NONLINEAR ACTIVE NOISE CONTROL USING LYAPUNOV THEORY AND RBF NETWORK

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A new approach in design an efficient algorithm for ANC system is proposed. The transversal filter-based controllers (FIR and IIR) are first considered. A Lyapunov function of the error is first defined and filter coefficients are then adaptively adjusted based on Lyapunov stability theory so that the error converges to zero asymptotically. The design is independent of the statistic properties of signals and its computation complexity is comparable to FXLMS. It has fast error convergence properties and the stability is guaranteed by Lyapunov stability theory. This scheme can be further extended to an efficient nonlinear ANC using RBF network for excellent performance. Simulation examples are demonstrated to show the degree of noise cancellation this scheme can achieve.

1. INTRODUCTION

Active noise control (ANC) [1],[4] has been successfully applied to HVAC (Heating, ventilating and air conditioning) systems [2], exhaust noise and motor noise [3]. In general, ANC is based on the principle of the destructive interference between a primary noise source and a secondary source, whose acoustic output is governed by a controller. The output of the secondary source has to be in exact anti-phase with the acoustic wave produced by the primary noise source. A typical ANC system in a duct, is shown in Figure 1 and its equivalent block diagram in Figure 2. Such systems are usually based on a feedforward control strategy. The noise from the primary source travels, from left to right, as plane waves through the dust. A microphone located upstream from the secondary source detects the incident noise waves and supplies the controller with an input signal. The controller sends a signal to the secondary source (i.e. loudspeaker) which is in anti-phase with the disturbance. A microphone located downstream picks up the residuals and supplies the controller with an error signal. The variance of the error signal is usually minimized by the LMS algorithm in the controller of ANC system.

However, for the adaptive filter involved in ANC system to properly converge to the desired solution, it is necessary to compensate for the distortion caused by the secondary-path effects. The reason is that introduction of the secondary-path transfer function, \(S(z)\) into a controller using standard LMS will generally cause instability. The error signal is not correctly ‘aligned’ in time with the reference signal, due to the present of \(S(z)\). Furthermore \(S(z)\) is normally non-minimum phase zero outside the unit circle, which on inversion could lead to instability. Therefore a modified version of LMS, the filtered-x LMS (FXLMS) was
developed. [4] by including the $S'(z)$ shown in Figure 2. This $S'(z)$ has to be at least accurate enough to match the phase within $90^\circ$ for the algorithm to be stable [4]. Although FXLMS offers a simple update strategy, it requires knowledge about the statistics of the input data in order to choose the proper step size, especially when on-line secondary path identification is employed. To ensure convergence, the step size is typically chosen to be smaller, causing the system to converge slowly and to exhibit poor performance. FXNLMS [4] is guaranteed to converge for a step size range that is independent of the data statistics and it converges faster than XLMS. However it is also affected by the random noise such as measurement noises due to the reference and error sensors, $u(k)$ and $v(k)$ which is added to the system.

In this paper, a new approach in design an efficient algorithm for ANC system is proposed. The transversal filter-based controllers (FIR and IIR) are considered and the adaptive algorithm is non-gradient based. A Lyapunov function instead of the cost function of the error is first defined. Filter coefficients are then adaptively adjusted based on Lyapunov stability theory so that the error converges to zero asymptotically. The design is independent of the statistic properties of signals and its computation complexity is comparable to FXLMS and less than FXRLS. It has fast error convergence properties and small steady state error. The stability is guaranteed by Lyapunov theory even if random noise or disturbance is added to the system. Due to the fact that many of the noise processes arise from nonlinear dynamical systems, this scheme can be further extended to an supervised RBF training algorithm for excellent performance. Simulation examples are performed to demonstrate the performance of this scheme.

Fig. 1: Single-channel broadband feedforward ANC system in a duct

Fig. 2: ANC using FXLMS algorithm

Fig. 3: ANC with Lyapunov theory & RBF
2. BROADBAND FEEDFORWARD ANC USING LYAPUNOV THEORY

Authors in [5]-[8] have proposed a range of Lyapunov theory-based algorithms. Those algorithms are the modification of RLS using LST. They are not gradient-based method and are independent of the stochastic properties of the signals. These ideas can be extended to design an ANC controller filter. If the FIR filter-based controller is implemented, the output of the adaptive controller filter is

\[ y(k) = H^T(k)X(k) \]  \hspace{1cm} (2.1)

where \( H(k) = [h_0(k), h_1(k), ..., h_{N-1}(k)]^T \), \( X(k) = [x(k), x(k-1), ..., x(k-N+1)]^T \)

An error sensor measures the error signal as modeled by the equation

\[ e(k) = d(k) + s(k)\cdot y(k) = d(k) + s(k)\cdot H^T(k)X(k) \]

\[ e(k) = d(k) + H^T(k)X'(k) \]  \hspace{1cm} (2.3)

where \( X'(k) = [x'(k), x'(k-1), ..., x'(k-N+1)]^T \) is the filtered reference signal vector with elements \( x'(k) = s'(k)\cdot x(k) \) and \( s'(k) \) is the impulse response of the secondary path \( S'(z) \). \( d(k) \) can be the estimated response from plant modeling.

