Growing dust grains in protoplanetary discs – III. Vertical settling

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ABSTRACT

We aim to derive a simple analytic model to understand the essential properties of vertically settling growing dust grains in laminar protoplanetary discs. Separating the vertical dynamics from the motion in the disc mid-plane, we integrate the equations of motion for both a linear and an exponential grain growth rate. Numerical integrations are performed for more complex growth models. We find that the settling efficiency depends on the value of the dimensionless parameter $\gamma$, which characterizes the relative efficiency of grain growth with respect to the gas drag. Since $\gamma$ is expected to be of the same order as the initial dust-to-gas ratio in the disc ($\approx10^{-2}$), grain growth enhances the energy dissipation of the dust particles and improves the settling efficiency in protoplanetary discs. This behaviour is mostly independent of the growth model considered as well as of the radial drift of the particles.

Key words: hydrodynamics – methods: analytical – planets and satellites: formation – protoplanetary discs.

1 INTRODUCTION

In this series of papers, we study the dynamics of growing dust grains in protoplanetary discs. In the two previous papers (Laibe et al. 2013a,b, hereafter Paper I and Paper II, respectively), we have shown how grain growth interplays with the radial drift of the grains and can lead to situations where the dust particles are accreted on to the central star (the so-called radial-drift barrier) or survive in the disc. These studies assumed that the radial and the vertical motion of grains can be decoupled since they occur on very different time-scales. Grains radial drift was therefore derived as if the grains motion occurred only in the disc mid-plane.

However, in addition to their radial evolution, grains experience a vertical motion that results from the balance between the vertical component of the central star’s gravity and of the gas drag. Dust particles settle more or less efficiently to the mid-plane of the disc depending on their size. This motion is therefore called vertical settling. By definition, vertical settling consists of the dust motion in a laminar flow. When the disc is turbulent, the particles are stirred out of the disc mid-plane in a process called vertical stirring. However, turbulence is not a purely diffusive noise since turbulent fluctuations are correlated. Studying laminar flows is therefore important, as it provides the limit at infinitely large correlation times.

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both a linear and an exponential growth model, as well as numerically for other physical growth models. The results are shown to be mainly independent of the growth model considered. Moreover, the studies of both the radial and vertical motion of growing grains performed, respectively, in Papers I, II and in this paper are based on the assumptions that the motions in the disc mid-plane and in the vertical direction are decoupled. We test the validity of this assumption in Section 4 and present our conclusions in Section 5.

2 VERTICAL SETTLING OF NON-GROWING GRAINS

The disc is a thin, non-magnetic, non-self-gravitating, inviscid perfect gas disc which is vertically isothermal. Its radial surface density and temperature are described by power-law profiles. The flow is laminar and in stationary equilibrium. Consequently, the gas velocity and density are described by well-known relations, which we presented in Paper I. Notations are described in Appendix A. The equation of motion for dust grains in protoplanetary discs are given by

$$\frac{d\phi}{dt} = \frac{\rho_0}{M_0} R^{(p+\frac{1}{2})} e^{-\frac{2}{1+\phi_0 Z^2}} + \frac{R}{(R^2+\phi_0 Z^2)} = 0$$

$$\frac{d^2\phi}{dt^2} + \frac{6\phi_0}{R} \frac{d\phi}{dt} + \frac{1}{\sqrt{\pi z_0}} R^{(p+\frac{1}{2})} e^{-\frac{2}{1+\phi_0 Z^2}} = 0$$

These equations depend on five control parameters ($\tau_0, S_0, \phi_0, C, q$) which are the initial dimensionless accelerations due to the pressure gradient, initial dimensionless grain size, initial disc aspect ratio and exponents of surface density and temperature profiles, respectively.

