log h. To calibrate the relation between L_{group} and L_{19.5} we first select all groups with z ≤ 0.09, which corresponds to the redshift for which a galaxy with 0.1 M_r = -19.5 + 5 log h has an apparent magnitude that is equal to the magnitude limit of the survey. For groups with z > 0.09 we use this ‘local’ calibration between L_{group} and L_{19.5} to estimate the latter. Detailed tests have shown that the result-

2.3 Estimating group masses

Following YMBJ we use the group luminosity to assign masses to our groups. The motivation behind this is that one naturally expects the group luminosity to be strongly correlated with halo mass (albeit with a certain amount of scatter). Since the group luminosity is dominated by the brightest members, which are exactly the ones that can be observed in a flux limited survey like the SDSS, the determination of the (total) group luminosity is more robust than that of the group’s velocity dispersion, especially when the number of group members is small (see Appendix B in paper I).

Clearly, because of the flux limit of the SDSS, two identical groups observed at different redshifts will have a different L_{group}, defined as the summed luminosity of all its identified members. To circumvent this bias we first need to bring the group luminosities to a common scale. A nearby group selected in an apparent magnitude limited survey should contain all of its members down to a faint luminosity. We can therefore use these nearby groups to determine the relation between the group luminosity obtained using only galaxies above a bright luminosity limit and that obtained using galaxies above a fainter luminosity limit. Assuming that this relation is redshift-independent, one can correct the luminosity of a high-z group, where only the brightest members are observed, to a empirically normalized luminosity scale.
ing group luminosities are significantly more reliable than those in which the correction for missing members is based on the assumption of a universal luminosity function (see YMBJ for details).

The final step is to obtain an estimate of the group (halo) mass from $L_{19.5}$. This is done using the assumption that there is a one-to-one relation between $L_{19.5}$ and halo mass. For each group we determine the number density of all groups brighter (in terms of $L_{19.5}$) than the group in consideration. Using the halo mass function corresponding to a ΛCDM concordance cosmology with $\Omega_m = 0.3$, $\Omega_{\Lambda} = 0.7$, $h = H_0/(100 \text{ km s}^{-1} \text{ Mpc}^{-1}) = 0.7$ and $\sigma_8 = 0.9$ we then find the mass for which the more massive haloes have the same number density. Although the masses thus derived depend on cosmology, it is straightforward to convert the masses derived here to any other cosmology.

Finally we note that not all groups can have a halo mass assigned to them. First of all, the mass estimator described above does not work for groups in which all members are fainter than $0.1M_r = -19.5 + 5\log h$. Secondly, the combination of $L_{19.5}$ and redshift may be such that we know that the halo catalogue is incomplete, which means that there is a significant number of groups at this redshift with the same $L_{19.5}$ but for which the individual galaxies are too faint to be detected. Since our mass assignment is based on the assumption of completeness, any group beyond the completeness redshift corresponding to its $L_{19.5}$ is not assigned a halo mass (see Yang et al. 2005b for details).

2.4 The SDSS group catalogue

Applying our group finder to the sample of SDSS galaxies described in Section 2.1 yields a group catalogue of 53,229 systems with an estimated mass. These groups contain a total of 92,315 galaxies. The majority of the groups (37,216 systems) contain only a single member, while there are 9220 binary systems, 3073 triplet systems, and 3720 systems with four members or more (see paper I for details)\(^1\). In what follows we refer to the brightest galaxy in each group as the ‘central’ galaxy, while all others are termed ‘satellites’.

3 THE SEMI-ANALYTIC MODEL

The semi-analytic model (hereafter SAM) to which we compare the SDSS group catalogue discussed above is based on an output of the Millennium Run N-body simulation (Springel et al. 2005b) and is described in detail in Croton et al. (2006; hereafter C06). The simulation is based on the cosmological parameters $\Omega_m = 0.25$, $\Omega_{\Lambda} = 0.75$, $\Omega_b = 0.045$, $h = 0.73$ and $\sigma_8 = 0.9$ and has a volume of $125 h^{-1}$ Gpc$^3$. Dark matter haloes are identified with a FOF group finder, and subsequently populated with galaxies following the semi-analytical model described in C06.

One of the relative novelties of this SAM is the inclusion of ‘radio mode’ feedback from AGN that lie at the center of a halo with a static corona of hot gas. As shown in C06, this feedback mode suppresses the cooling flow in massive haloes at relatively late times, which in turn yields luminosities, colours and stellar ages for massive galaxies in better agreement with observations. In particular, the inclusion of the radio-mode AGN feedback can explain the exponential cut-off at the bright end of the galaxy luminosity function, and the fact that the most massive galaxies are red and consist of old stellar populations (see De Lucia et al. 2006 for details). The model also predicts star formation histories, cold gas mass fractions and metallicities that are all in good agreement with observations. The model even predicts a Tully-Fisher zero-point that matches the data, as long as the rotation velocity of a disk galaxy is equal to the maximum circular velocity of the dark matter halo (however, see Dutton et al. 2006 for the unorthodox implications of this assumption).

