Maximum steepness of oceanic waves: Field and laboratory experiments

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[1] The breaking of waves is an important mechanism for a number of physical, chemical and biological processes in the ocean. Intuitively, waves break when they become too steep. Unfortunately, a general consensus on the ultimate shape of waves has not been achieved yet due to the complexity of the breaking mechanism which still remains the least understood of all processes affecting waves. To estimate the limiting shape of ocean waves, here we present a statistical analysis of a large sample of individual wave steepness. Data were collected from measurements of the surface elevation in laboratory facilities and the open sea under a variety of sea state conditions. Observations reveal that waves are able to reach steeper profiles than the Stokes' limit for stationary waves. Due to the large number of records this finding is statistically robust. **Citation:** Toffoli, A., A. Babanin, M. Onorato, and T. Waseda (2010), Maximum steepness of oceanic waves: Field and laboratory experiments, *Geophys. Res. Lett.*, *37*, L05603, doi:10.1029/2009GL041771.

1. Introduction

[2] The breaking of deep water surface waves is an intrinsic feature of the ocean and appears in the form of sporadic whitecaps. Beneath and above the surface of breaking waves, a mixture of air and water generates a turbulent flow which is responsible for the exchange of gasses, water vapor, energy and momentum between the atmosphere and the ocean [*Melville*, 1996; *Jessup et al.*, 1997]. These processes play a very important role in many physical, chemical and biological phenomena in the upper-ocean layer and lower atmosphere. Apart from being directly responsible for the dissipation of energy in the wave field [*Komen et al.*, 1994], the breaking generates marine aerosols [*Jessup et al.*, 1997] which influence cloud physics, atmospheric radiation balance and hurricane dynamics [*Melville and Matusov*, 2002], changes the sea-surface roughness which moderates the air-sea momentum and energy exchange [*Babanin et al.*, 2007b] and facilitates the upper-ocean mixing [*Babanin et al.*, 2009]. Hence, appropriate account for the wave breaking physics and statistics is a fundamental part of applications ranging from forecasting the waves [e.g., *Babanin*, 2009] to the estimation of the global weather and climate [*Csanady*, 1990]. Furthermore, owing to the violent nature of steep breaking waves, ships and offshore structures may suffer serious damages especially in harsh sea conditions.

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[3] Because the wave breaking (whitecapping) plays such a vital role at the air-sea interface, there is a need for accurate and quantitative estimates of its properties. Among them, the ultimate steepness that breaking waves can reach is of particular interest. Unfortunately, breaking is a very complicated process and such properties have been elusive for decades. A full understanding of this mechanism and the ability to quantify it have been hindered by the strong nonlinearity of the process, together with its irregular and intermittent nature (an extended review of the breaking mechanism can be found in work by *Babanin* [2009]).

[4] Intuitively, it is reasonable to assume that an individual wave may no longer sustain its shape and hence break when its height becomes too large with respect to its length, i.e., the wave becomes too steep. Over a century ago, *Stokes* [1880] predicted theoretically that a regular, stationary progressive one-dimensional wave would become unstable and break only if the particle velocity at the crest exceeded the phase velocity. In terms of wave profile, this corresponds to a wave having an angle between two lines tangent to the surface profile at the wave crest of 120° (i.e., 60° on each side). In deep water, *Michell* [1893] found that this particular shape implies that the wave height (H) is 0.14 times the wavelength (L), which corresponds to a wave steepness $kH/2 = 0.44$, where $k = 2\pi/L$ is the wavenumber.

[5] However, finite amplitude Stokes-like waves tend to be unstable to modulational perturbations [*Zakharov*, 1966; *Benjamin and Feir*, 1967]. Thus, an initially regular wave train develops into a series of wave packets. Within such groups, individual waves can then grow and eventually break [*Longuet-Higgins and Cokelet*, 1978; *Melville*, 1982]. Interestingly enough, recent numerical and laboratory experiments [*Dyachenko and Zakharov*, 2005; *Babanin et al.*, 2007a] revisited the process of modulational instability and consequent breaking for initial quasi-monochromatic one-dimensional wave trains with mean steepness well below the value for the limiting Stokes' wave. These studies showed that the wave steepness of unstable individual waves does grow up to the threshold value of $kH/2 = 0.44$, after which the irreversible process of breaking begins.

