A MATHEMATICAL, STATISTICAL, PROBABILISTIC AND STRATEGIC ANALYSIS OF TENNIS

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Abstract

The relatively complicated tri-nested scoring system used in tennis presents many opportunities for the mathematical, statistical, probabilistic and strategic analysis of the game. Summary data is now available for nearly all matches in professional tennis, but point-by-point data required for studies of independence between points are much harder to obtain and analyse. Fortunately, data were available for the 2011 Grand Slams and these provide the statistical data to go with the mathematical and probabilistic analysis. My own background in tennis as a player and administrator provides the accompanying strategic analysis.

The opportunity to have a second serve, if the first serve fails, invites an investigation into the optimal use of both serves. It is shown that if you don't serve some faults, and even some double faults, you are not taking enough risk on service to maximise the chance of winning the point.

Not all points are equally important and an alternative definition of importance is presented to challenge the traditional definition. If you can lift (increase the probability of winning the point) on one or two points in a game, the two definitions produce quite different solutions of when to lift.

Assuming all points in a game are independent is a basic assumption for mathematical and probabilistic analysis. This assumption is challenged. Four direct and four indirect tests of independence are developed and applied to the point-by-point data.

There is one clear difference between successive points and that is that one point is served to the first court and the other point is served to the second court. This

suggest that tennis could be considered as point pairs and this approach is discussed and applied to the data.

Tennis players have always endured the indefinite nature of the scoring system, but with tennis now professional, administrators have gradually introduced modifications to the scoring system to improve the efficiency of tennis as presented to spectators, especially on television. The efficiency of all the three and five set systems are calculated and discussed, as well as the efficiency of tournament structures.

But efficiency in scoring or tournament structure may not actually improve a tournament, so the final chapter looks at other strategic ways tournaments could improve. Lessons from the Australian Open can be modified for other tournaments.

Acknowledgements

I would like to acknowledge the contribution of my brother Professor Graham Pollard who encouraged me to get involved in academic research again after an interval of approximately twenty years when my full-time role as President, Chairman and CEO of Tennis Australia changed to Executive President and Chairman and I employed a CEO to run the day-to-day business. Graham and I co-authored 21 articles with an overall even split of responsibility, he being named first 11 times and me ten times. He also encouraged me to stick with the thesis, even at times when I could not understand why a person approaching retirement would bother.

I would also like to acknowledge the help and guidance from my supervisor Professor Denny Meyer and thank her for being so tolerant with my challenges and time and travel restraints that went with my national and International roles in tennis administration. She offered great encouragement throughout the long ordeal and never wavered in her commitment. My other supervisor Professor Stephen Clarke retired from Swinburne University, but was always there and was of great help as a new set of eyes to my original draft of this thesis.

Finally I would like to thank my wife of 47 years, Eleanor, who could never really understand why I was still studying, what interest there was in numbers and whenever would I finish and be able to clear the desk. We already have a list of things to do together now we have more time and we are looking forward to it. We have two great children and three lovely grandchildren and will also be able to spend more time with them. Thank you Eleanor for your love and support.

Declaration of Authorship

This thesis is presented to fulfil the requirements of a Doctor of Philosophy at Swinburne University of Technology. This thesis is a complete update and rewrite of ten years of part-time academic research, much of which has already been published in academic journals and conference papers and one is a chapter in a major encyclopaedia. Altogether there are 39 published articles and 21 of these were in conjunction with my brother, Graham. We each made substantial contributions to all of our joint publications. Four chapters are completely my own work and four chapters are the result of the collaboration with my brother. All the data analysis, which was substantial, is mine.

With this qualification, I hereby declare that this thesis and the work presented in it is entirely my own. No other person's work has been used without due acknowledgement in this thesis. All references and verbatim extracts have been quoted and the sources of the data used in the analyses have been specifically acknowledged.

Geoff Pollard

December 8, 2017

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CHAPTER 1.

INTRODUCTION AND OUTLINE

1.1. Introduction

This thesis combines the work and play attributes of a former academic, whose primary interest was in mathematical statistics applied to population data (demography), with his involvement in tennis as a player and later career as a leading national and international administrator in the sport. The attraction of combining an inherent leaning towards mathematics, statistics and probability with the love of strategic decision making when playing or administering tennis led to this thesis entitled “A MATHEMATICAL, STATISTICAL, PROBABILISTIC AND STRATEGIC ANALYSIS OF TENNIS.”

A typical best of five sets match, as played by male competitors at each of the four Grand Slams, the Australian Open, the French Open, Wimbledon and the US Open, consists of anywhere between a theoretical minimum of seventy-two points up to a maximum that is theoretically unlimited, but in practice can be a few hundred points. Various statistics are collated and published for each match, but point by point data, as recorded manually on the umpire’s scorecard, was not publicly available until recently, as scoring became computerised, so only statistical summaries of each set as well as the match totals were ever published. For just one year (2011) each Grand Slam in conjunction with technology partner IBM published point-by-point data and these data are utilised in the numerical examples to accompany the maths and probability analyses in this thesis.

In many sports the scoring system is the fastest time or longest throw or is a simple accumulation of the number of goals scored or runs scored by each team. Some sports such as squash or table tennis have a bi-nested scoring system of points and games (or sets, but not both). As far as I know, only tennis has a tri-nested scoring system. In tennis the best of six points (leading by two) makes up a game, the best of ten games (leading by two) becomes a set, and the best of three or five sets makes up the match. It also means that all points are not equal in importance, with some points being more important than others. This contributes to the important part that strategy can play in tennis.

The tri-nested scoring system magnifies the difference between the players so that a slight difference in the probability of one player winning a point against another player becomes a greater probability of that player winning a game or a set or a match against that opponent. In the simplest case of one advantage service game, the game is won by the first player to win at least four points with a majority of at least two points. If the server has a probability p of winning a point on service then the receiver has a probability $q = 1-p$ of winning a point, then if all the points are independent and identically distributed, the probability g that the server wins the game is given by

$$g = p^4 + 4p^4q + 10p^4q^2 + 20p^3q^3(p^2/(p^2+q^2)) = p^4(-8p^3+28p^2-34p+15)/(p^2+q^2) = p^4(1-16q^4)/(p^4-q^4)$$

and the mean duration of the match μ points is given by

$$\mu = 4[(p^4 + q^4) + 5pq(p^3 + q^3) + 15p^2q^2r^{-1} + 10p^3q^3(3 + r)] \qquad \text{where } r^{-1} = p^2 + q^2$$

These equations show that the scoring system is fair, in that when $p = 0$, $g = 0$, when $p = 0.5$, $g = 0.5$ and when $p = 1$, $g = 1$, but the relationship is not linear but rather S-shaped so that when $p = 0.6$, $g = 0.74$ and when $p = 0.7$, $g = 0.90$. Thus there is a multiplier effect in that if $p>0.5$, as is normal in tennis, $g > p$. This typical advantage to the server, $p > 0.5$, is magnified so that $g > p$, but strategically the benefit is greater in lifting p from 0.5 to 0.6 or even 0.7, than trying to lift it from 0.7 to 0.8.

From here the probabilistic and mathematical study gets more difficult because once the first game is completed, the other player, with a different p value, has a turn to serve. This alternating service game continues until one player wins the set, or if the score reaches six games all, a tiebreaker game is played. This playing and scoring format presents opportunities for analysis and strategies not available in many other sports.

As a further opportunity for statistical and strategic analysis, tennis scoring possibly uniquely allows the server a second chance if the first service does not go into play. To optimize the probability of winning a point, a player will normally use a faster and riskier (less chance of serve going into play) first service, which has a greater chance of winning the point if it goes in. He can afford this risk and reward first service as he knows that if he does serve a fault, he can then resort to a slower and safer second serve, but with less chance of winning the point. Players work out their own use of first and second serve by trial and error depending on their own ability, but we can do this mathematically and statistically.

The assumption behind most theoretical modelling in tennis (and many other sports) is that all points are independent and identically distributed (for each server). There is some evidence that this assumption is not true, but that it is a good approximation, and can be utilized to undertake theoretical modelling producing acceptable conclusions. To seriously analyse independence between points requires point by point data which was not readily available until recently. With these data one can actually look at the outcome of any point based on the outcome of the previous point, or which player is ahead in that game or the importance of the point. Specific and non-specific tests can be developed to test the accuracy of this basic assumption of independence.

One obvious potential cause of lack of independence between points is to recognise that the first point is served to the first (or forehand) court and the second point is served to the second (or backhand) court and this cycle is repeated for the third and fourth point and so on until one player wins the best of six points game (and leads by two). This characteristic suggests that tennis scoring could be considered as two points at a time (bi-points or point pairs) rather than individual points. This novel approach to looking at independence, and the game of tennis in general, and the methodology and data analysis are introduced in this thesis.

In yet another twist to the scoring system and tennis data analysis, the sport has introduced various alternatives to the scoring system that can be utilised at the choice of the tournament organisers. All four Grand Slams use best of five sets for men (three sets for women) where the first four sets (first two sets for women) are tie-break sets if the score reaches six games all. However, three Grand Slams use an advantage set (first to win six games and lead by two games) for the fifth set (third set for women) and one uses a tie-break set. There are further differences in scoring in men’s doubles, women’s doubles and mixed doubles. Options for these events include five or three sets, advantage or tie-break sets, replacing the entire final set completely with a longer tie-break game, no-ad scoring and other measures primarily developed to reduce the potential length of a match and also reduce the variability in that length. These options have advantages for television programming and match scheduling, but so far have been restricted to doubles and junior events, as the prime spectator and media interest is in singles.

The data analysis in this thesis is restricted to the men’s singles as the best of five set format produces a more reasonable volume of data than best of three sets used in women’s singles and all doubles except men’s doubles at Wimbledon which remains best of five sets. Further doubles data are not available on a point-by-point basis and players do not always play with the same partner in each tournament. The other attraction of the men’s singles data is the consistency of the top four players (Djokovic, Federer, Nadal and Murray) in being semi-finalists in each Grand Slam so there is also the possibility of considering them on a player by player basis for any differences.

The game of tennis is dominated by the four Grand Slam Tournaments (Australian Open, French Open (also known as Roland Garros), Wimbledon and the US Open) which are often described as “the Pillars of the Game.” They are the only remaining tournaments to play best of five sets for men’s singles, they clearly offer the highest level of prizemoney, they are awarded double the computer ranking points of the next level of tournament and always attract the greatest attendances, media and sponsor interest. Just as the players use strategic analysis to maximise their chance of winning, the Australian Open needed to strategically address its many geographic, population, financial, sponsor and player issues to be recognised as equal in all aspects with the other Grand Slams, which it now is.

1.2. Outline

Chapter 2 is a summary of the academic research on the mathematical, statistical, probabilistic, strategic and other scientific research into tennis. A much shorter version of this literature review was originally published by Pollard and Meyer (2010) as “An

Overview of Operations Research in Tennis" in the substantial eight volume Wiley Encyclopaedia of Operations Research and Management Science (2010). In this thesis it has been updated to cover more recent publications and expanded to cover additional topics and it is now nearly double the length of the original publication. In particular a section on Integrity and Betting in Tennis has been added as this has become a major issue in tennis, and indeed all sport. Likewise the Tennis Anti-Doping Programme has undergone substantial expansion and enhancements over this period. Through my roles as President of Tennis Australia, Vice President of the International Tennis Federation and Chairman of its Technical and Rules of Tennis Committees, I have been integrally involved in both integrity issues and the introduction of approved modified scoring options.

Chapter 3 looks at the differences between first and second serves and the search for the optimal way and service strategies to maximise the probability of the desired outcome. Tennis is possibly unique in the way it allows a player a second chance to serve if the first service does not go into play. This topic has long interested investigators and the percentage of first and second serves that go into play and the percentage of points won on first and second serves that do go into play have long been the key statistics published for all tennis matches (along with number of aces and double faults). Variation in service strategy is shown to be better than consistency. Elite players don't just have an aggressive first service and a conservative second service, but a full range of serves between these two extremes. It is also shown that players are not serving optimally if they put all their first serves into play, and also not serving optimally if they don't take any risk and put all their second serves into play. The optimal serving strategy varies from player to player and match to match and also depends on the returning characteristics of the receiver, but it is shown that the optimal strategy (see page 89) is to get between 50 and 60% of first serves into play and between 90 and 100% of second serves into play.

Chapter 4 looks at the importance of a point in tennis and presents a different way of defining importance to the classical definition by Morris (1977). This classic definition of the importance of a point in tennis is the difference between the probability that the server wins the game if he wins the next point and the probability that the server wins the game if he loses the next point. This definition can be extended to cover the importance of games in a set or sets in a match. While this classical definition has stood the test of time, Chapter 4 looks at an alternative way of defining importance and what that means to a tennis player. It has been shown that some players are able to lift at certain stages in a game or set or match. Given this, which are the points or games or sets that the player receives the greatest benefit by lifting? This alternative approach defines these points or games or sets as more important as they achieve the greatest return for lifting. Mathematically, points in a game or sets in a match are similar because there is only one probability factor to be considered, namely the probability that the server wins a point on service in the case of a game or the probability that a player wins a set in the case of sets in a match. However, in the case of games in a set or points in a tiebreak there are two probability factors to consider, namely the probability p_1 that one player wins a point on his service and the probability p_2 that the other player wins a point on his service and consequently the analysis is much more difficult. From this analysis we can determine mathematically on which points, games or sets a player should endeavour to lift his game, if possible. Not all the results are obvious, but the strategic benefits to players of this knowledge are important.

Chapter 5 investigates the assumption that points are independent and identically distributed. This assumption is not precisely correct, but it is a good first approximation and results obtained by adopting this assumption are also reasonable approximations. Tests to consider independence between points by looking at point-by-point data do not currently exist, primarily because point-by-point data are not readily available. Chapter 5 presents four specific tests for independence between points and four non-specific tests for independence that could be used if point-by-point data were available. After developing the eight tests, data for the top four male players are then analysed. The majority of the tests show no significant variation from independence, but a few significant variations are identified.

Chapter 6 looks at the efficiency of scoring systems. Given the potential length of a best of five sets men's singles match and the huge variation in the length, it is clear that tennis does not employ the most efficient scoring system. It is also an unfair system as players meeting in one round have probably not experienced the same length of match in the previous round. One player could have experienced an easy straight sets win, while his opponent in the next round could have played a long, energy draining five set thriller. For any scoring system, assuming independence between points, it is possible to calculate the mean, variance and skewness of the number of points played and the probability that the better player wins. If two different scoring systems have the same probability that the better player wins, then one system can be said to be more efficient than the other system if it has a smaller expected number of points played.

Primary interest is in the scoring systems used in the four Grand Slam Tennis Championships, but also considered are the others accepted by the International Tennis Federation (ITF) as alternative scoring systems and so far only utilised in doubles on the ATP (men's) and WTA (women's) Tours. More efficient alternatives have been suggested, but are unlikely to be accepted by conservative officials and players at this time. Nevertheless, modifications to the scoring system to reduce the mean and variance of the length of a match, without significantly changing the overall probability that the better player wins, are widely sought, especially for doubles. This approach to measure the efficiency of scoring systems is also modified to measure the efficiency of various tournament structures from the knock-out to the round-robin.

Chapter 7 introduces a new approach to the analysis of tennis data and the question of independence between points by recognising that there are two types of point, a serve to the first or forehand court and a serve to the second or backhand court. This pairing of two different types of point is repeated until the game is won or lost, the only exception is the last point when a game is won or lost at 40-15 or 15-40 respectively. This approach to tennis scoring considers tennis as a game of point pairs and without loss of generality this exception only eliminates about five percent of points played. Similar results for independence between points were obtained for the top four players using this approach as was obtained in Chapter 4 using point-by-point data, but it is hoped this different approach will encourage further analysis of tennis as a game of pairs of points.

Chapter 8 considers the strategic planning for major events, using the Australian Open as a role model. The Australian Open was in danger of losing its Grand Slam status in the 1980s, but has not only unquestionably retained that status, it now leads the other

Grand Slams in many characteristics. This chapter outlines a dozen strategic planning issues that the Australian Open had to address to achieve universal, unqualified acceptance as a Grand Slam again. The strategic planning principles outlined in Chapter 8 worked for the Australian Open and are adaptable for other events. This is an expanded and updated version of an unpublished presentation to an international conference where I explained the strategic high level decisions that needed to be made, and then successfully implemented, to substantially grow an event.

The majority of the work in this thesis was initially published in a collection of academic papers that were either published in refereed journals or accepted (after refereeing) and presented to academic conferences in Australia and overseas. The complete list of these thirty-nine papers is shown in Appendix 1. It should be noted that in five papers I am the sole author, in 21 papers I am the first named author and in 15 papers I am the second named author. Those in which I am the third named author and those in which I have not contributed at least fifty percent to the research have not been used in this thesis. Most of those used in this thesis have been rewritten and in many cases expanded to include additional material.

CHAPTER 2

AN OVERVIEW OF OPERATIONS RESEARCH AND MANAGEMENT SCIENCE IN TENNIS

In 2010 I was invited by the Editor-in-Chief of the Wiley Encyclopedia of Operations Research and Management Science to write a chapter on Operations Research and Management Science in Tennis. Scientific and technological developments have created both problems and opportunities for the game of tennis. This chapter is an updated and expanded version of the G Pollard and D Meyer (2010) chapter in the Wiley Encyclopedia. It considers some of the developments that have occurred in the game and highlights the past, current and potential future use of operations research and management science methods in the game of tennis and sets the stage for the more detailed mathematical, probabilistic and statistical analysis of certain aspects of the game that then follows in this thesis.

Briefly, the modern game of “tennis” began in 1874 as “lawn tennis” which was conceived as an outdoor version of the ancient game of tennis, which is now known as “royal tennis”. Over the next nearly one hundred years the equipment and the rules hardly changed. Originally designed as a portable game to be played on grassed areas using balls and stringed racquets made of wood, the courts soon became permanent and the surface extended from grass to clay and then various hardcourt and synthetic surfaces. Racquets and balls continued to improve in quality, but remained essentially the same. The three tiered scoring system of points, games and sets remained consistent throughout, along with the principle of winning by at least two points (in a game) and at least two games (in a set).

In 1968 the longstanding division between amateurs and professionals ended and tournaments were then opened to all players. The game expanded rapidly. The equipment and the scoring system that had served the amateur game for so long were repeatedly thrown into question by technical developments, especially in the manufacture of racquets and strings, and by the demands of television coverage, especially the problems associated with the undefined length of time it takes to play a match of tennis.

The professional nature of the game meant that playing tennis became a full-time occupation and players engaged an increasing entourage of coaches, trainers, physiotherapists, mental advisors, managers, etc. Meanwhile the ever-increasing level of prizemoney meant that tournaments had to be better planned, organized and managed to ensure their survival and financial success.

This chapter looks at the ways technological science and management is used or could be used to address the following issues for the professional game of today. Topics covered are:

2.1. Current and Potential Use of Operations Research and Management Science Methods in tennis

2.2. Developing Alternative and Optimal Scoring Systems.

2.3. Developing Optimal Match Strategies.

2.4. Surface Optimization.

2. 5. Optimization of Ball Characteristics.

2. 6. Optimization of Racket Characteristics.

2.7. Optimization from a Coaching Perspective.

2. 8. Optimizing Player Technique.

2. 9. Minimizing Player Medical Risk and performance enhancing drug taking.

2.10. Efficient Tournament Management.

2.11. Strengthening the Mental side of the game.

2.12. Accurate Ranking of Players

2.13. Improving Officiating

2.14. Integrity and Betting

2.15. Summary and Implications

2.1. Current and Potential use of Operations Research and Management Science in Tennis.

The following Table 1 illustrates how Operations Research and Management Science methods are currently used in tennis. However, modern technological developments mean that there are now ways to measure many more of the relevant variables, opening up many new uses for these methods. Under a range of topics the following table suggests various decision methods using appropriate objective functions and constraints. These topics are then discussed in detail in this chapter.

Current and Potential Use of OR/MS Methods in Tennis

Topic	**Objective Function**	**Constraints**	**Decisions**	**Appropriate OR/MS Methods**
Developing alternative and optimal scoring systems	Minimize the mean and variation in length of matches	Integrity of tennis' unique scoring system; better player should win	Alternative procedures and scoring systems	Deterministic optimization using mathematical models
Developing optimal match strategies	Maximize the probability of player success	Surface properties (e.g. court speed); Current scoring system.	Serving strategy, When to lift When to challenge	Deterministic optimization, Markov Chain and Dynamic Programming models.
Surface optimization	Minimize capital and maintenance costs Maximize income from court hire and the chance of winning	Weather. Player preferences in terms of speed and comfort	Choice of surface	Deterministic optimization, Multi-criteria optimization

Optimizing ball characteristics	Optimize ball speed, spin, trajectory, durability, bounce, etc. Minimize cost effects and maximize ease of play	Surface and racket properties; player ability, style and age; cloth texture	Internal pressure or pressureless, dynamic stiffness.	Stochastic optimization using mathematical modelling and computer simulation
Optimizing racket characteristics	Maximize ease of play and minimize cost effects. Optimize ball trajectory and power for any stroke	Preservation of the nature of tennis; ball, racket and surface properties; racket material; atmospheric conditions	Head size, weight and grip-size. Frame stiffness, balance point, sweet spots, swing-weight, stability, shock, power, vibration, feel	Stochastic optimization using mathematical modelling and computer simulation
Optimization from a coaching perspective	Optimize performance, minimize injury risk;	Physical and mental attributes of player, ball, racket and surface characteristics	Technical advice and short/long-term training (periodisation) plans	Pattern recognition using video footage; deterministic optimization
Optimizing playing technique	Optimize power and control combination	Equipment, surface, athletic and physical attributes of player, injury risk	Choice of style and technique for individual players	Multi-criteria programming.
Minimizing medical risk and performance enhancing drug taking	Minimize injury risk; Optimize rehabilitation and training; Integrity through Anti-Doping Programme	Player characteristics, opponent's strength; ambient conditions, other risk factors; Continually updating Anti-Doping programme	Tournament rules (e.g. minimum age of players, heat policies, racket design, etc.) Fully WADA compliant	Deterministic optimization using mathematical models
Efficient tournament management	Maximize the number of feasible matches	Availability of courts and players, Maximum time for matches	Match schedule	Constraint programming with heuristic search

Strengthening the mental side of the game	Minimize variation in playing ability through the control of heart rhythms and emotion	Physical and mental attributes of player; ambient conditions; surface, other risk factors	Choice of techniques for maintaining concentration (e.g. Breathing, Stalling, Grunting, Variation in play, crowd interaction).	Deterministic optimization using mathematical models
Accurate player rankings	Maximize accuracy of rankings in terms of performance	Different levels of tennis, singles/doubles, surface, effects of injury	Weightings for importance and closeness of a match, strength of opponent. Protective rankings. Average or “Best of x” results rules	Kalman filter, Monte Carlo simulation, Nonlinear optimization.
Improving officiating	Optimized accuracy of line calls and foot faults	Technology used, surface, human error	Choice of method, model, training of officials	Pattern recognition using video footage; deterministic optimization
Integrity and betting	Zero tolerance to betting related corruption	Betting industry, Technology used, Online issues, player and official temptation	Tournament rules and practices	Match Alert Data analysis

Table 2.1. Current and potential use of Operations Research and Management Science in tennis.

2.2. Developing Alternative and Optimal Scoring Systems.

Tennis has a unique scoring system, in that it is triple nested (points, games, and sets). One player serves and his/her opponent receives and the server has two chances to put the ball into play. The same player continues serving until a game is completed, which is won by the player winning the “best of six” points (i.e. first to four points) provided that one player leads by at least two points. In the following game the receiver becomes the server and the players continue to swap roles each game. The set is “best of ten” games (i.e. first to six games) provided the player leads by at least two games (called an “advantage set”). A match consists of the “best of three” sets (i.e. first to win two sets) or in Grand Slam men’s tournaments and Davis Cup the “best of five” sets (i.e. first to win three sets).

The first significant change of the open tennis era, which was intended to help reduce the mean and variation in the length of matches, was to introduce the option of a ”tie-break set” instead of an “advantage set”. The International Tennis Federation (ITF) Rules of Tennis (2015) now provide that if the score reaches six games all, the set can be decided by playing a tie-break game where the first player to win seven points wins the game and the set, provided there is a margin of two points within the tie-break game. If necessary the tie-break game continues until this margin is achieved. Thus, although the duration of a tie-break set is still unbounded, it clearly has a lower mean and variance than the advantage set. The tie-break set is now universally used with the exception of three of the four Grand Slams which require the deciding final set to remain an advantage set.

Tennis, pushed by television, searches for a solution to the reasonably unpredictable length of tennis matches, while maintaining the integrity of its unique scoring system. Appendix IV, of the ITF Rules of Tennis (2017) offers alternative procedures and scoring methods such as no ad scoring, short sets, a match tie-break (first to ten instead of first to seven and leading by two points) to replace the deciding final set and eliminating the let during service. In the Forward to the Rules of Tennis, the International Tennis Federation invites interested parties to submit applications to officially trial other scoring systems.

At the elite level, the major use of modified scoring has been in doubles and mixed doubles. Three of the Grand Slams now use a match tiebreaker in place of normal set for the third set of mixed doubles, but retain a tie-break for the third set of women’s doubles and the fifth set of men’s doubles. Only Wimbledon still uses an advantage final and deciding set in singles, doubles and mixed doubles. Both the ATP and WTA Tours use no ad scoring and match tie-break in doubles, but retain three tie-break scoring in singles.

There is a growing collection of computer simulation, probability and statistical analyses of the official scoring system and alternative more efficient scoring systems. The simplest stochastic models (unipoints) assume all points are independent and identically distributed. A more realistic but more complicated model for elite tennis recognizes that there are two types of point where the probability of the server winning the point depends on which player is serving (bipoints).

Kemeny and Snell (1960) modeled a single game of tennis using a Markov chain. Schultz (1970) examined the feasibility of using a Markov chain with stationary transition probabilities to evaluate scoring systems. Hsi and Burych (1971), taking a bipoints approach, evaluated the probability that a player wins an advantage set. Carter and Crews (1971), taking a unipoints approach, found the expected number of points in a game, games in a set and sets in a match. Miles (1988), using the bipoints approach, found the expected duration of a set in tennis. Pollard (1983) showed mathematically that the expected duration and variance of a tie-break set and match are always smaller than using advantage sets, but the probability the better player wins is slightly reduced.

Morris (1977) introduced the concept of importance for each point played and the more complex notion of “time-importance” where the importance is weighted by the expected number of times the point is played. Miles (1984) noted the link between sports scoring systems and symmetric sequential statistical hypothesis testing and examined the efficiencies of sports scoring systems such as tennis. Pollard (1986, 1992) linked Miles’ and Morris’ work and found an important relationship between the importance of points within a system and the efficiency of the system itself. He uses this relationship to find the optimally efficient tennis scoring system. Pollard (1990) showed how, in many cases, the relative efficiency of two different scoring systems could be found without calculating the efficiency of each system separately. Pollard and Noble (2004) suggested the 50-40 game (server has to reach 50 before the receiver reaches 40 so that each player has a game point at 40-30 and games consist of a maximum of six points) and showed that it leads to good scoring systems in terms of variance and efficiency, particularly for doubles.

Pollard and Pollard (2008a) generalized Miles formula from singles (two parameters) to doubles (four parameters) and identified very efficient doubles scoring systems. Pollard and Pollard (2008b) also found the moment generating function for the present three set tennis scoring system.

Elite players have shown little interest in modified scoring systems despite the mathematical evidence, but accepted some changes to the doubles scoring on the ATP and WTA Tours in return for more doubles matches being scheduled on Centre Court now that the mean and variation in the duration of the match was reduced for Tournament organisers.

2.3. Developing Optimal Match Strategies.

Croucher (1998) summarized much of the existing literature on developing strategies within current scoring systems in tennis and separated his analysis into firstly, serving strategies (given you have a first and second serve), secondly, the probability of winning a game, set or match and thirdly, the analysis of match data. More recently Klassen and Magnus (2014) looked at twenty two often stated hypotheses in tennis (e.g. It is an advantage to serve first in a set; the seventh game is the most important game in tennis; a player is as good as his or her second serve; new balls are an advantage to the server; etc.) and prove which hypotheses were correct and which were false. Many of these hypotheses had been tested in previous papers by the authors over a twenty year period, but were now consolidated in book form.

Gale (1971) used a simple mathematical model to identify the best first serve and the best second serve from a set of serves. Redington (1972) concluded that in the 1971 Wimbledon final, Newcombe would have done better if he had served two first serves rather than a first and second serve. George (1973) used a simple probabilistic model to show that a 'strong' first serve followed by a weaker second serve was generally but not always the best strategy. Using considerably more data King and Baker (1979) came to the same conclusion. Hannan (1976) proposed a game theory approach that factored in the opponents return while Norman (1985) used dynamic programming to decide when a first serve should be used. With even more data McMahon and de Mestre (2002) also confirmed that the 'strong weak" serve pattern was usually the best but were surprised at the number of matches where an alternative was better. Barnett and Clarke (2005) used server and receiver statistics to predict strategies and outcomes in any match between two players. Barnett and Pollard (2007) adjusted this analysis for the effect of court surface. Pollard and Pollard (2007) and Pollard (2008) looked at the mathematical relationship between the probability a serve goes in and the probability a player wins the point and thus determined the optimal strategy for first and second serves. Barnett, Meyer and Pollard (2008) used the large OnCourt database (www.oncourt.info) to calculate match statistics for each court surface and thus determine service strategies to improve performance.

There has been much research over the years, including work by Fischer (1980), Pollard (1983)and Croucher (1986) on the probability of winning a game, set (advantage and tie-break) and match where the probability of winning a point is constant for each player and each point is independent. For example Croucher (1986) calculated the probability that the server held serve from each of the 16 possible scorelines. Morris (1977) introduced the concept of the importance of a point defined as the difference between the two conditional probabilities that the server wins the game if he wins that point and the probability that he wins the game after losing that point. Pollard (1986) expanded and generalized this analysis of importance and Barnett (2006) showed numerically when a player should lift (increase) his/her probability of winning a point. Using ten years of Grand Slam men's singles data, Pollard, Cross and Meyer (2006) showed that the better player had the ability to lift his play. Pollard and Pollard (2007) used differential calculus to identify the points, games or sets when the player gets the most reward for lifting. All these mathematical analyses assume each point is independent of the previous points. However, using four years data from Wimbledon 1992-1995, Klaassen and Magnus (2001) showed that points in tennis are neither independent nor identically distributed. Nevertheless they claimed that these assumptions are "sufficient as a first order approximation”, which is similar to Croucher's (1998) comment that these assumptions "provided some interesting, if debatable, conclusions".

Selected statistics (aces, double faults, winners, errors, total points) were often collected for selected important matches such as Davis Cup Challenge Rounds but were primarily for media use. For these and some of the earlier analyses above, data was laboriously collected by pencil and paper by someone watching the match or a video replay. In 1982 Bill Jacobsen began to record his son’s matches on a micro-computer. This soon developed into a system Compu Tennis which became a useful tool for coaches. Today the umpire records each point live into the computer and organizers record additional statistics at all Grand Slam matches and many matches on the Association of Tennis Professionals (ATP) and Women’s Tennis Association (WTA) Tours, although only limited data is published and access to the data is restricted. By accessing Wimbledon data 1992-1995 Magnus and Klaassen (2008) were able to test and mostly refute some of the common hypotheses in tennis such as; any advantage in serving first, the first use of new balls, who has the advantage in the final set in a five set match, and even forecasting the winner before and at various stages during a match. The fact that their data are still being utilized demonstrates the limited access for researchers to officially collected data although individual matches are provided to the media and selected match statistics are available on the OnCourt database.

Klaassen and Magnus (2014) combined all their analyses of the commonly held hypotheses on tennis and published them in a book entitled “Analyzing Wimbledon- The Power of Statistics”. Twenty Two hypotheses are presented and ultimately accepted or rejected. I was pleased to be invited to referee the book and am acknowledged in the introduction.

2.4. Surface Optimization.

Originally designed to be played on quality grass surfaces, the weather and cost of maintenance requirements soon led to lawn tennis being played on alternative surfaces, initially on clay (crushed brick) followed by various hardcourt surfaces such as cement, asphalt, wood, etc. and as technology developed, an increasing range of synthetic hard and cushioned acrylic surfaces as well as artificial grass surfaces. Further, court surface has a very significant effect on the nature of the way the game is played, so that, for example, selection of court surface is a prized home ground advantage in Davis Cup. Selection of court surface for a venue is a multi-criteria optimization problem where the capital cost of construction has to be balanced against maintenance costs and player preferences in terms of court speed and comfort.

The rules of tennis were effectively silent on court surfaces, but to allow quantification of the speed of a court surface, the ITF developed a Court Surface Classification Scheme in 2000 which was superseded by the ITF Court Pace Classification Program in 2008 and updated annually. This system divides court pace as measured by the Court Pace Rating (CPR) into five categories:

Category 1	(slow pace)	CPR 0-29
Category 2	(medium slow)	CPR 30-34
Category 3	(medium)	CPR 35-39
Category 4	(medium fast)	CPR 40-44
Category 5	(fast)	CPR 45 & over

and recognizes ten court surface types:

A	Acrylic
B	Artificial Clay
C	Artificial Grass
D	Asphalt
E	Carpet
F	Clay
G	Concrete
H	Grass
J	Hybrid Clay
K	Other (e.g. modular tiles, wood, canvas)

Currently around 140 different court surface suppliers and around 300 courts are recognized by the ITF, and their products cover approximately 35 (Category 1), 50 (Category 2), 100 (Category 3), 65 (Category 4) and 50 (Category 5) surfaces. However few companies laying clay or grass courts register as their products are inherently variable, although in general they are recognized as slow (Category 1) and fast (Category 5) respectively.

Barnett and Pollard (2007) analysed the performance of players on grass, hard and clay courts used on the ATP and WTA Tours, and how the changing number of tournaments on each surface, especially the decline in grass courts, affects player rankings and also injuries. The decline in grass court tournaments was reversed in 2015 when Wimbledon moved its dates back by one week increasing the interval between the French Open and Wimbledon from two to three weeks, allowing additional grass court tournaments to be added to the ATP and WTA Tours.

Only Wimbledon (grass) adjusts official rankings when seeding its Championships, giving greater weight to its event and other tournaments played on grass and less weight to clay court tournaments. Rankings and medical risks are discussed later, but both would benefit from further operations research analysis factoring in the effect of court surface. At dispute is the issue of whether rankings are simply a reward for performance over the previous twelve months (the player view) or whether they are designed to distribute the best players for that tournament on that specific court surface evenly throughout the draw (the tournament view).

The development of a court pace classification programme has been an interesting technical, scientific and statistical exercise, especially since 2008 when the ITF introduced rules to limit the ability of home nations to select extreme synthetic surfaces for the Davis Cup Competition. This is outlined in the ITF Manual (2015 and earlier years) and in papers such as Spurr, Capel-Davies and Miller (2007). However, a player comfort classification system for court surfaces has been discussed but is yet to be developed and approved.

2.5. Optimization of Ball Characteristics.

Compared to the technological development of the golf ball over the past 100 years, the tennis ball used at most levels of play has remained almost unchanged except for the development of the pressurized can, to ensure longer shelf life, and the change in colour from white to yellow. Pressureless and high altitude balls are also available which differ from the "standard ball" by virtue of their internal pressure and bounce height. Larger and softer balls coloured green, orange and red have been introduced for starter players aged ten and under to help them enjoy playing while learning the game.

Over the last decade the ITF has tried to address the issues of court speed and style of play by encouraging manufacturers to make three types of ball, where ball type one (fast speed ball) is designed for use on slow paced court surface; ball type three (slow speed ball) is designed for use on fast paced court surfaces, while the standard ball type two (medium) is designed to be used on medium-slow, medium and medium-fast surfaces (i.e. the majority of courts). Whilst there is clear evidence that using the appropriate ball on the relevant surface makes the game easier to play (for both players), the use of balls other than ball type two in tournaments is very limited.

Brody, Cross and Lindsay (2002) have a number of chapters devoted to the tennis ball and the testing of ball properties for perfect bounce, ball spin, trajectories and similar technical characteristics. All ITF ball testing measures the properties of only new balls. The ITF Technical Department has now revolutionized ball approval by introducing the testing of ball durability into the process. A key challenge has been to accurately simulate in the laboratory the effects of play on the ball. This development is discussed in Spurr and Capel-Davies (2007) and the ITF now tests balls after a period of simulated use.

Another development discussed by Downing (2007) is the comparison between the static stiffness of tennis balls as measured in the current official ball testing process and the dynamic stiffness in real play, the potential outcome of which is the introduction of approval tests that are more representative of player perceptions of ball properties.

Special tennis balls have been developed to make it easier for starter players, especially ten and under, to play the game and to position tennis as easy and fun. These introductory balls are colour coded red (stage 3), orange (stage 2) and green (stage 1) and are also officially tested for approval by the ITF and are now mandatory worldwide for tournaments in the under ten age group under the rules of tennis.

The ITF currently recognizes 43 different manufacturers of tennis balls and approves around 165 Type Two balls (regular tennis balls), 10 high altitude balls, and between 20 and 30 each of green, orange and red tennis balls for starter players.

2.6. Optimization of Racquet Characteristics.

For nearly 100 years tennis racquets were made of wood and generally strung with strings made from sheep's gut. Aluminium frames appeared in the 1960s leading to the larger head size in the 1970s. Aluminium was replaced with graphite and then by composite rackets made mainly from carbon fibre reinforced with graphite. Titanium and more recently ceramic fibres, boron and other products, in many cases outcomes of the research and development in the aerospace industry, have also been used.

Comparing modern carbon fibre composite racquets to the best wooden racquets at the beginning of the open era shows they are much lighter (180-350g compared to 370-425g), longer (68-73cm compared to 66-68cm) with a larger head size (580-870 cm^2 compared to 420-450 cm^2). Amongst the top 20 male players Warinka uses a 271g racquet while the average racquet weight is 320g. Amongst the top 20 women players the Williams sisters use 270g rackets while the average racquet weight is 305g.

The modern composite racquets have a greater "sweet spot" and are considerably more powerful. The term "sweet spot" is the commonly accepted term for the area on the racquet face where its performance is maximized. In fact, the "sweet spot" is three spots; the Centre of Percussion where shock (i.e. acceleration) of the racquet is minimized, the node where vibration is minimized and, finally, the location of maximum ball rebound speed (although the impact at this point does not necessarily feel "sweet"). Inherently the material of which modern racquets are made are more powerful than wood in that less energy is lost (due to bending) during impact. On the assumption that a player puts constant energy into a stroke, a modern (lighter) racquet will have a greater impact speed and will therefore generate a higher ball speed, which is used by racquet manufacturers as the definition of “power”, although strictly it is just “velocity”.

At the social level larger racquets have made the game easier to play by being more “forgiving” to off-centre impacts. At the elite level they have changed the way the game is played (technique) and professional tennis is now a considerably faster and different game to that of the amateur era.

Strings and stringing have also contributed to the development of the modern racquet. Until the 1940s all racquets were strung with gut strings but gradually nylon strings appeared, although nylon was inferior to gut in the wood racquets. The development of the modern composite racquets and the considerable improvement in strings with nylon, polyester, Kevlar and other substances, means that today there is little difference between different string types and all are used by the top professional players, with polyester the most preferred string followed by gut and then nylon. Polyester strings do generate more spin.

But it was the method of stringing that lead the ITF to finally act in 1978 on the definition of the racquet when “double stringing” or “spaghetti stringing” allowed some players to generate so much spin that the ball was virtually unplayable. Such stringing was banned. In the 1980’s the increasing size of the racquet lead the ITF to finally limit the size of a tennis racquet in the Rules of Tennis and to define the parameters for stringing patterns.

In the 1990s the ITF established a research laboratory at its headquarters in London with the analysis of racquets and strings and the increasing “power” they can generate being a major issue for on-going research and for the possible introduction of limits into the Rules of Tennis. The measurement of spin is discussed in Goodwill, Douglas, Miller and Haake (2006) and the measurement of “power” in Goodwill, Haake and Miller (2007).

Brody, Cross and Lindsay (2002) have a number of chapters devoted to the tennis racquet and the technical measurement of relatively simple characteristics such as head-size, weight and grip-size, through to the more complicated characteristics of frame stiffness, balance point, sweet spot, swing-weight, stability, shock, vibration, feel and "power".

As part of its role to preserve the nature of tennis, the ITF Science and Technical Department has developed a software program which simulates the effects of ball, racquet and surface properties and atmospheric conditions on ball trajectory for any stroke. Not only is the software (known as Tennis GUT) able to establish the effects of any combination of equipment on the nature of the game, but it also allows the user to modify the properties of that equipment. Trends in equipment design and properties can be used, for example, to predict future properties and, as a consequence, to predict how these trends may affect the way the game is played.

2.7. Optimization from a Coaching Perspective.

Coaching is the method by which one generation passes on its considerable knowledge, experiences and expertise to the next generation, thus enabling the game to grow and develop. In tennis, coaches play a significant role at every level from introducing youngsters to the game and teaching them the basic rudiments of how to play, through to helping the world's best players to become even better and retain their high rankings. This includes analyzing each opponent to detect any potential weakness that might be exploited.

Traditionally tennis coaching involved either past champions describing their experience and how they played the game or other teachers, who may not have had the same on court success, but who appreciated and understood the style and technique of the champions and had the ability to describe and teach it. Whilst there is still a role for the traditional coach, the modern coach also makes considerable use of all the sports science and technology that has been introduced into the game over recent decades.

Coaching courses today include a significant component of sports science and technology and coaches are encouraged to access the various ITF coaching publications and the ITF coaching website www.tenniscoach.com to see the extent of modern technology available and to engage in on-line learning or upgrading. The ITF has recommended syllabi for level one, two and three coaches that are available in a number of languages and which are now used in over 80 countries.

Many coaches use video to record their pupil in practice or match conditions and then use software such as Dartfish to analyse their game. Other match and player analysis programs commercially available include the Swinger video analysis, ChartMate Pro and Stats Master video analysis. Tournaments often provide players with a video of their matches and players will often try to obtain videos of their next opponent for analysis prior to their next match.

For running their businesses coaches use specifically designed commercial programs such as Tennis Biz, Tennis Logic or Software Management Solutions. For marketing and communications purposes there is increasing use of SMS, email and in some cases their own website.

National associations use products such as Athlete Management System to track all players in their academies. Coaches record their analysis following matches, practice sessions, training regimes, technical advice, injury, periodisation plans, communication with other coaches and even video footage and analysis, etc.

Using Wimbledon Singles data 1992-1995, Klassen and Magnus (2003) converted player rankings into the probability a particular player would win a match between two players. They then calculated that probability as the match progressed. They suggested such information was probably more useful for live television commentators rather than coaches, as coaches were not permitted to have contact with their players during a match (except Davis and Fed Cups). The WTA Tour only (and not the ATP Tour or the Grand Slams) now allows on court coaching under specific guidelines. Obviously these changing probabilities are also useful for punters and betting agencies and these are discussed by Bedford and Clarke (2000) and Barnett and Clarke (2002).

2.8. Optimizing Player Technique.

Biomechanics is the study of human movement. In tennis this involves the search for the optimum playing technique, the most efficient and effective combination of power and control in both stroke and movement while minimizing the risk of injury.

Traditionally coaches performed this task simply by observing the world's best players and then trying to teach these observed techniques to their pupils. But tennis is such a multidimensional sport. It is played by men and women, young and old, clumsy and talented, serve/volley players and baseliners and with ever changing equipment on a wide range of surfaces from clay (slow) to hardcourts (medium) to grass (fast). There is no single optimum way to play and champions have displayed a wide range of styles and techniques. However, modern technology has made it easier to describe body movement, to quantify movement and to compare players' style.

Elliott and Reid (2008) give a good summary of the wide range of descriptive and objective technologies now available to the coach or the tennis biomechanical researcher. Generally they involve one or more high speed video cameras linked to a computer loaded with one of the increasing range of sport analysis software packages. However they repeat the warning of Elliott and Knudson (2003) that any recommendation by a coach to change a player's game first needs a comprehensive assessment of a wide range of other player characteristics.

For a complete study of the biomechanics of advanced tennis with respect to the effectiveness and efficiency of a player's on-court movement and stroke production, and the relationship with performance enhancement and injury prevention, the reader is referred to the ITF publication, Biomechanics of Advanced Tennis edited by Elliott, Reid and Crespo (2003) and its wealth of references.

2.9. Minimizing Medical Risk.

2.9.1. Sports Medicine

Sports Medicine has become a legitimate and recognized discipline within medicine. Basically it involves the prevention of athletic injuries or their detection, treatment and rehabilitation. It is closely linked to the other sports sciences, although in most countries only medical practitioners are licensed to approve drugs as part of the treatment. There are now doctors who specialize in injuries specific to tennis. The WTA and ATP are in the process of introducing an injury registration database which will provide accurate data on tennis injuries in the future.

The best known injury in tennis is the so called “tennis elbow”. Many people acquiring tendonitis of the elbow from other causes unrelated to tennis will still call the injury “tennis elbow”. The different injuries causing elbow pain and their diagnosis, treatment and rehabilitation are described by Renstrom (2002).

The International Olympic Committee and the International Tennis Federation, together with the Society for Tennis Medicine and Science and the ATP and WTA Tours, have combined to produce a Handbook of Sports Medicine and Science in Tennis edited by Per Renstrom (2002). The nature of the game means that injuries occur throughout the whole body with shoulder, back, ankle and knee being the most common sites of injury, but these are generally minor injuries that respond well to treatment. The majority of injuries are due to micro trauma repetitive overwork. Macro trauma, such as sprains or fractures, does occur in a minority of cases and are difficult to prevent with training activities. An online version was published in 2008.

Although injuries do occur, Pluim et al (2007) have conducted an extensive literature search of the health benefits of tennis and concluded that “people who choose to play tennis appear to have significant health benefits, including improved aerobic fitness, a lower body fat percentage, a more favourable lipid profile, a reduced risk for developing cardiovascular disease, and improved bone health”.

Other aspects of Sports Science and medicine currently undergoing research, analysis, progressive introduction or modification include minimum age limits for boys and girls for international tournament play, continuous registration of injuries and their treatment for international players, measurement of heat and humidity and appropriate rules affecting play in extreme conditions, and, finally, anti-doping.

Statistics is commonly used in Tennis Medicine. For example Hatch et al (2006) report an experiment in which sixteen asymptomatic Division I and II collegiate tennis players performed single-handed backhand ground strokes with rackets of 3 different grip sizes (recommended measurement, undersized ¼ in, and oversized ¼ in).

However, Pluim et al (2006) have concluded that there have been few longitudinal cohort studies that have investigated the relationship between risk factors and injuries in tennis. This together with the complete absence of randomized control trials concerning injury prevention means that this is an area that is desperately in need of more rigorous analysis in the future.

Other useful references to sports medicine in tennis include Pluim and Safran (2006) and Petersen and Nittinger (2006) while coaches are referred to the ITF Manual on Tennis Medicine for Tennis Coaches (2001). In addition the Society for Tennis Medicine and Science publishes the international pier reviewed journal “Medicine and Science in Tennis” in conjunction with the ITF, ATP, WTA, Grand Slams, and selected major National Tennis Associations.

2.9.2. Integrity of Tennis- The Anti-Doping Programme

Tennis has an Anti-Doping Programme that applies to all players who hold an ATP or WTA ranking in the professional game or who hold an ITF World Junior, Senior, Wheelchair or Beach Tennis ranking in the amateur game and compete in events organized, sanctioned or recognized by the ITF. This programme is comprehensive and internationally recognized by the World Anti-Doping Agency (WADA). The Tennis Anti-Doping Program is designed to achieve the dual objectives of maintaining the integrity of the game of tennis and protecting the health and rights of all tennis players. It has an educational component to ensure players are fully aware of the rules of the programme, the WADA Prohibited List of drugs and how to obtain a Therapeutic Use Exemption as well as the testing component and prosecution of players who test positive to a prohibited substance.

Drug testing was introduced in the late 1980s by the then Men's Tennis Council and focused on recreational drugs. When the ATP Tour was formed in 1990, it took over the testing and included performance-enhancing drug testing. The ITF (including the independent Grand Slams) and the WTA also conducted drug testing for players in their tournaments. In 1993 the different authorities agreed to a joint anti-doping programme administered separately by each authority. The ITF took over the management, administration and enforcement of the ATP events in 2006 and the WTA events in 2007 resulting in a unified Tennis Anti-Doping Programme across all tennis tournaments and events. An out-of-competition testing component was introduced in 2005 and became mandatory in 2009, so that there is now a roughly equal number of in-competition and out-of-competition tests. Likewise blood testing has increased so that there is also roughly an equal number of blood and urine tests conducted. Leading players are also required to maintain an Athlete Biological Passport (ABP) which records their blood characteristics and thus easily identify any significant changes. The Tennis Anti-Doping Programme is fully WADA compliant.

In 2016 the ITF conducted 4,899 doping tests as shown in the following Table 2.2 In considering these numbers it needs to be noted that players may also be tested by their National Anti-Doping Organisation and were also tested at the Olympic Tennis Event, but these additional tests are not included in Table 2.2. Seven violations were detected resulting in penalties ranging from no ban to a ban of 3 years and 9 months (plus forfeiture of prizemoney won and loss of computer ranking points earned.)

Type of Test	Men	Women	Total
In Comp. (Urine)	1,108	879	1,987
In Comp. (Blood)	118	134	252
In Comp. (ABP)	70	98	168
Out-of-Comp. (Urine)	361	268	629
Out-of-Comp (Blood)	660	542	1,202
Out-of-Comp. (ABP)	367	294	661
Total	2,684	2,215	4,899

Table 2.2. Drug tests conducted by the ITF in 2016 under the Tennis Anti-Doping Programme.

Further enhancements to the programme have been announced which in practical terms means 60% more In-Competition testing, double the current number of tournaments covered, 30% more players covered by out-of-competition testing, 60% more out-of-competition testing and more samples from more players being stored for potential future re-analysis if necessary.

Now that tennis has the one unified Tennis Anti-Doping Programme, the appropriate reference is www.itftennis.com/antidoping which includes the current programme, the Prohibited List, quarterly and annual reports, press releases including announcements of violations and penalties and even tests conducted for each player.

2.10. Efficient Tournament Management.

Most components of the work required for the running of a successful tournament have been gradually converted from long hours and substantial paperwork to computer assisted administrative systems such as CAT (Computer Assisted Tournaments), TMS (Tournament Management Systems) and TP (Tournament Planner). As explained by Della Croce et al (1999), the goal is to maximize the number of feasible matches given constraints in terms of court and player availability.

The world's first tournament to be managed with the aid of CAT was in Hay, Australia in October 1980. This first version of CAT was designed to produce the daily schedule for each player competing in up to five or six events in a three day country tournament. The current version handles virtually all aspects of tournament management.

In the 1990's the Americans introduced a similar product called TMS for which the US Tennis Association purchased a nationwide license. Many other countries followed the United States making TMS the most popular program for tournament management for some time.

The most recent tournament management software, TP, was introduced by the Dutch and adopted in 2007 by the ITF for management of all its tournaments worldwide. Its wide range of features and flexibility means that it has been adopted for small tournaments through to Grand Slams.

The ITF has recently introduced a compulsory International Player Identification Number (IPIN) and an associated online player entry system that in 2009 covers tens of thousands of players in well over one thousand tournaments on the ITF Junior Circuit, ITF Pro Circuit and ITF Seniors Circuit. This allows the online scheduling of a world-wide calendar of tournaments, permitting players to easily select their most appropriate tournament at any time according to their ranking and the entry decisions of other players.

2.11. Strengthening the Mental Side of the Game.

Sports Psychology deals with the mental side of tennis and covers the study of human behavior within the context of the game of tennis. Many matches are won or lost because of the mental differences between the two players rather than any technical or physical differences.

The role of Psychology in tennis is summarized in the comprehensive book "Tennis Psychology" by Crespo, Reid and Quinn (2006) including over 200 references on playing with confidence, playing in "the zone", "choking", lack of concentration, burnout, motivation, fear of losing or winning, or simply how to play matches the same way as you play in practice. This book published by the ITF gives over 200 on and off court psychological training techniques, drills and activities to strengthen the psychological approach of players to match play. The classic explanation and exploitation of the mental side of tennis is given in Gilbert and Jamison (1993) in which former Top Ten player Brad Gilbert recounts his own experience of "mental warfare in tennis-lessons from a master." The first "classic guide to the mental side of peak performance" or "the inner game of tennis" was by W. Timothy Gallway (1974).

But as noted by Forzoni (2008) and Calstedt (2007), because it deals with peoples' minds, sport psychology does not have the same degree of objective measurement as the physical and technical side of the game. But both these authors also recognize that with technological advances, decreasing costs and decreasing size of equipment, along with the ease of use of bio-feedback and neuro-feedback devices, it is becoming easier to measure and monitor over time the effectiveness of specific mental skills strategies.

Emotions being experienced during a game of tennis have an effect on the human body and some aspects can be measured. Forzoni (2008) strongly recommends the Heart Math Freeze Framer which measures an athlete's heart coherence under various situations, so that with various relaxation techniques (controlled breathing, visualizing positive images) you can learn to control your heart coherence and consequently your emotions. Calstedt (2007) suggests that the most reliable indication of psychological states in athletes is heart rate variability, or heart rhythms, which can be measured by electrocardiograms or pulse wave recordings. He recommends that players should learn techniques to control their heart rhythms and consequently their emotions. He has developed the Calstedt Protocol as one possible approach for athlete psychological assessment, mental training and interaction efficiency.

2.12. Accurate rankings of Players.

In 1973 the leading male tennis players formed their own union, the Association of Tennis Professionals (ATP), and one of their first acts was to introduce a 12 month weighted moving average computing rankings system to determine fairly which players gained entry into tournaments worldwide and to determine which players were seeded. A separate doubles ranking was introduced in 1976. The Women's Tennis Association (WTA) was also founded in 1973 and introduced its computer rankings system in 1975.

Prior to this time, entry into tournaments and seeding was done by the Tournament Director or Tournament Committee. National rankings were compiled annually by national tennis associations. World rankings were generally announced by various journalists, so there were many unofficial world rankings and no official world ranking.