Since the halo masses $M$ assigned to our SDSS galaxy groups are obtained by matching the abundances to the halo mass function, these are to be interpreted as the masses inside a radius with an overdensity of 180. As shown by Jenkins et al. (2001), for this definition of halo mass the analytical halo mass function of Sheth, Mo & Tormen (2001) used here is in good agreement with the halo mass function obtained from numerical simulations. The halo masses in the C06 catalogue, however, are defined as the masses inside a radius with a mean density that is 200 times the critical density, which we denote by $M_{200}$. In order to convert $M_{200}$ to $M$, we assume that dark matter haloes follow a NFW density distribution (Navarro, Frenk & White 1997). Using the relation between halo mass and halo concentration of Eke, Navarro & Steinmetz (2001), we find that the relation between $M$ and $M_{200}$ is well fit by

$$\frac{M_{200}}{M} = 0.745 - 0.0006 \log(M_{200}) - 7.0^{2.45}.$$  \(2\)

In what follows, we only consider the galaxies with $M_r \leq -16.72$, which reflects the magnitude completeness limit of the SAM, yielding a total of approximately 9 million model galaxies. The main information used in this paper are the $r$ band and $g$ band magnitudes of these galaxies, and the mass $M$ of the halo in which they reside. For comparison with the SDSS, we also compute the $g$ and $r$-band magnitudes k-corrected to $z = 0.1$, using

$$0.1g = g + 0.3113 + 0.4620 (g - r - 0.6102)$$  \(3\)

and

$$0.1r = g - 0.4075 - 0.8577 (g - r - 0.6102)$$  \(4\)

(Blanton et al. 2003b)\(^2\).

As we will see below, despite the AGN feedback the SAM still contains a significant number of very bright and blue galaxies that are not present in the SDSS. As mentioned in C06, these are mainly ULIRG-type starbursts for which the dust treatment of the model is inadequate; with a more proper dust model, these galaxies would be much more extincted, making them both fainter and redder. In order to suppress the impact of these galaxies on our SAM-SDSS comparison we remove all galaxies with $(g - r) < 0$ from both the SAM and the SDSS. In the case of the SAM

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\(^1\) This SDSS group catalogue is publicly available at http://www.astro.umass.edu/~xhyang/Group.html

\(^2\) These filter transformations are taken from a manuscript in preparation by Michael Blanton and Sam Roweis, available at http://cosmo.nyu.edu/blanton/kcorrect
Figure 1. A comparison of global statistics of the SAM-RS (right-hand panels) with the SDSS (left-hand panels). Panels in the first and second row show histograms of absolute $^{0.1}M_r-5\log h$ and $^{0.1}(g-r)$ colour for both redshift surveys. The contributions from blue and red galaxies are indicated by dashed and dotted lines, respectively. Note the good agreement between SDSS and SAM-RS. Panels in the third row show histograms of morphological parameters. In the case of the SDSS, we plot the distribution of the concentration parameter $c$, defined as the ratio of the radii containing 90 and 50 percent of the petrosian flux. In the case of the SAM-RS, we plot the distribution of the bulge-to-total mass ratio $B/T$ instead. Although these can not be compared directly, in general a more concentrated galaxy (higher $c$) will also have a larger $B/T$. Again, the contributions from red and blue galaxies are indicated. Finally, the fourth row of panels shows the colour-magnitude relations. The solid line indicates the bimodality scale given by equation (1), which we use to split the population of galaxies in red and blue subpopulations.
this only affects 0.5 percent of all galaxies, while in the case of the SDSS, this fraction is completely negligible.

We split the SAM model galaxies into ‘red’ and ‘blue’ subsamples using the same magnitude dependent colour-cut as for the SDSS, given by equation (1). In addition, we discriminate between ‘central’ galaxies, defined as the galaxy in a halo that is closest to the halo center, and ‘satellite’ galaxies. This differs from the definition used for the group catalogues, where the central galaxy is defined as the brightest group member. However, 97.5 percent of all central galaxies in the SAM are also the brightest galaxy in their halo, virtually independent of halo mass. We have verified that defining central galaxies in the SAM as the brightest halo members instead does not have a significant impact on any of our results.

3.1 Constructing a SAM redshift survey

In order to be able to compare the SAM to the SDSS results we need to mimic the construction of a galaxy redshift survey. We do so using the following steps. First, we construct a large virtual universe by replicating the periodic simulation box in a stack of $2 \times 2 \times 2$ boxes. This is required in order to be able to probe out to sufficiently high redshifts. Next we compute the redshift and apparent magnitude of each galaxy as seen by a virtual observer located in a corner of this virtual universe (who can thus see $\pi/2$ steradian of ‘sky’). We mimic the selection criteria of the SDSS discussed in Section 2.1 by only selecting those galaxies with $0.01 < z < 0.2$ and with $r < 17.77$. This leaves us with a grand total of 428,013 model galaxies. In what follows, we refer to this sample as the “SAM redshift survey” (hereafter SAM-RS).

Fig. 1 compares a number of statistics of these galaxies with those from the SDSS. The upper panels plot the distribution of absolute magnitudes, $M_r$, and their contribution due to red (dotted curves) and blue (dashed curves) galaxies. The agreement is very satisfactory, consistent with the fact that the SAM matches the observed luminosity functions (see C06). The second row of panels indicate the color distributions. Once again, the agreement is reasonable, although the bimodality in the SAM-RS seems somewhat more pronounced than in the SDSS, with a somewhat narrower ‘red peak’. Nevertheless, with blue fractions of 48 and 46 percent in the SDSS and SAM-RS, respectively, the overall agreement is very satisfactory.

