Dark-ages reionization and galaxy formation simulation – II. Spin and concentration parameters for dark matter haloes during the epoch of reionization

Paul W. Angel, Gregory B. Poole, Aaron D. Ludlow, Alan R. Duffy, Paul M. Geil, Simon J. Mutch, Andrei Mesinger and J. Stuart B. Wyithe

1 School of Physics, University of Melbourne, Parkville 3010, VIC, Australia
2 Institute for Computational Cosmology, Department of Physics, University of Durham, South Road, Durham DH1 3LE, UK
3 Centre for Astrophysics and Supercomputing, Swinburne University of Technology, PO Box 218, Hawthorn, VIC 3122, Australia
4 Scuola Normale Superiore, Piazza dei Cavalieri 7, I-56126 Pisa, Italy

Accepted 2016 March 28. Received 2016 March 23; in original form 2015 August 18

ABSTRACT

We use high-resolution N-body simulations to study the concentration and spin parameters of dark matter haloes in the mass range $10^8 M_\odot h^{-1} < M < 10^{11} M_\odot h^{-1}$ and redshifts $5 < z < 10$, corresponding to the haloes of galaxies thought to be responsible for reionization. We build a subsample of equilibrium haloes and contrast their properties to the full population that also includes unrelaxed systems. Concentrations are calculated by fitting both NFW and Einasto profiles to the spherically averaged density profiles of individual haloes. After removing haloes that are out of equilibrium, we find a $z > 5$ concentration–mass ($c(M)$) relation that is almost flat and well described by a simple power law for both NFW and Einasto fits. The intrinsic scatter around the mean relation is $\Delta c_{\text{vir}} \sim 1$ (or 20 per cent) at $z = 5$. We also find that the analytic model proposed by Ludlow et al. reproduces the mass and redshift dependence of halo concentrations. Our best-fitting Einasto shape parameter, $\nu$, depends on peak height, $v$, in a manner that is accurately described by $\nu = 0.0070 v^2 + 0.1839$. The distribution of the spin parameter, $\lambda$, has a weak dependence on equilibrium state; $\lambda$ peaks at roughly $\sim 0.033$ for our relaxed sample, and at $\sim 0.04$ for the full population. The spin–virial mass relation has a mild negative correlation at high redshift.

Key words: cosmology: theory – dark ages, reionization, first stars – early Universe.

1 INTRODUCTION

In the current cosmological paradigm cold dark matter collapses to form gravitationally bound structures within an expanding background universe. Known as dark matter (DM) haloes, these objects are initially small but undergo repeated merging to form ever larger systems. Galaxies form within these haloes as in-falling gas cools and converts to stars (e.g. White & Rees 1978). Their evolution and structural properties therefore underpin those of the embedded galaxies. These ideas have evolved into the field of semi-analytic modelling in which galaxies are grown within an evolving population of DM haloes extracted from purely N-body simulations (e.g. Croton et al. 2006; Lagos, Cora & Padilla 2008; Somerville et al. 2008, see Baugh 2006 for a review).

The characteristics of DM haloes have been the subject of extensive research. Mass determines the overall size of the halo, but several other important parameters have also been identified. For example, using N-body simulations (Navarro, Frenk & White 1997, henceforth NFW) found that the density profiles of virialized haloes can be well described by rescaling a simple formula (hereafter known as the NFW profile):

$$\rho(r) = \frac{\delta_c}{(r/r_s)(1 + r/r_s)^2}. \tag{1}$$

Here $r_s$ is the characteristic scale radius at which the logarithmic density slope is equal to $-2$; $\delta_c$ is the characteristic density contrast, and $\rho_c = 3 H(z)^2/(8\pi G)$ the critical background density at redshift $z$, where $G$ is Newton’s gravitational constant and $H$ the Hubble parameter. These parameters can be expressed in a variety of forms. One common approach is to use a virial mass,\textsuperscript{1} and concentration,
$c_{\text{vir}} \equiv R_{\text{vir}}/r_c$, defined as the ratio of the halo’s virial radius to its scale radius.

While the NFW profile is a common description, several recent studies (e.g., Navarro et al. 2004, 2010; Hayashi & White 2008) have shown that the density profiles of simulated haloes exhibit small but systematic deviations from equation (1). The Einasto profile (Einasto 1965), defined
\[
\ln \left( \frac{\rho_{\text{Ein}}(r)}{\rho_\odot} \right) = -\frac{2}{\alpha} \left[ \left( \frac{r}{r_\odot} \right)^{\alpha} - 1 \right],
\]
provides a better approximation to the radial density profile (Navarro et al. 2004; Ludlow et al. 2013). Like the NFW profile, equation (2) has two scaling parameters, $r_\odot$ and $\rho_\odot$, and an additional shape parameter, $\alpha$. Note that $r_\odot$ and $r_c$ are equivalent, and we will hereafter use the two interchangeably when defining the concentration of a halo.

At low redshift the concentration parameter decreases with increasing halo mass. NFW interpreted this finding as a result of hierarchical clustering: smaller haloes form earlier than more massive objects, when the universe was denser. They suggested that concentration, or equivalently the characteristic density, $\delta_c$, reflects the background density of the Universe at the halo’s formation time.

The same negative trend was also seen in subsequent N-body simulations (e.g. Eke, Navarro & Steinmetz 2001; Wechsler et al. 2002; Duffy et al. 2008), and lead to the development of analytic models attempting to explain the dependence of concentration on mass, redshift and cosmology. One approach relates $\delta_c$ to the past accretion history of the halo’s main progenitor. Wechsler et al. (2002), for example, calculated the mass assembly histories of simulated haloes and used a proportionality constant to relate the concentration to background density at the halo’s time of formation, defined by the point at which the logarithmic accretion rate falls below a specific value. The redshift dependence of the $c(M)$ relation was later studied by Zhao et al. (2003), who found a weakening of the relation for the highest mass haloes at any redshift. By $z \approx 3–4$ the negative trend is no longer present in the simulations of Gao et al. (2008), who focus on masses $M > 10^{11} \, M_\odot \, h^{-1}$. The flattening of the $c(M)$ relation was also reported by Zhao et al. (2009), who connected halo concentrations to the period at which halo growth transitions from a rapid to a slow phase. Although these models have met with limited success overall, they provide a clear interpretation of why concentration depends only weakly on mass for the most massive systems: because these haloes are forming today, they share the same formation time, and therefore concentration.

