Sliding Mode Learning Control
and its Applications

Manh Tuan Do

Submitted in total fulfilment of the requirements of the degree of
Doctor of Philosophy

Faculty of Science, Engineering and Technology
Swinburne University of Technology
Melbourne, Australia

2014
Abstract

With the rapid advancement of control technologies, there have been various intelligent control schemes established for complex systems with or without uncertain dynamics. Given a certain control problem, desirable qualities such as simplicity, applicability, adaptability, and robustness are the touchstones of control design so as to ensure excellent control performance against system parameter variations and unpredicted external disturbances.

Amongst many robust control techniques, Sliding Mode Control (SMC) has been increasingly receiving a great deal of attention in both theoretical and applied disciplines owing to its distinguishing features such as insensitivity to bounded matched uncertainties, order reduction of sliding motion equations, decoupling design procedure, and zero-error convergence of the closed-loop system, just to name a few. Nevertheless, the shortcomings inherent in conventional SMC approaches are yet to be fully addressed. For one, the chattering phenomenon has not been uprooted without compromising on the zero-error convergence. More importantly, from the control design perspective, there are certain constraints in the design of SMC, such as prior information about the bounds of uncertainties is often required, this in turn has greatly restrained the applications of SMC in many practical circumstances. Therefore, how to make the best use of SMC in order to develop a simple but effective SMC technique has remained a big challenge for both researchers and engineers in the areas of control engineering and the related technologies.

To tackle these issues, this thesis is concerned with the sliding mode based learning control technique and its applications. The sliding mode learning control (SMLC) developed in this research enjoys several overwhelming superiorities over its conventional counterparts: (i) since the learning algorithm is adopted, the knowledge of the uncertainty is no longer a prerequisite for controller design and thus (ii) the control input is completely chattering-free, and (iii) the SMLC scheme poses a strong robustness with respect to unmodelled dynamics. It is seen that the proposed SMLC not only inherits all the appealing characteristics of SMC, but also helps curb the drawbacks that befall conventional SMC approaches. Therefore, it is for this reason that the
proposed SMLC will potentially play an essential role in years to come, in terms of relaxing many constrains associated with the bounds of uncertain dynamics in conventional SMC schemes.

In this thesis, novel SMLC schemes will be developed for a wide range of uncertain dynamic systems. In particular, the concept of the most recently introduced SMLC technique associated with the so-called Lipschitz-like condition is extensively studied. First of all, the SMLC scheme is well examined with mathematical proofs and then developed for a class of uncertain dynamic systems in a continuous-time domain. Some concluding remarks are highlighted to boost the significant advantages of the proposed SMLC scheme over existing control ones. Numerical results are presented to verify the SMLC algorithm. Next, the SMLC technique is applied to address the stabilization of nonminimum phase nonlinear systems and congestion control of communication networks.

Following this development, the SMLC scheme is further tested and successfully deployed to control steer-by-wire systems of modern vehicles. The experimental results have confirmed the excellent performance of the proposed SMLC. Finally, the framework is further developed for a class of uncertain dynamic systems in a discrete-time domain followed by the application of congestion control in connection-oriented communication networks.
Declaration

This is to certify that this thesis:

- contains no material which has been accepted for the award to me towards any other degree or diploma, except where due reference is made in the text of the examinable outcome;

- to the best of my knowledge, contains no material previously published or written by another person except where due reference is made in the text of the examinable outcome; and

- where the work is based on joint research and publications, discloses the relative contributions of the respective authors.

________________________

Manh Tuan Do, 2014
Preface

This thesis is based on the research work conducted over the course of the past four years in the Faculty of Science, Engineering and Technology, Swinburne University of Technology, under the supervision of Prof. Zhihong Man, Prof. Cishen Zhang, and Dr. Jiong Jin.

As a result, a number of journal papers and international conference papers have been published or submitted for publication. The following summarizes the author’s publications and contributions pertaining to the relevance of each particular chapter of this thesis, and the complete list of the author’s publications can be found at the end of the thesis.

The work in Chapter 3 on the proposed SMLC scheme, the backbone of the SMLC concept developed in this thesis, presents the fundamental and conceptual theory of the proposed SMLC and the significance of the approach, as well as the so-called Lipschitz-like condition newly initiated by Man et al. [84].

The research outcome in Chapter 4 on Robust Stabilization of Nonminimum Phase Systems Using Sliding Mode Learning Controller has led to a journal paper submitted to *IEEE Transactions on Cybernetics* and a conference paper presented at *ICIEA 2013* (Do et al. [154]).

The work in Chapter 5 on Sliding Mode Learning Based Congestion Control for DiffServ Networks has resulted in a journal paper submitted to *IEEE Transactions on Control of Network Systems*.

The result of Chapter 6 on Robust Sliding Mode Based Learning Control for Steer-by-Wire Systems in Modern Vehicles has outputted a journal paper published in *IEEE Transactions on Vehicular Technology* (Do et al. [87]).

The work about Robust Sliding Mode Learning Control for Uncertain Discrete-Time MIMO Systems in Chapter 7 has yielded a journal paper published in *IET Control Theory and Applications* (Do et al. [88]) and a conference paper presented at *ICARV 2012* (Do et al. [107]).
Lastly, the research result of Chapter 8 on Discrete-Time Sliding Mode Learning Based Congestion Control for Connection-Oriented Communication Networks has led to a journal paper submitted to *IEEE Transactions on Communications Letters*.
Acknowledgement

First and foremost, I would like to thank my supervisory team Prof. Zhihong Man, Prof. Cishen Zhang, and Dr. Jiong Jin for their invaluable guidance and constant support throughout the past four years. They have made tremendous effort to offer me both academic and social advice to make my PhD life a noteworthy and rewarding one. Especially, I would like to express my deepest gratitude to Prof. Zhihong Man for his endless supervision of my doctoral research advancement, for giving me all the opportunities and motivation to pursue my PhD degree at Swinburne University of Technology, and for spurring me to endeavour for the best and to be a goal-oriented individual. I could not have asked for a more supportive and caring mentor who is always accessible and passionate in patiently coaching me about the much desired knowledge and skills that are indispensable for the accomplishment of this thesis.

I am grateful to Swinburne University of Technology for awarding me the SUPRA scholarship and catering me with a conducive and favourable working environment. Many thanks go to Melissa, Sophia and Adrianna from the research administration and finance group for promptly looking after any inquiries or concerns I had with a very warm welcome. I would also like to thank the senior technical staff, Walter and Krys, as well as the ITS members for their continual support in swiftly resolving many technical issues and providing resources and assistance whenever needed. Every little thing you did made my everyday life as a PhD candidate a whole lot easier.

To Jinchuan, Hai, Feisiang, and Kevin, I am very thankful to have you all as friends and research fellows. It has been my honour to have shared my research experience with all of you in some technical sessions, seminars, conferences, and even through our day-to-day discussions and chats. Thank you very much for your friendship and collaboration. You have made my PhD life in Melbourne much more vibrant and enjoyable.

Last but not least, I am truly indebted to my beloved family members who unrelentingly believed in me and encouraged me to follow my dreams. I cannot thank them enough for their endless love, care, and sacrifices. Without their support, neither my life nor my work would bring fulfilment.
To my wife Mai Tuyet Phung, my daughter Isabella Do, my mother Thin Thi Pham, and in memory of my father Cu Hong Do.
## Contents

1 Introduction ......................................................... 1
   1.1 Preliminaries ................................................................. 1
       1.1.1 Variable Structure Systems ........................................ 2
       1.1.2 Sliding Mode Control .................................................. 3
   1.2 Motivation ................................................................. 6
   1.3 Objectives and Major Contributions of the Thesis ............... 7
   1.4 Organization of the Thesis ............................................. 8

2 Background and Literature review .................................. 11
   2.1 Introduction ................................................................. 11
   2.2 Lyapunov Stability Theory ............................................. 11
       2.2.1 Stability of Equilibrium Points ................................. 12
       2.2.2 Lyapunov’s Direct Method ......................................... 13
   2.3 Basics of Sliding Mode Control Systems ......................... 14
       2.3.1 System Model and Sliding Mode Surface Design .......... 14
       2.3.2 Reaching Phase ....................................................... 16
       2.3.3 Reaching Laws ........................................................ 17
       2.3.4 Equivalent Controller Design ..................................... 18
       2.3.5 Robustness Property ............................................... 19
       2.3.6 Chattering Phenomenon ............................................ 20
   2.4 Sliding Mode Control Algorithms ................................... 23
       2.4.1 Second Order Sliding Mode Control ............................ 23
       2.4.2 Higher Order Sliding Mode Control ............................ 25
       2.4.3 Terminal Sliding Mode Control .................................. 26
       2.4.4 Integral Sliding Mode Control .................................... 28
       2.4.5 Sliding Mode Control with Perturbation Estimation ........ 30
   2.5 Discrete-Time Sliding Mode Control Systems ................... 31
       2.5.1 Overview ............................................................... 32
<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.5.2</td>
<td>Discretization of Sliding Mode Control Systems</td>
<td>32</td>
</tr>
<tr>
<td>2.5.3</td>
<td>Stability and Controller Design</td>
<td>34</td>
</tr>
<tr>
<td>2.6</td>
<td>Conclusion</td>
<td>35</td>
</tr>
<tr>
<td>3</td>
<td>Sliding Mode Learning Control Scheme</td>
<td>37</td>
</tr>
<tr>
<td>3.1</td>
<td>Introduction</td>
<td>37</td>
</tr>
<tr>
<td>3.2</td>
<td>Problem Formulation</td>
<td>39</td>
</tr>
<tr>
<td>3.3</td>
<td>Lipschitz-Like Condition</td>
<td>40</td>
</tr>
<tr>
<td>3.4</td>
<td>Convergence Analysis</td>
<td>42</td>
</tr>
<tr>
<td>3.5</td>
<td>Simulation</td>
<td>44</td>
</tr>
<tr>
<td>3.6</td>
<td>Conclusion</td>
<td>46</td>
</tr>
<tr>
<td>4</td>
<td>Robust Stabilization of Nonminimum Phase Systems Using Sliding Mode Learning Controller</td>
<td>49</td>
</tr>
<tr>
<td>4.1</td>
<td>Introduction</td>
<td>49</td>
</tr>
<tr>
<td>4.2</td>
<td>Problem Formulation</td>
<td>52</td>
</tr>
<tr>
<td>4.2.1</td>
<td>Input-Output Realization of Nonlinear Systems</td>
<td>52</td>
</tr>
<tr>
<td>4.2.2</td>
<td>Vanishing Perturbation</td>
<td>55</td>
</tr>
<tr>
<td>4.2.3</td>
<td>Sliding Mode Learning Controller</td>
<td>57</td>
</tr>
<tr>
<td>4.3</td>
<td>Convergence Analysis</td>
<td>58</td>
</tr>
<tr>
<td>4.4</td>
<td>Simulation Results</td>
<td>62</td>
</tr>
<tr>
<td>4.5</td>
<td>Conclusion</td>
<td>72</td>
</tr>
<tr>
<td>5</td>
<td>Sliding Mode Learning Based Congestion Control for DiffServ Networks</td>
<td>73</td>
</tr>
<tr>
<td>5.1</td>
<td>Introduction</td>
<td>73</td>
</tr>
<tr>
<td>5.2</td>
<td>Problem Formulation</td>
<td>75</td>
</tr>
<tr>
<td>5.2.1</td>
<td>Congestion Control for DiffServ Networks</td>
<td>75</td>
</tr>
<tr>
<td>5.2.2</td>
<td>Sliding Mode Learning Controller Design</td>
<td>77</td>
</tr>
<tr>
<td>5.3</td>
<td>Stability Analysis</td>
<td>79</td>
</tr>
<tr>
<td>5.4</td>
<td>Simulation Results</td>
<td>83</td>
</tr>
</tbody>
</table>
6 Robust Sliding Mode Based Learning Control for Steer-by-Wire Systems in Modern Vehicles

6.1 Introduction .................................................................................................. 91
6.2 Problem Formulation .................................................................................... 93
  6.2.1 Dynamics of SbW Systems .............................................................. 93
  6.2.2 Steering AC Motor Torque Perturbation .......................................... 97
  6.2.3 Sliding Mode Learning Control ....................................................... 100
6.3 Convergence Analysis ................................................................................. 101
6.4 Numerical Simulations ................................................................................ 101
6.5 Experimental Results ................................................................................... 105
6.6 Conclusion ................................................................................................... 112

7 Robust Sliding Mode Learning Control for Uncertain Discrete-Time MIMO Systems

7.1 Introduction ................................................................................................. 113
7.2 Problem Formulation ................................................................................... 116
  7.2.1 Discretization of Continuous-Time MIMO Systems ....................... 116
  7.2.2 Design of Sliding Manifold ............................................................. 119
  7.2.3 Design of Discrete-Time Sliding Mode Learning Controller ........ 120
7.3 Convergence Analysis ................................................................................. 122
7.4 Illustrative Examples ................................................................................... 127
7.5 Conclusion ................................................................................................... 133

