The thermal history of the intergalactic medium down to redshift $z = 1.5$: a new curvature measurement

Elisa Boera,1* Michael T. Murphy,1 George D. Becker2 and James S. Bolton3

1Centre for Astrophysics and Supercomputing, Swinburne University of Technology, Hawthorn, Victoria 3122, Australia
2Kavli Institute for Cosmology and Institute of Astronomy, Madingley Road, Cambridge CB3 0HA, UK
3School of Physics and Astronomy, University of Nottingham, University Park, Nottingham NG7 2RD, UK

Accepted 2014 March 30. Received 2014 March 29; in original form 2014 February 16

ABSTRACT

According to the photoheating model of the intergalactic medium (IGM), He II reionization is expected to affect its thermal evolution. Evidence for additional energy injection into the IGM has been found at $3 \lesssim z \lesssim 4$, though the evidence for the subsequent fall-off below $z \sim 2.8$ is weaker and depends on the slope of the temperature–density relation, $\gamma$. Here we present, for the first time, an extension of the IGM temperature measurements down to the atmospheric cut-off of the H I Lyman-$\alpha$ (Ly$\alpha$) forest at $z \sim 1.5$. Applying the curvature method on a sample of 60 Ultraviolet and Visual Echelle Spectrograph (UVES) spectra we investigated the thermal history of the IGM at $z < 3$ with precision comparable to the higher redshift results. We find that the temperature of the cosmic gas traced by the Ly$\alpha$ forest [$T(\hat{\Delta})$] increases for increasing overdensity from $T(\hat{\Delta}) \sim 22670$ to 33740 K in the redshift range $z \sim 2.8$–1.6. Under the assumption of two reasonable values for $\gamma$, the temperature at the mean density ($T_0$) shows a tendency to flatten at $z \lesssim 2.8$. In the case of $\gamma \sim 1.5$, our results are consistent with previous ones which indicate a falling $T_0$ for redshifts $z \lesssim 2.8$. Finally, our $T(\hat{\Delta})$ values show reasonable agreement with moderate blazar heating models.

Key words: intergalactic medium – quasars: absorption lines – cosmology: observations.

1 INTRODUCTION

Starting from a very hot plasma made of electrons and protons after the big bang, to the gas that now fills the space between galaxies, the intergalactic medium (IGM) has been one of the main ‘recorders’ of the different phases of evolution of the Universe. Changes in the thermodynamic state and chemical composition of this gas reflect the conditions for the formation and the evolution of the structures that we can observe today. In particular, the IGM thermal history can be an important source of information about reionizing processes that injected vast amounts of energy into this gas on relatively short time-scales. For this reason considerable efforts have been made to find any ‘footprint’ of either H I ($z < 6$) or He II ($z < 3$) reionization. Because the ionization potential of He II (from He II to He III) is 54.4 eV and fully ionized helium recombines more than five times faster than ionized hydrogen, this second reionization event should have begun later, after the reionization of hydrogen and H I ($11 \lesssim z \lesssim 6$; Becker et al. 2001; Fan, Carilli & Keating 2006; Larson et al. 2011) when quasars (QSOs) started to dominate the ultraviolet background (UVB; Miralda-Escudé, Haehnelt & Rees 2000). Theoretically, the much harder photons from QSOs would have been able to fully ionize He II around redshifts $3 \lesssim z \lesssim 4.5$ but these estimates change depending on assumptions about the abundance of QSOs and the hardness of their spectra (Meiksin 2005). While the direct observation, through the detection of the ‘Gunn–Peterson effect’, recently suggests the end of the He II reionization at $z \sim 2.7$ (e.g. Shull et al. 2010; Worseck et al. 2011; Syphers & Shull 2013, 2014), any current constraint on the physics of this phenomenon is limited by the cosmic variance among the small sample of ‘clean’ lines of sight, those along which the H I Lyman-$\alpha$ (Ly$\alpha$) transition is not blocked by higher redshift H I Lyman limit absorption. For this reason indirect methods have been developed to obtain a detailed characterization of the He II reionization.

It is predicted that the IGM should be heated by photoionization heating during He II reionization and, because its cooling time is long, the low density gas retains some useful memory of when and how it was reionized. In fact, at the mean density of the IGM the characteristic signature of reionization is expected to be at peak: a gradual heating followed by cooling due to adiabatic expansion (e.g. McQuinn et al. 2009). In the last decade, the search for this feature and the study of the thermal history of the IGM as a function of redshift have been the objectives of different efforts, not only to verify the theoretical prediction and constrain the timing of He II reionization, but also to obtain information on the nature of the ionizing sources and on the physics of the related ionizing mechanisms. To obtain measurements of the temperature of the IGM, studying the absorption features of the H I Ly$\alpha$ forest has proven to
be a useful method so far. The widths of these lines are sensitive to thermal broadening but are also affected by Hubble broadening and peculiar velocities. Cosmological simulations are therefore required to characterize the large-scale structure and bulk motion of the IGM (Meiksin, Tittley & Brown 2010), before the gas temperature can be determined.

Previous efforts to extract information on the thermal state of the cosmic gas from the Lyα forest can be divided into two main approaches: the study of individual absorption features and the quantification of the absorption structures with a global statistical analysis of the entire forest. The first method consists of decomposing the spectra into a set of Voigt profiles. Schaye et al. (2000) found evidence using this technique for an increase in the IGM temperature consistent with He II reionization at $z \approx 3$. In contrast, McDonald et al. (2001) found a constant temperature over $z \sim 2–4$. A characterization of the flux probability distribution function (PDF) based on pixel statistics has also been used to analyse the forest and extract information from the comparison with theoretical models (Bolton et al. 2008; Calura et al. 2012). However, the PDF is sensitive to a range of systematic effects, including the placement of the unabsorbed continuum. A further approach is to use wavelet analysis to characterize the Lyα line widths distribution in terms of discrete wavelets. Theuns et al. (2002) and Lidz et al. (2010) found evidence using this technique for He II reionization completing near $z \approx 3.4$, but with large statistical uncertainties. In the recent work of Garzilli et al. (2012), the PDF and the wavelet decomposition methods have been compared and tested on Lyα spectra at low redshift. While the results are in formal agreement with previous measurements, the uncertainties are still large and there is a mild tension between the two analyses.

Recently, Becker et al. (2011) developed a statistical approach based on the flux curvature. This work constrained the temperature over $2 \lesssim z \lesssim 4.8$ of a ‘optimal’ or ‘characteristic’ overdensity, which evolves with redshift. The error bars were considerably reduced compared to previous studies, partially at the expense of determining the temperature at a single density only, rather than attempting to constrain the temperature–density relation. Some evidence was found for a gradual reheating of the IGM over $3 \lesssim z \lesssim 4.5$ with no clear evidence for a temperature peak. Given these uncertainties, the mark of the He II reionization still needs a clear confirmation. Nevertheless, the curvature method is promising because it is relatively robust to continuum placement errors: the curvature of the flux is sensitive to the shape of the absorption lines and not strongly dependent on the flux normalization. Furthermore, because it incorporates the temperature information from the entire Lyα forest, this statistic has the advantage of using more of the available information, as opposed to the line-fitting method which relies on selecting lines that are dominated by thermal broadening.

An injection of substantial amounts of thermal energy is predicted to result in both an increase in the IGM temperature and a change in the temperature–density ($T$–$\rho$) relation. The detailed study of this process has to take into consideration the effects of the IGM inhomogeneities driven by the diffusion and percolation of the ionized bubbles around single sources, and currently constitutes an important object of investigation through hydrodynamical simulations (e.g. Compostella, Cantalupo & Porciani 2013). In the simplest scenario, for gas at overdensities $\Delta \lesssim 10 (\Delta = \rho/\bar{\rho})$, where $\bar{\rho}$ is the mean density of the IGM, the temperature is related to the density with a power-law relation of the form

$$T(\Delta) = T_0 \Delta^{\gamma-1},$$  

where $T_0$ is the temperature at the mean density (Hui & Gnedin 1997; Valageas, Schaeffer & Silk 2002). The evolution of the parameters $T_0$ and $\gamma$ as a function of redshift then describes the thermal history of the IGM. A balance between photoheating and cooling due to adiabatic expansion of the Universe will asymptotically produce a power law with $\gamma = 1.6$ (Hui & Gnedin 1997). During the reionization the slope is expected to flatten temporarily before evolving back to the asymptotic value. Possible evidence for this flattening at $z \approx 3$, seems to be consistent with He II reionization occurring around this time (e.g. Ricotti, Gnedin & Shull 2000; Schaye et al. 2000).

Some analyses of the flux PDF have indicated that the $T$–$\rho$ relation may even become inverted (e.g. Becker, Rauch & Sargent 2007; Bolton et al. 2008; Viel, Bolton & Haehnelt 2009; Calura et al. 2012; Garzilli et al. 2012). However, the observational uncertainties in this measurement are considerable (see discussion in Bolton et al. 2014). A possible explanation was suggested by considering radiative transfer effects (Bolton et al. 2008). Although it appears difficult to produce this result considering only He II photoheating by QSOs (Bolton, Oh & Furlanetto 2009; McQuinn et al. 2009), a new idea of volumetric heating from blazar TeV emission predicts an inverted temperature–density relation at low redshift and at low densities. According to these models, heating by blazar $\gamma$-ray emission would start to dominate at $z \approx 3$, obscuring the ‘footprint’ of He II reionization (Chang, Broderick & Pfister 2012; Puchwein et al. 2012). Even if in the most recent analysis, with the line-fitting method (Rudie et al. 2013; Bolton et al. 2014), the inversion in the temperature–density relation has not been confirmed, a general lack of knowledge about the behaviour of the $T$–$\rho$ relation at low redshift ($z < 3$) still emerges, accompanied with no clear evidence for the He II reionization peak. A further investigation of the temperature evolution in this redshift’s regime therefore assumes some importance in order to find constraints for the physics of the He II reionization and the temperature–density relation of the IGM.

