# Einstein-Podolsky-Rosen Entanglement Strategies in Two-Well Bose-Einstein Condensates

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Criteria suitable for measuring entanglement between two different potential wells in a Bose-Einstein condensation are evaluated. We show how to generate the required entanglement, utilizing either an adiabatic two-mode or a dynamic four-mode interaction strategy, with techniques that take advantage of *s*-wave scattering interactions to provide the nonlinear coupling. The dynamic entanglement method results in an entanglement signature with spatially separated detectors, as in the Einstein-Podolsky-Rosen paradox.

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One of the most important questions in modern physics is the problem of macroscopic spatial entanglement, which directly impinges on the nature of reality. Here we analyze how rapid advances in Bose-Einstein condensation (BEC) in ultracold atoms can help to resolve this issue. Recently, the observation of spin squeezing has shown that measurement beyond the standard quantum limit is achievable [1–3]. Spin squeezing is known to demonstrate entanglement between atoms [4] but not which subsystems have been entangled. An important step forward beyond this would be to realize quantum entanglement in the Einstein-Podolsky-Rosen (EPR) [5] sense, that is, having two spatially separated condensates entangled with each other [6]. This is an important milestone towards future experiments involving entanglement of macroscopic mass distributions, thereby demonstrating quantum Schrödinger-cat-type superpositions of distinct mass distributions.

In this Letter, we analyze achievable entangled quantum states by using a two-well BEC and the measurable criteria that can be used to signify entanglement. Demonstration of spatial entanglement is a first step towards demonstrating a true EPR paradox [7,8], in which an inferred uncertainty principle is violated for even stronger correlations. The types of quantum state considered include number anticorrelated states prepared by using adiabatic passage, as well as dynamically prepared spin-squeezed states. In particular, we focus on spin entanglement, as a particularly useful route for achieving measurable EPR entanglement. Spin orientation is easily coupled to magnetic forces to allow superpositions of different mass distributions, once spin entanglement is present. Note the related recent work on other scenarios of EPR entanglement generation [9].

We show that existing experimental techniques are capable of generating spatial entanglement, with relatively minor changes. What is needed to generate entanglement is a combination of nonlinear local interactions to generate a state that is not coherent—typically a squeezed state or number state—together with a nonlocal linear interaction to produce entanglement between two spatially distinct locations. In the case of a BEC, the *s*-wave scattering provides a nonlinear local interaction (attraction or repulsion), although it is sufficient to simply have an overall number state between the two wells. At the same time, quantum diffusion across a potential barrier acts like a beam splitter to provide a nonlocal linear interaction.

Our conclusion is that the criterion used to measure entanglement must be chosen carefully. Not all measures of entanglement are equivalent, and there is an important question as to what one regards as the fundamental subsystems, i.e., particles or modes. The appropriate choice of measure depends on the entangled state, how it is prepared, and what type of detection is technologically feasible. We choose here to analyze two- and four-mode models of a BEC, indicated schematically in Fig. 1, where $a_1$ and $a_2$ are operators for two internal states at $A$ and $b_1$ and $b_2$ are operators for two internal states at $B$. In all cases we include Poissonian fluctuations in the atomic number, as typically found in experiment.

In the limit of tight confinement and small numbers of atoms, this type of system can be treated by using a simple coupled mode effective Hamiltonian, of the form

$$\hat{H}/\hbar = \kappa \sum_i \hat{a}_i^\dagger \hat{b}_i + \frac{1}{2}\Big[\sum_{ij} g_{ij}\hat{a}_i^\dagger \hat{a}_j^\dagger \hat{a}_j \hat{a}_i\Big] + \{\hat{a}_i \leftrightarrow \hat{b}_i\}. \quad (1)$$

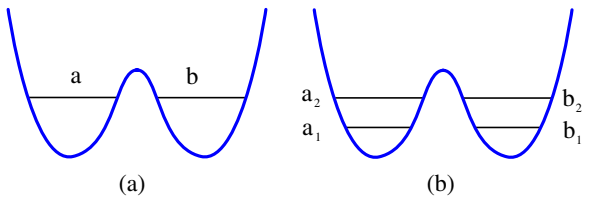


FIG. 1 (color online). (a) Two spatial modes $a$ and $b$; (b) two pairs of modes, each with two internal spin configurations, giving operators $a_1$, $a_2$ and $b_1$, $b_2$.

