Local Observation of Pair Condensation in a Fermi Gas at Unitarity

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We present measurements of the local (homogeneous) density-density response function of a Fermi gas at unitarity using spatially resolved Bragg spectroscopy. By analyzing the Bragg response across one axis of the cloud, we extract the response function for a uniform gas which shows a clear signature of the Bose-Einstein condensation of pairs of fermions when the local temperature drops below the superfluid transition temperature. The method we use for local measurement generalizes a scheme for obtaining the local pressure in a harmonically trapped cloud from the line density and can be adapted to provide any homogeneous parameter satisfying the local density approximation.

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Phase transitions and critical phenomena are central topics in low-temperature physics in settings ranging from the solid state [1] to superfluids [2] and cold atomic gases [3]. Clear identification of phase boundaries, however, can prove challenging in experiments. A key example is a Fermi gas with resonant interactions where bulk superfluidity was definitively shown via the observation of vortex lattices [4], yet detailed characterizations of the phase transition and superfluid fraction have taken much longer [5–8]. Superfluidity in three-dimensional (3D) Fermi gases occurs simultaneously with the formation of a Bose-Einstein condensate of fermion pairs. In a spin-balanced Fermi gas at unitarity, this pair condensation is difficult to observe directly, as it leads to only a very slight change in the atomic density [9]. Nonetheless, condensation has been verified using rapid sweeps of the effective attractive interaction during time-of-flight expansion, in which pairs are transformed into tightly bound molecules that preserve their center-of-mass momentum [10]. While effective, this method relies on the interplay of expansion and pairing dynamics [11] and, due to the necessity for expansion, is incompatible with obtaining local information.

An alternative signature of macroscopic order is the collective (Goldstone) mode [12], a long-wavelength bosonic excitation with linear dispersion and gradient equal to the sound velocity [13–15]. At large momenta, this mode evolves into a particlelike excitation with quadratic dispersion that, in two-component fermionic systems, physically represents the scattering of zero-momentum pairs from the condensate [16–19]. In this Letter, we study this mode in a trapped spin-balanced Fermi gas at unitarity using high-momentum Bragg spectroscopy and find that it provides a dramatic signature for pair condensation that can be studied locally.

For 3D atomic gases, absorption imaging provides only a 2D projection of inhomogeneous atom clouds which integrates over regions with different densities. Thus, local information, such as the precise density or temperature at a phase boundary, is generally not accessible in a standard image. Techniques such as the inverse Abel transform can reconstruct the local density, as was recently used for the measurement of the equation of state of the unitary Fermi gas [7]; however, one often wishes to know more than simply the density. For example, measuring dynamical variables generally requires perturbing the system with a probe particle or photon which can destroy the (elliptic) cylindrical symmetry necessary for the inverse Abel transform [7]. Recently, it was shown that probing a small region near the center of an inhomogeneous cloud can provide a good representation of a homogeneous system [20,21]. Here, we present an alternative scheme that does not require imaging of clouds with (elliptic) cylindrical symmetry and facilitates the measurement of dynamic variables, including the dynamic spin susceptibility [22], density-density response [17], as well as Tan’s universal contact [21,23,24]. The method generalizes a scheme for obtaining the local pressure from the 1D line density [6,25] and shows that this conceptual approach is more powerful than previously realized.

Consider the measurement of the imaginary part of the dynamic susceptibility (density-density response) \(\chi^{\imath}_{(k,\omega)}\) using Bragg spectroscopy [Fig. 1(a)], where \(k\) is the Bragg wave vector and \(\hbar \omega\) is the Bragg energy. A bulk Bragg spectrum \(\chi^{\imathB}_{(k,\omega)}\), representing the density-averaged response of an inhomogeneous atom cloud, is obtained by illuminating the atoms with two Bragg lasers intersecting at an angle of \(\theta = 84^\circ\) and measuring the total momentum imparted to the cloud as a function of \(\omega\) [26,27]. The atomic recoil frequency is defined as \(\omega_r = \hbar k^2/(2m)\), where \(m\) is the mass of a single atom. Bragg spectroscopy has previously yielded the bulk dynamic and static structure factors of trapped Bose [26,28] and Fermi gases [17,19], as well as Tan’s universal contact parameter [19,23].