The purpose of this work is to apply the curvature method to obtain new, robust temperature measurements at redshift $z < 3$, extending the previous results, for the first time, down to the optical limit for the Lyα forest at $z \approx 1.5$. By pushing the measurement down to such a low redshift, we attempt to better constrain the thermal history in this regime, comparing the results with the theoretical predictions for the different heating processes. We infer temperature measurements by computing the curvature on a new set of QSO spectra at high resolution obtained from the archive of the Ultraviolet and Visual Echelle Spectrograph (UVES) on the Very Large Telescope (VLT). Synthetic spectra, obtained from hydrodynamical simulations used in the analysis of Becker et al. (2011) and extended down to the new redshift regime are used for the comparison with the observational data. Similar to Becker et al. (2011), we constrain the temperature of the IGM at a characteristic overdensity $\Delta$, traced by the Lyα forest, which evolves with redshift. We do not attempt to constrain the $T$–$\rho$ relation, but we use fiducial values of the parameter $\gamma$ in equation (1) to present results for the temperature at the mean density, $T_0$.

This paper is organized as follows. In Section 2 we present the observational data sample obtained from the VLT archive, while the simulations used to interpret the measurements are introduced in Section 3. In Section 4 the curvature method and our analysis procedure are summarized. In Section 5 we present the data analysis and we discuss the strategies applied to reduce the systematic uncertainties. The calibration and the analysis of the simulations is described in Section 6. The results are presented in Section 7 for the temperature at the characteristic overdensities and the
temperature at the mean density for different values of $\gamma$. We discuss the comparison with theoretical models in Section 8, and conclude in Section 9.

2 THE OBSERVATIONAL DATA

In this work we used a sample of 60 QSO spectra uniformly selected on the basis of redshift, wavelength coverage and signal-to-noise ratio (S/N) in order to obtain robust results in the UV and optical parts (3100–4870 Å) of the spectrum where the Ly$\alpha$ transition falls for redshifts $1.5 < z < 3$. The QSOs and their basic properties are listed in Table 1. The spectra were retrieved from the archive of the UVES on the VLT. In general, most spectra were observed with a slit width $\lesssim$1.0 arcsec wide and on-chip binning of $2 \times 2$, which provides a resolving power of $R \approx 50,000$ (full width at half-maximum, FWHM, $\approx 7$ km s$^{-1}$); this is more than enough to resolve typical Ly$\alpha$ forest lines, which generally have FWHM $\gtrsim 15$ km s$^{-1}$. The archival QSO exposures were reduced using the European Southern Observatory (ESO) UVES Common Pipeline Language software. This suite of standard routines was used to optimally extract the echelle orders. The custom-written program UVES_POPLER was then used to combine the many archival QSO exposures were reduced using the European Southern Observatory (ESO) UVES Common Pipeline Language software. This suite of standard routines was used to optimally extract and wavelength-calibrate individual echelle orders. The custom-written program UVES_POPLER was then used to combine the many QSO spectra into a single normalized spectrum on a vacuum-heliocentric wavelength scale. For most QSOs, the orders were redispersed on to a common wavelength scale with a dispersion of 2.5 km s$^{-1}$ per pixel; for four bright (and high S/N), $z \lesssim 2$ QSOs the dispersion was set to 1.5 km s$^{-1}$ per pixel. The orders were then scaled to optimally match each other and then co-added with inverse-variance weighting using a sigma-clipping algorithm to reject `cosmic rays' and other spectral artefacts.

To ensure a minimum threshold of spectral quality and a reproducible sample definition, we imposed an ‘S/N’ lower limit of 24 per pixel for selecting which QSOs and which spectral sections we used to derive the IGM temperature. A high S/N is, in fact, extremely important for the curvature statistic which is sensitive to the variation of the shapes of the Ly$\alpha$ lines: in low S/N spectra this statistic will be dominated by noise and, furthermore, by narrow metal lines that are difficult to identify and mask.

The ‘S/N’ cut-off of 24 per pixel was determined by using the hydrodynamical Ly$\alpha$ forest simulations discussed in Section 3. By adding varying amounts of Gaussian noise to the simulated forest spectra and performing a preliminary curvature analysis like that described in Sections 4 and 5, the typical uncertainty on the IGM temperature could be determined, plus the extent of any systematic biases caused by low S/N. It was found that a competitive statistical uncertainty of $\approx 10$ per cent in the temperature could be achieved with the cut-off in ‘S/N’ set to 24 per pixel, and that this was well above the level at which systematic biases become significant. However, in order for us to make the most direct comparison with these simulations, we have to carefully define ‘S/N’. In fact for the Ly$\alpha$ forest the S/N fluctuates strongly and so it is not very well defined. Therefore, the continuum-to-noise ratio, C/N, is the best means of comparison with the simulations. To measure this from each spectrum, we had first to establish a reasonable continuum.

The continuum fitting is a crucial aspect in the QSO spectral analysis and for this reason we applied to all the data a standard procedure in order to avoid systematic uncertainties due to the continuum choice. We used the continuum-fitting routines of UVES_POPLER to

Table 1. List of the QSOs used for this analysis. For each object we report the name (column 1) based on the J2000 coordinates of the QSO and the emission redshift (column 2). The redshift intervals associated with the Ly$\alpha$ absorption are also reported with the corresponding C/N level per pixel (columns 3, 4 and 5). The sections of Ly$\alpha$ forest obtained from this sample were required to have a minimum C/N of 24 per pixel. The note below the table provides the ESO Programme IDs that contributed to the spectra.

<table>
<thead>
<tr>
<th>QSO</th>
<th>$z_{\text{em}}$</th>
<th>$z_{\text{start}}$</th>
<th>$z_{\text{end}}$</th>
<th>C/N</th>
</tr>
</thead>
<tbody>
<tr>
<td>J092913-021446</td>
<td>1.6824</td>
<td>1.263</td>
<td>1.6824</td>
<td>5-29</td>
</tr>
<tr>
<td>J051707-441055</td>
<td>1.71</td>
<td>1.286</td>
<td>1.71</td>
<td>10-54</td>
</tr>
<tr>
<td>J014333-391700</td>
<td>1.807</td>
<td>1.368</td>
<td>1.807</td>
<td>13-69</td>
</tr>
<tr>
<td>J222756-224302</td>
<td>1.891</td>
<td>1.439</td>
<td>1.891</td>
<td>15-73</td>
</tr>
<tr>
<td>J013857-255447</td>
<td>1.893</td>
<td>1.440</td>
<td>1.893</td>
<td>16-52</td>
</tr>
<tr>
<td>J113015-213446</td>
<td>1.9</td>
<td>1.446</td>
<td>1.9</td>
<td>2-35</td>
</tr>
<tr>
<td>J005824+040113</td>
<td>1.92</td>
<td>1.463</td>
<td>1.92</td>
<td>10-43</td>
</tr>
<tr>
<td>J034037-485523</td>
<td>1.94</td>
<td>1.480</td>
<td>1.94</td>
<td>2-35</td>
</tr>
</tbody>
</table>

1 UVES_POPLER was written and is maintained by M. T. Murphy and is available at http://astronomy.swin.edu.au/mmurphy/UVES_popler
Table 1 – continued

<table>
<thead>
<tr>
<th>QSO</th>
<th>(z_{\text{em}})</th>
<th>(z_{\text{Ly}\alpha})</th>
<th>C/N</th>
</tr>
</thead>
<tbody>
<tr>
<td>J132029−052335</td>
<td>3.70</td>
<td>2.965</td>
<td>3.70</td>
</tr>
<tr>
<td>J162116−004250</td>
<td>3.7027</td>
<td>2.967</td>
<td>3.7027</td>
</tr>
<tr>
<td>J014049−083942</td>
<td>3.7129</td>
<td>2.976</td>
<td>3.7129</td>
</tr>
</tbody>
</table>


To determine the final continuum for all our QSO spectra, we iteratively fitted a fifth-order Chebyshev polynomial to overlapping 10 000 km s\(^{-1}\) sections of spectra between the Ly\(\alpha\) and Lyman \(\beta\) (Ly\(\beta\)) emission lines of the QSO. The initial fit in each section began by rejecting the lowest 50 per cent of pixels. In subsequent iterations, pixels with fluxes \(\geq 3\sigma\) above and \(\geq 1\sigma\) below the fit were excluded from the next iteration. The iterations continued until the ‘surviving’ pixels remained the same in two consecutive iterations. The overlap between neighbouring sections was 50 per cent and, after all iterations were complete, the final continuum was formed by combining the individual continua of neighbouring sections with a weighting which diminished linearly from unity at their centres to zero at their edges. After this initial treatment of all spectra we applied further small changes to the fitting parameters after visually inspecting the results. In most cases, we reduced the spectral section size, the threshold for rejecting pixels below the fit at each iteration, and the percentage of pixels rejected at the first iteration to values as low as 6000 km s\(^{-1}\), 0.8\(\sigma\) and 40 per cent, respectively. In Fig. 1 we show examples of continuum fits for Ly\(\alpha\) forest regions at different redshifts obtained with this method. This approach allowed us to avoid cases where the fitted continuum obviously dipped inappropriately below the real continuum, but still defined our sample with specific sets of continuum parameters without any further manual intervention, allowing a reproducible selection of the appropriate sample for this analysis. Furthermore, as described in Section 5, to avoid any systematics due to the large-scale continuum placement, we re-normalized each section of spectra that contributed to our results.

The redshift distribution of the Ly\(\alpha\) forest (\(z_{\text{Ly}\alpha}\)) of the QSOs in our selected sample is shown in Fig. 2, where their distribution of C/N in the same region is also reported. Due to instrumental sensitivity limits in the observation of bluest part of the optical Ly\(\alpha\) forest, it is more difficult to collect data with high C/N for \(z_{\text{Ly}\alpha} < 1.7\). The general lower quality of these data and the lower number of QSOs contributing to this redshift region will be reflected in the results, causing larger uncertainties.

3 THE SIMULATIONS

To interpret our observational results and extract temperature constraints from the analysis of the Ly\(\alpha\) forest, we used synthetic spectra, derived from hydrodynamical simulation and accurately calibrated to match the real data conditions. We performed a set

Figure 1. Examples of continuum fit (green solid line) in Ly\(\alpha\) regions for the QSO J112442−170517 with \(z_{\text{em}} = 2.40\) (top panel) and J051707−441055 at \(z_{\text{em}} = 1.71\) (bottom panel). The continuum-fitting procedure used is described in the text.