[6] Another possible mechanism for the deep water wave breaking is linear dispersive focusing of waves [*Rapp and Melville*, 1990; *Pierson et al.*, 1992]. Such focusing will lead to a breaking onset also at a steepness of $kH/2 = 0.44$ [*Brown and Jensen*, 2001]. Most of the wave-focusing research has been conducted in quasi-one-dimensional environments [*Rapp and Melville*, 1990; *Pierson et al.*, 1992; *Brown and Jensen*, 2001], but the directional focusing has also been highlighted as a possible breaking cause of steep coherent wave trains [*Fochesato et al.*, 2007].

[7] One-dimensional studies deal with simplification of real ocean waves as they exclude effects related to the

directional properties of the wave fields. In this respect, laboratory experiments on the evolution of short-crested regular waves and wave groups [*She et al.*, 1994; *Nepf et al.*, 1998] suggested that the breaking onset is sensitive to wave directionality. In particular, breaking waves were observed to become bigger and with a steeper front as the directional spreading was increased; in contrast, the rear steepness was observed to be independent from wave directionality. A general quantitative consensus on the wave shape at the time of breaking, however, has not been achieved yet. Thus, a major question which still remains unanswered (and is the subject of the present Letter) pertains to the maximum (ultimate) shape that realistic ocean waves can exhibit. In order to provide an answer to the aforementioned question, here we present a statistical analysis of large samples of individual wave steepness which were collected from measurements of the surface elevation in laboratory facilities and open sea locations within a variety of sea state conditions.

2. Data Sets

[8] The advantage of using laboratory experiments is due to the fact that the wave conditions are under control. Two data sets from two independent directional wave basins were employed. One of the experiments took place at the University of Tokyo, Japan (Kinoshita Laboratory/Rheem Laboratory) [*Waseda et al.*, 2009]. The second one was conducted at the Marintek's ocean basin in Trondheim, Norway, which is one of the largest wave tanks in the world [*Onorato et al.*, 2009]. The experimental tests were carried out in a very simple way. A number of random wave fields were mechanically generated at the wave maker by imposing an input (initial) spectral energy density and randomizing the wave amplitudes and phases. A JONSWAP formulation was used to model the energy in the frequency domain and a $\cos^N(\vartheta)$ directional function was used for the directional domain [*Komen et al.*, 1994]. Different combinations of significant wave height, peak period and directional spreading (from unidirectional to directional sea states) were tested [*Onorato et al.*, 2009; *Waseda et al.*, 2009]. However, the peak period (≈1 s) was chosen to have deep-water waves only. We mention that the random tests were mainly performed to study the statistical properties of extreme waves. Therefore, the initial conditions were selected such that the occurrence of wave breaking was minimized; spectral conditions with steepness $k_pH_s/2 \leq 0.16$, where k_p is the spectral peak wavenumber and H_s is the significant wave height, were used to this end. A number of tests were also performed with higher steepness ($k_pH_s/2 > 0.2$) so that waves were forced to reach their breaking limit. In addition, a series of experiments specifically designed to study the wave breaking were performed by generating individual two-dimensional wave groups (only at the University of Tokyo). As the wave field propagated along the tank, the surface elevations were monitored by measuring time series at different locations with wire resistance wave gauges.

[9] The use of mechanically-generated waves provides a clear overview of effects related to the dynamics of the wave field not influenced by the wind forcing. Additionally, we also investigated field observations, i.e., time series of the surface elevations obtained in real directional wind-generated waves under a broad variety of conditions. Field measurements were collected at two distinctly different locations: one in the northwestern part of the Black Sea [*Babanin and Soloviev*, 1998] and a second one in the Indian Ocean off the North-West coast of Australia [*Young*, 2006]. The latter data set, which were collected by Woodside Energy Ltd. at the North Rankin A Gas Platform, contains observations of harsh sea conditions including several tropical cyclones between 1995 and 1999. Unlike the Black Sea data set which was recorded with wire resistance wave gauges, data at North Rankin were collected with directional wave buoys. A discussion on the differences between Lagrangian and Eulerian sensors can be found in work by *Longuet-Higgins* [1986].

