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The WiggleZ Dark Energy Survey: measuring the cosmic expansion history using the Alcock-Paczynski test and distant supernovae

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ABSTRACT
Astronomical observations suggest that today’s Universe is dominated by a dark energy of unknown physical origin. One of the most notable consequences in many models is that dark energy should cause the expansion of the Universe to accelerate: but the expansion rate as a function of time has proven very difficult to measure directly. We present a new determination of the cosmic expansion history by combining distant supernovae observations with a geometrical analysis of large-scale galaxy clustering within the WiggleZ Dark Energy Survey, using the Alcock-Paczynski test to measure the distortion of standard spheres. Our result constitutes a robust and non-parametric measurement of the Hubble expansion rate as a function of time, which we measure with 10-15% precision in four bins within the redshift range 0.1 < z < 0.9. We demonstrate that the cosmic expansion is accelerating, in a manner independent of the parameterization of the cosmological model (although assuming cosmic homogeneity in our data analysis). Furthermore, we find that this expansion history is consistent with a cosmological-constant dark energy.

Key words: surveys, distance scale, dark energy

1 INTRODUCTION

Observations by astronomers over the past fifteen years have suggested that the Universe is dominated by an unexpected component known as “dark energy”, yet we still have no physical understanding of its existence or magnitude. Determining the nature of dark energy is one of the most im-
portant challenges for contemporary cosmology because its presence implies that our understanding of the physics of the Universe is incomplete. Possible explanations are that the theory of General Relativity must be modified on cosmological scales to generate an effective repulsive gravitational force, the Universe is filled with a diffuse material that acts with an effective negative pressure, or our interpretation of cosmological observations must be changed to reflect inhomogeneities.

One of the most important consequences of the first two scenarios is that the Universe underwent a transition from decelerating to accelerating expansion within the last few billion years. In this paper we use new observations and methods to map this expansion history in a model-independent, non-parametric and robust manner. We do assume cosmic homogeneity in our data analysis, and therefore we do not address the third scenario.

In the standard cosmological analyses, involving datasets such as distant supernovae (Riess et al. 1998, Perlmutter et al. 1999, Amanullah et al. 2010, Conley et al. 2011), galaxy surveys (Giannantonio et al. 2008, Percival et al. 2010, Blake et al. 2011b) and the Cosmic Microwave Background radiation (Komatsu et al. 2011), the expansion rate of the Universe at different look-back times is not measured directly. Accelerating expansion is a model-dependent implication of fitting cosmological data with parametric models in which prior assumptions are made about the physical components of the Universe and their evolution with redshift. However, the unknown physical nature of dark energy implies that we cannot yet be certain that any particular parametric model is correct.

For example, measurements of distant supernovae provide some of the strongest evidence for the existence of dark energy. The observed quantities are the apparent magnitudes of the supernovae, which yield a relative luminosity distance as a function of redshift, i.e., $D_L(z)H_0/c$ where $D_L(z)$ is the luminosity distance, $H_0$ is the local value of the Hubble expansion rate and $c$ is the speed of light. These supernovae distances are well-fit by a model in which the physical contents of the Universe are divided into pressureless matter with a current fractional energy density $\Omega_m$, which dilutes with redshift as $(1+z)^3$, and a "cosmological constant" $\Omega_\Lambda$ whose energy density does not change with redshift. The best-fitting parameters assuming this model are $\Omega_m \approx 0.27$ and $\Omega_\Lambda \approx 0.73$, implying a transition from decelerating to accelerating expansion at $z \approx 0.6$ (e.g. Conley et al. 2011).

However, this conclusion is dependent on the assumed parameterized model. The acceleration of the expansion rate is not directly observed. When the supernovae data are subject to model-independent non-parametric analysis, the evidence for accelerating expansion is much weaker (Wang & Tegmark 2005, Shapiro & Turner 2006, Shafieloo 2007, Sollerman et al. 2009, Shafieloo & Clarkson 2010). This is primarily because the acceleration rate is obtained as the second derivative of the noisy observed luminosity distances. Moreover, obtaining the cosmic expansion rate from a luminosity distance requires an additional assumption about spatial curvature. Although recent cosmological data is consistent with a spatially-flat Universe (Percival et al. 2010, Komatsu et al. 2011), this conclusion is again reached within the assumption of a parameterized model (unless we invoke a strong prior from inflationary cosmological models).

The expansion history of the Universe as a function of redshift $z$ is described by the evolution of the Hubble parameter $H(z) \equiv (1+z)\frac{da}{dt}$, where $a(t)$ is the cosmic scale factor at time $t$. In this paper we present a new determination of the function $H(z)$ using a two-step approach. Firstly we apply an Alcock-Paczynski measurement (Alcock & Paczynski 1979) to the large-scale clustering of galaxies in the WiggleZ Dark Energy Survey (Drinkwater et al. 2010), thereby measuring the distortion parameter $F(z) \equiv (1+z)D_A(z)H(z)/c$, where $D_A(z)$ is the physical angular-diameter distance. Secondly we combine this measurement with the supernovae luminosity distances $D_L(z)H_0/c$, using the equivalence of distance measurements $D_L(z) = D_A(z)(1+z)^2$, to infer the value of $H(z)/H_0$. The result is an accurate non-parametric reconstruction of the cosmic expansion history, which is independent of the assumed cosmological model (including spatial curvature). We present our results in the form of both individual measurements of $H(z)/H_0$ in four redshift bins within the redshift range $0.1 < z < 0.9$, and a non-parametric reconstruction of a continuous cosmic expansion history using a modified version of the iterative methods introduced by Shafieloo et al. (2006).