Figure 2. Histograms showing our sample of QSO spectra with C/N > 24 in the Ly\(\alpha\) forest region. Top panel: redshift distribution referred to the Ly\(\alpha\) forest. Bottom panel: distribution of the C/N in the forest region. The redshift bins of the histograms have been chosen for convenience of \(\Delta z = 0.025\). The vertical lines divide the histograms in the redshift bins width of \(\Delta z = 0.2\) in which the measurements (at \(z \lesssim 3.1\)) will be collected in the following analysis.
of hydrodynamical simulations that span a large range of thermal histories, based on the models of Becker et al. (2011) and extended to lower redshifts (z < 1.8) to cover the redshift range of our QSO spectra. The simulations were obtained with the parallel smoothed particle hydrodynamics code GADGET-3 that is the updated version of GADGET-2 (Springel 2005) with initial conditions constructed using the transfer function of Eisenstein & Hu (1999) and adopting the cosmological parameters \( \Omega_m = 0.26, \Omega_\Lambda = 0.74, \Omega_b h^2 = 0.023, \)
\( h = 0.72, \sigma_8 = 0.80, n_s = 0.96, \) according to the cosmic microwave background constraints of Reichardt et al. (2009) and Jarosik et al. (2011). The helium fraction by mass of the IGM is assumed to be \( Y = 0.24 \) (Olive & Skillman 2004). Because the bulk of the Ly\( \alpha \) absorption corresponds to overdensities \( \Delta = \rho/\bar{\rho} \lesssim 10 \), our analysis will not be affected by the star formation prescription, established only for gas particles with overdensities \( \Delta > 10^3 \) and temperature \( T \lesssim 10^4 \) K.

Starting at \( z = 99 \) the simulations describe the evolution of both dark matter and gas using \( 2 \times 512^3 \) particles with a gas particle mass of \( 9.2 \times 10^7 \) M\(_\odot\) in a periodic box of 10 comoving \( h^{-1} \) Mpc. Instantaneous hydrogen reionization is fixed at \( z = 9 \). From the one set of initial conditions, many simulations are run, all with gas that is assumed to be in the optically thin limit and in ionization equilibrium with a spatially uniform UVB from Haardt & Madau (2001). However, the photoheating rates, and so the corresponding values of the parameters \( T_0 \) and \( \gamma \) of equation (1), were changed between simulations. In particular, the photoheating rates from Haardt & Madau (2001) \( (\epsilon_i^{\text{HM01}}) \) for the different species \( (i = [\text{H}\, I, \text{He}\, I, \text{He}\, II]) \) have been re-scaled using the relation \( \epsilon_i = \zeta \Delta^3 \epsilon_i^{\text{HM01}} \), where \( \epsilon_i \) are the adopted photoheating rates and \( \zeta \) and \( \Delta \) are constants that change depending on the thermal history assumed. Possible bimodality in the temperature distribution at fixed gas density, observed in the simulations of Compostella et al. (2013) in the early phases of the He\( \, II \) reionization, has not been taken into consideration in our models. We assume, in fact, that the final stages of He\( \, II \) reionization at \( z \lesssim 3 \), when the IGM is almost completely reionized, can be described in a good approximation by a single temperature–density relation and are not affected anymore by the geometry of the diffusion of ionized bubbles.

Our models do not include galactic winds or possible outflows from AGN. However, these are expected to occupy only a small proportion of the volume probed by the synthetic spectra and so they are unlikely to have an important effect on the properties of the Ly\( \alpha \) forest (see e.g. Bolton et al. 2008 and also Theuns et al. 2002 for a discussion in the context of the PDF of the Ly\( \alpha \) forest transmitted fraction where this has been tested).

A summary of the simulations used in this work is reported in Table 2. We used different simulation snapshots that covered the redshift range of our QSO spectra (1.5 \( \lesssim z \lesssim 3 \)) and, to produce synthetic spectra of the Ly\( \alpha \) forest, 1024 randomly chosen ‘lines of sight’ through the simulations were selected at each redshift. To match the observational data, we needed to calibrate the synthetic spectra with our instrumental resolution, with the same H\( \, I \) Ly\( \alpha \) effective optical depth and the noise level obtained from the analysis of the real spectra (see Section 6.1).

### 4 THE CURVATURE METHOD

The definition of curvature \( (\kappa) \), as used by Becker et al. (2011), is the following:

\[
\kappa = \frac{F''}{[1 + (F')^2]^{1/2}}. \tag{2}
\]

with the first and second derivatives of the flux \( (F, F') \) taken with respect to wavelength or relative velocity. The advantage of this statistic is that, as demonstrated in Becker et al. (2011), it is quite sensitive to the IGM temperature but does not require the forest to be decomposed into individual lines. In this way the systematic errors are minimized if this analysis is applied to high resolution and high S/N spectra. Its calculation is relatively simple and can be computed using a single b-spline fit directly to large regions of forest spectra. This statistic incorporates the temperature information from all lines, using more of the available information, as opposed to line fitting which relies on selecting lines that are dominated by thermal broadening. If calibrated and interpreted using synthetic spectra, obtained from cosmological simulations, the curvature represents a powerful tool to measure the temperature of the IGM gas, \( T(\Delta) \), at the characteristic overdensities \( (\Delta) \) of the Ly\( \alpha \) forest at different redshifts.

However, at low redshifts (\( z \lesssim 3 \)), the IGM tends to show characteristic overdensities \( (\Delta) \) much higher than the mean density \( (\bar{\rho}) \). Estimating the temperature at the mean density of the IGM \( (T_0) \) is then not straightforward. In fact, in the approximation of a gas collected into non-overlapping clumps of uniform density that have the same extent in redshift space as they have in real space, the Ly\( \alpha \) optical depth at a given overdensity \( (\Delta) \) will scale as

\[
\tau(\Delta) \propto (1 + z)^4 \Gamma^{-1} T_{0}^{-0.7} \Delta^{-0.9(1-\gamma)}. \tag{3}
\]

where \( \Gamma \) is the H\( \, I \) photoionization rate and \( T_0 \) and \( \gamma \) are the parameters that describe the thermal state of the IGM at redshift \( z \) in equation (1) (Weinberg et al. 1997). In general, if we assume that the forest will be sensitive to overdensities that produce a Ly\( \alpha \) optical depth \( \tau(\Delta) \simeq 1 \), it is then clear from equation (3) that these characteristic overdensities will vary depending on the redshift. At high redshift the forest will trace gas near the mean density while in the redshift range of interest here \( (z \lesssim 3) \) the absorption will be coming from densities increasingly above the mean. As a consequence, the translation of the \( T(\Delta) \) measurements at the characteristic overdensities into the temperature at the mean density becomes increasingly dependent on the value of the slope of the temperature–density relation (equation 1).
constrained parameter \( \gamma \), a degeneracy is introduced between \( \gamma \) and the final results for \( T_0 \) that can be overcome only with a more precise measurement of the \( T - \rho \) relation.

In this work we do not attempt to constrain the full \( T - \rho \) relation because that would require a simultaneous estimation of both \( T_0 \) and \( \gamma \). Instead, following consistently the steps of the previous analysis of Becker et al. (2011), we establish empirically the characteristic overdensities \(( \Delta )\) and obtain the corresponding temperatures from the curvature measurements. We define the characteristic overdensity traced by the Ly\( \alpha \) forest for each redshift as that overdensity at which \( T(\Delta) \) is a one-to-one function of the mean absolute curvature, regardless of \( \gamma \) (see Section 6). We then recover the temperature at the mean density \(( T_0 \) from the temperature \( T(\Delta) \) using equation (1) with a range of values of \( \gamma \) (see Section 7).

We can summarize our analysis in three main steps as follows.

(i) Data analysis (Section 5): from the selected sample of Ly\( \alpha \) forest spectra we compute the curvature \(( \kappa \) in the range of \( z \approx 1.5 - 3 \). From the observational spectra we also obtain measurements of the effective optical depth that we use to calibrate the simulations. We obtain the curvature measurements from the synthetic spectra following the same procedure that we used for the real data and we determine the characteristic overdensities \(( \Delta ) \) empirically, finding for each redshift the overdensity at which \( T(\Delta) \) is a one-to-one function of \( \log(\langle |\kappa| \rangle) \) regardless of \( \gamma \).

(ii) Simulations analysis (Section 6): we calibrate the simulation snapshots at different redshifts in order to match the observational data. We obtain the curvature measurements from the synthetic spectra following the same procedure that we used for the real data and we determine the characteristic overdensities \(( \Delta ) \) empirically, finding for each redshift the overdensity at which \( T(\Delta) \) is a one-to-one function of \( \log(\langle |\kappa| \rangle) \) regardless of \( \gamma \).

(iii) Final temperature measurements (Section 7): we determine the \( T(\Delta) \) corresponding to the observed curvature measurements by interpolating the \( T(\Delta) \)–log(\( \langle |\kappa| \rangle \)) relationship in the simulations to the values of \( \log(\langle |\kappa| \rangle) \) from the observational data.

5 DATA ANALYSIS

To directly match the box size of the simulated spectra, we compute the curvature statistic on sections of \( 10 h^{-1} \) Mpc (comoving distance) of ‘metal free’ Ly\( \alpha \) forest regions in our QSO spectra. Metals lines are, in fact, a potentially serious source of systematic errors in any measure of the absorption features of the Ly\( \alpha \) forest. These lines tend to show individual components significantly narrower than the Ly\( \alpha \) one (\( b \lesssim 15 \text{ km s}^{-1} \)) and, if included in the calculation, the curvature measurements will be biased towards high values. As a consequence, the temperature obtained will be much lower. For these reasons we need to ‘clean’ our spectra by adopting a comprehensive metal masking procedure (see Section 5.2).

However, not only metals can affect our analysis and, even if effectively masked from contaminant lines, the direct calculation of the curvature on observed spectra can be affected by other sources of uncertainties, particularly, noise and continuum errors. To be as much as possible consistent with the previous work of Becker et al. (2011) we adopted the same strategies to reduce these potential systematic errors.