In the experiments which follow, we use an evaporatively cooled cloud containing a balanced mixture of approximately \(N/2 = 250 000^6\)Li atoms in each of the lowest two
Bragg lasers for clouds. Bragg spectroscopy is performed by pulsing on the displacement \[17,19\]. The Bragg lasers are detuned from the cloud into a series of horizontal strips (typically 10 to 30 μm wide) and held for a further 500 ms for reequilibration or by releasing the atoms typically produce clouds with temperatures of \(\approx 500 \text{ MHz} \) after a Bragg pulse was applied with Bragg frequencies of \(\omega = \omega_r/2\) [Fig. 1(b)] and \(\omega = 0\) [Fig. 1(c), no Bragg kick], respectively. While these images appear nearly identical, subtracting them [Fig. 1(d)] reveals that the Bragg pulse not only displaces atoms from left to right, but that the strongest response comes from the center of the cloud. Furthermore, for different frequencies, the z dependence of the response changes. Figure 1(e) shows a difference image for \(\omega = \omega_r\), where the response is less intense but originates from a broader area of the cloud.

To analyze these images, we determine a \(z\)-dependent line response function \(\tilde{\chi}''_{(k,0)}(z)\) which quantifies the atom displacement as a function of \(z\). This is found by dividing the image into a series of horizontal strips and evaluating the (left to right) center-of-mass displacements within each strip. \(\tilde{\chi}''_{(k,0)}(z)\) is given by the density-weighted response function integrated over \(x\) and \(y\) [31]:

\[
\tilde{\chi}''_{(k,0)}(z) = \frac{1}{\tilde{n}(z)} \int_{-\infty}^{\infty} \chi''(\mu(x), T)n(\mu(x), T)dx\,dy, \tag{1}
\]

where \(\tilde{n}(z) = \int n(\mu(x), T)dx\,dy\) is the (doubly integrated) line density and \(\chi''(\mu(x), T)\) and \(n(\mu(x), T)\) are the local response and density of a cloud with chemical potential \(\mu(x)\) and temperature \(T\), respectively. Equation (1) assumes the local density approximation (LDA), where \(\mu(x) = \mu_0 - V(x)\), \(\mu_0\) is the chemical potential at the trap center, and \(V(x)\) is the confining potential. We expect the LDA to be valid for \(\chi''_{(k,0)}(\mu(x), T)\) at large \(k\), as the Bragg response is primarily determined by correlations on a length scale of \(\lesssim k^{-1}\). In our experiments, \(k^{-1} = 80 \text{ nm}\), which is much smaller than the mean harmonic oscillator quantization length \(l_{HO} = \sqrt{\hbar/(m\omega_{HO})} = 5 \mu m\). Thus, provided the atomic density also satisfies the LDA (i.e., \(\mu \gg \hbar\omega_{HO}\)), as has been validated experimentally \[6\], Eq. (1) will be valid here.

For a gas confined in a harmonic potential, Eq. (1) can be rewritten as an integral over the chemical potential using \(dx\,dy = -2\pi/(m\omega_x\omega_y)\,d\mu\), where \(\omega_x\) and \(\omega_y\) are the trapping frequencies in the \(x\) and \(y\) directions, respectively \[25\].

Making this substitution, differentiating with respect to \(z\), and rearranging \[31\], we extract the local homogeneous value of \(\chi''_{(k,0)}(\mu(z), T)\) along the axis of the trap approximately 600 MHz from the nearest atomic transition to probe the density-density response \[22\].

As the atom cloud is elongated along \(z\), and the Bragg lasers transfer momentum to the atoms in the \(x\) direction, it becomes possible to resolve the response from different \(z\) positions along the cloud, provided a short time of flight is used (compared to the time scale for dynamics along \(z\)). Figures 1(b) and 1(c) show optical density images \(\tilde{D}_{(k,0)}\) (averaged over 10 runs of the experiment under the same conditions) 300 μs after a Bragg pulse was applied with Bragg frequencies of \(\omega = \omega_r/2\) [Fig. 1(b)] and \(\omega = 0\) [Fig. 1(c), no Bragg kick], respectively. These images appear nearly identical, subtracting them [Fig. 1(d)] reveals that the Bragg pulse not only displaces atoms from left to right, but that the strongest response comes from the center of the cloud. Furthermore, for different frequencies, the \(z\) dependence of the response changes. Figure 1(e) shows a difference image for \(\omega = \omega_r\), where the response is less intense but originates from a broader area of the cloud.
\[ \chi''_{(k, \omega)}(\mu, z, T) = \frac{\partial [\chi''_{(k, \omega)}(z) \tilde{n}(z)]]}{\partial \tilde{n}(z)}, \]  