Other methods for determining the Hubble expansion rate have been previously attempted. Firstly, $H(z)$ may be inferred from the relative ages of passively-evolving galaxies (Stern et al. 2010). This is a promising technique although subject to many assumptions about the stellar populations and formation epochs of these galaxies. Secondly, the imprint of baryon acoustic oscillations in galaxy clustering has been applied as a standard ruler along the line-of-sight (Gaztanaga, Cabre & Hui 2009). However, the current availability of large-scale galaxy survey data restricts this latter technique to low redshift $z = 0.35$ and it is debatable whether the signal-to-noise of current measurements is in fact sufficient to ensure a robust measurement (Kazin et al. 2009). Finally, the physical presence of dark energy has been inferred from the late-time Integrated Sachs-Wolfe effect which correlates low-redshift galaxy overdensities with Cosmic Microwave Background (CMB) anisotropies (Giannantonio et al. 2008). However, this last technique does not allow recovery of the detailed expansion history and the achievable statistical significance is limited.

The Alcock-Paczynski test (Alcock & Paczynski 1979) is a geometric probe of the cosmological model based on the comparison of the observed tangential and radial dimensions of objects which are assumed to be isotropic in the correct choice of model. It can be applied to the 2-point statistics of galaxy clustering if redshift-space distortions, the principal additional source of anisotropy, can be successfully modelled (Ballinger, Peacock & Heavens 1996, Matsubara & Suto 1996, Matsubara 2000, Seo & Eisenstein 2003, Simpson & Peacock 2010). This method has been previously utilized with data from the 2-degree Field Quasar Survey (Outram et al. 2004), but the WiggleZ survey offers a superior measurement owing to the reduced importance of shot noise. Chuang & Wang (2011) recently performed fits to the tangential and radial clustering of Luminous Red Galaxies within the Sloan Digital Sky Survey (SDSS) at redshift $z = 0.35$, utilizing Alcock-Paczynski information. Indeed, a general analysis of the tangential/radial galaxy cluster-
ing pattern in the presence of baryon acoustic oscillations demonstrates how the information may be divided into an overall scale distortion, quantified by a distance parameter $D_L \propto (D_{A \mu}/H)^{1/3}$, and a warping, quantified by the Alcock-Paczynski distortion factor $D_A/H$, enabling the disentangling of $D_A$ and $H$ (Padmanabhan & White 2008; Taruya, Saito & Nishimichi 2011; Kazin, Sanchez & Blanton 2011). Such approaches are just becoming possible with the current generation of large-scale galaxy surveys, and will be very powerful when applied to future datasets such as the Baryon Oscillation Spectroscopic Survey (BOSS, Eisenstein et al. 2011). Another recent application of the Alcock-Paczynski test was presented by Marinoni & Buzzi (2010), who presented a study of the distribution of close galaxy pairs.

The WiggleZ Dark Energy Survey (Drinkwater et al. 2010) at the Australian Astronomical Observatory has recently provided a new large-scale galaxy redshift survey dataset, allowing low-redshift cosmological measurements in the SDSS to be extended to higher redshifts up to $z = 0.9$. In particular, the survey has yielded a measurement of the baryon acoustic peak in the clustering pattern at $z = 0.6$ (Blake et al. 2011b). In addition, Blake et al. (2011a) mapped the growth of cosmic structure in this dataset across the redshift range $0.1 < z < 0.9$ for a fixed background cosmological model. The new study presented in this paper constitutes a generalization of that analysis incorporating Alcock-Paczynski distortions arising in varying cosmological models.

Our paper is structured as follows: in Section 2 we introduce the WiggleZ survey dataset and power spectrum measurements. In Section 3 we describe our implementation of the Alcock-Paczynski test and our model for marginalizing over redshift-space distortions. In Section 4 we combine these Alcock-Paczynski measurements with supernovae data to deduce the expansion history $H(z)$. In Section 5 we present a non-parametric reconstruction of the continuous cosmic expansion history, and we summarize our findings in Section 6.

2 WIGGLEZ SURVEY POWER SPECTRA

2.1 The WiggleZ Dark Energy Survey

The WiggleZ Dark Energy Survey (Drinkwater et al. 2010) at the 3.9-m Anglo-Australian Telescope has provided the next step in large-scale spectroscopic galaxy redshift surveys, mapping a cosmic volume $\sim 1 \text{Gpc}^3$ over the redshift range $0 < z < 1$. By covering a total of about 800 deg$^2$ of sky the WiggleZ survey has mapped about 100 times more effective cosmic volume in the $z > 0.5$ Universe than previous galaxy redshift surveys. Target galaxies were chosen by a joint selection in UV and optical wavebands, using observations by the Galaxy Evolution Explorer satellite (GALEX; Martin et al. 2005) matched with ground-based optical imaging from the Sloan Digital Sky Survey (York et al. 2000) in the Northern Galactic Cap, and from the Red-sequence Cluster Survey 2 (RCS2) (Gilbank et al. 2011) in the Southern Galactic Cap. A series of magnitude and colour cuts (Drinkwater et al. 2010) was used to preferentially select high-redshift star-forming galaxies with bright emission lines, that were then observed using the AAOmega multi-object spectrograph (Sharp et al. 2006) in 1-hr exposures. In this study we analyzed a galaxy sample drawn from our final set of observations, after cuts to maximize the contiguity of each survey region. The sample includes a total of 162,323 galaxy redshifts, which we divided into four different redshift slices of width $\Delta z = 0.2$ between $z = 0.1$ and $z = 0.9$. The numbers of galaxies analyzed in each redshift slice were $N = (22072, 42160, 63737, 34354)$.

2.2 Power spectrum measurements

Our analysis is based on measurements of the power spectrum amplitude of WiggleZ galaxies as a function of the angle to the line-of-sight $\theta$, which we parameterize by the variable $\mu = \cos \theta$. By comparing the amplitude of power spectrum modes as a function of $\mu$ we can apply an Alcock-Paczynski measurement as proposed by Ballinger et al. (1996) and Matsubara & Suto (1996) and described below in Section 3. We measured galaxy power spectra in six independent survey regions, dividing each region into four redshift slices. At a given redshift we fitted models to the measurements in each different survey region, convolving with the respective survey window function.