Neto et al. (2007) studied the $z = 0$ $c(M)$ relation in the Millennium simulation (Springel et al. 2005), while Klypin, Trujillo-Gomez & Primack (2011) and Prada et al. (2012) extended the analysis to $z = 6$ using both the Millennium and Bolshoi simulations. In agreement with previous work, these authors each find a decline of concentration with mass. However, both Prada et al. (2012) and Klypin et al. (2011) have also reported an upturn of the $c(M)$ relation at the high-mass end. On the other hand, Ludlow et al. (2012) demonstrated that there is no upturn amongst relaxed haloes, and showed how the transient dynamical states of merging systems can result in a non-monotonic $c(M)$ relation. While the details continue to be debated, it is clear overall that the diversity of halo formation histories play a critical role in establishing the shape and evolution of the $c(M)$ relation (Ludlow et al. 2014; Correa et al. 2015).

While the low redshift mass–concentration relation is well studied, at high $z$ the relation is poorly constrained. For example, Prada et al. (2012) and Diemer & Kravtsov (2015) find a high-mass upturn (above a few times $10^{10} \, h^{-1} \, M_\odot$) at $z = 5$ amongst the full halo population. At similar masses, Dutton & Macciò (2014), find a relation with a slightly positive slope, whereas Hellwing et al. (2016) report a weak negative slope that flattens by $z = 9$. These authors, however, imposed different equilibrium cuts on their halo samples, which hampers a direct comparison with their results. Thus, there is uncertainty regarding the precise nature of the high-redshift $c(M)$ relation and the role played by unrelaxed haloes.

For example, a halo suffering a merger is unlikely to have a simple, smooth density profile, and will take time to settle back into equilibrium. This situation becomes increasingly important at high redshift due to the elevated merger rates of potentially star-forming haloes. These concerns led Neto et al. (2007) to introduce three physically motivated parameters to identify systems far from equilibrium: (1) the mass-fraction in substructure $f_{\text{sub}} = M_{\text{sub}}(<R_{\text{vir}})/M_{\text{vir}}$; (2) the offset between the halo’s centre of mass and its most-bound particle, $x_{\text{off}}$; and (3) the pseudo-viral-ratio of kinetic and potential energies, $\psi$. The effectiveness of these parameters in isolating relaxed DM haloes is further discussed in Gao et al. (2008) and Ludlow et al. (2012).

A detailed study of these parameters with regard to dynamical relaxation at high redshift is provided in the first paper of the DRAGONS\textsuperscript{2} series, Poole et al. (2016, hereafter referred to as Paper I), who examine the behaviour of these equilibrium diagnostics during the relaxation phase that follows a significant merger or accretion event during the epoch of reionization. Their results suggest that, across the mass range of our simulations, $10^8 < M_{\text{vir}}(<R_{\text{vir}}) < 10^{11}$, and for $z > 5$, standard relaxation values for $f_{\text{sub}}, x_{\text{off}}$ and $\psi$ obtained from low-redshift studies are very effective at identifying systems relaxing from halo formation or recent mergers at high redshift.

Concentration is not the only relevant halo property for galaxy formation. Halo spin also plays an important role in semi-analytic models, since angular momentum conservation determines the size of galactic discs (e.g. Mo, Mao & White 1999; Guo et al. 2011), which in turn determine their star formation rates (Schmidt 1959; Kennicutt 1998). A halo’s angular momentum is often expressed as a dimensionless spin parameter:
\[
\lambda = \frac{J_{\text{vir}}}{\sqrt{2}M_{\text{vir}}V_{\text{vir}}R_{\text{vir}}},
\]
where $J_{\text{vir}}$ is the total angular momentum within $R_{\text{vir}}$.

Most studies of the spin parameter have focused on the distribution of spins and its dependence on halo mass (e.g. Bett et al. 2007; Neto et al. 2007; Knebe & Power 2008; Muñoz Cuartas et al. 2011). At any redshift, halo spins are distributed approximately lognormally, and peak at $\lambda_0 \approx 0.03–0.04$. At low redshift, spins are approximately independent of mass but gain a slight negative correlation at higher redshifts (Knebe & Power 2008; Muñoz Cuartas et al. 2011). Recently, Dutton & Macciò (2014) measured the redshift evolution of the $\lambda$–$M_{\text{vir}}$ relation, reporting a weak negative correlation at $z = 5$.

In this work, we use the Tiramati simulation suite to extend the study of concentration and spin to redshifts $z > 5$. Our simulations were designed to investigate halo masses relevant for galaxy formation during this high-redshift epoch. The purpose of our study is to

\textsuperscript{2}The Dark-ages, Reionization And Galaxy-formation Observables Numerical Simulation project; http://dragons.ph.unimelb.edu.au/.
measure the structural and dynamical properties of haloes that are necessary for forthcoming semi-analytic models of reionization.

We organize the paper as follows. In Section 2, we describe the numerical simulations, including halo finding, analysis techniques, and our parametrization of concentration and spin. In Section 3, we present our concentration–mass relation and its redshift dependence, and in Section 4 the spin distribution, and its mass and redshift dependence. Finally, in Section 5 we summarize our main results.

## 2 Numerical Simulation

Our analysis focuses on DM haloes identified in three cosmological $N$-body simulations. These include a 2160$^3$-particle, 67.8 Mpc $h^{-1}$ cubed box (the Tiamat simulation) and two smaller but higher resolution volumes of 10 and 22.6 (Mpc $h^{-1}$)$^3$. Each run was carried out with GADGET-2 (Springel, Yoshida & White 2001a; Springel 2005) with RAM (random-access memory) consumption changes in accordance with those detailed in Poole et al. (2015). For each run, the Plummer-equivalent softening length was 1/50th of the mean Lagrangian interparticle spacing, and the integration accuracy parameter, $\varepsilon$, is set to 0.025, as motivated by the convergence study presented in Poole et al. (2015). Initial conditions were generated using second-order perturbation theory (using the code 2LPTIC$^3$; Crocce, Pueblas & Scoccimarro 2006) at $z=99$ and each simulation was run down to $z=5$; 100 snapshots of particle data were taken equally spaced in time from $z=35$ to 5 (one every 11 Myr). Cosmological parameters for each box were chosen to be consistent with the Planck 2015 data release (Planck Collaboration XIII 2015) ($h, \Omega_m, \Omega_b, \Omega_{\Lambda}, \sigma_8, n_s$) = (0.678, 0.308, 0.0484, 0.692, 0.815, 0.968). The relevant numerical parameters are summarized in Table 1. A more detailed discussion of these simulations can be found in Paper I.