Under the original ranking systems tournament importance or "strength" was determined by prize money and player performance was measured by the round reached. A schedule of points was agreed based on the above and a player's ranking was calculated as the average points earned for tournaments played in the previous 12 months. Utilizing the capacity of a computer to create a more rigorous ranking Musante and Yellin (1979) were able to rank the strength of tournaments by the ranking of all players entered in that event and, also, to measure performance using the ranking of defeated opponents, not just the round reached. The concept of bonus points for defeating a higher ranked player was used by the Women's Tennis Association (WTA) for some years, but was subsequently discontinued.

Subsequently Blackman and Casey (1980) recognized that previous rankings were useful for determining tournament entry for players and for tournament seedings, but not for determining match result probabilities, betting odds and equitable handicapping methods. Using the actual scores in all matches between the players being ranked, they developed a rating in numerical units similar to a golf handicap. The difference in these rating units for any two players was shown to be a very good indication of match result probabilities and could also be used to determine what handicap should be given to the weaker player to make the match more even.

In 1989 the ATP players union combined with the tournaments (excluding ITF and Grand Slams) to form the ATP Tour. Likewise in 1995 the WTA Players Association combined with the Women's Tennis Council (including ITF and initially the Grand Slams as well, but these are now separate) to form the WTA Tour. Since then both ATP and WTA computer rankings have rewarded quantity as well as quality by selecting a player's best results from a minimum number of compulsory and optional tournaments. The ITF rankings for Junior and Senior players all use different formulae, but only the ITF uses one ranking for each player based on both singles (weighted 80%) and doubles (20%) matches.

Mathematicians could clearly develop a ranking or rating system superior to any of those currently used by tennis governing bodies, but the players and tournaments prefer simplicity and are reluctant to make changes to their current systems.

In a Grand Slam tournament the singles draw consists of 128 players of which 104 players are accepted directly from their computer ranking, 16 earn their place through a qualifying competition for the next 128 ranked players and 8 players are accepted as "Wild Cards" at the sole discretion of the Tournament Committee. All tournaments except Wimbledon accept the players' request to use the computer rankings for seeding, which is seen as a reward for performance over the previous twelve months. Wimbledon argues that seeding should represent a spreading of the best players on grass spread evenly throughout the draw. As a compromise between Wimbledon and the ATP and WTA Tours, Wimbledon agreed to accept the top 32 players on the computer for a seeding, but to recalculate the rankings and hence the seeding for these players by using a different formula that gave greater weight to tournaments played on grass and less weight to tournaments played on clay.

2.13. Improving Officiating.

A game of singles in tennis involving two players can involve no umpire at all with the players calling the shots and keeping score, or one central chair umpire making all decisions and keeping score, or a full complement of up to 12 officials (centre chair plus up to eleven lines persons) making tennis one of the most highly officiated sports. The largest tournaments, the Grand Slams, require a team of 350 officials, gradually reducing in numbers as players (and therefore number of matches played simultaneously) are eliminated.

Clay courts have always had the advantage of the ball leaving a mark which can be examined if there is a dispute over a line call. Ball mark inspection procedures are included as an Appendix to the Rules of Tennis (2017).

The first technical development to assist line calling was the “Cyclops” machine which served tennis well for 20 years. It was only used on the service line and involved sending beams of light either side of the service line to detect whether the service was in or out of play.

The first attempt to cover the whole court was the TEL machine which involved an undetectable modification to the ball (iron particles in the cover) and placing wires in the court either side of the line to detect whether a ball that bounced close to the line was either in or out. It was successfully used for one year at the Hopman Cup with a centre chair and no linesmen, but the cost, the requirement for standby arrangements (still need to keep linemen on call in case of technical failure) and the need to dig up the court for installation, limited its use.

The Auto-Ref and Hawkeye optical tracking system take a different approach to line calling. These systems track the flight of a tennis ball with software being used to map the point of impact relative to the lines. As explained by Szimak et al (2003) the software uses pattern recognition and other algorithms to find the ball when it is hit over the net, to convert 2D to 3D images, estimating the ball trajectory and predicting where the ball will intersect with the court surface.

Originally introduced as an aid to television coverage, increasing the number of cameras and other technical improvements enabled the ITF Technical Department to approve Hawkeye's use for officiating in tournament play. Hawkeye is now utilized at all major tennis tournaments and events, at least on the show courts, and is well accepted by the players and has taken line calling arguments out of the game at the elite level. Electronic Review procedures are also included as an Appendix to the Rule of Tennis (2017).

Research continues into other technical systems to assist in line calling, including heat sensors (infra-red cameras recording the heat generated when a ball bounces).

Mathematical modeling and statistics have been used extensively in the development of the above and other line-calling systems. For example, Jonkhoff (2003) used experimental data to show that electronic line-calling using the TEL system is consistent and accurate, and also cheaper than line umpires. Marshall (2003) showed that the multi-beam LineHawk provided superior accuracy than a single master beam. However, Collins et al (2008) have expressed concerns at the way Hawkeye decisions are naively accepted although the technology is certainly not perfect. They suggest that measurement error needs to made salient and that confidence levels should be attached whenever ball impacts are reconstructed. The average accuracy is about 5mm but players accept the machine made decision over a human decision. Mather (2008) has used Monte Carlo simulation to show that, for the best fitting space constants, the condition for a challenge (player and umpire disagree) was met in 20.8% of all trials. In 39.6% of the simulated challenges, a line judge error was recorded, which is very close to the 39.3% of errors found in the actual challenge records.

At www.freepatentsonline.com/5908361.html details of a new line calling invention (Automated tennis line calling system: US Patent 5908361) were reported on 20/3/09. This invention uses mathematical models to merge information from loudspeakers, ball impact sensors close to boundary lines and pressure sensing devices, to make line calls and foot fault calls. Foot faults are detected by comparing (a) the time of occurrence obtained from signals induced by contact of the serving player's foot on a pressure sensor at the baseline with (b) the computed time of occurrence of racquet contact with the ball as derived from the racquet sounds received by three or more microphones. This system can also be used to monitor the progress of play and to keep score using the sequence of locations of ball bounces, net-cord hits, service foot faults and racquet hits. In particular player statistics such as ball speed on service, winners, errors and other measures of player accuracy and effectiveness can be compiled from this data. This is clearly an area of great potential for further research using management science and operations research methods along with technical development.

2.14. Integrity and Betting

The mathematical, statistical, probabilistic and strategic analysis of tennis assumes that all points in a game, set or match are independent and identically distributed. It assumes that all matches are contested by two players (or pairs in doubles) each playing to the best of their ability. Apart from the normal random fluctuations, variations can occur naturally through changes in playing conditions (court surface, weather conditions, ball wear, etc.) or player characteristics (personal mental circumstances and physical fitness changes during the match) but also through deliberate strategic tactical changes (lifting or relaxing at certain stages or scores).

Variation in player success during a match are an accepted part of the game (and indeed in all sports) and were not viewed with any suspicion until gambling in tennis became widespread with the introduction of virtually anonymous online gambling. Unlikely results (upsets), especially if accompanied by an unusually large amount of betting on the upset, threatened to challenge the integrity of the sport and thus the mathematical, statistical, probabilistic and strategic analysis of tennis.

Betfair, the world's largest online betting exchange, reports that tennis is the third most wagered upon sport, behind only racing and soccer. Tennis is an ideal sport for gamblers. It is played one-on-one. It is broken down into points, games, sets, matches and tournament importance. Tournaments are played everywhere, so that at any time of the day there is some match being played somewhere in the world that is either being televised live or live scoring is available on the internet. A wealth of statistics are available on each match and the odds are continually changing and bettors can make waging during play itself.

Initially the online betting agencies used the tennis data without permission of the various tournament organisers- Grand Slams, ITF, ATP and WTA. In 2003 the tournament organizers entered into an arrangement with Betfair and a number of other online gambling companies that allowed them to use the tennis tournament data in return for providing confidential account information whenever suspicious betting activity occurred. It is a major offence for players, their support personnel and indeed any person granted tournament accreditation and therefore possible access to players or restricted information (eg medical treatment being given to players), to bet on any tennis matches or to do anything that might affect the outcome of matches.

In August 2007 there was a match in Sopot, Poland between the Russian Nikolay Davydenko, ranked fourth in the world and Argentinian Martin Arguello, ranked 87th, in which the betting pool was ten times the normal size for such a match and most of the money was on Arguello to win, which he ultimately did. In an unprecedented move, Betfair voided all bets, and soon thereafter, the four tennis regulatory bodies-Grand Slam Committee (of which I was a member as President of Tennis Australia), ITF, ATP and WTA instigated an independent inquiry into the threat and the rules, regulations and procedures affecting gambling in tennis. The Report, by former Scotland Yard detectives Ben Gunn and Jeff Rees (2008), was entitled “Environmental Review of Integrity in Professional Tennis” and restricted in circulation, but can be found on the internet. It recommended, inter alia, the establishment of a Tennis Integrity Unit (September 2008) to enforce the sport’s unified Tennis Anti-Corruption Programme (adopted January 2009). Another good summary of the history of integrity, corruption and its impact on fair play in tennis and other sports is given by McLaren (2008), now an Independent Anti-Corruption Hearing Officer under the Tennis Anti-Corruption Programme. In September

2008 after an intensive investigation, the ATP cleared both Davydenko and Arguello of any involvement in match fixing.

The 2008 Environmental Review examined 73 matches from the previous five years that were identified as having suspect betting patterns. They examined more closely 45 of those matches where account holders had successfully backed a lower ranked player to defeat a higher ranked player. Twenty seven account holders in two countries had patterns of suspected betting activity that would warrant further review and suggest the persons involved had access to "inside information". But overall the independent reviewers were "confident of our principal assessment that professional tennis is not institutionally or systematically corrupt. There are strong intelligence indications, however, that some players are vulnerable to corrupt approaches and there are people outside tennis who seek to corrupt those within the sport".

The Review identifies the category of player who may be vulnerable to possible corrupt approaches and the list does not include top players who receive substantial prizemoney, endorsements and appearance fees and value the computer ranking points for official tournaments (not exhibitions). The Tennis Integrity Unit website www.tennisintegrityunit.com identifies 21 persons that are currently suspended including ten with life suspensions and a further 12 persons who have served their period of suspension. The list includes players, managers and officials. None of these involve top players. It follows that we can be confident of the integrity of the Grand Slam data for the Top Four players used in the statistical, probabilistic, mathematical and strategic analysis in this thesis.

The cost of maintaining the Tennis Integrity Unit (TIU) is shared between the four members who established the TIU, namely the ATP (25%), WTA (25%), ITF (10%) and the Grand Slam Board, previously known as the Grand Slam Committee, 10% for each Grand Slam. The TIU educates all players on the importance of integrity in the game and the dangers and penalties of betting on the sport as well as reviewing and conducting a detailed investigation of all matches that appear suspicious, primarily based on advice received from the betting firms recognized by the four members of the TIU. The TIU has formal relationships with 18 betting companies or regulatory authorities. Since its formation, the four TIU member organisations have battled with the fact that they meet the costs of ensuring that the integrity of the game is maintained, while the betting companies reap the substantial financial rewards of using reliable and safe point-by-point data which is available worldwide immediately each point is completed.

On any week of the year live scores are available for tournaments being conducted on every continent not just at ATP and WTA Tour level, but challenger level, futures level and ITF level as well. At any time of the day and any day of the year, tennis is being played professionally somewhere and access is available to live scoring and gamblers can bet on the outcome of the next point, game, set or match. Trying to make the betting companies meet some of the costs of providing this huge amount of reliable data that they use for running their gambling business is a work in progress for the TIU member organisations and no safe solution has been found. In countries where gambling is illegal there can be no legal involvement with illegal gambling companies. In countries where gambling is legal, it is possible for a tournament to enter into a sponsorship arrangement with a gambling company, but this could present the wrong image for a sport that prides itself on its integrity. In Australia for example, Tennis Australia entered into a sponsorship with one company that included on court signage during the 2016 Australian Open. This relationship was heavily criticized by tennis purists, and the second year of the sponsorship the on court signage was removed, but the sponsorship continued. Further a number of gambling companies advertise during the television broadcast including promoting in match odds. The Government is reviewing whether this sort of advertising should be allowed during live sport. Whatever, a long standing solution to the issue is yet to be found. Overall, the integrity of the sport at the elite level is unrivalled, but there have been cases at the lower prizemoney level tournaments where players and officials have been tempted.

Not all variations in probability of winning a point, game, set or match can be attributed to player actions. Some depend on the scoring system being used. As a simple example, if a player has probability $p = 0.6$ of winning a point on service, then the probability the player wins the game when the score is deuce is $p = 0.6$ if no ad scoring is being used (assuming p is constant and points are independent and identically distributed) whereas the probability of winning the game using advantage scoring is $p^2/(p^2 + q^2) = 0.6923$.

A more complicated example is that in the usual tennis example, where the probability of the server A holding service p > 0.5, then the probability of a particular first set score varies slightly depending on which player serves first. Simply expressed, to win 6-3, a player has to break twice if he serves second, but only once if he serves first.

Consider the case of a match between two players A and B, both of whom have a probability p of holding service. Then the probability A serves first and wins by various scores is given by

$P_1(6\text{-}0) = p^3q^3$

$P_1(6\text{-}1) = p(p^3 3pq^2 + 3p^2q q^3)$

$P_1(6\text{-}2) = q(p^4 3p^2q + 4p^3q 3pq^2 + 6p^2q^2q^3)$

$P_1(6\text{-}3) = p(p^4 4p^3q + 4p^3q 6p^2q^2 + 6p^2q^2 4pq^3 + 4pq^3q^4)$

$P_1(6\text{-}4) = q(p^5p^4 + 5p^4q 4p^3q + 10p^3q^2 6p^2q^2 + 10p^2q^3 4pq^3 + 5pq^4q^4)$

The probability the score reaches 5-5 is

$P_1(5\text{-}5) = p^5p^5 + (5p^4q)^2 + (10p^3q^2)^2 + (10p^2q^3)^2 + (5pq^4)^2 + q^5q^5 = P_2(5\text{-}5)$

which is the same value if A served second $P_2(5\text{-}5)$

Likewise the probability A receiving first and then wins or loses by each of these scores can written down and is given by

$P_2(6\text{-}0) = q^3p^3$

$P_2(6\text{-}1) = q(p^3 3pq^2 + 3p^2q q^3)$

$P_2(6\text{-}2) = p(p^3 6p^2q^2 + 3p^2q 4pq^3 + 3pq^2q^4)$

$P_3(6\text{-}3) = q(p^4 4p^3q + 4p^3q 6p^2q^2 + 6p^2q^2 4pq^3 + 4pq^3q^4)$

$P_2(6\text{-}4) = p(p^4 5p^4q + 4p^3q 6p^3q^2 + 6p^2q^2 10p^2q^3 + 4pq^3 5pq^4 + q^4p^4)$

It should be noted that the probability Player A serving first loses X-6 [i.e. $P_1(X\text{-}6)$] is the same as the probability player A serving second wins 6-X [i.e. $P_2(6\text{-}X)$]. For example

$P_1(2\text{-}6) = p(q^4 3pq^2 + 4pq^3 3p^2q + 6p^2q^2p^3) = p(p^3 6p^2q^2 + 3p^2q 4pq^3 + 3pq^2q^4) = P_2(6\text{-}2)$

The differences between the two corresponding probabilities whether serving first or second are

$$P_1(6\text{-}0) - P_2(6\text{-}0) = 0$$

$$P_1(6\text{-}1) - P_2(6\text{-}1) = 3p^2q^2(p-q)(p^2 + q^2) > 0$$

$$P_1(6\text{-}2) - P_2(6\text{-}2) = 3p^2q^2(q^4 - p^4) < 0$$

$$P_1(6\text{-}3) - P_2(6\text{-}3) = pq(4p^6 + 24p^4q^2 + 24p^2q^4 + 4q^4)(p-q) > 0$$

$$P_1(6\text{-}4) - P_2(6\text{-}4) = pq(4(q^8 - p^8)+ 20p^2q^2(q^4 - p^4)) < 0$$

The figures for a match between two equal players A and B who both have a probability $p = 0.6$ of holding service are shown in table 2.3.

Score A wins	P_1 A serves first	P_2 A serves second	Probability before service order known
6-0	0.013824	0.013824	0.013824
6-1	0.053914	0.035942	0.044928
6-2	0.071055	0.089026	0.080041
6-3	0.132744	0.088498	0.110621
6-4	0.102581	0.146829	0.124705
Subtotal	0.37412	0.37412	0.37412
5-5	0.12588	0.12588	0.12588

Table 2.3. The probability of various set scores for two equal players with probability of holding service p = 0.6 depending on which player serves first.

The probability of a score of 6-0, 0-6 and any score 5-5 and beyond is independent of which player serves first. However a score of 6-1 or 6-3 in favour of player A is greater if player A serves first than second. The probability of a score of 6-2 or 6-4 is greater if player A serves second than first. Before the toss for service is taken there is a probability of 0.155549 of a score of 6-1 or 6-3 and a probability of 0.204746 of a score of 6-2 or 6-4. If you know that player A is serving first the probability A wins 6-1 or 6-3increases to 0.186658 and if you know A is receiving in the first game the probability A wins 6-2 or 6-4 increases to 0.235855.

If you know that A is serving in the first game and the bookmaker does not, you can bet on A winning 6-1 or 6-3 plus losing 2-6 or 4-6 and increase the odds from 0.360295 to 0.422513. The time difference between knowing the outcome and it being recorded on the scoreboard represents an opportunity for the gambler to increase his odds of winning. The time interval between winning a point and entering it on the scoreboard has virtually been eliminated by requiring the umpire to enter the score immediately instead of waiting until after he has formally announced it. But there is always a brief interval between the toss and the umpire entering which player is serving first and this presents an opportunity for on-site gamblers to increase their odds. You only get one chance, before play starts, as you cannot accurately predict which player will serve first in the second and subsequent sets.

Finally, this example uses a very conservative value of 0.6 for the probability of winning service. In the men's game, the probability of winning a point on service is generally 0.6 or higher, and that produces a probability of holding service of 0.736 or higher. The same values for the probability of holding service of 0.8 produce the figures shown in table 2.4.

Score A wins	P_1 A serves first	P_2 A serves second	Probability before service order known
6-0	0.004096	0.004096	0.004096
6-1	0.041779	0.010445	0.026112
6-2	0.039567	0.070902	0.055235
6-3	0.187728	0.046932	0.117430
6-4	0.066954	0.207749	0.137351
Subtotal	0.340124	0.340124	0.340124
5-5	0.159876	0.159876	0.159876

Table 2.4. The probability of various set scores for two equal players with probability of holding service p = 0.8 depending on which player serves first

The differences in probability before and after the service order is known are now more extreme as is the likelihood of a 6-3 score if A serves first or a score of 6-4 if A serves second. The likelihood of a 7-5 set or a tiebreak is also higher. If you know that A is going to serve first before the odds are adjusted for service order, you can back A to win 6-3 or lose 6-4 where the probability is now 0.395477 instead of 0.254781, a healthy margin in any betting. However, you will find that bookmakers will stop you betting if you keep winning, or refer you or the players involved in the match to the Tennis Integrity Unit.

In 2016 the Tennis Integrity Unit began publishing the match alert data supplied under Memorandums of Understandings with 18 betting companies and government regulators. The results given in Table 2.5 show that 292 matches were queried, up from 246 in 2015, and the majority of alerts were clearly from the lower levels of professional tennis. As a result nine players and officials were convicted and sanctioned, with five receiving lifetime bans. The data show that only eight matches in total at the Grand Slams and ATP/WTA Tours were the subject of a betting alert and none were found to contain any corrupt activity. The learning gained from all the investigations is that unusual betting activity can be explained by many other factors such as incorrect odds-setting, player fitness, fatigue and form, playing conditions, personal circumstances, well-informed betting, etc. For the record, a total of 114,126 professional men's and women's matches were played in 2016.

	Grand Slam	ATP Tour	WTA Tour	ATP Challenger	ITF Men's Futures	ITF Women's	Davis & Hopman &Fed Cups	Total
Q1	1	0	1	12	24	10	0	48
Q2	0	2	0	14	33	24	0	73
Q3	2	0	0	31	54	9	0	96
Q4	0	0	2	23	41	9	0	75
Total	3	2	3	80	152	52	0	292

Table 2.5. Match alert data received by the Tennis Integrity Unit for each quarter of 2016 from 114,126 matches from which nine persons were convicted and sanctioned.

In January 2016 following media allegations of match fixing in tennis, the four governing bodies of international tennis (ATP, WTA, ITF, Grand Slams) jointly announced that an Independent Review Panel would be appointed to undertake a review of the integrity in the sport. The report is due in 2017 and will be made public. The four governing bodies have strenuously refuted the media allegations, and continue to do so, but have also committed to accept, implement and fund all of the recommendations from the report. For more information see the website of the Panel, www.tennisirp.com.

2.15. Summary and Implications

Some aspects of tennis have essentially never changed over the past century or even longer. The court and net are still the same size. The rackets are still a frame with a handle and strings. The ball is still a pressurized rubber ball covered in cloth. The scoring is still a three tiered system consisting of points, games and sets. But behind this appearance of consistency, there has been change. Massive change. Especially over the past fifty years since the amateur era ended in 1968 and the professional ("open tennis") era began. In some ways the changes are accelerating and a challenge for administrators to maintain the fairness and integrity in the game in a time of continued technical and medical advances in sport.

This chapter looks at the history, the current status and the potential future for all the scientific, technical, statistical and management aspects of the game.

Section 2 looks at the search for a better, or even an optimal, scoring system to the current system which has the beauty of a three tiered scoring system, but the ugliness of an uncertain and highly variable time taken.

Section 3 looks at how players can optimize their chance of winning within the scoring systems currently being used.

Section 4 looks at the way synthetic surfaces have increasingly replaced the natural surfaces of grass and clay, but leave organisers with the ability to vary the speed of the court which now needs to be controlled to maintain integrity.

Section 5 looks at ball characteristics that also need to be controlled to maintain integrity, even though both players use the same ball.

Section 6 looks at the huge changes in racket specification, particularly size and rackets stringing, both of which needed rule changes to maintain integrity and the way the game is played.

Section 7 looks at the increasing role being played by coaches in professional tennis, their professional development and future.

Section 8 looks at optimizing player technique and the challenges of trying to copy the best serve, the best forehand, the best backhand, etc.

Section 9 looks at minimizing player risk both through utilizing all the advances in avoiding injury and treating injury, age specific training programmes, development of a heat policy, etc., but also the never ending challenge of a Tennis Anti-Doping Programme for player safety and the integrity of the game.

Section 10 looks at the efficiency of the most commonly used knock-out and round-robin tournament systems as well as suggesting various hybrid tournament systems.

Section 11 looks at the importance of the mental side of the game of tennis and how at the elite level the difference between players is often more mental than physical.

Section 12 looks at the official rankings of players that are widely accepted and used to determine player entry into tournaments and for seedings, but are far from perfect as they combine the objectives of players and tournaments as they involve a combination of quality and quantity data.

Section 13 looks at the way on court officiating has changed from well-meaning umpires and linesmen in the amateur game to full and part time officials assisted by line calling machines.

Section 14 looks at the relatively new issue of betting in tennis and what is being done to maintain the integrity of the game. It shows how the Match Alert data received from betting companies can be due to a lot of other factors and not necessarily corrupt activity.

Technical development has already made and will continue to make a significant contribution to the development of the game and the way it is played, managed and viewed. The challenge is to maintain the integrity of the sport, whilst keeping it relevant in the 21st Century. In this introductory chapter it has been shown that in this context there is plenty of scope for Operations Research and Management Science methodologies because the technical developments are allowing more scientific measurement of the important decision-making variables and constraints.

For a more extensive list of scientific tennis articles than included here the ITF web site www.itftennis.com/coaching/publications contains hundreds of articles in categories including Sports Science, Biomechanics, Medicine and Conditioning, Psychology, and Technique and Methodology. This thesis concentrates on the mathematical, statistical, probabilistic and strategic issues affecting tennis and relevant articles are referenced in each chapter.

CHAPTER 3.

FIRST AND SECOND SERVICE

One special feature of tennis (possibly unique) is that it allows a player, without any penalty, to have a second service if the first service is a fault. This adds an additional dimension to the already complicated three tiered scoring system used in a match of tennis which consists of points, games and sets. The simplest of these is the point, but even the point needs to be split into two types of point depending on which player is serving, each having his or her own probability of winning a point on service. But for each server it can also be split into whether it is a first or second service, each of which have quite different characteristics.

Traditionally a player will serve faster and aim closer to the lines on a first service, hoping to achieve an ace or an unreturnable service, safe in the knowledge that if the service is a fault, the server has another chance to put the ball into play. The second service is likely to be slower, not so close to the lines and with more spin to ensure the ball goes into court and is not a double-fault. In mathematical terms the first service is less likely to go in to play, but the server is more likely to win the point if it does. The reverse applies to the second serve, which is more likely to go into play, but the server is less likely to win the point than on the first service.

Data on percentage of first and second serves successfully put into play and percentage of points won on first and second serve if the serve goes into play are the most readily collected statistics.

Of all the coaching advice I ever received, two pieces of advice stick permanently in my mind. The first is the often heard statement, usually applied to male players, that "a player is only as good as his second serve." The second came from Australian and International Tennis Hall of Fame inductee, Mervyn Rose, who told me that "you might as well serve a double fault as a second serve like that". No-one likes serving a double fault, but the message clearly was to be more aggressive on your second service so that you win a lot more points on your second service, despite the odd double fault that results from this aggression. Thus the objective was a combination of serving harder, closer to the line and with more spin or slice rather than continuing with the safe service and trying to ensure no double fault was served. In other words, the objective, often overlooked, is to win the point, not to avoid serving a double fault. This chapter looks at the probability and statistics behind first and second service strategy.

Data on the percentage of first and second serves successfully put into play and the percentage of points won on first and second serves if the serve goes into play are the most readily collected statistics on a tennis match (apart from the actual score). Unlike data on forced and unforced errors, and even backhand and forehand winners and errors, all of which require observation and interpretation by the person recording the data, data on the percentage of first and second serves into play and the percentage of points won on first and second service are readily available and accurate. As a result there is considerable literature on the subject of first and second serves, mostly assuming a player has two distinct serves, the first serve described as strong or aggressive or fast or risky and rapid and the second serve described as a conservative or weak or slow or safe. The literature is discussed in Section 3.2. Players are unlikely to have just two service types, (say strong and weak) but a range of serves with in between strengths and in between probabilities of winning the point if the service goes into play. Assuming the data for first and second serve are the two known data points, it can be assumed that a straight line between them approximately represents the relationship between service type and probability of winning the point over that range. Using maximum likelihood theory the best first and second serve for any player can be determined and this is discussed in Section 3.3.

Players who take more or less risk than the optimal risk on first and/or second service pay a penalty in the decrease in the probability of winning a point on service and this is discussed in Section 3.4.

While there is some merit in considering a straight line relationship between risk taken on service and the probability of winning the point if the service goes into play, it is clear that a curve is more likely to reflect the relationship. A quadratic relationship is discussed in Section 3.5.

There are four factors affecting the probability that a player wins a point on service, namely the percentage of first and second serves that go into play and the probabilities of the server winning the point if the serve goes into play. The relative importance of each and hence the relative reward for improving each is discussed in Section 3.6, followed by a general tennis discussion in Section 3.7.

3.1. The basic equation for serving

The probability of winning a point on service (P) is a combination of

p_1 = the probability that the first service goes in (i.e. is not a fault),

y_1 = the probability of winning the point given the first service went in,

p_2= the probability that the second service goes in (i.e. is not a double fault), and

y_2= the probability of winning the point given the second service went in.

These five probabilities are connected by the equation

$$P = p_1y_1 + (1 - p_1)p_2y_2 \, . \tag{3.1}$$

This can also be written

$$P(p_1,p_2) = w(p_1) + (1 - p_1)w(p_2) \tag{3.2}$$

where $w(p_1) = p_1y_1$ and $w(p_2) = p_2y_2$, which converts the conditional probabilities y_1 and y_2 of winning the point given that the service is good (in play) into the unconditional probabilities $w(p_1)$ and $w(p_2)$ of winning a point on that particular first or second service respectively. Equations (3.1) and (3.2), which are really the same relationship, are the basic equation for winning a point on service.

The social player may only have two types of service, most likely a strong first service and a weak second service. He or she is also less likely to be interested in optimum serving strategy and receives satisfaction from serving the odd ace or not serving double faults. On the other hand top players are interested in the optimum serving strategy to maximise the chance of winning their service game. Rather than having just two types of service, top players have a range of services (speed, spin, slice and closeness to the lines) with a high value of y generally associated with a low value of p and a low value of y associated with a high value of p. A player's selection of which service to use and when depends on various factors such as the score in that game, result of the previous point, opponent's returning ability, etc. Overall they will have an average which represents the probability of winning a point on service.

A player who wins (say) 70% of points on first service if the first service goes in with 60% success rate, but only wins 40% of points on second service when the second service goes in with 90% success rate, wins (0.60)(0.70) = 42% of points on his first service and (0.90)(0.40) = 36% of points on his second service and overall wins (0.60)(0.70) + (1 – 0.60)(0.90)(0.40) = 0.42 + 0.40(0.36) or 56.4% of points played on his service. However if the player with these probabilities were to serve two first serves instead of a first and second service, and assuming he could endure the indignity of 40% of his second serves being double faults, the probability this player wins the point is 0.42 + (0.40)(0.42) = 58.8% which is slightly higher than when serving a first and second serve. Of course, this difference between first and second service is quite large, so if his second serve was a little more aggressive, (say) the probability of the serve going in was 0.85 and the probability of him then winning the point was 0.5, then it would be slightly better to serve a second serve than another first service, namely 0.42 + (0.40)(0.85)(0.50) = 59%.

Assuming the player does not have just two types of serve, but a range of serves in between with intermediate risk and reward probabilities we can search for the optimum serving strategy. This approach to risk and reward with first and second serves is the core of this chapter.

3.2 Literature on first and second serves

The literature on first and second service statistics, probability and strategy begins in the early 1970s. Gale (1971) utilized equations (3.1) and (3.2) to recognise an optimal strategy for serving in tennis. Clearly for the second serve you need to maximize $w(p_2)$ =

p_2y_2. If this maximum is $w(p_2^*)$, then for the first serve you need to maximise $P = P(p_1,p_2^*) = w(p_1) + (1 - p_1)w(p_2^*)$ or since $w(p_2^*)$ is maximised, just maximise $w(p_1) - p_1w(p_2^*)$. In the usual case where $p_1<p_2$, then the service strategy (p_1,p_2) was shown by Gale to be an optimal strategy if and only if $w(p_2) \geq w(p_1) \geq w(p_2)(1 - (p_2-p_1))$. No data was utilised in this first analysis. Using a very limited data set, Redington (1972) concluded that in the 1971 Wimbledon final, John Newcombe would have done slightly better if he had served two first serves instead of using a softer second serve, and that his optimal second serve was somewhere between the two serves he used.

George (1973) considered a typical player with a "strong" and a "weak" serve, and, under very general assumptions (the probability that a weak service goes into play is greater than the probability that a strong service goes into play and the probability of winning the point if the service is good is greater for a strong service than a weak service) he identified the circumstances under which a "strong-strong" service is actually better than the usual "strong-weak." He also showed that, given his assumptions, the "weak-strong" option is always sub-optimal. His limited data set of just two men's best of three sets singles matches supported the common practice that the "strong-weak" option is typically the best.

Hannan (1976) continued the strong serve or weak serve analysis by representing the serving in tennis as a Markov Chain with probabilities attached to each possible event and outcome. From this, the server can determine the optimal mix of serves to maximise his probability of winning the point and ultimately the match. The original probabilities can be revised and updated during the match using Bayesian methods, presumably by a friend or coach on the sidelines rather than the player. An extension to a game theory approach taking into account the opponent's expectations is suggested and then dismissed as "possibly computationally infeasible." No actual data were collected and analysed.

King and Baker (1979) carried out the same analysis as George for a large set of women's singles data. For one player in their data set the "weak-weak" strategy was optimal, whilst for another the "strong-strong" strategy was best, but for the remaining ten players in the analysis it appeared that the "strong-weak" strategy was best. Gillman

(1985) extended the approach of Gale (1971) and George (1973) using the terminology r for “risky and rapid” in place of “strong” serve and s for “slow and safe” in place of “weak” serve. He showed that serving strategy sr (i.e. ‘slow and safe’ for first serve followed by ‘risky and rapid’ for second serve) is never the best strategy and determined the conditions under which serving strategies rs, rr and ss are each the best. Looking at the four finals between Borg and McEnroe at the 1980 and 1981 Wimbledon and US Championships he showed that both players used the conventional serving strategy rs, but Borg would have performed better if he had served ss in the 1980 Wimbledon and rr in the 1981 Wimbledon Finals.

More recently, McMahon and de Mestre (2002) addressed the “two service” problem by analysing 414 women’s and 444 men’s grand slam singles matches in the year 2000. They concluded that in a surprising number of matches, one or both players would have benefited from a departure from the usual “strong-weak” strategy. The most common beneficial change for women was to a “slow-slow” strategy, while for men it was to a “fast-fast” strategy (in spite of the additional double faults).

Norman (1985) used dynamic programming to determine whether a player should use a fast or slow service on first and second service.

Pollard and Pollard (2007) determined a formula for finding the optimal risks that should be taken by a player on his/her first and second serve, assuming a continuous linear and a quadratic relationship between the type of serve used and the probability of winning the point. The price paid by the server for not serving optimally was also quantified. Pollard (2008) extended this analysis and concluded that in order to improve the probability of winning a point on service a player should focus on improving his first serve outcome rather than second serve outcome and concentrate on winning the point after the serve has gone in rather than increasing the probability that the first serve goes in. These two papers form the basis of this chapter.

Barnett, Meyer and Pollard (2008) noted that a player’s service statistics also depend on the receiving qualities of his/her opponent and the court surface on which a match is played. Looking specifically at Andy Roddick serving to Rafael Nadal, they found that on grass Roddick might do better serving two first serves rather than the conventional first and second service, at least some of the time, but that this was not a good strategy on clay. Players could use this methodology in determining their serving strategy against specific players.

Magnus and Klassen (1999, 2008) used data from the men's singles at Wimbledon 1992-1995 to test the hypothesis that "a player is as good as his or her second serve" and concluded that this is not true. They added that it would be more realistic to say that "a player is as good as his or her first service". However, in a subsequent analysis of the same data, Magnus and Klassen (2014) concluded that "there is no compelling evidence against the hypothesis" and "there is some evidence in favour of the hypothesis" and so concluded that "a player is as good as his or her second service". Given that better players have better second serves, the suggested change to a one serve rule to reduce the domination of the service probably would result in most players predominantly using their second service and probably an increase in the difference between the better and weaker player.

For professional players the prime objective is to win the match and so players will be searching for the optimal strategy. Amateur players may have other objectives such as having fun, serving aces, hitting incredible shots (for them), not serving double faults, etc. and optimization is irrelevant. Magnus and Klassen (2014) developed a model to determine how difficult a player should make his serve to return in order to maximize the probability of winning a point on service. By comparing actual probability to optimal probability the "efficiency" of a player can be determined. Efficiency varies from player to player, but the average efficiency was 98.9% for men and 98% for women. This is pretty efficient, and higher ranked players were noted to be more efficient than lower ranked players.

3.3. Optimal risk taking on first and second serve.

Assume that the probability of a service being good (i.e. going into the service court), p, decreases as the server takes greater risk (speed and direction) with his/her service. The risk can be denoted by x on a scale from 0 (virtually no risk) to 10 (maximum risk) and the probability the serve goes into play p(for risk x) can be assumed to equal $p(x) = 1-0.1x$. Thus, x=1 represents the situation in which the player puts 90% of serves into play, x=2 represents the situation in which the server has probability 0.8 of putting the serve into play, and so on. When the server takes maximum risk (x=10) he or she typically serves a very fast serve aimed at a corner of the service court, and it has virtually no chance of going into the service court. When the server takes minimal risk (x=0), he/she serves a very "safe" and relatively slow serve often with spin and not directed near the lines, and it has a very high (assume 1) probability of going into the service court. The probability the server wins the point given that the service goes into the service court P(x) increases as the level of risk (x) increases. Initially we assume that this relationship is continuous and linear, and it may be linear over a short range, but we will subsequently assume it is curved as this would appear to be more likely. Hence in the linear case we assume $P(x) = a + bx$ where $a + bx$ is assumed to take values between 0 and 1 for the relevant values of x, a>0, b>0.

We begin by analysing just the second serve. The probability that the server wins the point given a second serve with risk x_2 is used, is given by $P_2(x_2)$ where

$$P_2(x_2) = p(x_2).P(x_2) = (1-0.1x_2)(a+bx_2) = a+bx_2-0.1ax_2-0.1bx_2^2 \qquad (3.3)$$

Taking the differential of $P_2(x_2)$ with respect to x_2, the maximum value of $P_2(x_2)$, which can be written $P_2^*(x_2^*)$, occurs when the differential of $P_2(x_2)$ in equation (3) with respect to x_2 is equal to zero, namely $b - 0.1a - 0.2bx_2 = 0$ or x_2^* is given by $x_2^* = 5 - 0.5a/b$, which can be written

$$x_2^* = 5 - 0.5r \quad \text{if } r\leq 10 \quad \text{or must be} \quad x_2 = 0 \quad \text{if } r\geq 10 \qquad (3.4)$$

where r = a/b and r>0

Hence $P_2^*(x_2^*) = (1 - 0.1x_2^*)(a + bx_2^*)$ is given by

$$P_2^*(x_2^*) = 0.025(10b+a)^2/b \quad \text{if } r\leq 10 \quad \text{or} \quad P_2^* = a \quad \text{if } r\geq 10 \qquad (3.5)$$

Next, we analyse the first serve, recognising that the server has a second serve if required. The probability that the server wins the point given a first service with risk x_1 is used and a second service with risk x_2 is used (if required) is given by

$$P_1(x_1,x_2) = (1-0.1x_1)(a+bx_1) + (0.1x_1)(1-0.1x_2)(a+bx_2). \quad (3.6)$$

The probability that the server wins the point given a first service with risk x_1 is used and a second service with optimal risk x_2^* is used (if a second service is required), $P_1(x_1, x_2^*)$ is given by

$$P_1(x_1,x_2^*) = (1-0.1x_1)(a+bx_1) + 0.1x_1P_2^*(x_2^*). \quad (3.7)$$

By taking the differential of $P_1(x_1,x_2^*)$ with respect to x_1 and equating to zero, the maximum value, $P_1^*(x_1^*,x_2^*)$ of $P_1(x_1,x_2^*)$ occurs when $b - 0.1a - 0.2bx_1^* + 0.1P_2^*(x_2^*) = 0$ or

$$x_1^* = 6.25-0.25r+0.0125r^2 \quad \text{if } r\leq10 \qquad \text{or } x_1^*= 5 \quad \text{if } r\geq10, \quad (3.8)$$

and the maximum value, $P_1^*(x_1^*, x_2^*)$, of $P_1(x_1, x_2^*)$ is given by

$$P_1^*(x_1^*,x_2^*) = a + 0.1b(6.25-0.25r+0.0125r^2)^2 \quad \text{if } r\leq10, \quad (3.9)$$

or $\quad a + 2.5b \quad \text{if } r\geq10$

Alternatively, the above expression (3.6) for $P_1(x_1,x_2)$ could be differentiated partially with respect to x_1 and partially with respect to x_2 to produce the same optimal values x_1^* and x_2^*. The partial differential of equation (3.6) with respect to x_2 is $0.1x_1(b – 0.1a – 0.2bx_2) = 0$ as before (leading to equation (3.4) for x_2^*) and the partial differential of equation (3.6) with respect to x_1 is $b – 0.1a - 0.2bx_1 + 0.1(1 – 0.1x_2)(a + bx_2) = 0$ leading to equation (3.8) for x_1^* as before.

Each player has his own values of a and b. These values are not constant but vary from match to match or even set to set depending on the current form of the player, the strength of the opponent, the court surface and other factors. In Grand Slam tournaments the average probability of winning a point on service for men is about 0.625 (Australian Open 0.622 in 2002-2005 and US Open 0.630 in 2002-2005) and slightly higher on the grass at Wimbledon being 0.652 in 2004 and slightly lower on the clay at Roland Garros being 0.592 in 2004. The average probability of winning a point on service has essentially not changed since these figures were recorded in 2007.

A player serving a very safe second serve (x_2=0) may still win 50% of points after the second serve goes in to play. Assume he wins about 80% of points when he serves a stronger first serve (x_1=6) and the first serve goes into play. With these two values and a linear relationship between P(x) and x, then a=0.5 and b=0.05 (and r = 10). A player who uses these two particular serves has an overall probability of 0.62 of winning the point, but the optimal probability, obtained by substituting in equation (9), occurs when x_1=5 and x_2=0 and is 0.625.

Tables 3.1 and 3.2 give some relevant values for a and b when the optimal probability is 0.625 or very close to it. Every player with an overall optimal probability of winning a point on service of $P_1^*(x_1^*, x_2^*)$ equal to 0.625 can be represented by one of the pairs of a and b values in Table 3.1 (where a/b ≥ 10) and Table 3. 2 (where a/b ≤ 10). The case with a = 0.5 and b = 0.05 represents a 'middle-of-the-road' situation, whereas the case with a =0.4 and b = 0.08 represents a situation in which the server gets a very large return from taking increased risks. It would appear that a majority of men professional players (and many other men as well) might be represented in most of their matches by the cases where a = 0.48, 0.5, 0.52 or 0.54.

a	0.50	0.52	0.54	0.56	0.58	0.60
b	0.050	0.042	0.034	0.026	0.018	0.010
x_2^*	0	0	0	0	0	0
$P_2^*(x_2^*)$	0.50	0.52	0.54	0.56	0.58	0.60
x_1^*	5	5	5	5	5	5
$P_1^*(x_1^*,x_2^*)$	0.625	0.625	0.625	0.625	0.625	0.625

Table 3.1. Various characteristics for a point when P(x) = a + bx and a/b ≥ 10

It can be seen from Table 3.1 and formulae (3.4) and (3.8) that when a/b is greater than 10, the optimal value of x_2^* is 0 and the optimal value of x_1^* is 5 and the maximum value of $P_1^*(x_1^*, x_2^*)$ is constant at a + 2.5b = 0.625.

In other words when the relationship between the risk taken on service and the probability of winning the point if the serve goes into play is linear and a/b is greater than 10, then the maximum chance of winning the point on service is achieved if a first serve going in with probability 50% is used and a second serve going in with probability 100% is used.

1	a	0.50	0.48	0.46	0.44	0.42	0.40
2	b	0.05	0.057	0.064	0.069	0.075	0.080
3	x_2^*	0	0.79	1.41	1.81	2.20	2.50
4	$P_2(x_2^*)$	0.5	0.484	0.473	0.463	0.456	0.450
5	x_1^*	5	5.03	5.10	5.16	5.24	5.31
6	$P_1^*(x_1^*,x_2^*)$	0.625	0.624	0.626	0.624	0.626	0.626

Table 3.2. Various characteristics for a point when P(x) = a + bx and a/b ≤ 10

However from Table 3.2 and formulae (4) and (8) where a/b is less than 10, the optimal value of x_2^* is greater than zero and the optimal value of x_1^* is slightly greater than 5 and the maximum value of $P_1^*(x_1^*, x_2^*)$ is approximately 0.625.

In other words when the above relationship is linear and a/b ≤ 10, the optimal strategy is to serve slightly less than 50% of first serves into play and slightly less than 100% of second serves into play. Surprisingly the optimal value of x_1^*in both tables is 5 or very slightly higher, so players who are serving a high proportion of first serves into play (say 70%) are not serving optimally and should be advised to take greater risks on their first service. Obviously those serving a low proportion of first serves into play (say 30%) are also not serving optimally and should be advised to take less risks on their first service.

For the second service the optimum is generally to aim for 100% success in putting the service into play, but where a/b is less than 10, the value of b, the slope of the linear relationship between risk and reward is high, the player should be advised to take greater risk on second service, even if it results in a number of double faults. The optimal second service strategy could be 90% or even a lower percentage of second services going into play.

3.4. Penalty for non-optimal serving

Players who take more risk or less risk than the optimal risk on first and/or second serve pay a penalty in the decrease in the probability of winning a point on service. For a player that takes a higher or lower risk x_2 instead of the optimum risk x_2^*, the difference between $P_2(x_2)$ and $P_2^*(x_2^*)$ can be shown to be

$$P_2^*(x_2^*) - P_2(x_2) = 0.1b(x_2-x_2^*)^2 \quad \text{if } r\leq 10 \tag{3.10}$$

or $$= b(0.1r-1)x_2 + 0.1bx_2^2 \quad \text{if } r\geq 10$$

Tables 3.3 and 3.4 give the penalty (decrease in probability) for non-optimal serving for some relevant values of a and b when the optimal probability is 0.625 or very close to it.

1	a	0.50	0.52	0.54	0.56	0.58	0.60
2	b	0.050	0.042	0.034	0.026	0.018	0.010
3	P_2^* - P_2	0.020	0.037	0.054	0.070	0.087	0.104
4	P_1^*- P_1	0.020	0.017	0.014	0.010	0.007	0.004
5	P_1^*- P_1	0.034	0.043	0.051	0.060	0.068	0.077
6	P_1^*- P_1	0.026	0.028	0.030	0.031	0.033	0.035

Table 3.3 Reduction in probability for non-optimal serving when P(x) = a + bx and r≥10

1	a	0.50	0.48	046	0.44	0.42	0.40
2	b	0.050	0.057	0.064	0.069	0.075	0.080
3	P_2^*- P_2	0.020	0.023	0.026	0.028	0.030	0.032
4	P_1^*- P_1	0.020	0.023	0.026	0.028	0.030	0.032
5	P_1^*- P_1	0.034	0.039	0.044	0.047	0.052	0.055
6	P_1^* - P_1	0.026	0.030	0.034	0.036	0.040	0.043

Table 3.4. Reduction in probability for non-optimal serving when P(x) = a + bx and r≤10

In Tables 3.3 and 3.4 above,

Row 3 gives $P_2^*(x_2^*) – P_2(x_2)$ where $x_2 = x_2^* + 2$

Row 4 gives $P_1^*(x_1^*, x_2^*) – P_1(x_1, x_2^*)$ when $x_1= x_1^*+ 2$ (or -2) and $x_2 = x_2^*$

Row 5 gives $P_1^*(x_1^*, x_2^*) – P_1(x_1, x_2)$ when $x_1= x_1^*+ 2$ and $x_2 = x_2^*+ 2$

Row 6 gives $P_1^*(x_1^*, x_2^*) – P_1(x_1, x_2)$ when $x_1= x_1^*- 2$ and $x_2 = x_2^* + 2$

Row 3 of Tables 3.3 and 3.4 considers the difference in probability of winning a point on second serve if the serve is not optimal. Whilst a deviation in x_2 of 2 from optimality for the second serve may be fairly high, it can be seen from row 3 in Table 3.3 that the difference in probability of winning the point can be quite large. Thus if b is small (r > 10), a player suffers a considerable decrease in probability in not serving optimally and would be advised to not take additional risks on the second serve. On the other hand if b is large (r < 10) it can be seen from row 3 of Table 3.4 that the deviation in probability from serving optimally is smaller and more consistent as b changes. It can also be seen from equation (10) that the difference in probability is symmetrical about x_2^* in the range 0 to $2x_2^*$ and has a minimum value when r=10 (i.e. b = 0.1a)

Row 4 of Tables 3.3 and 3.4 considers the difference in probability of winning the point if the second service is always optimal, but the first service is not optimal. It can be shown that

$$P_1(x_1 , x_2^*) – P_1^*(x_1^*, x_2^*) = 0.1b(x_1 - x_1^*)^2 \qquad \text{for all r} \qquad (3.11)$$

This difference is symmetrical about x_1^* and increases as b increases.

Rows 5 and 6 of Tables 3.3 and 3.4 consider the difference in probability if both the first and second serves are not optimal. The difference in probability is given by

$$P_1^*(x_1^*, x_2^*) – P_1(x_1, x_2) = 0.1b(x_1 – x_1^*)^2 - 0.1x_1(-0.1b(x_2 – x_2^*)^2) \qquad \text{if } r \le 10,$$

or

$$=0.1b(x_1 – x_1^*)^2 - 0.1x_1(-b(0.1r – 1)x_2 – 0.1bx_2^2) \qquad \text{if } r \ge 10 \qquad (3.12)$$

Row 5 of Tables 3.3 and 3.4 gives values of this difference in probability when $x_1 = x_1^*+ 2$ and $x_2 = x_2^* + 2$

Row 6 of Tables 3.3 and 3.4 gives values of this difference in probability when $x_1 = x_1^* - 2$ and $x_2 = x_2^* + 2$

It can be seen that this difference in probability can be relatively large. It can also be seen that the values in row 5 are always larger than the values in row 6. This is because the x_1 value in row 5 is larger than the x_1 value in row 6 and therefore the second serve is required more often. This leads to a greater contribution from the $P_2^*(x_2^*) – P_2(x_2)$ component shown in row 3. This indicates that if the second serve is not optimal, it is better if the server takes a smaller risk than optimal on the first service, $x_1^* - d$ than it is to take the corresponding greater risk on the first service $x_1^* + d$.

It can also be seen that the difference in probability in both rows 5 and 6 take the largest values when b is small or large and take the smallest values when b = 0.1a.

3.5. Extension to a quadratic model and other curves

While there is some merit in considering a straight line relationship (between risk taken on service and the probability that the server wins the point given that the service went into play) over some limited range, it is preferable and more flexible to consider some curvature such as a quadratic relationship. The linear relationship requires just two properties and we readily have these from the statistics for first and second serve. The quadratic relationship requires three parameters, and three suggestions to overcome this challenge are made here.

Pollard and Pollard (2007) suggested that a relevant quadratic relationship could be obtained by taking the straight line relationship determined by the two known properties and decreasing the value of the probability at either end of the line and increasing the probability at the middle. Pollard (2008) utilized this methodology to determine a realistic quadratic equation for the relationship between risk taken on service and the probability of the server winning the point over a typical range of risk values for elite players.

Klaassen and Magnus (2014) overcame the lack of a third parameter to obtain a quadratic (or cubic) equation by assuming the probability curve is of the form $y(x) = a - cx^n$ where $y(x)$ is the probability the server wins the point when using a service which has probability x of going in and n is assumed to be 2 or even 3, so only two parameters, a and c, need to be estimated.

A third and new approach in this chapter is to assume that elite players are very close to optimal servers and consequently it is possible to use this information as a third equation to determine the values of the constants a, b and c in the quadratic relationship between the probability of winning a point on service and the risk taken with the service.

3.5.1 The quadratic model

We assume that

$$P(x) = a + bx + cx^2 \tag{3.13}$$

Where, as in the linear case, $P(x)$ is the probability that the server wins the point given that the service with risk x goes into play. ($0 \le x \le 10$)

Consequently,

$$P_2(x_2) = p(x_2).P(x_2) = (1 - 0.1x_2)(a + bx_2 + cx_2^2)$$

$$P_1(x_1,x_2) = (1 - 0.1x_1)(a + bx_1 + cx_1^2) + (0.1x_1)(1 - 0.1x_2)(a + bx_2 + cx_2^2)$$

In the linear case it was seen from equation 8 that the optimal risk on the first service is 5 when $r = a/b$ is greater than or equal to 10 and is slightly greater than 5 when r is less than or equal to 10. Thus the optimal risk on first service can never be less than 5 when the relationship is linear. However the optimal risk on first service can be less than 5 when the above relationship is quadratic rather than linear.

The average risk taken by the players on first service in the Wimbledon Men's Singles Championships from 1992 to 1995 was 4.06 (Magnus and Klassen 1999) and thus for most male players a quadratic relationship is appropriate.

Using the quadratic relationships (3.13) and the formulae for $P_2(x_2)$ and $P_1(x_1,x_2)$ above, it can be shown that the optimal value of x for the second serve is

$$x_2^* = (c - 0.1b + \sqrt{(c^2 + 0.1bc + 0.01(b^2 - 3ac))})/0.3c \qquad (3.14)$$

if this expression is positive, or zero otherwise.

The associated value of $P_2^* (x_2^*)$ is given by

$$P_2^* (x_2^*) = (1 - 0.1x_2^*)(a + bx_2^* + cx_2^{*2}) \qquad (3.15)$$

and the optimal value x_1^* for the first service is given by

$$x_1^* = (c - 0.1b + \sqrt{(c^2 + 0.1bc + 0.01(b^2 - 3ac + 3cP_2^* (x_2^*)))})/0.3c \qquad (3.16)$$

if this expression is positive, or zero otherwise.

The associated value for $P_1^* (x_1^*, x_2^*)$ is given by

$$P_1^*(x_1^*, x_2^*) = (1 - 0.1x_1^*)(a + bx_1^* + cx_1^{*2}) + (0.1x_1^*)(1 - 0.1x_2^*)(a + bx_2^* + cx_2^{*2}) \qquad (3.17)$$

1	P(0) = a	0.5	0.5	0.499	0.495	0.485	0.47
2	P(3)	0.65	0.65	0.655	0.66	0.665	0.67
3	P(6)	0.8	0.799	0.794	0.785	0.774	0.75
4	b	0.05	0.0502	0.0548	0.0617	0.0718	0.0867
5	-c	0	0.00006	0.0009	0.0022	0.0039	0.0067
6	x_2^*	0	0.0164	0.3871	0.7474	1.1145	1.4261
7	P_2^*	0.5	0.5	0.4999	0.4995	0.4977	0.4973
8	x_1^*	5	4.99	4.78	4.51	4.24	3.91

1	P(0)	0.5	0.4	0.38	0.37	0.35
2	P(3)	0.65	0.64	0.65	0.66	0.665
3	P(6)	0.8	0.877	0.853	0.818	0.79
4	b	0.05	0.0805	0.1012	0.1187	0.1367
5	-c	0	0.0002	0.0037	0.0073	0.0106
6	x_2^*	0	2.48	2.54	2.48	2.51
7	P_2^*	0.5	0.4502	0.4573	0.4656	0.693
8	x_1^*	5	5.29	4.90	4.50	4.25

Table 3.5 The characteristics for a point when $P(x) = a + bx + cx^2$ and $P_1^*(x_1^*,x_2^*) = 0.625$

Table 3.5 is an example of the change from straight line relationship (column 3) to quadratic relationships (columns 4 to 8). The straight line relationship is the one with the lowest penalty (decrease in probability) in Tables 3.3 and 3.4 namely P(x) is equal to 0.5 when x=0, 0.65 when x=3 and 0.8 when x=6 and thus a=0.5 and b=0.05. Curvature is then introduced by decreasing P(x) when x=0 and x=6 and increasing it when x=3 with the requirement that the optimal probability of winning the point $P_1^*(x_1^*,x_2^*)$ remains constant at 0.625. Row 1 in Table 3.5 is equal to a in the quadratic relationship P(x) = a + bx +cx^2, row 4 is b and row 5 is –c.