where \( \mu = \mu_0 - V(0,0,z) \). This simple relation connects the local response along the axis of the trap to the derivative of the line response multiplied by the line density. We emphasize that this procedure is completely general and can be adapted to provide the local value of any quantity satisfying the LDA. The images required for Eq. (2) [Figs. 1(d) and 1(e)] no longer satisfy the symmetry requirements for performing an inverse Abel transform. The only requirement is that the cloud was initially confined in a harmonic potential.

We now proceed to the measurement of the homogeneous response \( \chi''_{(k, \omega)} \). At unitarity, this will be a universal function of the relative temperature \( T/T_F \) and wave vector \( k/k_F \), where \( T_F \) is the local Fermi temperature. While \( k \) and \( T \) are uniform across the cloud, this method allows us to probe a range of \( T/T_F \) and \( k/k_F \) values simultaneously using a single cloud. The local density sets the energy scale through the Fermi energy \( E_F(r) = k_B T_F(r) = (\hbar^2/2m)(3\pi^2 n(r))^{2/3} \). Similarly, the Fermi wave vector varies as \( k_F(r) = (3\pi^2 n(r))^{1/3} \). The local response along the trap axis will therefore span a range of \( T/T_F \) and \( k/k_F \) as the density along the trap axis changes. We find the local density \( n(0,0,z) \) either from the derivative of the line density \([25,32]\) or from the inverse Abel transform \([7]\) of a trapped cloud before Bragg scattering.

Figure 2 (main panel) shows the local (homogeneous) response functions, constructed using Eq. (2), for atoms below (blue and green lines and circles) and above (orange and red lines and circles) the superfluid transition temperature \( T_c = 0.167 T_F \) \([7]\). Local values of \( T/T_F \) and \( k/k_F \) contributing to each measurement are given in the text. Solid lines are a guide to the eye. Spectra below \( T_c \) are dominated by a sharp feature at \( \omega_0/2 \) corresponding to pair scattering, while the spectra above \( T_c \) show a rounder response peaked just below \( \omega_c \) corresponding to the continuum of single-particle excitations. Inset: Bulk response of a trapped unitary Fermi gas at a temperature of \( T = 0.087 T_F \) \([19]\) which shows a weaker pairing signature at \( \omega_0/2 \), as well as a broader peak centered near \( \omega_c \), due to the averaging over a range of relative temperatures and wave vectors in an inhomogeneous trapped cloud.

The spectra possess no sharp features and show a much weaker temperature dependence. Also plotted (inset, Fig. 2) is a bulk spectrum of a cold trapped gas which shows both pair and single-particle peaks due to the averaging over different densities \([19]\).

Next, we perform measurements of the Bragg response at a frequency of \( \omega_0/2 \) corresponding to the top of the pair peak. For this, we use a 200 \( \mu \)s Bragg pulse to increase the signal-to-noise ratio and improve spectral resolution. Several clouds with bulk temperatures ranging from \( 0.08 T_F \) to \( 0.6 T_F \) were used, and the measured (local) center-of-mass displacements were binned according to \( T/T_F \) and \( k/k_F \) to produce a false color image of the local response \( \chi''_{(k,\omega_0/2)} \) \([3a]\). Dashed white lines show the range of temperatures and wave vectors spanned by individual clouds (average of 20 experimental runs) used to construct the image. At high temperatures, the response is relatively flat, showing a weak dependence on \( T/T_F \) and \( k/k_F \); however, a sharp increase is observed below \( T/T_F \sim 0.2 \). Examining the response over the smaller range of wave vectors \( 4.5 < k/k_F < 5.5 \), indicated by the shaded region in Fig. 3(a), which includes data both above and below \( T_c \), we can more clearly see the temperature dependence \([3b]\). At high temperatures \((>0.2 T_F)\), the response increases slowly with decreasing
temperature. However, a rapid increase occurs when the temperature drops below 0.18$T_F$. Also shown is the calculated response of an ideal Fermi gas at $k = 5k_F$ (dashed black line).