In the third row of panels, we ‘compare’ two different morphology indicators. For the SDSS galaxies we plot the distribution of the concentration $c$, defined as the ratio between the radii that contain 90 and 50 percent of the Petrosian flux. For the SAM model galaxies, we plot the distribution of the bulge-to-total ($B/T$) stellar mass ratio instead. Typically, a galaxy with a large $B/T$ will also have a high concentration parameter. For both $c$ and $B/T$ there is a very significant overlap of red and blue galaxies. Therefore, our split in red and blue galaxies does not necessarily correspond to a morphological split in early and late-type galaxies, respectively, even though both are clearly correlated (see Fig. 1 in paper I).

Finally, the lower panels of Fig. 1 show scatter plots of the color-magnitude relations. The solid line corresponds to the bimodality scale given by equation (1). Again, there is reasonable overall agreement between the SAM-RS and the SDSS, although there are more galaxies with very blue colours in the SAM-RS, especially at the bright end. Recall that galaxies with $(g - r) < 0$ have already been excluded from these plots. As mentioned before, these bright, blue galaxies would appear significantly fainter and redder with proper dust modeling. Another apparent discrepancy concerns the red sequence, which at the bright end appears significantly tighter for the SAM than for the SDSS, which is also apparent from the histograms in the second row of panels.

To summarize, despite some small discrepancies, the global, statistical properties of the SAM model galaxies are in good agreement with the SDSS. However, this does not mean that the SAM also predicts the correct statistics as a function of halo mass. This is clearly a much tighter constraint for the model, and, as we argued in Section 1, may provide useful insights regarding the halo-mass dependence of various physical processes.

3.2 Construction of the SAM group catalogue

The main goal of this paper is to compare the fractions of red and blue galaxies as a function of both halo mass and luminosity in the SAM with those obtained from our SDSS group catalogue. In principle we could do so by comparing our SDSS group results directly with the SAM. However, our group finder, and in particular the algorithm used to as-
Figure 3. The conditional luminosity function obtained from the SDSS group catalogue (upper panel) from the SAM directly, using true halo masses and true halo members (middle panel) and from the SAM group catalogue, using assigned group masses and assigned memberships (lower panels). Results are shown for four different mass bins (masses in $h^{-1} M_\odot$), as indicated at the top of each column. Luminosities are in the $0.1r$-band. The dotted (blue) line marks the contribution from the satellite galaxies, the dashed (red) line the contribution from the central galaxies.

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Application of our group finder to the SAM-RS described above yields 98,130 groups with an assigned mass, which host a total of 206,076 galaxies. This amounts to an average of 2.10 members per group, which is significantly higher than for the SDSS, where the groups with an assigned mass have on average 1.73 members. However, the SDSS is not complete; in fact, the average completeness of our SDSS sample is 0.88, which largely explains the difference in the mean number of members per group. Another reason for this discrepancy is that, as we will see, massive haloes in the SAM contain too many faint satellites compared to the SDSS.

The fraction of blue galaxies in the SAM-GC is only 29 percent. Comparing this to the fraction of 46 percent of blue galaxies in the SAM-RS indicates that a red galaxy is much more likely to be associated with a group than a blue galaxy. At first sight this seems a logical consequence of the fact that (i) our group catalogue is limited to relatively massive haloes with $M \gtrsim 5 \times 10^{11} h^{-1} M_\odot$ and (ii) low mass haloes are more likely to host blue galaxies, see e.g. paper 1. However, the application of the group finder to the SDSS only reduces the fraction of blue galaxies from 48 percent to 41 percent. This reduction is much less severe than for the SAM. This is the first indication that the SAM and SDSS do not agree well when it comes down to details regarding the distribution of red and blue galaxies (see Section 4.2 below).

Since the SAM contains the full halo occupation information, we can check whether our group finder has assigned the correct galaxies to the same group, and whether the assigned mass is in agreement with the true halo mass. We have performed a large number of tests to investigate how well the group finder allows us to recover the true re-
lations between galaxies and their dark matter haloes. Several of these tests have been described in detail in Yang et al. (2005a,b), and show that the average occupation statistic of dark matter haloes are accurately recovered. However, higher-order moments of the occupation statistics, such as the scatter around a mean relation, are typically severely underestimated by the group catalogue, due to the fact that we assume a one-to-one relation between halo mass and halo luminosity (with zero scatter) when assigning masses to our groups. As an illustration of the accuracy of our group finder, Fig. 2 plots the average mass-to-light ratio of the dark matter haloes in the SAM (symbols with errorbars). Here \( M \) is the halo mass defined according to equation (2), and \( L \) is the total luminosity in the 0.1\( r \) band of all galaxies in that halo with \( 0.1 M_r - 5 \log h \leq -19.5 \). The solid line in Fig. 2 shows the average mass-to-light ratios obtained from the SAM-GC, which agree extremely well with the true \( \langle M/L \rangle_M \). This demonstrates that our group finder can accurately recover the average relation between mass and light.

In what follows, whenever we present any result obtained from the SAM-GC, we will also present the same results extracted directly from the SAM (using the true halo masses and the true halo memberships). A comparison among these results safeguards against potential problems with the group finder.