 

Here $\kappa$ is the interwell tunneling rate between wells, while $g_{ij}$ is the intrawell interaction matrix between the different spin components.

*Adiabatic preparation.*—We first consider two-mode states having a single spin orientation, with number correlations established by using adiabatic passage in the ground state. This makes them practical to prepare following earlier experimental approaches [1,10], as shown in Fig. 1(a). A recent multimode analysis shows that effects of other spatial modes may be relatively small [11]. In a two-mode analysis, we assume that $a_1$ and $b_1$ have been prepared in the many-body ground state of Eq. (1) with a mean number of atoms $N$, while the second pair of spin states $a_2$ and $b_2$ remain in the vacuum state, so that we can write $a \equiv a_1$ and $b \equiv b_1$. In these cases there is only one nuclear spin orientation, and there are existing experimental data on phase coherence and number correlations [1,10], with 10 dB relative number squeezing being maximally indicated.

A number of previous analyses have used entropic measures specific to pure states to study entanglement theoretically. These signatures cannot be readily measured and are not applicable to realistic mixed states that are typically created in the laboratory. Instead, one can demonstrate measurable spatial entanglement between the two wells $a$ and $b$ by using the non-Hermitian operator product criterion of Hillery and Zubairy [12]. This is also related to a recently developed continuous-variable Bell inequality criterion [13]. A sufficient entanglement criterion between $A$ and $B$ is the operator product measure: $|\langle \hat{a}^\dagger \hat{b}\rangle|^2 > \langle \hat{a}^\dagger \hat{a} \hat{b}^\dagger \hat{b}\rangle$. However, this measure is not robust against total number fluctuations. Instead, we propose using a modified operator measure, based on the normalized annihilation operator $\tilde{a} = \hat{a}\hat{N}^{-1/2}$, where $\hat{N} = \hat{N}_A + \hat{N}_B = \hat{a}^\dagger \hat{a} + \hat{b}^\dagger \hat{b}$. This corresponds to normalized fringe visibility measurements utilized in recent experiments, in order to remove technical noise due to number fluctuations [2,3].

A sufficient *phase-entanglement* (PE) criterion using the operator product measure is $|\langle \tilde{a}^\dagger \tilde{b}\rangle|^2 > \langle \tilde{a}^\dagger \tilde{a} \tilde{b}^\dagger \tilde{b}\rangle$. Using this inequality we can introduce a criterion for phase entanglement:

$$E_{\mathrm{PE}} = 1 - \frac{|\langle \hat{a}^\dagger \hat{b} \hat{N}^{-1}\rangle|^2 - \langle \hat{N}_A \hat{N}_B \hat{N}^{-2}\rangle}{\min[\langle \hat{N}_A \hat{N}^{-2}\rangle, \langle \hat{N}_B \hat{N}^{-2}\rangle]} < 1. \qquad (2)$$

The interference term in this criterion has the property that $\langle \hat{a}^\dagger \hat{b} \hat{N}^{-1}\rangle \sim \sqrt{N_A N_B} e^{i\phi}/N$. This is experimentally observed by expanding the two condensates and measuring the absorption imaging average fringe visibility, which has already been measured [1].

Theoretically, we find that two-well entanglement exists in the ground state with this criterion, although suppressed for increasingly strong repulsive interactions. Similar behavior is also known from previous studies using an entropic $\varepsilon(\rho)$ entanglement measure [6,14]. The strongest theoretical entropic entanglement is found when all atom numbers are equally represented. For fixed $N$, $\epsilon_{\max} = \log_2(N+1)$. We find that the closest state to this "superentangled" limit is obtained at a critical value of $Ng/\kappa \simeq -2$. This attractive interaction regime (as found in $^{39}$K and $^{7}$Li isotopes) gives rise to a maximal spread in the distribution of numbers in each well. Maximum entanglement results for this model have also been found [14] for entropic entanglement measures. In our calculations, we account for effects of finite temperatures by assuming a canonical ensemble of $\hat{\rho} = \exp[-\hat{H}/k_B T]$, with an interwell coupling of $\hbar\kappa/k_B = 50$ nK. Our results for the entropic and phase-entanglement signature are graphed in Fig. 2, showing that the PE measure is an excellent proxy for entropic entanglement. Our results show that two-well spatial entanglement is maximized for an attractive interatomic coupling. As shown in the figure, the effect is robust against Poissonian number fluctuations, even including thermal excitations.

