THE ESTIMATION OF IMPACT FORCES BASED ON FIRST PRINCIPLES

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ABSTRACT: Design provisions for resistance against impact forces generated by vehicular, or train, collision in codes of practices are typically prescriptive in nature. The basis of the prescriptive clauses is often difficult to trace whereas commonly used analytical models employing energy principles can be overly simplified. Consequently, engineers often find it challenging to make estimates that are outside the scope of coverage of the relevant code clauses. For example, the impact of a boulder rolling downslope is not covered by any loading code nor is the accidental dropping of a heavy object. Amid lack of confidence in predicting impact forces with a reasonable degree of accuracies full scale field trials are typically conducted to verify the performance of a built component when subject to certain impact hazard. Full scale experimentations serve the purpose of verifying the adequacy of the design of the component in withstanding a specific impact action, but it would be too costly to have such tests applied repetitively on different components, and to cover for different projected impact scenarios. This paper introduces closed form solutions for the prediction of impact forces in the context of civil engineering and construction. The presented solutions were derived from first principles and have been validated by laboratory experimentations thereby giving engineers, and product designers, confidence in adapting them to a diversity of engineering applications.

KEYWORDS: Impact forces, contact forces, hail, storm debris, barriers, momentum

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1 INTRODUCTION

The use of an equivalent static force to emulate the effects of an impact is the basis of most commonly adopted codified method of designing against impact actions. It is revealed by *energy* and *momentum* principles that the magnitude of the *quasi-static* force is not solely the function of *mass* and *incident velocity* of the impact but is also dependent on the self-weight of the target and its boundary conditions [1]. Interestingly, the amount of bending moment and shear force generated by the impact on a pole decreases with increasing height of the pole as illustrated in the paper by working through examples. In other words, the magnitude of the induced bending moment and shear forces become less with increasing span length which is counter intuitive from the perspectives of statics. In other words, smaller targets are more vulnerable to damage. The revealed phenomena provide explanations for wide spread damage to roof tiles in hail storms.

Quasi-static forces that are calculated from *momentum-energy* principles are not large enough to explain widespread denting to metal claddings and roof coverings by debris impact in extreme weather conditions. The phenomenon of contact force is then introduced to explain the anomalies [2,3].

In summary, a quasi -static force as derived from momentum- energy principled represents the impulsive effects of the impact causing bending and displacement (that can result in overturning) of the target. On the other hand, contact forces which are dependent on the hardness properties of the impactor object (in addition to its mass and velocity) are responsible for damage of a localised nature such as *denting* on metal surfaces and *perforation* of glazing panels. The predictions for both types of forces are introduced in the rest of this paper under separate headings.

2 QUASI-STATIC FORCES OF IMPACT BY LIGHT OBJECTS

The use of a *quasi-static* force to emulate impact conditions is based on deflecting the structure (or a member within the structure) by an amount equal to that predicted for a given impact scenario. To give such predictions *momentum* and *energy* principles are typically applied to a spring connected lumped mass analytical model (Fig. 1) which is a simplified representation of the targeted structural element [4]. The value of the generalised mass (αm) and stiffness (k) assumed for the target is dependent on the boundary conditions of the element (refer Fig. 2). Similar recommendations for plate elements can be found in the literature and are not presented herein [5].

For light ”impactor” objects such as windborne debris (and hailstones) interferences by gravity in the impact actions may be neglected. Thus, the same model can be applied to analyse impact actions in the horizontal and vertical directions. Impact analyses for heavy fallen objects are presented in a later section of this paper.