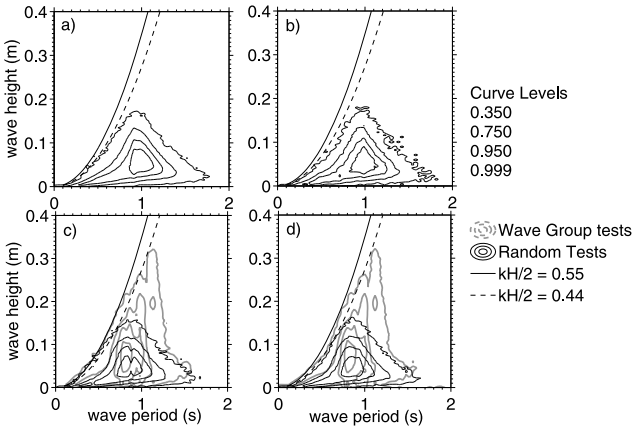

Figure 1. Joint cumulative distribution function of wave height and period. Wave fields with (top) $k_pH_s/2 \leq 0.16$: (a) downcrossing waves and (b) upcrossing waves. (bottom) Wave fields with $k_pH_s/2 > 0.2$: (c) downcrossing waves and (d) upcrossing waves. The curves represent the non-exceedance probability levels: lowest probability (inner curve); highest probability (outer curve). Curve of equal steepness are presented for comparison: $kH/2 = 0.55$ (solid line); $kH/2 = 0.44$ (dashed line).

[10] From the recorded surface elevations, we extracted individual waves by using zero-downcrossing and upcrossing detection which assume that an individual wave is the portion of a record between two consecutive zero-downcrossing or upcrossing points respectively. The wave height is then defined as the vertical distance between the lowest and the highest elevation, while the wave period is the time interval between two consecutive zero-downcrossing (or upcrossing) points. As aforementioned, the wave steepness is defined as the wavenumber times half the wave height. Because of the nonlinear nature of breaking waves, the wavenumber of individual waves is calculated from the wave period using a nonlinear dispersion relation [*Yuen and Lake*, 1982]. We mention that the downcrossing definition provides a measure of the steepness at the wave front, while the upcrossing definition provides a measure of the steepness at the wave rear. On the whole, about 5×10^5 individual waves were extracted from each set of observations; waves shorter than 0.5 times the peak period were excluded from the analysis though.

3. Limiting Steepness of Individual Waves

[11] An overview of the individual wave shape is provided by the joint cumulative distribution function of the local

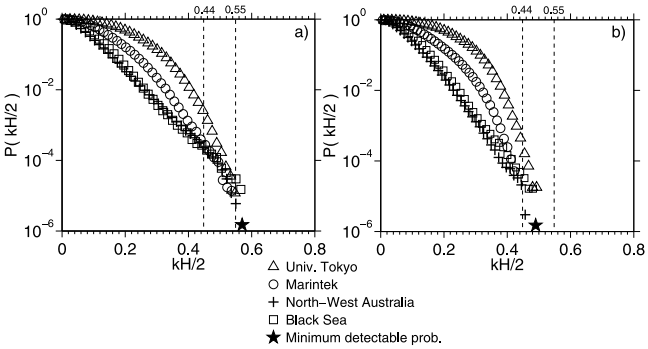

Figure 2. Wave steepness distribution for (a) downcrossing and (b) upcrossing waves. The star indicates the minimum possible level that could be detected with a sample of 5×10^5 observations.

(individual) wave height and period. This is presented in Figure 1 from data collected in the laboratory facilities only. From visual observations, we know that individual waves recorded under initial spectral conditions with $k_pH_s/2 \leq 0.16$ (hereafter test A, Figures 1a and 1b) seldom reached the breaking point, while for initial $k_pH_s/2 > 0.20$ (hereafter test B, Figures 1c and 1d) wave breaking was a distinctive feature.

[12] The distribution indicates that there exists an upper bound for the wave shape. This limit can be conveniently described by curves with constant steepness. For test A, where waves were generally far from breaking, the profile was rather symmetric. In this respect, the joint distributions of downcrossing and upcrossing waves show a similar upper limit slightly below 0.44, which corresponds to the breaking onset for unidirectional waves [see *Babanin et al.*, 2007a]. For the steeper sea states in test B, on the other hand, waves were more prone to breaking. At the point of breaking the waves are symmetric, but while already breaking they become asymmetric and with a steeper front [*Babanin et al.*, 2007a]. In general, the change of wave shape is more related to the reduction of the downcrossing wave period (shortening of the wave front) rather than to the increase of wave height. In the joint distribution, the increase of the front-face or downcrossing wave steepness is reflected by the enhancement of the upper bound, which rises up to the value of 0.55 (Figure 1c). A visual analysis of waves approaching this critical steepness, i.e., waves with $kH/2 > 0.44$, suggests that these waves are already breaking rather than imminent breakers [*Babanin et al.*, 2007a]. In this respect, although the final collapse of the wave structure can occur anytime after the breaking onset, waves do not appear to overcome a downcrossing steepness of 0.55. This is the maximal steepness that water surface waves seem to be able to reach.

