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A Robust MIMO Terminal Sliding Mode Control Scheme for Rigid Robotic Manipulators

Man Zhihong, A. P. Paplinski, and H. R. Wu

Abstract—In this paper, a robust multi-input/multi-output (MIMO) terminal sliding mode control technique is developed for n -link rigid robotic manipulators. It is shown that an MIMO terminal switching plane variable vector is first defined, and the relationship between the terminal switching plane variable vector and system error dynamics is established. By using the MIMO terminal sliding mode technique and a few structural properties of rigid robotic manipulators, a robust controller can then be designed so that the output tracking error can converge to zero in a finite time, and strong robustness with respect to large uncertain dynamics can be guaranteed. It is also shown that the high gain of the terminal sliding mode controllers can be significantly reduced with respect to the one of the linear sliding mode controller where the sampling interval is nonzero.

I. INTRODUCTION

Sliding mode control is one of the most important approaches to handling systems with large uncertainties, nonlinearities, and bounded external disturbances. Generally, in most of sliding mode control schemes for multi-input/multi-output (MIMO) systems, an MIMO linear sliding mode is first designed to describe the desired system error dynamics, a robust controller drives the switching plane

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variables to reach the sliding mode, and the asymptotic convergence of error dynamics can then be obtained on the linear sliding mode [1]–[6]. To get fast error convergence on the sliding mode, however, the sliding mode parameters must be chosen such that the poles of the sliding mode dynamics are far from the origin on the left-half of the s -plane. This will, in turn, increase the gain of the controller. Considering the saturation property of control input signals in practical robot control, a sliding mode controller with high gain is undesirable. On the other hand, a linear sliding mode technique can guarantee only the asymptotic error convergence on the sliding mode, and therefore error dynamics can not converge to zero in a finite time.

In this paper, a robust MIMO terminal sliding mode control scheme is developed for n -link rigid robotic manipulators based on [11]–[13]. It is shown that an MIMO terminal switching plane variable vector is first defined, and the relationship between the terminal switching plane variable vector and system error dynamics is investigated. By using the MIMO terminal sliding mode technique and a few structural properties of rigid robotic manipulators, a robust controller can then be designed. Unlike conventional linear sliding mode control schemes, the switching plane variable vector in this paper has a nonlinear term of the velocity error. By suitably designing the controller, the switching plane variables can reach the terminal sliding mode in a finite time, and the output tracking error can then converge to zero in a finite time on the terminal sliding mode. It is also shown that this scheme is more practical because the gain of the terminal sliding mode controller can be significantly reduced with respect to the high gain of linear sliding mode controllers in practical situations where the sampling interval is non zero.

Similar to the linear sliding mode technique, strong robustness with respect to large uncertain dynamics can be obtained by using the proposed control scheme. Also, the controller design is simple in the sense that only a few uncertain bounds based on structural properties of robotic manipulators are used in the controller parameter designs.

The paper is organized as follows: In Section II, an n -link rigid robotic manipulator model and its a few useful structural properties are formulated, and an MIMO terminal nonlinear sliding mode is defined to describe desired error dynamics. In Section III, a robust MIMO terminal sliding mode control scheme is developed for rigid robotic manipulators, the stability of error dynamics and robustness with respect to uncertain dynamics are discussed in detail, and an advantage of the proposed scheme in its practical applications is also remarked. In Section IV, a simulation for a two-link rigid robotic manipulators is performed in support of the proposed control scheme. Section V gives conclusions.

II. PROBLEM FORMULATION

The dynamics of an n -joint robotic manipulator can be described by the following second-order nonlinear vector differential equation

$$M(q)\ddot{q} + F(q, \dot{q})\dot{q} + G(q) = u(t) \quad (2.1)$$

where $q(t)$ is the $n \times 1$ vector of joint angular positions, $u(t)$ is the $n \times 1$ vector of applied joint torques, $M(q)$ is the $n \times n$ symmetric positive-definite inertia matrix, $F(q, \dot{q})\dot{q}$ is the $n \times 1$ vector of coriolis and centrifugal torques, and $G(q)$ is the $n \times 1$ vector of gravitational torques. Further, we assume that vectors u , q , and \dot{q} are measurable (see Fig. 1).