Noise. If applied directly to high resolution and high S/N spectra, the curvature measurements will be dominated by the noise in the flux spectra. To avoid this problem we fit a cubic \( b \)-spline to the flux and we then compute the curvature from the fit. In Fig. 3, top panel, is shown a section of normalized Ly\( \alpha \) spectra in which the solid green line is the \( b \)-spline fit from which we obtain the curvature. For consistency, we adopt the same specifics of the fitting routine of the previous work of Becker et al. (2011). We then use an adaptive fit with break points that are iteratively added, from an initial separation of \( 50 \text{ km s}^{-1} \), where the fit is poor. The iterations proceed until the spacing between break points reach a minimum value or the fit converges. With this technique we are able to reduce the sensitivity of the curvature to the amount of noise in the spectrum as we can test using the simulations (see Section 6.2).

Continuum. Equation (2) shows a dependence of the curvature on the amplitude of the flux, which in turn is dependent on the accuracy with which the unabsorbed QSO continuum can be estimated. The difficulty in determining the correct continuum level in the Ly\( \alpha \) region can then constitute a source of uncertainties. To circumvent this issue we ‘re-normalized’ each \( 10 h^{-1} \) Mpc section of data, dividing the flux of each section (already normalized by the longer range fit of the continuum) by the maximum value of the \( b \)-spline fit in that interval. Computing the curvature from the re-normalized flux, we remove a potential systematic error due to inconsistent placement of the continuum. While this error could be important at high redshifts, where the Ly\( \alpha \) forest is denser, at \( z \lesssim 3 \) we do not expect a large correction. In Fig. 3, bottom panel, is shown the value of the curvature computed from the \( b \)-spline fit of the re-normalized flux (applying equation 2) for a section of forest.

We next measure the mean absolute curvature \( \langle |\kappa| \rangle \) for the ‘valid’ pixels of each section. We consider valid all the pixels where the re-normalized \( b \)-spline fit \( (F_b) \) falls in the range \( 0.1 \leq F_b \leq 0.9 \). In this way we exclude both the saturated pixels, that do not contain any useful information, and the pixels with flux near the continuum. This upper limit is in fact adopted because the flux profile tends to be flatter near the continuum and, as consequence the curvature for these pixels is considerably more uncertain. This potential uncertainty is particularly important at low redshift because increasing the mean flux also increases the number of pixels near the continuum (Faucher-Giguère et al. 2008).

5.1 Observed curvature and re-normalized optical depth

The final results for the curvature measurements from the real QSO spectra are shown in the green data points in Fig. 4. In this plot the
values of $\langle|\kappa|\rangle$ obtained from all the $10\,h^{-1}\text{ Mpc}$ sections of forest have been collected and averaged in redshift bins of $\Delta z = 0.2$. The error bars show the $1\sigma$ uncertainty obtained with a bootstrap technique generated directly from the curvature measurement within each bin. In all the redshift bins, in fact, the mean absolute curvature values of a large number of sections ($N > 100$) have been averaged and so the bootstrap can be considered an effective tool to recover the uncertainties. It is important to note that the smaller number of sections contributing to the lowest redshift bin ($1.5 \leq z \leq 1.7$; see Fig. 2), is reflected in a larger error bar. For comparison are shown the results of the curvature from Becker et al. (2011; black triangles) for redshift bins of $\Delta z = 0.4$. In the common redshift range the results seem to be in good agreement even if our values appear shifted slightly towards higher curvatures. Taking into consideration the fact that each point cannot be considered independent from the neighbours, this shift between the results from the two different data samples is not unexpected and may also reflect differences in the S/N between the samples. At this stage we do not identify any obvious strong departure from a smooth trend in $\langle|\kappa|\rangle$ as a function of redshift.

From each section we also extract the mean re-normalized flux ($F_R$) that we use to estimate the re-normalized effective optical depth ($\tau_{\text{eff}}^R = -\ln(F_R)$) needed for the calibration of the simulations (see Section 6.1). In Fig. 5 we plot the $\tau_{\text{eff}}^R$ obtained in this work for redshift bins of $\Delta z = 0.2$ (green data points) compared with the results of Becker et al. (2011) for bins of $\Delta z = 0.4$ (black triangles). Vertical error bars are $1\sigma$ bootstrap uncertainties for our points and $2\sigma$ for Becker et al. (2011). For simplicity we fitted our data with a unique power law ($\tau = A(1 + z)^\alpha$) because we do not expect that a possible small variation of the slope as a function of redshift will have a relevant effect in the final temperature measurements. Comparing the least-squares fit computed from our measurements (green solid line) with the re-normalized effective optical depth of Becker et al. it is evident that there is a systematic difference between the two samples, increasing at lower redshifts: our $\tau_{\text{eff}}^R$ values are $\sim 10$ per cent higher compared with the previous measurements and, even if the black triangles tend to return inside the $\pm 1\sigma$ confidence interval on the fit (green dotted lines) for $z \gtrsim 2.6$, the results are not in close agreement. However, the main quantities of interest in this paper (i.e. the curvature, from which the IGM temperature estimates are calculated) derive from a comparison of real and simulated spectra which have been re-normalized in the same way, so we expect that they will not depend strongly on the estimation of the continuum like the two sets of $\tau_{\text{eff}}^R$ results in Fig. 5 do (we compare the effective optical depth prior the continuum re-normalization from the simulations calibrated with the $\tau_{\text{eff}}^R$ results in Section 6.1.2). The higher values shown by our re-normalized effective optical depth reflect the variance expected between different samples: a systematic scaling, of the order of the error bar sizes, between our results and the previous ones may not be unexpected due to the non-independence of the data points within each set. These differences will reflect different characteristic overdensities probed by the forest (see Section 6.3). Fortunately, the correct calibration between temperature and curvature measurements will wash out this effect, allowing consistent temperature calibration as a function of redshift (see Section 7 and Appendix A).

Even if the scaling between the data sets could be smoothed, considering the fact that, according to Rollinde et al. (2013), the bootstrap errors computed from sections of $10\,h^{-1}\text{ Mpc}$ (and then $\lesssim 25\text{ Å}$) could underestimate the variance, another possible cause could be differences in the metal masking procedures of the two studies. In the next section we explain and test our metal masking technique, showing how our results do not seem to imply a strong bias due to contamination from unidentified metal lines.
5.2 Metal correction

Metal lines can be a serious source of systematic uncertainties for both the measures of the re-normalized flux (F_R) and the curvature. While in the first case it is possible to choose between a statistical (Tytler et al. 2004; Kirkman et al. 2005) and a direct (Schaye et al. 2003) estimation of the metal absorption, for the curvature it is necessary to directly identify and mask individual metal lines in the Lyα forest. Removing these features accurately is particularly important for redshifts z \lesssim 3, where there are fewer Lyα lines and potentially their presence could affect significantly the results. We therefore choose to identify metal lines proceeding in two steps: an ‘automatic’ masking procedure followed by a manual refinement.

First, we use well-known pairs of strong metal-line transitions to find all the obvious metal absorbing redshifts in the spectrum. We then classify each absorber as of high (e.g. C IV, Si IV) or low (e.g. Mg II, Fe II) ionization. To understand if the proximity effect can bias the effective optical depth, we compare our results with the measurements of Faucher-Giguère et al. (2008). The relative metal correction to \( \tau_{\text{eff}} \) decreases with increasing redshift, as expected if the IGM is monotonically enriched with time (Faucher-Giguère et al. 2008) and in general is consistent with the previous results. In their work in fact, Faucher-Giguère et al. evaluated the relative percentages of metal absorption in their measurements of \( \tau_{\text{eff}} \) when applying two different corrections: the one obtained with the direct identification and masking method by Schaye et al. (2003), and the statistical estimate of Kirkman et al. (2005) in which they used measurements of the amount of metals redwards the Lyα emission line. At each redshift, Faucher-Giguère et al. (2008) found good agreement between the estimates of their effective optical depth based on the two methods of removing metals. In individual redshift bins, applying our final relative metal absorption percentages, we obtain a \( \tau_{\text{eff}} \) that agrees well within 1σ with the ones obtained after applying the corrections of these previous results, encouraging confidence that our metal correction is accurate to the level of our statistical error bar. However, our corrections are overall systematically larger than the previous ones. If we have been too conservative in removing potentially metal-contaminated portions of spectra in our second ‘by-eye’ step, this will bias the effective optical depth to lower values.

In Fig. 4 is also shown the effect of our metal correction on the curvature measurements: red points are curvature values obtained from the raw spectra while the green points are the final measurements from masked sections, with vertical bars representing the 1σ bootstrap errors. However, our corrections are overall systematically larger than the previous ones. If we have been too conservative in removing potentially metal-contaminated portions of spectra in our second ‘by-eye’ step, this will bias the effective optical depth to lower values.

In Fig. 6 is presented the correction for the metal-line absorption on the re-normalized effective optical depth measurements; from the raw spectra (red triangles) to the spectra treated with the first ‘automatic’ correction (yellow stars), to the final results double-checked by eye (green points). For all the three cases we show the least-squares fit (solid lines of corresponding colours) and the 1σ vertical error bars. In Table 3 are reported the numerical values for our metal absorption compared with previous results of Schaye et al. (2003) and Kirkman et al. (2005) used in the effective optical depth corrections. For each redshift (column 1) is reported the percentage metal absorption correction obtained in this work in the first, ‘automatic’ mask described in the text (column 2) and in the refinement by eye (column 3). For comparison, in the overlapping redshift range are presented the results of Faucher-Giguère et al. (2008) obtained applying the direct metal correction of Schaye et al. (2003; column 4) and the statistical one of Kirkman et al. (2005; column 5).

Table 3. Metal absorption correction to the raw measurements of \( \tau_{\text{eff}} \). For each redshift (column 1) is reported the percentage metal absorption correction obtained in this work in the first, ‘automatic’ mask described in the text (column 2) and in the refinement by eye (column 3). For comparison, in the overlapping redshift range are presented the results of Faucher-Giguère et al. (2008) obtained applying the direct metal correction of Schaye et al. (2003; column 4) and the statistical one of Kirkman et al. (2005; column 5).