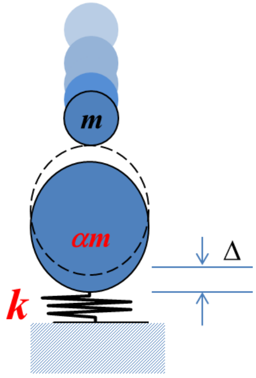


Figure 1: *Spring-connected lumped mass model*

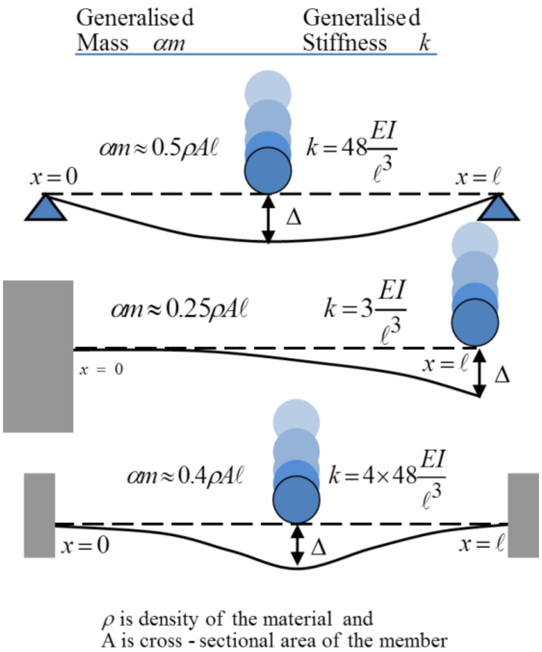


Figure 2: *Generalised mass and stiffness of structural elements*

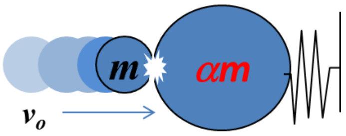
The amount of deflection (Δ) sustained by the target lumped mass in an impact is often calculated by simply equating the kinetic energy delivered by the impactor with the energy of absorption as defined by equation (2a -2b) as the use of such equality can be found in highway codes of practices for the design of barriers. The amount of quasi-static force to emulate the same amount of deflection is accordingly given by equation (2c)

$$\frac{1}{2}mv_o^2 = \frac{1}{2}k\Delta_o^2 \qquad (2a)$$

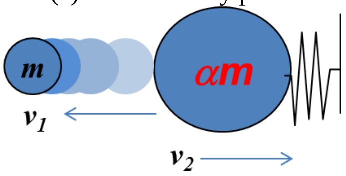
$$\Delta = \frac{mv_o}{\sqrt{mk}} \qquad (2b)$$

$$F_{qs} = k\Delta = v_o\sqrt{mk} \qquad (2c)$$

The use of equations 2b – 2c for analysing the impact actions have neglected energy losses occurring in the course of the impact. Such energy losses can be taken into account by incorporating the *coefficient of restitution* (COR) as a modelling parameter as shown by equations (3a – 3f). Equation (3a) is based on *equal momentum* principles as illustrated in Fig. 3a and 3b.



(a) Immediately prior to contact



(b) Immediately following contact

Figure 3: *Basis of equating momentum*

Algebraic manipulation of equation (3a) along with the use of the relationship defined by equation (3b and 3f) result in equations (3c – 3d). Details of the derivation of these equations can be found in a companion paper presented in this conference [6].

$$mv_o = -mv_1 + \alpha mv_2 \qquad (3a)$$

$$\frac{1}{2}mv_o^2\beta^2 = \frac{1}{2}k\Delta_o^2 \qquad (3b)$$

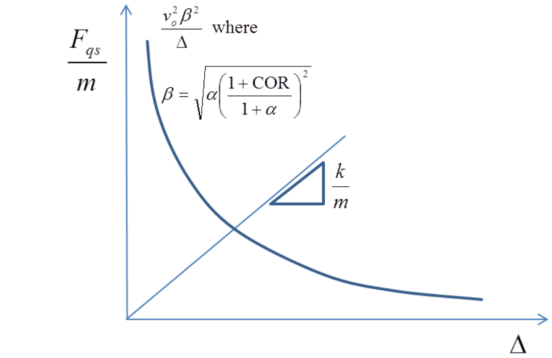
$$\Delta = \frac{mv_o}{\sqrt{mk}}\beta \qquad (3c)$$