### 2.1 Halo finding

Haloes were identified in each simulation snapshot using SUBFIND (Springel et al. 2001b). This produces two outputs: the first contains structures found by a friends-of-friends (FoF) algorithm (we adopt a linking length of 0.2 times the mean inter particle spacing); the second is obtained by dissecting each FoF group into its self-bound 'substructure'. This result is a central 'main halo', typically containing >90 per cent of its virial mass, and a group of lower mass subhaloes which trace the undigested cores of previous merger events.

Each main halo and its substructures were further analysed to catalogue their basic properties and to build their spherically averaged profiles. A (FoF or substructure) halo was required to have at least 32 particles, resulting in a minimum halo mass of $8.4 \times 10^7 M_\odot h^{-1}$ in Tiamat, $2.5 \times 10^7 M_\odot h^{-1}$ in Medi Tiamat and $2.2 \times 10^6 M_\odot h^{-1}$ in Tiny Tiamat. However, when estimating concentrations, a stricter limit of $n_p > 5000$ within the bound structure was imposed to ensure that the halo’s inner regions are well resolved. For the purpose of estimating spin parameters, this limit is relaxed to 600 particles. These parameter choices are further discussed and motivated in Section 2.3.

### 2.2 Density profiles and concentration estimates

For each main halo, spherically averaged density profiles were constructed in bins containing an equal number of particles. Only particles considered by SUBFIND to be bound to the central haloes were used. The number of bins was increased along with the number of halo particles. We imposed a minimum of 5 bins for the smallest haloes, and a maximum of 1000 bins for the largest (reached as $n_p$ tends to $10^8$). For example, haloes with $n_p = 5000$ have 25 bins, rising to ~125 bins for haloes containing $10^8$ particles.

Best-fitting NFW and Einasto profiles were obtained by minimizing

$$\psi = \frac{1}{N} \sum_i \frac{\Delta r_i}{r_i} \left( \log \rho_i - \log \rho_{\text{NFW(Ein)}} \right)^2,$$

where $N$ is the number of bins, $\rho_i$ is the density in bin $i$, and $\rho_{\text{NFW(Ein)}}$ is the corresponding density of the NFW (Einasto) model. The factor $\Delta r_i/r_i$ (the width of bin $i$ divided by the median radius of particles in it) appropriately weights bins of differing size. Note that only bins in the radial range $0.05R_{\text{vir}} < r_i < 0.8R_{\text{vir}}$ were used (we have verified that the minimum bin radius is always larger than the convergence radius defined in Power et al. 2003).

In Fig. 1, we show the ratio of the minimum bin size to the halo virial radius. For the smallest resolved haloes ($n_p = 5000$) this ratio falls to ~12, and thus scale radii corresponding to concentrations as high as 12 could be resolved even for the smallest haloes considered.

![Figure 1](https://example.com/fig1.png)

**Figure 1.** The ratio between the minimum bin size of the halo profile in dark blue and the size of the virial radius, plotted as a function of particle number for the Tiamat simulation. The solid line is the median while the shaded area is the scatter (68 per cent confidence interval). The largest bin size of 0.09 $R_{\text{vir}}$ indicates our bins begin well inside the halo scale radius for the halo masses considered here.

Table 1. Summary of simulation parameters. $N_p$ is the total number of particles, $L$ is the length of the box, $m_p$ is the mass of each particle and $\varepsilon$ is the gravitational force softening length.

<table>
<thead>
<tr>
<th>Simulation</th>
<th>$N_p$</th>
<th>$L$ (Mpc $h^{-1}$)</th>
<th>$m_p$ ($M_\odot h^{-1}$)</th>
<th>$\varepsilon$ (kpc $h^{-1}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tiamat</td>
<td>2160$^3$</td>
<td>67.8</td>
<td>$2.64 \times 10^6$</td>
<td>0.63</td>
</tr>
<tr>
<td>Medi Tiamat</td>
<td>1080$^3$</td>
<td>22.6</td>
<td>$7.83 \times 10^5$</td>
<td>0.42</td>
</tr>
<tr>
<td>Tiny Tiamat</td>
<td>1080$^3$</td>
<td>10.0</td>
<td>$6.79 \times 10^4$</td>
<td>0.19</td>
</tr>
</tbody>
</table>

3 Code was obtained from [http://cosmo.nyu.edu/roman/2LPT/](http://cosmo.nyu.edu/roman/2LPT/) with alteration to allow for greater than 1625$^3$ particles.

...
We find concentrations are almost always \(<10\). This indicates that our bins span a sufficient portion of the halo profile to provide reliable estimates of \(r_x\).

### 2.3 Resolution and virialization cuts

In defining our sample of equilibrium DM haloes, we impose both dynamical and resolution criteria following the procedure established by Neto et al. (2007), Gao et al. (2008) and Ludlow et al. (2012). The behaviour of these equilibrium diagnostics over the mass and redshift range probed by the Tiamat simulation suite is discussed further in Paper I. The criteria defining relaxed haloes include upper limits on the following three quantities:

(i) The fraction of mass found in satellite subhaloes, \(f_{\text{sub}} = M_{\text{sub}}(< R_{\text{vir}})/M_{\text{vir}} < 0.1\). As discussed in Neto et al. (2007) a high fraction of mass in substructure may be indicative of a recent merger. Such a halo will not, in general, have a smooth, spherically averaged density profile.

(ii) The offset between the position of the most-bound-particle, \(r_{\text{mbp}}\), and centre-of-mass, \(r_{\text{com}}\), in units of virial radius: \(\delta_{\text{off}} = |r_{\text{mbp}} - r_{\text{com}}|/R_{\text{vir}} < 0.07\). This is complementary to \(f_{\text{sub}}\) as it includes mass from unresolved subhaloes, or merging haloes that have their centre just outside the virial radius and consequently are not included in \(f_{\text{sub}}\).

(iii) The pseudo-virial ratio of kinetic and potential energies, \(\varphi = 2K/|U| < 1.35\). This criterion tends to be sensitive to haloes at the pericentre of a merger that may not be flagged by the above two parameters.

Neto et al. (2007) find that these restrictions, although arbitrary in value, provide a simple and physically motivated method to exclude haloes that are not well described by an NFW profile. In Paper I, these parameters were also shown to be discriminative between haloes that have either recently doubled in mass, or suffered a major or minor merger, within the last 1–2 dynamical times. We therefore adopt them as our standard equilibrium criteria, and use them to split our full halo sample into a relaxed subsample, which we analyse separately.