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1.6</td>
<td>13.8%</td>
<td>22.9%</td>
<td>n.a.</td>
<td>n.a.</td>
</tr>
<tr>
<td>1.8</td>
<td>13.2%</td>
<td>21.5%</td>
<td>n.a.</td>
<td>n.a.</td>
</tr>
<tr>
<td>2.0</td>
<td>12.6%</td>
<td>20.1%</td>
<td>13.0%</td>
<td>21.0%</td>
</tr>
<tr>
<td>2.2</td>
<td>12.0%</td>
<td>18.8%</td>
<td>12.3%</td>
<td>16.0%</td>
</tr>
<tr>
<td>2.4</td>
<td>11.5%</td>
<td>17.5%</td>
<td>11.4%</td>
<td>12.6%</td>
</tr>
<tr>
<td>2.6</td>
<td>11.0%</td>
<td>16.3%</td>
<td>10.4%</td>
<td>10.4%</td>
</tr>
<tr>
<td>2.8</td>
<td>10.5%</td>
<td>15.1%</td>
<td>9.7%</td>
<td>7.8%</td>
</tr>
<tr>
<td>3.0</td>
<td>8.8%</td>
<td>14.0%</td>
<td>9.0%</td>
<td>6.0%</td>
</tr>
</tbody>
</table>
obtained after masking the chunks of spectra that are potentially affected by the QSO radiation. Typically the ionizing UV flux of a bright QSO is thought to affect regions of \( \leq 10 \) proper Mpc along its own line of sight (e.g. Scott et al. 2000; Worseck & Wisotzki 2006). To be conservative, we masked the 25 proper Mpc nearest to each QSO Ly\( \alpha \) and Ly\( \beta \) emission lines and re-computed \( \tau_{\text{eff}}^R \). The final comparison is presented in Fig. 7: masking the proximity regions does not have any significant effect on \( \tau_{\text{eff}}^R \). In fact, the results obtained excluding these zones (light green circles) closely match the results inferred without this correction (green points), well within the 1\( \sigma \) error bars. We then do not expect that the inclusion in our analysis of the QSO proximity regions will affect significantly the temperature measurements.

6 SIMULATIONS ANALYSIS

To extract temperature constraints from our measurements of the curvature we need to interpret our observational results using simulated spectra, accurately calibrated to match the real data conditions. In this section we explain how we calibrate and analyse the synthetic spectra to find the connection between curvature measurements and the characteristic overdensities. We will use these results in Section 7 where we will interpolate the \( T(\Delta) \)–log(\( \langle |\kappa| \rangle \)) relationship to the value of \( \log(\langle |\kappa| \rangle) \) from the observational data to obtain our final temperature measurements.

6.1 The calibration

To ensure a correct comparison between simulation and observational data we calibrate our synthetic spectra to match the spectral resolution and the pixel size of the real spectra. We adjust the simulated, re-normalized effective optical depth \( \tau_{\text{eff}}^R \) to the one extracted directly from the observational results (see Section 5.1) and we add to the synthetic spectra the same level of noise recovered from our sample.

Figure 7. Effect of masking the proximity region on the re-normalized effective optical depth. The final results for \( \tau_{\text{eff}}^R \) without the masking of the proximity zones are shown as green points and those with this correction are shown as light green circles. Solid lines represent the least-squares fit of the data and vertical error bars are the 1\( \sigma \) statistical uncertainties.

Figure 8. Top panel: an example of noise distribution for a \( \Delta z = 0.2 \) redshift bin of real QSO spectra (in this case one with \( \bar{\varepsilon}_{\text{mean}} = 2.4 \)). On the x-axis of the histogram is shown the mean noise per section of \( 10h^{-1} \) Mpc while on the y-axis is shown the number of sections contributing to the particular bin. Bottom panel: the same distribution presented in the top panel but simplified, collecting the data in a noise grid of \( \Delta \bar{\sigma} = 0.01 \).

6.1.1 Addition of noise

To add the noise to the synthetic spectra we proceed in three steps: first, we obtain the distributions of the mean noise corresponding to the \( 10h^{-1} \) Mpc sections of the QSO spectra contributing to each redshift bin. As shown in Fig. 8 (top panel) these distributions can be complex and so to save computational time we simplify them by extracting grids of noise values with a separation of \( \Delta \bar{\sigma} = 0.01 \) and weights re-scaled proportionally to the original distribution (Fig. 8 bottom panel). At each redshift the noise is finally added at the same levels of the corresponding noise distribution and the quantities computed from the synthetic spectra, with different levels of noise, are averaged with the weights of the respective noise grid.

6.1.2 Recovered optical depth

The simulated spectra are scaled to match the re-normalized effective optical depth, \( \tau_{\text{eff}}^R \), of the real spectra. These can then be used to recover the corresponding effective optical depth (\( \tau_{\text{eff}} \); prior to the continuum re-normalization). In fact, the re-normalized effective optical depth cannot be compared directly with the results from the literature and to do so we need to compute the mean flux and then \( \tau_{\text{eff}} \) (\( \tau_{\text{eff}} = -\ln(\bar{F}) \)) from the synthetic spectra using the same procedure applied previously to the real spectra (see Section 5) but without the re-normalization. In Fig. 9 is shown the trend of the recovered effective optical depth for three different simulations, A15, G15 and C15 in Table 2. For clarity we do not plot the curves for the remaining simulations but they lie in between the curves of simulations A15 and G15. Depending on the different thermal histories, the recovered effective optical depths vary slightly but the separation of these values is small compared with the uncertainties about the trend (e.g. green dotted lines referred to the simulation C15).
In Fig. 9 we also compare our results with the previous studies of Becker et al. (2013), and Kirkman et al. (2005). The results of Becker et al. (2013), that for \( z \lesssim 2.5 \) have been scaled to the Faucher-Giguère et al. (2008) measurements, are significantly shifted towards lower \( \tau_{\text{eff}} \), presenting a better agreement with Kirkman et al. For \( z < 2.2 \) the effective optical depth of Kirkman et al. still shows values \( \sim 30 \) per cent lower than ours. In this case, again, part of the difference between the results could be explained by the non-independence of the data points within each set. Such an offset could also be boosted by a possible selection effect: the lines of sight used in this work were taken from the UVES archive and, as such, may contain a higher proportion of damped Ly\( \alpha \) systems; even if these systems have been masked out of our analysis, their presence will increase the clustering of the forest around them and so our sample will have higher effective optical depth as a consequence. The simulated \( \tau_{\text{eff}} \) presented in Fig. 9 were obtained by matching the observed \( \tau_{\text{eff}}^R \) (see Fig. 5) and do not represent one of the main results of this work, so we did not investigate further possible selection effects driven by our UVES sample. Being aware of this possibility, we decided to maintain the consistency between our curvature measurements and the simulations used to infer the temperature values, calibrating the simulated spectra with the effective optical depth obtained from our sample (see Appendix A). In the comparison between our results and the previous ones of Becker et al. (2011), the effect of a calibration with a higher \( \tau_{\text{eff}} \) will manifest itself as a shift towards lower values in the characteristic overdensities traced by the Ly\( \alpha \) forest at the same redshift (as we will see in Section 6.3).

Figure 9. The effective optical depth, prior to the re-normalization correction, recovered from the simulations, which matches the re-normalized effective optical depth, \( \tau_{\text{eff}}^R \), of our real spectra. The effective optical depth for three different simulations is shown: A15 (blue solid line), G15 (pink solid line) and C15 (green solid line). The recovered \( \tau_{\text{eff}} \) for the remaining simulations in Table 2 are not reported for clarity but they lie in between the trends of A15 and G15. The spread in values for different thermal histories is, in fact, small compared with the \( 1\sigma \) uncertainties about the trend of each of the effective optical depths (green dotted lines for simulation C15). Our results are compared with the effective optical depths of Becker et al. (2013; red points) and Kirkman et al. (2005; black points).

6.2 The curvature from the simulations

Once the simulations have been calibrated we can measure the curvature on the synthetic spectra using the same method that we used for the observed data (see Section 5). In Fig. 10 are plotted the values of \( \log(|\kappa|) \) obtained from our set of simulations in the same redshift range and with the same spectral resolution, effective optical depth and mix of noise levels of the real spectra. Different lines correspond to different simulations in which the thermal state parameters are changed. We can preliminarily compare our data points with the simulations, noticing that the simulation that has the values of the curvature close to the real observations is C15, which assumes the fiducial parameter \( \gamma = 1.54 \) at redshift \( z = 3 \). The variation in the properties of the real data alters the trend of the simulated curvature to be a slightly non-smooth function of redshift. As expected, the curvature values are sensitive to changes in the effective optical depth: as shown in Fig. 11 for the fiducial simulation C15, the \( 1\sigma \) uncertainty about the fit of the observed \( \tau_{\text{eff}}^R \) (see Fig. 5) is in fact reflected in a scatter about the simulated curvature of about 10 per cent at redshift \( z \sim 1.5 \), decreasing at higher redshifts. The next section shows how this dependence of the simulated curvature on the matched effective optical depth will imply differences in the recovered characteristic overdensities between our work and Becker et al. (2011).

6.3 The characteristic overdensities

The final aim of this work is to use the curvature measurements to infer information about the thermal state of the IGM, but this property will depend on the density of the gas. The Ly\( \alpha \) forest, and so the curvature obtained from it, in fact does not always trace the gas at the mean density, but, instead, at low redshift \( z \lesssim 3 \) the forest lines will typically arise from densities that are increasingly above the mean. The degeneracy between \( T_0 \) and \( \gamma \) in the temperature–density relation (equation 1) will therefore be significant. For this reason
6.3.1 The method

We determine the characteristic overdensities empirically, finding for each redshift the overdensities at which $T(\bar{\Delta})$ is a one-to-one function of $\log(|\kappa|)$ regardless of $\gamma$. The method is explained in Fig. 12: for each simulation type we plot the values of $T(\Delta)$ versus $\log(|\kappa|)$, corresponding to the points with different colours, and we fit the distribution with a simple power law. We change the value of the overdensity $\Delta$ until we find the one ($\bar{\Delta}$) for which all the points from the different simulations (with different thermal histories and $\gamma$ parameters) lie on the same curve and minimize the $\chi^2$. The final $T(\bar{\Delta})$ of our real data (see section 7) will be determined by interpolating the $T(\bar{\Delta})–\log(|\kappa|)$ relationship in the simulations to the value of $\log(|\kappa|)$ computed directly from the real spectra.