Because of energy and momentum conservation at the Bragg condition, the sudden increase in the response at $\omega_r/2$ below $T_c$ signifies the accumulation of zero-momentum pairs in the condensate. Despite the fact that the Bragg recoil energy is more than 10 times larger than the pairing gap $\Delta$ ($\Delta = 0.44E_F$ at unitarity [33]), two atoms can still scatter as a pair, provided any increase in their relative energy is less than $\Delta$. At $k \sim 5k_F$, this collective (paired) mode at unitarity lies within the continuum of single-particle excitations [15] yet remains highly visible at $\omega_r/2$ [16], so both pair and single-particle scattering contribute significant weight to our measurements. It is interesting to note that the sudden appearance of the pair peak does not coincide with a strong enhancement of the contact below $T_c$ [21,34]. This highlights the difference between the dynamic response, which reveals the pairing peak, and the integrated (static) response used to obtain the contact [19].

Empirically, we find that the data in Fig. 3(b) in the vicinity of $T_c$ are approximately linear, with two different slopes above and below the transition temperature. Fitting straight lines over the ranges $0.11 < T/T_F < 0.17$ and $0.2 < T/T_F < 0.6$ (dash-dotted blue lines), we estimate the critical temperature for pair condensation from the intercept to be $T_c = 0.18 \pm 0.03T_F$, in good agreement with the recent thermodynamic determination of 0.167$T_F$ (solid brown lines in Fig. 3) [7]. Our error bars include uncertainties arising from the finite time of flight and spatial averaging ($\sim 20 \mu m$) used in this measurement. We note that even though the peak associated with pair condensation is visible in the bulk Bragg spectrum (inset of Fig. 2), we cannot use bulk measurements to determine $T_c$, as density averaging necessarily includes a large spread of different relative temperatures and wave vectors in the bulk response. Instead of showing a sudden change at $T_c$, the bulk response at $\omega_r/2$ displays only a smooth and more gradual increase as the temperature is lowered.

In the normal phase, above $T_c$, the measured response displays the opposite temperature dependence to an ideal gas. This shows the buildup of short-range pair correlations as the temperature is lowered; however, as both pair and single-atom scattering are present, we cannot identify this as the scattering of noncondensed (bound) pairs [35]. Local measurements of the dynamic spin susceptibility [22] at $k \lesssim k_F$ could be used to clarify this issue of pseudogap pairing. At higher temperatures, the response should approach the ideal gas result which turns over near $2T_F$ and begins decreasing (not shown).

In summary, we have shown that a scheme developed to measure the local pressure using a (nonuniform) harmonically trapped quantum gas [25] is more powerful than previously realized and is capable of yielding local parameters not accessible by conventional methods such as the inverse Abel transform. We have used this technique to make the first measurements of the homogeneous density-density response function $\chi''(k,\omega)$ of a Fermi gas at unitarity using Bragg spectroscopy. Measuring the local response allows us to connect features in the Bragg spectrum with a specific local temperature, revealing a strong signature of pair condensation when the temperature drops below $T_c$. 

FIG. 3 (color online). (a) False color image of the local (homogeneous) response $\chi''(k,\omega)$ of a unitary Fermi gas as a function of the relative temperature $T/T_F$ and wave vector $k/k_F$. The image was constructed by measuring the local response for a range of clouds prepared at different initial temperatures and binned according to temperature and wave vector. Dashed white lines show the temperatures and wave vectors spanned by individual (averaged) clouds. A rapid increase in the response is observed for temperatures $T/T_F < 0.2$, due to the appearance of the pair condensate. (b) Local response versus relative temperature for wave vectors in the range $4.5 < k/k_F < 5.5$, using data indicated by the grey shaded region in (a). This smaller range of momenta shows the sudden increase in the response below $T_c = 0.167T_F$ [7] (solid brown lines in (a) and (b)). Also shown (dashed black line) is the calculated response for an ideal gas at $\omega_r/2$ for $k = 5k_F$ which shows the opposite temperature dependence to the data. Dash-dotted blue lines are straight line fits to the data over the temperature ranges $0.11 < T/T_F < 0.17$ and $0.2 < T/T_F < 0.6$ used to estimate the critical temperature.
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