8 Discrete-Time Sliding Mode Learning Based Congestion Control for Connection-Oriented Communication Networks

8.1 Introduction .................................................................................................. 135
8.2 Problem Formulation ................................................................................... 136
  8.2.1 Network Model ................................................................................ 136
  8.2.2 Design of Discrete-Time Sliding Mode Learning Controller ........ 138
8.3 Stability Analysis ................................................................. 139
8.4 Simulation Example ............................................................ 142
8.5 Conclusion ........................................................................... 144

9 Conclusion and Future Work ............................................. 145
  9.1 Summary of Contributions .................................................... 145
  9.2 Future Work ........................................................................ 146
    9.2.1 Time-Delayed Systems .................................................. 146
    9.2.2 Observations and Identifications ................................. 146
    9.2.3 Real-World Applications .............................................. 147

Appendix A .............................................................................. 149
  A.1 Proof of the Lipschitz-Like Condition in a Continuous-Time Domain Given in the Inequality (3.11) ................................................................. 149
  A.2 Proof of the Condition (3.12) ............................................... 150
  A.3 Verification of the Condition (3.13) ....................................... 151
  A.4 Verification of the Continuity of the Proposed SMLC \( u(t) \) Given in the Equation (3.6) ................................................................. 152

Appendix B .............................................................................. 153
  B.1 Validation of the Lipschitz-Like Condition in a Discrete-Time Domain Given in the Inequality (7.13) ................................................................. 153
  B.2 Validation of the Inequality (7.15) ........................................... 154

Author’s Publications ............................................................. 157

Bibliography .......................................................................... 159
List of Figures

1.1 The representation of a controlled VSS .............................................................. 2
1.2 Phase portrait of system (1.1) with controller (1.2) ............................................ 3
1.3 Phase trajectory of system (1.3) with controller (1.5) ........................................ 5
2.1 The two phases of ideal sliding mode ............................................................... 16
2.2 The chattering phenomenon .............................................................................. 21
2.3 Saturation function sat(s) ................................................................................. 22
3.1 Sliding mode variable (SMLC) ......................................................................... 45
3.2 System state responses (SMLC) ........................................................................ 46
3.3 Control input (SMLC) ....................................................................................... 46
4.1.a Virtual sliding variable σ (Conventional SMC) ............................................. 65
4.1.b Input-output states x_1 & x_3 (Conventional SMC) ....................................... 65
4.1.c Internal state x_2 (Conventional SMC) .......................................................... 65
4.1.d Control input (Conventional SMC) ................................................................ 66
4.2.a Virtual sliding variable σ (Proposed SMLC) ................................................ 66
4.2.b Input-output states x_1 & x_3 (Proposed SMLC) ........................................... 66
4.2.c Internal state x_2 (Proposed SMLC) .............................................................. 67
4.2.d Control input (Proposed SMLC) ................................................................... 67
4.3.a Input-output states x_1 & x_2 (Backstepping) ............................................... 69
4.3.b Internal states x_3 & x_4 (Backstepping) ....................................................... 70
4.3.c Control signal (Backstepping) ....................................................................... 70
4.4.a Virtual sliding variable (Proposed SMLC) ................................................... 70
4.4.b Input-output states x_1 & x_2 (Proposed SMLC) .......................................... 71
4.4.c Internal states x_3 & x_4 (Proposed SMLC) .................................................... 71
4.4.d Control signal (Proposed SMLC) ................................................................. 71
5.1 Proposed control scheme for DiffServ traffic ................................................... 76
5.2 Incoming rate of premium traffic ................................................................. 85
5.3.a Buffer length of premium traffic (Conventional SMC) ................................. 85
5.3.b Control signal of premium traffic (Conventional SMC) ............................... 86
5.3.c Buffer length of ordinary traffic (Conventional SMC) ................................. 86
5.3.d Control signal of ordinary traffic (Conventional SMC) ............................... 86
## List of Abbreviations and Acronyms

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>BL-SMC</td>
<td>Boundary layer sliding mode control</td>
</tr>
<tr>
<td>CT</td>
<td>Continuous-time</td>
</tr>
<tr>
<td>DiffServ</td>
<td>Differentiated service</td>
</tr>
<tr>
<td>DSMC</td>
<td>Discrete-time sliding mode control</td>
</tr>
<tr>
<td>DSMLC</td>
<td>Discrete-time sliding mode learning control</td>
</tr>
<tr>
<td>DT</td>
<td>Discrete-time</td>
</tr>
<tr>
<td>FTSM</td>
<td>Fast terminal sliding mode</td>
</tr>
<tr>
<td>HOSM</td>
<td>Higher order sliding mode</td>
</tr>
<tr>
<td>IntServ</td>
<td>Integrated service</td>
</tr>
<tr>
<td>LC</td>
<td>Learning control</td>
</tr>
<tr>
<td>LTI</td>
<td>Linear time invariant</td>
</tr>
<tr>
<td>MIMO</td>
<td>Multi-input multi-output</td>
</tr>
<tr>
<td>OF-SMC</td>
<td>Output feedback sliding mode control</td>
</tr>
<tr>
<td>PMSM</td>
<td>Permanent magnet synchronous motor</td>
</tr>
<tr>
<td>QoS</td>
<td>Quality of service</td>
</tr>
<tr>
<td>QSM</td>
<td>Quasi-sliding mode</td>
</tr>
<tr>
<td>SbW</td>
<td>Steer-by-wire</td>
</tr>
<tr>
<td>SISO</td>
<td>Single-input single-output</td>
</tr>
<tr>
<td>SMC</td>
<td>Sliding mode control</td>
</tr>
<tr>
<td>SMCPE</td>
<td>Sliding mode control with perturbation estimation</td>
</tr>
<tr>
<td>SMLC</td>
<td>Sliding mode learning control</td>
</tr>
<tr>
<td>SOSMC</td>
<td>Second order sliding mode control</td>
</tr>
<tr>
<td>TA</td>
<td>Twisting algorithm</td>
</tr>
<tr>
<td>TCP</td>
<td>Transport control protocol</td>
</tr>
<tr>
<td>TSM</td>
<td>Terminal sliding mode</td>
</tr>
<tr>
<td>VGRS</td>
<td>Variable gear ratio steering</td>
</tr>
<tr>
<td>VSC</td>
<td>Variable structure control</td>
</tr>
<tr>
<td>VSS</td>
<td>Variable structure system</td>
</tr>
<tr>
<td>ZOH</td>
<td>Zero order hold</td>
</tr>
</tbody>
</table>
Chapter 1
Introduction

The primary objective of control engineering is to ensure that an object or a system under control operates in a desired manner. The desirable operation of the system has to be achieved in real time despite unpredictable influences of the environment on all parts of the controlled system, including the system itself, and no matter whether a system designer knows precisely all the parameters of the system. Though the parameters may vary with time, load, and external disturbances, still the system should preserve its nominal properties and ensure the desired behaviour of the system. In other words, the underlying purpose of control engineering is to design control systems which are robust with respect to external disturbances and modelling uncertainties. On the other hand, almost all of the real-world systems are complex and uncertain in nature. As the complexity of control problems soars, strategic control designs for complex systems become more crucial.

Variable structure control (VSC) or sliding mode control (SMC) in particular has been treated as a powerful technique to cope with complex systems with unmodelled dynamics due to its simplicity and strong robustness with respect to system parameter variations and external disturbances. For this reason, in this thesis we aim to focus on designing novel intelligent control schemes based on SMC philosophy. In order to make the thesis self-contained, let us begin this chapter by presenting the main concepts commonly used in the field of variable structure systems (VSS), SMC designs and applications.

1.1 Preliminaries

In this section we introduce some basic concepts and fundamentals of VSS and SMC that will be used frequently throughout this thesis.
1.1.1 Variable Structure Systems

VSS concepts are of great importance in systems and control theory. Since the pioneering work of Emel’yanov and Barbashin in the 1960s, VSS theory has evolved at a rapid rate and attracted plenty of researches in both literature and applied aspects [1-14]. In principle, VSS can be represented by Figure 1.1, which consists of several different continuous subsystems or structures \((S_i, i = 1, n)\) that act one at a time through the input-output path. Study of VSC therefore involves the design of certain switching logic schedules according to the relevant structures.

![Figure 1.1. The representation of a controlled VSS.](image)

This control initiative may be illustrated by the following example. Let us consider a second-order system

\[
\begin{align*}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= u_i \\
\end{align*}
\tag{1.1}
\]

where \(x_1, x_2\) denote the system state variables, and the feedback control law \(u_i\) is defined as

\[
\begin{align*}
u_i = \begin{cases} 
-x_1 & \text{for } x_1x_2 < 0 \\
-5x_1 & \text{for } x_1x_2 \geq 0 
\end{cases}
\end{align*}
\tag{1.2}
\]

The performance of the system (1.1) controlled by (1.2) is shown in Figure 1.2. It is seen that by adopting the switching control law (1.2) the system (1.1) is guaranteed to
be asymptotically stable. This example presents the concept of VSC and stresses that the system dynamics in VSC is determined not only by the applied feedback controllers but also, to a large extent, by the adopted switching strategy.

![Phase portrait of system (1.1) with controller (1.2).](image.png)

**Figure 1.2.** Phase portrait of system (1.1) with controller (1.2).

### 1.1.2 Sliding Mode Control

VSC is inherently a nonlinear control technique and as such, it offers a variety of merits which can hardly be achieved using conventional linear controllers. However, the main benefit of the system is in fact obtained as soon as the controlled plant exhibits the so-called sliding motion [1-5, 15]. The idea of SMC is to employ different feedback controllers acting on the opposite sides of a predetermined surface (often called sliding surface) in the system state space. Each of those controllers drives the system trajectory to reach the sliding surface, and once it hits the surface for the first time it stays on it thereafter. The resulting motion of the system is confined to the surface, which graphically can be interpreted as “sliding” of the system states along that surface. The idea is illustrated by the following example.

Let us consider another second-order system
\[
\begin{align*}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= b\sin(mx_1) + u & |b| \leq \beta
\end{align*}
\] (1.3)

where \(b, m\) are possibly unknown constants and \(\beta > 0\) is the upper bound of \(b\).

We select the sliding surface in the state space as follows

\[
s = x_2 + cx_1
\] (1.4)

and apply the controller

\[
u = -cx_2 - \alpha \text{sign}(s)
\] (1.5)

where \(c\) is a positive constant, \(\alpha > \beta\), and the \(\text{sign}(\cdot)\) function is widely known as

\[
\text{sign}(s) = \begin{cases} 
1 & \text{for } s > 0 \\
0 & \text{for } s = 0 \\
-1 & \text{for } s < 0 
\end{cases}
\] (1.6)

The simulation result is shown in Figure 1.3 with the system parameters being \(c = 0.5, m = 15, \beta = 1.75, \alpha = 2\), and the initial condition \(x_1(0) = 5\) and \(x_2(0) = 2\).

The distinction of SMC systems is made up of two phases: the reaching phase which lasts until the controlled plant trajectory has reached the sliding surface, and the sliding phase. In the latter, the plant motion is governed by the sliding surface. This implies that neither modelling inaccuracies nor external disturbances affect the responses of the closed-loop dynamics that is a highly desirable property of SMC systems. Another immediate consequence of the fact that in the sliding mode, the system dynamic motion being restricted to the switching hypersurface (which is a subset of the state space) enjoys the reduction of the system order.

To put things in a nutshell, the major task of SMC system design is the selection of an appropriate control law in a way that alters the dynamics of a complex system by application of a discontinuous control input that drives the system states to reach the sliding hypersurface and slide along the surface for all subsequent time. The underlying feature of the method is that once the sliding mode is reached, the system dynamics, by
A proper choice of a predefined hypersurface, exhibit desirable behaviour which is inherently invariant to disturbances.

![Phase trajectory of system (1.3) with controller (1.5).](image)

Figure 1.3. Phase trajectory of system (1.3) with controller (1.5).

Despite some aforementioned benefits, SMC brings with it several disadvantages associated with conventional designs. For one, due to the discontinuous switching mechanism, the undesired chattering in the control input may excite high frequencies in system responses which are almost unbearable to operations of actuators and mechanical components. This phenomenon leads to deteriorations and potentially causes unpredictable instabilities of the closed-loop system. In addition, without knowing the information about the bounds of the uncertainties, it is hard, if not impossible, to design a robust SMC to ensure the robust stability of the closed-loop system. These drawbacks to a large extent have restricted the applications of conventional SMC schemes in many practical circumstances. Therefore, the quest for developing novel intelligent control solutions to tackle these issues appears to be a demanding challenge of our times.
1.2 Motivation

The foregoing problems of the conventional SMC approaches are mainly due to strongly nonlinear behaviours and lack of precise knowledge of complex systems. Therefore, advanced control techniques are much needed to cope with the detrimental effects of uncertainties and nonlinearities on dynamic systems. Since the SMC can provide an efficient method that can guarantee a strong robustness and an asymptotic convergence of the closed-loop system, many researchers are mostly concerned with developing advanced SMC strategies and SMC based ones for an uncertain dynamic model which best describes the dynamic system.