Here we derive approximate solutions for the vertical motion of dust grains. This motion depends on the radial distance from the central star and is thus coupled to the grains radial evolution. However, its general behaviour is well reproduced by separating both radial and vertical motions (i.e. all quantities are taken at $r = r_0$, i.e. $R = 1$, see below for the justification). The equation of grain dynamics on the $e, Z$ axis is given by the following Lienard equation:

$$\dot{Z} + \frac{e^2}{S_0} Z + \frac{Z}{(1 + \phi_0 Z^2)^{3/2}} = 0.$$ 

To $O(\phi_0^2)$ (for a thin disc) and $O(Z^2)$ (for particles close to the disc mid-plane), or equivalently, performing a linear expansion of this equation near its fixed point ($Z = 0, \dot{Z} = 0$) this becomes

$$\dot{Z} + \frac{1}{S_0} Z + Z = 0.$$ 

which implies that near this fixed point, grain dynamics are equivalent to the damped harmonic oscillator. Fig. 1 compares the vertical motion of the dust particles given the harmonic oscillator approximation (equation 3) and the general case (equation 2), and show that the harmonic oscillator approximation is justified. Moreover, given the sizes $S_0$ of the dust particles, the three characteristic regimes of the damped harmonic oscillator can then be distinguished as follows:

(i) $S_0 > 1/2$: underdamped oscillator

$$Z(T) = e^{-T/\lambda} \left[ A \cos(\lambda T) + B \sin(\lambda T) \right]$$

$$\lambda = \sqrt{1 - \left( \frac{1}{S_0} \right)^2}$$

$$A = Z(0)$$

$$B = \frac{i}{2} \left( \frac{1}{S_0} Z(0) + Z(0) \right) .$$

(ii) $S_0 = 1/2$: critical regime

$$Z(T) = (A + BT)e^{-T}$$

$$A = Z(0)$$

$$B = Z(0) + \dot{Z}(0) .$$

(iii) $S_0 < 1/2$: overdamped oscillator

$$Z(T) = A e^{\lambda_+ T} + B e^{\lambda_- T}$$

$$\lambda_\pm = \frac{-1 + \sqrt{1 - \left( \frac{1}{2S_0} \right)^2}}{2} \pm 1$$

$$A = Z(0) + \frac{1}{\lambda_-} Z(0) - Z(0)$$

$$B = \frac{1}{\lambda_-} Z(0) Z(0) - \lambda_+ .$$

The vertical settling of a grain occurs on a dimensionless time-scale that is shorter than the migration time-scale. From equation (6) (respectively, equation 4), for small (respectively, large) grains, the dimensionless settling time is $1/S$ (respectively, $S$). The fastest sedimentation occurs for the critical regime, i.e. for $S = 1/2$ (equation 5). A good approximation for the typical settling time $T_{sett}$ is therefore

$$T_{sett} \approx \frac{1 + S^2}{S} .$$
The typical migration time is obtained from equation 16 of Laibe, Gonzalez & Maddison (2012) estimating $R/\eta_0\kappa$ at $R = 1$, providing

$$T_{\text{mig}} \approx \frac{1 + S^2}{\eta_0 S},$$

(8)

where $\eta_0$ is the kinematic viscosity and $S$ is the scaleheight of the gas layer considered. As discussed in Paper II, the oscillations are sufficiently close to the mid-plane ($Z \ll 1$) during the evolution. For non-growing grains, we have seen that these assumptions have a negligible impact on the vertical evolution. Thus, equation (9) reduces to

$$\dot{S}(T) = S(T).$$

(10)

Substituting $S_0$ by $S$ in equation 3 provides the differential equation which governs the vertical motion of the grains:

$$\dot{Z} + \frac{1}{S(T)} \dot{S} + Z = 0.$$  

(11)

