Transmission Augmentation in an Oligopoly Electricity Market – Part I
(Mathematical Formulation)

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Abstract- This paper proposes a Three-Stage Model for transmission augmentation in restructured electricity markets. The mathematical formulation of the model is developed based on the game theory. Transmission Network Service Provider, TNSP, Generating Companies, GenCos, and Market Management Company, MMC, are placed in different stages of the model. These stages are linked to each other using the Leader-followers game and the concept of Nash equilibriums. An increase in transmission capacity can have two benefits for the electricity market; firstly, efficiency benefit in terms of improving the social welfare of the electricity industry, and, secondly, competition benefit which leads to increasing competition among generating companies. The introduced Three-Stage Model can capture both benefits of transmission projects in electricity markets. An effective numerical method is designed for solving the developed Three-Stage Model. A modified IEEE 14 example system is employed to show the effectiveness of the methodology. This paper has been organized in two parts. First part deals with the mathematical formulation of the algorithm and second part deals with the numerical studies. What follows is the first part of the paper.

I. INTRODUCTION TO PART I

The National Electricity Market, Australia, NEM, is a gross pool with Queensland, New South Wales, Victoria, South Australia, and Tasmania as participating jurisdictions. The National Electricity Market Management Company is the market operator.[1], [2]. The Queensland transmission system with the total line length of around 11902km and the maximum demand of 8232MW is one of the longest transmission systems in the NEM. Powerlink is a regulated monopoly business responsible for the augmentation of the high voltage transmission system in the state of Queensland to provide all NEM participants with open and non-discriminatory access.

Powerlink, as the Transmission Network Service Provider, TNSP, is in charge of transmission planning and operation within the state of Queensland. This central transmission entity bears any transmission investment cost. Unlike the reliability-based planning approach, the market-based augmentation approach has not been accommodated properly in the Queensland transmission planning methodology. In addition, the regulatory test introduced by the Australian Competition and Consumer Commission, ACCC, as a market-based metric can not capture the whole value of the transmission upgrade.

Several authors have examined the market-driven transmission augmentation. In [3], the authors have done a review on different aspects of transmission planning problem with a list of issues regarding this problem. The definition of the objective function to measure the goodness of a solution, and the decision criterion framework can be seen on top of their list.

References [4] and [5] have carried out a comparison between the transmission planning framework in traditional and competitive electricity markets. Bridging the gap between the economic and engineering considerations, and the proper addressing of market mechanisms in the transmission augmentation framework have been concluded in both. [6][7][8][9][10][11][12][13] have studied the different aspects of transmission augmentation and collected different challenges regarding this very complex and multi dimensional problem. More and less, all of them define a good transmission expansion plan as the one with good economic effect while meeting reliability requirement. The improving of the social welfare has been defined as the economic benefit of the transmission expansion projects in restructured electricity industry.

Using social welfare as the economic benefit of the transmission projects, [14] has designed a regulatory contract that induces TNSP to optimally expand the grid. In the proposed framework, the author has proved that the optimal transmission capacity is such that the expected marginal value of capacity equals the marginal cost for each line. [15] uses two heuristic procedures for measuring the effect of transmission capacity on social welfare. Congestion cost and congestion revenue as two by-products of the MMC dispatching module have been employed in [16] for transmission expansion in competitive power markets. [17], [18] [19][20][21][22] have measured the economic benefit of transmission projects only in terms of improving the social welfare.

On the other hand, [23] warned analysts against the practice of neglecting the transmission network, and showed numerically that transmission expansion reduces generators’ market power. Models of imperfect quantity competition have been developed in [24], [25], [26]. Models of imperfect price competition have been proposed in [27]. [28] has examined empirically the bidding behaviour of generators in England and Wales, including the impact of transmission constraints. Using a simplified version of the power network in California, [29] has quantified the impact of local market power. [30] and [27] show that generators benefit from a reduction in transmission capacity. [28] finds in England and Wales generators protected by transmission constraints bid significantly higher than those without this status. Obviously, generating companies’ strategic bidding is as an ultimate outcome of market power. This could result in a transfer of transmission rents from MMC or owner of the transmission assets to the GenCos.
Accordingly, [31] uses a simple three-node network to show the competition effect of transmission capacity. The competition effect of transmission capacity has been discussed mathematically in [32] by using a simple three-node network as a stylized version of North America. TEAM methodology introduced by the California ISO [33] can be acknowledged as a good model for market-based transmission augmentation. However, it has two drawbacks. Firstly, the strategic bidding of GenCos has been estimated through an tailor-made and empirical methodology which limits its application. Secondly, the whole framework does not have an integrated mathematical framework.

