THREE-DIMENSIONAL PERIODIC FULLY NONLINEAR POTENTIAL WAVES

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ABSTRACT
An exact numerical scheme for a long-term simulation of three-dimensional potential fully-nonlinear periodic gravity waves is suggested. The scheme is based on a surface-following non-orthogonal curvilinear coordinate system and does not use the technique based on expansion of the velocity potential. The Poisson equation for the velocity potential is solved iteratively. The Fourier transform method, the second-order accuracy approximation of the vertical derivatives on a stretched vertical grid and the fourth-order Runge-Kutta time stepping are used. The scheme is validated by simulation of steep Stokes waves. The model requires considerable computer resources, but the one-processor version of the model for PC allows us to simulate an evolution of a wave field with thousands degrees of freedom for hundreds of wave periods. The scheme is designed for investigation of the nonlinear two-dimensional surface waves, for generation of extreme waves as well as for the direct calculations of a nonlinear interaction rate. After implementation of the wave breaking parameterization and wind input, the model can be used for the direct simulation of a two-dimensional wave field evolution under the action of wind, nonlinear wave-wave interactions and dissipation. The model can be used for verification of different types of simplified models.

INTRODUCTION
The majority of the models designed for investigation of the three-dimensional wave dynamics are based on the simplified equations (see review [1]). Much closer to reality is an approach based on Zakharov’s equations [2]. He suggested using the boundary condition for the velocity potential written on an interface. The Laplace equation remains written in Cartesian coordinates according to this approach. For utilization of the boundary condition for the Laplace equation a procedure of interpolation of the potential from a free surface to a fixed level was suggested [3,4]. Later this method was titled as HOS (High Order Scheme) and used in many works (see, for example, [5,6,7]). An advantage of this approach is that the method is simple, computationally effective and robust. However, applicability of this method for simulation of highly nonlinear waves and for such delicate problem as the nonlinear wave-wave interaction remains unclear (see [8]).

The most developed methods are based on the full three dimensional equations and the surface integral formulation [9,10,11]. The method can be applied to the periodic and non-periodic flows. The main advantage of the method is exactness. The method does not impose any restriction on the wave steepness, so it can be used for simulation of waves approaching the breaking. However, the method seems quite complicated, so it is unlikely to be applied to the large-scale modeling of a long-term evolution of the real sea waves.

Another method for the 3-D waves includes an elliptic boundary layer problem solved by the finite-difference methods. Such approaches to simulation of the unsteady free surface flows based on full equations have been developed at least over three decades (see, for example [12.]). The related applications were later described in [13,14,15]. The main advantage of this method is that it is based on the initial equations transformed into the surface-following coordinate system. The Laplace-type equation obtained by transformation into the sigma-coordinate system, was solved in [16] by the iterative conjugate gradient method using the three dimensional finite element discretization. The finite-difference multi-grid model for 3D flow was developed in [17]. All papers of this group were mostly devoted to the technical application of the water wave theory, i.e., for the calculations of the dynamic load on the submerged bodies, or for simulation of wave dynamics in a domain with a complicated shape. A long-term evolution of a multi-mode wave field was not reproduced; this is why the exact
conservation of energy was not the priority target of such models.

In this work we suggest a new approach specifically targeted at simulation of a long-term multi-mode periodic wave field evolution in the deep ocean. It is well known that the nonlinear transformation and growth of waves occur over hundreds and thousands of wave periods. This property imposes tough restrictions on the model because such modifications of waves should not be obscured with the numerical errors. It means the model should be exact enough to reproduce a relatively slow spectrum evolution. This condition is well satisfied in the 1-D model in conformal coordinates. The 3-D waves represent a far more difficult object, since it is probably impossible to reduce the problem to the surface problem (in fact, the surface integral method cannot be referred to as the surface method, since it uses the Green function); this is why a velocity potential should be calculated from the elliptic equation. Such model is described in this paper. In section 1 the primary equations, as well as transformation of coordinates and the numerical scheme in the non-orthogonal and nonstationary curvilinear coordinates are given. The results of validation of the approach and the codes of the model are discussed in section 2. The results of the long-term simulations of a multi-mode three-dimensional wave field are described and section 3. The main results and prospects of the investigation are discussed in Conclusions.