The coefficient vector update equation is

\[ H(k) = H(k-1) - g(k)\alpha(k) \]  \hspace{1cm} (2.4)

where \( g(k) \) is the adaptation gain and \( \alpha(k) \) is a priori estimation error defined as

\[ \alpha(k) = d(k) + H^T(k-1)X'(k) \]  \hspace{1cm} (2.5)

The adaptation gain \( g(k) \) in (2.4) is adaptively adjusted using Lyapunov stability theory as (2.6) so that the error \( e(k) \) asymptotically converges to zero.

\[ g(k) = \frac{X'(k)}{\|X'(k)\|_2^2} \left( 1 - \kappa \frac{\|e(k-1)\|}{\|\alpha(k)\|} \right) \]  \hspace{1cm} (2.6)

where \( 0 \leq \kappa < 1 \).

The design of the ANC controller filter using Lyapunov theory is described by the following theorem:

**Theorem 2.1:** For the given desired response \( d(k) \), if the weight vector \( H(k) \) of the ANC controller filter \( y(k) = H^T(k)X(k) \) is updated as follows

\[ H(k) = H(k-1) - g(k)\alpha(k) \]

and

\[ g(k) = \frac{X'(k)}{\|X'(k)\|_2^2} \left( 1 - \kappa \frac{\|e(k-1)\|}{\|\alpha(k)\|} \right) \]

where \( 0 \leq \kappa < 1 \), then the error \( e(k) = d(k) + y(k)\cdot s(k) \) asymptotically converges to zero.

**Proof:** Define a Lyapunov function

\[ V(k) = e^2(k) \]

Then, \( \Delta V(k) = V(k) - V(k-1) \)
To prevent singularities problem, the adaptation gain may be modified as (2.6)

\[ g(k) = \frac{X'(k)}{\lambda_1 + \|X'(k)\|^2} \left( 1 - \kappa \frac{e(k - 1)}{\lambda_2 + \|\alpha(k)\|} \right) \]  

(2.9)

where \( \lambda_1, \lambda_2 \) are small positive numbers and \( 0 \leq \kappa < 1 \). According to [9], the tracking error \( e(k) \) will not converge to zero if the adaptive gain \( g(k) \) is adjusted using expression (2.9), but it will converge to a ball centred at the system origin. The radius of the ball depends on the values of \( \lambda_1 \) and \( \lambda_2 \). Generally, smaller \( \lambda_1 \) and \( \lambda_2 \) are, smaller the error \( e(k) \) is.

**Remark 2.1:** The design of this Lyapunov controller filter is similar to the design of Lyapunov-based filtering [5]-[7]. However, ANC controller produces exact anti-noise of equal amplitude and opposite phase to cancel the primary noise. In addition, an consideration of secondary path effects is necessary for the design of ANC algorithm.

**Remark 2.2:** Due to the secondary path, the instantaneous measurement of the gradient means square error is thus no longer an unbiased estimate of the true error. An extra filter \( S'(z) \) or estimate of \( S(z) \) is needed to ensure the stability. Hence LMS is modified to FXLMS or FXNLMS. The stability of FXLMS or FXNLMS is also affected by the accuracy of the filter \( S(z) \). [4] has shown that for pure tone reference signals the phase difference of \( S(z) \) and \( S'(z) \) has to be within \( \pm 90^\circ \) for the system to be converge slowly. Design the new ANC controller filter based on Lyapunov stability is a new approach. A Lyapunov function of the error between the desired response and filter output is first defined:

\[ V(k) = e^2(k) \]

The coefficients are then adaptively adjusted based on Lyapunov theory. According to Lyapunov theory [9], if a positive define function, \( V(k) = e^2(k) \) is found such that its discrete time difference taken along a trajectory is always negative \( \Delta V(k) < 0 \), then as time \( k \) increases, \( V(k) \) will finally converge to zero and therefore the error will also converge to zero asymptotically. However, if \( g(k) \) are adjusted using (2.9), then the \( e(k) \) will converge to a ball centred at the origin of the error space with radius of the ball depends on \( \lambda_1, \lambda_2 \). Hence the stability of the error dynamics between the desired response and filter output is guaranteed even with the introduction of additive noise or large bounded input disturbances in ANC system. Unlike FXLMS or gradient-based algorithm, this technique is independent of signals' statistic properties.
Remark 2.3: The convergence of FXLMS depends on the step size. Due to the secondary path $S(z)$ that affects limit the upper bound for the step size to ensure the algorithm's convergence and stability. Hence small step size is chosen and often leads to slow convergence. The proposed scheme has fast error convergence rate. It has been shown in [5]-[7], for an input signal $x'(k)$ and a desired response, $d(k)$, the convergence rate of error $e(k)$ computed with expressions (2.1)-(2.9) depends on the initial error, $e(0)$ and the constant $\kappa$.

