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The influence of magnetic field on convection in an inclined ferrofluid layer heated from below

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Abstract. Ferrofluids are strongly magneto-polarisable nanofluids. Their flows can be non-intrusively controlled by applying an external magnetic field. One of their prospective applications is as a heat carrier in thermal management systems operating in conditions where natural convection is suppressed due to extreme confinement (microelectronics) or reduced gravity (orbital stations). The linear and weakly nonlinear flow stability analyses that are presented here illustrate an intricate interplay between thermogravitational and thermomagnetic mechanisms of convection in one of the practically important geometrical setups, an inclined fluid layer heated from below. The low-dimensional amplitude evolution equations of Landau type are derived to model the physical phenomena of interest. The solutions of the so-obtained dynamical system show that the application of magnetic field can indeed trigger convection in regimes where natural convection cannot exist, thus enhancing heat transfer. At the same time in regimes where both magnetic and gravitational buoyancy mechanisms are active the competition between the two may suppress the overall heat exchange, which has to be kept in mind in designing practical heat management systems.

1. Introduction

Ferrofluids are colloidal liquids containing nanoscale ferromagnetic particles suspended in a carrier fluid (frequently, an organic solvent). Given their small size (average particle diameter ~ 10 nm [6]), particles in ferrofluids are permanently suspended by Brownian motion and a thermal agitation disperses them approximately uniformly within a carrier fluid under normal conditions so that they contribute to the overall magnetic response of the fluid.

Because of their unique properties, over the past few decades ferrofluids found a vast variety of applications. A specific motivation for the current study is the use of ferrofluids as magnetically controllable heat carrier agents. The basic physical principle behind it is the appearance of the so-called Kelvin force $M\nabla H$ driving a magnetised fluid in the direction of magnetic field gradient ∇H . A warmer fluid has a smaller magnetisation M [3, 1] and thus it tends to be displaced by a cooler and thus stronger magnetised fluid, which leads to a flow referred to as thermomagnetic convection. In contrast to natural convection arising due to the thermal expansion, thermomagnetic convection can be induced in the absence of the gravity thus defining a potential use of ferrofluids in the reduced gravity conditions such as those existing on orbital stations. The investigation of interaction between gravitational and thermomagnetic types of convection is the subject of the current study.



2. Problem formulation

We consider a layer of a ferromagnetic fluid that fills the gap between two infinitely long and wide parallel non-magnetic plates. The layer is inclined at an angle ε with respect to the horizontal. The distance between two plates is $2d$ and the plates are maintained at constant different temperatures $T_* \pm \Theta$. An external uniform magnetic field, \mathbf{H}^e such that $|\mathbf{H}^e| = H^e$ is applied normally to the layer. This field induces an internal magnetic field \mathbf{H} such that $|\mathbf{H}| = H$ within the layer. The fluid magnetisation \mathbf{M} such that $|\mathbf{M}| = M$ is assumed to be co-directed with the internal magnetic field: $\mathbf{M} = \chi_* \mathbf{H}$, where χ_* is the integral magnetic susceptibility of the fluid. We choose the right-hand system of coordinates (x, y, z) with the origin in the mid-plane of the layer in such a way that the plates are located at $x = \pm d$ and the y and z axes are parallel to the plates. Assuming that the temperature difference 2Θ between the walls is sufficiently small, we adopt the Boussinesq approximation of the continuity, Navier-Stokes and thermal energy equations that upon appropriate scaling are written in a nondimensional form as [3, 7, 5]

$$\nabla \cdot \mathbf{v} = 0, \quad (1)$$

$$\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{v} = -\nabla P + \nabla^2 \mathbf{v} - Gr\theta \mathbf{e}_g - Gr_m \theta \nabla H, \quad (2)$$

$$\frac{\partial \theta}{\partial t} + \mathbf{v} \cdot \nabla \theta = \frac{1}{Pr} \nabla^2 \theta, \quad (3)$$

$$\nabla \times \mathbf{H} = \mathbf{0}, \quad (4)$$

$$(1 + \chi)(\nabla \cdot \mathbf{H} - \nabla \theta \cdot \mathbf{e}_H) + \frac{(\chi_* - \chi)N - (1 + \chi)\theta}{H} (\nabla \cdot \mathbf{H} - \nabla H \cdot \mathbf{e}_H) = 0, \quad (5)$$

$$\mathbf{M} = [\chi H + (\chi_* - \chi)N - (1 + \chi)\theta] \cdot \mathbf{e}_H, \quad (6)$$

with boundary conditions

$$[(1 + \chi)(H \pm 1) + (\chi_* - \chi)N] \cdot \mathbf{e}_H - \mathbf{H}^e \cdot \mathbf{n} = 0, \quad (7)$$

$$\mathbf{v} = 0, \quad \theta = \mp 1 \quad \text{at} \quad x = \pm 1. \quad (8)$$