1. 3-D DEEP WATER WAVE MODEL

Let us introduce the non-stationary surface-following non-orthogonal coordinate system:

\[ \xi = x, \quad \vartheta = y, \quad \zeta = z - \eta(\xi, \vartheta, \tau), \quad \tau = t \] (1)

where \( \eta(x, y, t) = \eta(\xi, \vartheta, \tau) \) is the shape of the wave surface, which maps the original domain

\[-\infty < x < \infty, \quad -\infty < y < \infty, \quad -\infty < z < \eta(x, y, t) \] (2)

which maps the original domain (4) onto the layer

\[-\infty < \xi < \infty, \quad -\infty < \vartheta < \infty, \quad -\infty < \zeta < 0 \] (3)

with the periodicity conditions over ‘horizontal’ coordinates \( \xi \) and \( \vartheta \):

The coordinates (1) is constructed for the deep water case. As seen, the vertical fluctuation of ‘horizontal’ coordinates \( \xi \) and \( \vartheta \) does not attenuate with depth, the lower boundary condition is applied at the variable level \( H = \zeta + \eta \). Such fluctuations do not create any problems with approximation since all variables in the wave motion attenuate with depth exponentially. For depth \( H >> \eta \) the difference between the fixed and fluctuating levels becomes negligible.

The model formulated above, is integrated in the domain \( \{ 0 < \xi \leq 2\pi, 0 < \vartheta < 2\pi, H < \zeta \leq 0 \} \) . This corresponds to a moving periodic wave surface given by Fourier series

\[ \eta(\xi, \vartheta, \tau) = \sum_{-M \leq k, M \leq M} h_{k, l}(\tau) \Theta_{k, l} \] (4)

where

\[ \Theta_{k, l} = \begin{cases} \cos(k\xi + l\vartheta) & -M \leq k \leq M, \quad -M < l < 0 \\ \cos(k\xi) & -M \leq k \leq M, \quad l = 0 \\ \sin(k\xi) & 0 \leq k \leq M, \quad l = 0 \\ \sin(k\xi + l\vartheta) & -M \leq k \leq M, \quad 0 < l < M \end{cases} \] (5)

The main advantage of a surface-following coordinate system is that the position of a variable surface \( \eta \) is mapped onto the fixed plane \( \zeta = 0 \). The 3-D equations of potential waves in the system of coordinates (1) at \( \zeta < 0 \) take the following form:

\[ \eta_\tau = -\eta_\xi u - \eta_\vartheta v + w \] (6)

\[ \varphi_\tau = -\eta_\xi w - \frac{1}{2}(u^2 + v^2 + w^2) - \eta - p \] (7)

\[ \Phi_\xi + \Phi_\xi_\vartheta + \Phi_\xi_\zeta = \left( \eta_\xi W \right)_\xi + \left( \eta_\vartheta W \right)_\vartheta + \eta_\xi U_\zeta + \eta_\vartheta V_\zeta \] (8)

where \( \Phi \) is a three-dimensional velocity potential, \( p \) is an external pressure, \( U, V, W \) are expressions for the Cartesian components of velocity:

\[ U = \Phi_\xi - \eta_\xi W \] (9)

\[ V = \Phi_\vartheta - \eta_\vartheta W \] (10)

\[ W = \Phi_\zeta \] (11)

and \( \varphi, u, v, w \) are the values of \( \Phi, U, V, W \) at surface \( z = \eta \).

The equations (6) and (7) are written for a free surface which position in the surface-following coordinate system is fixed at \( \zeta = 0 \). These equations formally look as 2-dimensional, however, they include a vertical derivative of potential \( w = \Phi_\zeta \) which should be derived from an elliptic equation (8) with the following boundary conditions:

\[ \Phi(\zeta = 0) = \varphi, \quad \frac{\partial \Phi}{\partial \zeta}(\zeta \rightarrow -\infty) = 0 \] (12)

The second condition (12) in the numerical scheme is replaced by the condition at the finite depth:

\[ \frac{\partial \Phi}{\partial \zeta}(\zeta = H) = 0, \quad \text{where depth } H \text{ should be large enough to be considered as infinitely large.} \]

The previous calculations with the 1-D model showed that \( H \) can be defined by the formula \( H = 2\pi n / k_p \) where \( k_p \) is a wavenumber of the mode with the largest amplitude and \( n = 1 \pm \frac{1}{2} \).