The evolution of $S(T)$ is governed by the growth rate of the particles. Several models of grain growth have been introduced and studied in Paper II. Importantly, it is explained that for a cold disc at $Z \ll 1$ the growth rate of the particles is of the form

$$\frac{dS}{dT} = \gamma f(S),$$

(12)

where $f$ is a function of the grain size which depends on the models for the relative turbulent velocities between the particles and the scaleheight of the dust layer considered. As discussed in Paper II, $\gamma$ is of order $\epsilon$, the initial dust-to-gas ratio of the disc, which is $\approx 10^{-2}$ in protoplanetary discs. With the most recent models of dust and gas turbulence modelling (see Paper II for a discussion), $f$ is of the form

$$f(S) = \frac{S}{1 + S} \simeq S^{\gamma},$$

with $\gamma < 1$ for $S \ll 1$ and $\gamma = 0$ for $S \gg 1$. $f$ often reduces to a simple power law of exponent $\gamma$ when treating the small and the large grains separately. In this case, as discussed in Paper II, $\gamma$ can take values of order unity in the case of realistic growth rates and differs from the case $S \ll 1$ to the case $S \gg 1$. The size evolution is thus given by

$$S(T) = \left((-\gamma + 1)\gamma T + S_0^{-\gamma + 1}\right)^{-\frac{1}{\gamma - 1}},$$

(14)

if $\gamma \neq 1$ and

$$S(T) = S_0 e^{\gamma T},$$

(15)

if $\gamma = 1$. The case $\gamma = 0$ (linear growth, equation 19 of Paper I) corresponds to the limit of the large grains in equation (13) and the case $\gamma = 1$ to the limit of the small grains. It is also straightforward to derive the general expression of the size evolution to the power-law toy model (equation 23) used in Paper I.

3.2 Linear growth model

We investigate the coupling between the growth and the settling using the simplest linear growth model from equation (14) with $\gamma = 0$ giving

$$\dot{Z} + \frac{1}{S_0 + \gamma T} \dot{Z} + Z = 0.$$  

(16)

To solve this differential equation, we introduce the auxiliary function $\zeta(T)$ such that

$$Z(T) = \zeta(T) \times e^{-\frac{1}{S_0 + \gamma T}} = \zeta(T) \times \left(1 + \frac{\gamma T}{S_0}\right)^{-\frac{1}{\gamma}}.$$  

(17)

Hence, $Z(T)$ is the product of two functions: $1 + \frac{\gamma T}{S_0}$ and a function $\zeta$ which satisfies

$$\dot{\zeta} + I(T) \zeta = 0,$$  

(18)

with

$$I(T) = 1 - \frac{1}{4} - 2\gamma.$$  

(19)

The general solution of equation (18) with (19) is

$$\zeta(T) = C_1 \sqrt{S_0 + \gamma T} \gamma_{\frac{1}{\gamma} - 1} \left(\frac{S_0 + \gamma T}{\gamma}\right) + C_2 \sqrt{S_0 + \gamma T} \gamma_{\frac{1}{\gamma} - 1} \left(\frac{S_0 + \gamma T}{\gamma}\right),$$

(20)

where $\gamma_{\frac{1}{\gamma} - 1}$ are the Bessel functions of first and second kind of order $\frac{1}{\gamma}$, and $C_1$, $C_2$ are constants determined by the initial conditions. Therefore, the solution of equation (16) is

$$Z(T) = \left(1 + \frac{\gamma T}{S_0}\right)^{-\frac{1}{\gamma}} \left[C_1 \sqrt{S_0 + \gamma T} \gamma_{\frac{1}{\gamma} - 1} \left(\frac{S_0 + \gamma T}{\gamma}\right) + C_2 \sqrt{S_0 + \gamma T} \gamma_{\frac{1}{\gamma} - 1} \left(\frac{S_0 + \gamma T}{\gamma}\right)\right].$$

(21)

If $Z(T = 0) = Z_0$ and $Z(T = 0) = 0$, the constants $C_1$ and $C_2$ are given by

$$C_1 = -\frac{Z_0}{S_0^{\frac{1}{\gamma} - 1}} \gamma_{\frac{\gamma}{\gamma} - 1} \left(\frac{S_0}{\gamma}\right) J_{\frac{\gamma}{\gamma} - 1} \left(\frac{S_0}{\gamma}\right) - S_0 Y_{\frac{\gamma}{\gamma} - 1} \left(\frac{S_0}{\gamma}\right) J_{\frac{\gamma}{\gamma} - 1} \left(\frac{S_0}{\gamma}\right),$$