Although approaches developed to address this problem are varied, there has not been a perfect solution. Indeed, most of the control strategies developed for the complex systems require prior knowledge of uncertain system dynamics. In practice, this is often not possible as the information about uncertain system dynamics is not achievable. In consequence, such control strategies may not be applicable to large-scale systems. It goes without saying there is still an urgent need to focus on the development of a robust controller to deal with real-time complex systems without relying on the information related to the system uncertainties and disturbances. This has piqued more intense interests in the development of a robust SMC control scheme to overcome the following major issues:

- Zero-error convergence, finite-time stability, and strong robustness
- Lack of information about the bounds of system uncertainties and external disturbances
- Workability and applicability to real control problems
- Ease of implementation

In order to develop a simple but effective control scheme for uncertain dynamic systems, all the existing issues mentioned earlier will be addressed appropriately in this thesis. Inspired by the SMC and learning control (LC) theory, several sliding mode learning control (SMLC) algorithms will be proposed in an attempt to stabilize a large class of complex systems with uncertain dynamics. The proposed control algorithms not only guarantee an asymptotic stability of the closed-loop system, but also allow the closed-loop system to boast a strong robust property with respect to system uncertainties and disturbances. The huge advantages of the proposed control algorithms are that the
controller designs do not require prior information about uncertain system dynamic to be known, meanwhile, the chattering phenomenon that frequently appears in conventional SMC system is completely eliminated without deteriorating the robustness of the closed-loop system.

1.3 Objectives and Major Contributions of the Thesis

The goal of this thesis is to develop a new breed of robust intelligent control schemes based on the philosophy of sliding mode control for a large class of systems with uncertain dynamics. More specifically, the major contributions of the thesis are outlined as follows:

i. A novel SMLC scheme is proposed to address long-standing drawbacks existing in conventional SMC designs such as chattering, finite-time stability, robustness, and especially constraints on the bounds of uncertainties.

ii. The so-called Lipschitz-like condition is well studied and cleverly embodied in the proposed SMLC scheme, which helps to relax the constraints on the bounds of uncertainties.

iii. The SMLC approach is well examined and investigated through both rigorous mathematical approaches and numerical simulations to verify the effectiveness of the proposed control algorithms.

iv. The SMLC scheme is fully developed for robust stabilization of nonminimum phase nonlinear systems. Therefore, this thesis offers a practical solution to complex real-world control problems.

v. The newly developed SMLC technique is diversely disseminated and successfully applied to cross-disciplinary engineering fields including a mixed variety of practical applications in control of steer-by-wire systems in electric vehicles and congestion control of communication networks.

vi. The SMLC scheme is further developed for a class of uncertain discrete-time MIMO systems with an application in communication networks.

To sum up, the research conducted in this thesis offers both developments and implementations of the proposed SMLC methodology in the various fields of engineering such as control of steering systems in modern vehicles, congestion control
of communication networks, robotics, and power drive motor systems. It is highly believed that such SMLC algorithms are less conservative than conventional SMC ones, and hence can be potentially used to serve for the next generation of complex control systems with a wide range of applications in years to come.

1.4 Organization of the Thesis

This thesis explores the designs of novel sliding mode learning control scheme for a variety of complex systems that possess strong robustness with respect to uncertainties and disturbances. The rest of the thesis is organized as follows:

Chapter 2 provides a brief survey of the existing SMC and its development. Some important aspects in this area are discussed. Special attention is given to the SMC controller design methods, robustness analysis and key issues in conventional SMC theory and applications.

Chapter 3 presents the fundamental studies of proposed SMLC algorithms, which are a steppingstone for construction of the following chapters. The convergence analysis is discussed in detail with mathematical proofs. Some essential remarks are highlighted and numerical simulations are conducted to confirm the significance of the proposed control approach.

From an implementation perspective, we have designed a robust SMLC scheme for a class of nonminimum phase nonlinear systems in Chapter 4. The concept of system centre is used to design a learning controller capable of driving the sliding variable to reach the sliding surface in finite time and remain on it thereafter. The closed-loop dynamics of both observable and nonobservable states are then guaranteed to asymptotically converge to zero in the sliding mode. The stability analysis and simulation results illustratively show superior characteristics of the proposed SMLC over existing ones.

Chapter 5 further investigates the sliding mode learning scheme in congestion control of DiffServ networks. The proposed SMLC takes into account the associated physical network resource limits and is intensively devised to guarantee the stability of the closed-loop system with strong robustness against unknown and time-varying delays. Numerical simulations are presented to demonstrate the effectiveness and capabilities of the sliding mode learning based congestion control technique.
Chapter 6 is dedicated to the application of the proposed SMLC scheme in control of SbW systems in road vehicles. The SMLC has been successfully developed and implemented for an SbW system with uncertain system parameters and unknown external disturbance from interactions between the tires and the variable road surface. Both simulations and experiments are carried out to show an excellent steering performance achieved using the proposed SMLC in comparison with other conventional control schemes.

Chapter 7 is concerned with the concept of the sliding mode learning control in a discrete-time domain. The SMLC is extended to facilitate a larger class of discrete-time systems with uncertainties. A discrete-time sliding mode learning control (DSMLC) scheme is accordingly developed to guarantee the asymptotic convergence of the closed-loop dynamics. It is proven that the appealing attributes of the SMLC in a continuous-time domain are to be retained in a discrete-time framework.

Next, Chapter 8 presents an application of the DSMLC scheme developed in Chapter 7 for congestion control of connection-oriented communication networks. The problem of congestion control in communication network is addressed completely by adopting the DSMLC, which guarantees the closed-loop stability with strong robustness against uncertain dynamics.

Finally, Chapter 9 draws a reasoned conclusion that highlights the major contributions and suggests future work in this field. The author’s publications based on this thesis are given at the end. In addition, the relevant mathematical proofs regarding the SMLC and DSMLC algorithms developed in this thesis are provided in the Appendices.
Chapter 2

Background and Literature Review

2.1 Introduction

Variable structure control, particularly known as sliding mode control was originated by Russian scientists in the early 1960s and later reported in Utkin’s monograph in 1970s [1, 4-5]. SMC has since been studied extensively and successfully applied to many practical control problems due to its simplicity and robustness against system uncertainties and disturbances [16-33]. The appealing feature of SMC is its sliding motion. In the sliding mode, the dynamic motion of the system is effectively constrained to lie within a certain subspace of the full state space. The sliding motion is then achieved by altering the system dynamics along sliding mode surfaces in the states space. On the sliding mode surface, the system is equivalent to an unforced system of lower order, which is insensitive to both system uncertainties and disturbances.

This chapter presents a brief literature review of underlying concepts of SMC. The content of this chapter is organised as follows: In Section 2.2, we recall the fundamental of Lyapunov stability theory, and Lyapunov’s direct method in particular, which is widely accepted as the centrepiece in the study of dynamical control systems. In Section 2.3, the basic concepts of SMC systems including SMC design and SMC properties will be reviewed followed by the introduction of some advanced SMC algorithms presented in Section 2.4. Next, the overview of discrete-time SMC will be given in Section 2.5 before the conclusion is drawn in Section 2.6.

2.2 Lyapunov Stability Theory

Stability theory plays a central role in systems theory and control engineering. There are different kinds of stability, such as input-output stability and stability of periodic orbits. In particular, stability of equilibrium points is usually characterized in the sense of Lyapunov, a Russian mathematician and engineer who laid the foundation of the theory
Lyapunov stability theorems give sufficient conditions for stability, asymptotic stability, and so on.

As the nonlinearities and possible time-varying parameters exist in the nonlinear systems, linear stability criteria, e.g., Routh’s stability criterion or Nyquist stability criterion cannot be generalized and carried over into the systems for stability analysis. The Lyapunov stability theory introduced in this section is the most general approach to determine the stability of the linear or nonlinear dynamical systems.

### 2.2.1 Stability of Equilibrium Points

Consider a dynamical system which satisfies

\[
\dot{x} = f(x, t)
\]  

(2.1)

where \(x \in \mathbb{R}^n\) is the state variable vector, and \(n\) is the order of the system. \(f(x, t) \in \mathbb{R}^n\) is a set of functions of \(x(t)\).

Suppose \(x_e \in \mathbb{R}^n\) is an equilibrium point of system (2.1), that is, \(f(x_e, t) = 0\). Without loss of generality, we state all definitions and theorems for the case when the equilibrium point is at the origin of \(\mathbb{R}^n\), \(x_e = 0\).

**Definition 2.1**: The equilibrium point at the origin of (2.1) is

- stable, if, for any \(\epsilon > 0\), there exists a \(\delta > 0\) such that

\[
\|x(0)\| < \delta \Rightarrow \|x(t)\| < \epsilon, \quad \forall \ t \geq 0
\]  

(2.2)

- unstable, if the above condition is not satisfied

- asymptotically stable if it is stable and \(\delta\) can be chosen such that

\[
\|x(0)\| < \delta \Rightarrow \lim_{t \to \infty} x(t) = 0
\]  

(2.3)

It is important to note that the Definition 2.1 is about local stability, which only describes the behaviour of a system near an equilibrium point. Thus, it is not very useful in practice. In order to archive global stability, the Lyapunov direct method is represented in the following to handle this drawback.
2.2 Lyapunov Stability Theory

2.2.2 Lyapunov’s Direct Method

Lyapunov’s direct method (also called the second method of Lyapunov) allows us to determine the stability of a system without explicitly integrating the differential equation in (2.1). The concept of Lyapunov function originates from theoretical mechanics that in stable conservative systems “energy” is a positive definite scalar function which should decrease with time. Following this analogy, we can construct a generalised scalar “energy-like” function as a Lyapunov function to analyse the stability of any nonlinear system.

**Theorem 2.1**: Let $D \subset \mathbb{R}^n$ be a domain containing the system origin and that $V(x) : D \rightarrow \mathbb{R}$ is a continuously differentiable such that

$$V(0) = 0 \text{ and } V(x) > 0, \forall x \neq 0 \quad (2.4)$$

and

$$\dot{V}(x) \leq 0 \quad (2.5)$$

then $x = 0$ is stable. In addition, if

$$\dot{V}(x) < 0, \forall x \neq 0 \quad (2.6)$$

then $x = 0$ is asymptotically stable.

**Remark 2.1**: A continuously differentiable function $V(x)$ satisfying (2.4) and (2.5) is called the Lyapunov function. It is noted that the Lyapunov criterion above is not constructive as it does not give a prescription for determining the Lyapunov function. It is the task of the control designer to search for an appropriate Lyapunov function establishing stability of an equilibrium point. This often appears to be an arduous task in practice. Moreover, since the theorem only gives sufficient conditions, the converse of the theorem is necessarily not true.
2.3 Basics of Sliding Mode Control Systems

The two-step procedure for SMC design is described as follows:

(i) A sliding surface is predefined in a way that desired system dynamics are achieved during sliding mode.

(ii) A controller is then designed to drive the closed-loop dynamics to reach and be retained on the sliding surface.

2.3.1 System Model and Sliding Mode Surface Design

Without loss of generality, we consider the following linear time-invariant (LTI) system:

$$\dot{x}(t) = Ax(t) + Bu(t) \quad (2.7)$$

where $x \in \mathbb{R}^n$ is the system state vector, $u \in \mathbb{R}^m$ is the control input vector, $A \in \mathbb{R}^{n \times n}$ and $B \in \mathbb{R}^{n \times m}$ are constant system matrices.

**Assumption 2.1**: It is assumed that $n > m$, $B$ is of full rank $m$, the pair $(A,B)$ is completely controllable, that is, the controllability matrix $[B \ AB \ A^2B \ ... \ A^{n-1}B]$ has full rank $m$.

Define a sliding variable vector $s(t) \in \mathbb{R}^m$ passing through the state space origin

$$s(t) = Cx(t) \quad (2.8)$$

where $C \in \mathbb{R}^{m \times n}$ is the sliding mode parameter vector and $\|CB\| \neq 0$.

The system (2.7) is said to attain a sliding mode surface when the state variable vector reaches and remains on the intersection of the $m$ switching plane variables.

The method of equivalent control is a way to determine the system motion restricted to the sliding mode surface $s(x) = 0$. On the sliding mode surface, $s(x) = 0$ and $\dot{s}(x) = 0$, using expressions (2.7) and (2.8), we have

$$\dot{s}(t) = C\dot{x}(t) = 0 \quad (2.9)$$

$$C \left( Ax(t) + Bu_{eq}(t) \right) = 0 \quad (2.10)$$
where $u_{eq}(t)$ is viewed as equivalent control.