A further criterion is imposed to select only well-resolved haloes from both the relaxed and unrelaxed populations, in order to fit a reliable radial halo profile. For the halo concentration analysis a lower limit on particle number (\(n_p\)) of \(n_p > 5000\) is imposed. This is derived from work studying convergence of NFW and Einasto profile fits in Neto et al. (2007) and Gao et al. (2008). This ensures that the inner portion of the halo is well enough resolved to measure the scale radius \(r_s\). We impose a restriction of \(n_p > 600\) particles for the spin parameter measurement following Knebe & Power (2008), who obtain the limit by comparing the measured energy from Monte Carlo realisations of analytic NFW profiles.

The dynamics of hierarchical growth means that at the high redshifts studied here, many of our haloes will be far from equilibrium and thus have ill-defined values for concentration and spin. These sample cuts are designed to remove such systems and to keep transients out of our analysis as much as possible. For example, a halo which has just undergone a major merger may be comprised of two large, high-density clumps, and consequently have a high \(x_{\text{vir}}\) and poorly defined centre. The density profile of such a system cannot be captured by simple spherical averages, and structural properties estimated from NFW or Einasto fits are meaningless. We stress, however, that there is a continuum of values for \(x_{\text{vir}}, f_{\text{sub}}\) and \(\varphi\); the particular values chosen to separate relaxed haloes from unrelaxed are the result of extensive past investigations (see e.g. Neto et al. 2007; Ludlow et al. 2012, and Paper I). Our choices of resolution and dynamical classification also simplify comparison with previous studies.

To quantify the effects of our sample selection, we note that in Tiamat there are 14 391 haloes with more than 5000 particles at redshift \(z \sim 5\), but only 4433 (or \(~30\) per cent) of these satisfy our relaxation criteria; this reduces to \(~15\) per cent by \(z \sim 10\). These numbers underline the importance of the dynamical state of haloes at high redshift. Nevertheless parametrized fits to the entire population are useful for many semi-analytic calculations (e.g. Mack 2014; Schön et al. 2015) and for this reason we report fits to both our full halo sample as well as the relaxed subsample. Further details regarding the dynamical state of high-redshift haloes are presented in Paper I.

### 3 CONCENTRATION–MASS RELATION

#### 3.1 The concentration–mass relation for relaxed haloes

The \(c(M)\) relations for our equilibrium haloes at \(z = 5–7\) are shown in Figs 2 and 3, respectively. Both NFW and Einasto concentrations are plotted. The inner shaded region shows the bootstrapped 90 per cent confidence interval on the median for mass bins containing at least 20 haloes. The outer shaded area fills the 68 per cent scatter in individual concentration estimates. We find a weak trend of decreasing concentration with mass at \(z = 5\) for both NFW and Einasto fits. This trend becomes shallower as redshift increases. By redshift 9 there is no trend apparent for either set of fits (see Table A1). The \(c(M)\) relations obtained from both NFW and Einasto fits are similar over this mass range. We note that the systematic difference between NFW and Einasto concentrations is \(\Delta c_{\text{vir}} \sim 0.1\), which is smaller than the change in concentration from the lowest to highest masses studied here.

Best-fitting power laws at \(z = 5\) are \(3.8 \pm 0.4\ (M/10^{10}\ h^{-1}\ M_\odot)^{0.039 \pm 0.005}\) for NFW fits and \(3.8 \pm 0.4\ (M/10^{10}\ h^{-1}\ M_\odot)^{0.039 \pm 0.005}\) for Einasto fits. Best-fitting parameters are obtained using the Monte Carlo Markov Chain (MCMC) method implemented with the emcee package (Foreman-Mackey et al. 2013). The quoted errors are the 68 per cent confidence interval derived from the posterior distribution. We find intrinsic scatter in the \(c(M)\) relation of \(\Delta c_{\text{vir}} \sim 1.0\) (or 20 per cent) for fits to both NFW and Einasto profiles. Best-fitting power-law relations (for both NFW and Einasto fits) are provided in the appendix in Table A1 for a range of redshifts.

In addition to the two scaling variables, the Einasto profile has a shape parameter, \(\kappa\). Previous studies have found that \(\kappa\) depends in a complex way on both halo mass and redshift, but follows a simple relation when expressed in terms of the dimensionless ‘peak height’ mass parameter, \(v = \delta_{\text{c}}/\sigma(M, z)\). Here \(\delta_{\text{c}} = 1.686\) is the density threshold for the collapse of a spherical top-hat density perturbation, and \(\sigma(M, z)\) is the rms density fluctuation in spheres enclosing mass \(M\). Fig. 4 shows the \(\alpha-\nu\) relation obtained from our Einasto fits. We find a similar \(\alpha-\nu\) relation to the previous authors – the fit from Gao et al. (2008) is shown as a dashed line – although our simulations cover a different mass and redshift range. Our best fit for the \(\alpha-\nu\) relation is \(\alpha = 0.007\nu^2 + 0.1839\).

In order to check for any subtle bias in our fits we have also constructed \(c(M)\) and \(\alpha(v)\) relations using the median density profiles obtained by stacking haloes in narrow mass bins. This smooths out any unique features of individual systems and allows for a robust estimate of the median structural properties of haloes of...
Concentration–mass relation of relaxed central haloes at $z = 5$. Left- and right-hand panels are the NFW and Einasto concentrations, respectively. Inner shaded region denotes the bootstrapped 90 per cent confidence interval on the median. The outer shaded region shows the 68 per cent scatter. The line of best fit is fit to the median using the Monte Carlo Markov Chain method implemented in the PYTHON package EMCEE (Foreman-Mackey et al. 2013). This produces $c_{\text{vir}} = 3.8 \pm 0.4 (M/10^{10} M_\odot h^{-1})^{-0.035 \pm 0.005}$ at $z = 5$ for NFW fits and $c_{\text{vir}} = 3.8 \pm 0.4 (M/10^{10} M_\odot h^{-1})^{-0.039 \pm 0.005}$ for Einasto fits. The fits from Dutton & Macciò (2014) are also shown with thick solid lines representing fitting formula over their derived mass range, although we emphasize Dutton & Macciò (2014) employ different relaxation and resolution criteria for their sample. Thin extensions of these lines represent these fits extrapolated to lower masses. The Tiamat simulation is designed to study reionization and explores a mass range below previous studies. The dashed black line at $c_{\text{vir}} = 4$ is added for a point of reference. Solid black points represent the Ludlow et al. (2014) model where halo concentrations are calculated from the median accretion history in the mass bin. Error bars are derived from the 68 per cent scatter in accretion histories.