4 GALAXY ECeOLy

4.1 Conditional Luminosity Functions

We start our SAM-SDSS comparison by focusing on the conditional luminosity function (CLF), \( \Phi(L|M) \), which specifies the average number of galaxies of luminosity \( L \) that reside in a halo of mass \( M \) (Yang, Mo \& van den Bosch 2003; van den Bosch, Yang \& Mo 2003). The upper panels of Fig. 3 show the CLFs obtained from the SDSS group catalogue. Results are shown for four different mass bins, as indicated at the top of each column. Note that these masses are the assigned group masses. The dotted (blue) and dashed (red) lines indicate the contributions from the satellite and central galaxies, respectively. The distribution of central galaxies is well approximated by a log-normal distribution, consistent with previous findings (Zheng et al. 2005; Yang et al. 2005b). The panels in the middle row of Fig. 3 show the CLFs obtained directly from the SAM; here the masses are the true halo masses, and the true halo members are used to construct the CLFs. The overall agreement with the CLFs extracted from the SDSS group catalogue is very satisfactory, although the width of the CLF for central galaxies is significantly broader in the SAM than in the SDSS. To allow for a more meaningful comparison, the lower panels of Fig. 3 plot the CLFs obtained from the SAM-GC. The first thing to notice is that the width of the CLFs of the central galaxies are now in much better agreement with the SDSS; apparently, the group finder artificially ‘narrows’ the scatter in the relation between halo mass and the luminosity of the central galaxy. This simply owes to the fact that we use the group luminosity to determine the group mass. Other than that, the agreement between the CLFs obtained from the SAM-GC, and those extracted directly from the SAM is very good, indicating that our group finder allows an accurate recovery of the true \( \Phi(L|M) \) (see also tests in Yang et al. 2005b).

Except for the highest mass bin, the CLFs extracted from the SDSS and the SAM group catalogues are in good agreement with each other, indicating that the SAM not only fits the galaxy luminosity function, but it even does so as function of halo mass. However, at the massive end the SAM predicts significantly more relatively faint galaxies in massive haloes than observed. In the highest mass bin shown, the SAM overpredicts the number of faint satellites with \( L = 3 \times 10^9 h^{-2} L_\odot \) by a factor \( \sim 2 \). Since these are virtually all red, early-type galaxies (see below), this suggests that the SAM overpredicts the number density of faint, red galaxies. Indeed, as already shown in C06, the SAM overpredicts the luminosity function of red galaxies at the faint end. The analysis here suggests that this largely owes to an overabundance of satellite galaxies in massive haloes with \( M \gtrsim 3 \times 10^{14} h^{-1} M_\odot \).

We emphasize that this discrepancy is not due to the fact that we have ignored fiber collisions in the SDSS. Since the spectroscopic fibers of the SDSS have a minimum angular separation of 55′, the spectroscopic catalogue suffers from an incompleteness on small angular scales. This will impact on the multiplicity function of the groups in our catalogue. However, as shown by Berlind et al. (2006), the effect is relatively small, typically reducing the multiplicity of groups by \( \sim 10 \) percent, which is negligible compared to the factor 2 eluded to above.

4.2 Blue fraction as Function of Luminosity

The dotted line in the upper-left panel of Fig. 4 shows the fraction of blue galaxies, \( f_{\text{blue}} \), in the SDSS as a function of luminosity. Here all 184,425 galaxies in our SDSS sample defined in Section 2.1 are used. As is well known, the fraction of blue galaxies decreases drastically with increasing luminosity, dropping from \( \sim 95 \) percent at \( 0.1 M_r - 5 \log h = -16 \) to \( \lesssim 5 \) percent for galaxies with \( 0.1 M_r - 5 \log h < -22.5 \). The dashed line in the lower-left panel shows the blue fraction for the 428,013 model galaxies in the SAM-LS. Although this blue fraction also reveals an overall decrease with increasing luminosity, there are two marked differences with respect to the SDSS. First of all, at the bright end there is a sudden upturn in \( f_{\text{blue}} \); galaxies with \( 0.1 M_r - 5 \log h \approx -23 \) have a blue fraction of \( \sim 26 \pm 6 \) percent, compared to zero percent in the SDSS (note though that there are only 8 SDSS galaxies in this luminosity bin). As already mentioned in Section 3, these bright blue galaxies in the SAM are ULIRGs for which the dust modeling is inadequate (cf. lower panels of Fig. 1). The second discrepancy between SAM-LS and SDSS is more important: at the faint end the blue fraction in the SAM-LS never exceeds 62 percent, and is therewith much lower than the blue fraction of faint SDSS galaxies. Consistent with what we inferred above from the CLFs, this indicates that the SAM severely overpredicts the fraction of faint, red galaxies.

To investigate whether this mainly concerns central galaxies, satellite galaxies, or both, we now resort to the group catalogues extracted from the SDSS and SAM. The open circles with errorbars, connected by solid lines, indicate the blue fractions of galaxies that make it into the group catalogue. Comparing these for the SDSS to those obtained
from the full sample (dotted lines), we see that the blue fraction has become somewhat lower at the faint end. We can understand this by looking at the blue fractions of central galaxies (upper middle panel) and satellite galaxies (upper right panel). This shows that both blue fractions decrease with increasing luminosity, but that the luminosity dependence is more pronounced for the centrals. Since the galaxies that do not make it into the group catalogue are mainly isolated, and thus central, galaxies that live in haloes below the mass limit of the group catalogue, these are mainly blue. This explains why the fraction of faint blue galaxies is lower in the group catalogue than in the full redshift catalogue.