Defining $x = (q^T, \dot{q}^T)^T$, expression (2.1) can be written as

$$\dot{x} = \begin{bmatrix} \dot{q} \\ M(q)^{-1}(-F(q, \dot{q})\dot{q} - G(q)) \end{bmatrix} + \begin{bmatrix} 0 \\ I \end{bmatrix} M(q)^{-1} u(t). \quad (2.2)$$

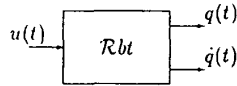
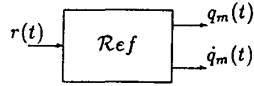

 Fig. 1. An n -link rigid robotic manipulator model.


Fig. 2. A desired reference model.

The reference model for the plant to follow can be represented as

$$\begin{bmatrix} \dot{q}_r \\ \ddot{q}_r \end{bmatrix} = \begin{bmatrix} 0 & I \\ P & Q \end{bmatrix} \begin{bmatrix} q_r \\ \dot{q}_r \end{bmatrix} + \begin{bmatrix} 0 \\ B_1 \end{bmatrix} r(t) = A_m x_m + B_m r \quad (2.3)$$

where $P = \text{diag}(P_i)$, $Q = \text{diag}(Q_i)$, and $B_1 = \text{diag}(b_i)$ ($1 \leq i \leq n$) are constant matrices which are chosen such that the reference model (2.3) is stable. Vectors r , q_m , and \dot{q}_m are assumed to be measurable (see Fig. 2).

Defining $\epsilon = q - q_r$, $e = [\epsilon^T, \dot{\epsilon}^T]^T$, and using expressions (2.2) and (2.3), we obtain the error differential equation as

$$\dot{e} = A_m e + B h(q, \dot{q}, u, r) \quad (2.4)$$

where $B = [0, I]^T$ and

$$h(q, \dot{q}, u, r) = M(q)^{-1} u + h_1(q, \dot{q}, r) \quad (2.5)$$

$$h_1(q, \dot{q}, r) = -Pq - Q\dot{q} - B_1 r + M(q)^{-1}(-F(q, \dot{q})\dot{q} - G(q)). \quad (2.6)$$

To design robust control system with the error convergence in a finite time, we define the following MIMO terminal switching plane variable vector

$$S = C\tilde{e} \quad (2.7)$$

where

$$C = [C_1 \ C_2] = \begin{pmatrix} c_{11} & & & 1 & & \\ & \ddots & & & \ddots & \\ & & & & & c_{nn} \\ & & & & & & 1 \end{pmatrix} \quad (2.8)$$

$$\tilde{e} = [\epsilon_1^{p_1} \cdots \epsilon_n^{p_n} \ \dot{\epsilon}_1 \cdots \dot{\epsilon}_n]^T. \quad (2.9)$$

Remark 2.1: In (2.9), $p = p_1/p_2$, where positive integers p_1 and p_2 are selected such that

$$p_2 = (2m + 1), \quad m = 1, 2, \dots \quad (2.10)$$

$$p_2 > p_1. \quad (2.11)$$

It is easy to shown that the selections of p_1 and p_2 in (2.10) and (2.11) can guarantee $0 < p < 1$ and the tracking error ϵ_i can then converge to zero on the terminal sliding mode in a finite time for all bounded initial conditions.

Vector \tilde{e} in expression (2.7) can also be written into the following form

$$\tilde{e} = e + \Delta\tilde{e} \quad (2.12)$$

where

$$\Delta\tilde{e} = [\epsilon_1^p - \epsilon_1, \dots, \epsilon_n^p - \epsilon_n, 0, \dots, 0]^T. \quad (2.13)$$

Remark 2.2: Using expressions (2.9), (2.12), and (2.13), the MIMO terminal switching plane variable vector in expression (2.7) can be written into the form

$$\begin{aligned} S &= C\tilde{e} \\ &= C(e + \Delta\tilde{e}) \\ &= Ce + C_1(\tilde{\epsilon} - \epsilon) \end{aligned} \quad (2.14)$$

where

$$\tilde{\epsilon} = [\epsilon_1^p \cdots \epsilon_n^p]^T. \quad (2.15)$$

It will be seen later that it is convenient to use expression (2.15) of switching plane variable vector S in controller design and stability analysis.