In order to measure the power spectrum and calculate this convolution we need to define the selection function of the survey which describes the mean galaxy density as a function of position. The angular and redshift dependences of the selection function for each WiggleZ region were determined using the methods described by Blake et al. (2010). This process models several effects including the variation of the GALEX target density with dust and exposure time, the incompleteness in the current redshift catalogue, the variation of that incompleteness imprinted across each 2-degree field by constraints on the positioning of fibres and throughput variations with fibre position, and the dependence of the galaxy redshift distribution on optical magnitude. The galaxy power spectrum was then measured using the optimal-weighting scheme proposed by Feldman, Kaiser & Peacock (1994). Our fiducial cosmological model which we use to convert redshifts and angular co-ordinates to distances is a flat $\Lambda$CDM model with matter density $\Omega_m = 0.27$. We therefore assume cosmic homogeneity in our data analysis.

Because the WiggleZ survey consists of a series of independent narrow-angle cones (of width $\approx 10$ degrees on the sky) a “flat-sky” approximation may be used in which the co-ordinate $x$-axis is aligned with the line-of-sight at the centre of each region; the wavevector components $(k_x, k_y, k_z)$ are then deduced from the three-dimensional Fourier vector $k = (k_x, k_y, k_z)$ as $k = \sqrt{k_x^2 + k_y^2 + k_z^2}$ and $\mu = k_z/k$. The observed power spectra were corrected for the small effect of redshift blunders (Blake et al. 2010). The covariance matrix of the power spectrum measurement was deduced using the methodology of Feldman et al. (1994), and the convolution matrices were determined from the window function as described in Section 2.2 of Blake et al. (2011a). Figure 1 displays the 2D power spectrum amplitudes as a function of $(k, \mu)$, stacked over the survey regions at each redshift.

The effective redshifts of the power spectrum measurements in each slice were $z = (0.22, 0.41, 0.6, 0.78)$, determined by weighting each pixel in the selection function by its contribution to the power spectrum error (Blake et al. 2010):
z_{\text{eff}} = \sum_{x} z \left( \frac{n_g(x)P_g}{1 + n_g(x)P_g} \right)^2,
\quad (1)

where \( n_g(x) \) is the galaxy number density in each grid cell \( x \) and \( P_g \) is the characteristic galaxy power spectrum amplitude, which we evaluated at a scale \( k = 0.1 \, h \, \text{Mpc}^{-1} \). We note that our results are not sensitive to the precise value of the effective redshift: the measured quantity is the scale distortion parameter \( F(z_{\text{eff}}) \equiv (1 + z_{\text{eff}})D_A(z_{\text{eff}})H(z_{\text{eff}}) \) relative to its value in the fiducial cosmological model, rather than the quantity \( F(z_{\text{eff}}) \) itself. In other words, if the effective redshift was in error by \( \Delta z_{\text{eff}} \), then the systematic error in \( F(z_{\text{eff}} + \Delta z_{\text{eff}})/F(z_{\text{eff}}) \) can be negligible even when the difference between \( F(z_{\text{eff}}) \) and \( F(z_{\text{eff}} + \Delta z_{\text{eff}}) \) is not.

3 ALCOCK-PACZYNSKI MEASUREMENT
3.1 The Alcock-Paczynski effect
An Alcock-Paczynski measurement (Alcock & Paczynski 1979) is a method for constraining the cosmological model by comparing the observed tangential and radial dimensions of objects which are assumed to be isotropic, i.e. possess equal comoving tangential and radial sizes \( L_0 \). The observed tangential dimension is the angular projection \( \Delta \theta = L_0/[(1 + z)D_A(z)] \). The observed radial dimension is the redshift projection \( \Delta z = L_0H(z)/c \). The intrinsic size \( L_0 \) does not need to be known in order to recover the observable \( \Delta z/\Delta \theta = (1 + z)D_A(z)H(z)/c \), which is independent of any assumption about spatial curvature. An Alcock-Paczynski measurement does not necessarily need to be applied to cosmological “objects” but is equally valid for an isotropic process such as the 2-point statistics of galaxy clustering (Ballinger et al. 1996, Matsubara & Suto 1996).

The relative tangential/radial distortion depends on the value of \( F(z) = (1 + z)D_A(z)H(z)/c \) relative to the fiducial model (which we take as a flat \( \Lambda \) Cold Dark Matter cosmology with \( \Omega_m = 0.27 \), although the results do not depend significantly on this choice). We applied the Alcock-Paczynski methodology to the WiggleZ survey clustering by measuring the relative amplitude of power spectrum modes as a function of their angle to the line-of-sight (Ballinger et al. 1996, Matsubara & Suto 1996, Simpson & Peacock 2010).
WiggleZ Survey: Alcock-Paczynski measurement

Figure 2. This Figure displays the joint likelihood of the Alcock-Paczynski scale distortion parameter $F(z)$ relative to the fiducial value $F_{\text{fid}}$, and the growth rate quantified by $f_{\sigma_8}(z)$, obtained from fits to the 2D galaxy power spectra of the WiggleZ Dark Energy Survey in four redshift slices. In order to produce this Figure we marginalized over the linear galaxy bias $b_2$ and the pairwise velocity dispersion $\sigma_v$. There is some degeneracy between $F$ and $f_{\sigma_8}$ but their characteristic dependence on the angle to the line-of-sight is sufficiently different that both parameters may be successfully extracted. The probability density is plotted as both greyscale and contours enclosing 68% and 95% of the total likelihood. The solid circles indicate the parameter values in our fiducial cosmological model.

assuming the underlying isotropy of these modes in the true cosmological model.