[13] It is interesting to note, however, that the upper limit of the joint distribution is reduced for period close to or greater than the initial peak wave period (≈1 s). Because waves are subjected to a shortening of the downcrossing period as they are about to break, it is not totally unexpected to observe a concentration of very steep waves at periods lower than the dominant. A similar result was also recovered from the independent set of wave group experiments.

[14] On the contrary, we observed that the wave rears did not modify substantially their shape, in agreement with previous three dimensional observations by *Nepf et al.* [1998]. As a result, despite the more frequent occurrence of breaking, the upper bound for upcrossing waves does not deviate from the limiting value of 0.44 (Figure 1d). Nonetheless, unlike the random tests, the wave group experiments show that upcrossing waves can actually exceed this threshold limit, at least within a short range of periods (see Figure 1d). Again, these must be the waves already breaking as the limiting rear-face steepness at the breaking onset is 0.44 [*Babanin et al.*, 2007a]. The distribution remains notably below the limit of 0.55 though. It is also important to mention that both limits (downcrossing and upcrossing) were not particularly sensitive to the variation of the directional spreading.

[15] It is now instructive to analyze the probability density function of the wave steepness. In Figure 2, the exceedance probability of the steepness is presented for the downcrossing and upcrossing definition respectively; all data sets, i.e., the laboratory and field observations, are displayed. Because the joint distribution of wave height and period is upper bounded by a limiting steepness, it is reasonable to expect that the tail of the probability density function would not extend farther than the aforementioned limits. In this respect, we saw that the distribution of the front-face steepness (Figure 2a) drops at a maximum value of about 0.55 and a probability level of 10^{-5}. Considering that the total number of observations in our sample is $N = 5 \times 10^5$, the minimum detectable probability level corresponds to $1/N = 2 \times 10^{-6}$ (see, e.g., Figure 2a). This level is about one order of magnitude lower than the one actually detected. Thus the maximum steepness can be regarded as a cut-off limit. Interestingly enough, this threshold also appears to be independent from the nature of the observations as it is in fact obtained from all the sets of measurements. Likewise, the distribution of the rear-face steepness (Figure 2b) drops at a limiting value closer to 0.44. Nonetheless, the field observations show a slightly higher limit than in the random laboratory experiments, in agreement with the finding in the wave group tests. It is important to stress that, while the laboratory and field probability density functions are essentially different, their cutoffs are close. This highlights the notion that the maximal possible steepness of deep water breaking waves is not a feature of wave-development conditions or environmental circumstances, but is rather a property of water surface in the gravity field.

[16] The probability distribution may suffer of statistical uncertainty, especially towards low probability levels (tail of the distribution). In this respect, an estimate of the 95% confidence intervals was calculated by means of bootstrap methods, which are based on the reproduction of random copies of the original data set [see, e.g., *Emery and Thomson* 2001]. Because of the large number of observations, the 95% confidence intervals remain rather small. At probability levels as low as 10^{-5} (i.e., exceedance probability for the maximum detected steepness), the degree of uncertainty is one order of magnitude smaller than the expected value of steepness. Thus, we can regard our estimate for the exceedance probability as statistically significant. It is however important to mention that maximum steepness can also be subject to uncertainty which derives from the fluctuation of the zero-crossing point due to short waves riding

on top of the long wave mainly and hence perturbs the wave period.

4. Conclusions

[17] We presented an analysis of the steepness of individual waves. Observations were collected from independent laboratory and field measurement campaigns under a broad variety of sea state conditions and mechanically generated directional wave fields. Despite the diversity of the observations, all data sets showed consistent results. Precisely, the findings indicate that there exists a well defined value for the wave steepness above which waves can no longer sustain their shape. In terms of front-face steepness, this ultimate threshold is equivalent to a steepness of 0.55, which is notably higher than the Stokes' limit for stationary waves. In terms of the rear-face steepness, however, the threshold values is slightly above 0.44, confirming a certain asymmetry of the ultimate shape. These limits were not significantly affected by the directional spreading. Moreover, due to the large number of observations involved, this finding is statistically robust.

[18] It is important to clarify that the aforementioned limits only represent the maximum steepness that water surface waves can reach. This implies that the structure of a breaking wave can collapse anytime after the onset of the process. In the course of the breaking, however, we can expect with high confidence that the steepness becomes higher than the onset threshold of 0.44 and likely reaches a value around 0.55. Nevertheless, the upper bound is subject to some uncertainty which originates from the fluctuation of the zero-crossing points due to short waves riding on top of the long wave and hence perturbs the wave period and not so much the wave height. The precise upper bound should be determined from hydrodynamic consideration in a more deterministic manner.

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