From expression (2.10), the equivalent control can be expressed as

$$u_{eq}(t) = -(CB)^{-1}CAx(t) \quad (2.11)$$

Substituting (2.11) into (2.7) yields the following differential equation

$$\dot{x}(t) = [I - B(CB)^{-1}C]Ax(t) \quad (2.12)$$

The system (2.12) is called the equivalent system which describes the dynamic motion of the system (2.7) on the sliding mode surface. The characteristics of the equivalent system can be summarised as below:

**Remark 2.2:** The dynamical behaviour of the equivalent system is independent of the control input. Thus, the determination of the matrix $C$ may be completed without prior knowledge of the form of control input. Generally, the sliding parameter $C$ is designed in a manner that the system response confined on the sliding mode surface (2.12) has a desired behaviour such as asymptotic stability and prescribed transient response. According to the linear control theory, in order to guarantee the solution of the differential equation in (2.12) to be asymptotically stable, the sliding mode parameter vector $C$ should be chosen, such that all the eigenvalues of the differential equation (2.12) have negative real parts. What’s more, though the sliding surface (2.8) is linear, it indeed could be any other forms with nonlinearity to ensure a finite time convergence of system dynamics in sliding mode. This will be looked at in the following part of this chapter.

**Remark 2.3 (system order reduction):** Since $s(t) = 0$ in the sliding mode, for the matrix $B$ with full rank $m$, there exist $m$ components of the state vector which are a function of the rest $(n - m)$ ones: $x_2 = s_0(x_1)$, $x_2, s_0 \in R^m$; $x_1 \in R^{n-m}$, and correspondingly the order of sliding mode equation may be reduced by $m$: $\dot{x}_2 = f_1(x_1, t, s_0(x_1))$, $f_1 \in R^{n-m}$ [2-3]. In order words, the equivalent system (2.12) is an $(n - m)^{th}$ order system, i.e., the system dynamic is simplified on the sliding mode surface.
2.3.2 Reaching Phase

SMC design includes reaching phase and sliding phase. The reaching phase is crucial in the sense that the system dynamics are guaranteed to reach the sliding surface and be retained on it thereafter. For a case in point, the idea of a sliding mode of a second order system can be depicted in Figure 2.1.

![Figure 2.1. The two phases of ideal sliding mode.](image)

The next important problem is how to design a controller to guarantee the reachability of the sliding variable to the sliding mode surface. Therefore, the task of the sliding mode controller is to drive the sliding variable \( s \) to converge to zero, and then the desired system dynamics prescribed in (2.12) will be obtained.

**Reaching Condition**

In fact, the condition for the switching plane variables to reach the sliding mode surface is a convergence problem. Therefore, the Lyapunov’s direct method has been widely used in SMC designs as a stability condition to ensure the convergence of the sliding mode variable onto the sliding surface during the reaching phase. All too often, the following Lyapunov function candidate is used in the sliding mode controller design:

\[
V(t) = \frac{1}{2} s^T(t)s(t)
\]  

(2.13)
In order to guarantee the asymptotic stability of the system (2.7) about the equilibrium point \( x(t) = 0 \), the following \textit{reaching condition} must be satisfied:

\[
\dot{V}(t) = s^T(t)\dot{s}(t) < 0 \quad \text{for} \quad s(t) \neq 0
\]  

(2.14)

\textbf{Remark 2.4:} The condition (2.14) indeed acts as a sufficient condition to ensure the existence of the sliding mode. It is worth noting that most of the sliding mode controllers are designed based on the reachability condition in (2.14) to ensure the sliding mode controller can drive the sliding variable \( s(t) \) to asymptotically converge to zero.

\subsection*{2.3.3 Reaching Laws}

SMC can be designed based on reaching laws to guarantee the existence of the sliding mode. Some possible types of reaching laws are given in [27]. In general, reaching law can be generalized in the following form

\[
\dot{s} = -\varepsilon \text{sign}(s) - f(s), \quad \varepsilon > 0
\]  

(2.15)

where \( f(0) = 0 \) and \( sf(s) > 0 \) when \( s \neq 0 \).

In practice, three special reaching laws commonly used can be derived from (2.15) as follows:

\textit{Constant rate reaching law}

\[
\dot{s} = -\varepsilon \text{sign}(s), \quad \varepsilon > 0
\]  

(2.16)

This law constrains the switching variable to reach the switching manifold at a constant rate \( \varepsilon \). The merit of this reaching law is its simplicity. However, as \( \varepsilon \) is too small, the reaching time will be too long. On the other hand, too large \( \varepsilon \) will cause severe chattering.

\textit{Exponential rate reaching law}

\[
\dot{s} = -\varepsilon \text{sign}(s) - ks, \quad \varepsilon > 0, \ k > 0
\]  

(2.17)
By adding the proportional rate term $-ks$, the states are forced to approach the switching manifold faster when $s$ is large.

**Power rate reaching law**

$$\dot{s} = -k|s|^\alpha \text{sign}(s), \ 1 > \alpha > 0, \ k > 0$$ \hspace{1cm} (2.18)

This reaching law increases the reaching speed when the states are far away from the switching manifold. However, it reduces the rate when the states approach the manifold.

It is evident that the above three reaching laws can satisfy the reaching condition (2.14), and thus ensure the existence of the sliding mode. It is worth noting that a reaching law method simultaneously takes care of ensuring the reaching condition, influencing the dynamic quality of the system during the reaching phase, and providing the means for controlling the chattering level. Thus, a reaching law method can be applied to both linear and nonlinear SMC systems with system perturbations and external disturbance, in order to improve the performance of the reaching phase and reduce the amplitude of chattering.

### 2.3.4 Equivalent Controller Design

In most of the VSC schemes, the control input usually consists of two components as follows

$$u(t) = u_{eq}(t) + u_s(t)$$ \hspace{1cm} (2.19)

where the linear component $u_{eq}(t)$ is defined as in (2.11) and the nonlinear signal incorporates the discontinuous component given below

$$u_s(t) = -\eta (CB)^{-1} \text{sign}(s(t))$$ \hspace{1cm} (2.20)

where $\eta>0$ is a constant control gain.

Substituting (2.11) and (2.20) into (2.14) leads to
\[
\dot{V}(t) = s^T(t)CAx(t) + s^T(t)CBu(t) \\
= -\eta |s(t)| < 0
\] (2.21)

From (2.21), we can conclude that the sliding mode variable is guaranteed to reach the sliding mode surface in finite time.

**Remark 2.5:** After the sliding variable vector \(s(t)\) is driven to zero, the closed-loop system dynamics are only determined by the desired dynamics in (2.12) and thus, the closed-loop system is insensitive to system uncertainties on the sliding mode surface. For this reason, SMC systems possess the property of robustness with respect to system uncertainties, that SMC becomes a powerful tool in the control of uncertain systems and significantly motivates the subsequent researchers in the area. However, it should be noted that the system remains affected by the perturbations during the reaching phase, that is to say, before the sliding surface has been reached.

### 2.3.5 Robustness Property

Robustness property is an important feature of SMC system. The system uncertainties and disturbances are always factored in an SMC controller design. With consideration of system uncertainties and disturbances, the LTI system in (2.7) can be generalized as

\[
\dot{x}(t) = (A + \Delta A)x(t) + (B + \Delta B)u(t) + d(t)
\] (2.22)

where \(\Delta A\) and \(\Delta B\) are the system uncertainties and \(d(t)\) is the external disturbances.

Equation (2.22) can be rewritten in the following form:

\[
\dot{x}(t) = Ax(t) + Bu(t) + f(t)
\] (2.23)

where \(f(t) = \Delta Ax(t) + \Delta Bu(t) + d(t)\) is the lumped uncertainty.

Following the concept of equivalent control in Section 2.3.4, one can design the controller form of (2.19), where \(u_s(t)\) is defined as in (2.20) and the equivalent control is given by the following equation
\[ u_{eq}(t) = -(CB)^{-1}C(Ax(t) + f(t)) \]  

(2.24)

Substituting (2.24) into (2.23) yields the equivalent system equation in sliding mode

\[ \dot{x}(t) = [I - B(CB)^{-1}C]Ax(t) + [I - B(CB)^{-1}C]f(t) \]  

(2.25)

If \( f(t) \) satisfies the matching condition, that is, \( f(t) = Bg(t) \), (2.25) becomes (2.12) which is completely insensitive to system uncertainties and external disturbances. In other words, the SMC system exhibits a strong robustness with respect to matched system uncertainty and disturbances. This invariance property makes SMC an efficient tool for controlling the uncertain systems and provides a strong motivation for the continuing research interest in the control area. However, the equivalent control action (2.24) is dependent on the unknown exogenous signal, therefore it cannot be realized in practice.

### 2.3.6 Chattering Phenomenon

**Zig-Zag Motion**

An ideal sliding mode shown in Figure 2.1 does not exist in practice since it would imply that the control commutes at an infinite frequency. As imperfections in switching devices, SMC suffers from *chattering*, the discontinuity in the feedback control produces a particular dynamic behaviour in the vicinity of the sliding mode surface as shown in Figure 2.2 [34-36].

In Figure 2.2, the system trajectory in the region \( s(t) > 0 \) heading toward the sliding surface \( s(t) = 0 \). It first hits the surface at point A. In ideal SMC the trajectory should start sliding on the surface from point A. However, due to a delay between the time the sign of \( s(t) \) changes and the time the control switches, the trajectory reverses its direction and heads again toward the surface. The repetition of this process creates the “zig-zag motion” which oscillating around the predefined sliding surface.
The chattering results in low control accuracy, high heat losses in electric power circuits and high wear of moving mechanical parts. It may excite unmodeled high-frequency dynamics, which degrades the performance of the system and may even lead to instability.

**Boundary Layer Technique**

Various techniques have been proposed to reduce or eliminate the chattering [37-40]. The boundary layer technique is one of the common approaches to eliminate the chattering.

It is seen that the discontinuous or switched component of SMC controller is designed as in (2.20). The boundary layer technique can be used to eliminate the chattering by replacing the sign function in (2.20) with a saturation function shown in Figure 2.3 as follows:

\[
    u_S(t) = -\eta(CB)^{-1}\text{sat}(s(t))
\]  

(2.26)

where \(\text{sat}(s)\) is the saturation function defined by

\[
    \text{sat}(s) = \begin{cases} 
    \frac{s}{\rho}, & \text{for } |s| \leq \rho \\
    \text{sign}(s), & \text{for } |s| > \rho 
    \end{cases}
\]  

(2.27)
and a positive constant $\rho > 0$ should be chosen in simulation or experiment to guarantee that the chattering can be eliminated and a reasonable control performance can be obtained.

![Saturation function](image)

Figure 2.3. Saturation function $\text{sat}(s)$.

This smoothing technique has been often employed in order to prevent chattering. However, although the chattering can be removed, the robustness of the sliding mode is meanwhile compromised. Such an approach might lead to a loss of asymptotic stability. Therefore, the boundary layer technique is not a perfect solution to eliminate the chattering.

Another solution to cope with chattering is based on a continuous approximation method (also called the pseudo-sliding mode method in the literature) in which the sign function in (2.20) is replaced by a continuous approximation as follows [36]:

$$
\dot{u}_s(t) = -\eta(CB)^{-1}\left(\frac{s(t)}{|s(t)| + \rho}\right)
$$

(2.28)

However, this approach gives rise to a high-gain control when the states are in the close neighbourhood of the sliding surface.
2.4 Sliding Mode Control Algorithms

We now concentrate on the development of sliding mode control algorithms which enforce the sliding variables to reach and be retained on the sliding surface and thus guarantee the existence of the sliding mode. This section is dedicated to briefly review the advancement of SMC techniques over the past few decades.

2.4.1 Second Order Sliding Mode Control

In early 80’s, the control community had experienced that the main drawback of SMC is the “chattering” effect. In order to combat this issue in the sliding mode, the second order sliding mode (SOSM) concept was introduced by A. Levant in [41-42].

The first and simplest SOSM algorithm is the so-called “twisting algorithm” (TA). Consider a dynamic system of the form:

\[ \dot{x} = a(t, x) + b(t, x)u \quad (2.29) \]

where \( x \in R^n \) is measureable state vector, \( u \in R \) is control input.

Define a proper sliding manifold in the state space

\[ \sigma = \sigma(t, x) \quad (2.30) \]

The relative degree of the system is assumed to be one, which implies that the first derivative of \( \sigma \) can be expressed as:

\[ \dot{\sigma} = h(t, x) + g(t, x)u, \quad h = \dot{\sigma}|_{u=0}, g = \frac{\partial}{\partial u} \dot{\sigma} \neq 0 \quad (2.31) \]

where \( g, h \) are some unknown smooth functions.

Suppose that the input-output termed conditions

\[ 0 < K_m \leq g \leq K_M, \quad |h| \leq C \quad (2.32) \]

hold globally for some \( K_m, K_M, C > 0 \).
The twisting algorithm

\[ u = -k\text{sign}(\sigma) \]  \hspace{1cm} (2.33)

with the condition

\[ 0 < K_m \leq g \leq K_M, \quad |h| \leq C \]  \hspace{1cm} (2.34)

can be used to solve the problem of establishing and keeping \( \sigma = 0 \). It is based on the knowledge of the sign of both \( \sigma \) and \( \dot{\sigma} \).