$$|e(k)| = \kappa^k |e(0)|$$

(2.10)

where $0 \leq \kappa < 1$. Notice that the error approaches to zero as $k$ approaches infinity if the selected $\kappa$ is between zero and one. Hence the proposed algorithm can be made to converge fast since the quantity $\kappa^k$ approaches zero quickly if smaller constant $\kappa$ is chosen. $\kappa$ can be any arbitrary number within 0 and 1.

Remark 2.4: Computational complexity is another essential quantity that measures the effectiveness of an adaptive algorithm. The FXLMS [4] requires $2N+M+1$ multiply/accumulates (MACs) per iteration [10], where $N$ is the filter length and $M$ is the secondary path impulse response length. Other modified filtered-X LMS algorithm requires $3N+2M+1$ [11], $2N+2M$ [12], $2N+5M+1$ [13]. The multiplication operation of the proposed scheme is about $3N+M+1$. Hence the computation of this new algorithm is equivalent to FXLMS modifications. This implementation is more computationally efficient compared to FXRLS [4] that requires about $N^2$ multiplication.

Remark 2.5: The propose scheme can be easily extended IIR based controller. Furthermore, this idea can be implemented to multi error Lyapunov ANC algorithm for multichannel active noise control.

3. ANC USING RBF NEURAL NETWORK

Normally the primary noise is usually assumed to be broadband random or periodic tonal noise. Based on this assumption, linear adaptive signal processing techniques are employed to estimate the anti-phase signal. However it is known that many of these noise processes arise from nonlinear dynamical systems. Therefore a nonlinear ANC is necessary. One of the major choices is RBF neural as RBF is becoming increasingly popular for adaptive nonlinear signal processing because of the distinctive properties of best approximation, simple network structure and efficient training procedures. If the feedforward RBF network shown in Figure 4 is implemented, the output of the RBF network can be described as

$$y(k) = \sum_{i=1}^{N} w_i(k) \phi_i(k)$$

(3.1)

The expression (3.1) can be rewritten as the expression (3.2).

$$y(k) = W^T (k) \Phi(k)$$

(3.2)
where \( W(k) = [w_1(k), w_2(k), \ldots, w_N(k)]^T \), \( \Phi(k) = [\phi_1(k), \phi_2(k), \ldots, \phi_N(k)]^T \)

\( \phi(k) \) can be the Gaussian type of functions or thin-plate function. The strategy for updating the network parameters involves supervised learning. At each iteration, the weights is updated using a modified Lyapunov ANC algorithm. The error considered the secondary path effects is given by

\[
e(k) = d(k) + s(k) \ast y(k) = d(k) + s(k) \ast W^T(k) \Phi(k)
= d(k) + W^T(k) \Phi'(k)
\tag{3.3}
\]

where \( \Phi'(k) = [\phi'(k), \phi'(k - 1), \ldots, \phi'(k-N+1)]^T \) is the filtered reference signal vector with elements \( \phi'(k) = s'(k) \ast \phi(k) \) and \( s'(k) \) is the impulse response of the secondary path \( S'(z) \).

To update the weight vector of the RBF neural network, the coefficients updated equation in the expression (2.4) in Lyapunov FIR filter can be replaced with the expression in (3.4).

\[
W(k) = W(k - 1) - g(k) \alpha(k)
\tag{3.4}
\]

where \( g(k) \) is the weight adaptation gain and \( \alpha(k) \) is a priori estimation error. The expression in (2.5) can be replaced with the following expression.

\[
\alpha(k) = d(k) + W^T(k-1) \Phi'(k)
\tag{3.5}
\]

The weight adaptation gain \( g(k) \) in (3.4) is adaptively adjusted based on Lyapunov stability theory to have error convergence to zero asymptotically.

\[
g(k) = \frac{\Phi'(k)}{||\Phi'(k)||^2} \left( 1 - \kappa \frac{|e(k-1)|}{|\alpha(k)|} \right)
\tag{3.6}
\]

where \( 0 \leq \kappa < 1 \).