In more detail: the true values of the angular diameter distance $D_A(z)$ and Hubble expansion parameter $H(z)$ may differ from our adopted fiducial values, $\tilde{D}_A(z)$ and $\tilde{H}(z)$, leading to two scaling factors $f_\perp \equiv D_A/\tilde{D}_A$ and $f_\parallel \equiv \tilde{H}/H$ which relate the apparent, observed tangential and radial wavenumber components $(k'_\perp, k'_\parallel)$ to their true values $(k_\perp, k_\parallel)$ via $k'_\perp = f_\perp k_\perp$ and $k'_\parallel = f_\parallel k_\parallel$. The apparent two-dimensional galaxy power spectrum is then related to the underlying fiducial galaxy power spectrum (neglecting redshift-space distortions) by

$$P_g(k', \mu') = \frac{b^2}{f_\perp^2 f_\parallel} P_m \left[ \frac{k'}{f_\perp} \sqrt{1 + \mu'^2 \left( \frac{f_\perp^2}{f_\parallel^2} - 1 \right)} \right]$$

(following Ballinger et al. 1996, Simpson & Peacock 2010)

where $k' = \sqrt{k^2_\perp + k^2_\parallel}$, $\mu' = k'_\parallel/k'$, $b$ is an unknown galaxy bias factor, and $P_m(k)$ is the underlying real-space, isotropic matter power spectrum. We generated this power spectrum shape using the CAMB software (Lewis, Challinor & Lasenby 2000) with cosmological parameters consistent with the latest observations of the Cosmic Microwave Background radiation (Komatsu et al. 2011): matter density $\Omega_m = 0.27$, cosmological constant $\Omega_\Lambda = 0.73$, baryon fraction $\Omega_b/\Omega_m = 0.166$, Hubble parameter $h = 0.71$, primordial scalar index of fluctuations $n_s = 0.96$ and total fluctuation amplitude $\sigma_8 = 0.8$. We included non-linear growth of structure in this model using the “halofit” prescription in CAMB (Smith et al. 2003). These assumptions do not impose a significant model dependence in our analysis, as discussed in Section 3.4 below.

If the underlying power spectrum $P_m(k)$ contains features at known physical scales, which we can clearly detect as a function of angle, then we can extract both distortion factors $(f_\parallel, f_\perp)$. This would be equivalent to knowing the absolute length scale $L_0$ introduced above, and may be achieved in principle using baryon acoustic oscillations as a standard ruler (Seo & Eisenstein 2003, Hu & Haiman 2003, Glazebrook & Blake 2005). However, the 2D WiggleZ power spectra do not possess a sufficiently high signal-to-noise ratio to resolve the imprint of baryon oscillations and they do not contribute any information to the fits presented here (although the baryon acoustic peak may be detected in the 1D correlation function, as described by Blake et al. 2011b).

If the power spectrum can be well-approximated by a scale-free power law over the range of scales of inter-
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\text{predicted by theory: (Percival & White 2009).}
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\text{liar velocity field in units of the co-moving Hubble velocity}
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\text{θ}
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\[
\text{application of the continuity equation implies that}
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\text{the range of scales relevant for this analysis.}
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\[
\text{correlation is consistent with deterministic, linear bias over}
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\text{underlying matter (Jennings et al. 2011). These equations are given in terms of}
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\text{both}
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\text{We checked that our results were not significantly changed}
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\text{expressible in terms of the growth factor}
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\text{we varied the value of}
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\text{scale factor}
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\text{redshift-space power spectrum of a population of galaxies}
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\text{the cosmic expansion history and the FRW metric, the}
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\text{Kaiser limit” (Kaiser 1987):}
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\[
\text{in Section 3.4 below.}
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\[
\text{As a further enhancement of the redshift-space distortion model, we also considered multiplying Equation 3 by an empirical damping function}
\]

\[
\text{D(k, µ) representing convolution with uncorrelated galaxy motions on small scales. The standard choice for this damping function is a Lorentzian}
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\[
\text{P(k, µ) = [1 + \left( \frac{k}{\sigma_b} \right)^2]^{-1}}
\]

\[
\text{following Ballinger et al. 1996, Simpson & Peacock 2010). Although there is some degeneracy between β and the Alcock-Paczynski distortion parameter F, their angular signatures in the clustering model are sufficiently different that both parameters may be successfully extracted after marginalizing over the other.}
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\text{However, we do not use the redshift-space distortion model of Equation 5 in our default analysis because the}
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\text{Kaiser-limit approximation is only appropriate at the largest}
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\text{scales (lowest values of k) due to the non-linear growth of}
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\[
\text{structure at smaller scales (Jennings et al. 2011, Okumura \\
\text{& Jing 2011). As a result, fitting Equation 5 to the galaxy}
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\[
\text{power spectra could produce a systematic error in the recovered}
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\[
\text{values of F and β; we include the equations simply to illustrate how F and β may be disentangled in model-fitting.}
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\[
\text{The signature of redshift-space distortions has been}
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\[
\text{modelled in more detail by many authors using various techniques including empirical fitting formulae (Hatton & Cole 1998), numerical N-body dark matter simulations (Jennings, Baugh \\
\text{& Pascoli 2011) and perturbation theory (Crocce \\
\text{& Scoccimarro 2006, Taruya, Nishimichi \\
\text{& Saito 2010). A full comparison of WiggleZ clustering data to}
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\text{these models was presented by Blake et al. (2011a).}
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\text{In the current study, we took as our fiducial redshift-space}
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\text{distortion model the fitting formulae provided by}
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\text{Jennings et al. (2011), which allow us to generate the functions}
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\[
\text{P_{θθ}(k) and P_{θθ}(k) in Equation 3 for given values of the}
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\text{growth rate f and the non-linear matter power spectrum}
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\text{P_{θθ}(k) = P_{θθ}(k) (which we obtained using “halofit” in}
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\text{CAMB). The Jennings et al. (2011) formulae have been}
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\text{constructed to be valid for a range of physical dark energy}
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\[
\text{models, and do not pre-suppose any specific continuity equation}
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\text{linking velocity and density. Unlike the Kaiser limit they do not}
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\text{assume a Friedmann-Robertson-Walker (FRW) metric,}
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\[
\text{as discussed by Samushia et al. (2011). We consider the}
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\text{effect of different choices of the redshift-space distortion model in}
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\[
\text{Section 3.4 below.}
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\[
\text{As a further enhancement of the redshift-space distortion model, we also considered multiplying Equation 3 by an empirical damping function}
\]