Remark 2.3: The i th element of S in expression (2.7) can be written into the following form

$$s_i = c_{ii}\epsilon_i^p + \dot{\epsilon}_i. \quad (2.16)$$

Similar to the conventional sliding mode control technique, if the controller is designed such that s_i ($i = 1, \dots, n$) converge to zero, then we say that the switching plane variables s_i ($i = 1, \dots, n$) reach the terminal sliding mode

$$c_{ii}\epsilon_i^p + \dot{\epsilon}_i = 0 \quad (i = 1, \dots, n). \quad (2.17)$$

It has been shown in Zak [12], [13] that $\epsilon_i = 0$ is the terminal attractor of the system (2.17). Let the initial value of ϵ_i at time $t = 0$ be $\epsilon_i(0)$ and parameter p be chosen as shown in Remark 2.1, then the relaxation time t_i for a solution of system (2.17) is given as

$$t_i = c_{ii}^{-1} \int_{\epsilon_i(0)}^{\epsilon_i^{-0}} \frac{d\epsilon_i}{\epsilon_i^p} = \frac{\epsilon_i(0)^{1-p}}{c_{ii}(1-p)}. \quad (2.18)$$

Expression (2.18) also means that, on the terminal sliding mode in (2.17), the output tracking error converges to zero in a finite time. The details on the terminal attractor and its applications can be found in [12] and [13].

Remark 2.4: For the simplified analysis, matrix C_2 in (2.8) is chosen as an unity matrix. Matrix C_2 , however, can be chosen as a different diagonal matrix for the improvement of the convergence of the error dynamics on the terminal sliding mode.

For further analysis, the following uncertain bounds for n -link rigid robotic manipulators in expression (2.1) are assumed to be known.

$$1) \lambda_{\min}(M(q)^{-1}) \geq a_1 \quad (2.19)$$

$$2) \|M(q)^{-1}\| \leq a_2 \quad (2.20)$$

$$3) \|F(q, \dot{q})\dot{q} + G(q)\| < b_1 + b_2\|q\| + b_3\|\dot{q}\|^2 \quad (2.21)$$

where a_1 , a_2 , b_1 , b_2 , and b_3 are positive numbers.

Remark 2.5: According to the characteristics of industrial robotic manipulators, the above assumptions are reasonable. Assumption A.2.1 has been used in Leung *et al.* [20], and Assumptions A.2.2) and A.2.3) have been used in Singh [8]. Also, an estimation method for the uncertain bound parameters in Assumptions A.2.1)–A.2.3) has been developed in Grimm [9] by neglecting some dynamics.

The objective of this paper is to design a robust tracking controller using the MIMO terminal sliding mode technique based on uncertain bounds in assumptions A.2.1)–A.2.3) instead of the upper and the lower bounds of all unknown system parameters so that, for any bounded uncertainties in parameters of robotic systems, the error dynamics can be driven into the terminal sliding mode in a finite time. On the terminal sliding mode, the error dynamics can then converge to zero in a finite time.

III. CONTROLLER DESIGN

Theorem 3.1: Consider the error dynamics in expression (2.4) with assumptions in (2.19)–(2.21). If the control input vector is designed such that

$$u = \begin{cases} -\frac{S}{a_1 \|S\|} w & \|S\| \neq 0 \\ 0 & \|S\| = 0 \end{cases} \quad (3.1)$$

where

$$w = \|Pq_r\| + \|Q\dot{q}_r\| + \|B_1 r\| + \|C_1 \epsilon_r\| + a_2(b_1 + b_2 \|q\| + b_3 \|\dot{q}\|^2) \quad (3.2)$$

$$\epsilon_r = \text{diag}(p\epsilon_1^{p-1}, \dots, p\epsilon_n^{p-1})\dot{\epsilon} \quad (3.3)$$

and

$$p = p_1/p_2 \quad \text{and} \quad p_2 > p_1 \geq (p_2 + 1)/2 \quad (\text{see Remark 3.3}) \quad (3.4)$$

then the output tracking error vector converges to zero in a finite time.