Note that the second derivative (2c) of all the quadratic equations is negative since c is negative and this is as expected in the tennis serving situation. It follows that the slope of the quadratic near x=0 (second service) is greater than the slope of the curve near x=5 (first service). The curvature of the quadratic increases from being very slight in column 4 to being moderate in column 8. Row 6 gives the optimal value of the risk being taken on the second serve, x_2^*. It can be seen that the optimal risk taken on the second service increases as the curvature increases. Row 8 gives the optimal value of the risk being taken on the first service, x_1^*. It can be seen that the optimal risk taken on the first service decreases as the curvature increases.

This method is then used in section 6 to obtain a potential quadratic equation for the relationship between the risk taken on service and the probability of winning the point.

3.5.2 Another form of the y curve

Recognising that some curvature in the y(x) probability equation is needed, Klaassen and Magnus (2014) suggested $y(x) = a - cx^n$ which requires three parameters a, c and n, but only two if you use n = 2 or 3 or even 4. Note that Klaassen and Magnus defined x as the probability that the service goes in and consequently lies between 0 and 1 whereas Pollard (2008) defined x as the risk taken by the server and lies between 0 and 10. Using the Pollard definition of x = risk, the relationship is $y(x) = a - [0.1(10 - x)]^2c$.

Assuming you have the data for the first serve (x_1, y_1) and the second serve (x_2, y_2), then

$Y_1 = a - cx_1^n$ and $y_2 = a - cx_2^n$ and hence

$a = (y_1 x_2^n - y_2 x_1^n) / (x_2^n - x_1^n)$ $c = (y_1 - y_2) / (x_2^n - x_1^n)$

Using the Klaassen and Magnus Wimbledon data, we have the average men's first serve as $(x_1, y_1) = (0.595, 0.740)$ and second serve $(x_2, y_2) = (0.864, 0.594)$ and hence two equations with two unknowns a and c and using n = 2, 3 or 4. When n = 2, the equation becomes $y = 0.872 – 0.372x^2$ and the optimal strategy $(x_1^*, x_2^*) = (0.567, 0.884)$ and $P^* = 0.649$.

When n = 3, the equation becomes $y = 0.811 – 0.336x^3$ and the optimal strategy $(x_1^*, x_2^*) = (0.605, 0.845)$ and $P^* = 0.649$.

When n = 4, the equation becomes $y = 0.782 – 0.338x^4$ and the optimal strategy $(x_1^*, x_2^*) = (0.630, 0.825)$ and $P^*= 0.651$.

The equation and the optimal strategy vary substantially with n = 2, 3, 4, but the optimal probability P^* is fairly constant regardless of the value of n.

It can also be seen that the actual probability for the average player P = (0.595)(0.740) + (0.405)(0.864)(0.594) = 0.648 which is almost identical with P^* suggesting players are surprisingly very efficient in their service strategy.

3.5.3 Recognising player efficiency

If we can recognise that elite players are efficient with their service, especially their second service, we can now obtain three equations with three unknowns, namely

$0.740 = a + bx_1 + cx_1^2 = a + 4.06b + 16.4836c$

$0.594 = a + bx_2 + cx_2^2 = a + 1.36b + 1.8496c$

$0 = \delta p(x_1,x_2)/\delta x_1\delta x_2= 0.3cx_2^2 + 2(0.1b – c)x_2 + (0.1a – b) =0.1(a – 7.28b – 21.6512c)$

From which we obtain a = 0.489, b = 0.086146 and c = -0.086146

These results are very close to the results obtained in section 3.6 using the methodology of that section.

3.6. Where the server might focus attention on increasing the probability of winning a point

We now return to the initial equation (1) for the probability P that the server wins a point, which is given by

$P = p_1y_1 + (1 - p_1)p_2y_2$, where

p_1 is the probability that the first service is in the court (i.e. is not a fault),

y_1 is the probability that the server wins the point given that the first serve is in the court,

p_2 is the probability that the second service is in the court, and

y_2 is the probability that the server wins the point given that the second serve is in the court.

It can be seen that there are four possible options for increasing the probability P, that a server wins a point. The server can do this by increasing one or more of the above four probabilities, assuming the other probabilities remain unchanged. One or two of these probabilities may be preferable to the others. This is now considered theoretically and then mathematically using the Wimbledon 1992-1995 singles data of Magnus and Klassen (1999).

Using this data, these four probabilities were estimated at p_1= 0.594, y_1= 0.733, p_2=0.864 and y_2= 0.594 respectively for male players and p_1= 0.608, y_1= 0.622, p_2=0.860 and y_2=0.541 respectively for women. For the male players the estimated probability of winning the point on the first service if the first serve was in play was 0.733 and the estimated risk taken (x_1) is 10(1 – 0.594) = 4.06. For the second serve the estimated probability of winning the point on the second serve if the second serve is in play is 0.594 and the estimated risk taken (x_2) is 10(1 – 0.864) = 1.36. Combined, the overall probability p of a male player winning a service point is (0.594)(0.733) + (1-0.594)(0.864)(0.594) = 0.644. In the case of women players the probability of winning a service point is (0.608)(0.622) + (1-0.608)(0.860)(0.541) = 0.561

Now consider a typical “average” male player with the above five average statistical characteristics (p_1=0.594, y_1=0.733, p_2=0.864, y_2=0.594 and P=0.644). It would appear reasonable to assume that these statistical estimates are close to optimal for the hypothetical “average” male player at Wimbledon. As mentioned in section 3 above these figures would be different for the other Grand Slams due to the different court surfaces with Wimbledon having the highest P value, Roland Garros the lowest P value and the Australian and US Opens being somewhere in between. The fact that the estimated risk for the first service at Wimbledon is 4.06, which is well below 5, suggests that the quadratic relationship will be better than the linear one. Fitting the above data to the quadratic model produces the following estimated values for the parameters as a= 0.491, b= 0.0837 and c= -0.00591 and thus the quadratic relationship for the probability the server wins the point given that he served with risk level x and the serve went into play is

$$P(x) = 0.491 + 0.0837x – 0.00591x^2.$$

This fitted quadratic relationship is shown in Figure 3.1. Note that the quadratic curve is a good fit for the normal range of risk from x = 0 to 6 or 7 but not for the even higher risk levels 7 to 10 which are not really physically practical anyhow and we would expect a levelling off, not a decline above x = 7

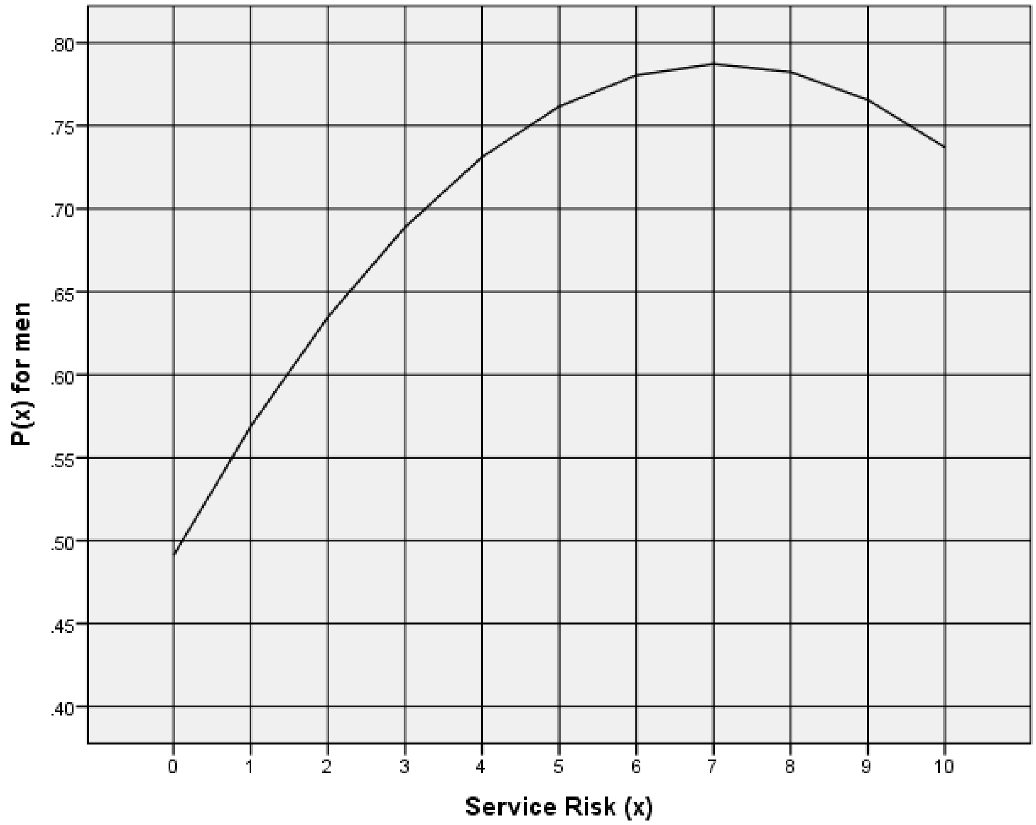

Figure 3.1: P(x) for service risk in men

The equivalent relationship for the "average" female player for the above Wimbledon data is

$$P(x) = 0.479 + 0.0487x - 0.00310x^2$$

This fitted quadratic relationship is shown in Figure3.2. Again the quadratic curve is a good fit for the normal range of risk levels from 0 to 7 but not for the even higher levels from 7 to 10 which are not physically possible.

It follows that the general discussion and conclusions for the men's game will also be applicable to the women's game.

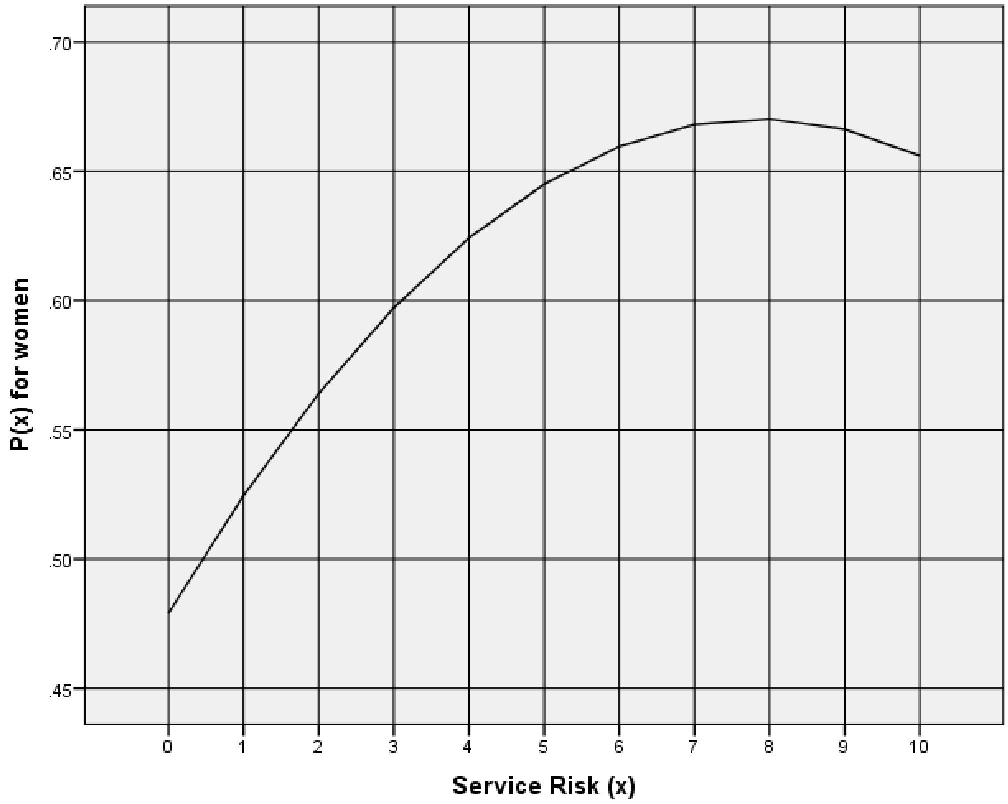

Figure 3.2: P(x) for service risk in women

We now look at the four first partial derivatives of P from equation (3.1) which are given by

$$\delta P/\delta p_1 = y_1 - p_2 y_2 \tag{3.18}$$

$$\delta P/\delta y_1 = p_1 \tag{3.19}$$

$$\delta P/\delta p_2 = (1 - p_1) y_2 \tag{3.20}$$

$$\delta P/\delta y_2 = (1 - p_1) p_2 \tag{3.21}$$

Partial derivative	Male	Female
Equation (3.18)	0.220	0.143
Equation (3.19)	0.594	0.608
Equation (3.20)	0.241	0.212
Equation (3.21)	0.351	0.337

Table 3.6 Partial derivatives of P (probability of server winning point)

Table 3.6 gives the values of the partial derivatives of equation (17) for both men and women using the 1992-1995 Wimbledon data of Magnus and Klassen (1999). It can be seen that for both men and women the largest of these partial derivatives is considerably bigger than the other partial derivatives. It follows that any male or female professional tennis player should firstly focus on improving his or her y_1 value, in other words increasing the probability of winning the point after the first serve has gone into play. This implies not only improving the difficulty, placement and speed of the first serve, but also the "follow up" to the first serve should it be returned until the point is won.

Secondly a player should concentrate on improving y_2, which implies similar improvements for the second service play as for first service play after the serve has gone into play. Thirdly a player should endeavour to improve p_2, the percentage of second serves into play, and lastly endeavour to improve p_1, the percentage of first serves into play. In other words, and somewhat surprisingly, increasing the percentage of first serves into play, which may be the easiest thing to do, is the least effective way of increasing the percentage of points won on service.

For example, suppose a male player with the above average statistics wishes to increase his probability of winning a point on service P by 0.05 (a rather large increase) from 0.644 to about 0.694. Compare what happens if both y_1 and y_2 are increased by 0.05 with what happens if both p_1 and p_2 are increased by 0.05. The former will increase P by 0.047 from 0.644 to 0.691 whereas the latter will only increase P by 0.023 from 0.644 to 0.667. However a tennis player will probably find it a little easier to increase the probability of getting the serve into play than increasing the probability of winning any ensuring rally because the former only depends on characteristics of the server whereas the latter depends on characteristics of the receiver as well as the server.

Alternatively, consider firstly as an example, what happens if an "average" player is able to improve his second serve by increasing just p_2 by 0.136 from 0.864 to (say) 1.0 (i.e. by eliminating all double faults) whilst leaving all the other three probabilities unchanged. If a player can achieve this improvement without any of the other probabilities changing, his overall probability of winning the point increases from 0.644 to 0.677. Secondly, consider increasing just p_1 and leaving all the other three probabilities unchanged. If p_1 is increased to achieve the same value of P of 0.677, then p_1 will need to increase by 0.149 from 0.594 to 0.743. Similarly it can be shown that to achieve this same increase in the overall probability of the server winning the point from 0.644 to 0.677 he would need to increase y_2 by 0.094 from 0.594 to 0.688 (assuming other three probabilities remain constant) or to increase y_1 by 0.055 from 0.733 to 0.788 (assuming other three probabilities remain constant).

This confirms that the average player gets the greatest return by increasing y_1 by (say) one percentage point, the next biggest return by increasing y_2 by (say) one percentage point, the next biggest return by increasing p_2 by one percentage point and the smallest return by increasing p_1 by one percentage point. Thus, for the above "average" player, the return for increasing y_1 is approximately 2.7 (i.e. 0.149/0.055) times the return for increasing p_1 by one percentage point. Although this example is for an "average" player, the conclusions are not sensitive to this assumption and apply to any player.

In practice a player will try to improve more than just one of these four probabilities at a time. For example he might focus initially on the first service by improving the first serve outcome y_1 and possibly simultaneously work on improving the percentage of first serves into play p_1.

3.7. General tennis discussion

Tennis is special in that it allows a server a second serve if the first serve does not go into play (called a fault). It follows that the probability that a player wins a point on service is a function of four probabilities, namely the probability that the first service goes into play, the probability that the server wins the point if the first serve went into play, the probability that the second serve goes into play and the probability that the server wins the point given that the second serve went into play.

Rather than have two specific serves, an aggressive first service with a low chance of the ball going into play and a softer second serve with a very high probability of going into play, players generally have a range of serves with some relationship between the aggressiveness of the service (or the risk taken on service) and the probability of winning the point if the serve went into play. This relationship between the probability of winning the point and the risk taken could be linear, or linear over a limited range, but is more likely to be quadratic (or curved).

To apply the results of this chapter, a player should acquire a general understanding of his/her own approximate p_1 and p_2 values (or p(x) values) and y_1 and y_2 values (or P(x) values), particularly in the domain where the risk x is near 4 and 5 for the first service and x is near 0 and 1 for the second service. This could be achieved by keeping some match statistics, actually doing some experimentation during matches, or doing some service experimentation during practice matches.

For a player with an approximate linear relationship between risk taken on service and the probability of winning the point if the serve goes into play, it can be optimal to serve only about 50% of first serves into play and close to 100% of second serves into play.

For a player with an underlying quadratic relationship, the slope of the curve near x =4 or 5 is important for the first service and the slope near x = 0 or 1 is important for the second serve. The optimal risk on the second serve x_2^* increases as the curvature of the quadratic increases. This is as expected as the server receives a greater reward from taking an increased risk near x = 0 or 1 in the quadratic case than in the linear case. Conversely the optimal risk on the first serve x_1^* decreases as the curvature increases.

Again this is as expected as the server receives less reward for taking a greater risk near x = 4 or 5 in the quadratic case than in the linear case. The optimal first serve risk is generally between 4 and 5 and the second serve risk between 0 and 1. This implies that the optimal strategy should be to put between 50 and 60 per cent of first serves into play and somewhere between 90 and 100 percent of second serves into play.

In order to serve optimally (or close to optimally) a player needs to know, at least approximately, the actual shape of the relationship between risk taken on service and the probability of winning the point if the service goes into play. This can be achieved through experimentation and data collection in practice or even in matches.

Unless the relationship between a player's underlying risk taken on service and probability of winning the point varies dramatically from a quadratic relationship, a player who is not putting between 50 and 60 percent of first serves and between 90 and 100 percent of second serves into play is not serving optimally. If the first serve percentage is less than 50% the player is relying too much on the weaker second serve. If the first serve percentage is above 60% the player is not taking enough risk on first service and not making enough use of the right to have a second serve if the first serve is a fault. If the percentage of second serves into play is near 100% the player is not taking enough risk on second serve by serving more aggressively, faster, closer to the line, etc. Below 90% is being too aggressive on the second serve.

Further, in order to improve the probability of winning a point on service, a player should concentrate firstly on improving the probability of winning the point if his serve goes into play, secondly on improving the probability of winning the point if the second serve goes into play, thirdly by increasing the percentage of first serves that go in to play and finally by increasing the percentage of second serves that go into play.

3.8. Summary and implications

The service in tennis, as in most sports, was introduced as the way to start a rally. But because of the possibility of too many faults and most service game being lost by both players, a second attempt was allowed without penalty. Gradually the service became more aggressive, knowing that there was always a second chance. Today the service is a real weapon and at the elite level it is normal for both players in a match to have a probability greater than 0.5 of winning a point on service and consequently a probability of holding service greater than (say) 0.6. This chapter looks at the mathematical characteristics of first and second serves and how players can use the free second chance to maximise their probability of winning a point on service. The literature on this subject begins in the early 1970s after Open Tennis replaced the amateur game and this chapter extends current knowledge in a number of ways.

It is integral to the mathematics and the tennis interpretations to recognise that the probability a player wins a point on service depends on four different probabilities, namely

1 The probability that the first service goes into play (i.e. is not a fault),

2 The probability that the server wins the point given the first serve was in play,

3 The probability that the second service goes into play (i.e. is not a fault), and

4 The probability that the server wins the point given the second serve was in play.

These four probabilities are readily available for most matches. The older literature recognised that each player had two distinct serves, a first and second serves with quite different probabilities. However the modern player has an almost continuous range of serves between a very aggressive first serve and a very safe second serve.

In Section 3.3 it is assumed that the relationship between the probability the serve goes in and the probability the server wins the point is linear and continuous over the range between the two extremes. We then find the optimal risk a player with these serving characteristics should take on first and second serve.

Section 3.4 measures the penalty (decrease in probability) for a non-optimal serve.

Section 3.5 extends the analysis from a straight line relationship between type of service used and probability of winning the point into a curved (quadratic) relationship.

Interestingly the analysis in Sections 3.3 to 3.5 show that a player should get about 50% of first serves into play and around 90% of second serves into play if serving close to optimal and players who get (say) 70% of first serves in, or (say) 100% of second serves in, are not serving optimally and maximising their chance of winning the point on service.

Section 3.6 looks at the relative importance of changes to each of the four probabilities that affect the probability of winning a point on service and hence where the player might get the greatest gain through improvement in that characteristic. Interestingly the greatest reward is through increasing the probability of winning the point after the serve has gone in (first serve rather than second) ahead of increasing the percentage of serves that go into play (and second serve better than first serve).

The above outcomes are not all intuitively obvious and so section 3.7 is a general tennis discussion of how coaches and players might use this information to improve their play.

As the key data on first and second serves is readily available at professional tournaments and often recorded by coaches at junior tournaments to assist their players, this chapter extends the way this information can be used to improve a player's performance.

CHAPTER 4

IMPORTANCE

It is well known to players, officials, media commentators and spectators that not all points in a game are equally important. Clearly the server winning the point at 30-40 (say) is much more important to the server than winning the point at 30-15. If he loses the point at 30-40 he also loses the game. If he wins the point the score becomes deuce and the game continues from an equal position. On the other hand, if he loses the point at 30-15 the score becomes even again at 30-all, whereas if he wins the point, the score advances to 40-15 and he is in a good position to hold service. Few players and spectators would know how to define importance and how to calculate the relative importance of the two scores, but most would know that winning the point when behind at 30-40 (say) is much more important to the server and to the receiver than winning the point at 30-15.

In the classical paper on the subject of importance, Morris (1977) defined importance of any game score as the difference between the probability that the server wins the game if he wins the next point and the probability that the server wins the game if he loses the next point. This definition has been adopted ever since by many researchers into the mathematical, statistical and probabilistic study of tennis, and used in other sports. Using a simplistic scoring notation where points are 0,1,2,3,4, etc. rather than the more complicated love (0),15,30,40,advantage,etc., the importance of a score (x,y) is the probability of the server (i) winning the game if he wins the next point $p_i(x+1, y)$ minus the probability he wins the game if he loses the next point $p_i(x, y+1)$. This can be written

$$\text{Importance }(x, y) = p_i\,(x+1, y) - p_i(x, y+1) \qquad (4.1)$$

If we consider the situation from the receiver's (j) point of view compared to the server (i)

$$\text{Importance to receiver }(x, y) = p_j(x, y+1) - p_j(x+1, y)$$

$$= (1 - p_i(x, y+1)) - (1 - p_i(x+1, y))$$

$= p_i(x+1, y) – p_i(x, y+1)$ = importance to server

It follows that the next point at the score (x, y) in a game is equally important to the server and to the receiver and equally important to the player who is ahead in that game as it is to the player who is behind. This result may be surprising to some and was one of a number of hypotheses or myths in tennis investigated by Magnus and Klaassen (1996, 2008, 2014) using the classical Morris (1977) definition of importance.

This definition of importance and the analysis that follows can be extended to determine the importance of each game in a set of tennis, but it is much more complicated because each player takes turns at serving and consequently you need to know firstly which player is serving first in the set, secondly whether it is a tiebreak or advantage set and thirdly you need to allow for the two probabilities p_i and p_j that the respective players (i) and (j) have of winning a point and hence a game on service. Similar challenges apply to the analysis of importance of points in a tiebreak game. However, the analysis of the importance of sets in a match is the simplest because you only need one probability namely the probability player (i) wins a set, and the match is first to win three sets in best of five sets (or two sets in best of three sets) without the need to lead by at least two which applies to ordinary games, tiebreak games and advantage sets. The classical definition of importance and the analysis of the scoring system under this definition is discussed in section 4.1 of this chapter.

This chapter then goes on to present in section 4.2 an alternative approach to the study of importance and in the remainder of the chapter utilizes this approach to analyse the various scoring systems used in the game of tennis. If a player is able to lift their play at some stage in some matches, and there is evidence that the better player can lift on one or more points (Pollard, Cross and Meyer (2006)) and Pollard (2004), then which are the points, games and sets within a match that the player receives the greatest benefit by lifting. These points/games/sets could be defined as more important as they achieve the greatest return for lifting. This different approach to importance demonstrated by Pollard and Pollard (2007a and 2007b) and used by Pollard (2008a) to look at any possible advantage for left handers when serving. Pollard (2008b) considered the best serving strategy using this approach.

Similar challenges in the analysis apply with this new approach to importance as applied with the classical definition, whereby points within a game and sets within a match only utilize one probability value, but games within a set and points within a tiebreak game require a probability value for each player when serving and also knowing which player is serving first.

Section 4.3 looks at sets within a match, both before the match commences and at any stage during the match. However, the conclusions on importance are complementary under this approach to importance. There are also some interesting conclusions on variability between sets.

Section 4.4 looks at points within a game and again the conclusions are complementary to those under the classical definition of importance. In particular, if you have the ability to lift your game, it is better to do so before deuce than after deuce.

Section 4.5 looks at points within a tiebreak game, where the answers depend on who serves first and the score. Interestingly, if you have one available lift, there are some scores where it pays to lift, others where you should not lift and some where it makes no difference whether you lift or not.

The final scoring component is games within a set and this is discussed in section 4.6. This section has similar scoring issues as in section 4.5 and similar conclusions as to when to lift, when to not lift, and when it makes no difference whether you lift or don't lift.

Section 4.7 looks at variability, say between the probability of winning a point on service to the first (forehand) court and the probability of winning a point to the second (backhand) court. Variability is shown to be better than the equivalent equal probability case and (perhaps surprisingly) if you can lift, it is better to lift when serving to the server's stronger side than to the weaker side.

4.1. The classical definition and analysis of Importance

4.1.1. Points within a game

If p is the probability of the server winning a point on service and q = 1 - p is the probability of the receiver winning a point on service and points are independent and identically distributed, then the probability of the server winning the game from any particular score is given in Table 4.1 and the importance of each point can be calculated using formula (4.1) by taking the difference between the two appropriate probabilities in Table 4.1 and is shown in Table 4.2. Using a typical serving probability of p = 0.6 and q = 0.4, the probability of the server winning the game and the importance of each point are also given in Tables 4.1 and 4.2 respectively.

Score	Probability of winning the game	when p=0.6
Deuce and 30-30	$P^2 / (p^2 + q^2)$	= 0.692
40-30 and Advantage Server	$(p^3 – p^2 + p) / (p^2 + q^2)$	= 0.877
30-40 and Advantage Receiver	$P^3 / (p^2 + q^2)$	= 0.415
40-15	$(2p – 4p^2 + 4p^3 – p^4) /(p^2+ q^2)$	= 0.951
15-40	$P^4 / (p^2 + q^2)$	= 0.249
40-00	$(3p - 8p^2 + 10p^3 – 5p^4 + p^5) / (p^2 + q^2)$	= 0.980
30-15	$(3p^2 – 5p^3 + 4p^4 – p^5) /(p^2 + q^2)$	= 0.847
15-30	$(p^3 + p^4 - p^5) / (p^2 + q^2)$	= 0.515
00-40	$P^5 / (p^2 + q^2)$	= 0.150
30-00	$(6p^2 -16p^3 + 19p^4 – 10p^5 + 2p^6) / (p^2 + q^2)$	= 0.927
15-15	$(4p^3 – 5p^4 + 2p^5) / (p^2 + q^2)$	= 0.714
00-30	$(p^4 + 2p^5 – 2p^6) / (p^2 + q^2)$	= 0.369
15-00	$(10p^3 – 25p^4 + 26p^5 - 12p^6 + 2p^7) / (p^2 + q^2)$	= 0.842
00-15	$(5p^4 -4p^5 – 2p^6 + 2p^7) / (p^2 + q^2)$	= 0.576
00-00	$(15p^4 – 34p^5 + 28p^6 – 8p^7) / (p^2 + q^2)$	= 0.736

Table 4.1. The probability that the server wins the game given the current game score. (p =0.6, q=0.4)

Score	Importance of next point	when $p=0.6$
Deuce and 30-30	$pq / (p^2 + q^2)$	$= 0.462$
40-30 and Advantage Server	$q^2 / (p^2 +q^2)$	$= 0.308$
30-40 and Advantage Receiver	$p^2 / (p^2 +q^2)$	$= 0.692$
40-15	$q^3 / (p^2 + q^2)$	$= 0.123$
15-40	$p^3 / (p^2 + q^2)$	$= 0.415$
40-00	$q^4 /(p^2 +q^2)$	$=0.049$
30-15	$(2p – 5p^2 + 4p^3 –p^4) / (p^2 + q^2)$	$= 0.258$
15-30	$(p^2 – p^4) / (p^2 + q^2)$	$= 0.443$
00-40	$p^4 / (p^2 + q^2)$	$= 0.249$
30-00	$(3p – 11p^2 + 15p^3 – 9p^4 + 2p^5) / (p^2 +q^2)$	$= 0.133$
15-15	$3p^2q^2 / (p^2 + q^2)$	$= 0.332$
00-30	$(p^3 + p^4 – 2p^5) / (p^2 + q^2)$	$= 0.366$
15-00	$(6p^2 – 20p^3 + 24p^4 – 12p^5 + 2p^6) / (p^2 +q^2)$	$= 0.213$
00-15	$(4p^3 - 6p^4 + 2p^6) / (p^2 + q^2)$	$= 0.346$
00-00	$10p^3q^3 / (p^2 + q^2)$	$= 0.266$

Table 4.2. Importance of each point in game at given score (p=0.6, q=0.4)

It is easily seen that the score 30-40 and advantage receiver (both have the same importance) are the most important point and 40-00 is the least important point for all $p>0.5$. The order of importance for the other points varies with the value of p. In the above case where $p=0.6$, the score 30-30 (and deuce) are next in importance after 30-40 followed by 15-30 and then 15-40. When $p = 0.618$ all four scores 30-30, deuce, 15-30 and 15-40 are equally important, but above this value of p, 15-40 is more important than 15-30 and 30-30 (and deuce). This follows from solving the equations $pq = p^3 =p^2 – p^4$ which all reduce to solving the quadratic $p^2 + p = 1$ giving $p = 0.618$.

4.1.2. Sets within a match

It is possible to determine the importance of each set in a match using the single probability p that one of the players (A) wins a set and the probability $q = 1 – p$ as the probability that the other player (B) wins the set. Table 4.3 gives the probability that player A wins the match at various stages (set scores) during the 5 set match. Using the definition of importance of the next set as the difference between the probability that player A wins the match if he wins that next set and the probability player A wins the match if he loses that next set, Table 4.4 gives the importance of that next set depending on the current set score.

Set Score	Probability Player A wins match	when p=0.6
0-0	$P^3 + 3p^3q + 6p^3q^2 \quad = p^5 + 5p^4q + 10p^3q^2$	0.68256
1-0	$P^2 + 2p^2q + 3p^2q^2 \quad = p^4 + 4p^3q + 6p^2q^2$	0.8208
0-1	$P^3 + 3p^3q \quad = p^4 + 4p^3q$	0.4752
2-0	$p + pq + pq^2 \quad = p^3 + 3p^2q + 3pq^2$	0.936
1-1	$P^2 + 2p^2q \quad = p^3 + 3p^2q$	0.648
0-2	$P^3 \quad = p^3$	0.216
2-1	$p + pq \quad = p^2 + 2pq$	0.84
1-2	$p^2 \quad = p^2$	0.36
2-2	$P \quad = p$	0.6

Table 4.3. Probability Player A wins the match from various set scores, where p is the probability player A winning a set, q=1-p, and the example where p=0.6.

Set Score	Importance of next set	p= 0.5	p = 0.6	p = 0.7
0-0	$6p^2q^2$	0.375	0.3456	0.2646
1-0	$3pq^2$	0.375	0.288	0.189
0-1	$3p^2q$	0.375	0.342	0.441
2-0	q^2	0.25	0.16	0.09
1-1	$2pq$	0.5	0.48	0.420
0-2	p^2	0.25	0.36	0.49
2-1	q	0.5	0.4	0.3
1-2	p	0.5	0.6	0.7
2-2	1	1	1	1

Table 4.4. Importance of the next set at a given set score for various values of p (the probability of player A winning a set)

As expected, the most important set is the fifth set which has an importance of 1 or 100%. A best of three set match is the same as a best of five set match once the first two sets have been shared. Thus, in a best of three set match the importance of the first set is 2pq and the second set is either q or p depending on whether Player A won or lost the first set. The importance of the third and final set is I or 100%.

Table 4.4 gives the varying importance of the next set for various probabilities of player A winning a set. The importance of each set clearly varies, but the fifth set remains the most important with importance of 1 or (100%).

4.1.3. Games within a set.

As described earlier, consideration of the importance of games within a set requires knowledge of which player serves first and whether the set is tiebreak or advantage. It also depends on the respective probabilities of each player winning a point on service, (from which can be derived the probability of each player holding service, as shown in the last line of Table 4.1). For simplicity, consider a tie-break set (the most common situation) between two equal players, who both have a probability 0.8 of holding service, with player A serving first. We then know that the probability that player A wins from any position where the scores are equal from 0-0 to 6-6 is 0.5. We also know that in the twelfth game A is receiving and the probability A wins the set from a score 6-5 is 0.2 + (0.8)0.5 = 0.6 whereas the probability A wins from a score 5-6 is (0.2)(0.5) = 0.1 In the eleventh game A is serving at 5-5 and the probability A wins the set is 0.8((0.2 + 0.8(0.5)) + 0.2(0.2)0.5 = 0.5. In the tenth game A is receiving and the probability A wins the set from 5-4 is 0.2 + 0.8(0.5) = 0.6 while from 4-5 it is 0.2(0.5) = 0.1. In general terms, when the match is between two equal players where each has a probability p of holding service and q=1-p of breaking service, then the probability P(a,b) that A wins the set from a score (a,b) where a≤6 and b≤6 depends on whether a+b is even and therefore player A is serving or a+b is odd and therefore A is receiving.

If a+b is even then A is serving and P(a,b) = p(P(a+1,b)) + q(P(a,b+1))

If a+b is odd then A is receiving and P(a,b) = q(P(a+ 1,b)) + p(P(a,b+1))

Continuing to work backwards from 6-6 to 0-0 using the above relationships we can determine the probability A wins the set from each possible set score from 6-6 to 0-0 and these are shown in Table 4.5. The importance of the next game at each possible set score is then calculated by taking the differences between the two probabilities of whether A won or lost the next game, and these importances are shown in table 4.6.

ScoreA/B	B = 0	1	2	3	4	5	6
a = 0	0.5	0.262	0.203	0.059	0.030	0.003	0
1	0.559	0.5	0.239	0.173	0.037	0.013	0
2	0.797	0.565	0.5	0.208	0.134	0.016	0
3	0.855	0.827	0.573	0.5	0.164	0.080	0
4	0.970	0.890	0.866	0.584	0.5	0.1	0
5	0.990	0.987	0.936	0.920	0.6	0.5	0.1
6	1	1	1	1	1	0.6	0.5

Table 4.5. Probability player A (serving first) wins the set from various scores against an equal player (probability of holding serve for each player p = 0.8)

Score A/B	b = 0	1	2	3	4	5	6
b = 0	0.297	0.297	0.180	0.143	0.035	0.013	
1	0.297	0.327	0.327	0.170	0.122	0.016	
2	0.290	0.327	0.366	0.366	0.148	0.08	
3	0.143	0.317	0.366	0.42	0.42	0.1	
4	0.010	0.122	0.352	0.42	0.5	0.5	0.5
5	0.013	0.064	0.08	0.40	0.5	0.5	0.5
6						0.5	1.0

Table 4.6. Importance of each game in a match between two equal players (probability of holding serve for each player p=0.8), A serving first

It can be seen that Player A starts with at probability 0.5 of winning the set which increases to 0.559 if he successfully holds his first serve to lead 1-0, but jumps dramatically to 0.797 if he breaks B service to lead 2-0. On the other hand the probability drops substantially from 0.5 to 0.262 if he drops his first service, but only drops further to 0.203 if B holds his first service so A trails 0-2. If the score becomes 1-1 the probability returns to 0.5. These differentials in the probability reflect that it is a more significant event to break service ($p = 0.2$) than to hold service ($p = 0.8$). But when you look at the importances of these games in Table 6 it can be seen that the importance of the first and second games are equal, reflecting the fact that both players have to take turns at serving. This pairing of equal importance of successive service games continues as the set advances from 0-0 to 6-6.

It can also be seen in Table 4.6 that the importance of the next pair of serves increases as the set score advances from 0-0 to 1-1 to 2-2 to 3-3 to 4-4 when it reaches an importance of 0.5 and remains 0.5 at 4-5, 5-4, 5-5, 6-5, 5-6 but jumps to 1.0 at 6-6 because the set must be won or lost in the next game (tiebreak). Interestingly, the seventh game at 3-3 is equal in importance (0.420) to the eighth game whether the score is 4-3 or 3-4 while the ninth game at 4-4 has a higher importance of 0.5 and every game thereafter has the same importance until the tiebreaker, being the deciding game, has an importance of 1.0. The value on the diagonal always equals the value to the right and the value below.

4.1.4. Tiebreak game

The same methodology of section 4.1.3 can be applied to the tiebreak game but we return to point probability $p = 0.6$, a different serving sequence of ABBAABBAABB…and the winner is the first player to reach 7 points (or more) leading by at least 2 points. Again, the probability of each player winning the tiebreak from a position where scores are equal, namely 0-0, 1-1,..., 6-6, 7-7,… is 0.5 as both players are equal and the scoring system is fair. The probability A wins the twelfth point when the point score is 6-5, which means A is serving, is $0.6 + 0.4(0.5) = 0.8$. If the score is 5-6 (A serving) the probability A wins is $0.6(0.5) = 0.3$. However at 7-6 or 6-7, B is serving and the probability A wins is $0.4 + 0.6(0.5) = 0.7$ and $0.4(0.5) = 0.2$ respectively. The probability A wins at 6-6 (A serving) is $0.6\ (0.7) + 0.4(0.2) = 0.5$ and the probability A wins at 5-5 (B serving) is $0.4(0.8) + 0.6(0.3) = 0.5$ both as expected since the two players are equal. This relationship continues to alternate as the tiebreak continues further until one player leads by two points. In general terms if P(a,b) is the probability player A wins the tiebreak game

P(a,b) = pP(a+1,b) + qP(a,b+1)   when a+b = 0,3,4,7,8,11,12

P(a,b) = qP(a+1,b) + pP(a,b+1)   when a+b = 1,2,5,6,9,10,

Knowing these probabilities and working backwards from 6-6 to 0-0 we can determine the probability A wins from 6-6 back to 0-0 and these probabilities are shown in Table 4.7 for two players of equal standard with the probability of winning a point on service p = 0.6. The importance of each point is then determined by calculating P(x + 1, y) – P(x, y + 1) and these are given in Table 4.8

ScoreA/B	b=0	1	2	3	4	5	6	7	8
a=0	0.5	0.362	0.270	0.184	0.085	0.028	0.007	0	
1	0.592	0.5	0.400	0.250	0.124	0.059	0.017	0	
2	0.730	0.650	0.5	0.333	0.222	0.121	0.029	0	
3	0.849	0.751	0.611	0.5	0.373	0.182	0.048	0	
4	0.915	0.843	0.778	0.691	0.5	0.272	0.12	0	
5	0.962	0.941	0.910	0.818	0.652	0.5	0.3	0	
6	0.993	0.988	0.971	0.928	0.88	0.8	0.5	0.2	0
7	1	1	1	1	1	1	0.7	0.5	0.3
8							1	0.8	0.5

Table 4.7. Probability Player A (serving first) wins tiebreak game from various scores against equal player (p=0.6)

ScoreA/B	b=0	1	2	3	4	5	6	7	8
a=0	0.230	0.230	0.216	0.164	0.097	0.052	0.017		
1	0.230	0.251	0.251	0.209	0.163	0.104	0.029		
2	0.199	0.251	0.278	0.278	0.252	0.154	0.048		
3	0.164	0.232	0.278	0.318	0.318	0.224	0.12		
4	0.119	0.163	0.219	0.318	0.38	0.38	0.3		
5	0.052	0.079	0.154	0.276	0.38	0.5	0.5		
6	0.012	0.030	0.072	0.12	0.2	0.5	0.5	0.5	
7							0.5	0.5	0.5
8								0.5	0.5

Table 4.8. Importance of each point in a tiebreak game between two equal players (p=0.6) A serving first

It can be seen that as the tiebreak progresses with each player winning the point on service, the probability A wins progresses as follows:

0.5, 0.592, 0.5, 0.4, 0.5, 0.611, 0.5, 0.373, 0.5, 0.652, 0.5, 0.3 0.5, 0.7, 0.5. 0.3, 0.5, 0.7, 0.5, ……..

and the importance of each point progresses as follows:

0.230, 0.230, 0.251, 0.251, 0.278, 0.278, 0.318, 0.318, 0.38, 0.38, 0.5, 0.5, 0.5, 0.5, 0.5, 0.5, ………

From Table 4.8 it can be seen that in a close tiebreak game, where both players are holding serve, the importance of each point rises in pairs (after both players have had the same number of serves) and reaches a peak of 0.5 at 5 points all (5-5) and thereafter remains at 0.5 for every point regardless of which player is serving or which player is receiving. Interestingly the importance of the points in such a match rises in pairs so the importance of 3-3 say is the same as the importance of 4-3 and 3-4 regardless of who is ahead and whether the server won or lost the point at 3-3. Apart from this idiosyncrasy, the importance of each point falls as one player draws ahead, e.g. 2-1,3-1,4-1,5-1,6-1 but makes a rise if the losing player wins one or more points, e.g. 4-1 to 4-2 to 4-3. In general, the closer the game, the more important the point.

However, as outlined in the introduction, the analysis of importance in this chapter will now look at importance in an alternative way. If a player is able to lift their play at some stage in some matches, and there is some evidence of this (Pollard, Cross and Meyer (2005)), then what are the points, games and sets within a match that the player receives the greatest benefit by doing so. Again the analysis of points within a game or sets within a match are the simplest as these involve just one probability, the probability of the player winning a point when serving in the first case and the probability of the player winning the set in the second case. The probability of winning a tiebreak game or the probability of winning a set involve two probabilities, one for each player.

4.2. A different approach to importance

Importance is a measure of the difference an event makes and in the mathematical, statistical and probabilistic study of tennis is traditionally defined as first suggested by Morris (1977) as the difference in the chance of winning a game, set or match if you win the next point, game or set respectively rather than if you lose it. But it is not necessarily the only possible definition of importance in tennis. For example, if a player is able to lift their play at some stage in a match, and there is some evidence that the better player can do this (Pollard, Cross and Meyer (2006)) and Pollard (2004), then what are the points, games and sets within a match that the player receives the greatest benefit by doing so? These points, games or sets could be defined as being more important than the others.

As explained previously the examination of sets within a match is the simplest to consider mathematically because there is only one probability involved, namely the probability of Player A winning a set in a best of five set match, and there is no requirement to lead by two as applies in all other scoring systems in tennis. Next is the examination of points within a game because there is still only one probability, namely the probability that the server wins a point on service, but there is the requirement to win four points and lead by at least two points. More difficult is the treatment of games in a set because there are two players taking turns to serve, possibly with different p-values of holding service, as well as resolution of the set winner by an advantage set or more commonly a tiebreak set is the score reaches 6-6. Finally, there is the tiebreak game with its own defined serving order for the two players and the need for one player to reach at least seven points and lead by two points. Importance, defined as when a player should lift his or her play is now considered for each of these four scoring systems that are used in tennis.

4.3. Importance of sets within a match

4.3.1. Sets within a match - Before the match commences

The first case to consider is sets within a match, for example best of five sets as used in men's Grand Slam Singles, because it is the simplest to calculate as it only involves one parameter, the probability of a player winning a set, and does not have the added complication of leading by at least two as applies to points within a game or the need to know which player is serving first. In other words it is a simple case of best of five (B_5) sets.

Assume player A has a probability p_i of winning set i (i = 1,2,3,4,5) and the outcome of each set is independent of every other set, then the probability player A wins the match is given by

$$P = p_1p_2p_3 + (1-p_1)p_2p_3p_4 + p_1(1-p_2)p_3p_4 + p_1p_2(1-p_3)p_4 + p_1p_2(1-p_3)(1-p_4)p_5 + p_1(1-p_2)p_3(1-p_4)p_5 + p_1(1-p_2)(1-p_3)p_4p_5 + (1-p_1)p_2p_3(1-p_4)p_5 + (1-p_1)p_2(1-p_3)p_4p_5 + (1-p_1)(1-p_2)p_3p_4p_5$$

$$= SP_{3,5} - 3SP_{4,5} + 6P_5 \quad (4.2)$$

Where $SP_{3,5}$ is the sum of all the products of the five p_i taken three at a time, namely

$$SP_{3,5} = p_1p_2p_3 + p_1p_2p_4 + p_1p_2p_5 + p_1p_3p_4 + p_1p_3p_5 + p_1p_4p_5 + p_2p_3p_4 + p_2p_3p_5 + p_2p_4p_5 + p_3p_4p_5$$

and $SP_{4,5}$ is the sum of all the products of the five p_i taken 4 at a time, namely

$SP_{4,5} = p_1p_2p_3p_4 + p_1p_2p_3p_5 + p_1p_2p_4p_5 + p_1p_3p_4p_5 + p_2p_3p_4p_5$

and P_5 is the product of all five p_i

$P_5 = p_1p_2p_3p_4p_5$

The coefficients of the sums of the products $SP_{3,5}$, $SP_{4,5}$ and P_5 in equation (4.2) are equal to the number of ways the score in sets reaches 2-0, 2-1 and 2-2.

It can be seen from equation (4.2) that the values of p_1, p_2, p_3, p_4 and p_5 can be re-arranged in any order without affecting the value of P. It can also be seen from equation (4.2) that if player A has the ability to lift in one or more sets, it makes no difference to P, the probability of winning the match, which set or sets the player lifts. Thus the player is just as well off lifting in the first (or any other set) than saving his lift for the fifth set. Of course, if he waits until the fifth set, he may never get to use the lift. If he uses the lift in the first three sets, the match will be shorter (if he is the better player) than if he saves the lift for the fourth or fifth set, but the probability of winning the match will be the same.

This characteristic can also be shown to be true by using partial differentiation and the Taylor's series expansion. If only one lift is possible, then one set has probability $p+\delta$ and the other four sets have probability p. The partial differential of P in equation (4.2) above with respect to p_1 (with p_2, p_3, p_4 and p_5 each equal to p) and where q = (1-p) is

$dP/dp_1 = 6p^2 - 3(4p^3) + 6p^4 = 6p^2(1-2p+p^2) = 6p^2(1-p)^2 = 6p^2q^2$

The second differential with respect to p_1 is zero.

Likewise, if the single lift occurs in the second set, the partial differential of P in equation (4.2) with respect to p_2, with p_1, p_3, p_4 and p_5 all put equal to p, is also equal to $6p^2q^2$, and so are the partial derivatives with respect to p_3, p_4 and p_5. The second and subsequent differentials with only one lift are all zero.

Under the Taylor series $f(x+\delta) = f(x) + \delta f'(x)/1! + \delta^2 f''(x)/2! + \delta^3 f'''(x)/3! + \ldots\ldots\ldots\ldots$

Similarly all of the corresponding second partial derivatives of P with respect to p_i and p_j (where $i \neq j$ and i, j = 1, 2, 3, 4, 5) are equal to $3p - 3(3p^2) + 6p^3 = 3p(1 - 3p + 6p^2) = 3p(1-2p)(1-p) = 3p(1-2p)q$. In the usual case where $p > 0.5$ the second partial derivatives are all equal and negative.

Under the Taylor's series expansion with two variables

$f(x+\delta,y+\delta) = f(x,y) + \delta(f'_x(x,y) + f'_y(x,y))/1! + \delta^2(f''_x(x,y) + f''_y(x,y) + 2f''_{xy}(x,y))/2! + \ldots\ldots\ldots$

Now consider what return a player receives if he can lift his or her play in one particular set and consequently one particular p_i value is increased. For example, suppose player A has a constant probability p = 0.6 of winning each set in a best of five sets match. From Table 4.3 player A has a probability 0.68256 of winning the match. Now suppose the player can lift his/her probability of winning one particular set by 0.1 from 0.6 to 0.7. It could be the first set due to better preparation or readiness to play than the opponent or sometimes the fifth set due to superior fitness than the opponent or the middle sets due to better concentration. But mathematically, it could be any of the sets and for any reason. Using the Taylor's Series expansion above with just one lift, Player A's probability of winning the match is now increased to $0.68256 + 6(0.6)^2(0.4)^2(0.1) = 0.71712$

If player A can lift the set-winning probability from 0.6 to 0.7 in any two of the five sets, then using the Taylor's Series expansion above for two lifts, the probability of Player A winning the match increases to $0.68256 + 6(0.6)^2(0.4)^2(0.1) + 6(0.6)^2(0.4)^2(0.1) + 2(3(0.6)(-0.2)(0.4)(0.1)^2)/2! = 0.75024$. Again, it does not matter in which two sets the player elects to lift.

Under this approach to importance, all sets in a match are equal in importance as it makes no difference which set or sets you elect to lift if able to do so. This approach is quite different to that of Morris (1977) in which the fifth set is the most important (because you either win or lose the match). Both approaches are interesting and important for players. Obviously you should lift in the fifth set if possible. But if you only have one or two possible lifts in a match, it is risky saving that lift for a fifth set and the match may be over before the need for a fifth set and you finish the match with an unused lift, which is unacceptable if you have lost. If you are winning without lifting, you can save the lifting until you fall behind. You may win without lifting, but the penalty is the match will be expected to be longer than if you had lifted (assuming you are the better player).

4.3.2. Sets within a match- During the match

The above characteristic, that it makes no difference to the probability P that a player wins the match, which of the five sets the player elects to lift, also applies to partially completed matches. As an example, suppose a player at the start of a match decides to lift in three sets of the match, including the first set, which he wins. He can now lift in any two of the remaining possible four sets. The probability that the player wins the match is then

$$P = p_2p_3 + (1-p_2)p_3p_4 + p_2(1-p_3)p_4 + p_2(1-p_3)(1-p_4)p_5 + (1-p_2)(1-p_3)p_4p_5 + (1-p_2)p_3(1-p_4)p_5$$

which can be written

$$P = SP_{2,4} - 2SP_{3,4} + 3P_4 \qquad (4.3A)$$

where $P_4 = p_2p_3p_4p_5$

$$SP_{2,4} = p_2p_3 + p_2p_4 + p_2p_5 + p_3p_4 + p_3p_5 + p_4p_5$$

$$SP_{3,4} = p_2p_3p_4 + p_2p_3p_5 + p_2p_4p_5 + p_3p_4p_5$$

The coefficients are the number of ways of reaching the set score 2-0, 2-1, and 2,2 starting from the set score 1-0. As the p-values in equation (4.3A) can be rearranged in any order without changing the result, it does not matter in which sets the player elects to lift.

Correspondingly, if the player had lost the first set, the probability of winning the match is

$$P = p_2p_3p_4 + (1-p_2)p_3p_4p_5 + p_2(1-p_3)p_4p_5 + p_2p_3(1-p_4)p_5$$

$$P = SP_{3,4} - 3P_4 \qquad (4.3B)$$

where the coefficients are the number of ways of reaching the set score 2-1 and 2-2 starting from the score 0-1. Again, the p-values in equation (4.3B) can be rearranged in any order without changing the result, so it does not matter in which of the remaining sets the player elects to lift.

4.3.3. Variability

Now consider the situation where the p-value is not constant from set to set but has value p_i in set i = 1,2,3,4 and 5 and p is the average value of all the $p_{i.}$ Let X_i be a variable which equals 1 when player A wins set i and 0 when player A loses set i, then the probability player A wins the match can be shown (Pollard (2005)) to be equal to the probability that the sum $X = X_1 + X_2 + X_3 + X_4 + X_5$ is greater than or equal to 3. The expected value of this sum is equal to $p_1 + p_2 + p_3 + p_4 + p_5$ and the variance is equal to $p_1q_1 + p_2q_2 + p_3q_3 + p_4q_4 + p_5q_5$ which can be shown to be equal to $5p(1-p) - \Sigma(pi-p)^2$. Since the last term must be positive, the variance of X is less when the pi are different than when all p_i are the same p. This is probably the opposite of what might be expected, but the smaller variance favours the better player (Pollard (1986). For example consider the three cases where (p_1,p_2,p_3,p_4,p_5) equals (0.6,0.6,0.6,0.6,0.6), (0.4,0.5,0.6,0.7,0.8) and (0,0,1,1,1,). In all three cases X = 3 but the variance is 1.2, 1.1 and 0 respectively and the probability player A wins the match is 0.68256, 0.69 and 1.0 respectively. This shows there is a reward for the better player for variability over a consistent value of p.

In another example of variability, suppose player A can lift his p-value from p to $p+\delta$ in one set and decrease his p-value to $p-\delta$ in another set so his average p-value remains at p. This can be done in either order where the effort of lifting leads to a drop in effort to recover or alternatively lowers effort in one set in order to lift in the following set. From the Taylor Series expansion above with two variables the increase in the probability of winning the match by doing so is $3pq(2p-1)\delta^2$ which is positive if p is greater than 0.5.