*Dynamic preparation.*—To proceed further, EPR entanglement as we define it requires using measurements $O_A$ and $O_B$ that are individually defined either at well $A$ or well $B$. Thus, entanglement is shown by performing a set of simultaneous measurements on the spatially separated systems, typically by measuring correlations $\langle O_A O_B\rangle$ or $P(O_A, O_B)$. This is necessary to justify Einstein's "no action at a distance" assumption, that making one measurement at $A$ cannot affect the outcome of another measurement at $B$. One could achieve EPR entanglement with this criterion by making quadrature amplitude measurements, that is, by expanding $a = X_a + iP_a$ and $b = X_b + iP_b$, where $X_a$, etc., are "quadrature amplitudes," so that the moment $\langle ab^\dagger\rangle$ is measured as four separate real correlations. Proposed methods for measuring entanglement in BEC experiments include time-reversed dynamics [15] and four-wave mixing [8]. This shows that, in principle, such a quadrature-based entanglement measurement is not

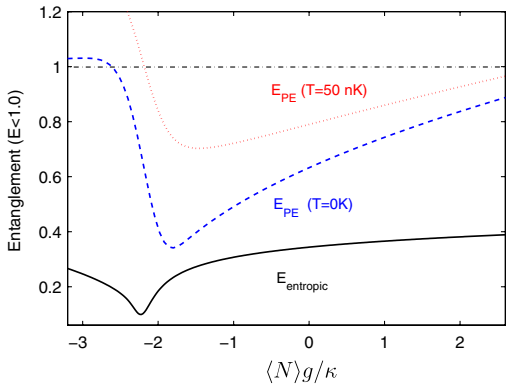


FIG. 2 (color online). Adiabatic entanglement with interactions in a two-well potential. Solid line: Entropic entanglement [$E_{\mathrm{entropic}} = 1 - \varepsilon(\rho)/\epsilon_{\max} < 1$] at $T = 0$ K, with a number state. Dashed and dash-dotted lines: Entanglement signature ($E_{\mathrm{PE}} < 1$) at $T = 0$ K and 50 nK, respectively, with a Poissonian mixture of numbers. In all cases, $\langle N\rangle = 100$.

impossible. However, while feasible optically, this type of measurement is nontrivial with ultracold atoms owing to interaction-induced phase fluctuations, and we propose a different strategy.

To get good EPR measurements we consider instead the intrawell "spins" $J^X$, $J^Y$, and $J^Z$ at sites $A$ and $B$. This means having at least four modes in total. To prove EPR entanglement by using these measurements, one can define the spin measurements at $A$ to be in terms of $a_1$ and $a_2$: $\hat{J}_A^X = (\hat{a}_1{}^\dagger \hat{a}_2 + \hat{a}_2{}^\dagger \hat{a}_1)/2$, $\hat{J}_A^Y = (\hat{a}_1{}^\dagger \hat{a}_2 - \hat{a}_2{}^\dagger \hat{a}_1)/(2i)$, $\hat{J}_A^Z = (\hat{a}_1{}^\dagger \hat{a}_1 - \hat{a}_2{}^\dagger \hat{a}_2)/2$, and $\hat{N}_A = \hat{a}_1{}^\dagger \hat{a}_1 + \hat{a}_2{}^\dagger \hat{a}_2$; also define raising and lowering operators as $\hat{J}_A^\pm = \hat{J}_A^X \pm i\hat{J}_A^Y$, and there is a similar definition for site $B$.