The four major dimensionless parameters involed in governing equations are

$$Gr = \frac{\rho_*^2 \beta_* \Theta g d^3}{\eta_*^2}, \quad Gr_m = \frac{\rho_* \mu_0 K^2 \Theta^2 d^2}{\eta_*^2 (1 + \chi)}, \quad Pr = \frac{\eta_*}{\rho_* \kappa_*}, \quad N = \frac{H_* (1 + \chi)}{K \Theta}. \quad (9)$$

Thermo-gravitational and magnetic Grashof numbers Gr and Gr_m characterise buoyancy and magnetic forces. Prandtl number Pr is the ratio of viscous and thermal diffusion and parameter N represents the strength of magnetic field at the reference location. In the above, $\rho_* = \rho(T_*)$ is the fluid density at the average temperature T_* and $\mu_0 = 4\pi \times 10^{-7}$ H/m is the magnetic constant. The thermal diffusivity κ_* , the dynamic viscosity η_* , the coefficient of thermal expansion β_* , the differential magnetic susceptibility $\chi = \partial M / \partial H|_{(H_*, T_*)}$ and the pyromagnetic coefficient $K = -\partial M / \partial T|_{(H_*, T_*)}$ of a working fluid are assumed to be constant. The gravity vector is $\mathbf{g} = g \mathbf{e}_g = g(-\cos \varepsilon, -\sin \varepsilon, 0)$.

Equations (1)–(8) admit steady solution of the form

$$\theta_0 = -x, \quad (u_0, v_0, w_0) = (0, \frac{Gr}{6}(x^3 - x) \sin \varepsilon, 0), \quad (10)$$

$$P_0 = -\frac{Gr}{2} x^2 \cos \varepsilon + Gr_m \int_0^x s e_{10}(s) DH_{x_0} ds + const. \quad (11)$$

with the cross-layer component of the magnetic field satisfying

$$((1 + \chi)(H_0 - \theta_0) + (\chi_* - \chi)N) H_{x_0} = H_x^e H_0. \quad (12)$$

Subsequently, we investigate linear and weakly nonlinear stability properties of this basic flow solution as detailed in Section 3.

3. Basic flow stability analysis

Next we investigate the stability of the basic state discussed in Section 2 with respect to small amplitude disturbances that are assumed to be periodic in the y and z directions. We apply a standard normal mode hypothesis for separation of variables and write the perturbed quantities at the leading order in amplitude as

$$W = W_0(x) + [A_1 e^{\sigma_1 t + i(\alpha_1 y + \beta_1 z)} W_1(x) + A_2 e^{\sigma_2 t + i(\alpha_2 y + \beta_2 z)} W_2(x) + c.c.] + H.O.T., \quad (13)$$

where $\sigma_{1,2}$ is the complex amplification rate, $\alpha_{1,2}$ and $\beta_{1,2}$ are wavenumbers in the y and z directions, respectively, and $c.c.$ denotes the complex conjugate of the expression in brackets. The above representation enables us to investigate flow patterns arising when basic flow becomes unstable with respect to up to two disturbance modes. If amplitudes $A_{1,2}$ are assumed to be infinitesimal, the problem reduces to standard linear stability analysis. On the other hand, due to the nonlinearity of the governing equations (2)–(6), the assumption that the amplitudes are small but finite leads to a hierarchy of equations involving higher order terms ($H.O.T.$) arising at progressively higher orders of amplitudes. They are too lengthy to be given here explicitly but their full derivation can be found in [4]. The systematic analysis of hierarchical equations reveals that the amplitudes must satisfy a system of coupled Landau equations

$$\frac{dA_1}{dt} = \sigma_1 A_1 + K_{11} A_1 |A_1|^2 + K_{12} A_1 |A_2|^2, \quad \frac{dA_2}{dt} = \sigma_2 A_2 + K_{21} A_2 |A_1|^2 + K_{22} A_2 |A_2|^2. \quad (14)$$