Consider \( v = \dot{u} \) as a new control, in order to overcome the chattering. Differentiating (2.31) achieve

\[ \dot{\sigma} = h_1 (t, x, u) + g(t, x)\dot{u} \]  \hspace{1cm} (2.35)

where

\[ h_1 = \dot{h} + \frac{\partial h}{\partial x} \dot{x} + \left( \dot{g} + \frac{\partial g}{\partial x} \dot{x} \right) u. \]

Moreover, define the function

\[ \Sigma = \dot{\sigma} + \beta|\sigma|^{\frac{1}{2}}\text{sign}(\sigma) \]  \hspace{1cm} (2.36)

Let

\[ v = \dot{u} = \begin{cases} -u & |u| > k \\ -\alpha\text{sign}\Sigma & |u| \leq k \end{cases} \]  \hspace{1cm} (2.37)

Then, with the sufficient large \( \alpha \), controller (2.37) provides for the establishment of the finite time stable SOSM \( \sigma = 0 \).

**Remark 2.6**: The main idea behind SOSM is to act on the second-order derivative of the sliding variable \( \sigma \) rather than the first derivative as in the standard sliding mode. In the SOSM, the time derivative of the controller \( v = \dot{u} \) is used as control input instead of the
actual input $u$. In other words, the new control is designed to be a discontinuous signal, but its integral is continuous, so that the chattering is completely eliminated.

### 2.4.2 Higher Order Sliding Mode Control

In 2001, the first arbitrary order SM controller was introduced in [43]. Such controllers allowed solving the finite-time enforcement of a $r$-th order sliding mode and uncertainties compensation.

Given the relative degree $r$ of the output, higher order sliding mode (HOSM) controllers are constructed using a recursion. The following is the recursion for the first reported kind of HOSM controller: the so-called nested ones [44-51]. Let $p$ be the least common multiple of $1, 2, ..., r$. Also let

$$
\varphi_{0,r} = \sigma, \quad N_{1,r} = |\sigma|^{\frac{r-1}{r}}
$$

$$
\varphi_{i,r} = \sigma^{(i)} + \beta_i N_{i,r} \text{sign} (\varphi_{i-1,r}), \quad N_{i,r} = \left( |\sigma|^{\frac{p}{r}} + |\sigma|^{\frac{p-1}{r-1}} + \cdots + |\sigma|^{\frac{r-i}{r-i+1}} \right)^{\frac{r-i}{p}}
$$

(2.38)

where $i = 1, ..., r - 1$, and the $r$-th order sliding mode controller

$$
u = -\alpha \text{sign} \left( \varphi_{i,r} (\sigma, \dot{\sigma}, ..., \sigma^{(r-1)}) \right)
$$

(2.39)

be applied to system (2.29). Then this algorithm provides for the finite-time stabilization of $\sigma, \dot{\sigma} = 0$ and therefore, of its successive derivative up to $\sigma^{(r-1)}$. Thus it guarantees the existence of an $r$-th order sliding mode

$$
\sigma = \dot{\sigma} = \cdots = \sigma^{(r-1)} = 0
$$

(2.40)

**Remark 2.7**: The HOSM approach allows us to solve the problem of finite-time output stabilization of a dynamic system. However, the following points of HOSM controllers remain to be unsolved: (i) the homogeneity features of the system, which are essential in the convergence proof, were destroyed if an adaptation of the gain of the controller was attempted. Thus, it is not possible to reduce the gain of the controller once the system approaches the origin. (ii) The time constant for finite-time convergence is tending to...
infinity together with the growing of the norm of initial conditions. (iii) Only asymptotic accuracy ensured by HOSM controllers and differentiator is proved. The constants for estimations of accuracy need to be computed [51].

### 2.4.3 Terminal Sliding Mode Control

As described in Section 2.3, SMC design with the linear sliding mode surface has been adopted for describing the desired performance of the closed-loop systems in detail, that is, the system state variables reach the system origin asymptotically in the linear sliding mode surface. When in the sliding mode, the closed-loop response becomes totally insensitive to both internal parameter uncertainties and external disturbances. Despite that the parameters of the linear sliding mode can be adjusted in order to obtain the arbitrarily fast convergence rate, the system states on the sliding mode surface cannot converge to zero in a finite time.

Recently, a new technique called terminal sliding mode (TSM) control has been intensively studied for achieving finite time convergence of the system dynamics in the terminal sliding mode [52-65]. In comparison with the linear sliding mode based SMC design, TSM possesses the superior characteristics of fast and finite time convergence, which particularly improves the high precision control performance by accelerating the convergence rate near an equilibrium point.

Consider the following second-order uncertain nonlinear system:

\[
\begin{align*}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= f(x) + g(x)u
\end{align*}
\]  

(2.41)

where \( x = [x_1, x_2]^T \) is the system state vector, \( f(x) \) and \( g(x) \neq 0 \) are smooth nonlinear functions of \( x \), and \( u \) is the scalar control input.

In order to obtain the terminal convergence of the system state variables, the first-order terminal sliding variable is defined as follows:

\[
s = x_2 + \beta x_1^p
\]  

(2.42)
where $\beta$ is a designed positive constant, $p$ and $q$ are two positive odd integers satisfying the following condition:

$$p > q$$

(2.43)

The sufficient condition for the existence of TSM is

$$\frac{1}{2} \frac{d}{dt} s^2 < -\eta |s|$$

(2.44)

where $\eta > 0$ is a constant. According to [52], for the case of $s(0) \neq 0$, the time $t_r$ for the system states to reach the sliding mode $s = 0$ is finite and satisfies

$$t_r \leq \frac{|s(0)|}{\eta}$$

(2.45)

In order to ensure the terminal sliding variable $s$ to reach the terminal sliding mode surface $s = 0$, we adopt the following sliding mode controller:

$$u = -g^{-1}(x) \left( f(x) + \beta \frac{q}{p} x_1^{p-1} x_2 + \eta \text{sign}(s) \right)$$

(2.46)

In the terminal sliding mode, the system dynamics are determined by the following nonlinear differential equation:

$$\dot{x}_1 = -\beta x_1^p$$

(2.47)

It has been shown in [52-55] that $x_1 = 0$ is the terminal attractor of the system (2.47). The finite time $t_s$ that is taken to travel from $x_1(t_r) \neq 0$ to $x_1(t_r + t_r) = 0$ is then given by
Expression (2.48) means that, in the terminal sliding mode, both the system states $x_1$ and $x_2$ converge to zero in finite time.

**Remark 2.8:** It can be seen from the analysis above that TSM offers a superior property which can ensure the zero-error convergence of the closed-loop dynamics in finite-time. This idea has been intensively studied for years in an attempt to enhance the convergence rate as in fast terminal sliding mode (FTSM) [61-62] and overcome singularity problems for TSM systems as in non-singular terminal sliding mode (NTSM) [53]. Although the TSM technique has been widely applied to the control of mechanical systems, electrical systems, aircraft systems and other complex systems [52-65], the development of this technique is still at its initial stage, and many theoretical researches need to be done in years to come.

### 2.4.4 Integral Sliding Mode Control

Integral sliding modes [13, 66] were suggested as a tool to reach the following goals:

- Compensation of matched perturbations starting from the initial moment, i.e., ensuring the sliding mode occurring from the initial moment. In other words, the reaching phase is eliminated.
- Preservation of the dimension of the initial system, i.e., saving the system dynamics previously designed for the ideal case without perturbation.

Suppose that a control law $u = u_0(x,t)$ achieving the control objective is already available for an ideal, nominal system

$$\dot{x} = f(x,t) + B(x)u, \quad x \in R^n, u \in R^m \quad (2.49)$$

Now suppose that one has a perturbed system

$$\dot{x} = f(x,t) + B(x)(u + \delta) + \phi \quad (2.50)$$
where $\delta$ is a matched disturbance and $\phi$ is an unmatched disturbance. Then, a sliding mode control law $u_1(x, t)$ can be easily included such that the closed-loop

$$\dot{x} = f(x, t) + B(x)(u_0 + u_1 + \delta) + \phi$$

(2.51)

is insensitive to $\delta$.

One begin by constructing the sliding variable

$$s(x, t) = g(x) - z(t), \ s(x, t) \in \mathbb{R}^m$$

(2.52)

where

$$z(t) = g(x_0) + \int_{t_0}^{t} G(x)[f(x, \tau) + B(x)u_0(x, \tau)] \, d\tau, \ G(x) = \frac{\partial g}{\partial x}(x)$$

(2.53)

$g(x)$ is a function such that $G(x)B(x)$ is invertible.

Notice that at $t = t_0$, we have $s = 0$, thus the system starts at the sliding surface (there is no reaching phase). Let us now compute the time derivative of $s$:

$$\dot{s} = G(x)[f(x, t) + B(x)(u_0 + u_1 + \delta) + \phi - f(x, t) - B(x)u_0]$$

$$= G(x)B(x)(u_1 + \delta) + G(x)\phi$$

(2.54)

It can be seen that if $\delta$ and $\phi$ are bounded by known functions, then it is possible to construct a unit control $u_1$ ensuring $\dot{s} = 0$. The equivalent control is

$$u_{eq} = -\delta - (G(x)B(x))^{-1}G(x)\phi$$

(2.55)

So the trajectories of the system at the sliding surface are given by
\[
\dot{x} = f(x, t) + B(x)u_0 + \left[ I - B(x)\left(G(x)B(x)\right)^{-1}G(x) \right] \phi \tag{2.56}
\]

which shows the insensitivity with respect to \(\delta\).

**Remark 2.9:** The effect of \(\phi\) is not eliminated and only can be mitigated. In other words, the projection matrix \(G(x)\) should not amplify the remnant perturbation in (2.56), but minimize it. Optimal control such as H-infinity techniques can be used to further attenuate \(\phi\).

### 2.4.5 Sliding Mode Control with Perturbation Estimation

The classical form of SMC has brought several setbacks (i) the designer should have prior knowledge of the bounds of the perturbations, which may be impossible to access in practice. (ii) The resultant robust control obtained using the bounds of perturbations yields over-conservative feedback gains.

Sliding mode control with perturbation estimation (SMCPE) was introduced by [67-69] in an attempt to alleviate these drawbacks using the “Time-delayed control” concept of “Youcef-Toumi 1990”. The strategy used is an online estimation of the contributions of perturbations based on the observations of the dynamics.

Take the dynamic system of the following form as an example:

\[
x(t) = Ax(t) + Bu(t) + f(t) \tag{2.57}
\]

where \(x(t) \in \mathbb{R}^n\) is the system state vector, \(u(t) \in \mathbb{R}\) is the control input, \(f(t)\) is lumped perturbation.

The sliding hyperplane is selected as Hurwitz polynomials of system states

\[
s(t) = Cx(t) \tag{2.58}
\]

From (2.57), the actual perturbation of the system at any given time is

\[
f(t) = \dot{x}(t) - Ax(t) - Bu(t) \tag{2.59}
\]
If all the components in the dynamics show slower variations with respect to the loop closure (sampling speed), the right hand side of (2.59) can be re-written with \( u(t - \delta) \) instead of \( u(t) \). This arrangement leads to up-to-date approximate knowledge of influences of all perturbations \( f(t)_{est} \). (2.59) can be re-expressed as

\[
 f(t)_{est} = \dot{x}(t) - Ax(t) - Bu(t - \delta) \tag{2.60}
\]

where

\[
 \dot{x}(t) = \frac{x(t) - x(t - \delta)}{\delta} \tag{2.61}
\]

It is shown that the controller form of

\[
 u(t) = (CB)^{-1}(-Ps - K\text{sign}(s)) - Ca(t) - Cf(t) \tag{2.62}
\]

guarantees the reachability condition (2.14), and yields desirable dynamics of \( s \)

\[
 \dot{s} = -Ps - K\text{sign}(s) + f - f_{est} \tag{2.63}
\]

If \( |f - f_{est}| \) remains within a boundary of \( \eta|f_{est}|, \eta > 0 \), the boundary attractivity condition of (2.15) is assured by selecting

\[
 K > \eta|f_{est}|_{max} \tag{2.64}
\]

### 2.5 Discrete-Time Sliding Mode Control Systems

According to the aforementioned discussion, the characteristic feature of a continuous-time SMC system is that sliding mode occurs on a prescribed manifold, where switching control is employed to maintain the state on the surface. When a sliding mode is realised, the system exhibits some superior robustness properties with respect to external matched uncertainties. However, the realization of the ideal sliding mode requires switching with an infinite frequency.
Control algorithms are now commonly implemented in digital electronics due to increasingly affordable microprocessor hardware though the essential framework of the feedback design still remains to be in the continuous-time (CT) domain. Discrete-time sliding mode control has been extensively studied to address some basic questions associated with the sliding mode control of discrete-time (DT) systems with relatively low switching frequencies. Having said that, the quest of in-depth understanding of the complex dynamical behaviours due to discretization of continuous-time SMC systems has to be further explored.