These are measurable locally by using Rabi rotations and number measurements, without local oscillators being required. The spin orientation measured at each site can be selected independently to optimize the criterion for the state used. One can then show EPR entanglement via spin measurements by using the spin version of the Heisenberg-product entanglement criterion [16]

$$E_{\text{product}} = \frac{2\sqrt{\Delta^2 \hat{J}_{AB}^{\theta\pm} \cdot \Delta^2 \hat{J}_{AB}^{(\theta+\pi/2)\pm}}}{|\langle \hat{J}_A^Y \rangle| + |\langle \hat{J}_B^Y \rangle|} < 1 \tag{3}$$

or the sum criterion [17]

$$E_{\text{sum}} = \frac{\Delta^2 \hat{J}_{AB}^{\theta\pm} + \Delta^2 \hat{J}_{AB}^{(\theta+\pi/2)\pm}}{|\langle \hat{J}_A^Y \rangle| + |\langle \hat{J}_B^Y \rangle|} < 1, \tag{4}$$

with general sum and difference spins $\hat{J}_{AB}^{\theta\pm} = \hat{J}_A^\theta \pm \hat{J}_B^\theta$ and $\hat{J}^\theta = \cos(\theta)\hat{J}^Z + \sin(\theta)\hat{J}^X$, respectively. Here the conjugate Schwinger spin operators $\hat{J}^\theta$ and $\hat{J}^{\theta+\pi/2}$ obey the uncertainty relation $\Delta\hat{J}^\theta \Delta\hat{J}^{\theta+\pi/2} \geq |\langle \hat{J}^Y \rangle|/2$.

In order to obtain ultracold atomic systems with four-mode entanglement, we consider a dynamical approach to EPR entanglement which utilizes phase as well as number correlations. This requires the BECs to evolve in time, in a similar way to successful EPR experiments in optical fibers [16,18]. This is very different from the previous scheme, as the atom-atom interaction appears explicitly as part of the time evolution. The best entanglement is obtained when the interaction between atoms of different spin is different from the interaction between the atoms of the same spin. In rubidium, this requires either using a Feshbach resonance to break the symmetry or else separating the two spin components spatially as in the successful fiber experiments [18] or in spin-squeezing atom-chip experiments [3]. At a Feshbach resonance, for alkali metals like $^{87}$Rb, the interactions between the different spin orientations are reduced compared to the self-interactions, thus generating this type of entanglement with both the spin orientations remaining *in situ* in the same trap potential.

To start with, we consider the conditions required to obtain the best squeezing of Schwinger spin operators by optimizing the phase choice $\theta$: $tg(2\theta) = 2\langle \hat{J}^Z, \hat{J}^X \rangle / (\Delta^2 \hat{J}^Z - \Delta^2 \hat{J}^X)$. Entanglement can be generated by the interference of two squeezed states on a 50:50 beam splitter with a relative optical phase of $\varphi = \pi/2$. This has been achieved experimentally in optical experiments [16].

Here we explicitly assume that $a_1$, $b_1$ and $a_2$, $b_2$ are initially in coherent states. This models the relative coherence between the wells obtained with a low interwell potential barrier, together with an overall Poissonian number fluctuation as typically found in an experimental BEC. We note that the coherent state also includes an overall phase coherence, which has no effect on our results. For simplicity, we suppose that the initial state is prepared in an overall four-mode coherent state by using a Rabi rotation: $|\psi\rangle = |\alpha\rangle_{a_1}|\alpha\rangle_{b_1}|\alpha\rangle_{a_2}|\alpha\rangle_{b_2}$.

Next, we assume that the interwell potential is increased so that each well evolves independently. Finally, we decrease the interwell potential for a short time, so that it acts as a controllable, nonadiabatic beam splitter [19], to allow interference between the wells, followed by independent spin measurements in each well. For dynamics, we assume a simple two-spin evolution per well, which is exactly soluble. We can treat this by using either Schrödinger or