In addition, the interaction of transmission network service provider, market management company and generating companies in the market-based augmentation of transmission system has not received enough attention.

Briefly speaking, the Lack of modelling of the competition effect, using empirical models for strategic bidding of GenCos, lack of integrated mathematical framework for transmission augmentation, and proper interaction modelling among TNSP, MMC, and GenCos are the main issues which are addressed throughout this paper.

Covering the aforementioned shortcomings, we propose a Three-Stage Model based on the leader-follower game. In stage 1, TNSP moves and expands the transmission system based on the efficiency benefit, competition benefit, and investment cost of the transmission expansion project. GenCos take the transmission planning schedule and compete for having the highest share in the electricity market. Nash equilibrium point is found in stage 2. Finally, in stage 3, MMC clears the market by solving a security-constrained economic dispatch.

The rest of paper is organised as follows. The mathematical formulation of the problem is detailed in section 2. Section 3 deals with the solution of the problem using the extended Leader-followers game using the Nash equilibrium point.

To show the effectiveness of the algorithm a modified IEEE 14-bus test system has been used and the results of study have been collected in section 4. Section 5 deals with further discussion on the propose solution and finally, section 6 concludes the paper.

II. THE THREE-STAGE MODEL OF TRANSMISSION AUGMENTATION

We propose a three-stage model for transmission augmentation. At stage 1, TNSP determines the optimal expansion of the transmission network, according to the objective function introduced in subsection A. The owner of transmission assets follows the planning schedule and receives revenue independent of the congestion of the transmission lines. At stage 2, each GenCo bids strategically to maximise its profit function. Nash game is used to find the equilibrium point of the market. Finally, at stage 3, the electricity market takes place. In this stage, MMC runs a security-constrained economic dispatch to settle the market.

Matched with NEM, Australia, we assume away strategic behaviours of the owners of the transmission assets, and retailers. All GenCos are assumed to be independent and accordingly there is no “multi-unit effect” [31]. Also, TNSP is a proactive planner who expands the transmission system based on the worst case scenario of the horizon year transmission system [34].

A. The Mathematical Formulation of Three-Stage Model
A.1 Transmission Network Service Provider (TNSP)

Suppose a TNSP has \( m \) upgrade options and \( n \) expansion options for the market-based augmentation of the high voltage transmission system in its given territory. For the \( m \) upgrade options, \( f_{1}^{iu}, l \in \mathbb{Z} \) and for the \( n \) expansion options, \( f_{l}^{ie}, l \in \mathbb{Z} \) are the vectors of maximum thermal capacity which can be built on the transmission corridors of upgrade and expansion, respectively. Similarly, \( tc_{l}^{iu}, l \in \mathbb{Z} \) and \( tc_{l}^{ie}, l \in \mathbb{Z} \) are the vectors of the investment cost for the transmission upgrade and expansion projects, respectively. Figure 1 shows this situation.

![Figure 1 TNSP's transmission projects](image)

Since the TNSP pays the investment cost of upgrade or expansion, it is desirable to upgrade and/or expand the transmission system with the minimum cost. Mathematically, the TNSP’s objective function can be formulated as (1).

\[
\begin{align*}
\text{Max } & \Pi = SS - \alpha (MR) - \left( \sum_{l=1}^{m} f_{1}^{iu} tc_{l}^{iu} + \sum_{l=m+1}^{m+n} f_{l}^{ie} tc_{l}^{ie} \right) \\
\text{s. t. } & 0 \leq f_{1}^{iu} \leq f_{1}^{ru} , l = 1, ..., m \ \\
& 0 \leq f_{l}^{ie} \leq f_{l}^{re} , l = m + 1, ..., m + n
\end{align*}
\]

(1)