Proof: Consider the following Lyapunov function

$$V = \frac{1}{2} S^T S. \quad (3.5)$$

Differentiating V with respect to time, we have

$$\begin{aligned} \dot{V} &= S^T \dot{S} \\ &= S^T (C\dot{\epsilon} + C_1 \epsilon_r - C_1 \dot{\epsilon}) \\ &= S^T [CA_m e + CBh(q, \dot{q}u, r) + C_1 \epsilon_r - C_1 \dot{\epsilon}] \\ &= S^T [-Pq_r - Q\dot{q}_r + M(q)^{-1}u - B_1 r + C_1 \epsilon_r \\ &\quad - M(q)^{-1}(F(q, \dot{q})\dot{q} + G(q))]. \end{aligned} \quad (3.6)$$

Let

$$\frac{S^T M(q)^{-1} S}{a_1 \|S\|^2} = k_1(t) \quad (3.7)$$

and considering Assumption A.2.1), we have $k_1(t) \geq 1$.

Using control law (3.1), expression (3.6) can then be written as

$$\begin{aligned} \dot{V} &= -[k_1 \|S\| (\|Pq_r\| + \|Q\dot{q}_r\| + S^T (Pq_r + Q\dot{q}_r)) \\ &\quad - [k_1 \|S\| (\|B_1 r\| + \|C_1 \epsilon_r\|) + S^T (B_1 r - C_1 \epsilon_r)] \\ &\quad - [k_1 \|S\| a_2 (b_1 + b_2 \|q\| + b_3 \|\dot{q}\|^2) + S^T M(q)^{-1} (F\dot{q} + G)] \\ &\leq -[k_1 \|S\| a_2 (b_1 + b_2 \|q\| + b_3 \|\dot{q}\|^2) + S^T M(q)^{-1} (F\dot{q} + G)] \\ &= -k_1 \|S\| a_2 [(b_1 + b_2 \|q\| + b_3 \|\dot{q}\|^2) \\ &\quad + \frac{1}{k_1} \frac{S^T M(q)^{-1}}{\|S\|} (F\dot{q} + G)] \\ &= -k_1 a_2 \delta(t) \|S\| \end{aligned} \quad (3.8)$$

where

$$\begin{aligned} \delta(t) &= \left[(b_1 + b_2 \|q\| + b_3 \|\dot{q}\|^2) + \frac{1}{k_1} \frac{S^T M(q)^{-1}}{\|S\|} (F\dot{q} + G) \right] \\ &\geq \left[(b_1 + b_2 \|q\| + b_3 \|\dot{q}\|^2) - \frac{1}{k_1} \frac{\|M(q)^{-1}\|}{a_2} \|F\dot{q} + G\| \right] \\ &> (b_1 + b_2 \|q\| + b_3 \|\dot{q}\|^2) - \|F\dot{q} + G\| > 0. \end{aligned} \quad (3.9)$$

Then

$$\dot{V} < -k_1 a_2 \delta(t) \|S\| < 0 \quad \|S\| \neq 0. \quad (3.10)$$

Considering the fact that $\delta(t)$ is greater than a positive number by suitably selecting the bound parameters b_1 , b_2 , and b_3 , expression (3.10) means that switching plane variable vector S converges to zero in a finite time.

On the terminal sliding mode

$$C\dot{\epsilon} = 0 \quad (3.11)$$

the error dynamics satisfy (2.17), then the output tracking error converges to zero in a finite time.

Remark 3.1: It can be seen that, unlike the conventional linear sliding mode control schemes in [1]–[6], the output tracking error can converge to zero in a finite time by using the proposed control scheme due to the fact that the output tracking error can be driven into the terminal sliding mode in a finite time, and the error dynamics can then converge to zero in a finite time on the terminal sliding mode.

Remark 3.2: The proposed terminal sliding mode control scheme has strong robustness with respect to large parameter uncertainties because only five control parameters are adjusted in the controller, and the adjustable parameters depend only on the uncertain bounds in expressions (2.19)–(2.21).