In the SAM, the group selection also causes a drop of the blue fraction of faint galaxies, but of a much larger amplitude. In fact, the fraction of ‘group’ galaxies with $-16 > M_{b,J} - 5 \log h > -18$ is $\sim 20$ percent in the SAM, compared to $\sim 60$ percent in the SDSS. The reason for this discrepancy is largely due to the satellite galaxies: as shown in the lower right-hand panel, the blue fraction of satellite galaxies in the SAM is much too low, especially at the faint end. In the case of the central galaxies (middle panels), the agreement between SAM and SDSS is much better, especially for centrals with $M_{b,J} - 5 \log h \simeq -21$. However, at the faint and bright ends, the SAM significantly overpredicts the blue fractions by $\sim 15$ and $\sim 25$ percent, respectively.

The dashed lines in the lower middle and lower right-hand panels show the blue fractions of centrals and satellites of all galaxies in the SAM-RS (including those that are not in the group catalogue). This shows that the group finder very accurately recovers the blue fraction of central galaxies, but slightly overpredicts that of satellites. This owes to the interlopers (group members that do not actually belong to the same halo), which tend to be isolated, central galaxies in low mass haloes, and which are thus preferentially blue. The contamination, however, is sufficiently small that it does not significantly affect any of our results.

In summary, although the SAM matches the overall blue fraction of galaxies almost exactly (see Section 3.1), when split according to luminosity or according to centrals and satellites, there are very significant differences between SAM and SDSS. The SAM overpredicts the blue fraction of central galaxies at both the bright and the faint end of the distribution, and dramatically underpredicts the blue fraction of (faint) satellite galaxies. In particular, the SAM predicts that virtually all ($\gtrsim 85\%$) satellite galaxies are red, whereas the SDSS indicates that the fraction of red satel-
Blue galaxy fractions as a function of absolute magnitude in the $0.1 r$-band. Results are shown for all galaxies (left-hand panels), central galaxies (middle panels) and satellite galaxies (right-hand panels), and for 6 different mass bins as indicated (the values in square brackets indicate the range of $\log(M/h^{-1} M_\odot)$). Results are only shown for mass-luminosity bins that contain at least 50 galaxies in total, and for clarity (Poissonian) errorbars are only shown for one mass bin. From top to bottom, results are shown for the SDSS group catalogue, the SAM redshift survey (SAM-RS), and the SAM group catalogue (SAM-GC). Note the poor agreement between SDSS and SAM-GC, indicating that the SAM is not correctly treating the physics responsible for determining whether a galaxy is red or blue. See Section 4.3 for a detailed discussion.

4.3 Blue Fraction as Function of Halo Mass

We now turn to the mass dependence of $f_{\text{blue}}$. We split the SDSS and SAM group catalogues in six logarithmic mass bins and determine how the blue fractions in each of these bins depend on luminosity. For each bin in mass and luminosity, the blue fraction is defined as the total number of blue galaxies in that bin, divided by the total number of galaxies in that bin.
galaxies in that bin (i.e., we do not average the blue fraction over individual groups or haloes).

The results for the SDSS group catalogue are shown in the upper panels of Fig. 5 and are listed in tabular form in Appendix A. The upper left-hand panel shows the result for all galaxies (centrals plus satellites). In each mass bin, the blue fraction decreases with increasing luminosity, but only very mildly. In fact, over the magnitude range $-19 < 5 \log h < -21.5$ the luminosity dependence is remarkably weak, for all six mass bins. At fixed luminosity, however, there is a clear mass dependence, with the blue fraction decreasing with increasing halo mass. Over the range $10^{12} h^{-1} M_{\odot} \lesssim M \lesssim 10^{15} h^{-1} M_{\odot}$, the blue fraction changes by $\sim 30$ percent, at all luminosities. This indicates that the colour of a galaxy is more strongly determined by the mass of the halo in which it resides than by its own luminosity (cf. Yang et al. 2005b; paper I). Consequently, the strong luminosity dependence of $f_{\text{blue}}$ seen in Fig. 4 is mainly a reflection of the fact that more luminous galaxies typically reside in more massive haloes. Note that some earlier work (e.g. Balogh et al. 2004, Tanaka et al. 2004) has found no correlation between galaxy properties and halo velocity dispersion. However, as shown in paper 1 this is most likely due to the smaller sample size and the fact that using velocity dispersion as a mass estimator tends to smear out the mass dependence.

The upper middle panel shows the blue fractions of central galaxies. Note that at a given halo mass, there is only a relatively small dynamic range of luminosities over which we can measure $f_{\text{blue}}(L)$. Nevertheless, at a given halo mass there is a clear indication that the blue fraction decreases with increasing luminosity. At the same time, at a given luminosity the blue fraction also decreases with increasing halo mass. Finally, the upper right-hand panel shows the results for the satellite galaxies in the SDSS group catalogue, which look similar to those for the full sample of galaxies shown in the upper left-hand panel.