The weak trend in concentration with mass found in our simulation is in qualitative agreement with previous work that found a negative trend at low redshift that becomes progressively shallower with increasing $z$. For example, both the shallow negative slope and a given mass. For our relaxed population we recover the $c(M)$ to within $\Delta c \sim 0.1$, for both NFW and Einasto fits. We choose to use the individual fits when computing the best-fitting $c(M)$ relation.
The dependence of concentration on $x_{\text{off}}$ for central subgroups in several mass ranges. From left to right, the ranges are $[10^{10.0} M_\odot h^{-1} < M < 10^{10.2} M_\odot h^{-1}]$, $[10^{10.3} M_\odot h^{-1} < M < 10^{10.4} M_\odot h^{-1}]$ and $[10^{11.0} M_\odot h^{-1} < M < 10^{11.3} M_\odot h^{-1}]$. Blue lines represent the median. The dashed red line denotes the relaxation criteria cut above which a halo is considered to be out of equilibrium (Neto et al. 2007; Gao et al. 2008; Ludlow et al. 2012, Paper I). It can be seen that lower $x_{\text{off}}$ parameters (i.e. more relaxed haloes) correlate with higher $c_{\text{vir}}$ for all mass ranges.
Figure 6. Concentration–mass relation for relaxed haloes at $z = 5$, but now with haloes containing $> 500$ particles. The inner shaded region represents the bootstrapped median value and the outer region the 68 per cent scatter. Dashed black line is the $c_{\text{vir}} = 4$ line for reference and the fiducial red line is the best fit to the relaxed population as in Fig. 2. The inclusion of haloes with particles number $< 5000$ introduces more haloes with lower concentrations at the low-mass end of each simulation. Einasto shape parameters, $\alpha$, for haloes with $< 500$ particles are also higher as seen in Fig. 8.

Figure 7. The same as Fig. 2 but now the case in which no non-equilibrium cuts are enforced and the full sample of haloes with $> 5000$ particles is analysed. The inner shaded region represents the bootstrapped median value and the outer region the 68 per cent scatter. The dashed black line at $c_{\text{vir}} = 4$ is added for a point of reference and the fiducial red line is the best fit to the relaxed population as in Fig. 2. The thick solid line represents previous authors’ fits in the mass range their simulations cover, while the thin solid line is the extrapolation to lower masses. The median magnitude of the concentrations has decreased by $\Delta c_{\text{vir}} \sim 1$ over all masses in our simulations.

In Fig. 7 the $c(M)$ relation is plotted for the entire halo sample. At all masses, the median concentrations decrease relative to those of our relaxed haloes. We also note that inclusion of unrelaxed haloes alters the $\alpha - \nu$ relation slightly. Our best fit to the full population is $\alpha = 0.0091\nu^2 + 0.1447$, which is slightly steeper than that found by Duffy et al. (2008) when they include unrelaxed haloes.

In the left-hand panel of Fig. 7 we also plot the models of Diemer & Kravtsov (2015) and Prada et al. (2012), where we have overlapped, we find a significant discrepancy between the simulations for masses corresponding to haloes with $5000 > n_p > 500$. In particular, lower particle numbers result in lower values of $c_{\text{vir}}$. For Einasto profiles the $\alpha - \nu$ relation also changes slightly, as shown in Fig. 8. Many of the $500 < n_p < 5000$ haloes have $\alpha \sim 0.25$ and do not follow the previous quadratic $\alpha - \nu$ relation. We thus caution against overinterpreting Einasto fit parameters at low particle number.
analytically adjusted the Prada et al. (2012) relation from the \(c_{200}, M_{200}\) definitions, which they use, to the \(c_{\text{vir}}, M_{\text{vir}}\) definitions used in this work by assuming an NFW profile. Diemer & Kravtsov (2015) use the phase space halo finder ROCKSTAR (Behroozi, Wechsler & Wu 2013), and do not enforce any relaxation criteria. They also utilize different resolution criteria. The mass range covered in Diemer & Kravtsov (2015) is equivalent to the mass range covered by Tiamat with halo particle numbers of \(n_p > 1200\). In this mass range we see slight evidence for an upturn in the Tiamat \(c(M)\) relation, which is consistent with their findings. Note that this upturn is a feature exclusive to our full halo sample, suggesting its connection to departures from equilibrium.

Our concentrations at these masses and redshifts are lower than those found by Prada et al. (2012). The \(\Delta c_{\text{vir}} \sim 1\) difference likely originates from a combination of: (a) the use of \(M_{\text{vir}}\) and \(c_{\text{vir}}\) in their definitions of mass and concentration, (b) the use of maximum circular velocity as a measure of concentration, which is shown in Meneghetti & Rasia (2013) to reconcile differences in the \(c(M)\) relation between Prada et al. (2012) and Duffy et al. (2008) (a difference as large as \(\sim 40\) per cent), (c) a different halo finder (Bound–Density–Maxima), and/or (d) a lower particle limit of 500. We also note a similar comparison with Klypin et al. (2016), who measure concentrations using circular velocity. When we relax our halo equilibrium criteria in Fig. 7, we obtain concentrations at \(M = 10^{10}\ M_\odot\) which are in better agreement with DM14 and Diemer & Kravtsov (2015). Our simulations do not show an upturn after unrelaxed haloes are removed, as pointed out in Correa et al. (2015). We note that the trend for both relaxed NFW and relaxed Einasto fits are consistent with being mass-independent at \(z > 8\), as seen in Tables A1 and A2. In Appendix A the power law fits to both NFW and Einasto \(c_{\text{vir}}-M_{\text{vir}}\) relations are provided, although we caution against extrapolating the fits, particularly the fits of unrelaxed haloes, beyond their mass range.

In the case of both the relaxed population and the full population, we find the weak trend in the \(M_{\text{vir}}-c_{\text{vir}}\) to be steady across this mass range. The redshift dependence of the relaxed population of NFW concentrations can be described by

\[
c_{\text{vir}} = 3.2 \left( \frac{M}{10^{10}\ M_\odot\ h^{-1}} \right)^{-0.03} \left( \frac{1 + z}{10} \right)^{-0.29}
\]

and Einasto concentrations by

\[
c_{\text{vir}} = 3.1 \left( \frac{M}{10^{10}\ M_\odot\ h^{-1}} \right)^{-0.03} \left( \frac{1 + z}{10} \right)^{-0.29}.
\]

The full population of NFW concentrations is described by

\[
c_{\text{vir}} = 2.6 \left( \frac{M}{10^{10}\ M_\odot\ h^{-1}} \right)^{-0.03} \left( \frac{1 + z}{10} \right)^{-0.21}.
\]

and Einasto concentrations by

\[
c_{\text{vir}} = 2.6 \left( \frac{M}{10^{10}\ M_\odot\ h^{-1}} \right)^{-0.03} \left( \frac{1 + z}{10} \right)^{-0.11}.
\]

Here we assume that the slope is constant across the mass range and only fit to the normalization of the \(c_{\text{vir}}-M_{\text{vir}}\) relation.