(22)

and

$$C_2 = \frac{Z_0}{S_0^{\frac{1}{\gamma} - 1}} \gamma_{\frac{\gamma}{\gamma} - 1} \left(\frac{S_0}{\gamma}\right) J_{\frac{\gamma}{\gamma} - 1} \left(\frac{S_0}{\gamma}\right) - S_0 Y_{\frac{\gamma}{\gamma} - 1} \left(\frac{S_0}{\gamma}\right) J_{\frac{\gamma}{\gamma} - 1} \left(\frac{S_0}{\gamma}\right).$$

(23)

The sign of the function $L(T)$, for which we have $dL/dT = \frac{1}{\gamma} \gamma(1 - 2\gamma)/(S_0 + \gamma T)^{\gamma}$, provides information on the oscillating behaviour...
of the solution. Thus, three cases can be distinguished, separated by the critical value for settling $\gamma_s = \frac{1}{2}$:

(i) $0 < \gamma < \gamma_s = \frac{1}{2}$ and $dI/dT > 0$. $I$ increases from its initial value $I(T = 0) < 1$ and $\lim_{T \to +\infty} I = 1$. If $S_0 > \frac{1}{2}\sqrt{1-2\gamma}$, then $I(T = 0) > 0$ and $I$ is positive at all times: the solution is always pseudo-oscillating. The dust particle is always decoupled from the gas as the size can only increase and the dust evolution follows the large grain regime. This is of minor interest in the context of growing grains. We therefore consider the interesting case $S_0 < \frac{1}{2}\sqrt{1-2\gamma}$, for which $I(T = 0) < 0$ and $I$ becomes positive for $T > \frac{1}{2}\sqrt{1-2\gamma}$; the solution transitions from a monotonic decay to a pseudo-oscillating regime, indicating that particles decouple from the gas.

(ii) $\gamma = \gamma_s = \frac{1}{2}$ and $dI/dT = 0$. In this limiting case, $I(T) = 1$ for all time. Phase lag due to damped oscillations is exactly counterbalanced by the decrease of the drag caused by grain growth.

(iii) $\gamma > \gamma_s = \frac{1}{2}$ and $dI/dT < 0$. $I$ decreases from its initial value $I(T = 0) > 1$ and $\lim_{T \to +\infty} I = 1$. Since $I$ is always greater than 1, the solution is pseudo-oscillating at a frequency larger than the Keplerian frequency.

The envelope of the solution, which determines the damping of the dust’s vertical motion, is given by the product of $(1 + \frac{\gamma T}{S_0})^{\frac{1}{2}}\sqrt{S_0 + \gamma T}$ and the envelope of the Bessel functions. While not transparent, it is qualitatively interpretable. The dust behaviour for different values of $\gamma$ are shown in Fig. 2 for $S_0 = 10^{-2}$. We focus on initially small grains ($S_0 \ll 1$) because they correspond to the grains which originate in the interstellar medium and are involved in planet formation. In the case of the slow growth regime ($\gamma < 1/2$), the vertical dust motion is damped efficiently: particles settle to the mid-plane of the disc before they have time to grow and decouple from the gas. However, as grains decouple slowly from the gas as they settle, drag becomes weaker. Thus, dust settling occurs faster than for non-growing grains and the settling rate increases for increasing values of $\gamma$. On the contrary, in the fast growth regime ($\gamma > 1/2$), dust particles grow fast enough to decouple from the gas before they feel the gas drag and their settling time-scale becomes much longer. In this case, the settling time of the grain increases dramatically with $\gamma$ since an asymptotic expansion of equation (21) for $\gamma \gg 1$

Figure 2. Vertical motion of a growing dust particle with time $T$ starting at $Z_0 = 1$ with $S_0 = 10^{-2}$ and $\gamma = 5 \times 10^{-3}, 5 \times 10^{-2}, 0.5, 2$ and 20. The most efficient settling is obtained for $\gamma = 1/2$.