Coefficients K_{11} and K_{22} are referred to as the first Landau constants. They describe saturation due to nonlinear self-interaction of instability modes. It results in the equilibrium amplitudes $|A_{ei}| = \sqrt{-\frac{\sigma_i}{K_{ii}}}$ provided σ_i and K_{ii} have opposite signs (of interest here are supercritical regimes with $\sigma_i > 0$ requiring $K_{ii} < 0$ to ensure the existence of a saturated state). Coefficients K_{ij} , $i \neq j$ describe mode interaction and classify mode i as an acceptor (donor) if $K_{ij} > 0$ ($K_{ij} < 0$).

4. Results and discussion

The results have been computed for a typical ferrofluid ($Pr = 55$, $\chi = \chi_* = 3$, $N = 100$) layer inclined at angle $\varepsilon = 10^\circ$ with respect to horizontal in the absence of magnetic field at $Gr_m = 0$ and when the layer is placed in a uniform normal magnetic field at $Gr_m = 5$. As seen from Figure 1(a), in the former case gravitational convection in the form of stationary rolls aligned with the slope ($\alpha = 0$) sets only when Grashof number exceeds the critical value of $Gr_c \approx 2.07$ (at this point the perturbation growth rate σ crosses zero, see the dashed line). In contrast, when magnetic field is applied, stationary magnetoconvection exists at any Gr including zero (the solid line). This means that magnetoconvection can exist even in the absence of gravity. The corresponding convection patterns are characterised by noticeably different wavenumbers, see Figure 1(b): at the onset thermogravitational convection rolls are much more sparsely spaced than their thermomagnetic counterparts (compare the solid and dashed lines). As Gr increases the second instability mode with a shorter wavelength arises shown by the thin lines. However, it has a considerably smaller growth rate and, as will be discussed below, does not contribute to the overall cross-layer heat transfer.

Weakly nonlinear consideration of convection patterns enables us to make a number of further conclusions. As evidenced by Figure 1(c) the values of the first Landau constants K_{11} and K_{22} remain negative for all convection modes. This means that convection always sets as a result of a supercritical bifurcation and its onset is accurately predicted by linear analysis. At the same time, mode coupling coefficients shown in panel (d) have opposite signs: $K_{12} > 0$, $K_{21} < 0$, meaning that the second mode serves as the energy donor for the first one appearing at smaller Grashof numbers. While hypothetically the modes shown by thin lines in Figure 1(e) can exist and achieve finite equilibrium amplitudes $|A_{2e}|$, such states are found to be unstable with