Contrary to the boundary integral method and the HOS method, a numerical scheme for the 3-D wave problem is written directly for the initial system of equations. The scheme combines the 2-D Fourier transform method in ‘horizontal surfaces’ and the second-order finite-difference approximation on a stretched staggered grid defined by the relation: \( \Delta \xi_{j+1} = \gamma \Delta \xi_j \) in the vertical (\( \Delta \xi \) is a vertical step, and \( j = 1 \) at the surface). The values of the stretching coefficient \( \gamma \) lie within the interval of 1.10-1.20. The finite-
difference approximations of Eq. (8) on a non-uniform vertical grid are quite straightforward. The number of levels $L_w$ depends on a shape of the spectrum. In the calculations represented below, $L_w$ varies within the limits $L_w = 40 \div 70$. The Fourier transform method assumes that all nonlinear terms are calculated on the extended grid $N \times N (N = 4M)$ in the physical space. The result is transformed into the Fourier space (in some calculations the number of modes and knots in $\theta$-direction were taken smaller than in $\zeta$-direction). The description of variables in terms of the Fourier components is more compact than that in terms of grid values. That is why the Fourier components are considered as a basic presentation, while the grid fields were calculated and stored only when and where they are required. The model was validated and the preliminary results were obtained with $2M^2 + 2M + 1 = 2051$ ($M=32$) and $8195$ ($M=64$) Fourier modes. The simulations of a multi-mode wave field were performed in the rectangular domain with a different number of modes in $x$ and $y$ directions: $M_x = 256$ and $M_y = 64$. In this case, the grid includes 564,288 knots.

The diagnostic Poisson equation for the velocity potential $\Phi$ was solved using simple iterations. At each iteration the right-hand side of (8) was calculated using values of $\Phi$ obtained at the previous iteration. The initial $\Phi$ was assigned on the basis of the linear theory. The iterations were continued until Eq. (8) reached accuracy of $\varepsilon = 10^{-9} - 10^{-7}$ depending on the parameters of the vertical grid. An accuracy of the solution was calculated as a residual error of Eq. (8). It should be noted that for all the calculations considered, the error decreased exponentially during the iterations; thus, a typical number of the iterations was 5 per time step. The speed of the calculations for a multi-mode wave field is reasonably high if the $rms$ steepness does not exceed the value of 0.1, while the convergence rate actually slows down when the local steepness becomes large at one point, at least. However, the model remains stable, even if the local steepness in certain points approaches 1. Note that the variables $u$, $v$ and $w$ calculated in the process of solution of eq. (8) were used directly in calculations of right-hand sides of Eqs. (6) and (7).

The model is mostly intended for simulating of a long-term evolution of a multi-mode wave field with a realistic spectrum. No matter how high the spectral resolution might be, for the long-term simulations of nonlinear waves one must parameterize the energy flux into the truncated part of the spectrum ($\sqrt{k^2 + l^2} > M$); otherwise, a spurious energy accumulation at large wave numbers always corrupt the numerical solution. In numerical solutions of the fluid mechanics equations this effect is suppressed by introducing different types of viscosity. A similar scheme was developed in previous articles describing the conformal method for direct wave modeling [1,2]. Following the scheme, the simple dumping terms were added to the right-hand sides of the Fourier form of Eqs. (6) and (7):

\[ \frac{\partial \eta_{k,l}}{\partial \tau} = E_{k,l} - \mu_{k,l} \eta_{k,l} \]  \hspace{1cm} (13)
\[ \frac{\partial \phi_{k,l}}{\partial \tau} = F_{k,l} - \mu_{k,l} \phi_{k,l} \]  \hspace{1cm} (14)

where $E_{k,l}$ and $F_{k,l}$ are Fourier coefficients for right-hand sides of Eqs.(6) and (7), and

\[ \mu_{k,l} = \begin{cases} \left( \frac{rM}{M - k_d} \right)^2 & \text{if } |k| > k_d \\ 0 & \text{otherwise} \end{cases} \]  \hspace{1cm} (15)

where $|k| = \sqrt{k^2 + l^2}$ and $k_d$ is the radius of domain which is not affected by smoothing. A value of $k_d$ was chosen in interval $(0.5M, 0.75M)$. The value of $r = 0.25$ was chosen for all the runs discussed below after it was found that the results were reasonably insensitive to variations of $k_d$ and $r$. The dissipation effectively absorbs the energy at the wavenumbers close to the truncation number $M$ while not affecting the energy at wavenumbers $|k| \leq k_d$. Note that an increase of the truncation number $M$ shifts the dissipation area to the higher wavenumbers (while with $M \to \infty$ the energy sink due to dissipation, tends to zero), so the scheme with the dissipation described retains approximation of the original (non-dissipative) system. Note also that the scheme (13) - (15) is introduced to describe a real physical process, i.e. dissipation of the wave energy due to a flux of energy into the truncated part of the spectrum. This process is usually very slow, but in absence of dissipation the numerical instability growing exponentially in the vicinity $k=M$, occurs and finally terminates the solution. A time step was chosen empirically. Its value depends on the model parameters and usually falls within the range $\Delta \tau = 0.001 - 0.01$.