6.3.2 The results

The characteristic overdensities for the redshifts of our data points are reported in Table 4, while in Fig. 13 is shown the evolution of $\Delta$ as a function of redshift: as expected, at decreasing redshifts the characteristic overdensity at which the Ly$\alpha$ forest is sensitive increases. Note that Fig. 13 also shows that while the addition of noise in the synthetic spectra for $z \leq 2.2$ has the effect of decreasing the values of the characteristic overdensities, that tendency is inverted for higher redshifts where the noise shifts the overdensities slightly towards higher values with respect to the noise-free results. In Fig. 13 is also presented a comparison between the overdensities found in this work and the ones obtained in Becker et al. (2011) in their analysis with the addition of noise. Even if the two trends are similar, the difference in values of the characteristic overdensities at each redshift is significant ($\sim 25$ per cent at $z \sim 3$ and increasing towards lower redshift). Because we used the same set of thermal histories and a consistent method of analysis with respect to the previous work, the reason for this discrepancy lies in the different data samples: in fact, the effective optical depth observed in our sample is higher than the one recorded by Becker et al. (2011; see Section 5.1). As we have seen in Section 6.2, the simulated curvature is sensitive to the effective optical depth with which the synthetic spectra have been calibrated and this is reflected in the values of the characteristic overdensities. It is then reasonable that for higher effective optical depths at a particular redshift we observe lower overdensities because we are tracing a denser universe and the Ly$\alpha$ forest will arise in overdensities closer to the mean density.

![Figure 11. Dependence of the simulated $\log(|\kappa|)$ on the effective optical depth with which the simulations have been calibrated. The curvature recovered using the thermal history C15 (green solid line) is reported with the 1σ uncertainties about the trend (green dotted lines) corresponding to the 1σ uncertainties about the fit of the observed $r_{E}$ in Fig. 5. The variation generated in the curvature is about 10 per cent for $z = 1.5$ corresponding to a scatter of $\pm 2 - 3 \times 10^3$ K in the temperature calibration, and decreases at higher redshift.](http://mnras.oxfordjournals.org/)

![Figure 12. Example of the one-to-one function between $\log(|\kappa|)$ and temperature obtained for a characteristic overdensity ($\bar{\Delta} = 3.7$ at redshift $z = 2.173$). Different colours correspond to different simulations. At each redshift we find the characteristic overdensity, $\Delta = \bar{\Delta}$, for which the relationship between $T(\bar{\Delta})–\log(|\kappa|)$ does not depend on the choice of a particular thermal history or $\gamma$ parameter.](http://mnras.oxfordjournals.org/)

in our work we are not constraining both these parameters but we use the curvature to obtain the temperature at those characteristic overdensities ($\bar{\Delta}$) probed by the forest that will not depend on the particular value of $\gamma$. We can in this way associate uniquely our curvature values to the temperature at these characteristic overdensities, keeping in mind that the observed values of $\kappa$ will represent anyway an average over a range of densities.

Table 4. Numerical values for the results of this work: the mean redshift of each data bin is reported (column 1) with the associated characteristic overdensity (column 2). Also shown are the temperature measurement with the associated 1σ errors obtained for each data bin at the characteristic overdensity (column 3) and at the mean density under the assumption of two values of $\gamma$ (columns 5 and 6). Finally, the values of $\gamma$, recovered from the fiducial simulation C15, are presented (column 4).

<table>
<thead>
<tr>
<th>$z_{\text{mean}}$</th>
<th>$\bar{\Delta}$</th>
<th>$T(\bar{\Delta})/10^3$ K</th>
<th>$\gamma \sim 1.5$</th>
<th>$T_0^{\gamma=1.5}/10^3$ K</th>
<th>$T_0^{\gamma=1.5}/10^3$ K</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.63</td>
<td>5.13</td>
<td>33.74 ± 3.31</td>
<td>1.583</td>
<td>13.00 ± 1.27</td>
<td>20.66 ± 2.03</td>
</tr>
<tr>
<td>1.82</td>
<td>4.55</td>
<td>27.75 ± 1.19</td>
<td>1.577</td>
<td>11.79 ± 0.51</td>
<td>17.61 ± 0.76</td>
</tr>
<tr>
<td>2.00</td>
<td>4.11</td>
<td>27.62 ± 0.84</td>
<td>1.572</td>
<td>12.60 ± 0.38</td>
<td>18.08 ± 0.55</td>
</tr>
<tr>
<td>2.18</td>
<td>3.74</td>
<td>29.20 ± 1.06</td>
<td>1.565</td>
<td>14.05 ± 0.51</td>
<td>19.66 ± 0.71</td>
</tr>
<tr>
<td>2.38</td>
<td>3.39</td>
<td>25.95 ± 1.22</td>
<td>1.561</td>
<td>13.20 ± 0.62</td>
<td>18.00 ± 0.85</td>
</tr>
<tr>
<td>2.60</td>
<td>3.08</td>
<td>23.58 ± 1.09</td>
<td>1.554</td>
<td>12.77 ± 0.59</td>
<td>16.83 ± 0.78</td>
</tr>
<tr>
<td>2.80</td>
<td>2.84</td>
<td>22.67 ± 0.89</td>
<td>1.549</td>
<td>12.90 ± 0.51</td>
<td>16.58 ± 0.65</td>
</tr>
</tbody>
</table>
7 TEMPERATURE MEASUREMENTS

The selection of the characteristic overdensities, $\bar{\rho}$, and the associated one-to-one function between temperature and curvature allows us to infer information about the temperature of the gas traced by the Ly$\alpha$ forest, $T(\bar{\rho})$. This measurement is independent of the choice of the parameter $\gamma$ for the $T-\rho$ relation (equation 1) and for this reason will represent the main result of this work. We also translate our temperature measurements to values at the temperature at the mean density, $T_0$, for reasonable values of $\gamma$. In this section we present our results and compare them with those of Becker et al. (2011) at higher redshift. A broader discussion, taking into consideration theoretical predictions, can be found in Section 8.

7.1 Temperature at the characteristic overdensities

The main results of this work are presented in Fig. 14 where we plot the IGM temperature at the characteristic overdensities traced by the Ly$\alpha$ forest as a function of redshift. The $1\sigma$ errors are estimated from the propagation of the uncertainties in the curvature measurements. In fact, the uncertainties in the measured effective optical depth are reflected only in a small variation in the temperature measurements that falls well within the $1\sigma$ uncertainties due to the errors in the curvature measurements. Our temperature measurements show good agreement with the previous work of Becker et al. and are estimated from statistical uncertainties in the curvature measurements.

In the overlapping redshift range. Differences in the characteristic overdensities, $\bar{\rho}$, at a particular redshift between the two studies will cause variation in the derived temperature at the mean density ($T_0$) because we will infer $T_0$ using the $T-\rho$ relation with the values of $\bar{\rho}$. However, this effect will be modest and will cause disparity at the level of the $1\sigma$ error bars of our results (see Section 7.2 and Appendix ). For comparison, in Fig. 14 we show the $z = 2.4$ line-fitting result of Rudie, Steidel & Pettini (2012), with their $T_0$ and $\gamma$ values re-calibrated and translated to a $T(\bar{\rho})$ value by Bolton et al. (2014). Even if the line-fitting method is characterized by much larger $1\sigma$ error bars, it represents an independent technique and its agreement with our temperature values gives additional confidence in the results.

In general, the extension to lower redshifts ($z < 1.9$) that our new results provide in Fig. 14 do not show any large, sudden decrease or increase in $T(\bar{\rho})$ and can be considered broadly consistent with the trend of $T(\bar{\rho})$ increasing towards lower redshift of Becker et al. (2011). The increasing of $T(\bar{\rho})$ with decreasing redshift is expected for a non-inverted temperature–density relation because, at lower-$z$, the Ly$\alpha$ forest is tracing higher overdensities: denser regions are much more bounded against the cooling due to the adiabatic expansion and present higher recombination rates (and so more atoms for the photoheating process). We consider this expectation further in Section 8 after converting our $T(\bar{\rho})$ measurements to $T_0$ ones, using a range of $\gamma$ values.

7.2 Temperature at the mean density

Using the $T-\rho$ relation, characterized by different $\gamma$ values, we translate our measurements of $T(\bar{\rho})$ to values of temperature at the mean density ($T_0$). In Fig. 15 we present $T_0$ under two different assumptions for $\gamma$: for $\gamma$ values measured from fiducial simulations (A15–G15) in the top panel and for a constant $\gamma = 1.3$ in the bottom...
lower redshifts suggests a tendency of flattening of the increase in $T_0$ that can be interpreted as a footprint for the completion of the reheating of the IGM by He II reionization. The least-squares linear fit of our data presented in Fig. 15 (top panel) for this particular choice of $\gamma$ shows, in fact, quantitatively an inversion in the slope of the temperature evolution at the mean density: the general trend of the temperature is therefore an increase from $z \sim 4$ to 2.8 with a subsequent flattening of $T_0$ around $\sim 12,000$ K at $z \sim 2.8$. The evolution of the temperature for $z \lesssim 2.8$ is generally consistent with a linear decrease of slope $a = 0.80 \pm 0.81(1\sigma)$ generally in agreement with the decrease registered in Becker et al. (2011) for the same choice of $\gamma$.

The situation is similar in the second case for a constant $\gamma = 1.3$. This choice, which is motivated by the numerical simulations of McQuinn et al. (2009), corresponds to a mild flattening of the temperature–density relation, as expected during an extended He II reionization process. The trend of $T_0$ again shows a strong increase in the temperature from $z \sim 4$ to 2.8 and then a tendency of flattening from $z \sim 2.8$ towards lower redshift. However, the temperature obtained in this case is higher, fluctuating around $\sim 17,000$ K. The scatter between our data points and the ones from Becker et al. (2011) is also smaller for this choice of $\gamma$, even if ours are slightly higher on average. In this case the linear fit of our data points at $z \lesssim 2.8$ suggests a change in the slope of the temperature evolution but, while in the previous case we register a positive slope, for this $\gamma$ choice we see only a slowdown in the increasing temperature, with a slope that assumes the value $a = -1.84 \pm 1.06(1\sigma)$.