Where the \( f_{1}^{iu} \) and \( f_{l}^{ie} \) are integer numbers corresponding to the number of circuits in corridor \( l \), \( f_{1}^{ru} \) and \( f_{l}^{re} \) are the maximum circuits which can be built in transmission corridor \( l \). Accordingly, \( f_{1}^{iu} \) and \( f_{l}^{ie} \) are the TNSP’s design parameters.
SW is the social welfare of the electricity industry defined as (2).

\[ SS = VOLL \cdot d - C' \cdot g \]  

Where in (1), \( VOLL \) is the value of lost load for each retailer, \( d \) is the served demand of the system, \( C' \) is the strategic bidding of each GenCo and finally \( g \) is the total generation of the GenCos.

\( MR \) is the monopoly rent of the electricity industry defined as

\[ MR = \sum_{i=1}^{N_G} \max \{ (\Omega'_i - \Omega_{i}^f), 0.0 \} \]  

In (3), \( \Omega'_i \) is the profit of the \( i \)th GenCo under strategic bidding and \( \Omega_{i}^f \) is the profit of the same GenCo when he bids its true marginal cost. \( N_G \) is the total number of the GenCos participating in the market. From the view point of economics a firm has market power if it can change the price by changing its output and in doing so it earns extra profit[35]. Accordingly, if the strategic bidding of a GenCo can lead to the extra profit, it will be accounted in the \( MR \) index else it will set to zero.

\( \alpha \) is the weighting factor of the competition effect of transmission capacity. \( \alpha \) is set by the electricity market regulator based on his judgement on the value of transmission investment compared with the efficiency value and competition value of the transmission capacity.

### A.2 Independent Generating Companies (GenCos)

Suppose a GenCo has a linear cost function of the form (4).

\[ C_i = C(g_i) = c_i g_i \]  

Where in (4), \( c_i \) is the generation cost coefficient and \( g_i \) is the generation output of generator \( i \). Considering (4), the GenCo objective function can be written as (5).

\[ \Omega'_i = \lambda_i g_i - C(g_i) \]  

Where in (5), \( \lambda_i \) is the price of electricity at the connection point of \( i \)th GenCo. \( \lambda_i \) is the by-product of the settlement process of the MMC.

Bertrand model, Cournot model, Leader-followers model, and Nash model could be used for modelling of competition among GenCos. The Bertrand and Cournot model of competition does not appear to be consistent with NEM in Australia, where generators quote price/quantity schedules, and the MMC determines the quantity for each GenCo. Although consistent with the electricity market, the Leader-followers model of competition is not well-tractable analytically. Using the Nash model of competition as a good compromise between the Bertrand and Cournot model, the equilibrium point of the electricity market is formulated through (6).

\[
\begin{align*}
\max_{s_i} & \quad \Omega'_i(s_i) \\
\text{s.t.} & \quad s_{i}^\text{min} \leq s_i \leq s_{i}^\text{max} \\
& \quad \Omega_{N_G}(s_{N_G}) \\
\end{align*}
\]

Where in (6), \( \downarrow \) is the Nash equilibrium of the GenCos, \( s_i \) is the strategic bidding factor which multiply by the real cost function to build the bidding strategy, \( s_i \) is bounded by a lower and upper limit which are set by the market regulator. Each Genco solves his maximisation problem considering an estimation of the other GenCos and reaction of the MMC. This process would be repeated until finding the Nash equilibrium point of the market.

### A.3 Market Management Company (MMC)

Considering the National Electricity Market in Australia, suppose that the electricity market is a double-sided gross pool. MMC collects the bids and uses a security-constrained economic dispatch process for clearing the market. With \( N_R \) registered generators and \( N_R \) registered retailers in the market, vectors \( C' \) and \( VOLL \) are the strategic bid of \( N_G \) generators and the value of lost load for the \( N_R \) retailers. Accordingly, the mathematical formulation of the security-constrained economic dispatch has been given in (7).

\[
\begin{align*}
\min_{\theta, g, d} & \quad C' \cdot g - VOLL \cdot d \\
\text{s.t.} & \quad [B_\theta'][\theta] = g - d \\
& \quad - (f_0^0 + f_i^u + f_i^e) \leq [H'_i][\theta] \leq f_i^0 + f_i^u + f_i^e \\
& \quad g \leq g \leq \bar{g} \\
& \quad d \leq d \leq \bar{d} \\
\end{align*}
\]