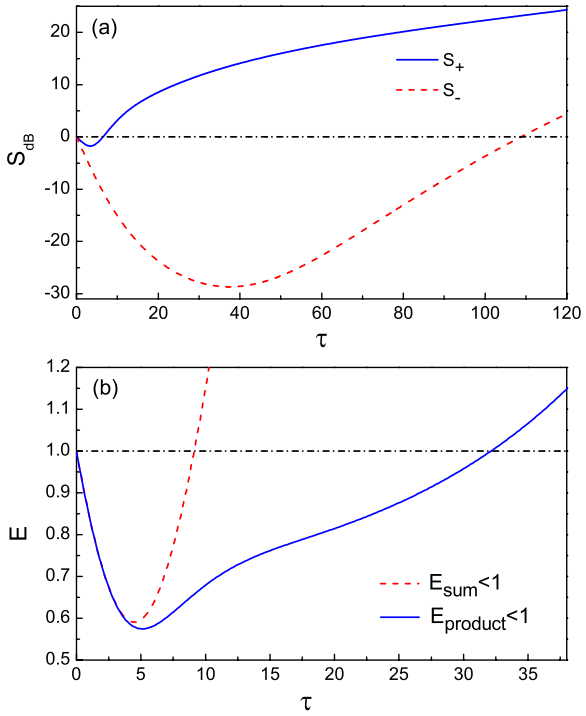


FIG. 3 (color online). (a) Squeezing of Schwinger spin operators $S_{\text{dB}}$: $S_+ = 10\log_{10}[\Delta^2(\hat{J}_A^\theta - \hat{J}_B^\theta)/n_0]$ (solid line), $S_- = 10\log_{10}[\Delta^2(\hat{J}_A^{\theta+\pi/2} + \hat{J}_B^{\theta+\pi/2})/n_0]$ (dashed line), and $n_0 = \frac{1}{2} \times (|\langle \hat{J}_A^Y \rangle| + |\langle \hat{J}_B^Y \rangle|)$ is shot noise (dotted line). (b) Entanglement ($E_{\text{product}}$) based on the criterion (3) by the solid curve and $E_{\text{sum}}$ in sum criterion (4) by the dashed curve. Here the parameters correspond to Rb atoms at magnetic field $B = 9.131$ G, with scattering lengths $a_{11} = 100.4a_0$, $a_{22} = 95.5a_0$, and $a_{12} = 80.8a_0$. $a_0 = 53$ pm. The coupling constant $g_{ij} \propto 2w_\perp a_{ij}$. Here $N_A = 200$ and $\tau = g_{11}N_A t$.

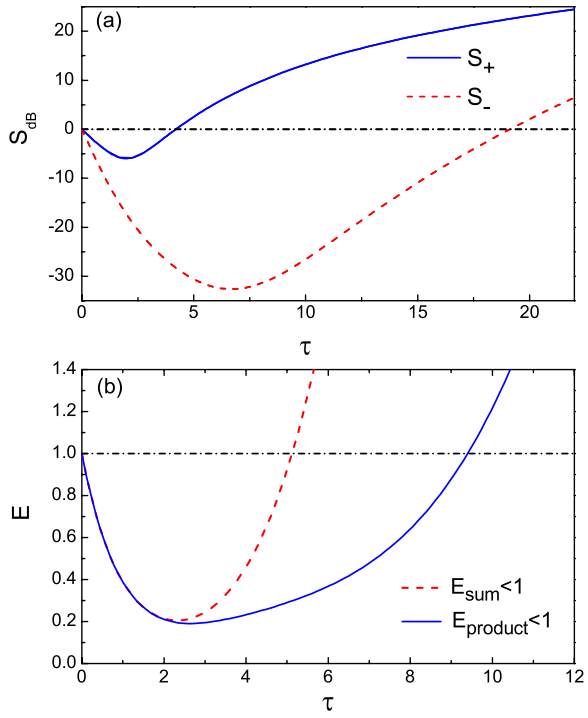


FIG. 4 (color online). The same as Fig. 3 but assuming $g_{12} = 0$.

Heisenberg equations of motion. In the Heisenberg case, since the number of particles is conserved in each mode, this has the solution

$$\hat{a}_i(t) = \exp\left[-i\sum g_{ij}\hat{N}_j t\right]\hat{a}_i(0), \tag{5}$$

where the couplings $g_{ij}$ are obtained from the known Rb scattering lengths at a Feshbach resonance.