Remark 3.3: On the terminal sliding mode in expression (2.15), the signal vector ϵ_r in expression (3.3) can be written as

$$\begin{aligned} \epsilon_r &= \text{diag}(p\epsilon_1^{p-1}, \dots, p\epsilon_n^{p-1})\dot{\epsilon} \\ &= \begin{bmatrix} p\epsilon_1^{p-1} & & \\ & \ddots & \\ & & p\epsilon_n^{p-1} \end{bmatrix} \begin{bmatrix} -c_{11}\epsilon_1^p \\ \vdots \\ -c_{nn}\epsilon_n^p \end{bmatrix} \\ &= [-c_{11}p\epsilon_1^{2p-1} \dots -c_{nn}p\epsilon_n^{2p-1}]^T. \end{aligned} \quad (3.12)$$

Expression (3.4) shows that, although mathematically the positive number p in (2.17) or (2.18) satisfies (2.10) and (2.11) to guarantee the terminal convergence of variable ϵ_r , the number p must satisfy (3.4) in this control scheme in the sense that the signal vector ϵ_r in (3.12) or (3.3) must be bounded as the output tracking error ϵ_i converge to zero on the terminal sliding mode.

Remark 3.4: In [19], a linear sliding mode control scheme using the same assumptions on uncertain bounds in A.2.1)–A.2.3) was developed. In [19, (7.16)], the term $\|C_1 \dot{\epsilon}\|$, on the terminal sliding mode, can be expressed as

$$\|C_1 \dot{\epsilon}\| = \left[\sum_{i=1}^n (c_{i1}^2 \epsilon_i^2) \right]^{1/2} \quad (3.13)$$

and the definition of the sliding mode parameter matrix C_1 can be found in (7.7) and (7.8) of [19], [19, (7.7), (7.8)]

Similarly, the term $\|C_1 \epsilon_r\|$ in expression (3.2), on the terminal sliding mode, can be expressed as

$$\|C_1 \epsilon_r\| = \left[\sum_{i=1}^n (c_{i1}^2 p \epsilon_i^{2p-1})^2 \right]^{1/2}. \quad (3.14)$$

When the sampling interval is nonzero, the output tracking errors will persist around the origin of the error space after the trajectories reach their vicinity, and therefore the ideal error convergence in both [19] and this paper cannot be obtained. If the tracking errors in these two control schemes are required to reach the vicinity of the origin at the same time, however, the linear sliding mode parameters c_{i1} in [19] must be chosen to satisfy the following relationship

$$c_{i1} \gg c_{ii} \quad (3.15)$$

and then the following inequality can often be satisfied

$$\|C_1 \dot{\epsilon}\| \gg \|C_1 \epsilon_r\|. \quad (3.16)$$

Therefore, by comparing expression (7.16) of [19] with (3.2) of this section, we can find that the control gain has been significantly reduced by using this scheme. This feature can also be seen from the simulations in following section and [19].