In (5), \( [B_\theta'] \) and \( [H'_i] \) are \( N_R \times (N_R - 1) \) and \( N_i \times (N_i - 1) \) matrices where the column related to the slack bus is omitted. \( N_R \) \( N_R \) and \( N_i \) are the total number of buses and total number of lines in the system. \( \theta \) is the vector of bus angles. \( g \) and \( d \) are the generation level of committed generators and the served demand of retailers. \( \theta, g, \) and \( d \) act as the decision variable of (7). These variables are bounded by their minimum and maximum values. Existing capacity of the transmission system has been modelled through vector \( f_i^0 \).

Section 3 discusses a mathematical solution of the three-stage model based on the extended Leader-followers game using Nash equilibrium concept.
III. THE THREE-STAGE MODEL OF TRANSMISSION AUGMENTATION

The mathematical formulation of the three-stage model of transmission augmentation is as (8).

As in (8), the mathematical formulation of the three-stage model is a three-level optimisation problem. The decision maker at one level may be able to influence the behaviour of a decision maker at another level but not completely control his actions. This is exactly what happens in the electricity market, say, TNSP decision has obvious effects on the GenCos’ strategic bidding but GenCos are completely independent from TNSP.

Programming problem (8) can be categorised as the non-linear multi level programming problem [36]. Generally, non-linear Multi level programming programs are intrinsically hard [36], [37]. The proposed numerical method for solving (8) can be explained in three partitions.

Partition 1 is a Mathematical Programming with Equilibrium Constraints, MPEC, which represents the profit maximisation of the GenCo. In this partition, each GenCo maximises its own revenue considering its estimation on bidding of other GenCos.

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The MMC’s central dispatch process can be generalized in (9).

\[
\text{Min } C^T \cdot x \\
\text{s.t.} \\
[B] x \leq D \\
[A] x = 0 \tag{9}
\]

The introduced vectors and matrices in (9) in terms of vectors and matrices in (8) are defined in appendix I.

Writing the Kuhn-Tucker (KT) optimality conditions for (9), and then simplifying and differentiating of KT optimality conditions with respect to \( s_i \) yield to the following sets of equations, [38], [39], [40], [41].

In (10), \( S \) is the vector of slack variables. Using the transpose properties of \( ([P] + [Q])^T = [P]^T + [Q]^T \) and \( ([P][Q])^T = [Q]^T[P]^T \), (10) can be written in the following matrix form;

\[
\begin{pmatrix}
\mu^T [B] & -S^T \\
[A] & 0 \\
0 & B^T & A^T
\end{pmatrix}
\begin{pmatrix}
\frac{\partial x}{\partial s_i} \\
\frac{\partial D}{\partial s_i} \\
\frac{\partial S}{\partial s_i} \\
\frac{\partial \lambda^T}{\partial s_i}
\end{pmatrix}
= \begin{pmatrix}
0 \\
0 \\
0 \\
-\frac{\partial C^T}{\partial s_i}
\end{pmatrix} \tag{11}
\]
\[
\frac{\partial \Omega_i}{\partial s_i} = \frac{\partial g_i}{\partial s_i} + (\lambda_i - c_i) \frac{\partial g_i}{\partial \lambda_i}
\] (12)

Using the gradient search, the algorithm starts with an initial guess for \( s_i \), and update the value of \( s_i \) based on the equation (13).

\[
s_i^{new} = s_i^{old} - \kappa \frac{\partial \Omega_i}{\partial s_i}
\] (13)

Where \( \kappa \) is the step length of movement to the new solution. If the new value of the \( s_i \) is not in the specified boundary, the algorithm sets the new value to the upper or lower bound of the \( s_i \) based on the violating of upper or lower bound, respectively. In each iteration of \( s_i \) update, the algorithm checks the variables of the (11) in terms of reaching their upper or lower limits. In either case, the algorithm gets back the previous value of \( s_i \) and terminates the iteration.