2. VALIDATION OF THE 3-D DEEP-WATER MODEL

A key point of any scheme for a full equation of potential waves is accuracy of calculation of a vertical velocity on the surface. We used several methods of validation of the scheme for solving of the equation for the velocity potential. The most simple is comparison of the solution for the 2-D problem for the iteration method with the solution obtained by conformal transformation of coordinates. The latter was transferred to the Cartesian coordinates with the high-order periodic spline interpolation providing accuracy $\sim 10^{-10}$ $|w|$ (where $|w|$ is a typical value for the vertical velocity on the surface). It was proved that the exact solution in conformal coordinates in terms of the vertical velocity $w$ coincides with the solution equations (8) in curvilinear coordinates, the accuracy being equal to $10^{-7}$. The number of levels $L_w$ was equal to 70, while the stretching parameter $\gamma = 1.2$, providing good approximation of the exponential attenuation of the velocity potential with
depth. Another way of validation of scheme is a use of analytical solution for equation (8). This method will be described in the next paper on 3-D model.

![Fig. 1](image)

**Fig. 1** Evolution of amplitudes of steep \( (ak = 0.35) \) Stokes wave \( A_k \) assigned initially at wavenumbers \( k = 1, 2, 3, ..., M \).

The most efficient method of the numerical scheme validation is comparison of the results of integration of the non-steady equations. This method developed in [18, 19] is also used in the current paper. The simulation of a very steep Stokes wave with the steepness \( ak = 0.35 \) (assigned as an initial condition) is performed. The time evolutions of amplitudes of the first 20 Stokes wave modes for the resolution \( M = 128 \) are shown in Fig. 1. As seen, the first several modes of the Stokes wave with the amplitudes as small as \( 10^{-3} \) remain practically unchanged, while the rest of the amplitudes fluctuate. The relative magnitude of the fluctuation increases with the wavenumber growth. However, the averaged values of amplitudes for each mode do not change on the average, and their values decrease monotonically with growth of the wave number. Note that in a case of instability, the high-order modes start to grow and the initial structure of solution disappears.

The calculations described above were performed for the Stokes waves propagating along \( x \)-axis with the wavenumbers \( k = 1, 2, 3, ..., M \). A similar validation was done for the Stokes wave propagating along the \( y \)-axis. This setting is the most appropriate for validation of the numerical scheme since there is no room for development of Benjamin-Feir instability [20]: all modes with wavenumbers \( k > 1 \) represent the components ('bound waves') of the Stokes wave.

3. SIMULATION OF A MULTI-MODE WAVE FIELD

The series of the calculations were performed for simulating a multi-mode wave field initially assigned as a superposition of linear modes with random phases corresponding to Pierson-Moskowitz spectrum [21], and with angular spreading proportional to \( \left( \text{sech} \left( \theta \right) \right)^4 \). The peak of the spectrum was placed initially at wavenumber \((k,l) = (64,0)\). The spectral resolution was \( M_x = 256, M_y = 64 \). The calculation with a time step \( \Delta \tau = 0.0025 \) was performed up to nondimensional time \( \tau = 250 \) (100,000 time steps), that corresponded to 318 wave peak periods.

Over such a long period of integration the energy of waves in absence of any energy input could decrease due to a flux of energy into the subgrid domain. In fact, the Pierson-Moskowitz spectrum describes a statistically steady wave regime when the total input of energy equals the total energy dissipation. This balance can be introduced using the energy input from wind, as formulated in [22], as well as the energy dissipation through breaking adjustment, as suggested in [19]. However, at the present stage implementation of the complicated physics seems premature. This is why the quasi-stationary regime is reproduced on the basis of a simple scheme designed to preserve the total energy. It was done by introduction of additional terms in Eqs. (6) and (7):

\[
\frac{\partial\eta_{k,l}}{\partial \tau} = H_{k,l} + (1 - \gamma)\eta_{k,l} \tag{16}
\]

\[
\frac{\partial\varphi_{k,l}}{\partial \tau} = F_{k,l} + (1 - \gamma)\varphi_{k,l} \tag{17}
\]