The middle and lower rows of panels in Fig. 5 show the results obtained from the SAM-RS and SAM-GC, respectively. In the former no group finder has been applied, and the mass binning is according to the true halo masses of the galaxies. A comparison with the results obtained from the SAM group catalogue is therefore indicative of the accuracy with which our group finder allows a recovery of the true underlying $f_{\text{blue}}(L,M)$. A number of differences are clearly apparent, which mainly owe to the impact of interlopers and, more importantly, to errors in the assigned group masses. Since the true $f_{\text{blue}}(L,M)$ has extremely steep gradients in both $L$ and $M$, even small errors in any of these quantities can have a significant impact on the blue fraction obtained from the group catalogue. This mainly causes errors in the absolute values of $f_{\text{blue}}(L,M)$. However, the relative relations of $f_{\text{blue}}(L,M)$ and $f_{\text{blue}}(M|L)$ are well recovered. Note that the blue fraction of central galaxies with $0.1 M_r - 5 \log h < -19.5$ is less than 10 percent for all mass bins shown. Yet, as can be seen from Fig. 4, the blue fraction of central galaxies in the group catalogue with that luminosity is $\sim 80$ percent. This indicates that virtually all blue centrals with $0.1 M_r - 5 \log h < -19.5$ reside in haloes with $M < 10^{12} h^{-1} M_{\odot}$.

Comparing the $f_{\text{blue}}(L,M)$ obtained from the SAM-GC with those obtained from the SDSS, one notices immediately that they have very little in common. Probably the most dramatic difference between the SDSS and the SAM-GC is the blue fraction of satellite galaxies (shown in the panels on the right hand side), which is much too low in the SAM-GC, especially for faint galaxies and for low mass haloes. A comparison with the SAM-RS results shows that this discrepancy can not be attributed to artefacts of the group finder. In addition, the SAM-GC predicts that the blue fraction of central galaxies increases with increasing luminosity, opposite to what is seen in the SDSS (middle panels). This effect is most severe for haloes with masses $M < 10^{14} h^{-1} M_{\odot}$. These two problems conspire to produce a blue fraction of the full galaxy population (centrals and satellites) in the SAM-GC which is very different from the SDSS (left hand panels), both quantitatively (mainly because of the too low number of blue satellites) and qualitatively (mainly because of the reversed relation between luminosity and the blue fraction of central galaxies).

We are thus led to conclude that although the SAM reproduces the overall blue fraction (when integrated over all galaxies), when it comes to $f_{\text{blue}}(L,M)$, there are dramatic differences between model and data. Note that the SAM of C06 fits the overall luminosity function and even yields CLFs, i.e., luminosity functions as function of halo mass, whose shapes are in remarkably good agreement with the SDSS data. This suggests that one of the main problems for the SAM is the treatment of the physics responsible for determining whether a galaxy is red or blue. This includes the star formation truncation mechanisms, such as strangulation and AGN feedback, but also the treatment of dust extinction. In the next section we present a more detailed discussion of the possible implications.

5 DISCUSSION

The above analysis of the fractions of blue galaxies as function of halo mass and luminosity has revealed several problems for the SAM of C06. In the following we discuss the possible implications for galaxy formation models.

5.1 Tidal Stripping

The first problem concerns the abundance of satellite galaxies. As shown in Section 4.1, the SAM overpredicts the number of faint satellite galaxies in massive haloes by up to a factor $\sim 2$. Most likely, this indicates that the luminosity evolution of the satellites is not properly accounted for. Satellite galaxies can become fainter due to star formation truncation followed by passive evolution, or due to tidal stripping of their stellar mass. Since most of the satellite galaxies in the SAM are already red, they can not become much fainter than they already are for their given stellar mass. In other words, there is little to gain from adding physical processes that may speed up the star formation truncation, such as ram pressure stripping. In fact, this will only increase the fraction of red satellites, which is already much too large. The most plausible explanation for the overabundance of satellite galaxies is the neglect of tidal stripping. It is well known that the tides in massive haloes can easily strip satellite galaxies and their dark matter haloes of large fractions of...
their mass. This is supported by the detection of intracluster light (e.g., Bernstein et al. 1995; Gonzalez et al. 2000) due to a diffuse population of intergalactic stars. Most likely, these stars, which contribute around 10 percent of the total cluster light (Zibetti et al. 2005), have been tidally stripped from satellite galaxies.

Formalisms to describe tidal stripping in the presence of dynamical friction in a background potential have been developed by, among others, Taylor & Bullock (2001, 2004) and Zentner & Bullock (2003). As shown in Benson et al. (2002), including such a formalism in semi-analytical models significantly reduces the abundance of satellite galaxies. Furthermore, since tidal stripping predominantly affects satellites with small pericentric radii, which are typically redder (blue galaxies have only recently been accreted by the halo, and have not yet experienced much dynamical friction), tidal stripping may also reduce the red fraction of satellites and thus help towards solving the problem with the red fractions of satellites being too large.

5.2 Strangulation

The second problem for the SAM of C06 is that the fraction of blue satellite galaxies is much too low, especially for faint galaxies and in low mass haloes. This suggests that ‘strangulation’, as incorporated in the SAM, is much too efficient. In virtually all semi-analytical models, strangulation is included and modeled in the same way: as soon as a galaxy becomes a satellite galaxy, its hot gas reservoir is ‘stripped’ (i.e., the hot gas that belonged to the satellite galaxy is added to the hot gas reservoir of the central galaxy). Consequently, after a delay time in which the new satellite galaxy consumes (part of) its cold gas, its star formation is truncated. The results presented here suggest that this formulation is too crude, as it predicts a blue satellite fraction that is much too low\(^3\). In particular, strangulation is modeled without any explicit halo mass dependence, which explains why the blue fraction of satellite galaxies in the SAM is virtually independent of halo mass. In the SDSS, however, the blue satellite fraction decreases with increasing halo mass, suggesting a clear mass scaling of the strangulation efficiency. The physical mechanisms thought to be responsible for the removal of the satellite’s hot gas reservoir are tides and ram pressure stripping. The latter requires that the parent halo has a sufficiently dense hot corona. Since this requirement is more likely to be fulfilled for more massive haloes, which in general have a larger fraction of hot gas, one actually expects that the strangulation efficiency increases with increasing halo mass. Indeed, using numerical simulations, Bekki et al. (2002) find that strangulation is significantly more effective in massive galaxy clusters than in lower mass groups. It remains to be seen whether a simple scaling along these lines can bring the red and blue fractions of satellite galaxies in SAMs in better agreement with the SDSS data presented here.