3.3 Other growth models

We can also integrate the vertical motion of the grains for several growth models: specifically power laws with $y_g = 1, -0.5, 0.5$ and growth rate given by the function $f$ of equation (13). For the exponential growth rate model ($y_g = 1$), we derive analytically the evolution of the dimensionless vertical coordinate, which is given by

$$Z(T) = \left[ B_1 e^{\frac{\gamma T}{\gamma S_0 e^{\gamma T}}} \frac{1}{\gamma} M \left( \frac{1}{\gamma}, \frac{2+i\gamma}{\gamma S_0}, e^{\frac{-\gamma T}{\gamma S_0}} \right) \right. + \left. B_2 e^{\frac{-\gamma T}{\gamma S_0}} \frac{1}{\gamma} M \left( \frac{1}{\gamma}, \frac{2+i\gamma}{\gamma S_0}, e^{\frac{-\gamma T}{\gamma S_0}} \right) \right],$$

where $i^2 = -1, B_{1,2}$ are constants which are determined by the initial conditions, and $M(a, b, z)$ is the M-Kummer confluent hypergeometric function of indices $a$ and $b$ with respect to $z$.

For the other growth models, we did not manage to derive the evolution analytically and therefore we must integrate the equations numerically.

Fig. 4 shows the vertical behaviour of the particle with $\gamma = 5 \times 10^{-2}$ (similar plots with $\gamma = 0.5$ and $\gamma = 5$ are shown in Appendix B).
and growth models is very similar. In particular, for $S > 5 \times 10^{-3}$, the linear growth provides a good approximation of the function $f$ in the case of small values of $\gamma$ (particles settle having mainly $S < 1$), whereas the exponential growth provides a good approximation of the function $f$ in the case of large values of $\gamma$ (particles settle having mainly $S > 1$). However, overall, the nature of settling for the different growth models is very similar. In particular, for $\gamma = 5 \times 10^{-2}$, the grains settle much faster than in the case without any growth. Therefore, the conclusion that grain growth enhances vertical settling’s efficiency in protoplanetary discs hold whatever the growth model considered.

4 COMBINING THE RADIAL AND THE VERTICAL MOTION

In the studies performed in Papers I, II and in this paper, we found two interesting values of the growth rate $\gamma$: $\gamma = \eta_0$ (giving $S = 1$) and $\gamma = 1/2$, corresponding, respectively, to the optimal values of $\gamma$ for the migration and the vertical settling process. The parameter space can therefore be divided in three regions as shown in Fig. 5: $\gamma < \eta_0$ (region 1), $\eta_0 < \gamma < 1/2$ (region 2) and $\gamma > 1/2$ (region 3), noting that $O(\eta_0) < 1/2$ for real discs. In each of these regions, the efficiency of the vertical (respectively, radial) motion is represented by the brightness of the top (respectively, bottom) colour bar. Importantly, this plot provides an indication of the relative efficiencies of the migration and settling processes, all the other parameters being fixed. It does not, however, predict the grains final state (decoupling at a finite radius, pile-up or accretion on to the star) but support the hypothesis of the decoupling between the radial and the vertical motions.

Until now, we have assumed that the grains radial and vertical motion are decoupled. We test this hypothesis by numerically integrating the combined vertical and radial equations of motion (equation 1) in three dimensions, substituting $S_0$ by $S(T)$. The resulting trajectories in the $RZ$ plane for $\gamma = 10^{-4}$, $10^{-3}$, $10^{-2}$, $1/2$, 10 and 100 are shown in Fig. 6. We studied the case of $\eta_0 = 10^{-2}$ with $p = 3/2$, $q = 3/4$, $n = 1$ and $\gamma = 10^{-4}$, $10^{-3}$, $5 \times 10^{-2}$, 1/2, 10, $10^2$ (from top to bottom).