$$F_{qs} = k\Delta = v_o\sqrt{mk}\,\beta \qquad (3d)$$

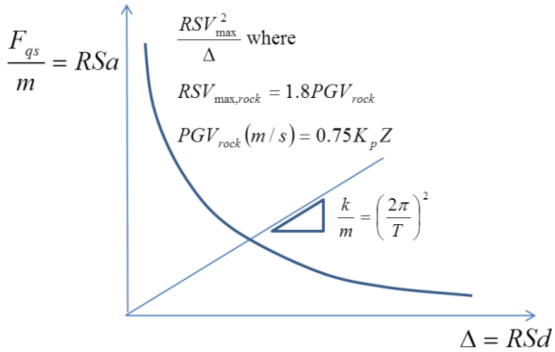
$$\text{where} \quad \beta = \sqrt{\alpha\left(\frac{1+\mathrm{COR}}{1+\alpha}\right)^2} \qquad (3e)$$

$$COR = \frac{v_1 + v_2}{v_o} \qquad (3f)$$

Importantly, experimental verifications of the proposed relationships can be found in Refs. [1] and [6]. Expressions that have been presented in the above for analysing the impact action of a solid object can be represented in the form of an *acceleration-displacement response spectrum* (ADRS) diagram of Fig. 4a which is shown alongside the ADRS diagram for seismic actions (Fig. 4b). The use of the latter diagram has been illustrated in the *Commentary* to the *Australia Standard for Seismic Actions* [7].



(a) ADRS diagram for impact action



(b) ADRS diagram for seismic action

Figure 4: *Acceleration-Displacement Response Spectrum (ADRS) Diagrams*

The use of equations (3a – 3f) for analysing the impact action of a flying object is illustrated in the following example of a steel pole which is struck by the flying object at the top. It is required to calculate the maximum horizontal deflection of the pole in order that the amount of *quasi-static* force (F_{qs}) representing the impact can be found. Once the value of F_{qs} is known the bending moment and shear force at the base of the pole can be calculated accordingly.

In this example, the flying object is of mass equal to **40** kg and velocity of impact equal to **30** m/s . The 3m tall steel pole has flexural rigidity (*EI*) of 9 x $\mathbf{10^4}$ kNm^2 and is fixed at its base. The spring stiffness (k) of the spring connected lumped mass model (Fig. 1) is estimated to be 10^4 kN/m (being $3EI/h^3$). The self-weight of the steel pole is **960** kg per m length which is translated to a total mass of 2880 kg and a target mass (αm) in the lumped mass model of 720 kg (being ¼ of the total mass according to Fig.2). The mass ratio α is accordingly equal to 18 (being 720/40). The impactor is assumed to rebound with a *Coefficient Of Restitution* of **0.2**. Linear elastic behaviour of the steel pole may be assumed.

The horizontal deflection of the pole (Δ) and the corresponding quasi-static force (F_{qs}) can be found by substituting various impact parameter values into equations (3c – 3e) as shown by the following:

$$\Delta = \frac{40\times 30}{\sqrt{40\times 10^{7}}}\sqrt{18\times\left(\frac{1+0.2}{1+18}\right)^{2}} = 0.016\text{m} \quad (4a)$$

$$F_{qs}(\text{kN}) = k\Delta = 10^{4}\times 0.016 = 160\text{kN} \quad (4b)$$

The *shear force* and *bending moment* at the base of the pole is accordingly 160 kN and 480 kNm respectively. The calculations have been repeated for poles of the same cross-section but with height varying between 0.5 m and 5 m to track the trend of change in the bending moment demand at the base. Results of the calculations as shown in Figure 5 demonstrates the interesting phenomenon of significant decrease in the bending moment value with increasing height of the pole which is contrary to predictions by the conventional approach (based on the notion of representing the impact by a prescribed equivalent horizontal force).