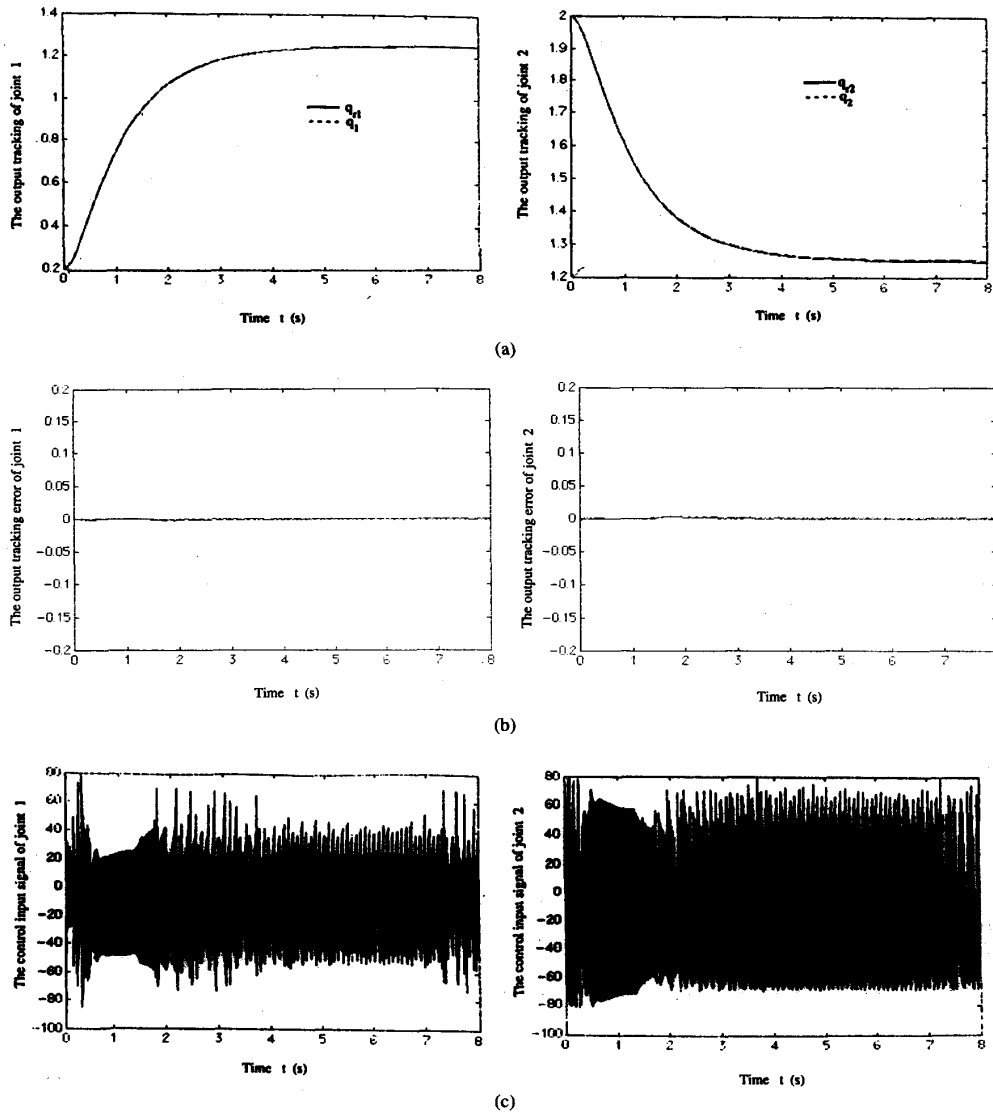


Fig. 3. (a) The output trackings of joint 1 and joint 2, (b) The output tracking errors of joint 1 and joint 2, (c) The control inputs of joint 1 and joint 2.

Remark 3.5: We have proved that the output tracking error can converge to zero in a finite time by the use of the proposed control scheme. The control law (3.1) is discontinuous across the sliding mode surfaces $S = 0$, however, which may excite undesired high frequency dynamics [4]. To eliminate the effects of the chattering, we use the following boundary layer control law in place of the discontinuous control law in expression (3.1)

$$u = \begin{cases} -\frac{S}{\alpha_1 \|S\|} w & \|S\| \geq \delta \\ -\frac{\delta}{\alpha_1 \delta} w & \|S\| < \delta \end{cases} \quad (3.17)$$

where $\delta > 0$.

By using the above boundary layer control law, we can guarantee the attractiveness of the boundary layer. For the region inside the boundaries, the ultimate boundedness of the error dynamics can be guaranteed to within any neighborhood of the boundary layer [3], [4].

IV. SIMULATION EXAMPLE

To illustrate the control schemes proposed in this paper, a simulation example for a two-link robotic manipulator is studied. The full

dynamic equations are given as [2]

$$\begin{bmatrix} \alpha_{11}(q_2) & \alpha_{12}(q_2) \\ \alpha_{12}(q_2) & \alpha_{22} \end{bmatrix} \begin{bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \end{bmatrix} = \begin{bmatrix} \beta_{12}(q_2)\dot{q}_1^2 + 2\beta_{12}(q_2)\dot{q}_1\dot{q}_2 \\ -\beta_{12}(q_2)\dot{q}_2^2 \end{bmatrix} + \begin{bmatrix} \gamma_1(q_1, q_2)g \\ \gamma_2(q_1, q_2)g \end{bmatrix} + \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

where

$$\begin{aligned} \alpha_{11}(q_2) &= (m_1 + m_2)r_1^2 + m_2r_2^2 + 2m_2r_1r_2 \cos(q_2) + J_1 \\ \alpha_{12}(q_2) &= m_2r_2^2 + m_2r_1r_2 \cos(q_2) \\ \alpha_{22} &= m_2r_2^2 + J_2 \\ \beta_{12}(q_2) &= m_2r_1r_2 \sin(q_2) \\ \gamma_1(q_1, q_2) &= -((m_1 + m_2)r_1 \cos(q_2) + m_2r_2 \cos(q_1 + q_2)) \\ \gamma_2(q_1, q_2) &= -m_2r_2 \cos(q_1 + q_2). \end{aligned}$$