\[
\text{D(k, µ) representing convolution with uncorrelated galaxy motions on small scales. The standard choice for this damping function is a Lorentzian}
\]

\[
\text{P_{θθ}(k) = [1 + \left( \frac{k}{\sigma_b} \right)^2]^{-1}}
\]

\[
\text{where \sigma_b is the pairwise velocity dispersion, which may be varied as a free parameter but is}
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\[
\text{set to zero for our default model. The effects of this choice on}
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\[
\text{the Alcock-Paczynski measurements are considered in}
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\[
\text{Section 3.4 below.}
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\[
\text{3.3 Model fits}
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\[
\text{We fitted our default Jennings et al. (2011) clustering model, parameterized by (F, f, b^2), to the WiggleZ Survey power spectra over the range of scales k < 0.2 h Mpc}^{-1}. Figure 2 displays the joint likelihoods of the scale distortion parameter and growth rate in each of the redshift slices. We quantify the growth rate in this Figure by f \sigma_b(z), where}
\]
$\sigma_s(z) = D(z) \sigma_s(0)$ quantifies the normalization of the matter power spectrum at redshift $z$. This is a more model-independent observable than the growth rate itself owing to the degeneracy between $\sigma_s(z)$ and the galaxy bias $b$ in determining the overall galaxy clustering amplitude. We note that there is a strong correlation between $F$ and $f \sigma_s$, with correlation coefficients $r = (0.83, 0.94, 0.89, 0.84)$ in the four redshift slices, but both parameters may be successfully extracted. The marginalized measurements of the growth rate are $f \sigma_s(z) = (0.53 \pm 0.14, 0.40 \pm 0.13, 0.37 \pm 0.08, 0.49 \pm 0.12)$. These measurements are consistent with those reported by Blake et al. (2011a) who considered fits of redshift-space distortion models assuming $F = F_{fid}$. The resulting measurements of $F$, marginalizing over $f$ and $b^2$, are plotted in Figure 3 as the solid black data points. Our result is $F(z) \equiv (1+z)D_A(z) H(z)/c = (0.28 \pm 0.04, 0.44 \pm 0.07, 0.68 \pm 0.06, 0.97 \pm 0.12)$ in the four redshift slices. All of the different measurements reported in Sections 3, 4 and 5 are collected together in Table 1 for convenience.

### 3.4 Systematics tests

Figure 3 presents some tests of the sensitivity of our results to the different assumptions in the model described above. Firstly, we checked that our measurement of $F$ was not significantly changed if we replaced our default model with a range of other implementations of redshift-space distortions discussed in the recent literature, including generating the density and velocity power spectra using Renormalized Perturbation Theory (Crocce & Scoccimarro 2006) and adding the correction terms proposed by Taruya et al. (2010). Indeed, using the simplication of Equation 5, where $F_{3D}(k)$ is the non-linear power spectrum obtained from CAMB, produced an unchanged result for $F$ (within the statistical errors), as illustrated in Figure 3 (the cyan data points).

We also considered including and excluding the pairwise velocity dispersion as a free parameter. If we fit for $F$ by varying $f_L$ and fixing $f_3$, then marginalizing over the additional free parameter $\sigma_v$ makes no difference to our results (illustrated by the blue data points in Figure 3). However, if we fit for $F$ by varying $f_3$ and fixing $f_L$ then the resulting error in $F$ is significantly increased by marginalizing over $\sigma_v$, due to the cross-talk between $k_{90}$ and $\sigma_v$ in the damping term.

Any potential systematic error in the redshift-space distortion model is likely to worsen at smaller scales (higher values of $k$) where the modelling becomes less accurate. We carefully compared different choices of the fitting range $0 < k < k_{\text{max}}$, in order to ensure that any systematic error in the derived distortion parameter was not significant. The results of some of these tests are plotted in Figure 3. Our fiducial choice, $k_{\text{max}} = 0.2 h \text{ Mpc}^{-1}$, was determined by the consideration that the measurements of $F$ should not differ systematically between different implementations of the redshift-space distortion model. Results for $k_{\text{max}} = 0.1$ and...
0.3 h Mpc$^{-1}$ are displayed as the red and green data points in Figure 3. As a further test we repeated the measurements of $F$ for independent scale ranges $k = 0 - 0.1, 0.1 - 0.2$ and $0.2 - 0.3 h$ Mpc$^{-1}$; the results were consistent within the statistical errors.

Our analysis uses an underlying isotropic matter power spectrum that is consistent with the latest observations of the Cosmic Microwave Background radiation (Komatsu et al. 2011). In order to show that this does not introduce any sensitivity to the model into our analysis we checked that our results were unchanged if we instead generated a function proportional to $P_m(k)$ using a polynomial fit to the observed, angle-averaged galaxy power spectrum in the fiducial model; this comparison is shown as the magenta data points in Figure 3. The similarity between these measurements and our fiducial results provides further evidence that the baryon acoustic oscillations (which do not appear in a smooth polynomial model) are not contributing any information to the Alcock-Paczynski distortion fits.

We conclude from these tests that the systematic error in $F$ induced from modelling redshift-space distortions is much lower than the statistical error in the measurement. Our results are therefore insensitive to the model adopted (to first order), even though a series of model assumptions are necessary to produce these fits.