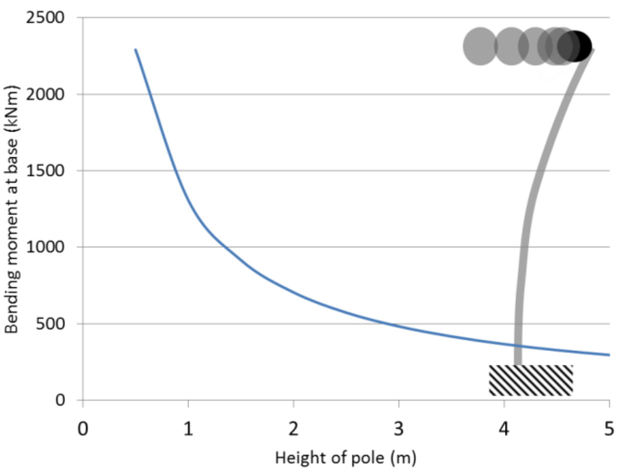


Figure 5: *Bending moment value at base of pole*

Clearly, the impact hazard can be overstated significantly by simply representing the impact action by an equivalent force without allowing for contributions by the inertial resistance of the target. The illustrated phenomenon explains how a large amount of resources have been wasted on the use of a prescribed equivalent horizontal force to represent an impact hazard when the self-weight of the target has not been factored into the design.

3 CONTACT FORCES OF IMPACT BY LIGHT OBJECTS

Expressions for determining the value of the quasi-static forces (F_{qs}) as presented in the previous section are for emulating deflection by a static analysis procedure. *Quasi-static* forces are not to be confused with *contact* force which is generated in the vicinity of the point of contact between the impactor and target, and is responsible for damage featuring perforation and denting. The distinction between the two types of forces in the context of impact actions is not widely understood and few guidelines have been developed for their estimation. The reason for their differences is explained in the schematic diagram of Fig. 6 which shows the significant influences of the inertia forces which can change the mode of failure from *bending controlled* in static conditions to *punching shear controlled* in an impact. The value of the *contact* force can be many times higher than that of the *quasi-static* force and lasts for only a few *milli-seconds* whereas the deflection of the target can evolve over a much longer time span (refer Fig. 7 for a sample of the measured results). The stiffer the impactor the shorter the period of time to deliver the impulsive action onto the target and consequently the higher the peak value of the contact force. Computer software such as LS-DYNA has the capability of accurately predicting contact force but the dynamic stiffness parameters of the impactor material which is required input into the program has not been documented for debris materials.

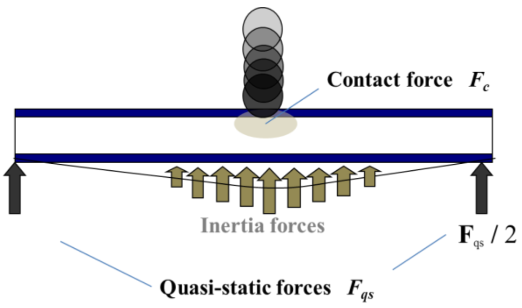


Figure 6: *Quasi-static force and contact force*

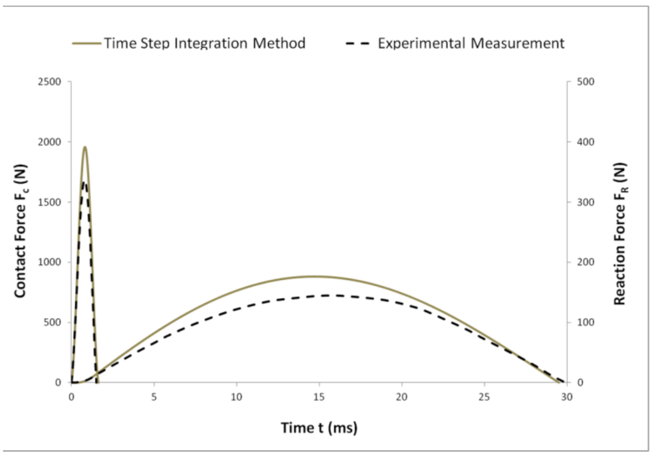


Figure 7: *Measured time-histories of quasi-static force and contact force*

The alternative, direct, approach for an accurate evaluation of the damage potential of an impact scenario is by physical experimentation. However, the amount of potentially useful information that can be retrieved from physical experimentations to guide design and to quantify risk is very limited. The authors have made use of their own custom-built apparatus for the measurement of contact force of a flying debris object along with a calibration procedure to determine the compressive stiffness parameters of a range of impactor material in order that contact force values can be computed for a given velocity of impact. Details of the apparatus has been reported in Refs. [2] and [3].