where \( \eta_{k,l} \) and \( \varphi_{k,l} \) are Fourier amplitudes for \( \eta \) and \( \varphi \), \( H_{k,l} \) and \( F_{k,l} \) are the right-hand sides of Eqs. (14) and (15) including the additional terms introduced in (14) and (15); \( \gamma \) is the coefficient

\[
\gamma = \left( \frac{E}{E_0} \right)^{1/2} \tag{18}
\]

where \( E_0 \) is the initial total wave energy equal to the sum of the kinetic and potential energies; \( E \) is the total energy at the previous time step. Since the coefficient \( (1 - \gamma) \) is very small (of the order of \( 10^{-6} \)), the algorithm (16)-(18) supports the total energy, the accuracy being of the order of \( 10^{-6} \) and it does not change the structure of the solution neither in Fourier nor in the physical spaces.

![Fig. 2](image)

**Fig. 2** The probability of surface elevation normalized by a significant wave height \( H_s \). The dashed line corresponds to the inverted probability distribution for the negative values \( z \), the solid line is the Gaussian distribution.
Since the coefficient $(1 - \gamma)$ is very small (of the order of $10^{-6}$), the algorithm (16)-(18) supports the total energy, the accuracy being of the order of $10^{-6}$, and it does not change the structure of the solution neither in Fourier nor in the physical spaces. The dashed line represents the reversed probability distribution for the negative values $\gamma$, the solid thin line is the Gaussian distribution.

For a typical peak wave length corresponding to the Pierson-Moscowitz spectrum a horizontal size of domain is of an order of several kilometers. After just one peak wave period the initially sinusoidal waves obtain a typical Stokes-like shape with sharp crests and gentle troughs. Probability of the surface elevation (normalized by a significant wave height) is shown in Fig. 2. For the calculations 655,360,000 values of $\eta_{i,j}$ were used. The dashed line represents a reflected part of the probability distribution for the negative $\gamma$. As seen, probability of the positive elevations (wave crests) is considerably larger than that of the negative ones (troughs), exactly as in 1-D modeling. Probability of the surface elevation (normalized by a significant wave height) is shown in Fig. 2. For the calculations 655,360,000 values of $\eta_{i,j}$ were used. The dashed line represents a reflected part of the probability distribution for the negative $\gamma$. As seen, probability of the positive elevations (wave crests) is considerably larger than that of the negative ones (troughs), exactly as in 1-D modeling.

The sum of the potential and kinetic energies is preserved with the accuracy of the order of $10^{-5}$. In the bottom panel of Fig. 3 an evolution of the skewness (solid curve) and kurtosis (dashed line) is given.

During the initial period a fast transformation of the elevation and surface potential assigned according to the linear theory, occurs. Then the system enters a quasi-stationary regime maintained for most of the integration time. Both the skewness and kurtosis are positive, which is a particular feature of nonlinear waves.

![Fig. 3](image-url)

**Fig. 3** The top panels represent evolution of the kinetic (solid curve) and potential (dashed curve) energies; bottom frames show the skewness (solid curve) and the kurtosis (dashed curve) of the wave surface. Left panels correspond to the first ten peak wave periods, while the right panels correspond to the last ten peak wave periods.

Evolution of the kinetic (solid curve) and potential (dashed curve) energies is shown in the top panels in Fig. 3 as a percentage of the total energy divided by two. To make the figure clearer, only initial and final intervals for the successive ten peak wave periods are shown. The potential and kinetic energies fluctuate considerably (up to 1%) over the initial period of adjustment of the linear initial conditions to nonlinearity. These fluctuations fall within the range of the order of 0.1% almost over the whole period of integration.

![Fig. 4](image-url)

**Fig. 4** Top panel corresponds to the initial Pierson-Moskowitz 2-D wave spectrum $\log_{10}(S)$. Bottom panel is the final spectrum after integration over 318 peak wave periods.