The exact redshift of the temperature maximum, reached by the IGM at the mean density approaching the tail-end of He II reionization, is then still dependent on the choice of $\gamma$, as already pointed out in Becker et al. (2011). Nevertheless, the extension at lower redshift of our data points gives stronger evidence about the end of this event. In fact, while an increase in the temperature for $z \sim 4–2.8$ has been recorded in the previous work, if $\gamma$ remains roughly constant, a tendency to a temperature flattening at lower redshift is suggested for both the choice of $\gamma$. A particularly important result is the suggestion of a decrease in $T_0$ in the case of $\gamma \sim 1.5$. In fact, according to recent analysis with the line-fitting method (Rudie et al. 2012; Bolton et al. 2014) at redshift $z = 2.4$ there is good evidence for $\gamma = 1.54 \pm 0.11(1\sigma)$ and, because we expect that at the end of He II reionization $\gamma$ will tend to come back to the asymptotic value of 1.6, indicating equilibrium between photoionization and cooling due to the adiabatic expansion, the possibility to have $\gamma \lesssim 1.5$ for $z \lesssim 2.4$ seems to be not realistic. Even if in this work we did not attempt to constrain the temperature–density relation, the scenario in the top panel of Fig. 15 seems likely to reproduce the trend in the evolution of the temperature, at least at low redshift, with our results reinforcing the picture of the reheating of the IGM due to He II reionization being almost complete at $z \sim 2.8$, with a subsequent tendency of a cooling, the rate of which will depend on the UVB.

### 7.2.1 The UV background at low redshifts

The tendency of our $T_0$ results to flatten at $z \lesssim 2.8$ seems to suggest that at these redshifts the reheating due to the He II reionization has been slowed down, if not completely exhausted, marking the end of this cosmological event. In the absence of reionization’s heating effects the temperature at the mean density of the ionized plasma is expected to approach a thermal asymptote that represents the balance between photoionization heating and cooling due to
the adiabatic expansion of the Universe. The harder the UVB is, the higher the temperature will be because each photoionization event deposits more energy into the IGM. In particular, under the assumption of a power-law ionizing spectrum, \( J_\gamma \propto \nu^{-\alpha} \), and that \( \text{He}\,\text{II} \) reionization no longer contributes any significant heating, the thermal asymptote can be generally described by (Hui & Gnedin 1997; Hui & Haiman 2003)

\[
T_0 = 2.49 \times 10^4 K \times (2 + \alpha)^{-\frac{4}{\alpha+1}} \left(\frac{1+z}{4.9}\right)^{0.53},
\]

where the parameter \( \alpha \) is the spectral index of the ionizing source. The observational value of \( \alpha \) is still uncertain. From direct measurements of QSO rest-frame continuua, this value has been found to range between 1.4 and 1.9 depending on the survey (e.g. Telfer et al. 2002; Shull, Stevans & Danforth 2012) whereas for galaxies the values commonly adopted range between 1 and 3 (e.g. Bolton & Haehnelt 2007; Ouchi et al. 2009; Kuhlen & Faucher-Giguère 2012) even if, in the case of the emissivity of realistic galaxies, a single power law is likely to be considered as an oversimplification.

Because our data at \( z \lesssim 2.8 \) do not show any strong evidence for a rapid decrease or increase in the temperature, here we assume that this redshift regime already traces the thermal asymptote in equation (4). Under this hypothesis we can then infer some suggestions about the expectation of a transition of the UVB from being dominated mainly by stars to being dominated mainly by QSOs over the course of the \( \text{He}\,\text{II} \) reionization (\( 2 \lesssim z \lesssim 5 \)). In Fig. 16 we show, as an illustrative example only, two models for the thermal asymptote: the first is the model of Hui & Haiman (2003) for the expected cooling in the absence of \( \text{He}\,\text{II} \) reionization, with \( \alpha \) scaled to 5.65 to match the flattening of the Becker et al. (2011) data at \( z \sim 4 - 5 \), while in the second case \( \alpha \) was scaled to 0.17 to match our results (for \( \gamma \sim 1.5 \)) at \( z \lesssim 2.8 \). These values are at some variance with the quantitative expectations: in the first case our UVB spectrum is significantly softer compared to typical galaxies-dominated spectrum while, after \( \text{He}\,\text{II} \) reionization, our value is somewhat harder than a typical QSO-dominated spectrum. We also emphasize that any such estimate of changes in the spectral index also involves the considerable uncertainties, already discussed, connected with the correct position of the peak in \( T_0 \) and the choice of \( \gamma \), and so we cannot make firm or quantitative conclusions here. However, in general, the observed cooling at higher temperatures at \( z \lesssim 2.8 \) seems to suggest that the shape of the UVB has changed, hardening with the increase in temperature during to the reionization event.

### 8 DISCUSSION

The main contribution of our work is to add constraints on the thermal history of the IGM down to the lowest optically accessible redshift, \( z \sim 1.5 \). These are the first temperature measurements in this previously unexplored redshift range. In this section we discuss the possible implications of our results in terms of compatibility with theoretical models.

Measuring that the low-redshift thermal history is important for confirming or ruling out the photoheating model of \( \text{He}\,\text{II} \) reionization and the new blazar heating models. According to many models of the former, the \( \text{He}\,\text{II} \) reionization should have left a footprint in the thermal history of the IGM: during this event, considerable additional heat is expected to increase the temperature at the mean density of the cosmic gas (\( T_0(z) \)) at \( z \lesssim 4 \) (Hui & Gnedin 1997). The end of \( \text{He}\,\text{II} \) reionization is then characterized by a cooling of the IGM due to the adiabatic expansion of the Universe with specifics that will depend on the characteristics of the UVB. However, even if some evidence has been found for an increase in the temperature at the mean density from \( z \sim 4 \) down to \( z \sim 2.1 \) (e.g. Becker et al. 2011), the subsequent change in the evolution of \( T_0 \) expected after the end of the \( \text{He}\,\text{II} \) reionization has not been clearly characterized yet and remains strongly degenerate with the imprecisely constrained slope of the temperature–density relation \( \gamma \) (see equation 1). This, combined with several results from PDF analysis which show possible evidence for an inverted temperature–density relation (Becker et al. 2007; Bolton et al. 2008; Viel et al. 2009; Calura et al. 2012; Garzilli et al. 2012), brought the development of a new idea of volumetric heating from blazar TeV emission (Chang et al. 2012; Puchwein et al. 2012). These models, where the heating rate is independent of the density, seem to naturally explain an inverted T–\( \rho \) relation at low redshift. Predicted to dominate the photoheating for \( z \lesssim 3 \), these processes would obscure the change in the temperature evolution trend due to the \( \text{He}\,\text{II} \) reionization, preventing any constraint on this event from the thermal history measurements.

A main motivation for constraining the temperature at lower redshifts than \( z \sim 2.1 \) is to confirm evidence for a flattening in the already detected trend of increasing temperature for \( z \lesssim 4 \). A precise measurement of a change in the \( T_0(z) \) slope, in fact, could bring important information about the physics of the IGM at these redshifts and the end of the \( \text{He}\,\text{II} \) reionization event. It is therefore interesting that our new temperature measurements in Fig. 15, which extend down to redshifts \( z \sim 1.5 \), show some evidence for such a change in the evolution for \( z \lesssim 2.8 \). However, in order to make a fair comparison with different heating models in terms of the temperature at the mean density, we must recognize the fact that we do not have yet strong constraints on the evolution of the T–\( \rho \) relation slope as a function of redshift: assuming a particular choice of \( \gamma \) for the translation of the temperature values at the characteristic overdensities to those at the mean density, without
considering the uncertainties in the slope itself, could result in an unfair comparison. Furthermore, the blazar heating models’ \( T-\rho \) relation at each redshift can be parametrized with a power law (of the form of equation 1) only for a certain range of overdensities that may not always cover the range in our characteristic overdensities (Chang et al. 2012; Puchwein et al. 2012). Therefore, to compare our results with the blazar heating model predictions, we decided to use directly the \( T(\bar{\Delta}) \) values probed by the forest.

In Fig. 17 we compare the model without blazar heating contributions, and the weak, intermediate and strong blazar heating models of Puchwein et al. (2012), with our new results for the temperature at the characteristic overdensities. The vertical error bars represent the 1\( \sigma \) errors on the temperature measurements.

\[
\text{Figure 17. Comparison of blazar heating models: the temperature values at the redshift-dependent characteristic overdensities, } T(\bar{\Delta}), \text{ inferred in this work (green points) are compared with the model without a blazar heating contribution (black solid line) and the weak (green dashed line), intermediate (yellow dashed line) and strong (red dashed line) blazar heating models of Puchwein et al. (2012). The blazar heating predictions were computed at the corresponding } \bar{\Delta}(z) \text{ in Table 4 in order to allow a fair comparison with our } T(\bar{\Delta}) \text{ measurements. Our observational results seem to be in reasonable agreement with the intermediate blazar heating model. The vertical error bars represent the 1\( \sigma \) errors on the temperature measurements.}
\]

\[
\text{The translation of the } T(\bar{\Delta}) \text{ measurements into values of temperature at the mean density, } T_0, \text{ depends on the slope of the temperature–density relation, } \gamma, \text{ which we do not constrain in this work. However, for reasonable, roughly constant, assumptions of this parameter, we do observe some evidence for a change in the slope of the temperature evolution for redshifts } z \lesssim 2.8, \text{ with indications of at least a flattening, and possibly a reversal, of the increasing temperature towards lower redshifts seen in our results and those of Becker et al. (2011) for } 2.8 \lesssim z \lesssim 4. \text{ In particular, for the minimum } T_0 \text{ case, with } \gamma \sim 1.5, \text{ the extension towards lower redshifts provided by this work adds to existing evidence for a decrease in the IGM temperature from } z \sim 2.8 \text{ down to the lowest redshifts observed here, } z \sim 1.5. \text{ This could be interpreted as the footprint of the completion of the reheating process connected with the He ii reionization.}
\]

9 CONCLUSIONS

In this work we have utilized a sample of 60 VLT/UVES QSO spectra to make a new measurement of the IGM temperature evolution at low redshift, \( 1.5 \lesssim z \lesssim 2.8 \), with the curvature method applied to the H i Ly\( \alpha \) forest. For the first time we have pushed the measurements to the lowest optically accessible redshifts, \( z \sim 1.5 \). Our new measurements of the temperature at the characteristic overdensities traced by the Ly\( \alpha \) forest, \( T(\bar{\Delta}) \), are consistent with the previous results of Becker et al. (2011) in the overlapping redshift range, \( 2.0 < z < 2.6 \), despite the data sets being completely independent. They show the same increasing trend for \( T(\bar{\Delta}) \) towards lower redshifts while, in the newly probed redshift interval \( 1.5 \lesssim z \lesssim 2.0 \), the evolution of \( T(\bar{\Delta}) \) is broadly consistent with the extrapolated trend at higher redshifts.
Following the additional hypothesis that our low redshift temperature measurements are already tracing the thermal asymptote, the cooling of $T_0$ inferred at $z \lesssim 2.8$ (assuming $\gamma \sim 1.5$) may suggest that the UVB has changed, hardening during the He II reionization epoch. However, the expectation for the evolution of $T_0$ following He II reionization will depend on the evolution in $\gamma$ and on details of the reionization model.