The *non-linear visco-elastic* contact model (as defined by equation (5) and illustrated in the schematic diagram of Fig. 8) was conceived for simulating the force-displacement behaviour of the frontal spring in a spring connected lumped mass model to emulate the stiffness behaviour of a piece of debris material when contact is made with the surface of the target.

$$\dot{\delta} \qquad (5)$$

where δ is the indentation and $\dot{\delta}$ is the velocity of indentation.

The first term in the expression involving the spring stiffness coefficient k_n and exponent p is consistent with *Hertz* law. The second term $\dot{\delta}$ represents the resistance force developed by the viscous damper to emulate energy losses on impact.

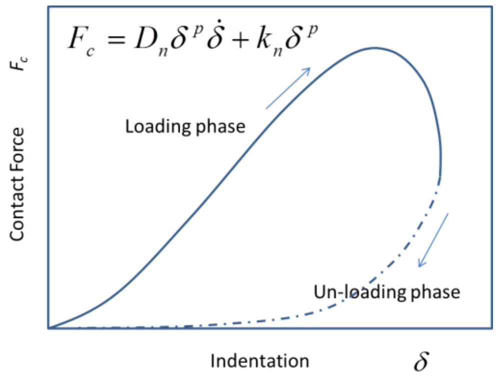


Figure 8: *hysteretic behaviour of non-linear visco-elastic model*

Several models have been proposed in the literature for characterising energy losses in the form of a damping term. The generalised relationship of equation (6) can be used to estimate the value of D_n for a given value of COR which is measurable by the high speed camera. The derivation of the relationship is presented in Ref [2].

$$(0 \qquad 3)\text{x}\left(\text{——}\right)\text{—} \qquad (6)$$

where COR and $\dot{\delta}_0$ are coefficient of restitution and initial indentation velocity respectively.

The *visco-elastic* contact model might appear not to be suitable for simulating interactions between objects which are brittle in nature. This apparent limitation of the *visco-elastic* material model is actually not an issue given the short duration nature of the contact force generated by the collision. It is noted that the non-linear visco-elastic contact model was successfully utilised in a study on the simulations of impact by hailstones which is well known to be brittle [2].

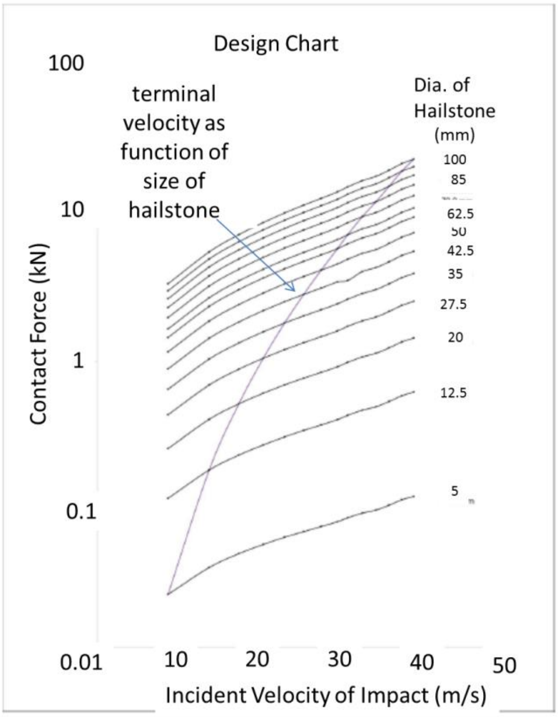


Figure 9: *Contact forces for hail impact*

The equations of motion for the non-linear visco-elastic 2DOF model have been formulated in accordance with the contact model introduced herein and implemented in the form of a time-step integration method [8]. A two-step calibration procedure has been employed for determining values of parameters k_n, p *and* COR by analysing results obtained from repetitive impact testing of a range of storm debris specimens (including simulated hail specimens) by the authors and co-workers [2,3]. The change in the values of these parameters with increasing velocity of impact has also been tracked. Design charts showing the value of contact force as function of mass and velocity of impact have been constructed accordingly for some common debris materials [2,3]. Figure 9 is a chart that can be used for estimating contact force produced by the impact of a hailstone of a given mass and velocity of impact.