Consider also the case where a player improves as the match progresses due to physical and mental fitness, then $0.5 \le p_1 < p_2 < p_3 < p_4 < p_5 \le 1.0$ where p_i (I = 1,2,3,4,5) is the probability player A wins set i, then the probability Player A wins the match P is given in equation (4.2) and can be partially differentiated with respect to p_i and p_{i+1} and it can be shown that $\delta P/\delta p_i < \delta P/\delta p_{i+1}$ (for i=1,2,3,4) .Hence if player A is able to increase his p-value in any set he should increase it in that set which has the highest p value, namely p_5. But due to the symmetry of the p values in equation (4.2) the various p_i-values can be in any order and thus it follows that p_5 is not necessarily the fifth set, but is the set with the highest p-value. Consequently, and maybe surprisingly to some players, if player A wishes to increase his chance of winning, he should increase his p-value in that set which already has the highest value of p. It should also be noted that player B receives the greatest increase in his probability of winning the match if he also lifts in the same set as player A, which is when player B's probability of winning the set is lowest.

Two results are emerging. Firstly that a variable p-value is more rewarding than a constant p-value equal to the average of the individual p-values for each set. Secondly, that if a player has the capacity to lift in one or more sets, he receives the greatest reward for lifting in the set or sets in which he already has the highest p-values. Variability is discussed further in section 8 after we have considered further the other importances of points in a game, points in a tie-break and games in a set.

4.4. Points within a game of tennis

Now consider points within a game of tennis where there is still only one p value, the probability that the server wins a point on service, but the game is not just won by the first player to win four points (best of seven points) but the player must also lead by at least two points. Should the score reach three all, (i.e. 40 all or deuce in tennis scoring terminology), then an advantage game is played until one player leads by two points.

Assume the outcomes of each point are independent and that the server has a probability p_i of winning the ith point in a game (i = 1, 2, 3, 4, 5, 6, 7, 8) and then a constant probability p of winning any further points played after the eighth, then the probability that the server wins the game is given by P where

$$P = p_1p_2p_3p_4 + (p_1p_2p_3(1-p_4)p_5 + \ldots\ldots + (1-p_1)p_2p_3p_4p_5) + p_1p_2p_3(1-p_4)(1-p_5)p_6 + \ldots\ldots$$

$$+ (1-p_1)(1-p_2)p_3p_4p_5p_6 + PD(p_7p_8 + (p_7q_8 + q_7p_8)p^2/(p^2 + q^2) \quad (4.4)$$

where PD is the probability the game score reaches deuce after six points (i.e. each player wins three points) and $q_i = l - p_i$ and the probability the server wins the game after deuce is reached for the second time (i.e. after eight points) p_d is given by

$$p_d = p^2 + 2pq(p_d) = p^2/(p^2 + q^2)$$

PD is the sum of all the possible different combinations of the product of three p_i values ($i = 1,2,3,4,5,6$) and three q_j values ($j = 1,2,3,4,5,6$) where $i \neq j$

Using the following terminology only involving p_i values and for example $SP_{4,6}$ is the sum of the products of the six probabilities p_1, p_2, p_3, p_4, p_5, p_6 taken four at a time

$$SP_{4,6} = p_1p_2p_3p_4 + p_1p_2p_3p_5 + p_1p_2p_3p_6 + \ldots\ldots + p_3p_4p_5p_6 \quad \text{(15 terms)}$$

$$SP_{5,6} = p_1p_2p_3p_4p_5 + p_1p_2p_3p_4p_6 + \ldots\ldots p_2p_3p_4p_5p_6 \quad \text{(6 terms)}$$

$$P_6 \quad = p_1p_2p_3p_4p_5p_6$$

then

$$P \quad = SP_{4,6} - 4SP_{5,6} + 10P_6 + PD(p_7p_8 + (p_7q_8 + q_7p_8)p^2/(p^2 + q^2)) \quad (4.5)$$

$$PD = SP_{3,6} - 4SP_{4,6} + 10SP_{5,6} - 20P_6 \quad (4.6)$$

Note that the coefficients of the sums of products in equation (4.5) and (4.6) are the number of ways the points score reaches 40-0, 40-15, 40-30 and deuce respectively.

Recognising the symmetry of the expression in equations (4.5), the values of p_1, p_2, p_3, p_4, p_5, p_6 can all be different and can be arranged in any order, but the expressions (4.5) and (4.6) remain the same. It follows that a player can lift one or more of his/her p_i values and the expressions in equations (4.4), (4.5) and (4.6) remain the same whichever points are selected to lift.

This can also be shown to be true using partial differentiation methods. The first partial derivative of P in equation (4.5) with respect to p_1, with p_2, p_3, p_4, p_5, p_6,p_7 and p_8 all put equal to p is given by

$$dP/dp_1 = 10p^3q^3/(p^2 + q^2) \quad (4.7)$$

Likewise the first partial differential of P with respect to p_2 with p_1, p_3, p_4, p_5, p_6, p_7 and p_8 all made equal to p is also given by equation (4.7). All first partial differentials with respect to p_i where i = 1, 2, 3, 4, 5 and 6 are given by equation (4.7).

The second partial differentials of P with respect to p_i and p_j ($i \neq j$ and i, j = 1, 2, 3, 4, 5 and 6) are all equal and given by

$$d^2P/dp_jdp_j = 6p^2q^2(q - p)/(p^2 + q^2) \quad (4.8)$$

In the normal tennis situation where p is greater than q it follows that the second derivative is negative.

The third partial derivatives with respect to p_i, p_j and p_k ($i \neq j \neq k$ and i, j and k = 1, 2, 3, 4, 5 and 6) are also all equal and can be shown to be

$$d^3P /dp_1dp_2dp_3 = pq(3 - 14p + 14p^2)/(p^2 +q^2) \quad (4.9)$$

which is negative in the range p = 0.3110 to 0.6890 and otherwise positive.

We can also consider the first two points played after deuce with p values of p_7 and p_8. The partial derivatives of P with respect to p_7 and the other seven p_i put equal to p and likewise the partial derivative of P with respect to p_8 and the other seven p_i put equal to p are both equal and given by

$$dP/dp_7 = dP/dp_8 = PDpq/(p^2 + q^2) = 20p^4q^4/(p^2 + q^2) \quad (4.10)$$

The ratio of expression (4.10) to expression (4.7) is equal to 2pq, which must be less than or equal to 0.5. This implies that it is better and the gain at least twice the size. Consequently it is at least as good for a player to lift on any of the first six points in a game as to wait and lift after deuce in the seventh or eighth points. This is primarily due to the fact that the seventh and eighth points occur much less frequently than the first six points.

So far it has been assumed that after the first eight points the value of p_i ($i>8$) is constant and equal to p. However it can be assumed that the ninth and tenth points have probabilities for the server winning the point of p_9 and p_{10} and thereafter p_i ($i >10$) is constant and equal to p. It can be shown that the partial derivatives of P with respect to p_9 and p_{10} are both equal to $40p^5q^5/(p^2 + q^2)$. This ratio to the previous value is also $2pq \leq 0.5$ and so it is at least twice as good to lift on the seventh or eighth points as the ninth or tenth points, and so on. Overall, it is better to lift in the first six points, i.e. before deuce, than to save the lift to after deuce. After deuce, if you still have lifts remaining, the sooner you lift the better.

Now consider a numerical example where a server has a constant $p_i = 0.6$ and an ability to lift by 0.1 on one or a couple of points in a game. Without any lifting, the server has a probability of winning the game of 0.7357. Substituting p = 0.6 in equations (4.7), (4.8) and (4.9), the first, second and third differentials are 0.2658, -0.1329 and -0.1662 respectively. If the player lifts on one point before deuce, the probability of winning the game increases by 0.02658 to 0.76228. If the player lifts on two points before deuce the increases 0.7357 + 0.02658 +0.02658 – 0.001329 = 0.7876, a rise of 0.0519 using the above Taylors Series expansion for two variables. If the player lifts on three points before deuce, there is a rise of 0.0756 to 0.8113 in the probability of the server winning the game, again using a Taylor's series expansion with 3 variables is 0.7357 + 3(.02658) – 3(0.001329) – 0.0001662 = 0.8113, a gain of 0.0756.

Under the Rules of Tennis, it is permitted to use an alternative scoring system where only one more point is played if the score reaches deuce so there is no requirement for one player to lead by at least two points. This scoring is used on the ATP and WTA Tours for doubles only and the receiver selects to which court side (first or second) the server must serve. This is a simple best of seven or first to four and the mathematics is similar to the best if five sets as discussed previously in section 4.4 and the conclusions are similar. Hence a player who is able to lift can select any of the seven points on which to lift with equal effect on the probability of winning the game. If he saves the lift for the fifth or later point, there is no difference in the probability of winning the game, but the game may be a little longer.

4.5. Points within a tiebreak game

4.5.1. The tie-break game

If the set score reaches six games each, in most cases a best of twelve points lead by at least two points tiebreak game is generally played to determine the winner of that set. The only exception where an advantage set is still played is the fifth set in men's singles (third set in women's singles) at three of the Grand Slams. The US Open plays a tiebreak in the fifth set as well as in all other sets. The ITF also played an advantage fifth set in Davis Cup and for women played an advantage third set in Fed Cup, but this changed in 2016 with both competitions now playing tiebreaks in all sets. The ATP and WTA play a tiebreak in all sets. In a match between Player A and Player B where Player A serves first in the set and therefore first in the tiebreak, the tiebreak service structure is (a, b, b, a, a, b, b, a, a, …..) and this continues until one player reaches seven points or more provided that the player leads by at least two points.

The tiebreak set was gradually introduced in the 1970s. Initially it was best on 9 points (first to reach five points) which meant that both players had set points if the score reached 4-4. This gave some advantage to the server and this one point sudden death aspect of this scoring system was not present in any other part of tennis scoring and was gradually replaced by the best of twelve lead by two system now universally adopted.

Over recent years the ATP and WTA Tours introduced a slightly longer match tiebreak to replace the third and final set in men's and women's doubles respectively. It has also been adopted by some Grand Slams for mixed doubles and by the ITF for the mixed doubles at Hopman Cup and in doubles for juniors. In the case the match at one set all is determined by a best of 18 points (first to 10) lead by at least two points extended game in place of a third tiebreak set. The serving order is the same as for the best of twelve lead by two tiebreak system used in singles and the mathematics and conclusions are similar so the best of twelve points tiebreak so the match tiebreak is not considered further here.

4.5.2. Modified short tiebreak

The mathematics of the tiebreak game is considerably more complicated than the previously considered sets in a match or points in a game, so first we consider a simplified short tiebreak where the winner is the first player to reach three points provided he/she leads by at least two points. If p_i is the probability player A wins point i and p_a is the probability Player A wins a point on service and clearly $(1 – p_a = q_a)$ is the probability player B wins that point, and similarly p_b and $(1 – p_b) = q_b$ are the probabilities that Player B and Player A respectively win a point when B is serving and the probability player A wins from score (2,2) is given by p_d where

$p_d = p_aq_b + (p_a p_b + q_aq_b)p_d = p_aq_b/(p_aq_b + b_bq_a)$, then the probability Player A wins the short tiebreak is given by

$$P = p_1p_2p_3 + p_1p_2q_3p_4 + p_1q_2p_3p_4 + q_1p_2p_3p_4 + (p_1p_2q_3q_4 + p_1q_2p_3q_4 + p_1q_2q_3p_4 + q_1q_2p_3p_4 + q_1p_2q_3p_4 + q_1p_2p_3q_4)p_aq_b/(p_aq_b + p_bq_a) \quad (4.11)$$

Using similar methodology and notation as in equations (4.3 and 4.5) above, and $p_1 = p_a$, $p_2 = q_b$, $p_3 = q_b$ and $p_4 = p_a$, this can be written

$$P = SP_{3,4} – 3P_4 + (SP_{2,4} – 3SP_{3,4} + 6P_4)p_aq_b/(p_aq_b + p_bq_a) \quad (4.12)$$

where $SP_{2,4} = p_1p_2 + p_1p_3 + p_1p_4 + p_2p_3 + p_2p_4 + p_3p_4$

$SP_{3,4} = p_1p_2p_3 + p_1p_2p_4 + p_1p_3p_4 + p_2p_3p_4$ and $P_4 = p_1p_2p_3p_4$

The coefficients of the sums of the products in equation (4.12) are equal to the number of ways the point score can reach 2-0, 2-1 and 2-2 respectively. It should be noted from the symmetry that P remains the same whether A serves first or second. As before, by looking at the first partial differentials of equation (4.12) with respect to p_5 and p_6 compared to those with respect to p_1, p_2, p_3 and p_4, it is best to lift on any point before the score reaches 2-2. But unlike the analysis of sets within a match or points within a game, where it was shown it did not matter in which set or on which point respectively the player elected to lift, the analysis of the tiebreak is much more complicated because the p value varies depending on which player is serving.

This can be seen by considering the various situations which arise in this short first to three leading by two tiebreak game. Assuming A serves first and the match is between two equal players A and B, both of whom have a probability p = 0.6 of winning a point on service and so the probability of each winning from 2-2 is therefore 0.5. Consider the situation where the score is 0-2 and player A can lift by δ = 0.1 on one point, which should be before the score reaches 2-2. The probability Player A wins from 0-2 down is P = $0.5p_3p_4$. If A lifts on the next (third) point P =(0.5)(0.5)(0.6) = 0.15. If Player A lifts on the following (fourth) point P =(0.5)(0.4)(0.7) = 0.14. This result can be confirmed by taking the partial differential of P with respect to p_3 which is $0.5p_4$ which is greater than the partial derivative of P with respect to p_4 which is $0.5p_3$ since in tennis p_4 (A serving) is greater than p_3 (A receiving). Hence A should lift on the next point, even though B is serving. Obviously it is most important to win the next point or there would be no further points.

On the other hand, if the score is 2-0, the probability Player A wins the game is P = $p_3 + q_3p_4 + 0.5q_3q_4 = 0.5(1 + p_3 + p_4 – p_3p_4)$ which is P = 0.5 +(0.5)(0.6) + (0.5)(0.5)(0.4) = 0.9 if he lifts on the next (third) point, whereas it is P = 0.4 +(0.6)(0.7) + (0.5)(0.6)(0.3) = 0.91 if he lifts on the fourth point. Hence it is better not to lift on the next point and to save the lift for the following (fourth) point, when A is serving. This is also confirmed by taking the partial differential of P with respect to p_4 which is $0.5(1-p_3)$ which is greater than the partial differential of P with respect to p_3 which is $0.5(1-p_4)$.

If the score is 1-1, the probability Player A wins is P = $p_3p_4 + (p_3q_4 + q_3p_4)0.5$ = 0.55 regardless of whether A utilizes the one remaining lift on the next (third) point or saves it for the following (fourth point). This is confirmed by the partial differential of P with respect to p_3 and p_4 are both equal to 0.5.

Thus it can be seen that the decision whether to lift or not to lift at any time during a short tie-break game depends on the score and who is serving (and also how many lifts are remaining if more than one). The full list of strategies for each of the possible scores below 2-2 with one lift remaining are given in Table 4.9.

Score	P (if don't lift this score)	P (if do lift this score)	Strategy
2-1	0.80	0.85	Lift
1-2	0.30	0.35	Lift
2-0	0.91	0.90	Don't Lift
1-1	0.55	0.55	Either lift or don't lift
0-2	0.14	0.15	Lift
1-0	0.694	0.690	Don't Lift
0-1	0.31	0.31	Either lift or don't lift
0-0	0.5404	0.538	Don't Lift

Table 4.9. The probabilities of winning a short tiebreak game by lifting or not lifting on the next point and hence the appropriate strategy when $p_a = p_b = 0.6$ and $\delta = 0.1$

The figures in Table 4.9 are not independent and can also be calculated recursively and backwards from the score 2-2 in the short tie-break, but also in the full tie-break game backwards from the score 6-6 where the probability of winning the tiebreak is 0.5 for two equal players. The optimum strategy of whether to lift or not to lift on a particular point can be determined by dynamic programming.

Dynamic programming was introduced by Richard Bellman (1957) to describe the process of solving problems where you need to find the best decisions one after the other. In the general case of optimal lifting under tennis scoring this can be written

$$P(i,j,n) = \text{Max}\{ (p_a P(i+1,j,n) + q_a P(i,j+1,n)) ; ((p_a+ \delta)P(i+1,j,n-1) + (q_a - \delta)P(i,j+1,n-1)) \} \quad (4.13)$$

where (i,j,n) is a point where the score is i points to player A and j points to player B and player A has n lifts left to use. The first expression in brackets in equation (4.13) corresponds to the strategy of not lifting at score i to j and the second expression corresponds to the strategy of lifting by δ at score i to j after which there will be n -1 lifts remaining.

Using known probabilities of winning (in this simple example P(2,2,0) = 0.5 since the two players are equal, the scoring system is fair and for optimal result players used their lift before score (2,2) is reached), you can calculate other probabilities as illustrated above and then these probabilities become known probabilities for calculating further probabilities, recursively.

4.5.3. Best of 12 Point Tiebreak

We now apply the approach of section 4.5.2 to the standard best of twelve lead by at least two points tiebreak game. Using an expanded equation (4.12), that when a standard tiebreak game is played (first player to win seven points and lead by at least two points) and using definitions for p_i, $q_{i,}$ P_{12} and $SP_{i,j}$ consistent with previous definitions in sections 4.4 and 4. 5, the probability Player A (serving first) wins the tiebreak game is given by

$$P = SP_{7,12} - 7SP_{8,12} + 28SP_{9,12} - 84SP_{10,12} + 210AP_{11,12} - 464p_{12} + (SP_{6,12} - 7SP_{7,12} + 28SP_{8,12} - 84SP_{9,12} + 210SP_{10,12} - 462SP_{11,12} + 924P_{12})(p_{13}p_{14} + (p_{13}q_{14} + q_{13}p_{14})p_aq_b/ (p_aq_b + p_bq_a)) \quad (4.14)$$

The coefficients of the sums of products are the number of different ways of reaching the point score (6, 0), (6, 1), (6, 2), (6, 3), (6, 4), (6, 5) and (6, 6) respectively.

Using first order partial differentials of equation (4.14) with respect to p_1 and p_2 compared to partial differentials respect to p_{13} and p_{14} it can be shown as before that it is better to lift before the score (6,6) is reached.

Now consider a standard twelve point tiebreak game between two equal players A and B who both have a probability of winning a point on service of 0.6 and without loss of generality assume A serves the first point of the tiebreak. The probability Player A wins the set from various scores from 0-0 to 6-6 were given in Table 4.7 in section 4.2. Using dynamic programming formula (4.13) and assuming A has the capacity to lift the p-value by 0.1 from 0.6 to 0.7 on one point during the first twelve points of the tiebreak, we can calculate the probability that A now wins the tiebreak game (Table 4.10) and determine at what scores A should lift or not lift if he has one lift remaining (Table 4.11).

A/ B	6	5	4	3	2	1	0
6	0.5	0.85	0.91	0.955	0.9784	0.99136	0.99482
5	0.35	0.55	0.705	0.8452	0.9251	0.951604	0.96889
4	0.15	0.32	0.5404	0.72328	0.80401	0.86308	0.92655
3	0.07	0.210	0.40824	0.53426	0.64215	0.77468	0.8658
2	0.036	0.1404	0.24754	0.36222	0.53015	0.6769	0.75245
1	0.0216	0.0691	0.14048	0.27352	0.4275	0.52726	0.61734
0	0.00864	0.0328	0.09742	0.2031	0.29286	0.38662	0.52505

Table 4.10. The probability that player A serving first and with the ability to lift his p-value by 0.1 once during the tiebreak wins the tiebreak against an equal player B from various scores (p=0.6)

A/B	6	5	4	3	2	1	0
6	N.A.	Lift	Not Lift	Not Lift	Lift or Not	Lift or Not	Not Lift
5	Lift	Lift or Not	Not Lift	Lift or Not	Lift or Not	Not Lift	Not Lift
4	Lift	Lift or Not	Not Lift	Not Lift	Not Lift	Not Lift	Not Lift
3	Lift or Not	Not Lift	Not Lift	Not Lift	Not Lift	Not Lift	Not Lift
2	Not Lift	Not Lift	Not Lift	Not Lift	Not Lift	Not Lift	Not Lift
1	Not Lift	Lift or Not	Not Lift	Not Lift	Not Lift	Not Lift	Not Lift
0	Lift or Not	Lift or Not	Not Lift	Not Lift	Not Lift	Not Lift	Not Lift

Table 4.11. The decision to Lift or Not to Lift at various point scores in a tiebreak game between two equal players A and B (p=0.6) with A serving first and able to lift once during the tiebreak.

Comparing Table 4.7 (not able to lift) with Table 4.10 (able to lift once) it can be seen that the probability A wins the tiebreak against an equal player (both have p=0.6) increases from 0.5 at the start of the set to 0.52505 if he uses his lift in accordance with Table 4.11. As the tiebreak continues through 1-1, 2-2 3-3, 4-4,5-5 and the lift is not used the probability A wins gradually increases to 0.55. At 5-5 he has the choice of lifting or not lifting and both decisions produce the same probability of A winning the tiebreak. Since B is serving at 5-5, if he lifts he increases his probability of winning the point from 0.4 to 0.5 but when he serves at 6-5 or 5-6 his probability of winning the point is 0.6. If he elects not to lift at 5-5 then his probability of winning the point is 0.4 and regardless of whether the score is 6-5 or 5-6 he must lift on this point which increases his probability of winning the point to 0.7. If he lifts at 5-5 the probability he wins the tiebreak is 0.5X0.6 + ((0.5X0.4) + (0.5X0.6))0.5 = 0.55. If he does not lift at 5-5 but lifts on the next point regardless of score, the probability of winning the tiebreak is 0.4X0.7 + ((0.4X0.3) + (0.6X0.7))0.5 = 0.55 confirming it makes no difference whether he lifts or not at 5-5. As it has been shown that it is better to lift before 6-6, it is assumed the probability of winning the tiebreak after 6-6 is 0.5.

It is also interesting to read Table 10 along the outside rows and columns as the score progresses from 0-0 to 6-0 to 6-6 or alternatively from 0-0 to 0-6 to 6-6 and note who is serving at each choice of Lift or Not Lift. In particular you do not lift at 6-3 and 6-4 while receiving but do lift at 6-5 when serving for the set. The other way around you do not lift until the score reaches 4-6 and do lift then whilst B is serving, having not lifted on B's other service at 3-6 and also B's service at 0-5 and 0-6 where it was equally good to lift or not to lift.

Overall you save the lift to be used towards the end of the tiebreak, to be used when serving for the tiebreak at 6-5 or when receiving at 4-6 or serving at 5-6, but there are some earlier points where it is just as good to lift or not to lift. Under this definition of importance the most important points for A are when serving for the set at 6-5 or when serving to save the set at 5-6 or when receiving at 4-6. These are also the most important points for B, namely when serving to win the set at 6-4 or when receiving to win at 6-5 or level from 5-6.

4.6. The most important games in a set.

Finally, consider a tiebreak set where the set is won by the first player to reach six games leading by at least two games and a tiebreak game is played to decide the set if the score reaches six games all. Let p_i be the probability Player A wins game number i (where i = 1, 2, 3, ….., 12), q_i = 1-p_i and p_{13} is the probability Player A wins the tiebreak game. Assume Player A serves first in the set so p_1, p_3, p_5, p_7, p_9 and p_{11} represent probabilities A wins the game whilst serving and p_2, p_4, p_6, p_8, p_{10} and p_{12} represent probabilities player A wins a game while receiving. $SP_{k,10}$ represents the sum of the probabilities p_1, p_2, p_3, …….p_{10} taken k at a time and P_{10} is the product of all ten probabilities. Then the probability Player A wins the tiebreak set (assuming player A serves first) is given by

$$P = SP_{6,10} - 6SP_{7,10} + 21SP_{8,10} - 56SP_{9,10} + 126P_{10} + (SP_{5,10} - 6SP_{6,10} + 21SP_{7,10} - 56SP_{8,10} + 126SP_{9,10} - 252P_{10})(p_{11}p_{12} + (p_{11}q_{12} + q_{11}p_{12})p_{13}) \quad (4.15)$$

The coefficients of the sums of products are the number of ways of reaching the score (5, 0), (5, 1), (5, 2), (5, 3), (5, 4), and (5, 5).

Now consider a set between two equal players A and B both of whom have a probability of holding service of p=0.7. Player A serves first and has the ability to lift once by 0.1 to 0.8. As shown previously, any lifting is best done before the score reaches 5-5. Table 4.12 shows the probability of player A winning the set from various scores where both players are equal and neither has the ability to lift. Table 4.13 shows the probability of player A winning the set from various scores when Player A has the ability to lift on one point. These probabilities are calculated using the dynamic programming formula (4.13). Table 4.14 shows the decision player A must make to lift or not to lift at each score up to 5-5.

A / B	5	4	3	2	1	0
5	0.5	0.65	0.893	0.9265	0.97795	0.984565
4	0.15	0.5	0.6135	0.8341	0,877255	0.952372
3	0.105	0.2235	0,5	0.60023	0.7901475	0.8388148
2	0.0315	0.1659	0.26613	0.5	0.5870442	0.7632835
1	0.02205	0.065205	0.2053525	0.2940967	0.5	0.578985
0	0.006615	0.0476282	0.0950953	0.2343961	0.3140772	0.5

Table 4.12. Probability Player A (serving first) wins a set between two equal players (p=0.7) for various game scores in the set.

A/ B	5	4	3	2	1	0
5	0.5	0.7	0.93	0.951	0.9853	0.98971
4	0.2	0.55	0.664	0.8649	0.90102	0.963103
3	0.14	0.263	0.5437	0.64006	0.822732	0.8648433
2	0.042	0.1967	0.3008	0.538282	0.623617	0.7924753
1	0.0294	0.07959	0.234437	0.3255905	0.534209	0.6116889
0	0.00882	0.058359	0.1111824	0.2612680	0.3431503	0.5311272

Table 4.13. Probability Player A (serving first) wins a set between two equal players (p=0.7) for various game scores in the set but A has the ability to lift his p-value once by 0.1 from 0.7 to 0.8.

A/B	5	4	3	2	1	0
5	N.A.	Lift	Lift	Not Lift	Lift or Not	Not Lift
4	Lift	Lift or Not	Lift or Not	Lift or Not	Not Lift	Not Lift
3	Not Lift	Lift or Not	Not Lift	Not Lift	Not Lift	Not Lift
2	Lift or Not	Not Lift	Not Lift	Not Lift	Not Lift	Not Lift
1	Not Lift	Lift or Not	Not Lift	Not Lift	Not Lift	Not Lift
0	Lift or Not	Not Lift	Not Lift	Not Lift	Not Lift	Not Lift

Table 4.14. The decision whether to lift or not lift for Player A in a set between two equal players (p=0.7) but player A (serving first) has the ability to lift his p-value once by 0.1 from 0.7 to 0.8.

Comparing Table 4.12 (not able to lift) with Table 4.13 (able to lift once) it can be seen that the probability Player A (serving first) wins the set against equal player B increases from 0.5 to 0.5311272 due to his ability to lift his p-value in one game by 0.1. As the set progresses from 0-0 to 1-1, 2-2, 3-3, 4-4, 5-5 and he retains the unused ability to lift once, the probability A wins the set continues to rise reaching 0.55 at score 4-4. At this score it makes no difference to the probability player A wins the set whether he lifts or does not lift, so presumably he does not lift, but he does lift in the next game whether the score is 5-4 or 4-5.

Again, it is interesting reading Table 4.13 along the outside rows and columns as the score progresses from 0-0 to 5-0 to 5-5 or alternatively from 0-0 to 5-0 to 5-5 and noting whether Player A is serving or receiving. As player A races to a 5-0 lead he does not lift and continues to refrain from using his lift option until serving for the set at 5-3. If he refrains from lifting then he will use the lift when leading but receiving at 5-4. In the alternative direction as he falls behind to 0-5 he refrains from lifting and continues to refrain holding the lift to be used if he recovers to 4-5. He could utilise the lift option at those scores marked Lift or Not Lift, but this makes no difference to his probability of winning the set and just might make the score look a little better.

4.7. Variation better than consistency.

Most of the leading male and female players appear to have a fairly consistent probability of winning a point on service, regardless of which side they are serving to and regardless of the score in the game or the result of the previous point. (See Chapter 5). As a result it is a reasonable assumption to assume that winning a point on service is an independent and identically distributed process (Klaassen and Magnus (2014)). But not all players serve equally well to both sides of the court. For example, there is some evidence that left handers serve better to the second or backhand court than they do to the first or forehand court. Using data from the 2008 French Open at Roland Garros, Pollard (2008) found that the winner, left-handed Rafael Nadal, won 71% of points when serving to the second court, but only 63% of points when serving to the first court. A lower ranked left handed player with a big service, Chris Guccione, won 63% of points when serving to the second court, but only 50% of points when serving to the first court. So the first question to be addressed is whether a player is advantaged by being able to serve better to one side of the court than the other compared to serving equally to both sides.

It needs to be noted ((Morris (1977) and Tables 4.2 and 5.1) that the more important points occur when serving to the second court (e.g. 30/40 and 15/30). On the other hand more points are played to the first court since a service can be won or lost in five points, but otherwise there are an equal number of points played to each side. So the second question is whether the reward is shared equally by having a greater success serving to the first court rather than the second court instead of the other way around.

It was shown by Pollard, Cross and Meyer (2006) that the better player has the capacity to lift on one or more points. Each server normally has a p-value greater than 0.5, so the server is the better player in their service game and has the capacity to lift on one or more points, so an interesting third question is whether a player should lift on their weakest or strongest side. A further fourth question is at what stage during the service game should the player lift.

Using the methodology and notation of section 4.3, the probability the server wins the game is given by equation (4.4), but this can be written in the format of equation (4.5) which enjoys symmetry with respect to the different p-values for each point. For the case where all the p-values are the same, the probability of the server winning the game is given by P where

$$P = 15p^4 – 24p^5 + 10p^6 + (20p^3 – 60p^4 + 60p^5 - 20p^6)(p^2/(p^2 + q^2))$$

$$= p^4(15 – 34p + 28p^2 – 8p^3)/(p^2 +q^2) \qquad (4.16)$$

On the other hand, if all the probabilities of winning the point to the first court are equal to p_1 and all the probabilities of winning the point to the second court are equal to p_2, and q_1 =1-p_1 and q_2 = 1-p_2, then the probability of the server winning the game is)

$$P = 3(p_1^3p_2 + 3p_1^2p_2^2 + p_1p_2^3) – 12(p_1^3p_2^2 + p_1^2p_2^3) + 10p_1^3p_2^3 + ((p_1^3 + 9p_1^2p_2 + 9p_1p_2^2 + p_2^3) – 12(p_1^3p_2 + 3p_1^2p_2^2 + p_1p_2^3) + 30(p_1^3p_2^2 + p_1^2p_2^3) – 20p_1^3p_2^3)(p_1p_2/(p_1p_2 + q_1q_2))$$

$$= ((3p_1^2 + 9p_1p_2 +3p_2^2) – (2p_1^3 + 15p_1^2p_2 +15p_1p_2^2 +2p_2^3) + (6p_1^3p_2 + 16p_1^2p_2^2 + 6p_1p_2^3) – (4p_1^3p_2^2 + 4p_1^2p_2^3))p_1p_2/(p_1p_2 + q_1q_2) \qquad (4.17)$$

which is the same equation as equation(4.16) above if $p_1 = p_2$

Now consider two players, one of whom is variable from one court side to the other with $p_1 = 0.5$ and $p_2 = 0.7$ while the second player has a consistent $p = (p_1 + p_2)/2 = 0.6$ to both sides. Using the above equations, the consistent player with p = 0.6 has a probability of holding service of P = 0.7357 whereas the variable player with $p_1 = 0.5$ and $p_2 = 0.7$ has a probability P = 0.7420 of holding service. Thus it can be seen that although both players have an average p-value of 0.6 of winning a point on service, the player with greater variability has a higher P-value if holding service than the consistent player.

It can also be seen that the greater the variability, the greater the chance of holding service. If $p_1 = 0.4$ and $p_2 = 0.8$ the probability of holding service P = 0.7643, while if $p_1 = 0.3$ and $p_2 = 0.9$ the probability of holding service P = 0.8199. In the theoretical extreme case where $p_1 = 0.2$ and $p_2 = 1.0$, then P = 1.0 and the server always holds service.

Previously it has been shown that the better player can lift and since the p value for a server is generally greater than 0.5, it is assumed that the server can lift on one or more points. Further, if the player is going to lift, it was shown in section 4.5 that if the p value is constant, it does not matter on which point (or points) before deuce the server elects to lift. But if the player has two p values, p_1 to the first court and p_2 to the second court, then on which court is it better to lift, the one with the higher p or the one with the lower p?

This can be determined by taking partial derivatives of P in equation (4.4) with respect to p_1 and with respect to p_2 and comparing. Note that the factor $p^2/(p^2 + q^2)$ at the end of equation (4.4) becomes $p_7p_8/(p_7p_8 + q_7q_8)$. The partial derivative of P with respect to p_1 is 0.2541 after substituting $p_1 = p_3 = p_5 = p_7 = 0.5$ and $p_2 = p_4 = p_6 = p_8 = 0.7$. The corresponding partial derivatives with respect to p_3 and p_5 are also equal to 0.2541. The partial differential of P with respect to p_2 (with the same substitutions) is equal to 0.2825, and is equal to the partial differentials with respect to p_4 and p_6. Thus, it can be seen that, in the case where $p_2 > p_1 > 0.5$, the partial differential of P with respect to p_2 is greater than the partial differential with respect to p_1. In comparison, the partial differential of P with respect to p_i (i =1, 2, 3, 4, 5 and 6) is equal to 0.2658 when all p_i = 0.6.

What this implies is that if a player performs better when serving to one side than to the other and the player has the capacity to lift by δ once or more during the game, it is better to lift before deuce and (perhaps surprisingly) lift when serving to the better side than when serving to the weaker side. It also means (again a surprising result) that if a player already has a more effective service to one side than to the other, there is more reward in focusing attention on further improving the serve to the stronger side by δ than improving the serve to the weaker side by δ. In practice this becomes harder to achieve as p gets closer to 1.

4.8. Summary and implications

In the traditional use of importance of points in tennis, the Morris (1977) definition of importance is used where importance is the difference between the probabilities of winning the game if you win the next point or if you lose the next point. In this chapter 4, importance is considered in a different way, namely, if a player has the capacity to lift at some stage during a game, and it has been shown by Pollard, Cross and Meyer (2006) that the better player has this ability, then on what points should the player exercise this lifting ability. This point or points can then be seen to be more important than other points. This approach can be extended to games in a set or sets in a match. When considering points within a game, or sets within a match there is only one probability to consider, namely the probability that the player wins a point on service or wins a set respectively. However, when considering games within a set, or points within a tie-break game, there are two players taking turns to serve and consequently two p values to consider.

Firstly, it was shown that in the conventional definition of importance, the importance of any point, game or set is equal for both players whether they are the one serving or receiving or the one ahead or behind. Under the alternative definition of importance, it was also shown that if a player has the ability to lift on one or more points, games or sets, the one to be chosen by the server for maximum return is the same one that should be chosen by the receiver. Any point, game or set is equally important to both players under either definition of importance.

Secondly, under the classical definition of importance, each point (except the known equal situations of 40-30 and advantage server, 30-40 and advantage receiver and 30-30 and deuce) has a different importance so some points, eg 30-40 are much more important than other points eg 40-30. Interestingly the sum of the importances to the forehand side before deuce equals the sum of the importances to the backhand side, but as there must be more points to the forehand side the average importance to the backhand side is greater. Further, for each pair of points after deuce, the importance of the point at deuce to the forehand court is less than the average of the importance of the two possible points to the second court (advantage server and advantage receiver). For this reason doubles pairs often encourage the better player to play on the second court.

In comparison, under the alternative approach to importance, all points before deuce are equally important and for each pair of points thereafter, the reward for lifting declines.

Thirdly, it was shown that similar conclusions apply for sets in a match. Under the classical definition of importance, each possible set score has a different importance, with the fifth and final set clearly having the highest importance of 100% since it must finish the match. The next most important set is the fourth set when down 1-2, but thereafter the importance depends on the p-value. However, under the alternative approach to importance, it does not matter which set you lift, so each set is equally important. Obviously you could save the lift to the fifth set if needed, which may increase the expected length of the match, but does not affect the probability of winning.

Fourthly, games in a set were considered. This introduces the complication of two p-values, one for each player. Under the classical definition of importance, in a match between two equal players, the total importance of each pair of games (at 0-0 and 1-0 or 0-1, 1-1 and 2-1 or 1-2, etc.) increases as the score progresses until 4-4 where it is 0.5 and remains 0.5 at each set score thereafter until 6-6 where it jumps to 1 or 100% as the set is determined by a tiebreak game.

In comparison, under the alternative approach to importance, it was shown that it is better to lift before the score reaches 5-5. If you have not already lifted, you should certainly lift when receiving at 5-4 or 4-5. When serving at 5-3 you should lift, but if serving at 3-5 you should not lift, but wait until receiving at 4-5. Using the method of dynamic programming, a full table of when to lift, when definitely not to lift and when it makes no difference whether you lift or not, so presumably don’t, was obtained for the case where a player can lift once in a set. This can be extended to the situation where a player has the ability to make two or more lifts.

Fifthly consider points in a tiebreak game under the two approaches to importance. In a match between two equal players, the importance gradually rises from

0-0 (equal to 0-1 and 1-0) to 1-1 (equal to 1-2 and 2-1) and so on to 5-5, where it is 0.5 and remains 0.5 thereafter until the tiebreak is completed.

Under the alternative approach to importance, a player should definitely lift before 6-6 and would definitely lift in the twelfth point when serving regardless of whether the score is 6-5 or 5-6. When receiving at 6-4 he should not lift, but hold the lift until serving at 6-5. On the other hand, when receiving at 4-6 he should lift. A complete table of when to lift, when not to lift and when it makes no difference whether you lift of don't lift was obtained. Again, this can be extended to the case where a player can make two or more lifts.

Finally, it was shown that if, at the commencement of a match, a player wishes to lift in one (or more) sets over a constant p value in the other sets, it does not matter to the overall probability P of winning the match which set(s) the player elects to lift. Under this approach, all sets are equally important. Further, if the decision is made at the end of one of the sets during the match, it does not matter to toe probability of winning the match which of the remaining sets are selected to lift.

And it was also shown that if, at the commencement of a match, a player decides to lift his p-value in one set from p to p + δ and decrease it to p - δ in another set, the overall probability P of winning the match increases by $3pq(2p - 1)\delta^2$ compared to maintaining a constant p value. In other words, it is better to have variations in p than a constant p value.

The conclusions outlined above are useful for players and coaches.

CHAPTER 5

STATISTICAL TESTS FOR THE INDEPENDENCE OF POINTS

The first and most important assumption made in the mathematical and probabilistic modelling and the statistical analysis of tennis and other sports is that points are independent of what happened the previous point or what the score is, or any other factor. In other words, the server has a constant probability (p) of winning a point on service over a defined period (game or set or match depending on the analysis being carried out) regardless of whether the previous point was won or lost (i.e. independence). The server also has the same probability of winning a point on service whether the score is 40-00 (say) or 15-40 (say) (i.e. identically distributed). Thus, service points are assumed to be independent and identically distributed.

Of course this assumption is not true. It reminds me of the basic assumption in economic analysis that perfect competition exists. It doesn't, but it is surprising how much reasonable economic analysis can be done despite the flaw in the basic assumption. Likewise in probabilistic modelling in tennis. Assuming Player 1 has a constant probability p_1 of winning a point on his/her service and the opponent Player 2 has a constant probability p_2 of winning a point on his/her service, then it is possible to calculate the probability each player wins a game, a set or a match, as well as a range of other characteristics, such as the mean, variance and skewness of the number of points, exactly.

As a general rule one might expect that the better the player the more likely the assumption of independence between points and identical distribution would apply as they would be playing "flat out" on every point. However, there is also some evidence (see Literature and Data in section 5.1) that the best players can lift their play on important points against weaker players, so points would not be independent in these matches. Weaker players are perhaps more likely to react to or modify their play depending on the outcome of the previous point (especially after an ace or a double fault or a long rally) or depending on the score or the importance of the point. The best players are much more likely to be able to shut these types of ideas out of their minds and thus make one point independent of the outcome of the previous point and to maintain a constant probability of winning a point on service throughout a game or set or even a match. However, as p does not depend solely on the characteristics of the server, but also the quality of the receiver, it is unlikely p will remain constant from match to match against different players. In fact, it is unlikely to remain constant over a five set match against the same player, but for the purposes of this analysis it would appear reasonable to assume the probability remains constant over a set. It also follows that any analysis for elite players should be carried out separately for matches between roughly equal players (say Top Ten) and matches between a top player and a slightly weaker player (say ranked 11 to 100). Also these are the matches where data is much more likely to be available.

In section 5.1 we consider the literature on independence between points including my own work which has been independently published and forms the basis of this chapter as well as the challenges of obtaining point by point data.

Section 5.2 outlines four direct point by point measures that can be used to test the hypothesis that points are independent.

Section 5.3 outlines four indirect measures that can be used to see whether there is some idiosyncrasy with the data that could not have arisen if points are independent.

Section 5.4 uses the four direct measures of section 2 to analyse the independence between points for Top 4 and sometime world number one ranked player

Rafael Nadal against Top Ten players and against other top 100 players using 2011 Grand Slam data.

Section 5.5 uses the same four direct measures to perform a similar analysis of independence between points for Nadal when receiving service.

Section 5.6 uses the four indirect measures of section 3 to see what they reveal about the independence between points for Nadal when serving.

Section 5.7 uses the same four indirect measures to perform a similar analysis to see what they reveal about the independence between points for Nadal when receiving.

Section 5.8 looks at the same eight tests for independence for the other three Top Four players when they are serving and when they are receiving. Data are given for the first three tests which are the most sensitive and other measures only given when a significant (at the 5% level) result was found.

5.1. Literature and data

Despite the importance of the assumption of independence and identically distributed points on service, there is surprisingly little published research to prove or disprove the hypothesis. This is primarily due to the fact that point-by-point data is voluminous and rarely available. Where it was available it was only available for matches played on the centre and other main courts. Point by point data is now available on the web site www.oncourt.info but the data has some deficiencies such as missing points and condensed data after deuce. My initial attempt to acquire data was to obtain the umpires scorecards at the Australian Open, but this became a challenge at other Grand Slams and the data was incomplete in that a number of matches were missing.

Then in 2011 through "IBM Pointstream" they provided point by point data for all matches. IBM had become the official technology partner for each of the Grand Slam tournaments and through "IBM Slamtracker" they published a live statistical analysis of each match providing the usual key measures of percentage of first and second serves that went into play, percentage of points won on first and second serves, number of aces, number of double faults, number of break points won and lost, total points won and lost, etc. But in 2011 they also provided point by point data, but discontinued this from 2012 onwards. So we only have one year of complete and reliable Grand Slam point-by-point data. This is most disappointing coming from the world leaders in computing and tennis.

Using data from 481 men's and women's singles matches played at Wimbledon from 1992 to 1995 (only from main courts where data was then collected, but with some qualifications data is now available from all courts), Klaassen and Magnus (2001, 2014) showed, *inter alia,* that winning a point on service increases the probability of winning the next point, but the probability of winning a point on service decreases as the importance of the point increases. Hence points are not independent and identically distributed. They went on to show that top ranked players are close to independence and identical distribution and as a player's ranking drops, the less likely this assumption will be true.

Pollard (2004) showed that over the seven matches Agassi played in winning the 2004 Australian Open, Agassi was able to lift on selected points and thus not all points on his service were independent and identically distributed.

Pollard, Cross and Meyer (2004) utilized data from best of 5 set men's singles matches at the Grand Slams from 1995 to 2004 to show that the pattern of the ten possible scores in 5 set matches could not have happened if the probability of winning a set was constant. Consequently, they concluded that the better player can lift his play some of the time and in certain circumstances.

Pollard and Pollard (2013) suggested four direct and four indirect methods to identify independence, or lack of independence, for a server. Direct methods look specifically at the score or the result of the previous point or the importance of the point. Indirect methods look at various other statistics and whether these results could be achieved if points were independent and identically distributed. As an example of these methods, they looked specifically at Nadal's performance against other Top Ten players in 2011 Grand Slam singles matches.

Pollard (2013) extended this analysis by looking at Nadal and the other top four players, Djokovic, Federer and Murray, looking at their matches against Top Ten and against other players and also considering their performance as receivers. This analysis is addressed in this chapter, starting with the introduction of the measures using hypothetical examples for illustration purposes and going on to consider actual data for one of the world's top four players.

The domination of the top four players in 2011 was such that we have a wealth of data for each of them. After starting the year as number one ranked player in the world, Nadal was surprisingly beaten in the quarter finals of the Australian Open, but reached the finals of the other three Grand Slams, winning the French at Roland Garros. Nadal actually played 26 matches (out of a possible maximum of 28 matches), but two early round matches ended prematurely due to injury to his opponent and two other early round matches were missing from the data set, so the total data available is 75 sets of data from 22 matches.

Djokovic had a great year winning three of the four Grand Slam tournaments, but was beaten by Federer in the semi-finals of Roland Garros. By the time of the US Open he was ranked number one in the world and provides data from 91 sets in 27 matches. Murray was in one final and three semi-finals and provided data from 85 sets in 25 matches. Federer was in one final, two semi-finals and one quarter-final to provide 83 sets from 24 matches. These four players were so dominant that Ferrer in Australia and Tsonga at Wimbledon were the only other players to make a single semi-final. Each of the four players provide over 2000 points of data, which is sufficient to provides a good analysis of their point-by-point performance, even when divided into sub-categories (in particular matches against other top ten players and matches against other players ranked outside the top ten.)

5.2 Direct Measures

Four direct measures of a server's actual performance in certain situations compared to the theoretical result if the points are independent and identically distributed are considered to see whether the assumption of independence between points is applicable. The set is the principle component of the analysis as games are too short and best of five set matches are too long not to expect some variability. Generally, all that is required is a simple chi-squared test on the difference between the observed and expected results to see whether the assumption is valid, but simple sign tests of the difference can also be employed.

For the purpose of demonstrating the various measures we utilise the first set of the Roland Garros 2011 Men's Final between Rafael Nadal and Roger Federer which Nadal won in four sets, 7-5, 7-6, 5-7, 6-1. The method can then be extended to cover the four sets of the match, and ultimately all matches played by Nadal at the four Grand Slams in 2011. In the first set Nadal served 44 points and Federer 36 points. In the match Nadal served 142 points and Federer 121 points. For all matches in Grand Slams each of the four top players considered served over 2000 points which is sufficient data to make sound conclusions about the independence between points for each of these players.

5.2.1. Direct Measure 1 (DM1). Server ahead, equal or behind.

Firstly, consider the state of the game, namely whether the server is ahead (e.g. 30-00), equal (e.g. 15-15) or behind (e.g. 00-30). If the probability of winning a point on service is independent of the score, then the actual number of points won in each of the three situations (Ahead, Equal, Behind) can be compared to the expected number if the probability of winning a point on service (p) for the whole set was applicable on each and every point in that set.

For example, a server who has a probability p = 0.6 of winning a point on service and the points are independent and identically distributed has a probability 0.7357 of holding service and the expected duration of the game is 6.4842 points. But if his probability of winning a point on service increases to p = 0.7 when behind (lifts his game when behind), remains at p = 0.6 when equal and drops to p = 0.5 when ahead in the game (relaxes when ahead), then his probability of winning the game increases to 0.7537 and the expected duration of the game increases to 7.5948 points.

In the first set of the Roland Garros 2017 Final, Nadal won 8/16 points when serving while ahead in a game, he won 10/17 points when serving with scores equal in a game and 8/11 points when serving from behind in a game. His overall percentage of points won on service in this set was p = 26/44 so although in this set he appears to serve better when behind than when equal and to serve better when point scores are equal than when he was ahead, the differences are not significant at the 5% level with χ_2^2 = 1.39.

5.2.2 Direct Measure 2 (DM2). Server won or lost previous point.

Secondly, consider which player won the previous point. If the probability of winning a point on service is constant and independent of the previous point, then it should make no difference who won the previous point, the server or the receiver. For the purpose of this analysis it makes sense to delete the first point of each game as there is no previous point in that game and to relate the first point of one game to the last point of the previous service game, which was two games earlier (as the opponent has served in between), would appear to be not appropriate as the points are not successive like all the others in the analysis.

For example, in this case consider a player whose probability of winning the first point is p = 0.6 as before, but increases to p = 0.7 if he loses a point (lifts his game) and decreases to p = 0.5 if he wins a point (relaxes). The probability of holding service increases to 0.7523 and the expected duration of the game rises to 7.2005 points compared to 0.7357 probability and 6.4842 points respectively if p remains constant at 0.6 (ie independence).

Looking (again)at the first set of the 2011 Roland Garros Final, Nadal won 10/21 points having won the previous point, 12/17 points having lost the previous point, with an overall percentage of 22/38 (the first point of each of his six service games is not counted). So, although in this set Nadal served better after losing the previous point than after winning the previous point, the difference is not significant at the 5% level with χ_1^2 = 2.04.

5.2.3. Direct Measure 3 (DM3). Combined state (DM1) and stepwise (DM2).

Thirdly, consider a combination of the two previous analyses with three states of the game outcomes (Ahead, Equal and Behind) and two stepwise outcomes (Won or Lost the previous point). There are thus six possible situations (AW, AL, EW, EL, BW, BL using the obvious notation) and we can compare the actual frequencies with the expected frequencies under the assumption p for each point is independent and identically distributed.

Now consider the example of a server whose probability of winning a point on service is 0.6 except when he is ahead and has just won the previous point but he then relaxes and the probability p drops to 0.5. Conversely when he is behind and just lost the previous point he lifts his game and the probability of winning the next point rises to 0.7. His probability of winning the game now rises to 0.7521 and the expected game length rises to 7.4026 points compared to probability 0.7357 and expected duration 6.4842 respectively if the points are independent and p is constant at 0.6.

In the first set of the 2011 Roland Garros Final Nadal's performance was AW = 6/13, AL = 2/3, EW = 3/6, EL = 3/5, BW = 1/2, BL = 7/9. The overall percentage is 22/38 as in 5.2.2 above. It is not appropriate to calculate a χ^2 where expected frequencies in some categories are small as in this case, but the method can be used when more data are available such as the whole match, all matches in the tournament and all matches in the 2017 Grand Slams as shown later.

The most interesting comparison is between AW (ahead and won previous point) = 6/13 and BL (behind and lost previous point) = 7/9, but even this difference is not significant at the 5% level (χ_1^2 = 2.24).

5.2.4. Direct Measure 4 (DM4). Importance of point.

Fourthly, consider the importance of the point (e.g. 30-40 or 40-30). Morris (1977) defined the importance of a point within a game of tennis as the probability that the server wins the game given that he wins that point minus the probability that he wins the game given that he loses that point. Obviously 40-30 and advantage server have the same importance, 30-40 and advantage receiver have the same importance, and 30-00 and deuce have the same importance., so there are fifteen different levels of importance as shown in Table 5.1. Points in decreasing order of importance (for a representative p-value for men's singles) are given in Table 5.1 and the numerical values displayed are for p=0.6, a typical value for the probability of winning a point on service, and it is assumed that all service points are independent and identically distributed.

Score	Importance
30-40; Advantage Receiver	0.6923
30-30, deuce	0.4615
15-30	0.4431
15-40	0.4154
Love 30	0.3665
Love 15	0.3456
15-15	0.3323
40-30; Advantage Server	0.3077
0-0 (First point of game)	0.2659
30-15	0.2585
Love 40	0.2492
15 love	0.2127
30 love	0.1329
40-15	0.1231
40 love	0.0492

Table 5.1. Importance of each point for server with constant probability of winning point p=0.6

In this case, consider the example of a server who lifts his game on the more important points (say 15-15 and above in Table 5.1) and in doing so raises his p-value from 0.6 to 0.7, but relaxes on the less important points (40-30 or Advantage Server and below in Table 5.1) thereby reducing his p-value to 0.5, then the probability of winning service rises to 0.7979 and the game duration increases to 7.4528 points, again compared to the independent and identically distributed p=0.6 values of probability 0.7357 and duration 6.4842 points respectively.

With 15 different possible score categories it is clear that a χ^2 test is not possible for a set or a match or even a number of matches, but becomes possible if we have just the two categories of more important and less important points as suggested in the previous paragraph. In the first set of the 2011 Roland Garros Final, Nadal won 15/23 of the above more important points on service and 11/21 of the above less important points on service. This difference is not significant at the 5% level (χ_1^2 = 0.74.)

5.3. Indirect Measures.

Four indirect measures are proposed by looking at various statistical features of the server or the set and comparing those with what is expected if the probability of winning a point on service p is independent and identically distributed and thus p is constant throughout the set. Indirect measures are generally less powerful (statistically) than direct measures but have the advantage that they may pick up evidence of lack of independence that is not picked up by direct measures that are generally looking at just one or two specific characteristics.

5.3.1. Indirect Measure 1 (IM1). Number of games won on service

Firstly, consider the number of games won on service in a set compared to the expected number if p is constant and each point is independent of the other points and identically distributed. For this we need the probability P that a server wins his service game when the probability p of winning a point on service is constant throughout the set. Pollard and Pollard (2010) quote Kemeny and Snell (1960) that P is given be

$$P = p^4 (1 - 16q^4)/(p^4 - q^4) \qquad \text{where } q = 1 - p \tag{5.1}$$

Klaassen and Magnus (2014) give the formula as

$$P = p^4 (-8p^3 + 28p^2 - 34p + 15)/ ((p^2 + (1 - p)^2) \tag{5.2}$$

but these two equations can be shown to be identical when p is not equal to q.

Thus, if a player serves n games in a set, the actual number of times he holds service can be compared to the number of times he is expected to hold serve (nP). If the difference between actual and expected is significant, this suggests lack of independence between points, but it does not identify the specific cause of the lack of independence. However n can only vary between a minimum of three service games in a set to a maximum of six service games (excluding a tie-break game and advantage sets beyond 6-6). It is unlikely that any single observed number of service games won in a set will be significantly different from the expected under the assumption of independence.

Thus, in the first set of the 2011 Roland Garros Final, Nadal had a p value 26/44 = 0.591 and a P value of 0.7165 whereas Federer had a p value of 21/36 = 0.583 and thus a P value of 0.6999. It follows that Nadal expected to win 4.2988 of his six service games and Federer 4.1997 of his six service games. With a set score of 7-5, Nadal won five service games and Federer won 4 service games. Nadal did better than expected and Federer worse than expected (but neither is significant) based on their p values which are only slightly different. This could have occurred by chance or possibly Nadal performed better than Federer on the important points in each service game.