For locating the global bidding of each GenCo, the bidding space is divided to a few segments and the optimal bid on each segment is calculated and saved. The best bid of these segments is selected as the best bidding strategy of the GenCo. The proper division of the bidding space is very effective in locating the global optimum of the GenCo optimisation problem as formulated in partition 1 of (8).

Partition 2 is an Equilibrium Problem with Equilibrium Constraint (EPEC) which finds the Nash equilibrium point of GenCos. Generally, the meaning of Nash equilibrium is that no GenCo can get extra profit by unilaterally deviating from the equilibrium, i.e.,

\[
\Omega_i(s^*_i, s^{*-}_i) \geq \Omega_i(s_i, s^{*-}_i) \quad i = 1, ..., N_G
\] (14)

Where in (14), \( s^{*}_i \) is the vector of optimal response of other GenCos. Diagonalization method and sequential nonlinear complementarity algorithm are used for solving Nash equilibrium problem. Nonlinear Jacobi and nonlinear Gauss-Seidel are two diagonalization methods. Nonlinear Gauss-Seidel is used for the solution of the partition 2 of (8) and is described as follows:

Step1. Initialization.
Chose a starting point \( s_i^0 \) for each GenCo, the maximum number of Gauss-Seidel iteration \( L \), and an accuracy tolerance \( > 0.0 \).

Step2. Loop over every MPEC (Partition 1 of (8)).
Suppose the current iteration point of \( s_i \) is \( s_i^l \). For each GenCo \( i \), the MPEC of partition 1 is solved while fixing \( s^{*}_{-i} = (s_i^{l+1}, s_i^{l+1}, s_i^{l+1}, ..., s_i^{l}) \)

Step3. Check convergence.
If \( l < L \), then increase \( l \) by one and repeat step 1.

TNSP starts with considering all the candidate transmission projects that are feasible. Set of feasible transmission projects can be produced by the experience of the TNSP of available corridors of upgrade or expansion. Iteratively, TNSP considers the lines in the set of candidate lines, still available at each iteration, he chooses the most beneficial ones based on the efficiency effect and competition effect of each transmission project. Accordingly, a transmission project can be approved if the objective function of the TNSP introduced in (8) for the transmission project is positive, otherwise the transmission project would be rejected. The exploration will continue until TNSP can not approve building of any more transmission line or the budget is run out.

The process of decision making by TNSP is illustrated in figure 2. Considering the above numerical solution for the introduced three-stage model, section 4 deals with application of the methodology to the modified IEEE 14-bus example system.

IV. APPLICATION TO THE MODIFIED IEEE 14-BUS EXAMPLE SYSTEM AND CONCLUSION OF PART I

To show the effectiveness of the algorithm, IEEE 14-bus example system has been firstly modified. At the next step, the algorithm has been applied to the developed test system.

There are five competing generators labelled as G1 to G5, and eleven competing retailers labelled as R1 to R11 in the 14-bus example system. The TNSP is responsible for the market-based augmentation of the system.

Two cases of monopoly rent and without monopoly rent have been studied. The results are compared and the conclusions are drawn.
The mathematical framework developed for augmenting of transmission systems, explained in part I, (1) can capture the real value of transmission projects in terms of efficiency benefit and competition benefit, (2) model the interaction of market participants in a proper and integrated mathematical framework and (3) produce the best transmission planning schedule in terms of the investment cost of expansions. It employs the equilibrium problems with equilibrium constraints to model the engaged parties in the transmission planning problem. The developed mathematical framework has been solved using a heuristic method combined with a gradient search method. The heuristic method is based on a forward methodology and the gradient search method is based on the Kuhn-Tucker optimality condition. Using the developed numerical solution, TNSP virtually approve a transmission project selected from the list of options. Then after, the Nash equilibrium point of Bertrand game among GenCos is calculated. Finally, the MMC clears the market using the security-constrained economic dispatch. The results of dispatch are used by the TNSP for evaluating of the transmission project option.

Since the whole scope of the methodology can not accommodate in one single paper, the application of the developed numerical methodology to an example system is detailed in a separate part. “Transmission Augmentation in an Oligopoly Electricity Market – Part II (Numerical Studies)” as the second part of this paper deals with the developed numerical platform and its application to an example system.

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