We also compared our $\langle T(\Delta) \rangle$ measurements with the expectations for the models of Puchwein et al. (2012) with and without blazar heating contributions. To allow a fair comparison with our observed values, the model predictions were computed at the corresponding (redshift-dependent) characteristic overdensities ($\Delta$). Our observational results seem to be in reasonable agreement with a moderate blazar heating scenario. However, to definitely confirm or rule out any specific thermal history it is necessary to obtain new, model-independent measurements of the temperature at the mean density.

With the IGM curvature now constrained from $z \sim 4.8$ down to $z \sim 1.7$, the main observational priority now is clearly to tightly constrain the slope of the temperature–density relation, $\gamma$, and its evolution over the redshift range $1.5 \lesssim z \lesssim 4$. This is vital in order to fix the absolute values of the temperature at the mean density and to comprehensively rule out or confirm any particular heating scenarios.

Finally, we note that, even though our new measurements have extended down to $z \sim 1.5$, there is still a dearth of QSO spectra with high enough S/N in the 3000–3300 Å spectral range to provide curvature information in our lowest redshift bin, $1.5 < z < 1.7$. We have searched the archives of both the VLT/UVES and Keck/High Resolution Echelle Spectrometer (HIRES) instruments for new spectra to contribute to this bin. However, the few additional spectra that we identified had relatively low S/N and, when included in our analysis, contributed negligibly to the final temperature constraints. Therefore, new observations of UV-bright QSOs with emission redshifts $1.5 \lesssim z_{\mathrm{em}} \lesssim 1.9$ are required to improve the temperature constraint at $1.5 < z < 1.7$ to a similar precision as those we have presented at $z > 1.7$.

ACKNOWLEDGEMENTS

We thank E. Puchwein for providing us with the temperature at our characteristic overdensity values within the various blazar heating scenarios. EB thanks E. Tescari for several useful discussions. We thank the referee, M. Shull, for helpful comments that clarified several points in the paper. The hydrodynamical simulations used in this work were performed using the Darwin Supercomputer of the University of Cambridge High Performance Computing Service (http://www.hpc.cam.ac.uk/), provided by Dell Inc., using Strategic Research Infrastructure Funding from the Higher Education Funding Council for England. MTM thanks the Australian Research Council for Discovery Project grant DP130100568 which supported this work. GDB gratefully acknowledges support from the Kavli Foundation. JSB acknowledges the support of a Royal Society University Research Fellowship.

REFERENCES


Churazov E. et al., 2007, A&A, 467, 529
Haardt F., Madau P., 2001, in Neumann D. M., Tran J. T. V., eds, Clusters of Galaxies and the High Redshift Universe Observed in X-rays. CEA, Saclay, 64

The IGM thermal history down to $z = 1.5$ 1931

APPENDIX A: THE EFFECT OF THE OPTICAL DEPTH CALIBRATION ON THE TEMPERATURE MEASUREMENTS

In this section we demonstrate that, in the curvature analysis of a particular sample of sight lines, calibrating the Lyα forest simulations with the effective optical depth of the sample provides robust measurements of the temperature at the characteristic overdensities. We will show how, even if different $\tau_{\text{eff}}$ calibrations produce different characteristic overdensities $\langle \bar{\Delta}(z) \rangle$; see Section 6.3], the temperature measurements, $T(\bar{\Delta})$, at each redshift will not be affected significantly by systematic effects related to possible biases in the sample selection. Instead, discrepancies in the characteristic overdensities will shift the derived temperature at the mean density. Nevertheless, this effect in the $T_0$ values will be modest, causing a disparity at the level of the observational $1\sigma$ error bars.

The test can be summarized as follows. We randomly select two subsamples of 300 spectral sections from the suite of simulations of one thermal history. One of the subsamples is selected in a biased way to result in a higher effective optical depth than the other; this difference is designed to be similar to that observed between the UVES sample used in this work and the sample adopted in the previous work of Becker et al. (2011). Treating the two subsamples as observational data, we analyse them separately with the curvature method presented in Section 4 and obtain the corresponding $T(\bar{\Delta})$ and $T_0$ measurements. The two sets of $T(\bar{\Delta})$ are found not to differ significantly, while a modest shift in the $T_0$ values is observed due to discrepancies in the recovered $\bar{\Delta}$ values at each redshift. The details of this test are described below.

A1 Selection of synthetic subsamples

We chose the 1024 synthetic sections of our fiducial simulation C15 (see Table 4) as the ‘global’ sample from which to select, at each redshift, two subsamples of $\sim$300 sections with two slightly different mean $\tau_{\text{eff}}$ that would simulate two random observational samples. We deliberately biased the mean optical depth of the second subsample towards higher values using the method explained in Fig. A1: we fit a Gaussian function to the global distribution of mean fluxes (at $z = 1.75$ in the example shown in the figure) from all 1024 sections of the C15 simulation and used this, and a shifted version of it, as the probability distributions for selecting sections randomly for the two subsamples of 300 sections each. By construction, the first subsample – which we call the ‘standard subsample’ for clarity – will have a mean $\tau_{\text{eff}}$ very close to the global mean. However, the mean optical depth of the second subsample – called the ‘biased subsample’ – is selected from the same probability distribution shifted slightly to lower mean fluxes, so it results in a higher mean $\tau_{\text{eff}}$. The shift in the probability distribution was tuned so that difference in the mean $\tau_{\text{eff}}$ at each redshift reflected the difference observed between our real UVES sample and the data used by Becker et al. (2011).

The results of the subsample selection are presented in Fig. A2 where we show the positions on the absolute curvature–mean $\tau_{\text{eff}}$ plane of all the sections from simulation C15 at $z = 1.75$. As
expected, the distribution of mean $\tau_{\text{eff}}$ and curvature in the standard subsample is very similar to the parent distribution. Also, while noting that, by construction, the biased subsample has a higher mean $\tau_{\text{eff}}$ than the standard subsample, we also see that the mean curvature of the biased subsample is very similar to the parent distribution. The subsample selection therefore should allow a test of the effect of selecting an observational sample with a higher mean $\tau_{\text{eff}}$ on the measured $T(\bar{\Delta})$ values.

### A2 Parallel curvature analysis and results

After the selection of the two subsamples we treated them as two separate observational data sets and we analysed the curvature following the steps presented in Section 4. That is, at each redshift and for each subsample, we computed the $\langle|\kappa|\rangle$ values and measured $T(\bar{\Delta})$ after calibrating all the simulations (from all thermal histories) with the mean $\tau_{\text{eff}}$ found in that particular subsample. Finally, using the $T-\rho$ relation, we derived the values of $T_0$ under the assumption of $\gamma = 1.54$ (corresponding to the chosen thermal history C15).

As expected, calibrating the simulations with the two different effective optical depths gave slightly different values for the $\bar{\Delta}$ at each redshift of the two subsamples. However, we find excellent agreement between the $T(\bar{\Delta})$ values for the two subsamples, as shown in the top panel of Fig. A3. There we also plot the difference between the $T(\bar{\Delta})$ values, $\Delta T$, from the two subsamples at each redshift in the lower panel. This difference is $\Delta T \lesssim 1100$ K at all redshifts and typically smaller than 800 K. Considering that the temperature measurements at the characteristic overdensities presented in this work have a minimum $1\sigma$ error bar of $\sim 1800$ K, these $\Delta T$ all fall inside the current statistical uncertainty budget. Also, we expect small, non-zero values of $\Delta T$, and small variations with redshift, due to the sample variance connected with the selection of the subsamples. In general, we can conclude that calibration of the simulations with the effective optical depth of the particular observational sample being analysed results in a self-consistent measurement of $T(\bar{\Delta})$.

In terms of the temperature values at the mean density, we find that the biased subsample produces a systematically higher temperature, as expected (top panel of Fig. A4). This discrepancy is due to the slightly different values of $\bar{\Delta}$ at each redshift in the two different subsamples. However, it is still modest and is generally below the minimum $1\sigma$ uncertainty of the measurements presented in this work (lower panel of Fig. A4).

Figure A3. Upper panel: $T(\bar{\Delta})$ values computed from the curvature analysis of the two synthetic subsamples. Lower panel: difference in $T(\bar{\Delta})$ between the standard and biased subsamples, $\Delta T$. The discrepancy between the temperature values is shown as a function of redshift (blue squares) and the minimum $1\sigma$ error bar observed in the UVES sample in this work (see Table 4) is given by the red dashed line for comparison.

Figure A4. Upper panel: $T_0$ values computed from the $T(\bar{\Delta})$ measurements of the two synthetic subsamples under the assumption of $\gamma = 1.54$. Lower panel: difference in $T_0$ between the standard and biased subsamples, $\Delta T_0$. The discrepancy between the temperature values is shown as a function of redshift (blue squares) and the minimum $1\sigma$ error bar observed in the UVES sample in this work (see Table 4) is given by the red dashed line for comparison.

This paper has been typeset from a 4pt/10pt file prepared by the author.