If we repeat these calculations for each set that Nadal played over the four Grand Slams in 2011, we can simply consider whether the difference is positive or negative and over a large number of sets consider the number of positive and negative variations against an expected equal number of 50% each if the points were independent.

5.3.2. Indirect Measure 2 (IM2). Average duration of a game

Secondly, consider the actual duration (number of points) of a game of tennis with the expected duration in points which is given by Pollard (1983)

$$\mu_1 = 4\,(p^4 + q^4 + 5s(p^3 + q^3) + 15s^2r^{-1} + 10s^3(3 + r))$$

$$\text{where } s=pq \text{ and } r^{-1} = 1-2pq \qquad (5.3)$$

The second non-central moment of the duration of a game of tennis is given by

$$\mu_2 = 16(p^4 + q^4) + 100s\,(p^3 + q^3) + 360s^2r^{-1} + 20s^3\,(36 + 24r + 4r^3\,(I + 2s)) \qquad (5.4)$$

and the variance of the duration of a game of tennis is given by $\mu_2 - \mu_1^2$

As expected the maximum value of μ_1 occurs when p = 0.5 giving $\mu_1 = 6.75$. Even when p = 0.6, a more reasonable value for a server, $\mu_1 = 6.484$

For Nadal in the first set of the 2011 Roland Garros Final, the estimated p value is 26/44 giving an estimated mean duration of 6.5284 and an estimated variance of 6.8698. Assuming the six service games are independent, the expected duration (number of points on Nadal's service) in the first set was 39.1701 and the variance was 41.2186. The actual duration was 44 points producing a standardized Z-value of 0.7523 which is not significant at the 5% level.

Given that the variance of the duration of a game of tennis is relatively large, a test for lack of independence based on the actual duration of a game or set will not be a powerful one. Instead a sign test can be applied (as in Indirect Measure 1) to a server's performance in each set indicating whether the set was longer or shorter than that expected assuming independence.

5.3.3. Indirect Measure 3 (IM3). Wald-Wolfowitz two sample runs test.

Thirdly, consider the number of runs of winning and losing points by applying the Wald-Wolfowitz two sample runs test. The number of runs has an approximate normal distribution (Seigel ,1955) with the mean given by

$$E(R) = 1 + 2n_1n_2/(n_1 + n_2) \quad (5.5)$$

and the variance given by

$$V(R) = 2n_1n_2(2n_1n_2 - n_1 - n_2) / (n_1 + n_2)^2 (n_1 + n_2 - 1) \quad (5.6)$$

where n_1 is the number of points won by the server and n_2 is the number of points lost by the server. The normal approximation is quite good when n_1 and n_2 are greater than 10, which generally applies in a set of tennis. Thus if R is the number of runs observed in the set, the normal test Z value is given by

$$Z = (R - E(R)) / V(R)^{1/2} \quad (5.7)$$

In the first set of the 2011 Roland Garros Final the relevant values for Nadal are $n_1 = 26$, $n_2 = 18$, $E(R) = 22.2727$, $V(R) = 10.0292$, $R = 26$ and $Z = 1.1769$ which is not significant at the 5% level. The equivalent figures for Federer are $n_1 = 21$, $n_2 = 15$, $E(R) = 18.5$, $V(R) = 8.25$, $R = 18$ and $Z = -0.1741$ which is not significant at the 5% level.

In virtually all sets the difference between observed and expected number of runs is not significant at the 5% level, but in a few cases the difference will be significantly above or below the expected. This is to be expected as each Top Four player is playing up to 28 matches over the four Grand Slams and around eighty sets. So again, this test is not likely to produce significant results applied this way. However, by applying a sign test to the difference between the expected and actual number of runs for the approximately eighty sets each player plays will be a stronger test.

5.3.4. Indirect Measure 4 (IM4) Distribution of set scores.

Fourthly, we can look at the distribution of set scores from 6-0, 6-1, 6-2 ……..7-6, 6-7 ……..1-6, 0-6. In this case we don't only need to know the p values of each player, but we also need to know which player served first in the set as this affects the likelihood of each set score. In simple terms and assuming the probability of winning a point on service is greater than 0.5 so the winning player effectively holds service throughout the set, the player who serves first is more likely to win 6- 1 (two breaks of service or a net result of two breaks) or 6-3 (one break of service or a net result of one break of service) than 6-2 (where the winning player must break at 5-2) or 6-4 (where the winning player must break at 5-4). The reverse applies where the winning player serves second and the score is more likely to be 6-2 or 6-4 than 6-1 or 6-3. Knowing whether the better player is serving first or second changes the odds for scores of 6-1, 6-2, 6-3 and 6-4 and so the five minute period between the toss for service and the start of play (the hit-up) represents an opportunity to bet on the score in the first set if the odds offered don't change during the hit-up. This is discussed in more detail in Section 2.15. With respect to evidence of independence or lack of it, when the observed median score exceeds the expected median score then the player has effectively lifted when it matters and thus the points are not independent.

5.4. Nadal Serving – Direct measures.

In sections 5.2 and 5.3 we developed four direct and four indirect measures to determine whether a points are independent and then calculated each measure for the first set of the final of Roland Garros 2011 between Nadal and Federer. Some differences were found, but as the data only covered one set of Nadal serving, it is not surprising that none of these differences were significant at the 5% level. We can expand this method to cover all sets of the match (same opponent), or all sets of the tournament (same court surface) or to maximise the data available, all sets played at the Grand Slams in 2011. This chapter now considers the eight measures for Raphael Nadal.

5.4.1. Direct measure 1-Nadal serving.

Over the four Grand Slams in 2011, point by point data are available for 22 completed matches for Nadal consisting of 75 sets, 729 games and 4376 points, of which 2132 points were on service and 2244 points were when receiving. As shown in Table 5.2, Nadal won 1410/2132 or 66.13% of points on service, ranging from 675/984 or 68.60% when ahead in the game, to 481/744 or 64.65% when points were equal down to only 254/404 or 62.87% when behind on service. Applying a chi-squared test, this slight progression is not significant at the 5% level ($\chi_2^2 = 5.32$).

PLAYERS	AHEAD		EQUAL		BEHIND		TOTAL		χ_2^2
	WIN	TOTAL	WIN	TOTAL	WIN	TOTAL	WIN	TOTAL	
TOP10	336	511	254	413	153	248	743	1172	2.17
	65.75%		61.50%		61.69%		63.40%		
OTHERS	339	473	227	331	101	156	667	960	2.85
	71.67%		68.58%		64.74%		69.48%		
TOTAL	675	984	481	744	254	404	1410	2132	5.32
	68.60%		64.65%		62.87%		66.13%		

Table 5.2. State dependent relative frequencies for Nadal when serving

There was some expectation that Nadal could lift against weaker players, so these data were split into 11 matches between Nadal and fellow top ten players (obviously the later rounds) and 11 matches between Nadal and lower ranked players (obviously the earlier rounds, but it is noted that these opponents would almost always still be an elite player ranked in the top 100, but occasionally could be a qualifier or a wild card ranked just outside the top one hundred). Over 11 matches against top ten players involving 40 sets and 1172 points on service, Nadal won 743 points or 63.4% of points. When ahead he won 65.75% of points, when equal 61.50%, and when behind 61.69% of points and these results are not significant at the 5% level (X_2^2 = 2.17). Over his 11 matches against other players involving 35 sets and 960 points, Nadal won 667 points or 69.48% of points played. When ahead he won 71.67%, when equal he won 68.58% and when behind he won 64.74% of points played, which is also not significant (χ_2^2 = 2.85). Overall this test of state of the game dependent relative frequencies does not support any significant variation from the assumption of independence between points by Nadal as a server.

5.4.2. Direct measure 2-Nadal serving.

The stepwise relative frequencies for Nadal when serving a point depending on whether he won or lost the previous point are shown in Table 5.3 for matches against fellow Top Ten players, against other players in the main draw of the 2011 Grand Slam tournaments and against all players in the Grand Slams. Over all matches (after eliminating the first point of the game as there is effectively no immediately preceding point on service), Nadal won 732/1114 or 65.7% of points after winning the previous point and 439/659 or 66.6% of points after losing the previous point, which is not significant at the 5% level. (χ_1^2 = 0.26)

Nadal won 66.0% of points on service against all players. He won 63.5% against Top Ten players and a significantly (χ_1^2 = 5.24) higher 69.2% against other players in the main draw of Grand Slams. There was no significant difference whether he had won the previous point (0.654) or lost it for both Top Ten (χ_1^2 =0.78) and for other players (χ_1^2 = 0.04)

Opponent	Previous	This Point	This Point	This Point	This Point
	Point	Win	Loss	Total	P Win
Top Ten	Win	373	224	597	0.625
$\chi_1{}^2$ = 0.78	Loss	254	136	390	0.651
	Total	627	360	987	0.635
Other players	Win	359	158	517	0.694
χ_1^2 = 0.04	Loss	185	84	269	0.688
	Total	544	242	786	0.692
All Players	Win	732	382	1114	0.657
χ_1^2 = 0.26	Loss	439	220	659	0.666
	Total	1171	602	1773	0.660

Table 5.3. Stepwise relative frequencies for Nadal when serving.

However, an alternative simple test is to consider for each of the 75 sets played set whether the percentage of points won after winning the previous point is greater than or less than the percentage of points won after losing the previous point. Under the null hypothesis you would expect roughly 50:50. Treating the rare cases where the percentages are equal as 0.5:0.5, there were 46.5 / 75 sets where Nadal won a greater percentage of points after losing the previous point and 28.5 / 75 sets where Nadal won a greater percentage of points after winning the previous point. This difference is significant (t_{74} = 2.05) and together the two tests suggest the difference must be small, but mostly positive in the direction of winning the next point after losing the previous point.

5.4.3. Direct Measure 3-Nadal serving.

The data are sufficiently large enough to consider the combined effects of state (ahead, equal or behind in the game) and stepwise (won or lost the previous point). There are six possible alternative outcomes and the relative frequencies for each category for Nadal when serving against Top Ten players, other main draw players and all players are shown in Table 5.4.

There was a significant difference between Nadal's winning percentages for the six possible situations against all players. (χ_5^2 = 11.45). In matches against the Top Ten players the differences are just significant at the 5% level (χ_5^2 = 11.08), but they are not significant against other players in the main draw (χ_5^2 = 7.45) .

Top Ten (χ_5^2 = 11.08)

State/Step	Win next point	Lose next point	Total	P Win
Ahead/Win	284	157	441	0.644
Ahead/Loss	51	20	71	0.718
Equal/Win	71	45	116	0.612
Equal/Loss	64	41	105	0.610
Behind/Win	18	22	40	0.450
Behind/Loss	139	75	214	0.650
Total	627	360	987	0.635

Other Players (χ_5^2 = 7.45)

State/Step	Win	Loss	Total	P Win
Ahead/Win	299	118	417	0.717
Ahead/Loss	39	16	55	0.709
Equal/Win	49	30	79	0.620
Equal/Loss	57	23	80	0.713
Behind/Win	11	10	21	0.524
Behind/Loss	89	45	134	0.664
Total	544	242	786	0.692

All Players (χ_5^2 = 11.45)

State/Step	Win	Loss	Total	P Win
Ahead/Win	583	275	838	0.679
Ahead/Loss	90	36	125	0.714
Equal/Win	120	75	195	0.615
Equal/Loss	121	64	185	0.654
Behind/Win	29	32	61	0.475
Behind/Loss	228	120	348	0.655
Total	1171	602	1773	0.660

Table 5.4. Combined state and stepwise relative frequencies for Nadal when serving.

5.4.4. Direct Measure 4-Nadal serving.

There are eighteen different point scores in tennis, but with only fifteen different levels of importance as shown in Tables 5.1 and 5.5. The relative frequency data for Nadal are considered insufficient to consider all fifteen different scores by importance, so a simple first alternative is to divide the scores into more important (say 00-15 and more important scores) and less important (say 15-15 and less important scores). Nadal won 234/385 = 0.608 more important points and 509/787 = 0.647 less important points, but the difference is not significant at the 5% level (χ_1^2 = 3.72). Only the Top Ten is shown in Table 5.5 as the tables for other players and all players were also not significant.

Top Ten

Score	Win	Loss	Total	P Win
30-40, adv. Receiver	49	26	75	0.653
30-30, deuce	84	57	141	0.596
15-30	26	21	47	0.553
15-40	17	12	29	0.586
00-30	13	11	24	0.542
00-15	45	24	69	0.652
15-15	51	33	84	0.607
40-30, adv. Server	65	39	104	0.625
00-00	119	69	188	0.633
30-15	54	23	77	0.701
00-40	9	3	12	0.750
15-00	79	41	120	0.658
30-00	53	25	78	0.679
40-15	44	28	72	0.611
40-00	35	17	52	0.673
Total	743	429	1172	0.634

Table 5.5. Importance of point relative frequencies for Nadal when serving.

5.5. Nadal serving – indirect measures

5.5.1. Indirect measure 1-Nadal serving

In his 40 sets against fellow Top Ten players, Nadal won more service games per set than that expected (if his probability of winning a point remained constant throughout the set i.e. points are independent and identically distributed)) a total of 28 times and 12 times he won less than expected. In 35 sets against other players in the main draw, Nadal won more games than expected in 23 sets and less than expected in 12 sets. Both these results are significant at the 5% level (t_{39} = 2.53 and t_{34} = 1.86 respectively), but over the 75 sets played against all opponents, Nadal won more games on service than expected in 51 sets and less in 24 sets and this is significant at the 5% level (t_{74} = 3.12). It follows that successive points are not independent and Nadal has the ability to lift his performance on service in certain games to outperform the result expected if he served consistently in each game in a set.

5.5.2. Indirect Measure 2- Nadal serving

In his 40 sets against fellow Top Ten players Nadal had three sets in which he played significantly ($z>1.96$) more points on service than expected on the assumption of independence, while there was no significant difference in the other 37 sets. Looking at the sign of the difference between the observed and expected there were 19 sets where the difference was positive and 21 sets where the difference was negative and this is not significant. (t_{39} = 0.32).

In his 35 sets against other players, there were 23 sets where the difference was greater than expected and 12 sets where the difference was negative and this is significant at the 5% level (t_{34} = 1.86). Combined for all players there were 42 positives and 33 negatives which is not significant (t_{74} = 1.04). So there is some evidence of a lack of independence against lower ranked players, but not against Top 10 players or overall against all players.

5.5.3. Indirect Measure 3- Nadal serving

In his forty sets against fellow Top Ten players Nadal had one set in which he had significantly ($z = 2.15$) more runs of winning points or of losing points than expected and one set in which he had significantly ($z = -2.13$) less runs than expected, but mostly there was no significant difference. Applying the sign test to the 40 sets there were 23 sets in which there were more runs than expected and 17 sets in which there were less sets than expected. This result shows no significant variation from independence. ($t_{39} = 0.95$) .

In his 35 sets against other players, there were 20.5 sets where the difference was greater than expected and 14.5 sets where the difference was negative and this is not significant ($t_{34} = 1.01$). Overall there were 43.5 out of 75 positive differences and this is not significant ($t_{74} = 1.38$)

5.5.4. Indirect Measure 4-Nadal serving

Looking at the first set of the 2011 French Open Final between Nadal and Federer, Nadal won 26/44 = 0.591 points on service and Federer won 21/36 = 0.583 points on service and Nadal won the set 7-5 and by two breaks of service to one. Federer served in the first game. Nadal normally elected to receive if he wins the toss whereas Federer and most other players normally elected to serve if they win the toss, so there was a high probability that Nadal would be receiving in the first game of this set of 2011 data. Over the last couple of years their preferences have changed and it is now common to see Nadal elect to serve if he wins the toss and Federer elect to receive if he wins the toss.

Using these estimates of p and the assumption of independence between points and applying the method of Pollard (1983), the probability of each set score can be calculated as well as the cumulative probability of the set score and these are shown in Table 5.6. It can be seen that the median score is 7-6 so that Nadal did slightly better by winning 7-5. Alternatively the end points for this outcome of 7-5 are 0.3852 and 0.4441 confirming that Nadal performed better than Federer when it mattered, despite the closeness of the two p-values.

This was repeated for the 40 sets against Top 10 players and Nadal exceeded expectation 25/40 = 0.625 times and this is not significant ($t_{39} = 1.58$). Likewise it was not significant for sets against other players and against all players.

Set Score	Probability	Cumulative Probability
6-0	0.0099	0.0099
6-1	0.0244	0.0343
6-2	0.0944	0.1287
6-3	0.0714	0.2002
6-4	0.1851	0.3852
7-5	0.0588	0.4441
7-6	0.0822	0.5263
6-7	0.0783	0.6046
5-7	0.0543	0.6589
4-6	0.0793	0.7382
3-6	0.1539	0.8921
2-6	0.0516	0.9437
1-6	0.0485	0.9922
0-6	0.0078	1.0000

Table 5.6. Probability and Cumulative Probability for the first set score in the Nadal-Federer 2011 French Open Final

5.6. Nadal Receiving – Direct Measures

5.6.1. Direct measure 1-Nadal receiving

The methodology used in considering Nadal's performance as a server can also be applied to Nadal's performance when receiving to see whether there is independence between points. Table 5.7 shows the data for Nadal when he is receiving and he is ahead, or the score is equal or he is behind. Nadal played slightly more points when receiving than when serving and overall won 960/2244 = 42.78%. When ahead in a game Nadal won 289/614 = 47.07% of the points, when equal he won 342/785 = 43.57% of the points and when behind he won 329/ 845 = 38.94% of points. Nadal performs significantly better (χ_2^2 = 9.92) at receiving when he is ahead in the game than when he is equal or behind in the game.

PLAYERS	AHEAD		EQUAL		BEHIND		TOTAL		χ_2^2
	WIN	TOTAL	WIN	TOTAL	WIN	TOTAL	WIN	TOTAL	
TOP10	125	272	157	403	173	481	455	1156	7.31*
	45.96%		38.96%		35.97%		39.36%		
OTHERS	164	342	185	382	156	364	505	1088	2.80
	47.95%		48.43%		42.86%		46.42%		
TOTAL	289	614	342	785	329	845	960	2244	9.92*
	47.07%		43.57%		38.95%		42.78%		

Table 5.7. State dependent relative frequencies for Nadal when receiving.

Again consider the independence when Nadal is receiving when ahead, equal or behind when playing against other Top 10 players and against lower ranked players. As expected Nadal performs better against the lower ranked players (wins 46.42% of points) than against Top 10 players (wins 39.39% of points). Interestingly he performs significantly better (χ_2^2 =7.31) against Top 10 players when he is ahead than when he is equal or behind, but there is no significant difference (χ_2^2 = 2.80) in his performance against other players when he is ahead, equal or behind.

5.6.2. Direct measure 2 –Nadal receiving

The stepwise relative frequencies from one point to the next for Nadal when receiving against Top Ten, other players in the main draw and against all players are given in table 5.8. Overall Nadal won 795/1879 = 0.423 points but he won significantly more points against other players 413/924 = 0.447 than he did against Top Ten players 382/955 = 0.400. (χ_1^2 = 4.25**).**

If he had won the previous point, Nadal won 361/830 = 0.435 of the next points against all players, whereas he won 434/1049 = 0.414 of the next points if he had lost the previous point, but the difference was not significant (χ_1^2 = 0.82). Likewise the results against Top Ten and against other players in the draw were not significant when measured against whether he won or lost the previous point (χ_1^2 = 0.14 and 0.77 respectively)

	Previous	This point	This point	This point	This point
	Point	Win	loss	Total	P win
Top Ten	Win	162	236	398	0.407
χ_1^2 = 0.14	Loss	220	337	557	0.395
	Total	382	573	955	0.400
Others	Win	199	233	432	0.461
χ_1^2 = 0.77	Loss	214	278	492	0.435
	Total	413	511	924	0.447
All players	Win	361	469	830	0.435
χ_1^2 = 0.82	Loss	434	615	1049	0.414
	Total	795	1084	1879	0.423

Table 5.8.Stepwise relative frequencies for Nadal when receiving.

5.6.3. Direct measure 3-Nadal receiving.

Table 5.9 gives the relative frequencies for the six possible state and stepwise outcomes for Nadal as a receiver. Overall Nadal won 795/1879 = 42.3% of points when receiving and performed significantly better against other players in the draw 413/924 = 44.7% than against Top Ten players 382/955 = 40.0% (χ_1^2 = 4.25).

Although Nadal averaged winning 42.3% of points overall while receiving, the figures for six possible state and stepwise situations vary between a minimum of 36.8% (BW) and a maximum of 50% (EL) and this is significant at the 5% level. (χ_5^2 = 16.86). Surprisingly the results for Nadal serving to both the Top Ten and to other opponents' show similar trends but were not significant at the 5% level (χ_5^2 = 8.81 and 7.49 respectively). The differences between observed and expected frequencies for all six categories were consistent and so the larger combined data produced a significant result. The Ahead/Win and Equal/Loss produced a positive difference between observed and expected number of wins, while the other four categories, especially Behind/Win, produced a negative difference. This applied to Nadal serving to Top Ten and to other players as well as the combined figures. The above result suggests that Nadal wins more points than expected when he is receiving and is ahead and just won the previous point or when the score is equal and he has just lost the previous point. He loses more point than expected in all other situations, but especially when he is behind and has won the previous point.

Top Ten (χ_5^2 = 8.81)

State/Step	Win	Loss	Total	P Win
Ahead/Win	100	121	221	0.452
Ahead/Loss	18	28	46	0.391
Equal/Win	41	69	110	0.373
Equal/Loss	48	52	100	0.480
Behind/Win	21	46	67	0.313
Behind/Loss	154	257	411	0.375
Total	382	573	955	0.400

Other players (χ_5^2 = 7.49)

State/Step	Win	Loss	Total	P Win
Ahead/Win	137	144	281	0.488
Ahead/Loss	23	35	58	0.397
Equal/Win	40	61	101	0.396
Equal/Loss	61	57	118	0.517
Behind/Win	22	28	50	0.440
Behind/Loss	130	186	316	0.411
Total	413	511	924	0.447

All players (χ_5^2 = 16.86*)

State/Step	Win	Loss	Total	P Win
Ahead/Win	237	265	502	0.472
Ahead/Loss	41	63	104	0.394
Equal/Win	81	130	211	0.384
Equal/Loss	109	109	218	0.500
Behind/Win	43	74	117	0.368
Behind/Loss	284	443	727	0.391
Total	795	1084	1879	0.423

Table 5.9. Combined state and stepwise relative frequencies for Nadal when receiving

5.6.4. Direct measure 4-Nadal receiving

This table is the same format as Table 5.5 and shows no significant variation from independence and would benefit from a larger data base because of the fifteen different score importances.

5.7. Nadal receiving – indirect measures

5.7.1. Indirect measure 1-Nadal receiving

In his 40 sets serving against fellow Top Ten players Nadal won more games as a receiver than expected (assuming that his probability of winning a point remained constant throughout the set) a total of 21.5 times and less than expected in 18.5 sets (t_{39} = 0.47) , assuming those sets where observed equals expected (e.g. p = 0.5 and he breaks 2/4 or 3/6 games). In his 35 sets against other main draw players under the same assumptions Nadal won 16 more games and 19 less games than expected (t_{35} = 0.510). Neither of these results are significant at the 5% level. Against all players Nadal won 27.5 more times than expected and also lost 27.5 more times than expected and this is clearly not significant. It follows that under this test there is no evidence of lack of independence between points when Nadal is receiving.

5.7.2. Indirect Measure 2-Nadal receiving

In his 40 sets receiving against fellow Top Ten players there was no set in which Nadal played significantly more or significantly less points than expected at the 5% level. However in just 12 of those sets was the observed number of points less than expected and in 28 sets was the number of point more than expected under the assumption of independence. This result is significant at the 5% level (χ_{39}^2 = 2.53) suggesting or rather confirming that Nadal is a great receiver, certainly against other Top Ten players in the later rounds of Grand Slams. In the 35 sets against other players, Nadal had more runs than expected in 20 sets and less runs than expected on 15 occasions and this is not significant at the 5% level (t_{34} = 0.90). In the total 75 sets, 48 had more runs than expected and 27 has less than expected and this is significant at the 5% level (t_{74} = 2.42).

5.7.3. Indirect Measure 3 –Nadal receiving

In his forty sets receiving against fellow Top Ten players Nadal had one set in which he had significantly more runs of winning points or losing points than expected and one set in which he had significantly less runs than expected, but mostly the difference was not significant. Applying the sign test to the 40 sets there were 19 sets where the observed exceeded the expected number of runs, 19 sets where the reverse applied and two sets where the observed equalled the expected. This result is clearly not significant and so shows no variation from the assumption of independence.

In his thirty five sets against other players there were 20 sets which had more runs than expected and 15 sets with less runs than expected and this is not significant at the 5% level(t_{34} = 0.85). The combined result 40 more and 35 less than expected assuming independence is also not significant (t_{74} = 0.58)

5.7.4. Indirect Measure 4-Nadal receiving

As this measure involves set scores it involves both players probabilities of winning a point on service and is the same as 5.6.4.

5.8. A look at the other Top Four players

The data available for Nadal was selected for applying the four direct and four indirect tests for independence because he was the number one ranked player in the world at the beginning of 2011 (the year when the Grand Slam data utilised in this analysis was available). However the analysis would not be complete without looking at the other three Top Four players, Federer, Djokovic and Murray, all of whom are right-handed players. The analysis is restricted to these four players because they have dominated the game for many years and their Grand Slam performance ensures there are over 20 matches and around 80 sets for each player (all during the same time period), which is way above that available for any other player from the point-by-point data required for this analysis.

Although all four direct and four indirect tests were applied for each player, this Section 5.8 only presents the data analysis for the first three direct tests, which are clearly the most sensitive tests because of their direct nature of measuring independence and are sufficient to show that points are not always independent, even for the top players. The direct test considering importance of the point would benefit from a larger data set because of the number of different point scores to be considered. Likewise, the four indirect tests would benefit from a greater data set as they are not directly measuring independence. Hence it is not surprising that they all showed no significant variation from independence and the results are not included here.

5.8.1. Murray

The 2011 Grand Slam data provided results for 24 matches for Andy Murray and 9 of these were against fellow Top Ten players and 15 were against other lower ranked players 11-100. Looking at Table 5.10 it can be seen that in matches against the Top 10 the value of χ_2^2 = 3.18 is not significant at the 5% level so points are independent. However, in matches against other players, the value of χ_2^2 = 8.56 is significant at the 5% level and so the points are not independent. It can be seen that Murray serves significantly better when ahead or equal than when behind. This is also true overall (χ_2^2 = 10.50) because of the greater amount of data for other players and because in matches against Top Ten the direction was similar, but not significant.

PLAYERS	AHEAD		EQUAL		BEHIND		TOTAL		χ_2^2
	WIN	TOTAL	WIN	TOTAL	WIN	TOTAL	WIN	TOTAL	
TOP 10	275	427	212	336	118	204	605	967	3.18
	64.40%		63.10%		57.84%		62.56%		
OTHERS	446	656	338	483	157	263	941	1402	8.56*
	67.99%		69.98%		59.70%		67.12%		
TOTAL	721	1083	550	819	275	467	1546	2369	10.50*
	66.57%		67.16%		58.89%		65.26%		

Table 5.10. State dependent relative frequencies for Murray when serving.

Next we look at Murray's performance depending on whether he won or lost the previous point in that game, noting that we eliminate the first point in each game as there is no relevant preceding point. All the values of χ_1^2 shown in Table 5.11 are not significant at the 5% level, and so we can conclude that Murray's performance is independent of whether he won or lost the previous point.

Opponent	Previous Point	This point Win	This point Loss	This point Total	This point P Win
Top Ten	Win	306	176	482	0.635
$\chi_1^2 = 0.93$	Loss	195	129	324	0.602
	Total	501	305	806	0.622
Other players	Win	498	237	735	0.678
$\chi_1^2 = 0.508$	Loss	279	145	424	0.658
	Total	777	382	1159	0.670
All Players	Win	804	413	1217	0.661
$\chi_1^2 = 1.48$	Loss	474	274	748	0.634
	Total	1278	687	1965	0.650

Table 5.11. Stepwise relative frequencies for Murray when serving.

Having looked at whether Murray won or lost the previous point and whether he was ahead, equal or behind, we can now look at his performance for the six joint state and stepwise situations. We can see from the χ_5^2 values given in the table that there is no significant difference when serving to Top Ten players, so points are independent, but there is when serving to other players and to all players. We already know from Table 5.12 that points are not independent and that Murray serves better when ahead or equal than when behind in a game, but we can now see that it is when he is both behind and lost the previous point that he really underperforms. In fact, observed exceeds expected in all five State/Steps except Behind/Loss.

Top Ten ($\chi_5^2 = 5.45$)

State/Step	Win next point	Lose next point	Total	P Win
Ahead/Win	233	132	365	0.638
Ahead/Loss	42	20	62	0.677
Equal/Win	50	34	84	0.595
Equal/Loss	58	33	91	0.637
Behind/Win	23	10	33	0.697
Behind/Loss	95	76	171	0.555
Total	501	303	806	

Other players ($\chi_5^2 = 13.69^*$)

State/Step	Win next point	Lose next point	Total	P Win
Ahead/Win	388	188	576	0.674
Ahead/Loss	58	22	80	0.725
Equal/Win	76	33	109	0.697
Equal/Loss	98	32	130	0.754
Behind/Win	34	16	50	0.680
Behind/Loss	123	91	214	0.575
Total	777	382	1159	0.670

All players ($\chi_5^2 = 17.65^*$)

State/Step	Win next point	Lose next point	Total	P Win
Ahead/Win	621	320	941	0.660
Ahead/Loss	100	42	142	0.704
Equal/Win	126	67	193	0.653
Equal/Loss	156	65	221	0.706
Behind/Win	57	26	83	0.687
Behind/Loss	218	167	385	0.566
Total	1278	687	1965	0.566

Table 5.12. Combined state and stepwise relative frequencies for Murray when serving

We can now repeat the three tests for Murray as a receiver. Again Murray's performance against Top 10 is not significant, but against others and overall is significant and thus points are not independent. Murray performs above average when he is ahead, average when points are equal and below average when he is behind.

PLAYERS	AHEAD		EQUAL		BEHIND		TOTAL		χ_2^2
	WIN	TOTAL	WIN	TOTAL	WIN	TOTAL	WIN	TOTAL	
TOP10	86	204	124	338	152	427	362	969	2.04
	42.16%		36.69%		35.60%		37.36%		
OTHERS	227	425	239	505	213	512	679	1442	13.05*
	53.41%		47.33%		41.60%		47.09%		
TOTAL	313	629	363	843	365	939	1041	2411	18.20*
	49.76%		43.06%		38.87%		43.18%		

Table 5.13. State dependent relative frequencies for Murray when receiving.

Direct tests 2 and 3 show the same result that against other Top 10 players Murray's returns of service are independent, but against other players (and against all players overall) the Chi-squared values are significant and thus his returns of service are not independent. For simplicity we just look at the performance against others. Using Test 2 we see that having won the previous point, Murray won 291/573 = 0.508 points, but having lost the previous point Murray won 276/633 = 0.436 points and this is significant at the 5% level (χ_1^2 = 6.23). Then looking at the six state/stepwise frequencies, we have AW = 195/360 = 0.542, AL = 32/65 = 0.492, EW = 63/132 = 0.477, EL = 0.467, BW = 33/81 = 0.407 BL = 180/431 = 0.418 and this is significant at the 5% level (χ_5^2 = 13.56). The major contributors to the lack of independence are AW and BL, so that Murray exceeds expectations (assuming independence) when he is ahead and won the previous point and below expectations (assuming independence) when he is behind and lost the previous point. Overall it seems that Murray is a very good "front-runner" whereas Nadal is a good "back to the wall" player, at least in 2011 when the data was collected.

5.8.2. Federer and Djokovic

The 2011 Grand Slam data provided results for 23 matches for Federer and a total of 2193 points. Twelve of these matches were against Top Ten players and eleven were in earlier rounds against other players who would have been ranked between 11 and 100. Looking at Table 5.14 it can be seen that the matches against Top Ten players were longer and tougher, noting that the matches against Top Ten players took 1254 points on service compared to 939 against other players and the win percentage of points was 68.34% against Top Ten compared to 73.9% against others for an overall win percentage of 70.73%.

Table 5.14 shows Federer's performance depending on the state of the service game, namely whether he was ahead, equal or behind in the game when the point was played. The data show a significant variation from independence with a χ_1^2 value of 8.85 overall, 7.44 for matches against the Top 10 and 9.64 for matches against other players, all of which are significant at the 5% level. We conclude that Federer performs better when ahead or at least equal in a service game and worse when behind, so points are not independent of the state of the game.

PLAYERS	AHEAD		EQUAL		BEHIND		TOTAL		χ_2^2
	WIN	TOTAL	WIN	TOTAL	WIN	TOTAL	WIN	TOTAL	
TOP 10	431	600	283	426	143	228	857	1254	7.44*
	71.83%		66.43%		62.72%		68.34%		
OTHERS	393	547	237	296	64	96	694	939	9.64*
	71.85%		80.07%		66.67%		73.91%		
TOTAL	824	1147	520	722	207	324	1551	2193	8.65*
	71.84%		72.02		63.89		70.73		

Table 5.14. State dependent relative frequencies for Federer when serving.

But this was the only test for Federer which produced a significant result. All other tests, direct and indirect, serving and receiving, were not significant at the 5% level, so for Federer the assumption of independence between points seems reasonable. For those who have watched Federer play, he somehow seems to play every point to his full capacity and it was not surprising to find this result.

Interestingly, the same can be said for Djokovic where all tests were not significant at the 5% level, so he demonstrates complete independence between points. This is also not surprising as in 2011 he won three of the Grand Slams and finished the year ranked number one in the world. His data contains only one loss, Nadal three losses while Federer and Murray had four losses on their data.

5.9. Summary and Implications.

In the mathematical and probabilistic modelling of tennis and other sports, the usual basic assumption is that all points are independent and identically distributed. It is also known that this is not true, but is a reasonable assumption to make good approximations for estimates of the probability a player wins a match or the mean number of points that will be required, the variance of the number of points and other characteristics. But to really look at the independence between points you need point-by-point data, and this is not readily available and only summary data is now published.

This chapter presents four direct ways and four indirect ways that independence between points, or lack of independence, can be investigated, and then conducts those tests for the four top male players in the world using point-by-point data for the Grand Slams 2011. All tests for Djokovic, who won three of the four Grand Slams that year, showed independence. Only one Test for Federer showed any lack of independence, while a number or tests showed lack of independence for Nadal and Murray. This is consistent with the generally held view that the better the player, the more each point is independent of what happened on the previous point or other factors that may be relevant for lower ranked players. More point-by point data for lower ranked players would be required to determine what their non-independent characteristics are and thus how you can use this information when playing against them, or betting on them.

CHAPTER 6

EFFICIENCY

In his very elegant paper in the Methodology Section of the Journal of the Royal Statistical Society, Miles (1984) set up a method for measuring the relative efficiency (as discussed in Section 6.1 and defined in Section 6.2) of various tennis scoring systems. He noted the link between singles tennis scoring systems and the statistical theory of sequential hypothesis testing, and he hypothesised that a particular scoring system using the "play-the-loser" structure would be optimally efficient in the singles tennis context. Pollard (1986, 1992) proved that this was in fact the case, whilst devising a 'spectrum' of tennis singles scoring systems.

Beginning in 2008, Pollard and Pollard have made several developments in the measurement of efficiency in sports scoring systems, namely

1. Pollard and Pollard (2008, 2010) extended Miles' theory of the efficiency to the tennis *doubles* situation in which there are basically 4 parameters, rather than just the 2 parameters in tennis singles. In this paper they indicated how this theory of efficiency could be extended even further to include two teams of doubles players, where there can be 8 or 16 parameters. The elements of this paper are discussed in Section 6.4 of this chapter.
2. Pollard and Pollard (2010) extended the theory of efficiency from the requirement of independence between points to the situation in which point probabilities can be dependent on the outcome (point won or lost) of the previous point. This is discussed in Section 6.5.
3. Pollard and Pollard (2012, 2015) extended the theory of relative efficiency to the situation in which the sport and the scoring system may comprise a sequence of 'points' which are possibly dependent and have varying probabilities. They did this by using an 'extrapolation' approach, rather than the 'interpolation' approach of Miles. An example of this might be two quite different types of game structure and associated scoring as for example in the game of darts. This extension is discussed in Section 6.6.

4. Pollard and Pollard (2013a, 2013b) analysed the ATP/WTA year-end events ("Masters") of eight players competing in two roughly equal round robins of four players followed by a four person knock-out between the two best players in each round robin. Using eight different definitions of "better" they found a marginally better system than that currently being used. In a second paper they also analysed the knock-out and round robin tournament structure and developed a new structure called a "partial round robin". This is discussed in Section 6.7.
5. Pollard, Meyer and Pollard (2015) considered the problem of measuring the efficiency of different tournament structures. They introduced the "draw and process" and the "partial round robin" to tennis and showed that the knockout format is the most efficient, followed by the draw and process, partial round robin and finally the round robin. This is discussed in Section 6.8.

As a further possible extension, Pollard (2017a, 2017b) has defined the excitement of a point in tennis, and has given as an example the famous fourth set between Borg and McEnroe in the 1980 Wimbledon Men's Final. As a further extension, he has noted a relationships between excitement, entropy, importance and the efficiency of hypothesis testing in statistics (Pollard, 2017b). This paper includes four new, interesting and very general theorems in sports scoring systems.

6.1. The search for efficiency in tennis scoring systems and tournaments

The probability player A defeats player B depends primarily on the probability player A wins a point on service and the probability player B wins a point on service, but it also depends on the scoring system being used. The probability player A goes on to win the tournament or event depends on the above factors for each match played, but also depends on the structure of the event which could be the traditional seeded knock-out single elimination tournament or a round robin event or some combination of both depending on the number of players involved. This chapter looks at the efficiency of various scoring systems and the efficiency of various tournament structures. Efficiency of the scoring system was never an issue in tennis until the sport became open to both amateurs and professionals in 1968 and became widely televised. Efficiency of the tournament structure was never really an issue in tennis because the traditional knock-out tournament was universally used and was already very efficient, but efficiency should always be considered in conjunction with maintaining fairness in the scoring system or the tournament structure.

Throughout the amateur era (up to 1968) tennis matches were either the best of five advantage sets or three advantage sets. Five sets were used at major events, such as the Grand Slams and Davis Cup, but were also used in many other important tournaments, particularly for the final, and for men only. The best of three advantage sets was used for women's matches at all levels of tournament and for men at lower level tournaments. Tennis has a three-tiered scoring system with points making up a game, games making up a set, and sets making up a match. Despite its scoring system of love, 15, 30, 40, deuce and advantage server or receiver, the game in tennis is effectively a best of six points, but if the score reaches 3-3 (deuce), play continues until one player leads by two points. A set is the best of ten games, but if the score reaches 5-5, play continues until one player leads by two games. A match is simply the best of three or five sets, with no requirement to lead by two. This scoring system lasted unchanged for nearly one hundred years and its efficiency was never queried. The length of a match was effectively unrestricted. The uncertain and quite variable length of a match was seen as an integral characteristic of the game of tennis to be enjoyed or tolerated (depending on your point of view) by players, officials and spectators.

In 1968 the distinction between amateur and professional players was eliminated and all tournaments were opened up to all players without restriction, with prizemoney an integral component of tournament structure and importance. Television coverage was an important component of a successful tournament as this led to sponsorship and continuing increases in prizemoney. For programming purposes, television wanted to know the approximate length of matches. This was available for other televised sports such as football, but not tennis. So began a series of changes or optional alternative scoring systems designed to reduce the mean and variance of the duration of a match in tennis, but also designed to have minimal effect on the probability the better player won the match. Tennis continues to address the efficiency of its tennis scoring systems but without making too many dramatic changes to the unique scoring system that remains a feature of the game.

The first significant change to the scoring system was the introduction of the tie-break when the game score in a set reached 6-6. This was initially introduced as a best of nine points or first to five with no requirement to lead by two points, but this was replaced by best of twelve points, but if the score reaches 6-6, play continues until one player leads by two points. Today all tournaments use best of three or best of five tie-break sets for singles, except three of the Grand Slams which retain an advantage set for the final third or fifth set respectively. On the other hand, men's doubles, women's doubles and mixed doubles have gone through a number of changes that continue to reduce the mean and variance in the length of matches. Seven different scoring systems are currently used in doubles or fourteen if you have a choice of playing an advantage game at deuce or a no advantage game (single point) at deuce. These scoring systems are considered further in Section 6.3. The efficiency of scoring systems was analysed by Miles (1984) and this analysis was extended by Geoff and Graham Pollard (2008, 2010) and by Graham and Geoff Pollard (2010, 2012, 2013 and 2015) and these extensions are considered in Sections 6.4 to 6.7.

Tennis tournaments are all played on a seeded, knock-out, single elimination draw. The only exception is the year-ending events on the women's WTA and men's ATP Tours (these events previously known as the Masters) which consist of eight players divided into two roughly equal groups of four who play a round-robin with the top two players in each group playing a four-person knock-out tournament to determine the winner. This is a compromise between the knock-out format, which would have too few matches for the tournament organisers, and the 8 person round-robin, which would have too many matches. The knock-out appears to be the most efficient structure, at some compromise to the probability that the best player wins. The round-robin has a higher probability that the best player wins, but involves the most number of matches and is clearly less efficient.

The challenges of considering the efficiency of various tournament structures were summarised by McGarry and Schultz (1997) as "……. while we may have an optimal scoring system to determine a game or match outcome between two players in a particular sport, we do not know the most efficient method to select ……… " They concluded that "no single best tournament structure exists." Appleton (1995) praised the seeded "draw and process" as possibly the best, but it has never been tried in tennis. Pollard and Pollard (2013) analysed the Masters tournament structure and developed a partial round robin method. Pollard, Meyer and Pollard (2015) extended the work of Miles (1984) for scoring efficiency to cover the relative efficiency of the various tournament structures mentioned above and these extensions are covered in Sections 6.7 and 6.8.

6.2. Defining Efficiency

For any scoring system and assuming independence between points, it is possible to calculate the mean, variance and skewness of the number of points played and the probability that the better player wins. If two different scoring systems have the same probability of correctly identifying the better player, one system can be said to be more efficient than the other system if it has a smaller expected number of points played. In tennis, despite the complexity of the scoring system, mathematically we effectively have just two types of scoring systems being used during a match. In looking at points within a game or sets within a match we only have one p-value to consider, namely the probability that the server wins a point on service or the probability that the better player wins a set, respectively. In looking at games within a set or points within a tie-break game there are two probabilities, respectively the probabilities player A and player B hold service and the probabilities player A and player B win a point on service. Miles (1984) considered the efficiency of such scoring systems and called the first category unipoints as they only involved one value of p (generally $p>0.5$ for the better player) and he called the second category bipoints as they involved two probabilities p_a and p_b when A and B respectively are serving, and if A is the better player $p_a > p_b$.

6.2.1 Unipoints

In his paper, Miles (1984) considered 'win-by-n' (W_n) scoring systems whereby the winner was the first player to lead by n points, which is to win n more points than the opponent. Assuming points are independent and player A has a constant probability p of winning each point, Miles (1984) showed that the probability P that player A wins by n points and the expected number of points played (μ) satisfy the (P, μ, n) equation

$$(P - Q)/\mu = (p - q)/n \tag{6.1}$$

and the ratio P/Q given by

$$(P / Q) = (p/q)^n \ , \tag{6.2}$$

where Q = 1 – P and q = 1 – p.

Equation (6.2) can also be written

$$P = p^n / (p^n + q^n) \tag{6.3}$$

Taking logs of equation (6.2), it follows that

$n = \ln(P/Q)/\ln(p/q)$ and hence

$$\mu = \frac{(P-Q)\ln(P/Q)}{(p-q)\ln(p/q)} . \tag{6.4}$$

Pollard (1986,1992) showed that these equations follow from the fact that for many W_n scoring systems, the ratio of the probability that player A wins in n + 2m points divided by the probability player A loses in n + 2m points is constant. He called this property the 'constant probability ratio' property. Wolfowitz (1948) showed that the optimally efficient statistical test for testing whether a Bernoulli probability was greater or less than 0.5 was the W_n scoring system and Miles (1984) gave this the optimally efficient family of W_n scoring systems an efficiency of unity (1 or 100%), and showed that the efficiency ρ of a general unipoints scoring system (SS) with the key characteristics P and μ is given by

$$\rho = \frac{(P-Q)\ln(P/Q)}{\mu(p-q)\ln(p/q)} \tag{6.5}$$

As described by Miles (1984), this efficiency measure of a unipoints scoring system SS is defined as the expected duration of the 'interpolated' W_n system with the same P-value as SS (as derived from the above '(P, μ, n) equation' divided by the expected duration of SS, which is μ. Utilizing the constant probability ratio of this W_n system, the value of n for this 'interpolated' W_n system is given by

$$n = \ln(P/Q)/\ln(p/q). \tag{6.6}$$

In comparing two different scoring systems with the same p (and q) values, the terms involving p (and q) can be ignored, so that the relative efficiency of scoring system one to scoring system two, using the obvious notation for each scoring system is

$$\frac{((P_1-Q_1)/\mu_1)\ln(P_1/Q_1)}{((P_2-Q_2)/\mu_2)\ln(P_2/Q_2)} \tag{6.7}$$

6.2.2. Bipoints

Miles (1984) also considered the tennis situation where there are two types of points depending on which player is serving. (This situation is also applicable to other sports such as volleyball.) Assume the probability player A wins a point on service is p_a and the probability player B wins a point on service is p_b. Player A is a better player than player B if $p_a > p_b$.

Noting that Wald (1947) recommended the use of paired trials when comparing two binomial probabilities, Miles (1984) used a point pairs set up W_n (point pairs) of playing pairs of points consisting of an A serving point and a B serving point and the winner is the first player to lead by n point pairs. This family of bipoint scoring systems, W_n (point pairs) where n = 1, 2, 3, …….. , which has efficiency of one, is the standard scoring system against which the efficiency of any tennis scoring system is compared. He showed that the efficiency ρ of any bipoints scoring system with expected number of points played equal to μ and the probability that the better player wins equal to P, is given by

$$\rho = \frac{2(P-Q)\ln(P/Q)}{\mu(p_a - p_b)\ln(p_a q_b / p_b q_a)} \tag{6.8}$$

where $Q = 1 - P$, $q_a = 1 - p_a$, and $q_b = 1 - p_b$

Again, when comparing two scoring systems with the same p_a and p_b values, the relative efficiency of scoring system 1 to scoring system 2 is given by

$$\frac{((P_1 - Q_1)/\mu_1)\ln(P_1/Q_1)}{((P_2 - Q_2)/\mu_2)\ln(P_2/Q_2)} \tag{6.9}$$

6.3. Characteristics of various men's doubles scoring systems including efficiency

While the interest in singles in tennis continues to rise, the interest in men's doubles has decreased and most of the top ten singles players never or rarely play doubles. Initially this was a feature of the men's game, but this trend is increasingly being replicated in the women's game. To address these issues, both the ATP and WTA Tours have utilised various alternative scoring systems designed to shorten the mean and variance of the length of matches and overall to make doubles shorter and more exciting. The request for change initially came from the tournament directors component of the ATP Tour who found doubles an expensive part of the tournament (in prizemoney and other costs such as players' per diem expenses and the need for extra courts, facilities, transport, officials and staff). These costs were not matched in financial return from spectators, sponsorship, television rights, etc.

However the shorter and more predictable length of matches through the use of modified scoring meant that tournaments could allocate more doubles matches to centre court, fill gaps arising by short singles matches, and schedule doubles before singles in the confidence the scheduled singles matches, particularly the final, would not be delayed by an extraordinary long doubles match. Modified scoring had benefits to all sectors of the game and ensured that doubles remained on the program at all tournaments. Modified scoring for doubles extended from the ATP Tour to the WTA Tour and is also used at the Grand Slams, primarily in the mixed doubles, and for ‘dead’ rubbers in Davis and Fed Cups.

Eight different scoring systems with three different points in a game systems, which represents 24 different systems overall were considered by Brown, Barnett, Pollard, Lisle and Pollard (2008). The analysis in this section considers only seven scoring systems (and excludes the tiebreak game in place of the fifth set which has never been used in singles tournament play). Also, it considers two different points in a game systems (and excludes the 50-40 game scoring system which has never been used) representing 14 different systems that have been used in the past or are currently being used in professional tennis.

Thus, the following seven scoring systems have been identified for the two different points in a game systems of advantage (lead by two points) or no advantage (sudden death) scoring if the score reaches deuce (or 3 – 3) in points:

1. The traditional men’s scoring system in major events was best of five advantage sets. Although this system is no longer in use, it is a useful benchmark against which to measure the various changes that have been made.

2. When the tiebreak was introduced some tournaments (Australian, French, Wimbledon and Davis Cup) changed to the first four sets being tiebreak sets, but they retained an advantage set for the fifth and final deciding set, if required. Only Wimbledon and Davis Cup now use five sets in doubles, and the ITF is considering reducing singles (not doubles) to three sets.

3. Other tournaments (US Open and those ATP tournament that retained a five set final) changed to best of five tiebreak sets. The ATP no longer plays five sets, while the Davis Cup changed from an advantage fifth set to a tiebreak fifth set in 2016.

4. The traditional women’s scoring system was best of three advantage sets and this was also used in some men’s tournaments for singles and doubles. This system is no longer used in women’s or men’s tennis, but it is also a useful benchmark against which to measure the various alternatives that are now used.

5. Similar to 2 above, the women's events at some tournaments (Australian, French, Wimbledon, Fed Cup) changed to the first two sets being tiebreak, but the third set, if required, remained an advantage set. Men's doubles were also played under this format, but subsequently changed to system 6 below.

6. The majority of tournaments on the ATP and WTA Tours plus women's events at the US Open utilize best of three tiebreak sets. The Fed Cup changed to this format in 2016. Doubles events on the ATP and WTA Tours have since changed to system 7 below.

7. In doubles only, the ATP and WTA Tours, and also mixed doubles at some Grand Slams, now play the first two sets tie-break, but the third set, if required, has been replaced by a match tie-break game, which is an extended tie-break best of eighteen points (first to ten) with an advantage (lead by two) if the score reaches 9-9. The recently introduced Laver Cup trialled this match tie-break system in singles in the October 2017 event.

More recently the ATP and WTA Tours for doubles and some Grand Slams for mixed doubles have introduced no advantage (sudden death) scoring at deuce. In practice this is only being used for scoring systems 6 and 7, but it is considered here for all other systems for benchmark purposes.

Of the seven different scoring systems above three are best of five sets and four are best of three sets. Allowing for the two alternatives at deuce (advantage or no advantage) six scoring systems are best of five sets and eight are best of three sets. The key values to be considered for the different scoring systems and for the selected values of p_a and p_b are:

(i) the probability player A wins P(A),

(ii) the expected number of points played,

(iii) the standard deviation of number of points played, and

(iv) the efficiency of the scoring system.

The results for two ‘big’ servers and two ‘average’ servers (and $p_a - p_b = 0.04$) are shown in Tables 6.1 and 6.2.

Scoring System and measures	Advantage p_a=0.77 p_b=0.73	No ad scoring P_a=0.77 p_b=0.73	Advantage p_a=0.62 p_b=0.58	No ad scoring p_a=0.62 p_b=0.58
1 (I)	0.788	0.759	0.752	0.728
(ii)	484.3	366.7	274.2	239.1
(iii)	218.2	149.6	73.8	61.6
(iv)	0.34	0.38	0.61	0.57
2 (i)	0.723	0.721	0.743	0.723
(ii)	290.3	259.0	262.1	230.7
(iii)	99.5	76.7	63.6	54.2
(iv)	0.34	0.38	0.59	0.55
3 (i)	0.708	0.712	0.741	0.721
(ii)	272.0	248.9	261.0	229.9
(iii)	60.7	55.7	61.6	52.7
(iv)	0.332	0.36	0.58	0.55

Table 6.1. Four measures (i) P(A wins), (ii) mean number of points, (iii) standard deviation of the number of points, and (iv) efficiency, for three best of five sets scoring systems.

Note: The three scoring systems are (1) five advantage sets, (2) four tiebreak sets and fifth set advantage, (3) five tiebreak sets, and each calculated for advantage and no ad scoring, and for high serving (p_a=0.77) and average serving (p_a=0.62) values and $p_a - p_b$ =0.04

Scoring system and measures	Advantage p_a=0.77 p_b=0.73	No ad scoring p_a=0.77 p_b=0.73	Advantage p_a=0.62 p_b=0.58	No ad scoring P_a=0.62 p_b=0.58
4 (i)	0.730	0.713	0.707	0.687
(ii)	298.2	225.3	168.3	146.5
(iii)	165.0	112.1	51.6	42.8
(iv)	0.36	0.40	0.65	0.60
5 (i)	0.690	0.686	0.701	0.683
(ii)	192.1	166.4	161.6	141.8
(iii)	93.1	66.8	44.6	37.6
(iv)	0.37	0.41	0.63	0.59
6 (i)	0.669	0.672	0.697	0.681
(ii)	166.3	155.2	160.0	140.7
(iii)	40.3	37.0	41.4	35.3
(iv)	0.34	0.38	0.62	0.58
7 (i)	0.656	0.658	0.670	0.658
(ii)	142.8	131.5	137.8	122.0
(iii)	21.8	20.5	24.5	20.5
(iv)	0.33	0.37	0.52	0.51

Table 6.2. Four measures (i) P (A wins), (ii) mean number of points, (iii) standard deviation of the number of points, (iv) efficiency, for four best of three sets scoring systems.

Note: The four scoring systems are (4) three advantage sets, (5) two tiebreak sets and third set advantage, (6) three tiebreak sets, (7) two tiebreak sets and third set one extended tiebreak game, each calculated for advantage and no ad scoring, and for high serving (p_a=0.77) and average serving (p_a=0.62) values and p_a – p_b = 0.04.