The discretization behaviours of SMC systems as well as some intrinsic properties of discretised SMC systems are investigated in this section.

2.5.1 Overview
Digitized control is implemented by “freezing” the control force during the sampling period. This very feature may deteriorate the elegant invariance property enjoyed by most, if not all continuous-time SMC systems. For DT systems, it is often assumed that the sampling frequency is sufficiently high to assume that the closed-loop system is continuous-time [21]. However, the actual closed-loop cannot be driven into true sliding mode but quasi sliding mode which was defined in [70]. Obviously, the most apparent difference between a DT system and its CT counterpart is the limited switching speed of the discontinuous control part. In DT SMC, because of the zigzagging behaviours, exact sliding on the intersection of predefined switching manifolds to some extent is impossible. To compensate for this disadvantage of DSMC, a new concept, sliding sector, was brought in and has been studied for quite a while [71-74].

2.5.2 Discretization of Sliding Mode Control Systems
Discretization is a major approach for industry applications of control systems. In many cases, control design is based on continuous-time system models due to their simplicity over their discrete-time counterparts, and the practical implementation is commonly done by using digital microprocessors or computers. There is a gap between the ideal dynamical performance anticipated based on the design from the theory for the continuous-time system models and the actual dynamical performance when the control system is discretised. The time delay in delivering control signals due to discretization is
the key factor affecting the control performance. This is particularly so, when the control is discontinuous by its nature, such as the SMC. The "disruptive" switching may possibly cause incorrect actions due to the delay of delivering timely control signals. These behaviours may likely cause severe damage to industrial control devices such as actuators. In addition, the deteriorated invariance property may worsen the reliability of SMC systems, hence making controlled industrial processes vulnerable to unexpected environmental changes. The detail of this phenomenon has been intensively studied in [75-76].

There are two main methods for discretization, Euler discretization and ZOH discretization. In industries, simulations of control systems are usually done via Euler discretization while their implementation in practice is commonly done via ZOH discretization.

**Euler Discretization**

In [75-76], several important issues with regard to the discretization of SMC were discussed. A mathematical formulation of discretization using Euler’s approximation was undertaken. It was shown that the solution trajectory must be attractive, so that the Euler’s and the exact solutions can be consistent, as the sampling period decreases. In comparison with other control methods, the sampled SMC suffers more from the sampling process, as it would lose the high gain property near the vicinity of the switching surface. To compensate for this, disturbance prediction is indispensable, which is feasible under the hypothesis that the disturbance is slow time-varying. In [77], the discretization behaviours of the most popular SMC systems using the Euler discretization were studied. It was shown that if the discretized SMC system is asymptotically stable then every trajectory converges to a period–2 cycle. Some symmetric features of the trajectory in steady state were explored and boundary conditions for the steady states were derived.

**ZOH Discretization**

Zero-order-holder is the most commonly discretization method used in industrial automatic control systems. Through ZOH, \( u(t) = u(k) \) over the time interval \( [kT, (k + 1)T) \), where \( T \) is a sampling period.
Under ZOH, the continuous-time system (2.7) is converted into the following discrete form

\[ x(k + 1) = \Phi x(k) + \Gamma u(k) \]  

(2.65)

where \( \Phi = e^{AT} \) and \( \Gamma = \int_0^T e^{AT} d\tau B \).

The most popular discrete time sliding mode control strategy is to steer the states towards and maintain them on the surface \( s \) at each sampling instant such that

\[ s(k) = Cx(k) = 0 \]  

(2.66)

During the sampling interval \( kT \leq t < (k + 1)T \), the state may deviate from \( s(k) = 0 \).

**2.5.3 Stability and Controller Design**

The stability of SMC systems has been studied for many years. Different from the continuous-time SMC systems, the discrete-time SMC system has its own properties. In the continuous-time SMC system, the sliding mode existence condition is \( \dot{s} s < 0 \), however, in the discrete-time SMC system, that is no longer the case.

In [78], the discrete SMC problem was first considered and the equivalent form of the continuous-time sliding mode existence condition to create a discrete-time sliding mode existence condition

\[ (s(k + 1) - s(k))x(k) < 0 \]  

(2.67)

In [70], the concept of the quasi-sliding mode (QSM) was suggested by Milosavljevic. The QSM is the motion that satisfies the following conditions:

(i) Starting from any initial state, the trajectory will move monotonically towards the switching plane and cross it in finite time.

(ii) Once the trajectory of the system first crosses the switching plane, it will cross it again at every successive sampling time, resulting in a zigzag motion about the switching plane.
(iv) The size of each successive zigzagging step is not increasing and hence the trajectory stays within a specified band.

The condition (2.67) was found out not to be sufficient for the existence of a QSM. In [79], a new sufficient condition was given:

\[ |s(k + 1)| < |s(k)| \] (2.68)

which was decomposed into the following inequalities:

\[ (s(k + 1) - s(k))\text{sign}(x(k)) < 0 \]
\[ (s(k + 1) + s(k))\text{sign}(x(k)) > 0 \] (2.69)

A more expedient approach was derived by Gao [80] which is called the reaching law approach. This method can be described as

\[ s(k + 1) - s(k) = -qTs(k) - \epsilon T\text{sign}(s(k)), \epsilon > 0, q > 0, 1 - qT > 0 \] (2.70)

For the control law design, Drakunov and Utkin [81] proposed a definition of discrete time equivalent control that directs the states onto the sliding surface in one sampling period. To remain on the surface, the associated control appears to be non-switching. Subsequently, the theoretical basis was furnished with a formal definition of sliding mode for discrete-time in the context of semigroups [82]. Su et al. [83] soon developed a control strategy which maintains the states on the switching surface at each sampling instant. Between samples, the states are allowed to deviate from the surface instead of being constantly and exactly on the switching surface, even the equivalent states travels within a boundary layer of that surface.

2.6 Conclusion

The theory of variable structure control has been briefly surveyed in this chapter. Since SMC exhibits many superiorities, it can be preferably embodied to control linear or nonlinear systems with uncertain dynamics. Although the robustness can be achieved without the exact knowledge of the control system, the system performance and control
quality depend very much on the choice of the sliding parameters and the estimate of bounding functions of the unknown components. In practice, excessive control input and severe control chattering which may excite unmodelled high frequency dynamics are highly undesired. Therefore, how to capitalize on the SMC’s merits to develop more intelligent control techniques for the purpose of improving the performance of the SMC systems or SMC based ones as well as relaxing the constraint on the bound information of the uncertain dynamics has become a demanding topic which will be thoroughly studied in the subsequent chapters.
A novel sliding mode based learning control technique for a class of uncertain dynamic systems is introduced in this chapter. It will be seen that the robust stability of the closed-loop system is guaranteed by employing an intelligent sliding mode controller. The working principle of the designed controller is as follows: The stability status of the closed-loop system is always checked based on the most recent information of the first-order derivative of the Lyapunov function and (i) if the closed-loop system is stable, the correction term in the controller will continuously adjust the control signal to drive the closed-loop dynamics to reach the sliding surface in finite time; (ii) if, however, the closed-loop is unstable, the correction term is capable of altering the control signal to reduce the value of the derivative of the Lyapunov function from positive to negative and then enforces the closed-loop trajectory to reach the sliding surface and thereafter ensures the desired closed-loop dynamics. The core merits of this control scheme are that no chattering occurs in the sliding mode control system because of the recursive learning algorithm; the system uncertainties and external disturbances are all embedded in the so-called Lipschitz-like condition and thus, no prior information on the upper and/or lower bounds of the uncertainties is required for the controller design.

3.1 Introduction

Over the past few decades, sliding mode control has been intensively investigated and successfully applied for controlling complex systems with uncertain dynamics. Generally speaking, if the knowledge about the bounds of the system uncertainties and external disturbances is known, a high speed switching sliding mode controller can be designed to drive the closed-loop dynamics to reach the sliding surface and be retained on it thereafter to ensure the desired closed-loop dynamics with zero-error convergence. However, the complete knowledge of the unmodelled dynamics is not always achievable beforehand and thus insufficient to design the conventional
sliding mode controllers. Even if the extremely large bounds of uncertainties may be selected, it may cause high gain in the control signal and subsequently go beyond the actuators’ capability.

Switching or chattering of sliding mode control signals crossing the sliding mode surfaces is yet another essential feature in all current sliding mode control systems. It has been well judged that this setback has largely constrained the applications of the sliding mode control methodology in practice since the high-speed chattering control signals may excite some undesired high frequency mode in the closed-loop system. Although the boundary layer technique has been widely used to eliminate the chattering, the property of the zero-error convergence is compromised as the replacing sigmoid function can only guarantee the stability of the closed-loop system within the prescribed boundary layer.

In fact, for years, the researchers in the area of sliding mode control technology have been constantly exploring the possibility of developing a novel sliding mode control technique which ensures both the zero-error convergence and the chattering-free characteristic for the sliding mode control systems. Inspired by this thought, a new sliding mode controller with an intelligent recursive-learning mechanism is developed and first reported in [84-85]. Soon after, the initiative has been further developed and disseminated quickly in a number of applications addressing the real problems in practice [86-88]. The distinguishing characteristic of the novel sliding mode control algorithm is that the Lipschitz-like condition, describing the key dynamic property of the closed-loop system with or without uncertain dynamics, is introduced, which helps to relax the constraint on the bound information often required in the conventional sliding mode controller designs.

The rest of this chapter is organised as follows: In Section 3.2, an SISO dynamic system with uncertainties is modelled and the novel sliding mode learning controller is proposed. In Section 3.3, the Lipschitz-like condition, an important property of the continuity of uncertain dynamic systems, is investigated in detail. The convergence analysis of the closed-loop system equipped with the new sliding mode control strategy is studied in Section 3.4, meanwhile, the chattering-free attribute and the robustness are also addressed in this section. In Section 3.5, an illustrative simulation example is presented to show the effectiveness of the proposed control scheme. Lastly, Section 3.6 draws a conclusion and the significances of this control scheme.
3.2 Problem Formulation

Consider a \( n \)-order SISO nonlinear system described as:

\[
x^{(n)} = f(x, t) + bu(t) + d(t) \tag{3.1}
\]

\[
x = [x \ x \ ... \ x^{(n-1)}]^T \tag{3.2}
\]

where \( b > 0, x \in \mathbb{R}^n, u \in \mathbb{R} \) and \( d(t) \) denotes external disturbance and uncertainty.

A sliding variable is then defined as:

\[
s(t) = Cx(t) \tag{3.3}
\]

where \( C \in \mathbb{R}^{1 \times n} \) is the sliding mode parameter matrix, selected such that the dynamics of \( s(t) = 0 \) are Hurwitz.

Thus, the time derivative of (3.3) is expressed as:

\[
\dot{s}(t) = C\dot{x}(t) = c_1\dot{x}(t) + c_2\dot{x}(t) + \cdots + c_{n-1}x^{(n-1)}(t) + x^{(n)}(t)
\]

\[
= \sum_{i=1}^{n-1} c_ix^{(i)}(t) + f(x, t) + bu(t) + d(t)
\]

\[
= f(t) + bu(t) \tag{3.4}
\]

where

\[
f(t) = \sum_{i=1}^{n-1} c_ix^{(i)}(t) + f(x, t) + d(t) \tag{3.5}
\]

In this paper, the sliding mode learning controller is proposed as follows:

\[
u(t) = u(t - \tau) - \Delta u(t) \tag{3.6}
\]

with the correction term \( \Delta u(t) \):
\[ \Delta u(t) = \begin{cases} \frac{1}{b s(t)} (\alpha \hat{V}(t - \tau) + \beta |\hat{V}(t - \tau)|) & \text{for } s(t) \neq 0 \\ 0 & \text{for } s(t) = 0 \end{cases} \quad (3.7) \]

where \( \alpha, \beta > 0 \) are controller parameters, \( \hat{V}(t - \tau) \) is the approximation of \( \dot{V}(t - \tau) \), \( \tau \) is the time delay, and \( \dot{V}(t) \) is the first-order derivative of the Lyapunov function candidate \( V(t) = 0.5s(t)^2 \), chosen for the closed-loop system, and computed as follows:

\[ \dot{V}(t) = s(t)\dot{s}(t) = s(t)(f(t) + bu(t)) \quad (3.8) \]

Before we proceed further, let us define

\[ \hat{V}(t, t - \tau) \triangleq s(t)(f(t) + bu(t - \tau)) \quad (3.9) \]

### 3.3 Lipschitz-Like Condition

In (3.7), \( \hat{V}(t - \tau) \), the estimate of \( \dot{V}(t - \tau) \), is computed as:

\[ \hat{V}(t - \tau) = \frac{V(t) - V(t - \tau)}{\tau} \quad (3.10) \]

The minimal value of the time delay \( \tau \) is equal to the sampling period, chosen to be sufficiently small in the sense that there exist \( M \gg 1 \) and \( 0 < \mu \ll 1 \), such that the following inequalities are held:

\[ |\hat{V}(t, t - \tau) - \dot{V}(t - \tau)| < \frac{1}{M} |\hat{V}(t - \tau)| \quad (3.11) \]

where \( \hat{V}(t, t - \tau) \) is defined as in (3.9), and

\[ |\hat{V}(t - \tau) - \hat{V}(t - \tau)| < \mu |\hat{V}(t - \tau)| \quad (3.12) \]
for $\dot{V}(t, t-\tau) \neq 0, \dot{V}(t-\tau) \neq 0, \hat{V}(t-\tau) \neq 0$.