The design of (3.6) is described in theorem 3.1:

**Theorem 3.1:** For the given desired response \( d(k) \), if the weight vector \( W(k) \) of the ANC controller filter \( y(k) = W^T(k)\Phi(s(k)) \) is updated as follows

\[
W(k) = W(k - 1) + g(k) \alpha(k)
\]

and

\[
g(k) = \frac{\Phi'(k)}{||\Phi'(k)||^2} \left( 1 - \kappa \frac{|e(k-1)|}{|\alpha(k)|} \right)
\]

where \( 0 \leq \kappa < 1 \), then the error \( e(k) = d(k) + s(k) \ast y(k) = d(k) + W^T(k) \Phi'(k) \)

asymptotically converges to zero.

**Proof:** Define a Lyapunov function

\[
V(k) = e^2(k)
\]

Then,

\[
\Delta V(k) = V(k) - V(k - 1)
= e^2(k) - e^2(k-1)
= (d(k) + W^T(k)\Phi'(k))^2 - e^2(k-1)
\]
\[ \begin{align*}
  &=(d(k) + (W^T(k-1) + g^T(k)\alpha(k)))\Phi'(k))^2 - e^2(k-1) \\
  &=(d(k) + W^T(k-1)\Phi'(k) - g^T(k)\alpha(k)\Phi'(k))^2 - e^2(k-1) \\
  &=(\alpha(k) - g(k)\alpha(k)\Phi'(k))^2 - e^2(k-1) \\
  &=\alpha^2(k)(1 - g^T(k)\Phi'(k))^2 - e^2(k-1) \\
\end{align*} \]

Using expression (3.6) in expression (3.7), we have

\[ \Delta V(k) = -(1 - \kappa^2)\bar{e}^2(k-1) < 0 \]

The \( g(k) \) may then be modified as equation (3.9) to elude the singularities due to zero values of \( \alpha(k) \).

\[ g(k) = \frac{\Phi'(k)}{||\Phi'(k)||^2}\left(1 - \kappa \frac{\bar{e}(k-1)}{\lambda_1 + ||\alpha(k)||}\right) \]

where \( \lambda_1 \) is a small positive number.

**Remark 3.1:** The stability of the error dynamics is still guaranteed for \( S'(z)\neq S(z) \) with certain range. However, it is suggested that this scheme is used for on-line secondary path estimation instead of off-line estimation so that the ANC system has excellent performance.

**Remark 3.2:** The proposed scheme can be further extended to the recurrent RBF with the realization follows the concepts introduced for the NARMAX IIR design that included the input and past output.

\[ \text{Fig. 4: Feedforward RBF ANC filter} \]

### 4. SIMULATION EXAMPLES

Simulation examples illustrate the performance of the proposed ANC filter. Segments of broadband noises used are sampled and then are applied to the input of the ANC system. The secondary path model used is \( S(z')=z^2(1-2z^2) \) which is a delayed bandpass and has zero outside the unit circle. Effects of measurement noises, \( u(k) \) shown in Figure 2 is also considered. The additive noise is white normal random noise \( \{0, 1\} \). To compare the performance of the proposed scheme, simulation with FXLMS algorithm is also presented.
Lyapunov ANC controller filter vs FXLMS filter—Fig. 7a show the incident primary noise, $x(k)$. The output of the Lyapunov ANC including the controller filter and secondary path transfer function is illustrated in Fig. 7b. The residual error, $e(k)$ is revealed in Fig. 7c. For the same setup, simulation results for FXLMS including the secondary path estimation are illustrated in Fig. 7d and 7e. It is noticed that the proposed controller filter can tolerate the additive noise, $u(k)$ and perform better than FXLMS. Thus the theoretical and simulation results have shown this scheme has fast convergence speed, good tracking property, robustness to additive noise and is highly stable.

Lyapunov ANC controller filter using RBF—For the same setup, nonlinear ANC is performance using a RBF network that is trained by the algorithm based on Lyapunov theory. The results are revealed in Fig. 8a and 8b. Fig. 8a shows the output of the ANC system including the RBF network and $S(z)$. Fig. 8b indicates the residual error, $e(k)$. By introducing the RBF network into the ANC, the system gives excellent results, compared to Fig. 7b-7e.

5. CONCLUSION

A new implementation of ANC algorithms using Lyapunov stability theory is proposed in this paper. This ANC algorithm is independent of the signals’ statistic properties. Its computation complexity is comparable to FXLMS and less than that of FXRLS. This scheme has fast error convergence properties and the error convergence is guaranteed by Lyapunov stability theory. This idea is further extended to a nonlinear ANC using RBF network. Theoretical and simulation results have demonstrated good performance of the proposed scheme.

6. REFERENCES


Fig 7d: The desired response and the secondary noise produced by the ANC-LMS

Fig 7c: ANC-LMS, the residual error

Fig 8a: The desired response and the secondary noise produced by the ANC-RBF

Fig 8b: ANC-RBF, the residual error, $y$-axis: $x10^{-4}$