In region 1 of Fig. 5, particles with small growth rates ($\gamma < \gamma_{c,m}$) settle to the mid-plane and then migrate towards the central star. Growth is not efficient enough to make the particles decouple from the gas before they reach the inner region of the disc. The migration efficiency is optimal for $\gamma = \gamma_{c,m}$. In region 2 of Fig. 5, grains with intermediate growth rates ($\gamma_{c,m} < \gamma < \gamma_{c,s} = 1/2$) grow as they settle to the mid-plane, radially migrate, but they decouple from the gas before reaching the central star and therefore experience an extremely slow migration motion. When $\gamma = \gamma_{c,s} = 1/2$ (the border of regions 2 and 3 of Fig. 5), the growth is optimal and particles decouple from the gas just as they reach the mid-plane.
Thus, particles migrate slightly while efficiently settling to the mid-plane (the envelope of vertical oscillations decreases very quickly). In region 3 of Fig. 5, the larger growth rates ($\gamma > \gamma_{cr} = 1/2$) ensure that particles grow very efficiently and rapidly decouple from the gas. They do not settle to the mid-plane (as there is no gas damping once they decouple from the gas phase) and they experience a very small migration motion. In all cases, the vertical motion occurs much faster than the radial motion and the predictions done assuming that both motions are decoupled hold.

To pedagogically illustrate the effect of the linear growth rate discussed in Section 3.2 on the resulting dust distribution in protoplanetary discs, we also run simple simulations with the 3D two-phase equations of motion (see Fig. 6) and in our SPH simulations (see Fig. 8). We find that the assumption of decoupling the radial and the vertical motion is verified both in our direct integration of the equations of motion and in our SPH simulations (see Fig. 8).

![Figure 8: Trajectories in the (r, z) plane of individual SPH dust particles for $\gamma = 10^{-2}$, 1/2 and 10. Top: the migration is highly efficient (the growth time-scale is of order the migration time-scale), contrary to the settling. Centre: the settling efficiency is optimal (the growth time-scale of order the optimal settling time-scale), but the migration is no longer efficient. Bottom: particles are almost instantaneously decoupled from the gas and are Keplerian orbits.](http://mnras.oxfordjournals.org/)

3.2.1. Growth

In this section, we will analytically determine the settling regime as a function of the time-scale ratio $\eta = \gamma/c_s$, which we define as the ratio of the growth time-scale $\gamma$ and the sound time-scale $c_s$. For $\eta > 1$, the settling is most efficient and the dust disc is thinnest. As a conclusion, we find that the assumption of decoupling the radial and the vertical motion of the grain is verified both in our direct integration of the equations of motion (see Fig. 6) and in our SPH simulations (see Fig. 8).

5 CONCLUSIONS AND PERSPECTIVES

In this paper, we have studied the vertical settling of growing dust grains in protoplanetary discs, using different rates for the grain growth and integrating the equations of evolution both analytically and numerically. The main results of the study are as follows.

(i) The vertical motion of growing dust grains is governed by the value of the dimensionless parameter which represents the relative efficiency between the growth and the drag, which we denote by $\gamma$. 

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**Figure 7.** Radial grain size distribution obtained with our SPH code after 10$^5$ yr for $\gamma = 10^{-4}$, $10^{-2}$, 1/2 and 10. Dark blue: gas. Light blue: dust. Thinner disc distributions are obtained when the ratio between the growth time-scale is the same as the optimal settling time-scale ($\gamma = 1/2$).
In protoplanetary discs, $\gamma$ is of the same order as the initial dust-to-gas ratio of the disc $\epsilon_0$, which is of order $10^{-2}$. This implies that the growth is not too efficient (the effective $\gamma$ being much smaller than the critical value of 1/2) enabling particles to settle towards the disc mid-plane where they concentrate.