Table 6.1 provides a comparison of the three different best of five sets scoring systems using advantage and no ad scoring, although to date no add scoring has not been used in five set matches. The first column of data shows that in a match between two big servers, five advantage sets has a very high mean number of points of 484.3 and also has a very high standard deviation of 218.2 points and a relative low efficiency of 0.34. Changing the scoring system to four tiebreak sets and fifth set advantage reduces the mean length to 290.3 points and the standard deviation to 99.5 points with little if any change to the efficiency. However the probability that the better player wins drops from 0.778 to 0.723. If the fifth set is also tiebreak there is a further reduction in the mean and standard deviation of the number of points played to 272.0 and 60.7 respectively. However, it is not a dramatic reduction because not all matches go to five sets, but the few extreme ones are eliminated resulting in a reasonable reduction in the standard deviation. The efficiency drops slightly as there is a further reduction to 0.708 in the probability the better player wins. Potential introduction of no ad scoring would further reduce the mean and standard deviation of the number of points played and there would be an increase in efficiency to 0.38, but this appears insufficient to change scoring from advantage to no ad scoring in five set matches.

In five set matches between players with less aggressive services, there is much less change in the probability the better player wins, the mean and the standard deviation of the number of points played as the number of sets using tiebreaks instead of advantage sets is less, but they are all lower than for the bigger servers. The efficiency of the scoring system however is higher being around 0.60 for the weaker servers compared to 0.34 for the bigger servers.

Table 6.2 shows the same comparison for the four different best of three sets scoring systems and for advantage and no-ad scoring. Interestingly, a best of three advantage sets with good servers has a mean length of 298.2 points and standard deviation 165.0 points which is a slightly higher mean and standard deviation than for a best of five sets match with four sets tiebreak and fifth set advantage. The mean and standard deviation of three set matches reduces to 192.1 and 93.1 points respectively if the first two sets are tiebreak and to 166.3 and 40.3 points if all three sets are tiebreak sets. The probability the better player wins is clearly lower than best of five sets, being 0.730 for best of three advantage sets and 0.669 for best of three tiebreak sets. The means and standard deviations are lower when no ad scoring is used for each scoring system. For a match between average servers, the mean and standard deviation are lower than for better servers. The efficiency of each scoring system is generally 0.02 higher for best of three sets than best of five set matches, with less points being played but lower probability that the better player wins.

Finally we look at format 7 which is the best of three sets with the first two sets being tiebreak and the third set replaced by a first to ten points lead by two points tiebreak game. The match between two good servers now has a mean length of 142.8 points and standard deviation a low 21.8 points. The probability the better player wins drops to 0.656 and the efficiency drops to 0.33. If no ad scoring is used the mean length of matches drops further to 131.5 points with standard deviation just 20.5 points. Similar results are obtained for players with weaker serves, but the efficiency increases to 0.52. It is clear why the ATP and WTA Tours adopted format 7 as it has a low mean and standard deviation, a lower but acceptable probability the better player wins and approximately the same efficiency as the other formats.

These results are depicted in Figure 6.1 which clearly shows that the probability that the better player wins is greatest when 5 advantage sets are played, a bit smaller when 4 tiebreak sets are played and the fifth set advantage, very slightly lower if all five sets are tiebreak. This trend continues as you move from three advantage sets through two tiebreak and third set advantage, three tiebreak sets down to two tiebreak sets and third set replaced by a match tiebreak. The only exception is that with strong serving players, three advantage sets has a higher probability that the better player wins than five tie-break sets.

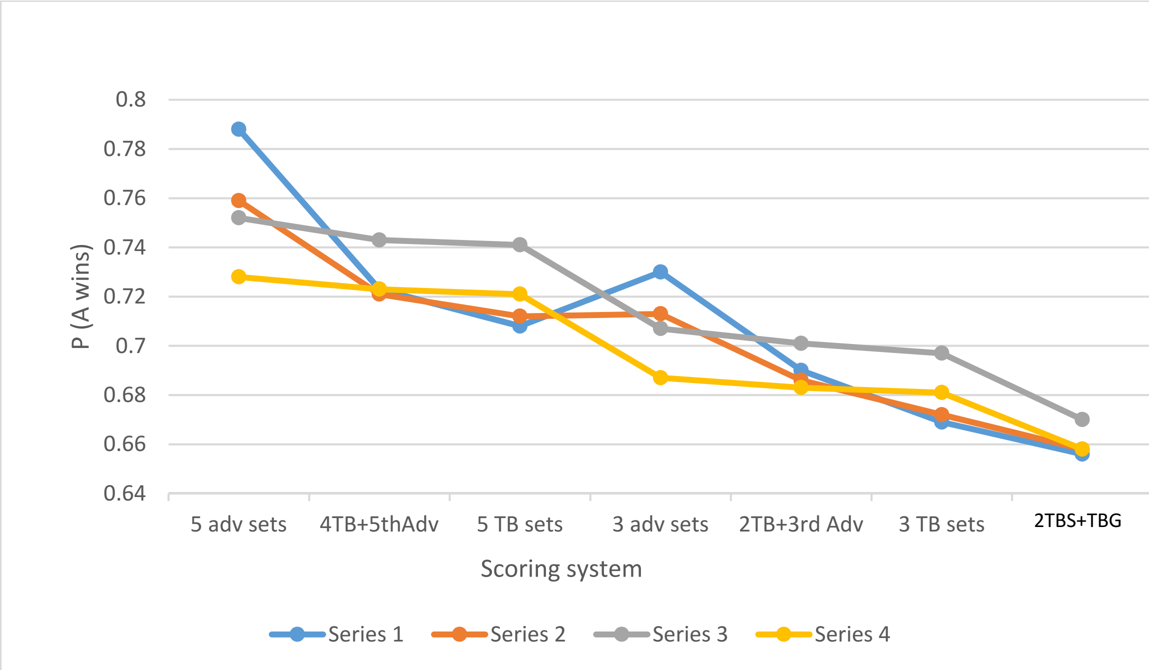

Figure 6.1. P(A wins) under various scoring systems

Note

2TBS+TBG is 2 tiebreak sets plus third set replaced by a match tiebreak game.

Series 1 Advantage games, $p_a = 0.77$. $p_b = 0.73$

Series 2 No Ad games, $p_a = 0.77$, $p_b = 0.73$

Series 3 Advantage games, $p_a = 0.62$, $p_b = 0.58$

Series 4 No ad games, $p_a = 0.62$, $p_b = 0.58$

6.4. Extension to Quadpoints for doubles.

The traditional method of analysing doubles is to give one pair A the average probability p_a of winning a point on service for its two players and the other pair B the average probability p_b of its two players. By doing this, the analysis of doubles is identical to singles (bipoints) as there are just the two probabilities p_a and p_b. This is clearly an approximation to the true situation where each of the four players in a doubles match have their own individual probabilities p_{a1}, p_{a2}, p_{b1} and p_{b2} of winning a point on service, where the notation is obvious, as is the terminology calling this parameterization 'quadpoints'.

Pollard and Noble (2004) noted the potential for the tie-break game to be unfair in doubles and proposed an alternative, whilst Pollard (2005) noted a solution to an aspect of unfairness in tennis doubles. Pollard (2006) outlined a fair scoring system for tennis doubles in the presence of sun and wind effects. Despite their potential unfairness characteristics the current systems are used on the ATP and WTA Tours, particularly in doubles, where the first two sets are tie-break (first to seven points in the tie-break game) and the third set is replaced by a tie-break game (first to ten points).

We now consider the extension of efficiency to quadpoints for doubles scoring systems as outlined in the papers by Pollard and Pollard (2008, 2010). In order to establish the relative efficiency of a (quadpoints) doubles scoring system, we need to set up an appropriate structure so that P and μ do not depend on the order of the four different points being played. Such a structure is the family of scoring systems (n = 1, 2, 3, 4,…..)

$$[W_n\{W_1(W_2AL(a_1, b_1), W_2AL(a_2, b_2)),\ W_1(W_2AL(a_1, b_2), W_2AL(a_2, b_1))\}]$$

which becomes the scoring system against which the efficiency of any doubles scoring system can be measured. This structure is explained below.

The first of the four components in the above formula, namely $W_2AL(a_1, b_1)$ consists of a pair of alternating points a_1 and b_1 which are played repeatedly until team A wins both (or loses both). It has characteristics P_1 and Q_1 and μ_1 given by

$$P_1 = (p_{a1}q_{b1}) / (p_{a1}q_{b1} + q_{a1}p_{b1}) \tag{6.10}$$

$$Q_1 = 1 - P_1 \qquad \text{and}$$

$$\mu_1 = 2(P_1 - Q_1) / (p_{a1} - p_{b1}) \tag{6.11}$$

where P_1 is the probability pair A win this first component, μ_1 is the expected number of points played in this component, $q_{a1} = 1 - p_{a1}$ and $q_{b1} = 1 - p_{b1}$. Note that the expected number of points played in this component conditional on A winning is also μ_1, as is the expected number of points played conditional on B winning.

The second component $W_2AL(a_2, b_2)$ is now played. The corresponding expressions for P_2, Q_2 and μ_2 for this second component can be written for the second component.

The first half of the above scoring system is now considered. This first half consists of playing each of the two components until A wins both (or loses both). If the two components are won by different pairs, the process is repeated until one pair wins both components.The probability pair A wins this half of the above scoring system follows from equations (2) and (3) and is given by

$$P_{1,2} = (P_1P_2) / (P_1P_2 + Q_1Q_2) \quad (6.12)$$

and the expected number of points in this first half is given by

$$\mu_{1,2} = (\mu_1 + \mu_2)/ (P_1P_2 + Q_1Q_2) \quad (6.13)$$

which can be written

$$\mu_{1,2} = (\mu_1 + \mu_2) / (1 - R_{1,2}) \quad (6.14)$$

where $1 - R_{1,2} = P_1P_2 + Q_1Q_2$

The second half of the above scoring system can be analysed similarly giving

$$P_{3,4} = (P_3P_4) / (P_3P_4 + Q_3Q_4) \quad \text{and} \quad (6.15)$$

$$\mu_{3,4} = (\mu_3 + \mu_4) / (1 - R_{3,4}) \quad (6.16)$$

where $1 - R_{3,4} = P_3P_4 + Q_3Q_4$

Combining the two halves of the scoring system, we have

$$P_{1,2,3,4,} = (P_1P_2P_3P_4) / (P_1P_2P_3P_4 + Q_1Q_2Q_3Q_4) \quad (6.17)$$

$$\mu_{1,2,3,4} = (\mu_{1,2} + \mu_{3,4}) / (1 - R_{1,2,3,4}) \quad (6.18)$$

where $1 - R_{1,2,3,4} = (P_1P_2P_3P_4 + Q_1Q_2Q_3Q_4) / ((P_1P_2 + Q_1Q_2)(P_3P_4 + Q_3Q_4))$

The complete scoring system possesses the constant probability ratio property and thus the probability pair A wins the complete Wn{…..} system is given by

$$P_n / Q_n = (P_{1,2,3,4} / Q_{1,2,3,4})^n \tag{6.19}$$

It follows that $2n = \ln(P_n/Q_n) / \ln((p_{a1}p_{a2}q_{b1}q_{b2}) /(p_{b1}p_{b2}q_{a1}q_{a2}))$ (6.20)

And the mean number of points played, μ_n , is given by

$$\mu_n = (n(P_n - Q_n) / (P_{1,2,3,4} - Q_{1,2,3,4}))\mu_{1,2,3,4} \tag{6.21}$$

A tennis doubles scoring system with mean number of points μ but the same probability that pair A wins equal to $P = P_n$ has an efficiency equal to μ_n / μ that is

$$\rho = ((P - Q)\ln(P/Q)\mu_{1,2,3,4}) / (2\mu(P_{1,2,3,4} - Q_{1,2,3,4})\ln(p_{a1}p_{a2}q_{b1}q_{b2} / p_{b1}p_{b2}q_{a1}q_{a2})) \tag{6.22}$$

Expressing $\mu_{1,2,3,4}$ and $(P_{1,2,3,4} - Q_{1.2.3.4})$ as functions of p_{a1}, p_{a2}, p_{b1}, p_{b2}, we obtain

$$\rho = \frac{f(p_{a1}, p_{a2}, p_{b1}, p_{b2})(P-Q)\ln(P/Q)}{\mu\ln(p_{a1}p_{a2}q_{b1}q_{b2} / p_{b1}p_{b2}q_{a1}q_{a2})} \tag{6.23}$$

where

$$f(p_{a1}, p_{a2}, p_{b1}, p_{b2}) = ((p_{a1} + p_{a2})(q_{b1} + q_{b2}) + (p_{b1} + p_{b2})(q_{a1} + q_{a2})) / (p_{a1}p_{a2}q_{b1}q_{b2} - p_{b1}p_{b2}q_{a1}q_{a2}) \tag{6.24}$$

This equation for the efficiency ρ is the general expression for the efficiency of a doubles scoring system.

It should be noted that if $p_{a1} = p_{a2} = p_a$ and $p_{b1} = p_{b2} = p_b$, this equation for efficiency ρ is the same as the expression for bipoints singles scoring systems, namely equation (6.8)

$$\rho = \frac{2(P-Q)\ln(P/Q)}{\mu(p_a - p_b)\ln(p_aq_b / p_bq_a)}$$

The reader is referred to the paper mentioned immediately above for some further results such as

1. A natural extension of Wn(point pairs) as set up by Miles (1984) is Wn(point quads). It was shown that the efficiency of Wn(point quads) must be less than unity.
2. By using the structure of 'PL_{teams}' in which the PL mechanism is used between teams but points are alternated within each team, along with the Wn structure, efficiencies slightly greater than unity can be achieved in the tennis context.
3. As noted earlier, the methods described above can be extended to teams of two doubles pairs, four doubles pairs, etc.

6.5. Extension to cover outcome dependent points

The assumption that the probability the server wins a point on service remains constant throughout the game, set or match and that all points are independent and identically distributed is the basic assumption for the mathematical and probabilistic study of scoring systems, and thus for determining the mean and variance of the number of points in a set or match and the probability that each player wins. There is some evidence that points in tennis are not independent (Klaassen and Magnus (2001), Pollard, Cross and Meyer (2006)), but there is also evidence (Klassen and Magnus(2014)) that it is a reasonable assumption to use in the analysis of scoring systems and probability calculations allowing the researcher making reasonable conclusions.

Pollard and Pollard (2010) extended the measurement of the relative efficiency of different scoring systems for independent points ((Miles (1984)) to cover scoring systems where points are not independent but depend on the outcome of the previous point. For example, some players with intrinsic probability p of winning a point on service have the ability to lift their play by δ to $p + \delta$ if they have lost the previous point and conversely to relax by δ to $p - \delta$ if they have won the previous point, resulting in a decrease in probability that they win the point. The analysis in this section looks at unipoints (as applies to points within a game or sets within a match where there is only one p-value) and bipoints (as applies to games within a set or points within a tiebreak game where there are two probabilities p_a and p_b depending on whether player A or player B is serving.) Again, we build on the work of Miles(1984) and in particular equations (1), (2), (5) and (8).

6.5.1. Unipoints with dependencies

We begin this section by noting that the standard bipoints scoring system $W_1(W_nPW^a, W_nPW^b)$ has the properties

$P/Q = (p_a^{2n-1}q_b) /(p_b^{2n-1}q_a)$, and

$(P-Q)/ \mu = (p_a-p_b) / (2(1+(n-1)(q_a+q_b)))$,

and that the efficiency of a general biformat is given by

$\rho = (2(P-Q)\ln(P/Q) / (\mu(p_a-p_b)\ln(p_aq_b/p_bq_a))$.

To consider the efficiency of unipoints (with dependencies) scoring systems we first set up a family of scoring systems against which the efficiency of any unipoints (with dependencies) scoring system can be measured. Noting that only the first point has a probability p of A winning the point and the remaining points have probability of A winning of either p_w if A won the previous point or p_l if A lost the previous point, we can consider the family of scoring systems $W_1(W_n^{pw}, W_n^{pl})$ where the superscript refers to the probability of A winning or losing the very first point in that section of the scoring system. This scoring system is a good one against which other one-step dependent scoring systems can be compared and in particular because it has the constant probability ratio property and thus allows a general formula for efficiency to be constructed. In fact, this scoring system is just the PW system noted above with an a-point replaced by an A having won point with a probability of pw and a b-point replaced with an A having lost point with a winning probability of pl. Thus we can study a unipoints system with one step dependencies by considering a structure such as the $W_1(W_nPW^a, W_nPW^b)$ system.

It follows (by replacing p_a by p_w, q_b by p_l, q_a by q_w and p_b by q_l) that the system $W_1(W_n^{pw}, W_n^{pl})$ has the key characteristics

$$P/Q = (p_w^{2n-1} p_l) / (q_l^{2n-1} q_w) , \tag{6.25}$$

$$\mu = 2(P - Q)(1 + (n - 1)(q_w + p_l)) / (p_w - q_l) , \text{ and} \tag{6.26}$$

$$\rho = ((2(P\text{-}Q)\ln(P/Q))/ \mu)*f(p_w, p_l) , \tag{6.27}$$

where $f(p_w, p_l)$ is a function of the parameters p_w and p_l.

It follows that we can compare the efficiency of any one step dependent unipoints systems with another by using the relative efficiency expression

$$((P - Q)/\mu)\ln(P/Q).$$

Note that in the above it would have been possible to use the corresponding results for the $W_1(W_nPL^a, W_nPL^b)$ system, and make the substitutions $p_a = p_l$, $q_b = p_w$, $p_b = q_w$ and $q_a = q_l$

6.5.2. Bipoints with dependencies

This approach can now be extended to consider the efficiency of bipoints (with dependencies) scoring system. Suppose player A has a probability p_a of winning the first point and then becomes $p_{aw} = p_a + \delta$ if he won the previous point and $p_{al} = p_a - \delta$ if he lost the previous point. Similarly assume player B when serving has a probability p_b of winning the first point and thereafter $p_{bw} = p_b + \delta$ or $p_{bl} = p_b - \delta$ depending on whether he won or lost the previous point respectively.

We then need to set up a scoring system against which the efficiency of any bipoints with one step dependencies with evaluated. Such a system is $Wn(W_1(W_2^{aw} , W_2^{al}) , W_1(W_2^{bw} , W_2^{bl}))$. This scoring system consists of two components, the first (A serving) with probabilities p_{aw} and p_{al} and the second (B serving) with probabilities p_{bw} and p_{bl}, and each component has the cpr probability. It follows that, where P_1 is the probability of A winning the first component and P_2 is the probability of B winning the second component, then

$P_1 / Q_1 = (p_{aw}{}^3 p_{al}) / (q_{al}{}^3 q_{al})$,

$\mu_1 = 2(P_1\text{-}Q_1)(1+(q_{aw}+ p_{al}))/(p_{aw}\text{-} q_{al})$,

$P_2 / Q_2 = (p_{bw}{}^3 p_{bl})/(q_{bl}{}^3 q_{bw})$, and

$\mu_2 = 2(P_2\text{-}Q_2)(1 + (q_{bw} + p_{bl}))/(p_{bw} – q_{bl})$.

But P_2 is the probability B wins and $Q_2 = 1 – P_2$ is the probability A wins the second component. When n = 1, the probability A wins both components is

$$P_{1,2} = (P_1Q_2) / (P_1Q_2 + Q_1P_2) \qquad (6.29)$$

and the expected duration is

$$\mu_{1,2} =(\mu_1 + \mu_2) / (1 - P_1P_2– Q_1Q_2) \qquad (6.30)$$

The full scoring system for any value of n has the constant probability ratio property, so

$$P_n / Q_n = (P_{1,2}/Q_{1,2})^n \qquad (6.31)$$

$$\mu_n = n(P_n – Q_n)\mu_{1,2}/(P_{1,2} – Q_{1,2}) \qquad (6.32)$$

and thus the efficiency of a general bipoints (with dependencies) scoring system with key characteristics P and μ is

$$\rho \;=\; \frac{(P-Q)\ln(P/Q)\mu_{1,2}}{\mu(P_{1,2}-Q_{1,2})\ln(P_{1,2}/Q_{1,2})}$$

Noting that the terms in this equation with the subscripts 1,2 are simply functions of the underlying p and q values, it can be seen that, when comparing the relative efficiencies of two scoring systems with the same dependent bipoint structure, it is simply a matter of comparing the values of the expression

$$((P – Q)/\mu)\ln(P/Q).$$

Pollard and Pollard (2010) noted

1. Just as the very efficient scoring systems for unipoints and bipoints without dependencies have points that are equally important points for two equal players, this is also the case when one-step dependencies exist.

2. In the tennis context, the play-the-loser service exchange mechanism is slightly more efficient than the alternating mechanism, with or without the dependencies considered in this section.
3. A relationship between the solutions to the PL and PW scoring systems with large mean durations and using Wn stopping rules could be derived. A relationship between these two systems and the corresponding AL one was observed.
4. The importance of a point (with outcome win or loss) could be generalized to the point-pair situation (win, draw or loss) and they made of use this generalisation in bipoints with dependencies.They noted its application to golf, and that the method could be extended to where there were 4 or more possible outcomes.

6.6. A general measure for the relative efficiency of any two scoring systems

As outlined in section 6.2, Miles (1984) showed that the efficiency of a unipoints scoring system with independent points was given by equation (6.5) namely

$$\rho = \frac{(P-Q)\ln(P/Q)}{\mu(p-q)\ln(p/q)}$$

and thus in comparing two scoring systems with the same p (and q = 1 – p) values, the measure for the relative efficiency for each scoring system is given by

$((P - Q)/\mu)\ln(P/Q)$

In the case of bipoints with independent points and p values p_a and p_b for player A and player B respectively, the efficiency of the bipoints scoring system is given by equation (6.8), namely

$$\rho = \frac{2(P-Q)\ln(P/Q)}{\mu(p_a-p_b)\ln(p_a q_b / p_b q_a)}$$

Thus in comparing two scoring systems with the same p_a and p_b values , the measure for the relative efficiency for each scoring system is also given by

$$((P - Q)/\mu)\ln(P/Q)$$

It follows that in comparing any two unipoint or bipoint scoring systems with constant p-values and independent points, the relative efficiency of System 1 to System 2 is given by equation (6.9) namely

$$\frac{((P_1 - Q_1)/\mu_1)\ln(P_1/Q_1)}{((P_2 - Q_2)/\mu_2)\ln(P_2/Q_2)}$$

Pollard and Pollard (2008), as outlined in section 6.4, showed this is also the measure for relative efficiency for independent quadpoints (as in doubles where there are four servers and therefore four p-values). Pollard and Pollard (2010), as outlined in section 6.5, further extended this conclusion for unipoints and bipoints where the points were not independent but were one-step dependent. That extension required the underlying point probability structure to have both a (P, μ, n) equation and the constant probability ratio property. Pollard and Pollard (2015) further extended this analysis to show that the need for a (P, μ, n) equation and a constant probability ratio relationship was not required and this extension is now discussed.

6.6.1. Nested scoring systems

A nested scoring system is one where there is an inner scoring system and an outer scoring system, such as in tennis, where the inner nest is playing a game and the outer nest is playing a set. In fact tennis is triple nested as best of three or five sets make up a match. The expected number of points in a set of tennis is approximately equal to the expected number of points in a game (which is different for each player) multiplied by the expected number of games in a set. It is further complicated by the fact that the number of service games for each player are not necessarily equal, and the expected duration of the set depends on who serves first.

The efficiency of the inner (i) nest ρ_i is given by

$$\rho_i = \frac{(P_i - Q_i)\ln(P_i / Q_i)}{\mu_i(p-q)\ln(p/q)} \tag{6.34}$$

The efficiency of the outer (o) nest ρ_0 is given by

$$\rho_0 = \frac{(P_o - Q_o)\ln(P_o / Q_o)}{\mu_o(P_i - Q_i)\ln(P_i / Q_i)} \tag{6.35}$$

where p is the probability of winning a point in the inner nest, P_i is the probability of winning a game in the inner nest and hence is the 'p-value' for the outer nest and P_o is the probability of winning the outer nest.

The efficiency of the total nested (n) scoring system ρ_n is given by

$$\rho_n = \frac{(P_n - Q_n)\ln(P_n / Q_n)}{\mu_n(p-q)\ln(p/q)} \tag{6.36}$$

Where P_n is the probability of winning the total game , which is the same as the probability of winning the outer nest P_o (and $Q_n = Q_o$) and hence it follows that

$$\rho_n = \rho_1^* \rho_0 \quad \text{if } \mu_n = \mu_i^* \mu_0 \tag{6.37}$$

In other words, the efficiency of a nested scoring system is exactly multiplicative if the expected duration of the the nested scoring system is exactly equal to the product of the expected durations of the nests. This also applies to triple nesting, etc.

To illustrate using a simple unipoints example, suppose player A has a constant probability p=0.6 of winning a point and that points are independent. Consider the simple nested system B3(B3) where the inner nest is the best of three points and the outer nest is the best of 3 games. Clearly player A can win (or lose) in as few as 4 points or at most 9 points. The inner nest or game has a mean of 2[(.6)(.6) + (.4)(.4)] + 3[(.6)(.4)(.6) +(.4)(.6)(.6) + (.4)(.6)(.4) + (.6)(.4)(.4)] = 2.48 points, a probability player A wins the game of [(.6)(.6) + (.6)(.4)(.6) + (.4)(.6)(.6)] = 0.648 and hence an efficiency of (0.648 – 0.352)ln(0.648/0.352) / 2.48(0.6 – 0.4)ln 0.6/0.4) = 0.8981959879. The outer nest uses the same equations with a p-value of 0.648 (q = 0.352) and has a mean duration of 2.456192 games and a probability of A winning of 0.715516416 and hence an efficiency of 0.8960404689. Writing down the probabilities the game lasts from 4 to 9 points, the mean length of the match can be calculated as 6.09135616, the probability A wins is 0.715516416 and the efficiency is 0.8048199542. It will be noted that the product of the game and set mean durations and the product of the game and set efficiencies are the same as the match duration and match efficiency respectively. Of course, the match probability that A wins is the same as the probability A wins the outer nest.

Other unipoints scoring systems where the mean length and the efficienciy are exactly multiplicative include B3(B5) or in general any B2n-1(B2m-1) systems W2(B3) or in general any Wn(B2n-1) system, W2(W3) or in general any Wn(Wm) system. It also applies in bipoints and quadpoints when the means are multiplicative as the form of the efficiency expressions remain the same.

Consider the nested scoring system Wn(SS) where SS is a scoring system with the probability that A wins is equal to p, mean duration is equal to μ points, mean duration conditional on player A winning is equal to μ_w points and mean duration conditional on A losing is equal to μ_l points. Then if D_z is the expected number of points remaining in the nested system when z is the score in the outer nest ($-n \le z \le n$) and an inner nest is about to begin, we have the recurrence relationship

$$D_z = p(D_{z+1} + \mu_w) + q(D_{z-1} + \mu_l) = pD_{z+1} + qD_{z-1} + \mu$$

where q = 1 – p and the boundaries are $D_n = 0$ and $D_{-n} = 0$. Then from Feller (1957, Ch. 14).

$$D_z = \frac{\mu(z-n)}{q-p} + \frac{2n\mu((q/p)^n - (q/p)^z)}{(q-p)((q/p)^n - (q/p)^{-n})} \tag{6.39}$$

Putting z = 0, the expected duration of the nested system is given by D_0 where

$$D_0 = \frac{P-Q}{p-q} n\mu \tag{6.40}$$

Where $P = p^n/(p^n+q^n)$ and $Q = 1 - P$

Thus the mean of the nested system Wn(SS) is equal to the mean of the inner nest SS (i.e. μ) multiplied by the mean of the outer nest (i.e. the mean of Wn at p which is $\frac{P-Q}{p-q}$ n .

Also, the efficiency of Wn(SS) is equal to the efficiency of SS, since the efficiency of the Wn system is unity.

6.6.2. Extended definition of Relative Efficiency.

Consider two scoring systems SS1 and SS2 that have identical underlying probabilistic structures with SSi (i = 1, 2) having a probability A wins of p_i and an expected duration of μ_i points. Consider the nested scoring systems $Wn_1(SS_1)$ and $Wn_2(SS_2)$. The probability A wins $Wn_i(SS_i)$ where i = 1, 2 is P_i which satisfies the equation

$P_i/Q_i = (p_i/q_i)^{ni}$

where $P_i + Q_i = 1$ and $p_i + q_i + 1$

Taking logs $n_i = (\ln(P_i/Q_I))/\ln(p_i/q_i)$

Using the above equation (6.40) for D_o , the expected duration of Wn_i(SSi) is equal to

$((P_i - Q_i)/(p_i - q_i))n_i\mu_i$

Now if n_1 and n_2 are two (possibly very large) values such that player A has the same probability of winning under either nested system, that is

$P_1 = P_2$ and hence $P_1/Q_1 = P_2/Q_2$ and $P_1 - Q_1 = P_2 - Q_2$

it follows that $n_1/n_2 = \ln(p_2/q_2) / \ln(p_1/q_1)$

Using the underlying concept of efficiency, the efficiency of the system Wn_1(SS1) relative to the system Wn_2(SS2) is given by the mean duration of Wn_2(SS2) divided by the mean duration of Wn_1(SS1) and is given by the expression

$$\frac{((p_1 - q_1)/\mu_1)\ln(p_1/q_1)}{((p_2 - q_2)/\mu_2)\ln(p_2/q_2)} \qquad (6.41)$$

In general, the efficiency of Wn(SS) is equal to the efficiency of SS, so the efficiency of SS1 compared to SS2 is also given by the above expression. It can be seen that this expression where a (p, μ, n) equation does not necessarily exist is identical to equation (6.9) where the (p, μ, n) equation and the constant probability ratio property do exist. It follows that the relative efficiency of two systems can now be measured in a much broader range of situations than before, and using the same expression. In comparison to the "interpolation approach" to relative efficiency used previously, this approach might be called the "extrapolation approach" to relative efficiency.

Finally, consider an example that will show that the interpolation and extrapolation methods give identical results. Suppose player A's probability of winning a point is 0.6 and points are independent. Then consider two scoring systems, SS1 is best of three points (B3) and SS2 is best of 5 points (B5). For SS1 we obtain $p_1 = 0.648$ and $\mu_1 = 2.48$ points and for SS2 we obtain $p_2 = 0.68256$ and $\mu_2 = 4.0656$ points. Substituting these values in the above equation we have

$$n_1/n_2 = 1.254485322$$

Thus player A has the same probability of winning W1,000,000,000(B5) as he does of winning W1,254,485,322(B3). We don't need these particular nested scoring systems to have such large values of n_1 and n_2 except to compare the interpolation and extrapolation approaches to the same degree of accuracy. Using the above equation for $D_0 = \frac{P-Q}{p-q}$ $n\mu$ we have that

W1,254,485,322 (B3) has expected duration

$((P_1\text{-}Q_1)/(0.648\text{-}0.352))^*1{,}254{,}485{,}322^*2.48$ and

W1.000.000.000 (B5) has expected duration

$((P_2\text{-}Q_2)/(0.68256\text{-}0.31744))^*1{,}000{,}000{,}000^*4.0656$

But $P_1\text{-}Q_1 = P_2\text{-}Q_2$ and both are extremely close to unity.

Thus the efficiency of W1,000,000,000 (B5) relative to W1,254,485,322 (B3), and hence the efficiency of B5 relative to B3, where both have the same point probability of 0.6, is the ratio of the above two expected durations, which is 0.943922915.

Using the interpolation approach, the relative efficiency expression ((P-Q)/μ)ln(P/Q) is equal to 0.07283742667 for B3 and 0.06875291605 for B5, and so the efficiency of (B5) relative to (B3) is given by the ratio of these two numbers and is equal to 0.943922915. Thus the extrapolation and interpolation methods produce the same result for relative efficiency.

In summary, suppose two players or teams are playing a sport and there are two scoring systems under consideration, SS1, where the better player or team has a probability p_1 of winning with expected duration μ_1, and SS2, where the better player or team has a probability p_2 of winning with an expected duration of μ_2. Then, under these very general assumptions, the efficiency of SSI relative to SS2 is given by expression (6.41) above.

Pollard and Pollard (2012, 2015)

1. Noted that this approach to relative efficiency was also applicable to the situation in which a scoring system could result in a draw as well as a win and loss. They related this to earlier work on the asymptotic efficiency of some (statistical) sequential probability ratio tests (Pollard (1986, 1990)).
2. Noted that, for example, it was possible to compare the efficiency of playing of two advantage games (player A's services followed by player B's) with say the efficiency of playing two 50-40 games (player A's service followed by player B's).

6.7. Comparing the Masters and the Knock-out scoring systems

Virtually all tennis tournaments are conducted under the knock-out (or single elimination) system in which matches are organised in rounds. The losers are eliminated, while the winners move on to play another round. Thus a Grand Slam tournament with 128 players in the main draw will take 7 rounds (or 127 matches) to reduce the field from

128 down to 1 undefeated player, who is the winner. The round-robin format requires each player to play every other player in the draw and this is clearly impractical, but is useful for tournaments with a smaller numbers of players such as four or eight players, where the number of matches would be 6 or 28 respectively, compared to 3 and 7 respectively under the knock-out formula. The year ending event for the top eight players on the WTA and APT Tours are conducted under a Masters (the original name of the ATP Finals) format where the eight players are divided in two "roughly equal" groups of four players who each play a round-robin event with the best two in each section progressing to the semi-finals, who then play a knock-out to determine the winner. This requires a total of 15 matches (the finalists will have played 5 matches) which is a good compromise between the two formats of knock-out and round-robin.

Scarf and Bilbao (2006) and Scarf, Yusuf and Bilbao (2009) stated that there are really only two tournament structures, namely the round-robin and the knock-out tournament, and all other designs are simply variations of these two formats. Ryvkin and Ortmann (2006, 2008) pointed out that there is a third type of contest or race where all players compete against each other as in athletics or golf, but this structure is not relevant to tennis. The round robin structure has a higher probability that the best player wins than do other structures and also provides a ranking of all the players in the event, but it is at the cost of requiring the most number of matches played. In comparison, the knockout requires the least number of matches, but there is a lower probability that the best player wins, but this can be partly offset by using a seeded draw. Tournament organisers can choose between the two formats and a range of hybrid alternatives that are a combination of the round robin and knockout systems. Pollard and Pollard (2013) analysed the Masters tournament system and showed the system could be improved . In another paper Pollard and Pollard (2013) compared the characteristics of the principal two tennis formats, masters and knockout formats, and suggested another hybrid, which they called the "partial round-robin" format, which increases the efficiency of the round robin component of masters formats by eliminating "dead" matches.

The efficiency of any tournament structure is increased by seeding players and then placing them evenly throughout the draw. This practice is always used in tennis, so only seeded draws are considered in this analysis of efficiency of the different formats. Under the Masters format with two 'roughly equal' round robins of four players, the players ranked 1 and 2 are placed in different groups, so are (each drawn randomly) players ranked 3 and 4, then 5 and 6 and finally 7 and 8. After the round robins have been played the two leading players in each group progress to the semi-finals and the winner is determined by knock-out. In the semi-finals, the player who comes first in the first group plays the player who comes second in the second group and the player who comes second in the first group plays the player who comes first in the second group.

The equivalent approach for the knock-out draw is to place players 1 and 2 in different halves of the draw and (by lot) place players 3 and 4 in different halves. Then for the first round and again by lot, player 1 will draw either player 7 or 8 and player 2 will play either player 8 or 7, player 3 will draw either player 5 or 6 and player 4 will play either player 6 or 5. There will be eight equally likely different draws for the Masters format and eight equally likely draws for the knock-out format and the characteristics and efficiency for each of these can be calculated.

6.7.1. Improving the Masters Tournament System

The Masters format outlined above begins with eight players divided into two "roughly equal" pools of four players who first play each other in round robin format. The objective of the format is equality and fairness, but this could be measured by (say)

1. the probability that the best ranked four players reach the semi-finals, or
2. the probability that the best ranked two players reach the finals, or
3. the probability that the best ranked player is the overall winner.
4. Since some of these probabilities are reasonably small, and recognising that there are eight players, it makes sense to consider enlarged outcomes, such as
5. the probability that at least three of the four best ranked players reach the semi-finals, or
6. the probability that at least one of the two best ranked players reaches the final, or

7. the probability that the final is between any two of the four best ranked players, or
8. the probability that the best ranked player is in the final, or
9. the probability that the best or second best ranked player is in the final.

Any system A could be considered better than System B if it has higher probabilities for more of these probabilities than the other system.

Consider the example where the highest ranked player is denoted as player i = j = 1 and the lowest ranked player is player i = j = 8 and the probability player i beats player j is equal to 0.5 + (j – i)d where d > 0 and in this example (given in Table 6.3 and used in Tables 6.4 to 6.9) we use d = 0.04. Player i is better than player j if the probability player i beats player j is greater than 0.5. Further the above probability relationship is transitive, namely that if i is better than j and j is better than k, then i is better than k. In practice this is not always true, but it is a good approximation for comparing the scoring systems. These probabilities are shown in Table 6.3.

Probability (i beats j)	j = 1	j = 2	j = 3	j = 4	j = 5	j = 6	j = 7	j = 8
i = 1	X	0.54	0.58	0.62	0.66	0.70	0.74	0.78
i = 2	0.46	X	0.54	0.58	0.62	0.66	0.70	0.74
i = 3	0.42	0.46	X	0.54	0.58	0.62	0.66	0.70
i = 4	0.38	0.42	0.46	X	0.54	0.58	0.62	0.66
i = 5	0.34	0.38	0.42	0.46	X	0.54	0.58	0.62
i = 6	0.30	0.34	0.38	0.42	0.46	X	0.54	0.58
i = 7	0.26	0.30	0.34	0.38	0.42	0.46	X	0.54
i = 8	0.22	0.26	0.30	0.34	0.38	0.42	0.46	X

Table 6.3. Probability player I beats player j equal to 0.5 + (j – i)d where d = 0.04

As explained above and shown in table 6.4, there are eight equally likely possible combinations for round robin group 1 (including the player ranked number 1) and the associated eight possible combinations for group 2 (including the player ranked number 2). Table 6.4 shows the eight possible draws and the probability rounded off (to four decimal places) of each of the eight possible outcomes suggested above as a measure of the equality and fairness of the draw. Note that without loss of generality in the conclusions reached, all ties between two or even three players at the round robin stage, have been resolved fairly (at random and with equal probabilities) between the players involved, although in practice they are determined by countback on sets won/lost and if still equal by games won/lost.

Draw > Outcome	1357 2468	1368 2457	1358 2467	1367 2458	Aver-age 4	1457 2368	1468 2357	1458 2367	1467 2358	Aver-age 4	Aver-age 8
1	.1193	.1179	.1184	.1183	.1185	.1183	.1184	.1181	.1181	.1182	.1184
2	.1074	.1065	.1068	.1068	.1069	.1068	.1066	.1066	.1066	.1066	.1067
3	.2807	.2900	.2858	.2845	.2853	.2825	.2925	.2880	.2866	.2874	.2863
4	.5344	.5370	.5337	.5347	.5350	.5347	.5337	.5328	.5327	.5335	.5342
5	.7414	.7422	.7414	.7415	.7416	.7416	.7412	.7414	.7407	.7412	.7414
6	.4879	.4893	.4874	.4874	.4880	.4874	.4873	.4861	.4862	.4868	.4874
7	.4530	.4673	.4613	.4584	.4600	.4529	.4668	.4613	.4580	.4598	.4599
8	.5067	.5066	.5063	.5062	.5065	.5060	.5072	.5064	.5061	.5064	.5064

Table 6.4. The eight outcome probabilities for the eight possible Masters draws plus the overall average and the averages for the four draws with players 1 & 3 in same group and likewise 1 & 4.

For outcomes 1 and 2, the draw with the highest probability is column 2 of Table 6.4, namely (1357)(2468). For outcomes 4,5,6 and 7, the draw with the highest probability is column 3, namely (1368)(2457). For outcomes 3 and 8, the draw with the highest probability is column 8, namely (1468)(2357).

Looking at the average for the four draws with players 1 and 3 in the same group (column 6) and the average for the four draws with players 1 and 4 in the same group (column 11), it can be seen that all the probabilities in column 6 are greater than the probabilities in column 11 except for outcome 3. In other words, if your measure is the probability that the best player is the overall winner, it is preferable to have players 1 and 4 in the same group, presumably because this increases the probability that player 1 is in the finals. If your measure is any of the other seven outcomes, it is preferable to have players 1 and 3 in the same group. Overall it is preferable to have players 1 and 3 (rather than 1 and 4) in one group and 2 and 4 (rather than 2 and 3) in the other group.

As a further extension we consider the importance (after placing players 1 and 3 in the same group) of placing players 5 and 6 in different groups and players 7 and 8 in different groups rather than just drawing two players at random from the four players ranked 5 to 8. In this case there are six possible groupings, namely the four in table 6.4 columns 2 to 5 plus (1378)(2456) and (1356)(2478). Table 6.5 gives these two additional columns of the probabilities for the eight outcomes previously identified as well as the average (column 6 of table 2) for the four outcomes of table 2, plus the average for five outcomes (adding (1378)(2456)) and six outcomes (adding (1356)(2478)).

Draw> Outcome	1378 2456	1356 2478	Average (4)	Average (5)	Average (6)	Overall average (8)
1	0.1181	0.1209	0.1185	0.1184	0.1188	0.1184
2	0.1065	0.1083	0.1069	0.1068	0.1071	0.1068
3	0.2954	0.2767	0.2853	0.2873	0.2855	0.2863
4	0.5427	0.5376	0.5350	0.5365	0.5367	0.5342
5	0.7435	0.7419	0.7416	0.7420	0.7420	0.7414
6	0.4936	0.4909	0.4880	0.4891	0.4894	0.4874
7	0.4754	0.4464	0.4600	0.4631	0.4603	0.4599
8	0.5077	0.5079	0.5065	0.5067	0.5069	0.5064

Table 6.5. The eight outcome probabilities for two more groupings and the average for four, five and six groupings all with player 1 and player 3 in the same group and compared to the overall average with all eight possible groupings under the current ATP/WTA Masters system.

For outcomes 1,2 and 8 the draw with the highest probability is now (1356)(2478), while for outcomes 3 to 7 the draw with the highest probability is now (1378)(2456). Both these draws are excluded from the current Masters system.

Looking at column 4 in table 6.5, which is the same as column 6 of table 6.4, we can see that, as before, if players ranked 1 and 3 are in the same group, seven of the eight measures are better than the present system which randomly draws players 3 or 4 to be in the same group as player 1 and also allocates players 5 and 6 to separate groups and players 7 and 8 to separate groups. If we eliminate the second condition, so the remaining four players are allocated at random to either group, we obtain column 6 of table 6.5, which still has 7 of the 8 measures better than the current system. But if we eliminate system (1356)(2478) on the grounds it is intrinsically unfair as group 1 (1356) has better players than group 2 (2478), we have column 5 in Table 6.5 where all 8 measures are equal to or better than the current system. In practical terms, this can be achieved simply by redrawing if the unacceptable outcome occurs.

Thus we have shown that although the current system is a good system, it can be marginally improved if players ranked 1 and 3 are placed in one group and players ranked 2 and 4 are placed in the other group and players ranked 5 to 8 are then drawn at random to fill the two groups of four players provided the outcome (1,3,5,6)(2,4,7,8) is not allowed and redrawn as obviously the two groups are not roughly equal.

Pollard and Pollard (2015) reconfirmed this conclusion with a different matrix of probabilities to that in Table 6.3.

6.7.2. Comparing the Masters and Knock Out Tournament Systems

Both the Masters and Knock-out systems involve reducing the number of players remaining in the tournament sequentially from eight to four and then four to two and finally from two to one. The purpose of the seeded draw is to distribute the best players evenly throughout the draw so that the better players are not scheduled to meet one another until later in the tournament and are less likely to be eliminated in the early rounds. This is accepted as being in the interest of players and spectators and preferable to a random unseeded draw. Accordingly one variable worth considering is the number X of the best four players that reach the semi-finals and the probability that two or more of the top four players reach the semi-finals. Likewise another variable worth considering is the number Y of the best two players who reach the finals and the probability that one or more of the top two players reach the finals. The probability that the best player wins or the best player reaches the final is not considered as this is not the objective of the draw process.

In the Masters format there are two groups with one of the top two and two of the top four in each group. If X_1 and X_2 are the number of players ranked 1 to 4 who reach the semi-finals in group one and group two respectively, X_1 and X_2 are independent variables and $X = X_1 + X_2$. The round-robin format can produce situations in which two or even three players can record the same number of wins and need to be split to determine which player progresses to the semi-final. In practice ties are resolved by a count back of sets won and lost or even games won and lost, but this information is not available in this mathematical exercise and so the tie is resolved at random with equal probability for each player involved.

Using the same probability matrix in Table 6.3 of section 6.7.1, Table 6.6 shows the eight possible Masters draw formats and for each of these draws, the probability for the number of top 4 players in the semi-finals X = 0, 1, 2, 3 and 4, together with the mean and variance of this distribution. Also shown is the probability that at least half the top four players reach the semi-finals and for all eight possible draws, these probabilities are all reasonably high indicating that the Masters is a good scoring system. There is little difference between the eight possible draws and the average of 0.9327, representing the overall performance of the Masters format since all eight draws are equally likely. Also some columns are identical because although they are based on different groupings, the distribution of the d values within the group is identical.

Dist^n Of X	(1,3,5,7) (2,4,6,8)	(1,3,6,8) (2,4,5,7)	(1,3,5,8) (2,4,6,7)	(1,3,6,7) (2,4,5,8)	(1,4,5,7) (2,3,6,8)	(1,4,6,8) (2,3,5,7)	(1,4,5,8) (2,3,6,7)	(1,4,6,7) (2,3,5,8)	Masters overall
P(X=0)	0.0029	0.0028	0.0028	0.0028	0.0028	0.0028	0.0028	0.0028	0.0028
P(X=1)	0.0646	0.0651	0.0647	0.0643	0.0643	0.0647	0.0641	0.0641	0.0645
P(X=2)	0.3981	0.3951	0.3987	0.3981	0.3981	0.3987	0.4002	0.4004	0.3984
P(X=3)	0.4150	0.4191	0.4153	0.4165	0.4165	0.4153	0.4147	0.4146	0.4159
P(X=4)	0.1193	0.1179	0.1184	0.1183	0.1183	0.1184	0.1181	0.1181	0.1184
Mean	2.5833	2.5843	2.5819	2.5830	2.5830	2.5819	2.5811	2.5810	2.5825
Variance	0.6284	0.6256	0.6264	0.6254	0.6254	0.6264	0.6247	0.6248	0.6258
P(X=2,3,4)	0.9325	0.9321	0.9325	0.9328	0.9328	0.9325	0.9330	0.9331	0.9327

Table 6.6. Distribution of X (number top 4 players in semis) for the eight possible Masters draws.

Table 6.7 calculates the same probabilities, mean and variance for the knock-out format. As the knock-out format involves just four matches, whereas the round-robin involves twelve matches to reach this semi-final stage, the central limit theorem comes into play and it is not surprising that the knock-out has much higher variance than the round-robin format (average variance 0.8688 compared to 0.6258) and lower probability that at least half the top four players reach the semi-finals (0.8688 compared to 0.9327). In the knock-out format the mean number of top 4 players in the semis is 2.6400 for all eight possible draws because the sum of the four probabilities that the higher ranked player wins each match is 2.64 for all eight draws and an expansion of the formula for the mean can be reduced to the sum of these four probabilities. The eight round-robin formats produce slightly different means and the overall mean is slightly less (both are 2.6 to one decimal place) is partly due to the random method used here rather than use a countback system (which favours the better player).

Dist^n Of X	(1,7,3,5) (2,8,4,6)	(1,8,3,6) (2,7,4,5)	(1,8,3,5) (2,7,4,6)	(1,7,3,6) (2,8,4,5)	(1,7,4,5) (2,8,3,6)	(1,8,4,6) (2,7,3,5)	(1,8,4,5) (2,7,3,6)	(1,7,4,6) (2,8,3,5)	K-O overall
P(X=0)	0.0119	0.0115	0.0117	0.0118	0.0118	0.0117	0.0115	0.0119	0.0117
P(X=1)	0.1008	0.1002	0.1006	0.1004	0.1004	0.1006	0.1002	0.1008	0.1005
P(X=2)	0.3068	0.3078	0.3076	0.3071	0.3071	0.3076	0.3078	0.3068	0.3073
P(X=3)	0.3962	0.3977	0.3965	0.3974	0.3974	0.3965	0.3977	0.3962	0.3970
P(X=4)	0.1842	0.1828	0.1837	0.1833	0.1833	0.1837	0.1828	0.1842	0.1835
Mean	2.6400	2.6400	2.6400	2.6400	2.6400	2.6400	2.6400	2.6400	2.6400
Variance	0.8720	0.8656	0.8688	0.8688	0.8688	0.8688	0.8656	0.8720	0.8688
P(X=2,3,4)	0.8873	0.8883	0.8878	0.8878	0.8878	0.8878	0.8883	0.8873	0.8878

Table 6.7. Distribution of X (number top 4 players in semis) for the eight possible Knock-out draws

Tables 6.8 and 6.9 look at the variable Y, the number of top two players in the finals, for the eight possible draws of eight seeded players under the Masters and Knock-out formulae. Again the central limit theorem applies and the Masters format involves more matches and has a lower variance in the number of top two players in the finals compared to the Knock-out format (0.3424 compared to 0.4973) and a higher probability that at least half the top two players are in the final (0.7414 compared to 0.7027). The eight individual distributions are all slightly different.

Dist^n of Y	(1,3,5,7) (2,4,6,8)	(1,3,6,8) (2,4,5,7)	(1,3,5,8) (2,4,6,7)	(1,3,6,7) (2,4,5,8)	(1,4,5,7) (2,3,6,8)	(1,4,6,8) (2,3,5,7)	(1,4,5,8) (2,3,6,7)	(1,4,6,7) (2,3,5,8)	Masters Overall
P(Y=0)	0.2586	0.2578	0.2586	0.2585	0.2584	0.2588	0.2586	0.2593	0.2586
P(Y=1)	0.6339	0.6357	0.6346	0.6346	0.6347	0.6345	0.6348	0.6341	0.6346
P(Y=2)	0.1074	0.1065	0.1068	0.1068	0.1068	0.1066	0.1066	0.1066	0.1068
Mean	0.8488	0.8487	0.8483	0.8483	0.8484	0.8478	0.8480	0.8473	0.8482
Variance	0.3432	0.3414	0.3424	0.3423	0.3423	0.3423	0.3421	0.3426	0.3424
P(Y=1,2)	0.7414	0.7422	0.7414	0.7415	0.7416	0.7412	0.7414	0.7407	0.7414

Table 6.8. Distribution of Y (number of top 2 players in final) for eight possible Masters draws

Dist^n Of Y	(1,7,3,5) (2,8,4,6)	(1,8,3,6) (2,7,4,5)	(1,8,3,5) (2,7,4,6)	(1,7,3,6) (2,8,4,5)	(1,7,4,5) (2,8,3,6)	(1,8,4,6) (2,7,3,5)	(1,8,4,5) (2,7,3,6)	(1,7,4,6) (2,8,3,5)	K-O Overall
P(Y=0)	0.2976	0.2974	0.2992	0.2980	0.2934	0.2962	0.2990	0.2972	0.2973
P(Y=1)	0.4980	0.4970	0.4958	0.4958	0.5019	0.4996	0.4963	0.4975	0.4977
P(Y=2)	0.2044	0.2056	0.2050	0.2062	0.2047	0.2041	0.2047	0.2053	0.2050
Mean	0.9068	0.9081	0.9058	0.9081	0.9113	0.9079	0.9058	0.9081	0.9077
Variance	0.4933	0.4946	0.4954	0.4958	0.4902	0.4919	0.4948	0.4940	0.4973
P(Y=1,2)	0.7024	0.7026	0.7008	0.7020	0.7066	0.7038	0.7010	0.7028	0.7027

Table 6.9. Distribution of Y (number of top 2 players in final) for eight possible Knock-out draws

Finally, Pollard and Pollard (2015) suggested consideration be given to using a "partial round robin" rather than a full round robin in the Masters format. Under this system the matches between the two highest and the two lowest ranked players in each of the two groups of four players are not played, thus reducing the total number of matches from 15 to 11. In other words the best two players in each group play the worst two players in their group, but not themselves. Likewise the two worst players in each group play the two best players in their group, but not themselves. This eliminates the two matches in each group that have the greatest chance of being "dead" matches (matches which have no impact on which two players from that group qualify for the semi-finals and which two players are eliminated). The "partial round-robin" will produce a similar mean value of the variable Y but the variance will lie between the variance of the knock-out and full round robin as will the probability at least half the top four players make the semis. Its practical relevance is reduced by the increased likelihood of draws in the round robin stage requiring "count-backs" of some type (or the "live" discarded matches needing to be played at some possible disruption to the scheduling of the tournament). Further analysis of the efficiency of the round robin, knock-out and partial round robin and another interesting alternative ("draw and process") are discussed in the next Section 6.8.

6.8. Measuring the relative efficiency of tournaments.

As shown in section 6.7, the Round-robin tournament format has a higher probability that the best players reach the semis, finals or win a tournament than a Knock-out tournament, but it is at the cost of playing considerably more matches. The knock-out tournament requires relatively few matches, but it has a lower probability that the best players reach the semis, finals or win the tournament. There are various hybrid tournament structures, such as the ATP and WTA Tour finals, which are a compromise between knock-out and round-robin structures that require an intermediate number of matches and have an intermediate probability that the best players reach the semis, final or win the tournament. Pollard, Meyer and Pollard (2015) utilised the approach of Miles

(1984) for determining the absolute and relative efficiency of scoring systems to determine the absolute and relative efficiency of various tournament structures.

We have seen earlier in this chapter that we can determine the efficiency of various scoring systems used to determine the winner of a game, set or match between two players or two teams, and now the methodology can be extended to determine the relative efficiency of different tournament structures. But whereas with two players the key characteristic is the probability that the better player wins, there are a range of possibilities to consider in tournaments, not just the probability that the best player wins the tournament. Other options include maximising the probability that the best two players meet in the final or the best four reach the semi-finals or other considerations such as at least one of the top two players reaches the final.

McGarry and Schultz (1997) concluded that “we do not know the most efficient method to select” and that “no single best tournament structure exists”. They also observed that the round robin “is the most accurate tournament in ranking all the competitors, but it does so at a high cost in that it requires over twice as many games as a single knock-out tournament”. Further they concluded that the (seeded) knock-out structure “is probably the most suitable tournament structure in most cases, given its ranking ability of all players, its promotion of the stronger players and the relatively few games required”.