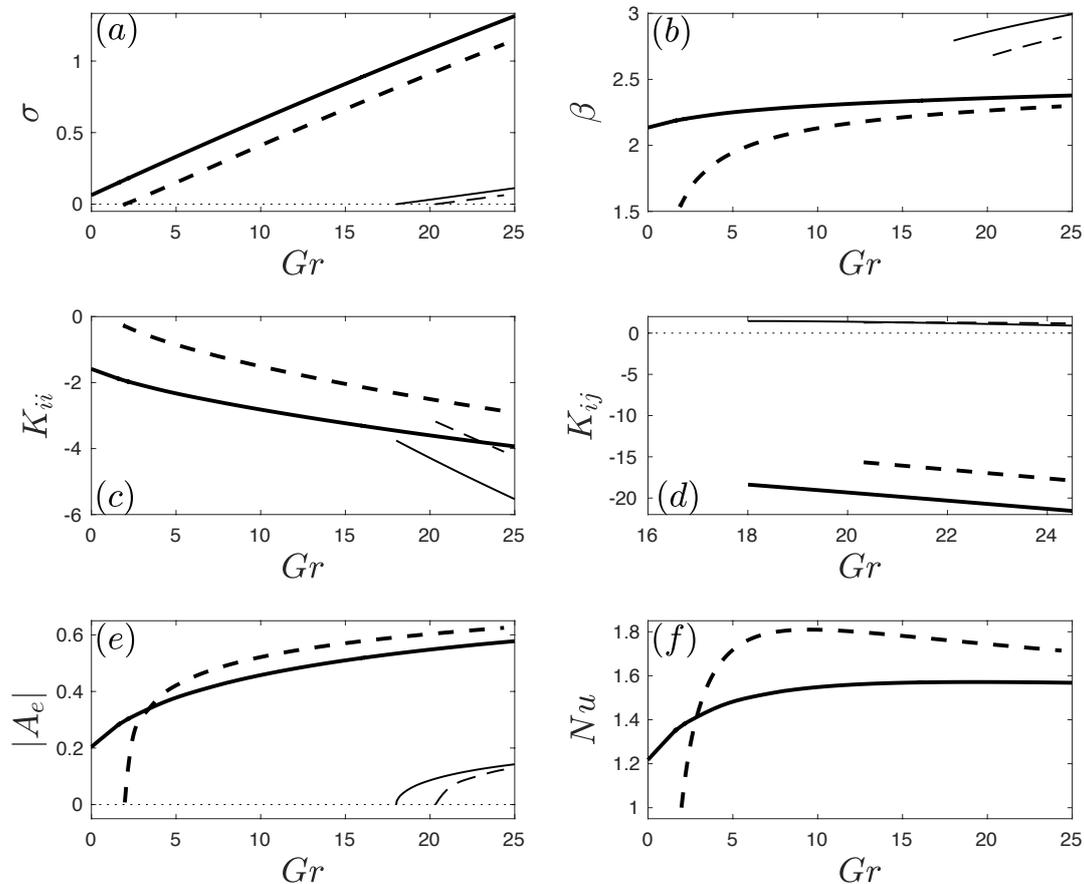


Figure 1. Comparison of thermomagnetic and thermogravitational convection pattern characteristics in a slightly inclined layer of ferrofluid: (a) linear growth rate, (b) wavenumber of longitudinal rolls, (c,d) Landau constants, (e) equilibrium convection roll amplitudes, (f) Nusselt number. Solid and dashed lines correspond to thermomagnetic and mixed thermomagneto-gravitational convection, respectively. Thick and thin lines denote stable and unstable convection modes in panels (a – c), (e) and (f) and donor and acceptor modes in panel (d), respectively.

modes shown by thick lines suppressing them completely. This is illustrated in Figure 2, where the temporal evolution of mode amplitudes is illustrated indicating that as long as the initial value $A_1(0) \neq 0$ the amplitude $A_2(t)$ of the second convection mode decays to zero. Therefore, it is expected that in convection experiments in a slightly inclined layer convection patterns will always be represented by a single instability mode corresponding to longitudinal rolls with wavenumbers shown by thick lines in Figure 1(b). The structure of such rolls is illustrated in Figure 3. The distortion of thermal field θ leads to the variation of the magnetisation field M : warmer fluid becomes less magnetised [3, 1, 7, 2]. Because of Maxwell's condition prescribing that the component of magnetic induction $B = \mu_0(H + M)$ normal to the layer must be preserved, the magnetic field H remains stronger wherever M is smaller. This leads to an inherently unstable situation when stronger magnetised cool fluid near the right wall is drawn towards the regions with a stronger magnetic field near the hot wall. Such a (Kelvin) force field accelerates the fluid across the layer as seen in the leftmost panel in Figure 3 and convection rolls are formed. It is also noteworthy that the application of a magnetic field creates magnetic pressure inside