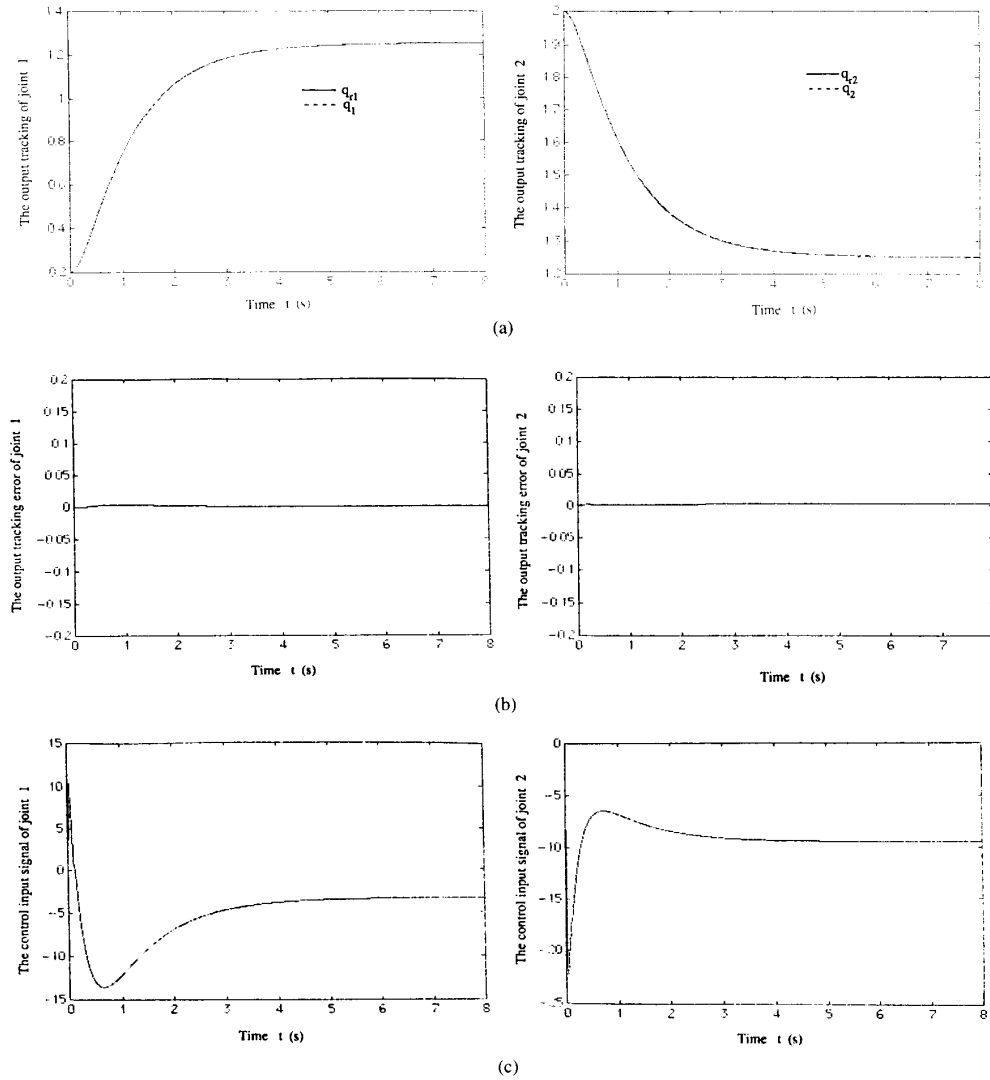


Fig. 4. (a) The output trackings of joint 1 and joint 2, (b) The output tracking errors of joint 1 and joint 2, (c) The control inputs of joint 1 and joint 2 using boundary layer controller.