After dynamical evolution from an initial coherent state, we find spin squeezing in each well, prior to using the beam splitter as shown in Fig. 3(a).

After using the beam splitter, entanglement can be detected in principle as $E < 1$, as shown in Fig. 3(b), which assumes the couplings between spins found at the rubidium Feshbach resonance. Note that Fig. 4 shows that assuming no cross couplings, i.e., $g_{12} = 0$, gives much better results still. This would require spatially separated condensates for each spin orientation, in order to eliminate cross couplings, as recently demonstrated by using magnetic gradient techniques [3]. In all cases the final squeezing or entanglement data are obtained from number difference measurements, as in recent intrawell interferometer experiments [2,3].

In summary, we have shown two feasible techniques for measuring EPR-type spatial BEC entanglement, by using currently available double-well BEC approaches combined with available atomic detection methods. The simplest method employs an attractive ground-state adiabatic method, with a single spin orientation. This requires an essentially nonlocal detection strategy, in which the two BECs are expanded and interfere with each other. To obtain a spatially separated EPR entanglement strategy for investigating questions of local realism, we propose a four-mode, dynamical strategy that employs two distinct spin orientations in each well.

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[1] J. Esteve, C. Gross, A. Weller, S. Giovanazzi, and M. K. Oberthaler, Nature (London) **455**, 1216 (2008).
[2] C. Gross, T. Zibold, E. Nicklas, J. Esteve, and M. K. Oberthaler, Nature (London) **464**, 1165 (2010).
[3] M. F. Riedel, P. Böhi, Y. Li, T. W. Hänsch, A. Sinatra, and Philipp Treutlein, Nature (London) **464**, 1170 (2010).
[4] A. Sorensen, L.-M. Duan, J. I. Cirac, and P. Zoller, Nature (London) **409**, 63 (2001).
[5] A. Einstein, B. Podolsky, and N. Rosen, Phys. Rev. **47**, 777 (1935).
[6] Andrew P. Hines, Ross H. McKenzie, and Gerard J. Milburn, Phys. Rev. A **67**, 013609 (2003).
[7] M. D. Reid, Phys. Rev. A **40**, 913 (1989); M. D. Reid *et al.*, Rev. Mod. Phys. **81**, 1727 (2009).
[8] A. J. Ferris, M. K. Olsen, E. G. Cavalcanti, and M. J. Davis, Phys. Rev. A **78**, 060104 (2008).
[9] Nir Bar-Gill, Christian Gross, Gershon Kurizki, Igor Mazets, Markus Oberthaler, preceding Letter, Phys. Rev. Lett. **106**, 120404 (2011).
[10] G. B. Jo *et al.*, Phys. Rev. Lett. **98**, 030407 (2007).
[11] A. J. Ferris and M. J. Davis, New J. Phys. **12**, 055024 (2010).
[12] M. Hillery and M. S. Zubairy, Phys. Rev. Lett. **96**, 050503 (2006).
[13] E. G. Cavalcanti, C. J. Foster, M. D. Reid, and P. D. Drummond, Phys. Rev. Lett. **99**, 210405 (2007).
[14] Q. Xie and W. Hai, Eur. Phys. J. D **39**, 277 (2006).
[15] M. Zhang, Kristian Helmerson, and L. You, Phys. Rev. A **68**, 043622 (2003).
[16] V. Giovannetti, S. Mancini, D. Vitali, and P. Tombesi, Phys. Rev. A **67**, 022320 (2003); R. Dong *et al.*, New J. Phys. **9**, 410 (2007).
[17] W. P. Bowen, N. Treps, R. Schnabel, and P. K. Lam, Phys. Rev. Lett. **89**, 253601 (2002).
[18] P. D. Drummond, R. M. Shelby, S. R. Friberg, and Y. Yamamoto, Nature (London) **365**, 307 (1993); R. Dong, J. Heersink, J. F. Corney, P. D. Drummond, U. L. Andersen, and G. Leuchs, Opt. Lett. **33**, 116 (2008).
[19] T. J. Haigh, A. J. Ferris, and M. K. Olsen, Opt. Commun. **283**, 3540 (2010).