Interestingly, in a comparison of semi-analytical models and hydrodynamical SPH simulations of galaxy formation, Zheng et al. (2005) have shown that the later predicts that haloes with \(10^{12} \, M_\odot \lesssim M \lesssim 10^{13} \, M_\odot\) have a significantly higher fraction of young satellites (similar to the blue satellites discussed here) than SAMs, in much better agreement with the SDSS results presented here (see their Fig. 4). Similarly, Cattaneo et al. (2006b) find that their semi-analytic model produces too few blue satellite galaxies compared to their SPH simulation. Since these SPH simulations follow the actual hydrodynamical processes leading to strangulation, this indeed suggests that a more realistic treatment of strangulation in the SAMs may solve the problem with the colors of satellite galaxies. SPH simulations such as those described in Zheng et al. (2005) and Cattaneo et al. (2006b) may prove useful in calibrating such a new and improved strangulation model.

5.3 Dust Extinction and AGN Feedback

The SAM also has problems with the blue fractions of central galaxies, which are too high, especially at the bright and faint ends. In addition, for a given halo mass, the blue fraction of central galaxies increases with luminosity, contrary to what is seen in the SDSS. Which aspect(s) of the semi-analytical model are responsible for these problems is not entirely clear. They can indicate a problem with the modeling of dust extinction, a problem with the treatment of AGN feedback, or both. The former is almost certainly responsible for the overproduction of bright and blue centrals. As already discussed in C06, this population of galaxies is reminiscent of the ULIRG population, for which the oversimplified treatment of the dust extinction is certainly inadequate: real starburst are likely to be accompanied by additional extinction which would make these galaxies both fainter and redder. This would help to suppress the strong increase of \(f_{\text{blue}}\) with increasing luminosity, though it remains to be seen whether it can result in a blue fraction that decreases with increasing luminosity, as observed. Furthermore, in order to suppress the impact of these starbursting model galaxies on the comparison presented here, we already removed all galaxies with \((g - r) < 0\) from the SAM. Despite this, however, the SAM still significantly overpredicts the fraction of blue centrals with \(0.1 \, M_r - 5 \log h \simeq -23\). Although improper dust modeling is likely to be the cause for this discrepancy, it remains to be seen whether this can also explain the overprediction (by ~15 percent) of the blue fraction of faint centrals.

Alternatively, the blue fractions of central galaxies may be modified by changing the AGN feedback description. As discussed in the introduction, it is still largely unknown how AGN impact on their surroundings, and thus feed back on the process of galaxy formation. It should therefore not come as a surprise if the parameterization of C06 is not entirely correct or complete. The purpose of this paper, however, is not to find an improved formulation of AGN feedback. Rather, we have presented data, in the form of blue galaxy fractions as function of both halo mass and luminosity, which we believe to be useful in discriminating between different AGN feedback models. Future SAMs can test and/or calibrate their particular parameterizations against these data.
6 CONCLUSIONS

It has become clear that a successful reproduction of the galaxy luminosity function, and of the bimodality in the galaxy distribution, requires a mechanism that can truncate the star formation in massive galaxies at relatively late epochs. At the same time, the fact that red, passive galaxies preferentially reside in overdense regions, such as clusters and groups of galaxies, suggests that star formation truncation acts preferentially in massive haloes. Current models consider two such truncation mechanisms: strangulation, which acts on satellite galaxies, and AGN feedback, which predominantly affects central galaxies.

Typically, galaxy formation models are tuned to reproduce the global properties of the galaxy distribution, such as the luminosity function and the total fraction of blue and red galaxies. However, even when two models predict exactly the same global statistics, their statistics as function of halo mass may be very different. The latter is clearly more constraining for the model, and furthermore, holds important information regarding the mass-dependence of the various physical mechanisms associated with galaxy formation. In particular, since star formation truncation causes a galaxy to become red, the relative fractions of red and blue galaxies as a function of halo mass holds important clues regarding the halo mass dependence of the efficiencies of AGN feedback and strangulation.

To provide a test-bed for models of galaxy formation, we have used a galaxy group catalogue extracted from the SDSS for which we have computed the fraction of blue and red galaxies as function of both galaxy luminosity and group (halo) mass. To illustrate the potential constraining power of this data we have compared these fractions to those in the semi-analytical model for galaxy formation of Croton et al. (2006), which includes both `radio mode‘ AGN feedback and strangulation. To allow for a fair comparison between the SDSS group catalogue and the SAM, not influenced by potential inaccuracies associated with the group finder (i.e., interlopers, incompleteness, errors in assigned group mass), we have applied the same group finder over a mock redshift survey constructed from the SAM. The \( f_{\text{blue}}(L, M) \) obtained from this SAM group catalogue is in fair agreement with the true \( f_{\text{blue}}(L, M) \) obtained directly from the SAM, indicating that our group finder allows a reliable recovery of the blue fraction as a function of both galaxy luminosity and halo mass. Although interlopers and errors in the group masses may cause some errors in the absolute values of \( f_{\text{blue}}(L, M) \), the relative scalings of \( f_{\text{blue}}(L|M) \) and \( f_{\text{blue}}(M|L) \) are well recovered.