It is not surprising then that the knock-out tournament structure was the first system used in the inaugural tournament, the Wimbledon Men’s Singles in 1887 and has been adopted by all tournaments worldwide since that date. Interestingly Wimbledon in 1888 adopted a Challenge Round whereby the winner in 1887 (S.W Gore) waited to play the winner of the Men’s “All-Comers Singles” knock-out tournament containing all the other players to ultimately determine the winner of the 1888 Championships. The Challenge Round remained until 1922 when the rules were changed requiring the previous year’s winner to play from the first round. Further the knock-out draws were initially unseeded and remained unseeded until seeding the top 8 players (now top 32 players, but likely to go back to 16 players in 2019) was introduced in 1927 and the seeds were strategically placed to avoid them meeting each other in the early rounds. The

Challenge Round concept remained in Davis Cup until 1972 when the previous year's winner was required to play throughout.

Appleton (1995) carried out simulations to determine the relative ability of each system to produce the best player as the winner of the tournament. He noted that " a strong contender when it is necessary to play relatively few games is the seeded draw-and-process." This is effectively two knock-out tournaments where the seeds are placed differently in the second tournament. For example if the first draw has seeds one and four in the top half and seeds two and three in the bottom half, then the second tournament (redraw or process) will have seeds one and three in the top half and two and four in the bottom half. If two different players win the draw and redraw, they play off in a final. Despite its relative efficiency it has never been used in tennis.

Pollard and Pollard (2013) suggested another format called the "partial round-robin". They noted that if the objective of the round robin was to select the best two of four players, it often happened that the last two matches between the top two ranked players and between the bottom two ranked players had no bearing on selecting the best two players and were consequently "dead" matches and need not be played, thus increasing the efficiency.

The next Section 6.8.1 looks at the efficiency and the relative efficiency of the knock-out, round robin, draw-and-process and partial round robin tournament structures.

6.8.1. The efficiency of four player tournament structures

Four different tournament structures are considered. Three of these were outlined in the previous Section 6.7, namely the conventional Knock-out, the Round-robin and the "partial round-robin' suggested by Pollard and Pollard (2013). The fourth structure considered is the "Draw and Process" (D & P) which is effectively two Knock-out tournaments where the draw is changed so that if the first Knock-out (KO-1) has seeds 1 and 4 in one half and 2 and 3 in the other half, then the second Knock-out (KO-2) will have 1 and 3 in one half and 2 and 4 in the other half (and similarly for the other seeds in larger draws). If the same player wins both knock-outs, that player is the overall winner.

If different players win the two Knock-out tournaments, they then play each other in the Final. This interesting format has not been used in tennis,(perhaps making it less relevant), because it is rare to have the same entry and rankings/seedings for successive tournaments, but could be useful in junior or local events where the same players are involved. Draw and Process is the recognised name of this format, but Pollard, Meyer and Pollard (2015) suggested that "Draw and Re-draw" was a better description of the process, but stuck with the original definition.

Consider the efficiency of a four person tournament. We can consider a tournament won by either of the best two players as a favourable outcome, and a tournament won be either of the weakest two players as an unfavourable outcome. This is referred to as a (1,2 : 3,4) outcome in Table 6.10. Alternatively we can consider a tournament won by the best player as a favourable outcome, a tournament won by the worst player as an unfavourable outcome and a tournament won by either of the other two players as a neutral outcome. This is referred to as a (1 :4) outcome in Table 6.10.

As discussed in section 6.2, Miles (1984) showed that the relative efficiency of two scoring systems is given by equation (6.9) namely

$$((P_1 - Q_1)/\mu_1)\ln(P_1/Q_1) \;\;/\;\; ((P_2 - Q_2)/\mu_2)\ln(P_2/Q_2)$$

where the subscripts refer to scoring systems 1 and 2 respectively and P refers to the probability player A wins and Q refers to the probability player B wins and μ_1 and μ_2 are the mean number of points played. This formula can be used to determine the relative efficiency of any two tournament structures where P refers to the probability of the favourable result as discussed in the the previous paragraph and Q refers to the probability of the unfavourable result.

First consider the efficiency of the four (five if you count KO-1 as being different to KO-2) different tournament structures In the case of a tournament with just four players. Using the notation p_{ij} is the probability player i beats player j and for simplicity assume that the probability the better player wins is 0.55 if the difference in rankings is one, 0.60 if the difference in rankings is two and 0.65 if the difference in rankings is three. We can then calculate P and Q for the four different tournament structures and thus the relative efficiency for the four tournament structures Knock-out (KO-1), Draw and Process (D & P), Partial Round-robin (Partial RR) and Round-robin (RR) using the two different definitions of favourable and unfavourable results. These are shown in table 6.10.

	Tournament Structure	P	Q	Number Matches μ	Relative Efficiency Formula (1)	Relative Efficiency Ratio
1 : 4	KO-1	0.3721	0.1479	3	0.0690	2.53
	D & P	0.4045	0.1237	6.7240	0.0495	1.83
	Partial RR	0.3541	0.1559	4	0.0407	1.50
	RR	0.3562	0.1572	6	0.0271	1
	KO-2	0.3540	0.1560	3	0.0541	2.00
1,2 : 3,4	KO-1	0.6485	0.3515	3	0.0606	2.54
	D & P	0.6937	0.3063	6.7240	0.0471	1.97
	Partial RR	0.6482	0.3518	4	0.0453	1.90
	RR	0.6322	0.3678	6	0.0239	1
	KO-2	0.6480	0.3515	3	0.0605	2.53

Table 6.10. Two relative efficiency measures for four (or five) tournament structures that reduce from 4 players to 1 player

Note that the number of matches is a whole number for all formats except D & P because the final between the winner of KO-1 and KO-2 is not required if the same player wins the first draw and the second draw. The probability player 1 wins KO-1 is 0.65((0.55 X 0.55) + (0.45 X 0.60)) = 0.3721. The probability player 1 wins KO-2 is 0.60((0.60 X 0.55) + (0.40 X 0.65)) = 0.354. The probability player 1 wins both tournaments is 0.3721 X 0.354 = 0.13173. Similarly the probability player 2 wins both tournaments is 0.2764 X 0.2940 = 0.08126, the probability player 3 wins both tournaments is 0.2036 X 0.1960 = 0.03991 and the probability player 4 wins both tournaments is 0.1479 X 0.1560 = 0.02307. Cumulatively, the probability the same player wins both tournaments is 0.27597. Hence the expected number of matches is 0.27597(6) + 0.72403(7) = 6.7240.

Given the small number of matches involved in each tournament structure it is possible to easily calculate the probability that player 1,2,3 and 4 wins the tournament under each structure and hence the relevant P and Q values in Table 6.10. From this the relative efficiency of each tournament structure can be calculated from formula (6.5) and hence the Relative Efficiency Ratio for each tournament structure compared to the Round-robin structure, which is the least efficient.

For the first measure 1 : 4, P is the probability Player 1 wins and Q is the probability Player 4 wins (P + Q is approximately 0.5) . For the second measure 1,2 : 3,4, P is the probability players 1 or 2 win and Q is the probability Players 3 or 4 win (P + Q = 1)

Another way to consider the efficiency of four person tournaments is to look at the efficiency of reducing the draw from four players to two players. In this case a good result is to have players 1 and 2 remaining, and the opposite "bad" result is to have players 4 and 3 remaining. In the case of the D & P tournament structure, one player is the winner of the draw (I,4:3,2) and the other is the winner of the process or redraw (1,3:4,2). It is possible that the same player wins both components, so the D & P compares 11 and 12 with 44 and 43 results, whereas the various knockout and round robin structures all compare 12 with 43 results for the remaining two players. The results are shown in Table 6.11.

Tournament Outcome Measure	Tournament	P	Q	Number of Matches μ	Relative Efficiency Formula (1)	Efficiency Relative to RR
12 : 43	K-O 1	0.3575	0.1575	2	0.0820	2.40
11,12 :44,43	D & P	0.3390	0.0838	6	0.0594	1.74
12 : 43	Partial RR	0.3208	0.1208	4	0.0488	1.43
12 : 43	RR	0.2712	0.0882	6	0.0342	1
12 : 43	K-O 2	0.3600	0.1600	2	0.0811	2.37

Table 6.11. Relative efficiency measures for the reduction from four to two players.

Again it can be seen that the knockout is the most efficient tournament structure, followed by the D & P, then the partial round Robin, while the full round robin is the least efficient.

Other Tournament Outcome measures could also be considered such as player 1 with either player 2 or 3 but not player 4 as a successful outcome (12,13 : 43, 42) (Note Not possible with K-O 2). See Pollard and Pollard for further details.

6.8.2. The efficiency of eight player tournament structures

Pollard, Meyer and Pollard (2015) extended this analysis to consider the efficiency of tournament structures for eight players. In this case players are ranked 1 to 8 and the K-O 1 knockout draw (7 matches) is ((1,w), (4,x)), ((3,y), (2,z)) where players 1 and 4 are drawn to meet in one semifinal and players 2 and 3 are drawn to meet in the other semifinal. The players ranked 5 to 8 are represented by w, x, y and z and are drawn at random. K-O 2 (also 7 matches) is ((1,z), (3,x)), ((4.y), (2,w)) and is the process (alternate) in D & P tournament (14 or 15 matches if a final is required.) In the round robin each of the eight players plays the other players resulting in 28 matches, while in the partial round robin the best four players play the weakest four players resulting in 16 matches. This analysis is much more complicated because of the much greater number of possible measures of efficiency and the much greater number of possible draws with four players drawn at random and was performed by carrying out 1,000,000 simulations of tournaments.

There are four possible measures of efficiency for reducing a tournament from eight to one player, and these are obviously (1 : 8), (1,2 : 8,7), (1,2,3 : 8,7,6) and (1,2,3,4 : 8,7,6,5). There are six possible measures for the efficiency of reducing a tournament from eight to two players and eight possible measures for reducing from eight to four players. Rather than write down all the possible six combinations, e.g. (12 : 87), (13,22 : 86,77),…… etc and likewise the eight possible combinations of four players, they simplified the recording by specifying the sum of the player rankings with the Favourable outcome having a specified maximum and the unfavourable outcome having a specified minimum sum of ranks. These are shown in tables 6.12 and 6.13.

Measure	1	2	3	4	5	6
Maximum Sum of Favourable Ranks	3	4	5	6	7	8
Maximum Sum of Unfavourable Ranks	15	14	13	12	11	10

Table 6.12 Definition of measures for a favourable and unfavourable outcome for the reduction from eight to two players.

Measure	1	2	3	4	5	6	7	8
Maximum Sum of Favourable Ranks	10	11	12	13	14	15	16	17
Maximum Sum of Unfavourable Ranks	26	25	24	23	22	21	20	19

Table 6.13. Definition of measures for a favourable and unfavourable outcome for the reduction from eight to four players.

The methodology for determining the relative efficiency of tournament structures developed here is a general one and can be used for all different tournament structures and for other sports, not just tennis. For all measures considered here, the order of efficiency from most efficient to least efficient was Knock-out, Design-and-Process, Partial Round Robin, Round Robin. The Knock-out structure is clearly the most efficient and the Round robin clearly the least efficient. This is partly because the seeded Knock-out uses information on player rankings, which the Round robin does not. In the Knock-out tournament structure, K-O 1 ((1,4) : (3,2)) is slightly more efficient than K-O 2 ((1,3) : (4,2)), but both are more than twice as efficient than the round robin, whether you are considering the probability the best player wins or one of the top two players wins, or whether you are looking at the probability the top two players make the final. If the Knock-out was not seeded, you have the possibility of the structure ((1,2) : (3,4)) which has a relative efficiency when looking at the first measure of chance the top player wins of 0.0503704 or ratio 1.86 compared to Round robin, but there is no possibility that the top two players can be in the final.

Finally it needs to be noted that there is no single way of measuring efficiency of tournament structure, but the various alternatives have been discussed. This is similar to measures of central tendency where the mean, median and mode are all considered.

In the above-mentioned pa[per, Pollard and Pollard demonstrated that the knock-out structure was very efficient in absolute terms, rather than just relative to other commonly used tournament structures. Thus, there is no particular need in terms of efficiency to think further about tournament structures that could possibly be more efficient than the knockout structure.

6.9. Summary and implications.

Assuming independence between points it is possible for any scoring system to calculate the mean, variance, skewness of the number of points played and the probability that the better player wins. If two different scoring systems have the same probability of correctly identifying the better player, the one which requires the smaller number of points played could be said to be more efficient than the other. Miles (1984) established a method for measuring the relative efficiency of various tennis scoring systems. He developed formulae for the efficiency of unipoints (one probability of player A winning each point or game or set) scoring systems and bipoints (two probabilities of player A winning a point depending on whether Player A or Player B is serving.

In this Chapter 6, a number of new developments have been made to the measurement of efficiency by:

1. Extending the theory for doubles from bipoints (both players in each pair have the same p value) to quadpoints (each of the four players have their own p value).
2. Removing the requirement for independence between points so that point probabilities can be dependent on the outcome of the previous point.

3. Further extended the theory of relative efficiency by developing an "extrapolation" approach rather than the "interpolation" approach of Miles (1984).
4. Using eight possible measures of "better", devised a better structure for the year-end ATP/WTA Finals, currently a hybrid round robin followed by a knock-out.
5. Extending the measurement of efficiency from scoring systems to also include tournament structure and showed that the knock-out format was the most efficient, followed by the "draw and process", partial-round-robin and finally the Round robin.

In the amateur days (until 1968) tennis officials and players have had little or no interest in efficiency and stuck with the traditional best of three or five advantage sets and the knock out tournament structure. The knock out tournament is still the most efficient tournament structure, but the best of five or even three advantage sets certainly is not a particularly efficient scoring system. The introduction of open tennis in 1968, along with professionalism, sponsorship, television coverage and worldwide participation saw the scoring system being continually questioned by players, officials and the media, especially television broadcasters that struggled to accommodate the uncertain length of matches. With little statistical information, changes were introduced starting with the tie-break set instead of the advantage set. Greatest pressure came on men's doubles, which the top singles players stopped playing, television stopped covering and spectators stopped watching. The result is a range of scoring systems currently being used and the most important characteristics of these alternatives are discussed in section 4 above.

As Chairman of the International Tennis Federation Rules of Tennis Committee I encouraged testing of alternative scoring systems and the following was inserted in the Foreword to the Official Rules of Tennis

"Appendix IV lists all known and approved alternative procedures and scoring methods. In addition, on its own behalf or on application by interested parties, certain variations to the rules may be approved by the ITF for trial purposes only at a limited number of tournaments or events and/or for a limited time period. Such variations are not included in the published rules and require a report to the ITF on the conclusion of the approved trial."

The alternative procedures and scoring methods listed in Appendix IV include No-Ad scoring, no-let rule, first to seven or first to ten match tie-breaks, change of ends during the tie-break and short sets (first to 4 lead by 2 tiebreak at 4 all or first to 4 with a tiebreak at 3 all). The short sets was thought promising and was trialled at lower ranked ATP tournaments by replacing best of three tiebreak sets with best of five short sets but was soon rejected by the players as an early break could lead to a player being down 0-3, with only one chance at breaking back before the set was lost. However short sets are sometimes used in exhibitions and Tennis Australia is promoting "Fast Four" with no let and no ad scoring as well.

Given the range of alternatives already available and the conservative nature of players and officials, change is more likely to come by players or officials requesting tournaments to move down the list of current options, as has already happened in doubles. Grand Slams and Davis Cup are at the greatest risk because they still play five sets for the men's singles (and also men's doubles at Wimbledon including an advantage fifth set) whereas the women play three sets yet receive equal prizemoney. Any attempts to address the inequality between men and women in greater work for equal pay is likely to be opposed by the better players, whose voice is significant, due to the decline in probability the better player wins. The analysis of efficiency in this chapter can contribute to the debate on both sides.

In another example of future trends, the ITF Board is trying to eliminate the obstacles to top players competing in Davis Cup by making the structure more efficient. The Board proposed making the four singles matches the best of three sets, instead of the current five sets, but retaining five sets for the doubles, which is the only match on the second (middle) day. At the 2017 AGM this received a majority vote of nearly 64%, but not the required two-thirds majority, so the matter is most likely to resurface. All groups except the World Group are now using best of three sets and this has allowed two days play instead of three days to also being trialled.

Efficiency is an important, but not the sole determinant of a scoring system being adopted. History and tradition, fairness and excitement are other important factors. Graham Pollard (2017) has recently devised an approach to measuring excitement in tennis and has developed a relationship between excitement, efficiency, importance and entropy.

CHAPTER 7

TREATING TENNIS SCORING AS POINT PAIRS

In almost all probabilistic modelling of tennis, the assumption is made that all points for each server are independent and identically distributed. In testing this assumption, one usually looks at the independence of one point from the previous point, although there can be some indirect consideration of more than just the previous point, for example when considering whether the player is ahead, equal or behind in a particular service game or when considering the importance of the point being played. In Chapter 5 four direct tests and four indirect tests of independence were developed and then applied to each of the top four players to search for independence when they are serving and also when they are receiving. Communication with my brother G. Pollard (2013) concerning further aspects of independence or lack of independence led to the suggestion that point-by-point data could be analysed by treating the data as two points at a time, or even as triplets, and the mathematical issues involved are discussed in G. Pollard (2013).

The points in tennis are not identical, at least to the extent that the first point is served to the first or forehand court and the second point is served to the second or backhand court. The third point is served to the first court again and the fourth point is served to the second court. This process of alternating service courts continues until one player reaches at least four points and also leads by at least two points. This player then wins that game and the other player has his/her turn to serve. Technically there are some differences between serving to the first court and serving to the second court. Likewise for the receiver there are technical differences receiving from the first and second courts. For example, if you are playing a right-handed player with a weaker backhand than forehand, you are more likely to serve down the middle to the first court, but wide to the second court. Players can have different serving and returning capabilities depending on which side of the court the server has served to begin the point. This is particularly true in doubles, where most players have a preferred side of the court to receive, the first or second court, and rarely play from the other side.

This integral difference in successive points on service suggests that tennis could be considered as consisting of point pairs involving the first point served to the first court and the second point served to the second court, which is then repeated again and again until the game is won. Therefore a game can consist of 4, 5, 6, 8, 10, 12, 14, etc. points. In only one case, namely in games containing precisely five points, the sixth point (served to the second court) is not required because the service game has been won four points to one or lost one point to four. Rather than consider a set of tennis on a point-by-point basis for each server throughout the whole set, in which case every five point game would change the structure of a point pair from (first court, second court) to (first court in one five point game and first court in next service game) to (second court, first court) thereafter until another five point game occurs and the original order starts again. The procedure adopted here is to disregard the last point of a five point game as it is not part of a completed point pair. It is shown that disregarding this last point in a five point game eliminates approximately five percent of the points played in a match and consequently has minimal effect on the generality of the analysis and conclusions made on independence between successive points..

In Section 7.1 we look at the literature on the subject, which is minimal, because considering tennis as point pairs or bi-points is a different or alternative approach to the analysis of point-by-point data discussed in Chapter 5.

In Section 7.2 we look at the mathematics behind point pairs, first considering the usual assumption where points are independent, second considering the situation where the second point depends on the outcome of the first point and third the situation where a player has a different probability of winning a point served to the first and second courts.

In section 7.3 we consider two potential biases in the data, firstly the elimination of the fifth point in a game won in just 5 points, and secondly recognizing that a long service game could have a large number of win-loss or loss-win point pairs after deuce and must end with a win-win or loss-loss point pair. These effects are shown to be minimal and have little or no affect on the generality of the conclusions.

In Section 7.4 and Section 7.6 we use Grand Slam data to look at the performance characteristics of the top four players, namely Nadal, who is left handed, and Federer, Djokovic and Murray, who are all right handed.

The obvious method of analysis used in Sections 7.4 and 7.6 is the Chi-squared test for the difference between observed and expected outcome under the assumption of independence, but in Section 7.5 the method of maximum likelihood is introduced which enables us to calculate maximum likelihood estimators for probability values and their standard error.

7.1. Literature

Probabilistic and statistical studies of tennis generally assume that each player (i and j) has their own specific probability of winning a point on their own service, p_i (player i) and p_j (player j), and this probability remains constant throughout the game, the set or even the match. Specifically the assumption is that the probability of winning each point on service is constant and both independent (of the outcome of the previous point) and identically distributed (regardless of the score or importance of the point). Various studies such as Klassen and Magnus (2001, 2014), Pollard, Cross and Meyer (2006), Pollard and Pollard (2012) and Pollard (2013) and Chapter 5 of this thesis have shown that this assumption of independence is not correct, but it is a good approximation and acceptable for studying a range of characteristics such as the probability of winning a game, a set or a match and calculating the mean, variance and skewness of the number of points played.

Klassen and Magnus (2001, 2014) also showed that the more professional the players are, the less dependence one would expect. Pollard and Pollard (2012) developed four specific and four general tests for looking at the independence between points. Pollard (2013) specifically looked at the independence between points for the top four male players (Nadal, Djokovic, Federer and Murray) at Grand Slams in 2011 and found some evidence of a lack of independence between points for three of these four players (not Federer). In a personal communication G H Pollard (2013) first suggested treating tennis as bi-points or even tri-points and the mathematics involved was included in G N

Pollard (2013), but no statistical analysis was undertaken. As every service game contains at least four points, there is also some merit in looking at quad points and seeing how each player performs over the first four points of any game. This Chapter 7 looks at the analysis on a point pairs' basis (first court, second court) rather than simply using successive bi- points (two successive points regardless of service court order) as discussed in Chapter 5. Although this eliminates the fifth point in a five point game, it has a clear application for the game of tennis and how it is actually played, serving alternately to the first and then the second court, and then repeating the process until the game is won or lost. There is some evidence that some players may have a different probability of winning a point according to which service court they are serving (Pollard, 2008) and thus this is a new approach to looking at the independence between two successive points which strictly uses a (first court, second court) pair, rather than any other pairing.

7.2. Method

If the probability of winning a point on service (p) is constant and independent of the previous point, then the four possible point pairs are Win Win (WW), Win Loss (WL), Loss Win (LW), and Loss Loss (LL), with respective probabilities p^2, pq, qp and q^2. In the typical case where p=0.6 and therefore q=0.4, then using the obvious notation, P(WW)=0.36, P(WL)=0.24, P(LW)=0.24 and P(LL)=0.16. Significant deviations from this distribution will suggest that the points are not independent and identically distributed. In the next Sections 7.2.1 and 7.2.2, a couple of possible departures from this independence assumption are considered. Testing the difference between the observed number of point pairs in each of the four outcomes and the expected under the assumption of independence can be carried out by a simple Chi-squared test with one degree of freedom. However, an interesting alternative approach is to use maximum likelihood theory. The likelihood function that a particular outcome occurred can be easily written down but is a complicated product to analyse. However the log-likelihood function converts a product of probabilities to a sum of the natural logs of probabilities that can be much more easily analysed. This is discussed in more detail in Section 7.5.

This independence of points logic outlined above can be extended to triplets where there are eight possible combinations and assuming independence, P(WWW) = p^3, P(WWL) = P(WLW) = P(LWW) = p^2q , P(WLL) = P(LWL) = P(LLW) = pq^2 and P(LLL) = q^3 . Any significant variance from this binomial distribution represents a lack of independence. But there is virtually no tennis logic to consider points as triplets, whereas there is a clear tennis logic in considering point pairs, where one point is served to the forehand court and the next point is served to the backhand court, so the analysis in this Chapter 7 is restricted to point pairs of the form (first court, second court). A point pair of (second court, first court) could also be considered, but you are then eliminating the first point of every game as well as the last point of every game with an even number of points, which is every game except those that take just five points. Consequently the only point pairs considered are (first court, second court) in that order.

7.2.1. Second point depends on outcome of first point.

Suppose the probability of winning a point on service having won the previous point is p_w, and the probability of winning a point on service having lost the previous point is p_l, then the steady state probability of winning a point, P_w is given by $P_w = P_w p_w + (1 - P_w)p_l$ or $P_w = p_l/(1 + p_l - p_w)$. In the case of the possible situation where a player can lift after losing a point but relaxes after winning a point so that for the second point of the point pair $p_l = 0.7$ and $p_w = 0.5$, the equivalent steady state probability of winning a point $P_w = 7/12 = 0.5833333$. Thus in a (long) sequence of points, the probability of WW, WL, LW, and LL is $P_w p_w$, $P_w(1-p_w)$, $(1-P_w)p_l$ and $(1-P_w)(1-p_l)$ respectively. Hence in the above example, P(WW) = 0.2917, P(WL) = 0.2917, P(LW) = 0.2917 and P(LL) = 0.1250. Alternatively, one can express these results as the number of wins across the two points as given in the following Table 7.1:

Number of Points won	Probability (dependence)	Binomial (independence)
0	$(1-P_w)(1-p_l) =$ 0.1250	$(1-P_w)^2 =$ 0.1736
1	$P_w(1-p_w) + (1-P_w)p_l =$ 0.5833	$2P_w(1-P_w) =$ 0.4861
2	$P_wp_w =$ 0.2917	$P_w^2 =$ 0.3403

Table 7.1. Point pair probabilities under assumptions of independence (P_w= 7/12)) and dependence (p_l= 0.7 and p_w = 0.5)

Thus it can be seen that in the more common situation where a player can lift the value of p after losing a point on service but relaxes after winning a point so that p declines, i.e. $p_l > p_w$, the above distributions for independent and dependent probabilities have the same mean number of points for each point pair, 1.1667, but the dependent probability has a smaller variance than under the corresponding binomial (independent) distribution.

Conversely, in the less likely situation where a player lifts after winning the first point in the point pair, but declines after losing the point, so $p_l < p_w$, the variance is higher than under the corresponding binomial distribution. This can be seen using the example $p_l = 0.5$ and $p_w = 0.7$, the equivalent steady state probability of winning a point is 5/8 = 0.625 and P(WW) = 0.4375, P(WL) + P(LW) = 0.1875 + 0.1875 = 0.3750 and P(LL) = 0.1875, which clearly has a higher variance than the independent binomial probabilities of 0.3906, 0.4688 and 0.1406 respectively, although both have the same mean number of points won , namely 1.250 points. Variation from the binomial distribution indicates a departure from independence between the two points in the point pair, but the variance decreases if $p_l > p_w$ and increases if $p_l < p_w$.

Alternatively, suppose player A has a probability $p_l = p + d$ of winning a point having lost the previous point, but otherwise it is p so $p_w = p$. Then the equivalent steady state probability of winning a point in a point pair is P_w where $P_w = P_wp + (1 – P_w)(p + d)$ so that $P_w = (p + d)/(1 + d)$ which equals 0.6363636 when p = 0.6 and d = 0.1. We also have, corresponding to the above, the point pair distribution in the following Table 7.2A. Note that, similar to the above, there is an increase in the probability of (WL + LW) and a decrease in the probability of (WW + LL) when d = 0.1 (some dependence) relative to when d = 0 (independence) and although the mean number of points remains constant at 1.2726 points, there is a decrease in variance. Conversely if $p_w = p + d$ and $p_l = p$, then $P_w = P_w(p + d) + (1 – P_w)p = p/(1-d)$ which is 0.6666667 when p = 0.6 and d = 0.1. Again, the mean in the dependent probabilities is the same as in the independent probabilities, namely 1.3333, but the variance is higher in the dependent distribution than in the corresponding binomial distribution. Again, variation from the binomial indicates a departure from independence between the two points in the point pair as shown in Table 7.2B.

Point pair	p = 0.6363636 and d = 0 (independence)	p = 0.6 and d = 0.1 (dependence)
WW	$p^2 = 0.4050$	$P_wp = 0.3818$
WL	p(1-p) =0.2314	$P_w(1-p) = 0.2545$
LW	(1-p)p = 0.2314	$(1-P_w)(p+d) = 0.2545$
LL	$(1-p)^2 =0.1322$	$(1-P_w)(1-p-d) = 0.1091$
Total	1	1

Table 7.2A. Point pair probabilities under assumptions of independence (p=0.6) and dependence ($p_l = p + d$)

Point pair	p = 0.6666667 and d = 0 (independence)	p = 0.6 and d = 0.1 (dependence)
WW	$P^2 = 0.4444$	$P_w(p+d) = 0.4667$
WL	p(1-p) = 0.2222	$P_w(1-p-d) = 0.2000$
LW	(1-p)p = 0.2222	$(1-P_w)p = 0.2000$
LL	$(1-p)^2 = 0.1111$	$(1-P_w)(1-p) = 0.1333$
Total	1	1

Table 7.2B. Point pair probabilities under assumptions of independence (p=0.6) and dependence ($p_w = p+ d$)

7.2.2. Varying probability to first and second court.

Now consider a completely different situation where a server's performance does not depend on whether he won or lost the previous point, but does depend on which court side he is serving towards (first or second court). Assume the server has a probability p_1 of winning a point on service to the first court and a probability p_2 of winning a point to the second court, then $P(WW) = p_1p_2$, $P(WL) = p_1(1-p_2)$, $P(LW) = (1-p_1)p_2$ and $P(LL) = (1-p_1)(1-p_2)$. Consider the different distribution for a player with constant p to both courts $p_1 = p_2=0.6$ and a player with $p_1=0.5$ and $p_2=0.7$ giving the following probability distribution (Table 7.3). Again, the mean number of points won is 1.2 points under both assumptions, but there is a slight decrease in variance in the number of points won under the dependence assumption. It is also interesting to note that if you consider the probability P of winning a game from deuce, then $P = p_1p_2 + (p_1(1-p_2) + (1-p_1)p_2)P$ from which $P = p_1p_2/(p_1p_2 + q_1q_2) = 0.35/0.52 = 0.6923$ if independence applies and $p_1=p_2=0.6$, but if $p_1=0.5$ and $p_2 =0.7$ so there is some dependence, then $P= 0.35/0.50 = 0.7$ which is slightly higher. So although both players have an average p value of 0.6, having a higher p value to one court and a lower p value to the other court gives a slightly higher probability of winning the game than maintaining the same average probability when serving to the both courts.

Point pair	$p_1=p_2=0.6$ (independence)	$p_1=0.5$, $p_2=0.7$ (dependence)
WW	0.36	0.35
WL	0.24	0.15
LW	0.24	0.35
LL	0.16	0.15
Total	1	1

Table 7.3. Point pair probabilities under assumptions of independence ($p_1=p_2=0.6$) and dependence ($p_1=0.5$, $p_2=0.7$)

7.3. Data analysis and potential bias

In 2011, each of the four Grand Slam tournaments (Australian Open, French Open, Wimbledon and US Open,) in conjunction with IBM Pointstream, provided point-by-point scoring on their web site, but this innovation was discontinued the following year. The Top Four male players, Djokovic Nadal, Federer and Murray, were clearly the most dominant players over that period and each played over twenty best of five sets Grand Slam singles matches (over 2000 points or over 1000 point pairs each player) for a reasonable statistical analysis to be made of their performance.

As mentioned in the introduction to this Chapter 7, all game scores, except serves won at the next point from 40-15 (i.e. fifth point of the game) or lost at the next point discrete point pairs. In the case of five point games, the fifth point can be eliminated as it is not part of a completed point pair. It can be shown that approximately five percent of points or less are eliminated and this does not affect the generality of the conclusions.

The probability of a four point game (win or lose serve to love) is $p^4 + q^4$.

The probability of a five point game (win or lose serve to 15) is $4(p^4q + pq^4)$.

The probability of a six point game (win or lose serve to 30) is $10(p^4q^2+ p^2q^4)$.

The probability of an 8,10,12,14,… point game (i.e. game goes beyond deuce) is $20p^3q^3$ and the average number of points in a game that goes beyond deuce is $6 + 2(p^2+q^2)/(1-2pq)^2$ or $6 + 2/(1-2pq)$.

If p = 0.5, the respective probabilities are 0.125, 0.25, 0.3125 0.3125, the average length of a game that goes beyond deuce is 10 points and the average length of a game is 6.75 points. The average proportion of points discarded if the last point of a five point game is discarded is 0.25/6.75 = 3.704%. Any effect on the analysis of point pairs by deleting the final point of a five point game is clearly minimal.

Using a more typical value of p = 0.6 and q = 0.4 the respective probabilities are 0.1552, 0.2688, 0.29952 and 0.27648. The average length of a game that goes beyond deuce is 9.84615 points and the average length of a game is 6.48418 points, so the proportion of points discarded is 0.2688/6.48418 or 4.145%. Any effect on the analysis of point pairs by deleting the final point of a five point game is still clearly minimal.

Using a higher value of $p = 0.7$ and $q = 0.3$, the respective probabilities are 0.2482, 0.31080, 0.25578 and 0.18522. The average length of a game that goes beyond deuce is 9.4483 points and the average length of a game is 5.83149 points. It follows that the percentage of points discarded is 0.31080/5.83149 or 5.330%. Any effect on the analysis of point pairs by deleting the final point of a five point game is still small and does not affect to any great extent the generality of the conclusions.

The other potential bias to an analysis by point pairs is to note that after deuce the game must end with a win-win or a loss-loss pair, but until that final pair all preceding point pairs after deuce must be win-loss or loss-win. However, this (bias) would appear to be counterbalanced by the fact that approximately half the games after deuce end with just two more points, so the combined biasing effect on the statistics of point pairs would appear to be small. Further, nearly half again end after two point pairs, one of which must be win-loss or loss-win and one of which must be win-win or loss-loss. Having one of each type is not inconsistent with point pairs before deuce, so it is only when there are many point pairs after deuce that the balance is affected, but this only occurs infrequently. As before, we can make a mathematical estimate of the effect.

The number of occurrences after deuce of a win-win or loss-loss point pair is

$$(p^2 + q^2) / (1 – 2pq)$$

which equals 1 (obviously since the game ends with the first such pair after deuce).

The average number of occurrences after deuce of a win-loss or a loss-win point pair is

$$0(p^2+q^2) + 1(2pq)(p^2+q^2) + 2(2pq)^2(p^2+q^2) + 3(2pq)^3(p^2+q^2) + \ldots\ldots\ldots\ldots\ldots$$

which equals $2pq / (1 – 2pq)$

It can be seen that when $p = q = 0.5$ the number of occurrences of WL or LW also averages one, so the data on point pairs after deuce can be used with no bias, as this is consistent with that before deuce.

When $p = 0.6$ and $q = 0.4$, the average number of occurrences of a win-loss or loss-win point pair after deuce is 0.92307692. It follows that after deuce there are slightly more win-win or loss-loss point pairs (one) than win-loss or loss-win point pairs (0.92307692), but the effect is only about (0.27648)(1-0.92307692) or 2.127%. However before deuce the expected proportion of win-win and loss-loss point pairs is 0.52 and the proportion of win-loss or loss-win point pairs is 0.48, so the ratio is 0.48/0.52 = 0.92307692. The ratio is the same after deuce as before deuce so there is no difference and all the data on point pairs before and after deuce can be used without any bias for the fact that after deuce the game must end with a win-win or loss-loss outcome. Mathematically this arises because the ratio $2pq/(p^2+q^2)$ before deuce is equal to the ratio of $2pq/(1-2pq)$ to 1 after deuce.

It should also be noted that the data can be analysed from the point of view of each of these top four players when serving and when receiving as in Chapter 5. Further, the data are sufficiently large that they can be divided roughly equally into matches against other top ten players (generally quarter finals onwards) and matches against other players (generally earlier rounds against players who would be ranked somewhere between 10 and 100). It is fortunate that one of the top four male players is left-handed (Nadal) as there is some suggestion (Pollard, 2008) that left-handers perform better when serving to or receiving from the second court. For this reason the data analysis begins with Nadal.

7.4. Nadal

We begin by looking at Nadal's performance In the 2011 Grand Slams. The available data showed that Nadal served a total of 1010 point pairs resulting in the following outcome shown in Table 7.4:

First Point Second Point	W	L	Total
W	416	224	640
L	259	111	370
Total	675	335	1010

Table 7.4. Nadal point pair performance when serving.

This produces a Chi-squared with 1 degree of freedom of 2.77 which is not significant at the 5% level. When serving to the first court, Nadal won 640/1010 points giving a p_1 value of 0.634. When serving to the second court, Nadal won 675/1010 points giving a p_2 value of 0.668. If the serves to the first and second court are independent, the probability of achieving a WW result is $p_1p_2 = 0.423$. In 1010 point pairs the expected number of WW pairs is 427.7 while the actual number was 416. Likewise, the expected number of the other combinations WL, LW and LL can be determined. This produces a Chi-squared value of 2.77 which is not significant and thus the data confirm that the two different serves are independent and there is no significant difference between his serve to the first and second court.

But the assumption of independence should include the total data of 1010 point pairs or 2020 points and thus the overall probability of winning a point on service is $p= (640 + 675)/2020 = 0.65099$. If all the serves are independent, $P(WW) = p^2$, $P(WL) = P(LW) = pq$ and $P(LL) = q^2$. This produces the following outcome (table 7.5) which has a Chi-squared value of 5.44 with 2 df which is not significant at the 5% level.

Outcome	Observed	Expected
WW	416	428.0
WL	224	229.5
LW	259	229.5
LL	111	123.0
Total	1010	1010

Table 7.5. Nadal observed and expected (assuming independence) point pair performance when serving.

However, it can be seen from Table 7.5 that Nadal has a better performance than expected when serving to the second court after losing the previous point which was served to the first court (LW), but is this difference significant? This can be tested as a specific hypothesis where LW has an observed 259 against an expected of 229.5 while the three others combined have an observed of 751 against an expected of 780.5 giving a Chi-squared value of 4.80 with 1 df which is significant at the 5% level. Thus, there is evidence that Nadal serves better to the second court after losing the point served to the first court than his other serves.

As outlined in Chapter 5, performance by the various opponents while serving against Nadal is also a measure of Nadal's performance as a receiver. Thus, the same analysis can be performed on Nadal when receiving. Even the best players in the world are more likely to lose the point when receiving and the results for Nadal when receiving are given in the following tables 7.6, which are the receiving equivalents of tables 7.4 and 7.5.

First Point Second Point	W	L	Total
W	223	251	474
L	251	364	615
Total	474	615	1089

Outcome	Observed	Expected
LL	364	347.3
LW	251	267.7
WL	251	267.7
WW	223	206.3
Total	1089	1089

Table 7.6. Nadal observed and expected (assuming independence) point pair performance when receiving.

In Table 7.6 there are 1089 point pairs with Nadal receiving. With the observed number of LW and WL outcomes equal, the two tests applied above are identical ($p_1 = p_2 = 474/1089 = 0.43526 = 948/2178 = p$) and produce the same value of Chi-squared of 4.23 with one degree of freedom for the first test, which is significant, and Chi-squared with 2 degrees of freedom for the second test, which is not significant at the 5% level. It can be observed that the WW and LL results are above expectation and the LW and WL results below expectation. Consequently, Nadal as a receiver appears more likely than expected to win or lose two points in a row than to achieve one win and one loss. Again, this can be specifically tested with an observed WW and LL combined of 587 against an expected of 553.6 producing a chi-squared with 1 df of 4.10, which is significant. So Nadal is more likely to win both points or lose both points than to win one and lose one.

It is also interesting to consider the difference between the more important matches played by Nadal (defined as matches against other top ten players, generally quarter finals and better, but could include a round of 16) and matches played against other players (being Grand Slam data these players would be ranked between 10 and 100). The results for Nadal serving were

Nadal Serving	Top 10 players	Top 10 players	Other players	Other players
Outcome	Observed	Expected	Observed	Expected
WW	206	217.8	210	211.0
WL	129	130.2	95	98.5
LW	155	130.2	104	98.5
LL	66	77.8	45	46.0
Total	556	556	454	454

Table 7.7. Nadal observed and expected (assuming independence) point pair performance serving against Top 10 and against other players.

These results for matches against other Top 10 players are significant at the 5% level with a chi-squared value or 4.37 with 1 df for the first test assumption and a chi-squared value of 6.98 with 2 df for the second test assumption of independence. Clearly against these top players Nadal has a significantly greater number of LW results than expected, which represents a combination of at least three possible factors, namely that Nadal is a left hander serving to the second court, or he is serving after losing the previous point or possibly because he is serving when behind in the score. Specifically testing an observed LW of 155 against an expected of 130.2 produces a chi-squared 1 df of 6.16 which is significant at the 5% level. Interestingly, the results against the lower (but still top 100) players are not significant at the 5% level (chi-squared 1 df of 0.46). From this point pair analysis it appears that Nadal can lift his play when required against the top players, but does not, or does not need to, against lower ranked players.

Finally we can consider the results for Nadal receiving against Top 10 and against other players. The results are as follows:

Nadal Receiving	Top 10 players	Top 10 players	Other players	Other players
Outcome	Observed	Expected	Observed	Expected
LL	207	196.4	157	151.9
LW	124	134.1	127	132.6
WL	123	134.1	128	132.6
WW	102	91.5	121	115.9
Total	556	556	533	533

Table 7.8. Nadal observed and expected (assuming independence) point pair performance receiving against Top 10 and against other players

None of these results against top ten players and against other players ranked 10 to 100 are significant at the 5% level, despite earlier (Table 7. 6) being shown to be just significant at the 5% level when combined to cover matches against all players. This is because against both top ten and against other players WW and LL are higher than expected and LW and WL are below expected, but not significantly so at the 5% level, but combined and therefore with double the data, they are significant.

7.5. Likelihood

An alternative form of analysis of the data to the Chi-squared test utilized above is to apply likelihood testing. Likelihood is the hypothetical probability that an event that has already occurred (the observed data) would have yielded that specific outcome. The likelihood function for these data is the probability for that particular outcome and with over one thousand point pairs is a detailed and very small probability. However, the log-likelihood function, which is simply the natural logarithm of the likelihood function, converts a probability that is the product of a number of probabilities into one which is the sum of a number of logs of probabilities that can be much more easily analysed without loss of generality, for example differentiated to determine maximum values.

Using Table 7.4 as an example, let p be the service probability to the first court, p+s is the service probability to the second court and p+s+e is the service probability to the second court having lost the point to the first court, then the likelihood function L(p,s,e,) is

$[1010!p^{640}(1-p)^{370}/640!370!]X[640!(p+s)^{416}(1-p-s)^{224}/416!224!]X[370!(p+s+e)^{259}(1-p-s-e)^{111}/259!111!]$

And the natural logarithm of the likelihood function LnL(p,s,e) is

640Ln(p) + 370Ln(1-p) + 416Ln(p+s) + 224Ln(1-p-s) + 259Ln(p+s+e) + 111Ln(1-p-s-e) + Ln(K)

Where K = [1010!/640!370!][640!/416!224!][370!/259!111!]

Next we can take partial derivatives of LnL(p,s,e) with respect to p, s and e and set equal to zero to determine maximum likelihood and thus solve the three simultaneous equations to find expected values of p, s and e

Differentiating LnL(p,s,e) with respect to e gives

259/(p+s+e) – 111/(1-p-s-e) =0   so that p+s+e = 259/370 =0.7

Differentiating LnL(p,s,e) with respect to s gives

416/(p+s) – 224/(1-p-s) + 259/(p+s+e) – 111/(1-p-s-e) = 0 so that p+s = 416/640 = 0.65

Differentiating LnL(p,s,e) with respect to p gives

640/p + 370/(1-p) + 416/(p+s) – 224/(1-p-s) + 259/(p+s+e) – 111/(1-p-s-e) =0

so that p= 640/1010 = 0.633663366 and therefore

s = 0.016336633 and e = 0.05

The maximum likelihood estimators for p, s and e are asymptotically distributed normally and we can calculate the standard error of e by firstly taking the second differential with respect to e for the first equation so that second differential with respect to e equals

$-259/(p+s+e)^2 – 111/(1-p-s-e)^2= -259/(0.7)^2- 111/(0.3)^2=-1761.904762$

Hence the Standard Error of the estimator of e is the square root of 1/1761.904762 = 0.023824

Likewise, the second differential with respect to s for the second equation is

$-416/(p+s)^2 – 224/(1-p-s)^2 – 259/(p+s+e)^2-111/(1-p-s-e)^2$

$= -416/0.65^2-224/0.35^2- 259/0.7^2 – 111/0.3^2 = -4575.091575$

And the standard error of the estimator of s is the square root of 1/4575.091575 = 0.014781.

Testing e and s against the null hypotheses e=0 and s=0 we have

the z value for e is z = 0.05/0.023824 = 2.0987 which is significant at the 5% level, and

the z value for s is z = 0. 016336633/0.014781 = 1.1052 which is not significant at the 5% level.

In other words, e (the lift to the second court after losing the point to the first court) is significant, whereas s (the lift because he is serving to the second court rather than the first court) is not significant.

Finally, we can look at the second differential with respect to p in the third equation which equals

$-640/p^2 + 370/(1-p)^2 – 416/(p+s)^2 – 224/(1-p-s)^2 -259/(p+s+e)^2 – 111/(1-p-s-e)^2$

= -3411.9706

The standard error is of the estimator of p is the square root of 1/3411.9708 = 0.01712

It follows that the estimated value of p = 0.63366 with the 95% confidence limits being 0.60011 to 0.66721.

7.6. Other top 4 players

7.6.1. Federer

The results for Federer serving against other top ten players, against other players ranked in the top 100 and against all players are given in the following table. None are significant at the 5% level with Chi-squared 1 df values of 1.34, 1.36 and 2.79 respectively.

FEDERER	Top 10	Top 10	Others	Others	All players	All players
SERVING	Observed	Expected	Observed	Expected	Observed	Expected
WW	280	274.0	239	234.1	519	507.7
WL	123	131.5	86	91.9	209	223.8
LW	128	131.5	88	91.9	216	223.8
LL	69	63.0	41	36.1	110	98.7
Total	600	600	454	454	1054	1054

Table 7.9. Federer observed and expected (assuming independence) point pair serving performance

The results for Federer as a receiver against other top ten players, against other players ranked in the top 100 and against all players are given in the following table. Again, none are significant at the 5% level with Chi-squared 1 df values of 0.32, 0.38 and 0.11 respectively.

FEDERER	Top 10	Top 10	Others	Others	All players	All players
RECEIVING	Observed	Expected	Observed	Expected	Observed	Expected
LL	243	243.1	146	149.2	389	390.7
LW	161	155.9	130	128.8	291	286.3
WL	151	155.9	134	128.8	285	286.3
WW	100	100.1	108	111.2	208	209.7
Total	655	655	518	518	1173	1173

Table 7.10. Federer observed and expected (assuming independence) point pair receiving experience

This analysis by point pairs confirms the results in Pollard (2013) and Chapter 5 that Federer performs equally well on the next point regardless of whether he won or lost the previous point, whether he is ahead or behind, which side he is serving to and the ranking of his opponent.

7.6.2. Djokovic

We now look at Djokovic who became number one ranked player during the year 2011 and won three of the four Grand Slam Championships, only losing the French when he was beaten by Federer in the semi-final. The following Table 7.11 gives the results for Djokovic as a server against top ten players, other players and all players. The results for Djokovic serving against Top Ten players and against other players ranked in the Top 100 are not significant at the 5% level producing Chi-Squared 1df values of 3.51 and 2.2 respectively. However when considering Djokovic service against all players by combining both sets of results, the results are just significant at the 5% level producing a Chi-Squared value 1 df of 4.18. This is due to Djokovic obtaining more WW and LL pairs than expected and less WL and LW pairs than expected.

DJOKOVIC	Top 10	Top 10	Others	Others	All players	All players
SERVING	Observed	Expected	Observed	Expected	Observed	Expected
WW	205	195.8	305	302.3	510	497.1
WL	97	107.2	134	125.7	231	233.9
LW	99	107.2	112	125.7	211	233.9
LL	68	58.8	55	52.3	123	110.1
Total	469	469	606	606	1075	1075

Table 7.11. Djokovic observed and expected (assuming independence) point pair serving experience

We now consider Djokovic's performance as a receiver against the same sets of players. When playing against fellow Top Ten players the results are significant at the 5% level producing a Chi-Squared value 1df of 4.92. There are more LL and WW than expected and less LW and WL, but particularly less WL.

DJOKOVIC	Top 10	Top 10	Others	Others	All players	All players
RECEIVING	Observed	Expected	Observed	Expected	Observed	Expected
LL	173	161.6	199	196.6	372	357.7
LW	112	118.4	170	170.9	282	289.8
WL	102	118.4	167	170.9	269	289.8
WW	98	86.6	151	148.6	249	234.7
Total	485	485	687	687	1172	1172

Table 7.12. Djokovic observed and expected (assuming independence) point pair receiving experience

7.6.3. Murray

Finally we consider Murray who was then ranked number four in the world and whose performance while serving is shown in the following Table 7.13. Like Djokovic, the results for Murray while serving against Top Ten players and against other players are not significant at the 5% level producing Chi-Squared 1 df values of 3.65 and 1.48 respectively, but when combined for all players is significant with a Chi Squared value 1 df of 4.40. Murray, like Djokovic, has more WW and LL than expected and less WL and LW than expected. Overall, Federer appears to play each point independently, Nadal exhibits a slight short term "back to the wall) effect, whilst Djokovic and Murray exhibit a slight short term "front runner effect".

MURRAY	Top 10	Top 10	Others	Others	All Players	All Players
SERVING	Observed	Expected	Observed	Expected	Observed	Expected
WW	184	178.9	299	293.5	483	467.4
WL	100	109.2	157	155.5	252	265.1
LW	99	109.2	143	155.5	242	265.1
LL	78	68.3	88	82.5	166	150.4
Total	461	461	687	687	1148	1148

Table 7.13. Murray observed and expected (assuming independence) point pair serving experience.

The results for Murray when receiving are given below and none are significant at the 5% level producing Chi-Squared values 1 df of 1.92 (Top Ten), 0.47 (Others) and 1.53 (All Players)

MURRAY	Top 10	Top 10	Others	Others	All Players	All Players
RECEIVING	Observed	Expected	Observed	Expected	Observed	Expected
LL	183	178.9	206	202.1	389	378.4
LW	111	111.1	178	185.4	289	299.1
WL	103	111.1	185	185.4	288	299.1
WW	73	68.9	174	170.1	247	236.4
Total	470	470	743	743	1213	1213

Table 7.14. Murray observed and expected (assuming independence) point pair receiving experience.

7.7. Conclusions and implications

The probabilistic analysis of tennis generally assumes that points are independent and identically distributed, whereas there is one clear difference in the way the game is played that justifies reconsideration of this assumption. This is the fact that one point is served to the first court and the next point is served to the second court. For some players, the probability of winning a point on service is not the same to both service courts, although the general assumption is that the better the player the more likely the probability remains the same under all conditions (Klassen and Magnus 2014). Thus, tennis is normally analysed as a series of points, whereas it could be analysed as a series of point pairs, preferably a series of point pairs where the first point is served to the first court and the second point to the second court (eliminating without loss of generality the last point only of those games which consist of 5 points).

The analysis shows that Nadal has a significantly better performance when serving to the second court after losing the point which was served to the first court. As a receiver, Nadal is more likely than expected to win or lose two points in a row than to achieve one win and one loss. On the other hand Federer showed no significant variation from the pure binomial assumption of independence between points. The other two top players Djokovic and Murray were close to independence between points with some evidence on service that they were more likely to have two wins or two losses than one of each. When receiving Djokovic showed a similar difference, but only against Top Ten players.

The analysis is eminently suited to a maximum likelihood approach which similarly indicates that Nadal can lift significantly to the second court after losing the point to the first court, but he has no intrinsic greater probability of winning the point when serving to the second court rather than the first court. This method also allows confidence limits to be determined for the probability of winning a point on service, so we know Nadal has an estimated value of $p = 0.634$ when serving, and the 95% confidence limits are 0.600 to 0.667.

This analysis of independence by considering point pairs rather than the more complicated point by point data is a step forward as the data collection and analysis requires just the four categories (WW, WL, LW, LL).

CHAPTER 8

STRATEGIC EVENT MANAGEMENT.

For a player, strategy is the skilful use of his or her tennis playing ability to get the better of an opponent and ultimately win a point, game, set, match and tournament. In this thesis we have considered this in various ways such as the optimal use of first and second serves or the importance of the score and when a player should lift, if possible. For a tournament, or indeed any event, strategy is skilful management in attaining a desired outcome. There are textbooks on event management, but not on strategic event management. This chapter addresses this issue by looking at all the strategic issues I had to address to transform the Australian Open from a good event, but one whose Grand Slam status was in doubt, into a great event, in many ways a leader in the tennis world. None were easy to achieve, but all were important. Any event can achieve success if it follows the principles outlined here.

Sporting and cultural events come in all shapes and sizes, level of importance, locations and venues. Events range from local community fairs to mega events such as the Olympic Games and the FIFA World Cup. There is no clear definition of events and ways of categorising them by size and impact. Likewise there is no clearly defined management formula and over the years each event has developed its own "modus operandi" ranging from the entirely volunteer organisation to the highly professional in-house or external management.

Most communities stage one or more annual festivals, fairs or sporting events generally designed to be a family-fun event and encourage a sense of belonging to the local community. They usually receive some form of local government funding and rely heavily on volunteers to survive. These events are generally not-for-profit or if there is some profit, it is likely to be donated to a local charity.

Events of national rather than local significance can be classified as major events. These events attract spectators and media from around the country or even overseas and deliver economic benefits to the city or region staging the event.

Hallmark events have widespread national and international recognition, awareness and visitations. They are not only part of the fabric of the society where the event is held, but encourage international tourism and worldwide media exposure and bring substantial economic benefits to the host city or region.

At the highest level there are the mega events, such as the Olympic Games and the FIFA world Cup, which because of their size and significance in terms of attendance, media coverage and economic impact on a city are generally held only every four years (but the NFL Super Bowl is held annually) and most likely held in a different city and country after a lengthy bidding process.

Owners and organisers of events generally want to improve the organisation of their event and dream of upgrading their event, at least to increase attendance, media coverage and profitability. But for a variety of reasons, not all events may want to move into the next category. For example, tennis has four Grand Slams that are each Hallmark events. They are deliberately equal in terms of computer ranking points and are equal or approximately equal in terms of all other measures of status (eg prize money, quality of venue facilities, two weeks play, 128 male and 128 female players plus the same in qualifying, attendance, media coverage, etc.).