**Remark 3.1:** The inequality (3.11) is called the **Lipschitz-like condition** [84]. It describes that, for a large class of systems with the continuity of their $\dot{V}(t)$, the difference between the current value of the gradient of the Lyapunov function and its most recent value is infinitesimal as the time interval $\tau$ is sufficiently small (see the proof given in Appendix A.1). The significances of the Lipschitz-like condition for control system designs are in two folds: First, the uncertain system dynamics are all embedded in the left-hand side of (3.11), and thus, for controller design with the aid of the Lipschitz-like condition, knowledge of the upper and lower bounds of the system uncertainties is no longer required. Second, the Lipschitz-like condition provides a strategy for us to design a learning controller that recursively updates the control signal based on the most recent stability history of the closed-loop system. This point can be seen from the following convergence proof of the closed-loop system. It is expected that, in the next generation of control system design, the Lipschitz-like condition will play a very essential role to relax many constraints on uncertain system dynamics in existing robust control technologies.

**Remark 3.2:** The inequality (3.12) implies that the difference between the value of the gradient of the Lyapunov function and its approximation is diminutive as the time interval $\tau$ is sufficiently small (see the proof given in Appendix A.2). Moreover, we can reasonably assume that

- $\dot{V}(t-\tau)$ is nonzero when the closed-loop dynamics are not constrained on the sliding surface $s(t) = 0$.
- $\dot{V}(t-\tau)$ and $\dot{V}(t-\tau)$ have the same sign for $\dot{V}(t-\tau) \neq 0$ (refer to the proof given in Appendix A.3), that is,

$$\text{sign}\left(\dot{V}(t-\tau)\right) = \text{sign}\left(\dot{V}(t-\tau)\right)$$  \hspace{1cm} (3.13)

**Remark 3.3:** It is seen from (3.6) and (3.7) that the control signal $u(t)$ is continuous for $s(t) \neq 0$. However, it can be verified that $\Delta u(t)$ is also continuous at all the points of
s(t) = 0. Thus, the proposed SMLC in (3.6) is continuous at every time instant in the state-space (refer to the proof given in Appendix A.4).

In the next section, the asymptotic convergence and the stability analysis of the proposed SMLC are discussed in detail.

### 3.4 Convergence Analysis

**Theorem 3.1:** Consider the uncertain dynamic system in (3.1), if the control input in (3.6) with the correction term in (3.7) is used, the system dynamics \( x(t) \) will asymptotically converge to zero.

**Proof:** Differentiating the Lyapunov function \( V(t) = 0.5s(t)^2 \) with respect to the time \( t \) and using (3.6) and (3.7), we have

\[
\dot{V}(t) = s(t)\dot{s}(t)
\]

\[
= s(t)(f(t) + bu(t))
\]

\[
= s(t)[f(t) + bu(t - \tau)] - s(t)b\Delta u(t)
\]

\[
= \dot{V}(t, t - \tau) - \alpha \dot{V}(t - \tau) - \beta \dot{V}(t - \tau)
\]

where \( \dot{V}(t, t - \tau) \) is defined as in (3.9).

Adding the term \( \dot{V}(t - \tau) - \dot{V}(t - \tau) \) to (3.14), we can express \( \dot{V}(t) \) as:

\[
\dot{V}(t) = \dot{V}(t, t - \tau) - \dot{V}(t - \tau) + \dot{V}(t - \tau) - \alpha \dot{V}(t - \tau) - \beta \dot{V}(t - \tau)
\]

\[
\leq |\dot{V}(t, t - \tau) - \dot{V}(t - \tau)| + \dot{V}(t - \tau) - \alpha \dot{V}(t - \tau) - \beta \dot{V}(t - \tau)
\]

(3.15)

Substituting (3.11) into (3.15) yields

\[
\dot{V}(t) < \frac{1}{M}|\dot{V}(t - \tau)| + \dot{V}(t - \tau) - \alpha \dot{V}(t - \tau) - \beta \dot{V}(t - \tau)
\]

(3.16)

- **For the case that** \( \dot{V}(t - \tau) > 0 \):

Re-writing (3.16), we have
3.4 Convergence Analysis

\[ \dot{V}(t) < \dot{V}(t - \tau) + \left( \frac{1}{M} - \alpha \right) |\dot{V}(t - \tau)| - \beta |\dot{V}(t - \tau)| \]  
(3.17)

If the control parameter \( \alpha \) is chosen such that

\[ \frac{1}{M} < \alpha < 1 - \frac{1}{M} - \mu \]  
(3.18)

(3.17) satisfies

\[ \dot{V}(t) < \dot{V}(t - \tau) \]  
(3.19)

The inequality (3.19) indicates that the learning controller (3.6) always makes the value of \( \dot{V}(t) \) decrease when \( \dot{V}(t - \tau) > 0 \).

Suppose that, at time \( t = t_0, \dot{V}(t_0) = 0 \). Then at the time \( t = t_0 + \tau \), (3.14) can be expressed as:

\[ \dot{V}(t_0 + \tau) = \dot{V}(t_0 + \tau, t_0) - \alpha \dot{V}(t_0) - \beta |\dot{V}(t_0)| \]  
(3.20)

If the control parameter \( \beta \) is chosen to satisfy the following condition:

\[ |\dot{V}(t_0 + \tau, t_0) - \alpha \dot{V}(t_0)| < \beta |\dot{V}(t_0)| \]  
(3.21)

(3.20) becomes

\[ \dot{V}(t_0 + \tau) < 0 \]  
(3.22)

The discussions from (3.17) to (3.22) have shown that the proposed learning controller in (3.6) is capable of reducing \( \dot{V}(t) \) from positive to negative, that is, the closed-loop trajectory can be driven from the unstable region (\( \dot{V}(t) > 0 \)) to the stable region (\( \dot{V}(t) < 0 \)).
• **For the case that** $\dot{V}(t - \tau) < 0$:

One can rewrite (3.16) as:

$$\dot{V}(t) < \frac{1}{M} \left| \hat{V}(t - \tau) \right| + \dot{V}(t - \tau) - \alpha \hat{V}(t - \tau) - \beta \left| \hat{V}(t - \tau) \right|$$  \hspace{2cm} (3.23)

Using (3.12) in (3.23), we can obtain

$$\dot{V}(t) < \frac{1}{M} \left| \hat{V}(t - \tau) \right| + \dot{V}(t - \tau) + \mu \left| \hat{V}(t - \tau) \right| - \alpha \hat{V}(t - \tau) - \beta \left| \hat{V}(t - \tau) \right|$$

$$< \left( \frac{1}{M} - 1 + \mu + \alpha \right) \left| \hat{V}(t - \tau) \right| - \beta \left| \hat{V}(t - \tau) \right|$$  \hspace{2cm} (3.24)

With the chosen parameter $\alpha$ in (3.18), (3.24) satisfies

$$\dot{V}(t) < 0$$  \hspace{2cm} (3.25)

Therefore, the closed-loop SbW system with the SMLC in (3.6) is asymptotically stable and the SMLC control law ensures that both the sliding variable $s(t)$ and the closed-loop dynamics $x(t)$ asymptotically converge to zero.

### 3.5 Simulation

Consider one-link inverted pendulum with its kinetic equation as follows [86]:

$$\begin{cases} 
\dot{x}_1 = x_2 \\
\dot{x}_2 = \frac{g\sin(x_1) - am\omega^2 \sin(x_1) \cos(x_1)}{4l/3 - am\cos^2(x_1)} - \frac{acos(x_1)}{4l/3 - am\cos^2(x_1)} u + d 
\end{cases}$$

(3.26)

where $x_1$ is the swing angle of the pendulum from the vertical, $x_2$ is the angular velocity, $g = 9.81 \text{m/s}$ is the gravity constant, $m$ is the mass of the pendulum rod, $M$ is the mass
of the cart, \(2l\) is the length of the pendulum, \(u\) is the control force applied to the cart, parameter \(a = 1/(m + M)\).

In this simulation, the system parameters are as follows: \(m = 2.0\text{kg}, M = 8.0\text{kg}, 2l = 1.0\text{m}\), the initial values of the state variables are \(x_1(t_0) = -\pi/3\) and \(x_2(t_0) = 0\), respectively, and \(d(t) = 0.3\sin(2t)\).

Figures 3.1, 3.2 and 3.3 show the sliding variable \(s(t)\), system output \(x(t)\) together with its derivative \(\dot{x}(t)\), and the control input \(u(t)\), respectively, where the sliding mode parameter matrix is chosen as \(C = [25 \quad 1]\), the sampling period is \(\Delta T = 0.001\text{ s}\), the time delay \(\tau = \Delta T\), and the control parameters in (3.7) are set to \(\alpha = 0.93\) and \(\beta = 0.92\).

![Figure 3.1. Sliding mode variable (SMLC).](image)
3.6 Conclusion

A novel sliding mode control technique with a learning control mechanism has been developed. The theoretical analysis and the simulation results have shown that the proposed SMLC can not only drive the closed-loop dynamics to reach the sliding surface in finite time and guarantee the desired closed-loop dynamics with the zero-error convergence on the sliding mode surface, but also exhibit the chattering-free
characteristic. More importantly, the prior information of the bounds of the uncertainties is no longer required in the proposed sliding mode learning controller design. The superior performance of the newly developed SMLC makes it clear that the proposed learning control technique provides an alternative of efficient robust control for a wide range of uncertain dynamic systems and will potentially play an essential role in control technology in years to come. This powerful control methodology will now be further developed and applied to different fields of engineering in the following chapters.
Chapter 4

Robust Stabilization of Nonminimum Phase Systems Using Sliding Mode Learning Controller

In this chapter, a robust sliding mode learning control scheme is newly developed for a class of nonminimum phase nonlinear systems with uncertain dynamics. It is shown that the proposed sliding mode learning controller, designed based on the most recent information of the stability status of the closed-loop system, is capable of adjusting the control signal to drive the sliding variable to reach the sliding surface in finite time and remain on it thereafter. The closed-loop dynamics including both observable and non-observable ones are then guaranteed to asymptotically converge to zero in the sliding mode. The developed learning control scheme exhibits many appealing characteristics including chattering-free and strong robustness against uncertainties. More significantly, the prior information of the bounds of uncertainties is no longer a prerequisite for the proposed controller design. Simulation examples are presented in comparison with the conventional sliding mode control and backstepping control approaches to illustrate the effectiveness of the proposed control methodology.

4.1 Introduction

Nonlinear systems from theoretical and especially practical points of view have become focal and real-life control objects that have been intensively studied for years. Generally, nonlinear systems can be classified into two categories in terms of the stability of internal dynamics: minimum phase and nonminimum phase systems. Since a minimum phase has stable internal dynamics, one may only need to design a controller to stabilize the linear subsystem after performing the input-output linearization for such a system. In fact, control designs for minimum phase nonlinear systems have been well developed in the literature such as global stabilization [89], nonlinear output regulation
unknown disturbance rejection [91-92], just to name a few. On the other hand, a dynamical system could be a nonminimum phase if its internal or zero dynamics are unstable. Although the nonminimum phase system could be controlled with a robust input-output stabilisation, the internal dynamics can hardly be stabilized completely. This adverse behaviour of the nonminimum phase system by its nature has restricted the applications of conventional control techniques in practice. Thus, the control of the nonminimum phase systems needs to be further studied.

Many research proposals have been established for a class of nonminimum phase nonlinear systems. These include, for instance, a feedback controller [93-94], that applied the concept of the stable inversion of a system. The drawbacks of this approach are that the nonlinearities should be well-known and the stable inversion of a system must exist. In other attempts [95], backsteppings, based on a recursive technique that interlaces the appropriate choice of a Lyapunov function with the design of feedback control, have been introduced to deal with such nonminimum phase phenomena. Nevertheless, this stepwise control often requires a very complex procedure and heavily depends on the feasibility of choosing a Lyapunov function. Another effective methodology in this area is output reconstruction, that is, a new virtual output which is a linear combination of state variables is defined such that the induced zero dynamics are stable, and thus some traditional control methods used to control the minimum phase systems can be applied to guarantee the asymptotic stability of nonminimum phase counterparts [96-97].