The most curious property of surface waves is demonstrated in Fig. 4. The spectrum assigned in the initial conditions is smooth (top panel in Fig. 4). However, after just several peak wave periods the spectrum starts transforming, i.e., sharp peaks and deep depressions appear. Finally, a continuous spectrum transforms into a discrete spectrum consisting of the individual peaks. It is tempting to explain this phenomenon on the basis of a resonance mechanism assuming that the resolution is not high enough to regard all possible resonant combinations of wavenumbers and frequencies. This explanation, however, should be based on the assumption that an exact dispersion relation is valid. Meanwhile, a phase velocity of each wave mode is fluctuating due to many reasons such as nonlinearity, Doppler effects the presence of bound waves [23] etc. Consequently, the resonant conditions are blurred over the finite area, which means that such an explanation is not valid. In this case a spectrum should be continuous. If the resolution had been a problem, the spectrum would have been
approaching the continuous spectrum similar to that in the top frame of Figure 4. Note that similar results were obtained using a simplified model based on the equations obtained by expansion of the Hamiltonian up to the fourth order [24]. The simplified approach allowed the authors to use the resolution several times higher than that used in the current work. Nevertheless, the simulation of an evolution of the initially homogeneous spectrum yielded a strictly discrete spectrum closely reminding the spectrum in Fig. 4. It should be noted that the discretization effect can be visible in the 2-D Fourier space only, while it can never manifest itself in a single-point frequency spectrum because of the floating frequencies and the angle spreading.

The data on the spectrum evolution are used for the calculations of the nonlinear spectrum transformation rate

$$\Delta S \frac{\Delta t}{\Delta k_{l}} = \overline{N}_{k,l}$$

In Fig. 5 the spectrum $\overline{N}_{k,l}$ integrated over the lateral wavenumbers $k_y$ is shown. As seen, the shape of $N_{k,l}$ is qualitatively similar to the results of the calculations based on the Hasselmann’s integral. The energy in the front slope of the spectrum increases, while the energy in the back slope of the spectrum decreases causing downshifting. Unfortunately, all available schemes for the Hasselmann’s integral do not allow us to perform any calculations for the high resolution used in the present work. It is unlikely though that the Hasselmann’s integral converges to a reasonable limit with increase of resolution.

4. CONCLUSIONS

In this paper a straightforward method for the numerical solution of the three-dimensional potential wave equations is suggested. The method uses a surface-following coordinate system. In the new coordinates the kinematic and dynamic conditions on the surface become more complicated, but if we consider them as evolutionary equations for the surface potential and elevation, we arrive at the conclusion that the equations can be easily integrated in the same way as the similar one-dimensional equations in the conformal coordinates. However, the calculation of the vertical derivative of potential on the surface becomes more complicated, since the Laplace equation for the 3-D velocity potential turns into an elliptic equation that should be solved at every time step, which requires a use of the extensive computer resources. However, it should be noted that this problem is still much simpler than the standard simulations of the 3-D Navier-Stokes equations or the LES (Large Eddy Scale) equations in the curvilinear coordinates when a problem of solving an elliptic equation for pressure arises. This method has an evident advantage compared with the HOS method, where an extrapolation of the full velocity potential with the Taylor series is used. The potential is a superposition of the exponentially decaying modes. The largest modes still allow extrapolation, while the high wavenumber amplitudes attenuate completely and cannot be restored correctly. So, the extrapolation of the potential regardless of its spectral structure suppresses the high-wavenumber modes and decreases nonlinearity. This is why the HOS method is so robust.

The potential wave problem gives a unique opportunity for validation of the full nonlinear model by comparison with the exact stationary solution obtained in a moving coordinate system. This solution is obtained with a completely different algorithm, hence, such validation can be considered as full, non-trivial and exact. Since the model uses a finite-difference approximation in a vertical direction, we can’t expect a perfect agreement between the exact and approximate solutions, though the results of such comparison are quite convincing. The structure of the Stokes wave was supported over a long interval of integration. Had the numerical scheme not been accurate enough, the evolution of modes would have exhibited a chaotic behavior, the Stokes wave quickly disintegrating because of the numerical instability. The scheme is consistent, since its accuracy increases with increase of the resolution.

The model is created for investigation of the 3-D wave dynamics. First of all, the adiabatic version of the model can be used for a long-term integration and calculation of the 2-D nonlinear interaction spectrum. The model is also designed for investigation of the nonlinear two-dimensional surface wave dynamics and generation of extreme waves.
After implementation of the energy input scheme and wave breaking parameterization, the model can be used for direct simulations of the two-dimensional wave field evolution under the action of wind, nonlinear interactions and dissipation. This model can be combined with the 3-D LES model of the atmospheric wave boundary layer, formulated in the same coordinate system. Such approach can be considered as the ultimate solution of the wind-wave interaction problem.

All numerical results presented in the current work were obtained using a standard one-processor Dell computer with a speed of 3.00GHz. More than 90% time is spent for the FFT calculations. This is why parallelizing of just 2-D FFT ‘by layers’ greatly increases a performance of the model.

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