4 DETERMINATION OF THE COSMIC EXPANSION HISTORY

We converted our Alcock-Paczynski measurements of the scale distortion parameter $F(z) = (1+z)D_L(z)/H(z)$ into a determination of the cosmic expansion history $H(z)$ by using Type Ia supernovae (SNe Ia) data to fix the cosmic distance-redshift relation. We used the “Union-2” compilation by Amanullah et al. (2010) as our supernovae dataset, obtained from the website http://supernova.lbl.gov/Union. This compilation of 557 supernovae includes data from Hamuy et al. (1996), Riess et al. (1999, 2007), Astier et al. (2006), Jha et al. (2006), Wood-Vasey et al. (2007), Holtzman et al. (2008), Hicken et al. (2009) and Kessler et al. (2009).

Given that the normalization of the supernova Hubble diagram $M - 5 \log_{10} h$ is treated as an unknown parameter, the supernovae data yields the relative luminosity distance $D_L(z)/H_0$ of a model-independent value of this quantity in each redshift slice by fitting the distance-redshift relation as a 3rd-order polynomial in $z$ (over the redshift range $0 < z < 0.9$) and marginalizing over the values of the polynomial coefficients. We used the full covariance matrix of the supernovae measurements including systematic errors, and checked that our results were not significantly changed by assuming a 2nd-order or 4th-order polynomial instead. Our results at the four redshifts $z = (0.22, 0.41, 0.6, 0.78)$ were $D_L(z)/H_0 = (0.210 \pm 0.001, 0.376 \pm 0.005, 0.526 \pm 0.010, 0.655 \pm 0.015)$, where we have converted the luminosity distances to angular diameter distances assuming $D_L(z) = D_A(z)(1+z)^2$. We note that the inclusion of the supernovae systematics covariance matrix (compared to uncorrelated errors excluding systematics) increases the errors in these measurements of $D_A H_0/c$ by a factor of two. The supernovae data points and best-fitting 3rd-order polynomial model are displayed in Figure 4. This model provides a good fit to the data, with a chi-squared statistic of 486.6 for 519 degrees of freedom.

Combining the Alcock-Paczynski and supernovae measurements in the four redshift slices we find that $H(z)/H_0(1+z) \equiv \dot{a}/a_0 = (1.11 \pm 0.17, 0.83 \pm 0.13, 0.81 \pm 0.08, 0.83 \pm 0.10)$ where $a_0 \equiv H_0$. These results are plot-

Figure 4. This Figure displays the best-fitting 3rd-order polynomial to the Union-2 compilation of supernovae data, normalized as a plot of angular-diameter distance versus redshift.

![Figure 4](image_url)

Figure 5. This Figure displays our measurement of the evolution of the cosmic expansion rate using Alcock-Paczynski and supernovae data. The expansion rate is displayed using the value of $\dot{a}/a_0 = H(z)/(H_0(1+z))$; accelerating expansion implies a decrease in the value of this quantity with increasing redshift. The black data points are obtained by combining Alcock-Paczynski measurements of $(1+z)D_A(z)H(z)/c$ in four independent redshift slices with supernovae distance determinations of $D_L(z)H_0/c$ at these redshifts, and are independent of curvature. The thicker, blue data points result from applying the distance reconstruction method of Shafieloo et al. (2006) to both the supernovae and Alcock-Paczynski data, producing optimal errors at both low and high redshift but making the additional assumptions of zero spatial curvature and that $D_A(z)$ may be expressed in terms of an integral over $1/H(z)$. Predictions are plotted for three different models: a fiducial ΛCDM model with $\Omega_m = 0.27$ (solid line), an Einstein de-Sitter model with $\Omega_m = 1$ (dashed line), and a “coasting” model where $\dot{a} = \text{constant}$ (dotted line).
ted as the black error bars in Figure 5 and compared to three expansion history models: a ΛCDM cosmology with matter density $\Omega_m = 0.27$, an Einstein-de-Sitter (EdS) model with $\Omega_m = 1$ and a “coasting” model for which $da/dt = H_0$ at all times. We find that our measurements are consistent with the ΛCDM model. Figure 5 constitutes our model-independent measurement of the cosmic expansion rate $da/dt$, with 10-15% precision in each of the four redshift slices. We note that the systematic errors in the supernovae measurements are insignificant in the determination of $H(z)/[H_0(1+z)]$ because the error propagation is dominated by the error in the Alcock-Paczynski measurement.

Accelerating expansion implies a decrease in the value of $da/dt$ with increasing redshift. This is directly observed in our data: if we fit for a constant value of $a/a_0$ we find that the range $a/a_0 < 1$, containing accelerating-expansion models, contains 99.86% of the probability. Hence in this assessment, accelerating-expansion models are preferred with a statistical significance of more than 3-σ. We stress that this is a non-parametric result, insensitive to the redshift-space distortion model adopted, based solely on combining Alcock-Paczynski measurements of $D_A(z)H(z)$ with supernovae measurements of $D_A(z)$.

5 FULL DISTANCE-REDSHIFT RECONSTRUCTION

In this Section we use our datasets to map out the full cosmic expansion history via the non-parametric reconstruction method introduced by Shafieloo et al. (2006). We note that, unlike the preceding analysis, this approach assumes zero spatial curvature and that $D_A(z)$ can be expressed in terms of an integral over $1/H(z)$ (i.e., the FRW metric). Hence we expect to derive tighter constraints based on more assumptions.

5.1 Method for reconstructing $D_A(z)$

We first describe the application of the Shafieloo et al. (2006) distance reconstruction method to SNe Ia data. We then outline its generalization to enable the inclusion of the Alcock-Paczynski measurements.

The methodology of Shafieloo et al. (2006) is an iterative approach that deduces a distance-redshift curve $D_A(z)$ from an initial guess model by smoothing the residuals between the model and measurements of $D_A(z)$ and using these residuals to modify the guess. Iteration proceeds until the chi-squared statistic between model and data is minimized. The only parameter required by the method is the width of the smoothing kernel in redshift, $\Delta$ (our fiducial value is $\Delta = 0.1$, other choices are considered below). For the initial guess model to commence the iteration, we chose a flat ΛCDM model with $\Omega_m = 0.27$. Our results have no sensitivity to the choice of initial guess (as illustrated by the offset between the reconstructed and fiducial distance curve in the top left-hand panel of Figure 6). We defined the model on a grid of redshift steps spaced by $dz = 0.01$ from $z = 0$ to $z = 1$.