(ii) All the growth models we have tested give essentially the same behaviour as the linear growth model. We therefore suggest that the results of this study are generalizable and that the solution we derive provides a good analytic prescription for the vertical evolution of the particles.

(iii) Simultaneously integrating both the radial and the vertical motion of the particles shows that the vertical settling of the particles occurs much faster than the radial drift of the particles, justifying the assumption of separating the radial drift and the vertical settling. This is a standard and well-known result for non-growing grains (see e.g. Garaud et al. 2004), we have shown that it also holds for growing grains.

(iv) Combining the results for the radial drift with the study of the vertical settling of dust grains, we distinguish three major regimes for growing grains: $\gamma < \eta_0$, $\eta_0 < \gamma < 1/2$, $1/2 < \gamma$, the first two being the most relevant for the context of planet formation. Initially, small grains grow as they settle to the mid-plane, the settling motion being faster than for non-growing grains. Varying $\gamma$ results in distinct profiles for the grain size distribution as well as their spatial distributions.

Importantly, dust concentration of growing grains in the disc mid-plane has been proven to occur when the disc is laminar. If not, turbulent fluctuations from the gas may spread the dust particles out of the disc mid-plane. This vertical stirring is widely supposed to prevent grains to concentrate in the disc mid-plane and form planet by gravitational instability for non-growing grains. This issue will be addressed in the case of growing dust grains in a forthcoming paper.

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**REFERENCES**


**APPENDIX A: NOTATIONS**

The notations and conventions used throughout this paper are summarized in Table A1.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Meaning</th>
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<tbody>
<tr>
<td>$M$</td>
<td>Mass of the central star</td>
</tr>
<tr>
<td>$g$</td>
<td>Gravity field of the central star</td>
</tr>
<tr>
<td>$r_0$</td>
<td>Initial distance to the central star</td>
</tr>
<tr>
<td>$\rho_0$</td>
<td>Initial gas density</td>
</tr>
<tr>
<td>$\rho_g(r)$</td>
<td>Gas density</td>
</tr>
<tr>
<td>$\rho_s(r)$</td>
<td>Symmetric density</td>
</tr>
<tr>
<td>$c_s$</td>
<td>Gas sound speed</td>
</tr>
<tr>
<td>$\Sigma_0$</td>
<td>Gas surface density at $r_0$</td>
</tr>
<tr>
<td>$T$</td>
<td>Dimensionless time</td>
</tr>
<tr>
<td>$T_0$</td>
<td>Gas temperature ($T_0$; value at $r_0$)</td>
</tr>
<tr>
<td>$\phi_0$</td>
<td>Square of the aspect ratio $H_0/r_0$ at $r_0$</td>
</tr>
<tr>
<td>$\eta_0$</td>
<td>Sub-Keplerian parameter at $r_0$</td>
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<tr>
<td>$s$</td>
<td>Grain size</td>
</tr>
<tr>
<td>$S$</td>
<td>Dimensionless grain size</td>
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<td>$\gamma$</td>
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<td>$w_g$</td>
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<tr>
<td>$v$</td>
<td>Grain velocity</td>
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<tr>
<td>$\rho_d$</td>
<td>Dust intrinsic density</td>
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<tr>
<td>$m_d$</td>
<td>Mass of a dust grain</td>
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<td>$t_s$</td>
<td>Drag stopping time</td>
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<tr>
<td>$t_{s0}$</td>
<td>Drag stopping time at $r_0$</td>
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APPENDIX B: SETTLING WITH DIFFERENT GROWTH MODELS

Figs B1 and B2 show the vertical evolution of particles for different growth models. No significant differences are found between the models.

**Figure B1.** Vertical motion of a growing dust particle starting at $Z_0 = 1$ with $S_0 = 10^{-2}$ and $\gamma = 1/2$ for different growth models. No significant differences are found between the different models.

**Figure B2.** Vertical motion of a growing dust particle starting at $Z_0 = 1$ with $S_0 = 10^{-2}$ and $\gamma = 5$ for different growth models. No significant differences are found between the different models.

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