## 4 SPIN PARAMETER

In Fig. 9, we show the distribution of spin parameters for relaxed haloes in the Tiamat simulation; results are shown for \(z = 5\). The mass range covered is now increased so that \(n_p > 600\) (as in Knebe & Power 2008). Rather than a lognormal, the solid black lines show the best-fitting function of Bett et al. (2007): 

\[
P(\lambda) = \frac{3 \ln(10)}{\Gamma(\alpha)} \alpha^{\alpha-1} \left( \frac{\lambda}{\lambda_0} \right)^\alpha \exp \left[ -\alpha \left( \frac{\lambda}{\lambda_0} \right)^{3/\alpha} \right].
\]

This function has been shown to better describe the low spin tail. Also shown are lognormal fits by Knebe & Power (2008) and Muñoz Cuartas et al. (2011). Although evaluated at different redshifts from...
The spin–virial mass relation for the full population of haloes at $z=0.17$ from Knebe & Power (2008) at redshift $z=2011$ shows the relation between spin and virial mass at $z=5$. Best-fitting parameters to equation (10) with a small dependence on redshift. At $z=5$ we have $(\lambda_0, \alpha) = (0.033 \pm 0.0002, 2.25 \pm 0.04)$ changing to $(\lambda_0, \alpha) = (0.029 \pm 0.0004, 2.36 \pm 0.1)$ at $z=10$. Best-fitting parameters and errors are again derived using the MCMC method described in Section 3. Parameters for all redshifts are shown in Table B2 along with the numbers of haloes in each sample.

Both Muñoz-Cuartas et al. (2011) and Knebe & Power (2008) fit a lognormal to their spin distribution, with best-fitting parameters given by $\sigma_0 = 0.57$ (variance) and $\lambda_0 = 0.031$ (mean), and $\sigma_0 = 0.53$ and $\lambda_0 = 0.035$, respectively. Both have slightly higher spins overall, and a more extended tail at the upper end of the distribution. As found by Bett et al. (2007), equation (10) provides a better fit to our low spin distribution than does the lognormal. We find that unrelaxed haloes have a noticeable impact on our spin distribution, which is shown in Fig. 10. Without removing these haloes we find the best-fitting parameters to be $(\lambda_0, \alpha) = (0.042 \pm 0.0002, 2.70 \pm 0.04)$.

Fig. 11 shows the relation between spin and virial mass at $z=5$, with a power law best fit of the form

$$\lambda = A \left( \frac{M_{\text{vir}}}{10^{10} M_\odot h^{-1}} \right)^B.$$ 

(11)

A slight negative slope of $B = -0.01 \pm 0.0006$ at redshift $z=5$ decreases to $-0.023 \pm 0.016$ by $z=10$. Fits from previous work that study the Bullock spin parameter at high redshift (Knebe-Power at $z=10$ and Muñoz-Cuartas at $z=2$) are also shown. We find the scatter in the spin to be roughly constant in each mass bin. Parameter fits for all our redshifts are provided in Table B1 for relaxed haloes as well as the full population.

The existence of a small negative trend of spin parameter with mass at these redshifts is in qualitative agreement with Knebe & Power (2008), who find no trends at $z=0$ but an emerging trend at $z=10$. As noted, the virialized halo cut has an effect on our results. However, we find a small negative trend in both the full and relaxed population. For example, the $\lambda-M_{\text{vir}}$ relation for the full sample at redshift 5 is shown in Fig. 12. Without making cuts we find a relation with slope $A = -0.009$ at $z=5$, changing to $B = -0.029$ at redshift 10. Our spin mass relation with sample cuts is in agreement with the result of $B = -0.06 \pm 0.17$ from Knebe & Power (2008).

5 DISCUSSION AND CONCLUSION

We used N-body simulations to study concentrations and spins of DM haloes at $z=5-10$ and across the mass range $10^8 M_\odot h^{-1} < M < 10^{11} M_\odot h^{-1}$; corresponding to the regime relevant for studies of structure formation during the epoch of reionization. The dependence of these parameters on equilibrium state was investigated by...
splitting our halo sample into two populations which include (i) only relaxed haloes and (ii) the full population. We find qualitatively similar results to previous studies. However, we find quantitative differences between our derived $c(M)$ relations and spin-mass relations and those of previous studies, which we attribute to each author’s use of different halo finders, to our higher simulation resolution, and to the different relaxation criteria used for our sample. We find that the model proposed by Ludlow et al. (2014) reproduces both the slope and redshift evolution of our $c(M)$ relation.

Our key results are as follows.

(i) Our best-fitting concentration–mass relations at $z = 5$ have a slightly negative slope that becomes more shallow towards $z = 9$. Limiting our analysis to equilibrium haloes has a strong impact on the derived $c(M)$ relation due to unrelaxed haloes having lower concentrations at all masses and redshifts. Haloes with larger centre-of-mass offset ($\chi_{\text{CM}}$) typically have lower concentrations.

(ii) The slope of the $c(M)$ relation becomes shallower at higher redshift, although concentrations decrease at all masses. However, at high redshifts the number of haloes passing the equilibrium criteria is low: $\sim 30$ per cent of haloes in the 67.8 Mpc $h^{-1}$ box pass our resolution and relaxation cuts at $z = 5$. Such a high proportion of unrelaxed haloes at the mass scales studied here is a distinct property of the high-redshift universe, as discussed in Paper I. We find concentrations of relaxed haloes at $z > 5$ to be well described by the relation $c_{\text{vir}} = 3.2 (M/[10^{10} M_\odot h^{-1}])^{-0.03} [(1 + z)/10]^{-0.29}$ for NFW fits and $c_{\text{vir}} = 3.1 (M/[10^{10} M_\odot h^{-1}])^{-0.03} [(1 + z)/10]^{-0.29}$ for Einasto concentrations. The intrinsic scatter around the $c(M)$ relations is $\Delta c_{\text{vir}} \sim 1$ (or 20 per cent).