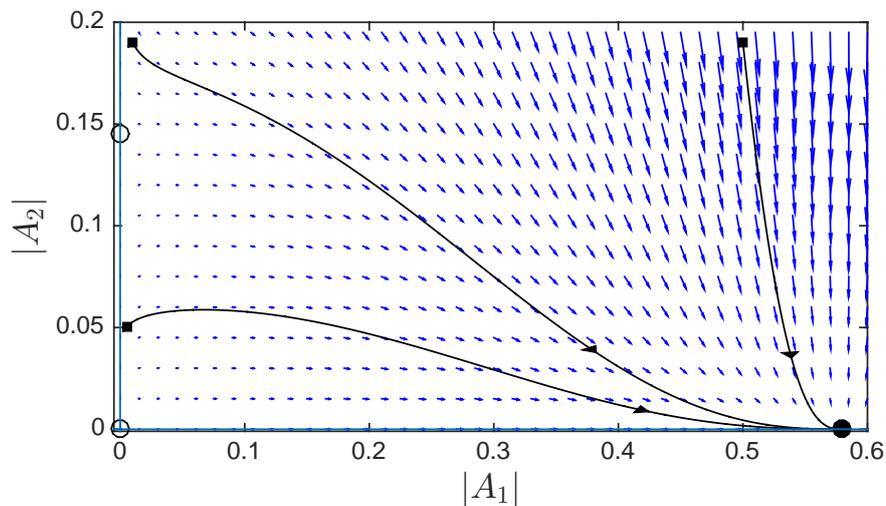


Figure 2. Amplitude flow diagram demonstrating the interaction of two instability modes with amplitudes A_1 (dominant energy acceptor) and A_2 (enslaved energy donor). The empty and filled circles denote unstable and stable fixed points (equilibria), the squares mark initial conditions for trajectories of system (14) shown by the solid lines.

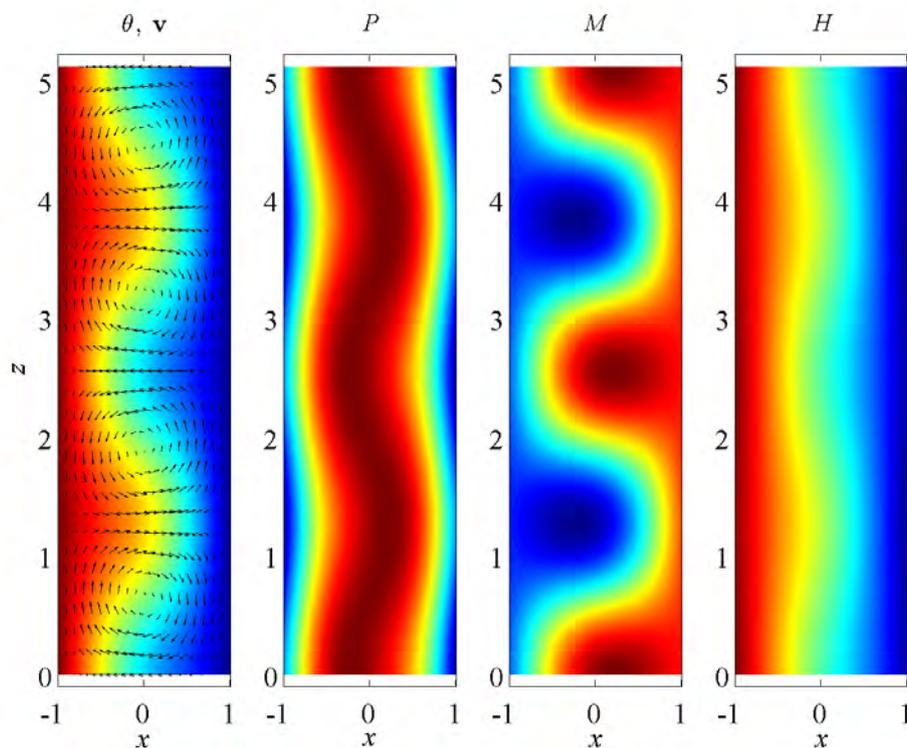


Figure 3. Cross-section view of vertical thermomagnetic rolls at $(Gr_m, Gr) = (5, 0)$. Red and blue regions correspond to large and small values of the depicted fields, respectively.

the layer that may be sufficiently strong to bulge the walls of a working chamber containing

ferrofluid when it is placed in an external magnetic field. This fact has to be taken into account when designing practical heat management systems using ferrofluids.

The knowledge of convection amplitudes shown in Figure 1(e) enables us to estimate Nusselt number $Nu = -\frac{d\theta}{dx}$ characterising the cross-layer heat flux and this leads to a somewhat unexpected observation. While magnetoconvection can set and significantly enhance heat flux in the absence of gravity, when both gravitational and magnetic forcing are present, that is when magnetic field is applied to the fluid layer in normal gravity conditions, the heat flux enhancement due to the arising mixed thermomagneto-gravitational convection is weaker than that observed in the same thermal and gravitational conditions in the absence of the field, compare the solid and dashed lines in Figure 1(f). Such an effect needs to be kept in mind when designing heat management system using magnetically controllable heat carrier media in normal gravity conditions.

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