The equation generating iteration $i+1$ from iteration $i$, in terms of the supernovae data points $j$, is given in Shafieloo & Clarkson (2010):

$$\ln D_L(z)^{i+1} = \ln D_L(z)^i + N(z) \times \sum_j \frac{[\ln D_L(z_j) - \ln D_L(z_j)]}{\sigma_j^2} \exp \left[ -\frac{(z_j - z)^2}{2\Delta^2} \right]$$

(6)

where the redshift-dependent normalization is

$$N(z)^{-1} = \sum_j \frac{1}{\sigma_j^2} \exp \left[ -\frac{(z_j - z)^2}{2\Delta^2} \right]$$

(7)

For the inverse-variance weighting $\sigma_j$, we followed Shafieloo & Clarkson (2010) and used the error in each supernova luminosity distance.

We then modified the distance reconstruction method to utilize the residuals with respect to the Alcock-Paczynski data points. We initially generated an iterative correction to the quantity $F(z) = (1+z)D_A(z)H(z)/c$ using an analogous method to Equation 6:

$$\Delta F(z) = M(z) \sum_k \frac{F(z_k) - F(z_k)}{\sigma_k^2} \exp \left[ -\frac{(z_k - z)^2}{2\Delta^2} \right]$$

(8)

where $k$ labels the Alcock-Paczynski measurements at redshifts $z_k$, with errors $\sigma_k$ in $F(z)$, and the normalization factor is given by

$$M(z)^{-1} = \sum_k \frac{1}{\sigma_k^2} \exp \left[ -\frac{(z_k - z)^2}{2\Delta^2} \right]$$

(9)

We then converted the iteration in $F(z)$ into an iteration in $D_L(z) = (1+z)D_A(z)$ using

$$D_A(z)^{i+1} = D_A(z)^i - \sum_{z'} (1+z')D_A(z')\frac{\Delta F(z')}{F(z')}\Delta z',$$

(10)

where $\Delta z' = 0.01$ is the redshift interval of the gridding of the model curves. The right-hand side of Equation 10 is derived by expressing $D_A(z) = \sum_{z'} [c/H(z')]\Delta z'$, substituting $c/H(z') = (1+z')D_A(z)/F(z')$, and taking the derivative with respect to $F$. At each step in the iteration we applied the residual distance correction for both the supernovae and Alcock-Paczynski data points, proceeding until the chi-squared statistic was minimized.

We estimated the error in the distance-redshift reconstruction by repeating the iterative process using bootstrap re-samples of the datasets. The range of resulting distance-redshift curves across the re-samples defines the error in the reconstruction. We performed extensive tests using Monte Carlo realizations of synthetic datasets to verify that this procedure produced bias-free reconstructions with reliable error ranges. We note that this procedure differs from that of Shafieloo et al. (2006) and Shafieloo & Clarkson (2010), who used the set of distance curves $[\ln D_L(z)]$ generated by a single application of the iteration process of Equation 6 to estimate the error in the reconstruction. In order to obtain the error in the reconstructed curves we constructed separate bootstrap re-samples of the supernovae and Alcock-Paczynski data points.
Figure 6. This Figure shows our non-parametric reconstruction of the cosmic expansion history using Alcock-Paczynski and supernovae data. The four panels of this figure display our reconstructions of the distance-redshift relation $D_A(z)$, the expansion rate $\dot{a}/H_0$, the $\Omega_m(z)$ statistic and the deceleration parameter $q(z)$ using our adaptation of the iterative method of Shafieloo et al. (2006) and Shafieloo & Clarkson (2010). The distance-redshift relation in the upper left-hand panel is divided by a fiducial model for clarity, where the model corresponds to a flat $\Lambda$CDM cosmology with $\Omega_m = 0.27$. This fiducial model is shown as the solid line in all panels; Einstein de-Sitter and coasting models are also shown defined as in Figure 5. The shaded regions illustrate the 68% confidence range of the reconstructions of each quantity obtained using bootstrap resamples of the data. The dark-grey regions utilize a combination of the Alcock-Paczynski and supernovae data and the light-grey regions are based on the supernovae data alone. The redshift smoothing scale $\Delta = 0.1$ is also illustrated. The reconstructions in each case are terminated when the SNe-only results become very noisy; this maximum redshift reduces with each subsequent derivative of the distance-redshift relation [i.e. is lowest for $q(z)$].

5.2 Method for reconstructing $H(z)$, $\Omega_m(z)$ and $q(z)$

We used the reconstructed distance-redshift curves to determine three other diagnostics that describe the cosmic expansion history. Firstly, assuming a flat Universe we can differentiate the reconstructed distance-redshift curves to determine the expansion rate $\dot{a}/a_0 = H(z)/[H_0(1+z)]$. Secondly, a useful diagnostic of the expansion history that can be derived from $H(z)$ is the “Om” statistic (Sahni, Shafieloo & Starobinsky 2008):

$$\Omega_m (z) \equiv \frac{[H(z)/H_0]^2 - 1}{(1+z)^3 - 1}. \quad (11)$$

In a spatially-flat $\Lambda$CDM model this statistic is constant at different redshifts and equal to today’s value of the matter density parameter $\Omega_m$. In universes with different curvature, or containing dark energy with different properties to a cosmological constant, $\Omega_m(z)$ would evolve with redshift. Finally, by a second differentiation of the distance-redshift curves we can obtain the deceleration parameter $q(z) \equiv -\ddot{a}/\dot{a}^2$:

$$q(z) = (1 + z) \frac{dH(z)/dz}{H(z)} - 1. \quad (12)$$

The confidence ranges for the reconstruction of each quantity are established using bootstrap re-sampling.