(iii) Without imposing equilibrium cuts on our sample, the concentrations found in Tiamat have similar values to those reported by DM14, Diemer & Kravtsov (2015) and Hellwing et al. (2016). Concentrations of haloes in Tiamat are a factor of $\Delta c_{\text{vir}} \sim 0.5$–1 lower than reported by Prada et al. (2012) and Klypin et al. (2016). The shallow negative trend in the $c(M)$ relation that flattens from $z = 5$ to 10, and the overall decrease in the magnitude of our Einasto concentrations agree well with Hellwing et al. (2016).

(iv) The distribution of Bullock spin parameters for relaxed haloes at $z \geq 5$ is found to be well fitted by equation (10) with $(\lambda_0, \alpha) \sim (0.033 \pm 0.0002, 2.25 \pm 0.04)$, with little evolution with redshift. Including unrelaxed haloes results in a spin distribution with a higher mean of $\lambda_0 = 0.042$.

(v) As in previous studies, we find a spin–virial mass relation with a slight negative correlation at high redshift. The trend found here has a slope $d \log \lambda / d \log M \sim -0.02$ at $z = 10$. The exclusion of unrelaxed haloes also has the effect of increasing the peak of the spin distribution while the slope of the $\lambda-M_{\text{vir}}$ relation remains slightly negative. Our best-fitting power-law relation for relaxed haloes at $z \geq 5$ is $\lambda = (0.029 \pm 0.004)(M/[10^{10} M_\odot h^{-1}])^{-0.009 \pm 0.006}$ and $\lambda = (0.038 \pm 0.002)(M/[10^{10} M_\odot h^{-1}])^{-0.009 \pm 0.003}$ for the full halo population.

The growth of DM haloes drives high-$z$ galaxy formation (Tacchella, Trenti & Carollo 2013), while the concentration and spin of haloes are key ingredients for semi-analytic models of galaxy formation (e.g. Croton et al. 2006). This study of these properties for haloes corresponding to the galaxies responsible for reionization will provide a valuable resource for understanding the framework of early galaxy formation.

ACKNOWLEDGEMENTS

This research was supported by the Victorian Life Sciences Computation Initiative (VLSCI), grant ref. UOM0005, on its Peak Computing Facility hosted at the University of Melbourne, an initiative of the Victorian Government, Australia. Part of this work was performed on the gSTAR national facility at Swinburne University of Technology. gSTAR is funded by Swinburne and the Australian Government’s Education Investment Fund. This research programme is funded by the Australian Research Council through the ARC Laureate Fellowship FL110100072 awarded to JSBW. ADL is financed by a COFUND Junior Research Fellowship. We thank Volker Springel for making the GADGET2 and SUBFIND codes available. We also thank N. Gnedin for useful comments on our manuscript.

REFERENCES

APPENDIX A: FITS FOR THE c(M) RELATION.

Table A1. Best-fitting values from the relaxed population for NFW-derived and Einasto-derived c(M) relations $c_{\text{vir}} = A(M_{\text{vir}}/10^{10} M_\odot h^{-1})^B$. $N_{\text{sample}}$ denotes the number of haloes in the sample for Tiamat, Medi Tiamat, Tiny Tiamat. Fits and errors are the median and 68 per cent confidence interval using the MCMC package quoted previously.

<table>
<thead>
<tr>
<th>z</th>
<th>Relaxed NFW best fit</th>
<th>Relaxed Einasto best fit</th>
<th>$N_{\text{sample}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>$c_{\text{vir}} = (3.8 \pm 0.4) \left( \frac{M}{10^{10} M_\odot h^{-1}} \right)^{-0.035\pm0.005}$</td>
<td>$c_{\text{vir}} = (3.8 \pm 0.4) \left( \frac{M}{10^{10} M_\odot h^{-1}} \right)^{-0.099\pm0.005}$</td>
<td>4443,738,864</td>
</tr>
<tr>
<td>6</td>
<td>$c_{\text{vir}} = (3.6 \pm 0.5) \left( \frac{M}{10^{10} M_\odot h^{-1}} \right)^{-0.023\pm0.006}$</td>
<td>$c_{\text{vir}} = (3.5 \pm 0.5) \left( \frac{M}{10^{10} M_\odot h^{-1}} \right)^{-0.022\pm0.007}$</td>
<td>2303,412,652</td>
</tr>
<tr>
<td>7</td>
<td>$c_{\text{vir}} = (3.4 \pm 0.6) \left( \frac{M}{10^{10} M_\odot h^{-1}} \right)^{-0.019\pm0.008}$</td>
<td>$c_{\text{vir}} = (3.3 \pm 0.6) \left( \frac{M}{10^{10} M_\odot h^{-1}} \right)^{-0.018\pm0.008}$</td>
<td>1043,221,417</td>
</tr>
<tr>
<td>8</td>
<td>$c_{\text{vir}} = (3.3 \pm 1.0) \left( \frac{M}{10^{10} M_\odot h^{-1}} \right)^{(0.005\pm0.013)}$</td>
<td>$c_{\text{vir}} = (3.3 \pm 1.2) \left( \frac{M}{10^{10} M_\odot h^{-1}} \right)^{(0.013\pm0.016)}$</td>
<td>391,109,267</td>
</tr>
<tr>
<td>9</td>
<td>$c_{\text{vir}} = (3.2 \pm 1.2) \left( \frac{M}{10^{10} M_\odot h^{-1}} \right)^{(0.003\pm0.016)}$</td>
<td>$c_{\text{vir}} = (3.2 \pm 1.5) \left( \frac{M}{10^{10} M_\odot h^{-1}} \right)^{(0.003\pm0.021)}$</td>
<td>134,49,155</td>
</tr>
</tbody>
</table>

Table A2. Best-fitting values for the full population of NFW and Einasto-derived c(M) relations parametrized with the form $c_{\text{vir}} = A(M/10^{10} M_\odot h^{-1})^B$. $N_{\text{sample}}$ denotes the number of haloes in the sample for the Tiamat, Medi Tiamat and Tiny Tiamat simulation, respectively.