Although the SAM fits the overall luminosity function, reproduces the overall color distribution of the SDSS galaxies, and even predicts a conditional luminosity function whose shape is in excellent agreement with the data, its prediction of \( f_{\text{blue}}(L, M) \) is in poor agreement with the SDSS data. In particular, we have identified four problems:

(i) In massive haloes the abundance of faint satellite galaxies is too high by up to a factor \~{}2.

(ii) The fraction of blue satellite galaxies is much too low, especially for faint galaxies and in low mass haloes.

(iii) The fraction of blue central galaxies is too high, especially at the bright and faint ends.

(iv) For a given halo mass the blue fraction of central galaxies increases with luminosity, contrary to what is seen in the SDSS.

The first of these problems is likely to owe to the fact that the SAM does not model the tidal stripping of the stellar mass of the satellite galaxies as they orbit the parent halo. As shown in Benson et al. (2002), inclusion of this effect significantly reduces the abundance of satellite galaxies at a fixed luminosity. The second problem is most likely due to an oversimplification of the treatment of strangulation. In the SAM, strangulation occurs instantaneously, independent of halo mass. However, based on the SDSS data, we have argued that the strangulation efficiency has to scale with halo mass, such that more massive haloes strangulate their satellites on a shorter time scale. The physical motivation for such a scaling is that the ram pressure stripping of the hot gas reservoir of newly accreted satellites requires the parent halo to have a sufficiently dense corona of hot gas. Since the fraction of hot gas is typically an increasing function of halo mass, this may introduce a mass dependence in the strangulation efficiency as required. Indeed, hydrodynamical SPH simulations, which automatically take this into account, seem to predict blue satellite fractions that are significantly higher than in the semi-analytical models (Zheng et al. 2005, Cattaneo et al. 2006b). Finally, the third and fourth problem listed above, both of which concern central galaxies, are likely to reflect shortcomings of the modeling of dust extinction and/or AGN feedback.

In summary, galaxy formation models are often tested and calibrated against global properties of the observed galaxy distribution. Recently, with the inclusion of AGN feedback, numerous studies have claimed success in reproducing these global statistics, even though very different formulations for the various physical processes have been used. In order to discriminate between these models more specific data is required. In this paper we have presented the fractions of blue and red galaxies as function of luminosity, halo mass, and separately for central and satellite galaxies. Clearly this data is far more constraining, and thus more challenging, than the global fractions of red and blue galaxies, or the overall luminosity function. We have shown that indeed these data provide valuable new insights regarding the physics of galaxy formation, and we hope that they will provide a useful test-bed for future models of galaxy formation.

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Science Foundation, the U.S. Department of Energy, the Japanese Monbukagakusho, and the Max Planck Society. The SDSS Web site is http://www.sdss.org/. The SDSS is managed by the Astrophysical Research Consortium (ARC) for the Participating Institutions. The Participating Institutions are The University of Chicago, Fermilab, the Institute for Advanced Study, the Japan Participation Group, The Johns Hopkins University, Los Alamos National Laboratory, the Max-Planck-Institute for Astronomy (MPIA), the Max-Planck-Institute for Astrophysics (MPA), New Mexico State University, University of Pittsburgh, Princeton University, the United States Naval Observatory, and the University of Washington. The Millennium Run simulation used in this paper was carried out by the Virgo Supercomputing Consortium at the Computing Centre of the Max-Planck Society in Garching. Semi-analytic galaxy catalogues from the simulation are publicly available at http://www.mpagarching.mpg.de/galform/agnpaper.

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APPENDIX A: BLUE GALAXY FRACTIONS IN THE SDSS

The following three tables list the fraction of blue galaxies for all galaxies (Table A1), for satellite galaxies (Ta-
Table A1. Blue fractions of all galaxies

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Table A2. Blue fractions of satellite galaxies

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<td>0.55 (9366)</td>
<td>0.44 (1673)</td>
<td>0.39 (1445)</td>
<td>0.31 (1172)</td>
<td>0.23 (771)</td>
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<tr>
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<td>0.50 (7890)</td>
<td>0.45 (3181)</td>
<td>0.39 (2556)</td>
<td>0.32 (1673)</td>
<td>0.24 (1058)</td>
<td>0.23 (380)</td>
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<tr>
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<td>0.42 (4752)</td>
<td>0.35 (3657)</td>
<td>0.28 (2164)</td>
<td>0.26 (1068)</td>
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<td>0.32 (7327)</td>
<td>0.23 (2231)</td>
<td>0.21 (1001)</td>
<td>0.17 (326)</td>
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<tr>
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<td>-</td>
<td>0.26 (464)</td>
<td>0.14 (1181)</td>
<td>0.11 (476)</td>
<td>0.08 (143)</td>
</tr>
<tr>
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<td>-</td>
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<td>0.06 (108)</td>
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### Table A3. Blue fractions of central galaxies

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<th>[14, 14.5]</th>
<th>[14.5, 15]</th>
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<td>0.46 (2792)</td>
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<td>0.11 (444)</td>
<td>0.07 (95)</td>
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<td>0.06 (108)</td>
</tr>
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