The parameter values are

$$\begin{aligned} r_1 &= 1 \text{ m}, & r_2 &= 0.8 \text{ m} \\ J_1 &= 5 \text{ kg.m}, & J_2 &= 5 \text{ kg.m} \\ m_1 &= 0.5 \text{ kg}, & m_2 &= 1.5 \text{ kg}. \end{aligned}$$

A reference model for the manipulator to follow is given by

$$\dot{x}_m = A_m x_m + B_m r$$

where

$$x_m = [q_{r1} \ q_{r2} \ \dot{q}_{r1} \ \dot{q}_{r2}]^T$$

$$A_m = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -4 & 0 & -5 & 0 \\ 0 & -4 & 0 & -5 \end{bmatrix}, \quad B_m = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\text{and } r(t) = [5 \ 5]^T \text{ for } t > 0.$$

Since we are interested in trajectory tracking and hope that the transient response is determined entirely by the sliding motion, we consider a situation characterized by the same initial values of both the reference model state and the plant state. In this simulation, we pick up the initial values of $x(t)$ and $x_m(t)$ to be

$$x(0) = x_m(0) = [0.2 \ 2 \ 0 \ 0]^T.$$

The parameters of the uncertain bounds in (2.17)–(2.19) are chosen as

$$\begin{aligned} a_1 &= 0.1, & a_2 &= 2 \\ b_1 &= 2, & b_2 &= 1, & b_3 &= 2. \end{aligned}$$

Terminal sliding mode is prescribed as

$$\begin{aligned} \epsilon_1^{0.6} + \dot{\epsilon}_1 &= 0 \\ \epsilon_2^{0.6} + \dot{\epsilon}_2 &= 0. \end{aligned}$$

Fig. 3 shows the output trackings, tracking errors, and input torques by the use of the control law (3.1). It can be seen that the effects of

system uncertainties are eliminated, and good tracking performance is achieved. To eliminate chattering, we implement the boundary layer control law (3.17). Here we take $\delta = 0.01$. Good system performance is shown in Fig. 4. As can be seen from these figures, the chattering is eliminated.

V. CONCLUSIONS

In this paper, a robust control scheme for rigid robotic manipulators using the MIMO terminal sliding mode technique has been proposed. The main contributions of this paper are that an MIMO terminal sliding mode is defined, and a robust terminal sliding mode control scheme for n -link rigid robotic manipulators is developed with the result that the output tracking error can converge to zero in a finite time. In addition, the robot control systems using the proposed scheme have a strong robustness property not only because on the sliding mode, the error dynamics is insensitive to uncertain dynamics, but also because only three uncertain bounds based on the structure properties of rigid robotic manipulators are used in controller design. It has also been remarked that this scheme is more practical in the sense that the gain of the terminal sliding mode controller can be significantly reduced with respect to the ones of linear sliding mode control schemes developed in [19], where the sampling interval is nonzero. A few problems for the practical implementation of this scheme, however, have been noted. Like all other control techniques, the ideal error convergence cannot be obtained in practical control systems where sampling interval is nonzero. To implement this scheme, some nonlinear electronic hardware to deal with the nonlinear function e_i^p needs to be built. The advantage that the proposed terminal sliding mode controller has smaller control gain with respect to linear sliding mode controller, however, cannot be counteracted by the above factors. In this paper, the terminal sliding mode technique is used to control only the second order robotic systems. The research on design of high-order terminal sliding mode control systems, however, is under author's investigations based on [12] and [13].

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A Common Lyapunov Function for Stable LTI Systems with Commuting A-Matrices

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Abstract—The paper demonstrates that a common quadratic Lyapunov function exists for all linear systems of the form $\dot{x} = A_i x$, $i = 1, 2, \dots, N$, where the matrices A_i are asymptotically stable and commute pairwise. This in turn assures the exponential stability of a switching system $\dot{x}(t) = A(t)x(t)$ where $A(t)$ switches between the above constant matrices A_i .

I. INTRODUCTION

In recent years, the scope of control theory is being enlarged to include intelligent control systems. One of the main features of such intelligent control systems is the systematic application of the idea of switching between different controllers [1], [2]. One of the first questions to be resolved in this context is that of the stability of the overall system.

Many of the stability problems that arise in intelligent control systems can be addressed by considering the following basic problem:

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