<table>
<thead>
<tr>
<th>z</th>
<th>Full population NFW best fit</th>
<th>Full population Einasto best fit</th>
<th>$N_{\text{sample}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>$c_{\text{vir}} = (3.0\pm0.3) \left( \frac{M}{10^{10} M_\odot h^{-1}} \right)^{-0.026\pm0.004}$</td>
<td>$c_{\text{vir}} = (3.0\pm0.3) \left( \frac{M}{10^{10} M_\odot h^{-1}} \right)^{-0.024\pm0.005}$</td>
<td>18 696,3014,3652</td>
</tr>
<tr>
<td>6</td>
<td>$c_{\text{vir}} = (2.8\pm0.3) \left( \frac{M}{10^{10} M_\odot h^{-1}} \right)^{-0.015\pm0.004}$</td>
<td>$c_{\text{vir}} = (2.7\pm0.4) \left( \frac{M}{10^{10} M_\odot h^{-1}} \right)^{(0.000\pm0.006)}$</td>
<td>10 057,1918,2838</td>
</tr>
<tr>
<td>7</td>
<td>$c_{\text{vir}} = (2.7\pm0.3) \left( \frac{M}{10^{10} M_\odot h^{-1}} \right)^{-0.002\pm0.005}$</td>
<td>$c_{\text{vir}} = (2.7\pm0.5) \left( \frac{M}{10^{10} M_\odot h^{-1}} \right)^{(0.013\pm0.008)}$</td>
<td>4772,1068,2069</td>
</tr>
<tr>
<td>8</td>
<td>$c_{\text{vir}} = (2.7\pm0.4) \left( \frac{M}{10^{10} M_\odot h^{-1}} \right)^{(0.002\pm0.007)}$</td>
<td>$c_{\text{vir}} = (2.7\pm0.7) \left( \frac{M}{10^{10} M_\odot h^{-1}} \right)^{(0.027\pm0.011)}$</td>
<td>1978,565,1347</td>
</tr>
<tr>
<td>9</td>
<td>$c_{\text{vir}} = (2.7\pm0.5) \left( \frac{M}{10^{10} M_\odot h^{-1}} \right)^{(0.019\pm0.009)}$</td>
<td>$c_{\text{vir}} = (2.8\pm0.8) \left( \frac{M}{10^{10} M_\odot h^{-1}} \right)^{(0.040\pm0.013)}$</td>
<td>678,268,829</td>
</tr>
<tr>
<td>10</td>
<td>$c_{\text{vir}} = (2.7\pm0.9) \left( \frac{M}{10^{10} M_\odot h^{-1}} \right)^{(0.034\pm0.016)}$</td>
<td>$c_{\text{vir}} = (2.9\pm1.0) \left( \frac{M}{10^{10} M_\odot h^{-1}} \right)^{(0.057\pm0.015)}$</td>
<td>247,142,499</td>
</tr>
</tbody>
</table>
APPENDIX B: FITS FOR THE SPIN DISTRIBUTION AND SPIN–MASS RELATION

Table B1. Best-fitting values spin–mass relation for relaxed, and the full population of haloes, \( \lambda = A(M/10^{10} M_\odot h^{-1})^B \). \( N_{\text{relaxed(total)}} \) denotes the number of haloes in the relaxed (full population) sample for the Tiamat, Medi Tiamat and Tiny Tiamat simulation, respectively.

<table>
<thead>
<tr>
<th>( z )</th>
<th>Relaxed haloes best fit</th>
<th>( N_{\text{relaxed}} )</th>
<th>Full population best fit</th>
<th>( N_{\text{total}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>(0.029±0.004) ( \frac{M}{10^{10} M_\odot h^{-1}} ) (−0.009±0.006)</td>
<td>61 854,8510,8539</td>
<td>(0.038±0.002) (−0.009±0.003)</td>
<td>234 992,32 174,32 208</td>
</tr>
<tr>
<td>6</td>
<td>(0.028±0.004) ( \frac{M}{10^{10} M_\odot h^{-1}} ) (−0.013±0.007)</td>
<td>41 034,6156,6843</td>
<td>(0.036±0.003) (−0.014±0.004)</td>
<td>163 111,24 791,27 899</td>
</tr>
<tr>
<td>7</td>
<td>(0.028±0.005) ( \frac{M}{10^{10} M_\odot h^{-1}} ) (−0.016±0.008)</td>
<td>25 384,4258,5381</td>
<td>(0.035±0.003) (−0.020±0.004)</td>
<td>104 335,17 832,22 982</td>
</tr>
<tr>
<td>8</td>
<td>(0.027±0.006) ( \frac{M}{10^{10} M_\odot h^{-1}} ) (−0.021±0.010)</td>
<td>13 996,2673,4094</td>
<td>(0.034±0.004) (−0.021±0.005)</td>
<td>61 194,12 049,18 065</td>
</tr>
<tr>
<td>9</td>
<td>(0.025±0.007) ( \frac{M}{10^{10} M_\odot h^{-1}} ) (−0.026±0.013)</td>
<td>6607,1519,2832</td>
<td>(0.033±0.004) (−0.027±0.006)</td>
<td>31 159,7194,13 227</td>
</tr>
<tr>
<td>10</td>
<td>(0.025±0.009) ( \frac{M}{10^{10} M_\odot h^{-1}} ) (−0.023±0.016)</td>
<td>3393,836,1980</td>
<td>(0.032±0.005) (−0.029±0.008)</td>
<td>16 132,4303,9621</td>
</tr>
</tbody>
</table>

Table B2. Best-fitting values for distribution of relaxed population spin parameters according to equation (10). \( N_{\text{sample}} \) is the number of haloes in the sample from Tiamat.

<table>
<thead>
<tr>
<th>( z )</th>
<th>( \lambda_0 )</th>
<th>( \alpha )</th>
<th>( N_{\text{sample}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>0.033 ± 0.0002</td>
<td>2.25 ± 0.04</td>
<td>61 857</td>
</tr>
<tr>
<td>6</td>
<td>0.032 ± 0.0002</td>
<td>2.30 ± 0.05</td>
<td>41 035</td>
</tr>
<tr>
<td>7</td>
<td>0.031 ± 0.0002</td>
<td>2.34 ± 0.06</td>
<td>25 384</td>
</tr>
<tr>
<td>8</td>
<td>0.030 ± 0.0003</td>
<td>2.30 ± 0.07</td>
<td>13 996</td>
</tr>
<tr>
<td>9</td>
<td>0.030 ± 0.0003</td>
<td>2.30 ± 0.08</td>
<td>6607</td>
</tr>
<tr>
<td>10</td>
<td>0.029 ± 0.0004</td>
<td>2.36 ± 0.10</td>
<td>3393</td>
</tr>
</tbody>
</table>

This paper has been typeset from a TeX/LaTeX file prepared by the author.