5.3 Results of the reconstruction

Figure 6 displays our non-parametric reconstructions of the four quantities $D_A(z)$, $\dot{a}/dt$, $\Omega_m(z)$ and $q(z)$. The light-grey regions display the 68% confidence range of reconstructions using the supernovae data alone, and the inner dark-grey region derives from the combination of supernovae and Alcock-Paczynski measurements.

The addition of the Alcock-Paczynski data does not
strongly enhance the reconstruction of the distance-redshift relation \(D_A(z)\), which is very well-measured by supernovae alone. However, the direct sensitivity of \(F(z)\) to \(H(z)\) produces dramatic improvements in the reconstruction of the expansion-rate history, with a factor of 3 shrinking of the confidence ranges in \(H(z)\) for \(z > 0.2\). At the four redshifts \(z = (0.22, 0.41, 0.6, 0.78)\) the reconstruction method gives us \(H(z)/[H_0(1+z)] = (0.91 \pm 0.02, 0.88 \pm 0.03, 0.88 \pm 0.08, 0.80 \pm 0.08)\). These measurements are correlated, but the level of correlation is low because the fiducial choice of the smoothing length \(\Delta = 0.1\) is somewhat smaller than the bin width \(\Delta z = 0.2\).

Our measurements establish the constancy of \(\Omega_m(z)\) with redshift for the first time with reasonable precision, a key prediction of a flat \(\Lambda\)CDM model. At the four redshifts listed above the reconstruction method produces \(\Omega_m(z) = (0.28 \pm 0.05, 0.29 \pm 0.07, 0.31 \pm 0.13, 0.22 \pm 0.09)\). If we fit these values for the matter density in a flat \(\Lambda\)CDM model we find \(\Omega_m = 0.29 \pm 0.03\), consistent with fits to the CMB radiation (Komatsu et al. 2011).

We also overlaid in Figure 5 the non-parametric measurements of \(da/dt\) obtained by this reconstruction method in the four redshift slices. Comparison of the errors with the direct combination of supernovae distances and Alcock-Paczynski data shows that at low redshift the supernovae data alone place the most powerful constraint on the expansion history, greatly improving the results presented in Section 4, but at higher redshifts the Alcock-Paczynski data are crucial in order to obtain accurate measurements.

Figure 7 illustrates the effect of changing the smoothing length on the reconstructed expansion history \(\dot{a}/a_0\), considering values \(\Delta = 0.05, 0.10, 0.15\) and 0.20. A flat \(\Lambda\)CDM model with \(\Omega_m = 0.27\), shown as the solid line, continues to be a good description of the expansion history regardless of the smoothing scale. We checked that the error in fitting \(\Omega_m\) to these data was insensitive to \(\Delta\) when covariances between different redshifts were taken into account (estimating these covariances using the bootstrap resamples).

6 CONCLUSIONS

We summarize our study as follows:

- We have performed joint fits for the Alcock-Paczynski scale distortion parameter \(F(z) = (1+z)D_A(z)H(z)/c\) and the normalized growth rate \(f \sigma_8(z)\), using 2D power spectra measured from the WiggleZ Dark Energy Survey in four redshift bins in the range 0.1 < \(z < 0.9\). We can separate the contributions of Alcock-Paczynski and redshift-space distortions, and our result is insensitive to the range of models we examined for non-linear redshift-space effects when fitted to the wavenumber range \(k < 0.2h\) Mpc\(^{-1}\).

- By combining the Alcock-Paczynski fits with luminosity-distance measurements using Type Ia supernovae, we determined the cosmic expansion rate \(\dot{a}/a_0 = H(z)/[H_0(1+z)]\) in four redshift slices with 10-15\% accuracy. Our results for redshifts \(z = (0.22, 0.41, 0.6, 0.78)\) are \(\dot{a}/a_0 = (1.11 \pm 0.17, 0.83 \pm 0.13, 0.81 \pm 0.08, 0.83 \pm 0.10)\). These measurements are independent of spatial curvature.

- Our measurements show that the value of \(\dot{a}\) was lower at high redshift, demonstrating that the cosmic expansion has accelerated. If we fit for a constant value of \(\dot{a}\) in the redshift range 0.1 < \(z < 0.9\) we find that accelerating-expansion models are favoured with a statistical significance of more than 3-\sigma.

- We used an adapted version of the reconstruction method of Shaferlo et al. (2006) to model the continuous cosmic expansion history since \(z = 0.9\) using both the Alcock-Paczynski and supernovae datasets. The Alcock-Paczynski measurements enable a much more accurate determination of the expansion history than the supernovae data alone. We demonstrate that the quantity \(\Omega_m(z) \equiv \{[H(z)/H_0]^2 - 1]/(1+z)^3 - 1\}\) is constant with redshift, as expected in a spatially-flat \(\Lambda\)CDM model; fitting for the value of this quantity allows us produce an estimate of the matter-density parameter \(\Omega_m = 0.29 \pm 0.03\), consistent with fits to the CMB radiation (Komatsu et al. 2011).

We conclude that accelerating cosmic expansion can be recovered from cosmological data in a non-parametric manner and hence is a real physical phenomenon that must be accounted for by theory. Our results are independent of the cosmological model adopted, although our data analysis assumes a homogeneous Universe. The measured expansion history is well-fit by a cosmological constant which grows in relative importance with cosmic time. The combination of Alcock-Paczynski and supernovae data is a novel approach that enables direct observation of the Hubble expansion rate and will be strengthened in the future by the availability of new galaxy redshift survey data.

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Figure 7. In this Figure we repeat the reconstruction of $\dot{a}/H_0$ using both supernovae and Alcock-Paczynski data for different choices of the smoothing length $\Delta$, the only free parameter in the reconstruction algorithm. The solid band shows the 68% confidence range spanned by the reconstructions and the solid red line is the prediction of a flat $\Lambda$CDM model with $\Omega_m = 0.27$. 

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