The Australian Open Tennis Championships has moved from a major event in the 1980s to a Hallmark event today alongside the other three Grand Slams. Tennis is fortunate in having four Hallmark events each year which is effectively four annual World Championships. There is no attempt to upgrade one of these four events into a mega event or to introduce a new mega event. Likewise, there is no intention of upgrading any tournament on the ATP/WTA Tours to Grand Slam status. The International Tennis Federation supports the four Grand Slams and its Constitution prohibits the introduction of a new mega World Championships. The Grand Slams have legally protected the term Grand Slam in tennis and only an unlikely unanimous vote of the four Slams would allow a fifth Slam. Further, the current ATP and WTA Tours are so full of tournaments that it is unlikely the Tours could find sufficient weeks or a venue of sufficient size and quality to match the four Grand Slams and a new event would never satisfy the past history requirement of Grand Slam status. However there are ways and means within the rules of the ATP and WTA Tours that tournaments can be moved or sold or upgraded or downgraded and there are plenty of examples of this with changes occurring annually.

This Chapter 8 is not about the management of events, but looks at the strategic decisions the Australian Open had to make during my period as President to grow in all aspects from a major event to a Hallmark event alongside the other Grand Slams. The principles are applicable to any event that wants to grow and in particular upgrade from one category of event to the next higher category.

After looking at the limited Literature on the subject in Section 8.1 and giving a brief History of the Australian Open in Section 8.2, the key principles that any event must follow if it is to succeed and grow are covered in the following sections:

- 8.3. A clear vision of where you want the event to be in 5, 10, or even 20 years' time.
- 8.4. A plan to achieve 10% growth each and every year for the foreseeable future.
- 8.5 Legal protection for your lease or ownership of the event.
- 8.6. Fixed dates and venue to make the event part of the fabric of the community.
- 8.7. Treat all participants well so they will return and encourage others to come.
- 8.8. Treat gender equity as a positive and not a challenge or a negative.
- 8.9. Develop a close working relationship with the venue owner/manager.
- 810. Never underestimate the significant contribution governments at all levels can play.
- 8.11. Multiple sponsors may be a better arrangement than a naming rights sponsor.
- 8.12. Television coverage often defines the status of the event so work on expanding it.
- 8.13 The role of options for both parties and other contract issues.

- 8.14. Plan to grow from local to regional to state to national to Asia/Pacific to world event.
- 8.15. Work to grow Australian participation and international participation.
- 8.16. One-off events will always require a subsidy of some type.
- 8.17. Volunteers
- 8.18. Summary and Future Direction.

8.1. The Literature

Some events just come and go. Others have lasted over one hundred years. None have published the secrets of their ongoing success, and still don't. Player management companies like Octagon and International Management Group (IMG) became event management companies as well and steered their players into their tournaments. These professional event managers obviously developed good event management techniques, but were reluctant to share this intellectual property with others, preferring to use this knowledge to acquire the management rights from the tournament owners, or even buy the tournament itself. The only exception to this lack of publication by experts in the field is "An Insider's Guide to Managing Sporting Events" by Jerry Solomon (2002), the former President of Proserv until establishing his own sport and entertainment company, StarGames. Unfortunately this book is not readily available and covers management rather than strategic planning.

There was no literature on organising events until sports management became a university course in the 1980s and 1990s and professors produced lecture notes that were subsequently published as textbooks. Some like Strategic Sports Event Management by Guy Masterman (2014) and Events Management by Glenn A J Bowden (2010) have gone into a third edition and are good textbooks for students studying sports management. Others include Event Management in Sport, Recreation and Tourism: Theoretical and Practical Dimensions by Cheryl Muller and Lorne Adams (2012), Event Management: An Introduction by Charles Braden (2012), The Event's Manager's Bible: The Complete Guide to Planning and Organising a Voluntary or Public Event by D G Conway (2009), Event Management for Dummies by Laura Capell (2013), Successful Event Management: a Practical Handbook by Anton Shone and Bryn Parry (2010), Essential Event Planners Kit by Godfrey Harris (2004) and Applied Sports Management Skills by Robert N Lussier and David C Kimball (2014). All these books are good references for students or persons being introduced to Event Management. This Chapter 8 is much more advanced as it discloses the key strategic measures taken to promote what was already a major event into a hallmark event.

In addition to the above student textbooks, most event organisers now have a check list of activities to be performed in staging their event and a time line for carrying out those activities. This was not always so, but has become a standard feature as events become more professional in their management and ensure there is a legacy left after the event that can be reviewed, updated and improved for the following year's event. This is particularly important when staff members leave, so their knowledge is not lost to the event. It is also important when events move each year from (say) one State of Australia to another, as used to happen in the amateur days, but still happens in many sports today.

At the highest level the International Olympic Committee (IOC), in conjunction with the Sydney Olympic Organising Committee, established a transfer of knowledge program, and in 2002 the IOC established an independent company called Olympic Games Knowledge Services employing many former Sydney 2000 employees. In 2005 the IOC took the transfer of knowledge program in house and called it the Olympic Games Knowledge Management. Its service is available to all bidding cities and all successful Games organising committees, who will probably only organise one Olympic Games in their lifetime, and no longer have to start from scratch in their planning.

This Chapter 8 is not about planning and organising events. In fact the existence of time lines and check lists has the danger of simply ensuring an event remains basically the same each year, well organised, but does not expand and grow. Using my principles and methods as Chairman of the Australian Open Tennis Championships from 1989 to 2010 (and vice president of Tennis Australia in the 1980s}, this Chapter 8 is about strategic planning and growing an event. By sticking to the principles outlined in this chapter, the

Australian Open over this period has grown from an event that was failing to match the other Grand Slam tournaments into the largest sporting event in Australia, the largest in the world in January each year and an equal partner with the other Grand Slams. The Australian Open has gone from a major event in name only into one of the world's hallmark events. The principles outlined in Sections 8.3 to 8.18 can be applied to any event of any size that is wishing to grow.

8.2. Brief Event History

The Australian Championships is the youngest of the four Grand Slams tournaments having commenced in 1905. The modern game of Lawn Tennis was introduced in 1873 and Wimbledon was the first ever tournament being held in 1877, followed by USA in 1881 and the French in 1891. Melbourne held the first Victorian Championships in 1880 (since discontinued) followed by the NSW Championships in Sydney in 1885, making it one of the four oldest tournaments in the world still being held today.

In late 1904 the six State Lawn Tennis Associations and their New Zealand counterpart met in Sydney for the purpose of establishing the Lawn Tennis Association of Australasia (LTAA) for the prime purpose of entering an Australasian team in the Davis Cup (introduced in 1900 with a challenge match between USA and Great Britain but subsequently other countries were invited to compete as well) and secondly for creating and organising an Australasian Lawn Tennis Championships for men to assist in selection of the Davis Cup Team. In fact, the LTAA's responsibilities were limited to selecting the Davis Cup team, selecting the State or New Zealand to host the Australasian Championships each year and if Davis Cup ties were scheduled to be played in Australasia, selecting the city to host the tie. However the selection was more rotation than selection as there was no bidding process. The Australian Women's Championships was not added until 1922, while the Federation Cup, the women's equivalent of the Davis Cup, was only introduced in 1963. New Zealand exited the arrangement in the early 1920s and the title became the Australian Championships from 1926.

Thanks primarily to Norman Brookes (Australia), who won Wimbledon in 1907 and 1914 and Anthony Wilding (NZ), who won Wimbledon in 1910-1913, Australasia won the Davis Cup in 1907 and again in 1908,1909, (no challengers in 1910), 1911 and 1914. In 1913, thirteen representatives of fifteen national tennis associations, including the LTAA, met in Paris to establish the International Lawn Tennis Federation (ILTF), primarily (by mutual consent) to take over the management of the Davis Cup competition from the individual nations and to become the final determiner of the rules and regulations of tennis. The ILTF recognised the Championships of the four best tennis nations, Australia, France, Great Britain (Wimbledon) and the United States as Official Championships of the ILTF and under its constitution, the ILTF was prohibited from conducting a separate World Championships. These four nations remained the only nations to win the Davis Cup until 1974, by which time tennis had changed from amateur to professional (1968) and the Davis Cup Challenge Round abolished (1972).

In 1933 Australian Jack Crawford won the Australian, French and Wimbledon titles and reached the final of the US Championships against Englishman Fred Perry. In a preview of the final, the New York Times wrote that “if Crawford beats Perry today, it would be something like scoring a Grand Slam on the courts, doubled and vulnerable.” Crawford lost in five sets, but the term “Grand Slam” in tennis stuck. Fred Perry went on to be the first player to win all the four Grand Slam tournaments during his career, sometimes called a “career Grand Slam”, but in 1938 the American Don Budge won all the four titles in the same year and thus achieved the first true same calendar year “Grand Slam”. Australian Rod Laver remains the only other man to win a Grand Slam, which he did twice, once as an amateur in 1962 and then as a professional in 1969. In women’s singles, American Maureen Connelly (1953), Australian Margaret Court (1970) and German Steffi Graff (1988) also won same calendar year Grand Slams.

The split between amateur and professional players and tournaments ended in April 1968 (after the amateur Australian Championships had been held in January that year) and all the major tournaments introduced prize money. The Australian Open struggled to match the other tournaments in prize money, but the quality of the Australian male and female players meant the Australian Open player field was initially world class, but declined as the Golden Era ended. The new computer ranking points systems for male and female players were introduced in 1973 and were initially based solely on prizemoney. Consequently the Australian Open became less attractive and top player support declined and the event became a Grand Slam in name only. After poor Australian Opens in Brisbane in 1969, Sydney in 1970 and 1971 and Melbourne 1972, the Australian Open was allocated to Melbourne long term, initially for five years and then indefinitely. This move was positive. But other radical measures were tried to address the lack of status problem affecting the Australian Open, such as moving the dates from January back to December and splitting the men's and women's championships into separate events held weeks apart. They may have delayed the agony of a declining Australian Open, but the end result was inevitable unless further action was taken. By the 1980s moves were afoot to strip the Australian Open of its Grand Slam status and replace it with the ATP and WTA Players Championships in Miami, which was offering more prize money than the Australian Open, received more computer ranking points and attracted all the players.

In Australia we still called the Australian Open a Grand Slam tournament, because by the historical definition it still was, but it failed all the modern measures of Grand Slam status such as prize money, computer ranking points, format, quality of the venue, top player participation, spectator attendance, international television and media coverage, naming rights sponsorship issues, etc.

If there was a turning point it probably came in 1983 when Tennis Australia decided to make the Honorary President, Brian Tobin, an Executive President. It broke the usual corporate governance rules in Australia that the President was both Chairman and CEO, but for the next 27 years under Brian Tobin until 1989 and Geoff Pollard from 1989 to 2010, Tennis Australia had strong leadership and a full time focus on restoring the Grand Slam status of the Australian Open, not just in name, but in reality. What follows are the principles we used and the strategic planning issues that had to be addressed. These principles are relevant for any event wanting dynamic growth.

8.3. Vision

The first and most important factor is to have a vision of where you want your event to be, not just next year, but five, ten and even twenty years' time. Then you can draw up a pathway for achieving that vision. This is not as easy as it sounds and too many events proceed one year at a time, working their way through the previous year's check list and timeline and thus producing an event almost identical to the previous year.

Because the Australian Open was so far behind the other Grand Slams, the Australian Open's vision was relatively simple to state. We had to get the Australian Open up to the same status and quality as the other Grand Slams as quickly as possible. But we were not heading for a fixed target because the other three Grand Slams were also growing and not standing still waiting for the Australian Open to catch up. It was clear that we needed to grow at double their rate of growth, and even then it could take twenty years to bridge the huge gap.

We were fortunate that the Victorian Government shared our vision for the Australian Open and in 1988 provided Tennis Australia with a wonderful new National Tennis Centre at Melbourne Park that matched the other Grand Slam venues, and in some aspects set the new standard for tennis by incorporating a retractable roof that enabled the event to proceed regardless of the weather. The new venue was supposed to last 25 years and in return Tennis Australia committed the Australian Open to Melbourne Park for that period. But by 1995 the Australian Open was bursting at the seams and the first of many extensions and improvements to the venue were undertaken by successive Governments. This support continues as the event grows and meets all the targets for tennis and for tourism.

We were also fortunate that the other Grand Slams shared our vision and helped in many joint strategic decisions that enabled our vision for the Australian Open to be achieved. For example, the Grand Slam Cup was introduced in 1990 and offered huge financial benefits to the best male players in the four Grand Slams. For entry into this event, each Grand Slam was treated equally and not based on prizemoney, as applied to the computer rankings. The winner received US$2million, the biggest pay day in tennis at the time. For ten years until 1999 this significant Grand Slam Cup prizemoney bonus encouraged the male players to play all four Grand Slams, but the event was finally merged with the ATP Finals when the ATP agreed with the Grand Slam Committee that the four Grand Slams were equal and worth double the computer ranking points of the next level of tournament. This strategy and resultant upgrading in status only applied to the men's game because Grand Slam prizemoney and computer points for women were already double the next level of tournament on the WTA Tour.

From the first professional Australian Open in Brisbane in 1969 until the move to Melbourne Park in 1988, the story of the Australian Open was one of survival from one year to the next rather than any strategic planning. The State Associations reluctantly allowed Kooyong to host the Australian Open in 5 year stints, basically tied to sponsor requirements. The Australian Open moved from January to December in 1977 and back to January in 1987, the last tournament on grass at Kooyong. The men's and women's championships were separated from December 1977 until they were reunited in January 1987. It was the French and ITF President Phillipe Chatrier who addressed an historic Tennis Australia Council Meeting in 1985 with the advice that if you want to be a Grand Slam, you have to at least look like a Grand Slam. That meant putting the separate men's and women's tournaments back together again played over two weeks. It meant increasing the women's qualifying and main draw from 64 and 96 respectively to 128 players for both. It meant ending cigarette major sponsorship. It meant ending naming rights sponsorship. It meant a venue capable of holding 15,000 spectators instead of less than 10,000 at Kooyong. But it was the first sign of a vision for a true Grand Slam Australian Open rather than simply surviving as a tournament. It also convinced the Victorian Government under Premier John Cain that Tennis Australia had a vision and could deliver that vision and so he committed the build the new venue in return for Tennis Australia's commitment to Melbourne Park for 25 years. But by following the principles outlined in this Chapter 8, the event has grown well beyond the original expectations and successive Governments have committed to develop and grow the facilities at Melbourne Park.

8.4. Plan for Growth

Commit yourself to growth. Any event, especially those starting from a low base, should be able to grow by 10% pa. All this requires is to repeat everything you did the previous year and find a way to add 10% more activity. In the case of the Australian Open, the spectator capacity and attendance doubled when it moved from Kooyong to Melbourne Park (initially known as Flinders Park) and the attendance has grown by an average of 10% every year since. In addition the Open has upgraded the spectator experience each year to justify above Consumer Price Index (CPI) increases in ticket prices, so the financial growth is closer to 20% pa, which is similar to what most commercial companies would expect to achieve in their strategic plans. Why not events?

Spectator attendance is only one aspect of an event's success. Similar growth was achieved in the other key sources of income, namely domestic television, international television, corporate packages and sponsorship. From the growing spectator and media interest and increasing player support and prize money, it was clear that commercial income sources needed to grow at the same rate. Interestingly, Formula 1 supremo Bernie Ecclestone uses the same 10% pa growth principle to all his income sources, including sanction fees paid by the host cities. Fortunately no such sanction fee is payable by the Australian Open for its Grand Slam status. This status was earned by the Australian Open through the historical performance of its players. However each of the Grand Slams do make substantial voluntary payments to the International Tennis Federation and the Grand Slam Development Fund to support the development of tennis worldwide and especially in less developed countries.

Significantly, Tennis Australia shared the growth with the players through better facilities and increased prize money. In 1988 The Australian Open offered $1.5million in prize money. This was less than half the other Grand Slams and even less than the Miami event. This prize money effectively increased by an average $1million pa for at least the first decade at Melbourne Park, very high percentage increases that were required as much to address a declining Australian dollar against the US dollar, the universal currency in prize money comparisons, as to bridge the gap with the other Grand Slams. Whenever the other Grand Slams increased their prize money by around 5% pa, the Australian Open needed to increase its prize money by around 10% pa to bridge the gap, or even more as the Australian dollar declined in value against the US dollar, the official currency of tennis.

By 2010 the Australian Open had reached parity with the other Grand Slams with each offering around $25 million in prizemoney. In fact, when the Australian dollar reached an exchange rate of 1.1 US dollars, the Australian Open was briefly the number one event in terms of prize money. Even though the Australian dollar has now dropped back to about 0.75 US dollars, the Australian Open is not the lowest in prize money amongst the four Grand Slams. Recent substantial increases in prize money by all the Grand Slams have taken the Australian Open to $50 million prize money, split equally between the men and women. Grand Slam status for the Australian Open is no longer an issue.

From a low starting base you can achieve better than 10% growth per annum. As the numbers grow, it becomes harder to continue to achieve 10% pa growth, but an average of 10% pa growth over a ten year period is a worthwhile objective. With such compound growth, an event will be three times its current size in just twelve years.

8.5. Understand your event

It does not matter whether you are creating a new event or leasing or acquiring an existing event, or whether you are obtaining a national or international sanction from a governing body, you must understand all the issues of ownership or lease, the parameters of the lease or sanction, and so on. To be considered a Grand Slam in everyone's eyes, not just our own interpretation of history, we had to look like a Grand Slam and act like a Grand Slam in every possible way.

We had to make many changes. We had to overcome state rights and stop moving the event around Australia. We had to stop changing the dates of the event and stick to fixed dates, the traditional last two weeks of January (awful dates for any Northern Hemisphere competition). We had to combine the separate men's and women's tournaments and look like a two week Grand Slam. We had to stop selling naming rights to the tournament so that we looked like a Grand Slam and not like all the other lower ranked tournaments who all sold naming rights sponsorship. We had to match the other Grand Slams in prize money, quality of venue, computer ranking points, media interest and exposure, player support, etc. None of these changes were easy, and each took years to achieve, but unless all were in place, the Australian Open would not be universally accepted as a true Grand Slam.

It is also important to understand what you own and protect it legally. When the Australian Open moved to Melbourne Park, it actually became a Melbourne and Olympic Parks Trust event, with Tennis Australia taking a variable percentage of the profits. Tennis Australia managed the venue and the event on behalf of the Trust and received the first $500K of any surplus while the Trust took the next $4.5 million to help pay for the venue. The split was 40:60 over the next $5 million surplus and 60:40 for $10 million surplus and above. Tennis Australia did not have the reserves to take the risk of ownership. But when this financial position improved, Tennis Australia acted to regain ownership of the Australian Open in return for surrendering its rights to manage the venue throughout the year and now pays an annual venue hire to the Trust which is expressed as a percentage of the gate, similar to other events at Melbourne Park.

The other legal issue that needed to be addressed internationally was who owns the concept and the use of the term "Grand Slam" as used in tennis. The term was descriptive based on the outstanding history of the four great tennis nations for most of the twentieth century. The ITF could make a claim to ownership having originally identified the four as the major international tournaments. The ATP and WTA, using their control of the computer ranking systems, considered moving the status of Grand Slam from the Australian Open to the Players Championships in Miami. More recently China, India and the Middle East wanted to organise tournaments with the same format and prize money and call themselves Grand Slams. So the four Grand Slams had to move quickly and register worldwide the term "Grand Slam" as used in tennis to protect it legally from potential other commercial users, particularly other tournaments.

There are many examples of tournaments that have ended up in legal disputes over who owns the event or disputes over their franchise rights. For example the dispute between the ATP Tour and the German Tennis Federation (in partnership with the Qatar Federation) over the downgrading of the German Open in status (without recompense) and the change of dates (from before Roland Garros to after Wimbledon) has cost both parties many millions of dollars in legal fees alone.

8.6. Location and Dates

The ultimate non-financial goal for any event is to become a tradition for that particular date, in that particular venue and that particular city. The objective is to become part of the fabric of the local community, who then become loyal supporters of the event. International visitors are an important bonus that adds significantly to the value of the event, but becoming a fixture on the local calendar is the first criteria. The Australian Open has now achieved that in Melbourne in January, as did the Melbourne Cup in November, the AFL Grand Final in September and more recently the Boxing Day Cricket Test, now permanently played at the Melbourne Cricket Ground. Significantly, none of these events require sanction fees to some over-riding sanctioning body, whereas the Formula 1 Grand Prix does and thus faces large annual sanction fees and annual confirmation of dates plus a costly battle to renew the contract every five years. Nevertheless it has also become part of the fabric of Melbourne life, even if it, unlike the others, has some vocal opponents. In comparison, the Australian Open Golf Championships, which was once seen as golf's fifth Major, has never been able to lock in fixed dates or venues and today is no bigger than it was in 1988. Likewise the National Championships for most of the Olympic Sports continue to rotate around the states and have seen little change in status and profitability over the same period.

As an amateur event, the Australian Championships moved around the nation's major cities and briefly continued to do so when the game went open, until the economic realities quickly set in. However Tennis Australia also dabbled with date changes and format changes until these were also seen as folly. Since 1988 Tennis Australia has successfully resisted all attempts, from insiders as well as outsiders, to move the event to March and has locked in to Melbourne Park for another 20 years to at least 2036, with options to extend. The move from Kooyong to Melbourne Park (both in Melbourne) was justified because the new venue was not only much bigger with much better facilities, but it was also in a much better central location in the city.

By sticking to the same venues and the same dates each year, events can become part of the fabric of that society and locked in to everyone's diary year after year. Sticking to the same venue year after year also makes it possible to justify capital investment in constructing permanent facilities rather than hiring and installing temporary facilities. This is further amplified if the venue is multi-purpose and can be used throughout the year for other events.

8.7. Participants

Care of your athletes or artists is partly management and partly strategic planning, and should always be near the top of the list in event management. Try to understand the needs and desires and expectations of the participants in your event and deliver on all reasonable requests. Since moving to Melbourne Park, the Australian Open has topped the world in the category of player satisfaction and is known as the "Friendly Slam". This category is not just prize money, where the Australian Open was well behind the other Slams, but includes local player transport, hotel, food/beverages, gifts, trophies, player services, doctor, physiotherapist, towels, practice facilities, player party, etc. If players are treated properly, not only will they return the following year, but they will bring other players with them. It was critical that the Australian Open deliver on all these player services whilst prize money was low, but remains important today. No player misses the Australian Open now and many describe it as the best.

In 1989 there was a second revolution in men's tennis. The first post open tennis revolution (male player boycott of Wimbledon) was in 1973, which lead to the formation of the Association of Tennis Professionals (ATP), who introduced and thereafter has controlled the player ranking points used for tournament entry purposes and for seeding players. The ITF, which controlled tennis in the amateur days, lost control of men's tennis and the world tour was initially managed by the Men's International (Professional) Tennis Council consisting of three representatives of the ITF/Grand Slams, three representatives of the other tournaments and three representatives of the players (ATP).

At the 1989 US Open the ATP announced that they would never enter a partnership in which the players did not control at least 50% of the vote. The ATP announced a 50:50 partnership with the tournaments, called the ATP Tour, and were determined to make the Tour bigger than the Grand Slams, which were deliberately left out of the action. It was a difficult time for the Grand Slams, and the Australian Open in particular, as it was the first Grand Slam tournament after the breakaway and then the weakest Grand Slam financially.

The first action by the Grand Slams, who all valued their independence, was to get together for the first time and form the Grand Slam Committee (now Board) to plan their future. One thing missing from the new ATP Tour was a bonus pool and so the second action was to accept a proposal, floated by the ITF, to hold a Grand Slam Cup in Munich for the best eight players at the Grand Slams. The winners of each of the Grand Slams plus the next best performed players at the Grand Slams only were invited to compete. Prize money was a record US$6million plus a further $2 million to the Grand Slams. Instead of keeping the money, the Grand Slams gave the $2 million per annum to the ITF to underwrite the development of tennis in the developing world. The level of prize money ($2 million to the winner, $1 million to the other finalist, $500K to the other 2 semi-finalists and $250K to the other 4 quarter-finalists) was incredible at the time. These levels of prizemoney for the top eight players was only reached by the Grand Slams themselves decades later and some players initially refused to play the new year-ending event claiming the prizemoney was obscene. But for the 1990 Australian Open, a couple of top ten players, who were originally not coming to Australia, changed their mind and entered the tournament. In 1995 the four Grand Slams were made equal in status and each awarded double points. Player participation was no longer an issue. The Grand Slam Cup had served its purpose and was merged with the ATP Finals in 2000 and called the Tennis Masters Cup, now the ATP Tour World Finals.

In the case of the women, player participation and double computer ranking points was not an issue. Back in 1973 the issue for the women was the huge discrepancy between men's and women's prize money and status at tournaments where they played together. With the strong support of Virginia Slims as tour sponsor, many of the players broke away in 1973 and formed their own tour and created the Women's Tennis Association (WTA). But they did play the Grand Slams and encouraged the Australian Open to split into separate men's and women's events which it did. Differences were reconciled and the players, tournaments and ITF agreed to the formation of a Women's International Professional Tennis Council with similar objectives to the men's equivalent. This has gone through a number of changes over the years, all resolved by negotiation, and now is known as the WTA Tour with a Board consisting of four player representatives, four tournament representatives and one ITF representative. As the size of Grand Slam prize money for women increased, participation was not an issue. What was an issue was equal prize money for men and women at the Grand Slams. Although there was much greater spectator, television and sponsor interest in men's tennis, the top women players had been much greater supporters of the Australian Open than the men. Consequently the Australian Open quickly followed the US Open in awarding equal prize money, whereas the French and Wimbledon, who unlike Australia both enjoyed full player participation, continued to maintain a small differential until recently.

As part of the negotiations with the ATP over double and equal points for the Grand Slam, Tennis Australia had to increase men's prize money in 1995 by 30% but could not afford to increase women's prize money by the same amount and only offered a 20% increase. This caused an uproar at the WTA, but by firmly explaining the economic situation and our desire to return to equality as soon as we could afford it, proposed protest action did not eventuate and equal prize money was restored a couple of years later.

The Grand Slams are the most successful and most profitable tournaments in the world today. Players have been asking for a greater share of revenues and substantial increases have been awarded over the past two years and foreshadowed for the next three years. This is all consistent with the important principle of being fair and reasonable with the players.

8.8. Gender equity

Equal prize money in sport is a very rare phenomena. Equal status and exposure are nearly as unlikely, even in amateur sport where there is no prize money. The Australian Open is unique in this country as it offers equal prize money, equal status, exposure and other important criteria that are denied women in other sports. Proudly, the Australian Open was second only to the US Open in introducing equal prize money. The French and Wimbledon took another fifteen years before agreeing to equal prize money, while virtually all other tournaments on the ATP and WTA Tours are still well short of equality, although both Sydney and Brisbane joint ATP and WTA Tournaments offer equal prizemoney as well.

When the (amateur) Australian Championships went open to professionals in 1969 and became the Australian Open, the winner of the men's singles took home $7,500, while the winner of the women's singles received $1,500, a factor of five to one. The differential decreased over the next two decades until finally the winners of the Australian Open Men's and Women's singles both received the same in 1987. But the women's draw was only 96 main draw players and 64 in qualifying compared to 128 players in both main draw and qualifying for the men, so total prize money was not actually equal. Equal draw sizes and equal prize money was achieved in 1988 when the tournament moved to the larger Melbourne Park (initially known as Flinders Park) and has been maintained ever since, (apart from the abovementioned brief period of negotiations over achieving double computer points for the men at Grand Slams over the next level of tournament, having already achieved double points in the women's game).

The arguments against equal prize money are well known, are primarily economic rationalism, and were clearly voiced by the French and Wimbledon during the 15 years they did not offer equal prize money, whereas the US Open and the Australian Open did. There is more interest in the men's matches, TV interest and ratings are higher for the men, ticket prices are higher for the men's finals and of course the men play best of five sets and the women just best of three sets (a legal justification for higher men's prizemoney). It can be argued that effectively equal prize money is a subsidy from the men's game to the women's game. But it can also be argued that the Grand Slams have that elite status in the game partly because they do have both men and women players competing, whereas many other tournaments only conduct single sex events.

Tennis Australia is the national association for both male and female players and always has been. Many other sports had separate national (and state) associations, but most have now combined (generally because it became a funding requirement of the Australian Sports Commission along with other governance changes and acceptance of the WADA Code of Conduct for Drug testing). There is a clear case for all sports to specify equal prize money as a vision for their future and to allocate funds accordingly. For example, in Australia, the four football codes (AFL, Soccer, Rugby and Rugby League) could begin by requiring all their clubs to have a women's team as well as a men's team and schedule the women to play the curtain raiser before the men's match each weekend, possibly with modified rules similar to the three and five set differential in Grand Slam tennis. History shows that Tennis was a reluctant acceptor of equal prizemoney, but it is now enjoying the financial and exposure benefits of having a game where men's and women's tennis are integral to the overall attraction of the game as a player, spectator, sponsor or television viewer. It is interesting to watch how other sports, and in particular the wealthy sports of cricket and football each try to include and grow their professional women's version.

8.9. Venue

The selection of a site or venue for an event is not quite the "location, location, location" rule of real estate investment, but it comes close to it. The venue can make or break an event, either through location or poor or conflicting management. The Australian Open was clearly fortunate to move from an aging Kooyong Club in suburban Melbourne to a state-of-the-art Flinders Park, now named Melbourne Park, in central Melbourne. In doing so Tennis Australia sacrificed grass courts for cushioned acrylic courts and accepted a State owned multi-purpose entertainment centre instead of a tennis club, provided it was designed primarily for tennis and the Australian Open had unfettered access in January and for any home Davis Cup Final, of which there have been only two, in 2000 and 2003. There was also a semi-final played at Melbourne Park in 2003 while some earlier rounds have been played in Melbourne using Kooyong. Proudly, Tennis Australia developed the world's first portable grass court that could be temporarily installed over existing hard courts and was successfully used in these three matches and another tie in Sydney in the early 2000s. After lying idle for over a decade it re-surfaced for a Davis Cup tie at Kooyong against the old foe USA in March 2016.

When your event only occupies a couple of weeks of the year there are benefits of not owning the venue (e.g. little maintenance and no depreciation to be provided for and sharing the development costs with the owner) provided you have unrestricted access for a reasonable price and can solve the conflicts of interest in management issues. It is important that venue management and event management are all on the same wavelength with respect to dates and positioning of the event and commitment to growth of the event. Issues that can arise and need to be resolved between the event and the venue include supply rights, ticketing rights, catering rights, bump-in and bump-out, naming rights, maintenance, capital development, signage, security, sponsorship, corporate boxes, parking rights, liability and insurance, and more. It is a long list and there is no simple formula and little alternative than to negotiate between the event organisers, who obviously want all these rights during their event, and the venue owners or managers, who clearly want to negotiate year round contacts with suppliers.

There have been many difficult negotiations on each of the above issues throughout the 30 years that the Australian Open has been played at Melbourne Park, and especially whenever the contract to play the Open at Melbourne Park is renewed, in which case all items are on the agenda. The discussions have generally been cordial and a win-win outcome sought. The Australian Open is in the privileged position that, unlike many other events, the venue was built primarily for the Australian Open and remains the primary source of income for the Venue. Also, compared to other events, the Australian Open makes a huge economic contribution to Melbourne and Victoria through international visitations and media coverage. The venue is profitable and has never had to seek an operating grant from the Victorian Government, but the Government does make substantial contributions to the capital development of Melbourne Park. The most recent renewal, which I signed in 2010, involves a twenty year extension from 2016 to 2036 and includes a commitment from the Government to invest approximately one billion dollars in capital improvements on the venue, which it owns. The first five years is now completed and involved $365 million improvements including a third court with a retractable roof as well as a new National Tennis (Training) Centre. The second five year plan is well underway and discussions are nearly finalised for the third five year plan.

Venues, whether they are parks, halls, arenas or stadiums, can be privately owned or operated, but are often owned by government, generally State or Local Government, so ultimately, dealing with venue operators often also involves dealing with the relevant government officials.

8.10. Government

Never overlook or underestimate the role Government can play in your event, whether it be Local, State or Federal Government. Government may be involved in the ownership of the venue where the event is held, but will also be involved in numerous operational activities (e.g. traffic control, security, liquor licence, catering licence) and approvals (e.g. signage, hours of operation, construction of temporary facilities). From the selection in 1985 of the fantastic site close to the city centre through to the construction of the venue which was opened in 1988 and all the following major expansions by successive Victorian Governments from both sides of politics, what remains constant is that the venue was built and subsequently developed primarily as a tennis centre and secondly as a multipurpose sports and entertainment centre. This contribution from the Victorian Government in building, maintaining and developing Melbourne Park, which the Government owns but is administered by the government appointed Melbourne and Olympic Parks Trust (including Tennis Australia representatives), has been instrumental in the growth of the Australian Open. In return the Australian Open pays a premium venue hire compared to other hirers and delivers to Victoria and Australia an event that contributes over $250 million annually to the Victorian economy. Properly negotiated in good faith by both sides can produce a win-win situation.

In 2010, following two years of negotiation, the Victorian Government agreed to three five year development plans for Melbourne Park with the first costing $365million and each of the next two expected to cost similar amounts (not all for tennis facilities). Proudly, this was the last contract I signed before retiring as President of Tennis Australia after 21 years at the helm. In return Tennis Australia agreed to extend the contract to stage the Australian Open at Melbourne Park for another twenty years from an existing commitment of 2016 to at least 2036. The development of Melbourne Park to accommodate the growth required to maintain the Australian Open as a Grand Slam Event is assured. The growth of the Australian Open to match the other Grand Slams, together with this long term contract that ensures the Open will continue to grow is my legacy to Australian tennis. Proudly, the Australian Open continues to grow since my retirement and the Government has delivered on its commitment to capital investment.

In comparison, the assistance to the Australian Open from the Federal and local Governments has been minimal. Attempts to obtain a financial contribution from the Federal Government (or its sporting and tourism agencies) for the Australian Open or Melbourne Park were unsuccessful. The Federal Government seemed to take the view that the Australian Open had to be held in Australia, so it was a matter for the State Governments to assist so as to win selection to stage the event. However they do assist in matters like expediting player visas and player taxation returns as well as contributing to player development. Likewise the City of Melbourne assists in expediting approvals for all the construction of temporary facilities to stage the event and expediting approvals for other operational activities.

The Australian Government will assist the cost of bidding, and if successful, the subsequent costs of staging the Olympic Games, Commonwealth Games and World Cups, but remains reluctant to assist other major events. Local Government has become much more pro-active in recent years in helping stage smaller events in their area which can prove they will contribute to local tourism. Tennis Australia also combines with local government and tennis clubs and centres to fund improvements to the playing and other facilities. All States have a Major Events Unit with a budget to help attract events to their respective State or Territory. They also provide assistance through the Departments of Sport and Recreation, Tourism, Arts and the Premier's Department along with organisational help from Police, Transport, etc. Working with the State Government may be the most significant contribution to the success of any event, as has occurred with the Australian Open. Similarly Tennis Australia has received great assistance from the Queensland Government in building the Queensland State Tennis Centre and growing the ATP/WTA event in Brisbane and the Western Australian Government in building the new Perth Entertainment Centre with a retractable roof to replicate Australian Open conditions and staging the Hopman Cup there. On the other hand the NSW Tennis Centre was built for the 2000 Sydney Olympic Games and assistance from the NSW Government since then has been minimal, but positive discussions are now underway.

Whereas the Grand Slams have remained in the same country and city for over one hundred years (except the Australian Open which has remained in Melbourne since 1972), the ATP and WTA World Tennis Tours are a moveable feast of tournaments that survive in one city or country only if financially successful. If unsuccessful they are sold and/or move to another city or country. The new tournaments in Asia and the Middle East mostly had their origins in the USA or Europe, but without strong government support failed to survive and were sold to interests in Asia and the Middle East, who had strong political and financial support mostly from their National Government. Formula 1 motor car racing has shown a similar directional movement, primarily through government support, or lack of it.

8.11. Sponsorship

Sponsorship support is crucial to the financial success of any event. And any event worth organising should be able to achieve sponsorship. The challenge always is to develop the right level of financial and promotional packages to achieve the maximum sponsorship. The Australian Open had a wonderful naming rights sponsor in Marlboro for ten years, but replaced them with the Ford Motor Company in 1987, long before cigarette sponsorship was outlawed in Australia, as Tennis Australia expected this would eventually happen. Ford continued as major sponsor for sixteen years, but the Australian Open's future as a Grand Slam event, rather than just another major tournament, meant strategically doing away with naming rights sponsorship to achieve consistency with the other Grand Slams, none of which sell naming rights to their event. Nor do the mega events (Olympic Games, Soccer World Cup, Rugby and Cricket World Cups), and most of the other hallmark events (Super Bowl, Golf Majors, Tennis Grand Slams). So when Ford came to a renewal in 1994, a package was presented to them that transitioned the event from the Ford Australian Open to the Australian Open, sponsored by Ford in association with Heineken (who took up the two associate sponsor's positions). Of course many people continued to call the event the Ford Australian Open, and Ford enjoyed this bonus, but technically it was now, and still is, the Australian Open. The other Grand Slams were most appreciative of this significant change, which brought the Australian Open in line with the other Grand Slams. The new sponsorship model enabled Ford to keep involved and Tennis Australia to receive greater sponsorship than the previous sole naming rights sponsorship model. Today there are three associate sponsors at the Australian Open, sponsored by Kia in association with ANZ, Jacobs Creek and Rolex, as well as a number of other companies with on court signage rights and a host of other corporate opportunities. Sponsor activation continues to develop dramatically with on-site, live site and off site activities. There were significant risks in changing the model, but the smooth and successful transition with Ford and Heineken is something I did personally and am proud of this achievement.

The days when sponsorships were determined by personal relationships with the Chairman or CEO are long gone. Detailed proposals must be developed, detailed contracts signed and all promises delivered. But you are still dealing with people and nothing can replace a strong working relationship with the people in the company handling the sponsorship as well as the Chairman and CEO. Good communications builds relationships that survive the inevitable change in personal or even change in marketing strategies. The

Australian Open is proud of the quality of its sponsors and the long term relationships that have occurred.

Most sponsors will generally seek a one, three or five year sponsorship and request an option to automatically continue if they are happy. Options are valuable and can only come back to hurt you and you should always charge a premium for any option given to a sponsor. The reverse of this is to give yourself an option to continue after the expiry of the contract term, but this is rarely achievable, although Tennis Australia does have an option to continue staging the Australian Open at Melbourne Park under current terms and conditions when the contract expires in 2036, unless a new contract can be negotiated. Options giving the sponsor (or television network) first and last rights to renew look harmless, but my experience is that it is hard to get another sponsor to make an offer when it is known that the incumbent has the right to match their offer. Nevertheless, somehow the AFL got Channel 7 to pay a substantial premium for first and last rights to the football. And then at renewal managed to get Channel Nine to make a substantial bid that Channel 7 ultimately matched, but this is the exception rather than the rule. I have no problem with granting first rights only, because it makes sense to go to the existing sponsor or television network first, if both sides are happy with the relationship, and try to strike a mutually satisfactory arrangement, but beware of first and last rights. This is discussed further in Section 8.13 on contracts.

8.12. Television

In many ways the TV coverage achieved is a measure of the success of an event. It is also financially significant and helps determine the level of sponsorship that can be obtained. In fact, it is hard to sell major sponsorships until the level of TV coverage is known. Free-to-air television provides the greatest exposure and is the first preference for major events, and may even be required to do so under Australian Government Anti-Siphoning legislation. The Australian Open and the other Grand Slams achieve a good mix of free-to-air and pay-tv (or the secondary free-to-air channels now available with digital television) because of the volume of quality matches and the number of courts being used to play matches simultaneously over around twelve hours of play for most of the fourteen consecutive days of the tournament except the finals. In Australia, Network 7 has been a loyal supporter of tennis for over forty years, but the coverage has changed from Network Seven only, to a combination of Network Seven on Rod Laver Arena plus Foxtel on the other courts, to the current situation where the main Channel 7 free-to-air station covers Rod Laver Arena, while Seven Two covers Margaret Court Arena and Seven Mate covers Hisense Arena. All three courts have retractable roofs so play and coverage is guaranteed, whatever the weather (rain or extremely hot and play on uncovered is abandoned). Further, cameras now cover all courts in use and with digital technology every match on every court is available through the computer. International television networks can access any court and so televise their national players, even if they are not playing on the main televised courts. International coverage of the Australian Open has gone from a minor financial contributor to the event when it was played at Kooyong to a major contributor today. The Australian Open is now covered worldwide for fourteen days on a mix of free-to-air and pay television. Television access to all courts plus the extended hours of play from 11am to 11pm or later and the guarantee of live play whatever the weather has made the Australian Open a much more attractive proposition for national and international television and this is reflected in the ratings and the rights fees.

The move from grass at Kooyong to cushioned acrylic at Melbourne Park enabled matches to be played at night, but initial player, spectator and television reaction was negative. Players refused to play more than once at night during the tournament, the stadium was only half full on the seven nights play took place and television would not commence coverage until 8.30pm. Gradually all three groups, (players, spectators and television) could see the benefits of playing at night and now there is play on all fourteen nights, including the finals. The best players want to play at night in front of enthusiastic full houses and large live television audiences. The best match of the day will draw double the Australian television audience if played at night than it would if played during the day. Further it is much better timing for television coverage in Asia and Europe. When you play each day from 11.00am to midnight, there is hardly a country in the world that can’t show the Australian Open live in good viewing times.

Smaller events often have to pay for television coverage (rather than the reverse), whether it be on free-to-air or pay-tv, but the television coverage may be essential for securing sponsorship and government support. This is a delicate balance to be worked out by event owners and organisers.

8.13. Contracts

It is one thing to negotiate heads of agreement with sponsors, television, venues, athletes, etc., but it is quite another to convert this handshake or even written agreement into a detailed contract. Of course lawyers will be engaged by both sides, but this is no guarantee you won't be left with clauses you will come to regret at some stage in the future. These are not the key issues of money, length of contract, etc., but often the apparently innocuous clauses that lawyers often add in a detailed contract that were never discussed when the heads of agreement were reached, e.g. renewal options, promotional activities, ticket allocation, parking rights.

Generally the sponsor, television, venue, or athlete and event management companies will offer to draw up the first draft of the contract and this is often accepted by event organisers as a money saving exercise. It isn't. They will produce a document which has everything they want and you will be on the back foot trying to make change after change to rescue the situation. In the worst possible case, and this happened to me, there were changes needed in virtually every paragraph and even items that the lawyers for the other side thought should be different to the heads of agreement. They were forced to apologise, sacked the lawyers, and started again. But even this took a long time to finalise and more time to recover the goodwill between the two organisations.

One item that is rarely discussed and agreed until the final contract stage is the question of options to renew. As discussed in Section 8.11 on Sponsorship, any options granted to the other side can only cost you money and hence a premium should be charged for giving an option. Conversely, insert as many options as required for yourself into the agreement. The obvious example is in life assurance where many policies include an option to increase the coverage (usually up to double) without the need for a health examination. If a healthy person exercises this option, they could have achieved the same increase in coverage with a health check. But without the option clause, the unhealthy ones could only achieve the increase in coverage at a higher premium, if at all. Hence there is a cost to giving the option which cost should be added to the initial contract.

In the event management business the usual options requested are options to renew under current terms and conditions (or CPI increases), which can be below current market rates in a rising market, or first and last rights (being the right of the incumbent to match any other offer you may receive). The problem with the first is that you don't get current market rates, which could be higher, especially if you are growing the event as suggested in Section 8.4. The problem with the second is that you may get a low first offer from the incumbent knowing they will always have the opportunity to match any other offer made and it could be impossible to get competing bids if it is known that the incumbent has the rights to match their offer. This was the situation with most of the Australian Open contracts I inherited for sponsorship, television, etc. and it was only on renewal that I was able to eliminate this option clause from the contract and never signed such an option again. Conversely, the Tennis Australia has an option to continue to play the Australian Open at Melbourne Park under current terms and conditions if it and the Government can't agree to new terms when the renewal occurs.

If you have to give an option without getting a premium for it, then it should be for first right of negotiation only. If you have a happy relationship with a company, then you are probably going to negotiate with them in the first place anyway and it is certainly a sign of good faith, but leaves you free and unencumbered if those negotiations don't work out.

In general terms, the better the contract is for you, the longer the term you should try to achieve. The less satisfactory the terms, the shorter the contract. At time of signing a contract, presumably both sides are happy with the contract and its terms and conditions. The longer the contract, the more likely one side will gradually become unhappy with the terms and conditions and it is generally in the best interests of both parties if the advantaged party voluntarily agree to review the terms and conditions in return for an extension in the length of the contract. At Tennis NSW I inherited a 20 year contract whereby Ampol leased a petrol station site on New South Head Road for a fixed payment of $5000 per annum. Towards the end of the lease it was clear that the payment was ridiculously low, but at no stage did Ampol offer to renegotiate, but milked the agreement for all it was worth until it expired. Tennis NSW obtained professional advice that it was a difficult site to value but a lease of $50,000 per annum was not unreasonable. When the lease expired, Tennis NSW went to the market and achieved $210,000 pa plus CPI from Shell. Ampol complained bitterly at the lack of support after all they did for tennis (at the beginning they sponsored the NSW Open for just two years), but never came forward with a win-win early proposal. Likewise the Australian Open was contracted to Melbourne until 2016, but during 2008-10 Tennis Australia negotiated a 20 year extension to 2036 in return for substantial improvements to the site, commencing immediately, rather than waiting to 2016.

8.14. Asia/Pacific Market

As a Grand Slam, the Australian Open, played in Melbourne with a population of 4 million and Australia with a population of 23 million, is competing with Roland Garros and Wimbledon with access to over 8 million persons in Paris and London respectively, and over 50 million in France and UK, but more significantly over 500 million Europeans living in an area the same size as Australia. The US Open is held in New York with a population of 20 million plus and ready access to the total US market with over 300 million persons and over 500 million in North and Central America. To compete, the Australian Open had to access the Asian market with over 3 billion persons, or at least the South East Asian market with a population of 2 billion persons. It is not just the Grand Slams but other tournaments in Europe and USA have easy access to much greater populations than the Australian Open does. To reinforce this position, the marketing tag-line for the Australian Open changed from “Australia’s Largest Sporting Event” in the early 1990s to “Grand Slam of Asia/Pacific” which was introduced in 1995, well before other Australian events tried to move into the Asian market. It also reinforced to the emerging Asian powerhouses of China, India and the Middle East, whose Governments were all keen to put up the money for a Grand Slam in Asia, that there could only be four Grand Slams in the world and the Australian Open would be the closest Asia went to hosting a Grand Slam. Money alone does not make a Grand Slam, rather over one hundred years of tennis history as a major tennis nation plus a commitment to continue to present the best tournaments in the world. Unable to snare a Grand Slam, the Asian Countries have turned to the ATP and WTA tours and have been the successful bidders for the year-end ATP and WTA Finals, while an increasing number of other tournaments have moved from the US or Europe to Asia through sale or lease. There are now ATP or WTA Tournaments in 8 and 14 Asian cities respectively. Back in 1988 when the Australian Open moved from Kooyong to Melbourne Park only Japan had men's and women's tournaments at this level. The most recent development for Asian tennis is the Asian Premier League, developed to replicate the Indian Premier League in cricket.

Asians love racquet sports, and excel in badminton, squash and table tennis, but have really only turned to tennis since that sport was re-instated into the Olympics in 1988. China PR led the charge with the objective of winning gold medals in Beijing 2008. It surprised by winning the women's doubles gold in Athens 2004. The only Asian player to win a Grand Slam Singles Championships is the Chinese player Li Na, who won the women's singles in Paris in 2013 and then Melbourne in 2014, but has now retired. Chinese, Indian, Taipei and Japan players have been successful in Grand Slam doubles. Japan and India have reached the World Group in Davis Cup, but no Asian nation has ever won the Davis Cup.

The focus by the Australian Open on developing relations with Asia lead to the replacement of Ford by Korean car maker Kia in 2002, while the three Associate Sponsors, ANZ, Jacobs Creek and Rolex, were all attracted to the Grand Slam of Asia/Pacific to help achieve their growth targets in Asia. Asia is not an easy market to break into, in fact it is a conglomerate of separate national markets, and each requires its own individual approach. This is a long and difficult process, but the existence of Pan-Asian television in addition to national television helps accelerate the process. The strategic issue for Australia is to access Asia and its large and increasingly wealthy population.

Implementation is challenging and a huge learning curve and the approach can vary from country to country.

8.15. Australian success

Tennis is unique in that it presents a hallmark event in Australia with no guarantee that an Australian will be involved in the final stages of the tournament. Pat Cash played the first final at Melbourne Park in 1988, but since then only Lleyton Hewitt in 2002 has reached the final. No Australian women have reached the final over the same period. The success of the event despite the lack of success by Australian players is remarkable. Australian success on the court will contribute greatly to the growth of the Australian Open domestically as much as the participation by the world's best players. Lleyton Hewitt's Australian Open men's singles final against Marat Safin remained for many years the highest rated event or show on Australian television since the Sydney Olympics Opening Ceremony. For this reason the development plans for Melbourne Park include a state-of –the-art National Tennis Centre and Tennis Australia has employed a number of former champions and the best coaches to deliver an outstanding player development program. There are many exciting players on the horizon and currently Australia has both a male and female player ranked in the top twenty in singles.

8.16. One-off events

Some comments must be made about one-off events because there are many of them held, but with varying levels of success. By definition they obviously fail many of the principles outlined in this Chapter 8, namely fixed location, fixed dates, vision for the future, plans for growth, etc. They may satisfy the venue requirement if they can use an existing venue, but are unlikely to justify investment in temporary facilities. They can presumably buy participation and even achieve sponsorship and television coverage, but almost certainly will only exist with Government support. This could be local government support for (say) a local centenary celebration, State Government support for bringing an international event or show to its major city, or Australian Government support bidding for, and if successful bringing to Australia a mega event such as the Olympics or FIFA world Cup. International bodies generally retain the television and the prime sponsorship opportunities, and may even charge a substantial sanction fee, leaving the local organisation with primarily the public and corporate gate to meet the costs of providing the venues and running the event. This requirement can even apply to annual sanctioned international events, but primarily applies to one-off events where there is international competition to stage the event. Governments can justify this contribution to the event provided there are measurable tourism and other flow-on benefits in excess of their contribution.

8.17. Volunteers

No discussion of the Australian Open would be complete without special consideration given to the important role volunteers can play in staging an event. many ways it was the volunteers who made the 2000 Sydney Olympics so successful and friendly and set a new standard for future Olympics. All Australian Opens, including the last one at Kooyong in 1987, depended almost entirely on volunteers, with only the President, General Manager and Tournament Director and a few other staff in paid positions. The tournament could not afford paid staff.

The move to Melbourne Park in 1988 started a change to professional staff that has continued, so that today there are few volunteers left. The first changes arose because Melbourne Park was a multipurpose arena that operated throughout the year. It had its own staff as ushers, security, ground staff and similar positions and they wanted to work the Australian Open. Most were part-time and many said they only worked the rest of the year to ensure they could work the Australian Open. Umpires, linespersons, ball-kids, drivers, court services, statisticians, player services and the like remained voluntary initially. Umpires, followed later by linespersons, were the next to go professional because the quality of umpiring was an issue worldwide. Gradually, and with minimum disruption, all the other volunteer positions moved to partly paid and finally fully paid positions. Managing a work force containing a mix of volunteers and paid staff can be a challenging matter and requires goodwill on both sides. For information on collecting statistics at the Australia Open see Clarke and Norton (1994).

But the friendly and welcoming culture remained and is such an important feature of the Australian Open, often called “the friendly slam” by the players “. In many ways it was the volunteers who made the 2000 Sydney Olympics so successful and welcoming and set a new standard for the Olympics, replicated at London 2012.

8.18. Summary and Future Direction

In summary, the key strategic ingredients to a successful, well organised and growing event, apart from good management, staff and the annual time-line and check list for the event’s organisation, are (in no particular order):

1. a vision of where you want the event to be in 5,10 or even 20 years’ time,
2. a plan to achieve 10% growth pa for the foreseeable future,
3. legal protection for your ownership or lease of the event,
4. fixed location and dates so the event becomes part of the fabric of the community,
5. treat all participants well so they will return and encourage others to come,
6. gender equity is a strength and not a costly weakness,
7. develop a close working relationship with the venue owner/manager,
8. never underestimate the significant contribution government can play,
9. multiple sponsors may be a better arrangement than naming rights sponsorship,
10. television coverage defines the status of an event, so work on expanding it,
11. be wary of options and other contract small print ,
12. plan to grow from local to regional to state to national to Asia/Pacific to world event,
13. work to increase Australian participation as well as international participation and note

14. One-off events fail the above requirements and only survive with Government support.

The future of the Australian Open as one of the four Grand Slams has now been assured through the participation of all the world's best players, arguably the best organised tournament in the world, and the contracts Tennis Australia has entered into with the Victorian Government for the development of Melbourne Park as the Home of a growing Australian Open for the next twenty years, until at least 2036. The first of three five year development plans has been completed and the second five year plan is well under way. The first added more seats and a retractable roof on Margaret Court Arena (the third arena with such a roof) as well as more courts and the National Tennis Training Centre. The second delivered a new management building for Tennis Australia and Melbourne Park staff as well as substantial improvements to Rod Laver Arena and the total site. Planning for the third stage is well under way. While the Australian Open has three courts with retractable roofs, the US Open has recently built its first retractable roof, and Wimbledon has nearly completed its second roof. The French are committed to a roof, but facing challenges from local residents.

The Grand Slam status has been protected legally through the registration of Grand Slam by the Grand Slam Board, rather than the ITF, ATP or WTA, and can only be changed by unanimous vote. Further the relativity with other tournaments has been locked in by making the four Grand Slams equal in status and receiving double the number of computer ranking points as the next level of tournaments on the ATP and WTA Tours. This ensures all players will compete in the Australian Open. The January dates are secure and the shortening of the tennis season was achieved by finishing the circuit earlier and not by starting later and thus forcing the Australian Open into February or March. Although much more can be done to grow in Asia, the pathway has been established and a number of significant steps taken.

If Tennis Australia can achieve the desired growth in Asia, there is no reason why the Australian Open can't be the Grand Slam of the 21st century. To return to the first point, this must be the Vision for the next twenty or more years.

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Note. This list of references excludes my own and joint author publications (39) which are listed separately in Appendix 1.

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Note. There are another 39 reference articles in which I am author or joint author listed in Appendix 1

APPENDIX 1

GEOFF POLLARD ARTICLES

1. Geoff Pollard and Graham Pollard, “Optimal risk taking on first and second serves”, Tennis Science and Technology 3, 2007, edited by S Miller and J Capel-Davies, International Tennis Federation, Roehampton, London, 273-280.

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