4 QUASI-STATIC FORCES OF IMPACT BY AN HEAVY FALLEN OBJECT

To analyse vertical impact which involves dropping an *impactor* onto the target the *left-hand-side* of equation (7a) representing the amount of energy to be absorbed should contain the *kinetic energy* term and the term representing the *loss of potential energy* resulted from the vertical deflection of the target. Meanwhile, the *right-hand-side* of the equation takes into account the (prestressing) effects generated by the self-weight of the supporting structural element. Equation (7a) can be re-arranged into equations (7b–7d) for calculating the displacement demand (Δ) on a structural element responding within the elastic limit [1].

$$\frac{1}{2}mv_o^2\left(\frac{1}{1+\alpha}\right)+(1+\alpha)mg\Delta=\alpha mg\Delta+\frac{1}{2}k\Delta^2 \quad (7a)$$

where $v_o=\sqrt{mgh}$

$$\Delta=\Delta_s+\sqrt{\Delta_s^2+\delta^2} \quad (7b)$$

$$\Delta_s=\frac{mg}{k} \text{ where} \quad (7c)$$

$$\delta=\beta\frac{mV_0}{\sqrt{km}} \quad \text{where} \quad \beta=\sqrt{\frac{1}{1+\alpha}} \quad (7d)$$

As the intensity of impact is increased, the supporting rear spring (representing the structural element) will have surpassed the limit state of yielding. The non-linear force-displacement behaviour of the spring can be simplified herein into the *elasto-plastic* model which is characterised by the yield displacement and yield strength [1]. The conditions of no *re-bounce* is assumed accordingly given that more energy is dissipated in a higher intensity impact. The energy balance is represented by equation (8a) and the displacement demand of the impact can be predicted using equation (8b).

$$\frac{1}{2}mV_o^2\left(\frac{1}{1+\alpha}\right)+(1+\alpha)mg\Delta_m= \frac{(\alpha mg+F_y)}{2}\left(\Delta_y-\frac{\alpha mg}{k}\right)+F_y\left(\Delta_m-\Delta_y\right) \quad (8a)$$

$$\Delta_m\approx\frac{mV_o^2}{2(F_y-(1+\alpha)mg)}\left(\frac{1}{1+\alpha}\right)+\frac{F_y}{F_y-(1+\alpha)mg}\frac{\Delta_y}{2}$$

$$\text{for } \alpha mg \langle\langle F_y \quad (8b)$$

where Δ_m is the displacement demand of the impact, Δ_y is the displacement at notional yield and F_y is the resistance of the target element at yield.

5 ON GOING RESEARCH

The use of momentum principles to predict deflection of the target responding within the elastic limit has been extended to the prediction of the displacement of barriers that are without a foundation. A free-standing rockfall barrier which relies on its self-weight for stability is currently a subject of research interests because of significant potential savings derived from waiving away the need to build a foundation for the barrier in a hilly terrain [6].

Simulated time-histories of contact forces can be used in quasi-static analyses, or physical load testing, to predict damage to the target [9]. Complex localised damage to the surface of a target can be predicted with good accuracies using this approach without the need of the much more costly analytical or experimental procedures involving the simulation of the impact action. Bypassing such costly procedures will facilitate even more innovations in the design and manufacturing of products in countering impact hazards.

6 CONCLUSIONS

With the introduction of the fundamentals of impact actions the shortcomings of current approaches employed for the engineering of impact hazards have been brought to light. For example, the effects of a projected impact actions on a target can be grossly overstated by the use of a prescribed equivalent static force in the design of a barrier, or that of the affected structure. Closed form expressions that have been derived to predict the performance of a barrier, a structure or an element within a structure, when subject to severe impact hazards have been presented and illustrated by worked examples. Ongoing R&D work in support the development of new methodologies which waive away the need for rigorous analyses or costly experimentations to predict damage has been outlined.

ACKNOWLEDGEMENT

Contributions by Dr Carlos Lam and Dr Julian Kwan of the Geotechnical Engineering Office of the Hong Kong Government, Dr Mahdi Miri Disfani, Dr Lihai Zhang, AProf Ruan Duan along with former, and current, research higher degree students working on this topic under the supervision of the authors are gratefully acknowledged.

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