In addition, it is well-known in the field of robust and nonlinear control that a sliding mode control (SMC) has been intensively receiving a great deal of attention for its high levels of robust performance in terms of dealing with uncertain dynamics [2, 8, 22, 98-100]. Among these methodologies, the SMC via the concept of the stable system center [101], the terminal SMC [102], and the virtual compensated SMC [103] have been effective control techniques to tackle the zero dynamics of the nonminimum phase nonlinear systems. In these designs, however, the concerns in chattering phenomenon have not been completely addressed without degenerating the robustness of the closed-loop system. More recently, advanced SMC schemes such as higher order SMC, integral SMC, and adaptive fuzzy SMC techniques have been reported in literature [23, 104-106] as alternative control methods to effectively address the chattering issue and further enhance robustness of the closed-loop system. Yet, the designs of these
controllers still require knowledge of the bounds of the uncertainties. Therefore, how to make use of the significances of SMC to enhance the control performance of the nonminimum phase nonlinear systems still remains a challenging topic.

In this chapter, we propose a novel sliding mode learning control technique for a class of causal nonminimum phase nonlinear systems with uncertain dynamics. First of all, a nonlinear system via input-output linearization method [22] is decomposed into an input-output model and internal dynamics which are approximately linearized soon after. Next, a virtual sliding variable inspired by [101, 103] is introduced to stabilize the unstable zero dynamics and realize the stabilization of the closed-loop system. More importantly, the novel sliding mode learning controller (SMLC) can be designed based on the concept of the recently developed Lipschitz-like condition [85-87, 107] so as to drive the virtual sliding variable to converge to zero in finite time, the closed-loop dynamics are therefore guaranteed to asymptotically converge to zero on the sliding mode surface. It will be shown later that the proposed controller synthesizes the adaptive learning control algorithm by which the control input is continuously corrected to remove the effects of uncertain dynamics. Especially, the SMLC is able to drive the system dynamics from an unstable region to a stable one. Thus, the developed controller exhibits an excellent performance of both strong robustness with respect to uncertain dynamics and the zero-error convergence. Also, since the iterative learning mechanism is adopted in the controller design, the control signal is completely chattering-free. Moreover, the uncertainties, with the help of the Lipschitz-like condition, are all embedded in this condition and thus the information of the bounds of uncertainties is not required for the controller design any longer.

The remainder of this paper is organized as follows: in Section 4.2, the control problem including the input-output realization of a nonminimum phase nonlinear system and the proposed sliding mode controller are first formulated. Next, the asymptotic convergence and the stability analyses of the closed-loop system are discussed in Section 4.3. In Section 4.4, simulation results are presented in comparison with the conventional SMC and backstepping control methods to validate the effectiveness of the proposed control technique. Finally, Section 4.5 draws a conclusion and future work on this trend.
4.2 Problem Formulation

In this section, we will discuss some mathematical preliminaries associated with nonminimum phase nonlinear systems as well as the design of the proposed SMLC.

4.2.1 Input-Output Realization of Nonlinear Systems

Without loss of generality, let us consider a single-input single-output (SISO) affine nonlinear system expressed as:

\[\begin{align*}
\dot{x} &= f(x, \theta) + g(x)u \\
y &= h(x)
\end{align*}\]  \hspace{1cm} (4.1)

where \(x \in \mathbb{R}^n\) is the state variable vector, \(h(x)\) is the output function, \(f(x)\) and \(g(x)\) are sufficiently smooth functions, and \(\theta\) denotes the lumped perturbation.

**Assumption 4.1**: The system (4.1) is locally input-output linearizable, and has a well-defined relative degree \(r, 1 \leq r \leq n\) at an equilibrium point \(x_0\). More precisely, \(L_g L_f^k h(x) = 0\) for all \(k < r - 1\), and \(L_g L_f^{r-1} h(x) \neq 0\) for all \(x\) in a neighbourhood of \(x_0\), where the notation \(L_f^i h(x), i \geq 1\), represents the Lie derivative of the scalar function \(h(x)\) with respect to the field \(f(x)\), and \(L_f h(x) = (\partial h / \partial x) f(x)\).

**Assumption 4.2**: All the system states and the perturbation term in (4.1) are assumed to be locally Lipschitz in the domain of interest [22].

The nonlinear system (4.1) can be transformed into the Byrnes-Isidori normal form by a local diffeomorphism \(T(x)\) as follows [10]:

\[
T(x) = \begin{bmatrix}
h(x) \\
\vdots \\
L_f^{r-1} h(x) \\
- \frac{\partial}{\partial x} \\
\vdots \\
\eta_i(x) \\
\eta_{n-r}(x)
\end{bmatrix} \equiv \begin{bmatrix}
\zeta \\
- \\
\eta
\end{bmatrix}
\]  \hspace{1cm} (4.2)

where \(\zeta \in \mathbb{R}^r\) is the vector of input-output dynamics, \(\eta \in \mathbb{R}^{n-r}\) is the vector of internal states, \(L_g \eta_i(x) = 0\) for \(1 \leq i \leq n - r\).
The dynamics of the system (4.1) can be decomposed into two parts in the new coordinate: the input-output model (4.3.a) and the internal dynamics (4.3.b), described by the following equations:

\[
\begin{align*}
\dot{\zeta}_i &= \zeta_{i+1}, \quad i = 1, 2, \ldots, r - 1 \\
\dot{\zeta}_r &= a(\zeta, \eta, \theta) + b(\zeta, \eta)u \\
y &= \zeta_1 \\
\dot{\eta} &= q(\zeta, \eta)
\end{align*}
\] (4.3.a)  
\[
\zeta_0 &= \Theta(\theta, \omega, a) + \Phi(\omega, a) \\
(4.3.b)
\]

where \(a(\zeta, \eta, \theta) = L^T_f h(T^{-1}(\zeta, \eta))\) and \(b(\zeta, \eta) = L_g L^{-1}_f h(T^{-1}(\zeta, \eta)) \neq 0\).

Taking the linear approximation of the internal dynamics (4.3.b) around the equilibrium point \((\zeta, \eta) = 0\), yields the linearized internal dynamics as:

\[
\dot{\eta} = E\eta + F\zeta + \delta(\zeta, \eta)
\] (4.4)

where

\[
E = \left[ \frac{\partial q}{\partial \eta} \right]_{(\zeta, \eta)=0} \in R^{(n-r) \times (n-r)}, \quad F = \left[ \frac{\partial q}{\partial \zeta} \right]_{(\zeta, \eta)=0} \in R^{(n-r) \times r}
\]

are known matrices for which the pair \((E, F)\) is controllable and \(\delta(\zeta, \eta)\) is the higher order term.

**Remark 4.1:** The \((n - r)\) dimensional subsystem given by (4.4) is completely unobservable and therefore is termed as the internal dynamics or zero dynamics of the nonlinear system. It is noted that different choices of the output function \(y = h(x)\) lead to different internal dynamic models. Thus, the behaviour as well as the stability of the internal dynamics has to be addressed carefully. In this chapter, a class of nonminimum phase systems is considered, that is, the internal dynamic equation (4.4) is unstable. More precisely, the matrix \(E\) has eigenvalues on the right half of the complex plane.

The sliding variable is defined as:

\[
s = \zeta_r + \alpha_{r-1}\zeta_{r-1} + \cdots + \alpha_1\zeta_1
\] (4.5)
where the parameters $\alpha_{r-1}, \ldots, \alpha_1$ are chosen such that $\lambda^r + \alpha_{r-1} \lambda^{r-1} + \cdots + \alpha_1 \lambda$ is a Hurwitz polynomial.

Let us define the new variable $\xi = \begin{pmatrix} z \\ \eta \end{pmatrix}$, where $z = [\zeta_1, \zeta_2, \ldots, \zeta_{r-1}]^T$, $\xi \in R^{n-1}$.

It is obvious to obtain the following relation:

$$ \begin{pmatrix} s \\ z \end{pmatrix} = \begin{bmatrix} \alpha_1 & \alpha_2 & \alpha_{r-1} & 1 \\ 1 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & 1 \end{bmatrix} \zeta = G \zeta \quad (4.6.a) $$

where $G = \begin{bmatrix} \alpha_1 & \alpha_2 & \alpha_{r-1} & 1 \\ 1 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & 1 \end{bmatrix} \in R^{r \times r}$.

One can get a diffeomorphism $\mathcal{D}: (\zeta, \eta) \rightarrow (s, \xi)$, which is explicitly expressed by the following equation

$$ \begin{pmatrix} s \\ \xi \end{pmatrix} = \begin{pmatrix} s \\ z \end{pmatrix} = \begin{bmatrix} G & \theta_{(n-r) \times r} \\ \Theta_{r \times (n-r)} & I_{n-r} \end{bmatrix} \begin{pmatrix} \zeta \\ \eta \end{pmatrix} \quad (4.6.b) $$

where $I$ and $\Theta$ are identity matrix and zero matrix, respectively, with appropriate dimensions.

Thus, in the $(s, \xi)$ coordinate, the dynamics of the system (4.3), (4.4) can be written as:

$$ \dot{s} = \dot{\zeta}_r + \alpha_{r-1} \dot{\zeta}_{r-1} + \cdots + \alpha_1 \dot{\zeta}_1 $$

$$ = \tilde{a}(s, \xi, \theta) + \tilde{b}(s, \xi)u \quad (4.7.a) $$

$$ \dot{\xi} = \tilde{E} \xi + \tilde{F} s + \tilde{d}(s, \xi) \quad (4.7.b) $$

where $\tilde{a}(s, \xi, \theta)$ denotes the expression of $\tilde{a}(\zeta, \eta, \theta) = \sum_{i=1}^{r-1} \alpha_i \zeta_{i+1} + a(\xi, \eta, \theta)$ under the coordinate $(s, \xi)$; also $\tilde{b}(s, \xi), \tilde{d}(s, \xi)$ denote the expressions of $b(\zeta, \eta), d(\zeta, \eta)$, respectively in the coordinate $(s, \xi)$. The matrix $\tilde{E} \in R^{(n-1) \times (n-1)}$ and $\tilde{F} \in R^{n-1}$ can be derived accordingly, and the pair $(\tilde{E}, \tilde{F})$ remains controllable since the nonsingular
4.2 Problem Formulation

linear transformation does not change this property of systems and the matrix \( \tilde{E} \) is still non-Hurwitz.

The virtual sliding variable is defined to stabilize the uncontrollable internal variables [103]:

\[
\sigma = s - L \xi
\]  \hspace{1cm} (4.8)

where \( s \) is defined as in (4.5) and the constant vector \( L \in \mathbb{R}^{1 \times (n-1)} \) is to be designed later.

When the internal mode of system reaches the designed sliding surface \( \sigma \to 0 \) or \( s \to L \xi \), (4.7.b) can be expressed as:

\[
\dot{\xi} = (\tilde{E} + \tilde{F}L)\xi + \delta(L \xi, \xi) \\
= A\xi + \Delta(\xi)
\]  \hspace{1cm} (4.9)

where \( A = (\tilde{E} + \tilde{F}L) \) and \( \Delta(\xi) \) is the higher order perturbation term.

**Remark 4.2**: As the pair \( (\tilde{E}, \tilde{F}) \) is controllable, one can design \( L \) such that the eigenvalues of \( (\tilde{E} + \tilde{F}L) \) is arbitrarily assigned in the left half plane. Hence, the closed-loop dynamics of system (4.9) will be asymptotically stable in the sliding mode.

**4.2.2 Vanishing Perturbation**

It is seen from (4.9) that \( L \) is designed to make the nominal system asymptotically stable. However, in the presence of the perturbation term \( \Delta(\xi) \), it is crucial to show the perturbed system (4.9) is asymptotically stable eventually in the sliding mode. Detail of this convergence is shown in Theorem 4.1 below.

**Theorem 4.1** [22]: Consider the homogeneous dynamic system (9) where both the system dynamics \( \xi \) and the perturbed dynamics \( \Delta \) satisfy the Assumption 4.2. Suppose the nominal system \( \dot{\xi} = A\xi \) is asymptotically stable at \( \xi = 0 \) and the perturbation term satisfies the following growth bound

\[
\|\Delta(\xi)\| \leq \rho(t)\|\xi\|
\]  \hspace{1cm} (4.10)
where \( \rho(t): R \rightarrow R \) is nonnegative and continuous for all \( t \geq 0 \), and \( \Delta(0) = 0 \).

Let \( Q = Q^T > 0 \) and solve the Lyapunov equation \( PA + A^T P = -Q \) for a unique positive definite solution \( P \). If \( \rho(t) < \bar{\rho} < \frac{\lambda_{\min}(Q)}{2\lambda_{\max}(P)} \), the perturbed dynamic system (4.9) is exponentially stable.

**Proof:** Let \( V(\xi) = \xi^T P \xi \) be a Lyapunov function candidate of the perturbed system (4.9). One can derive the following inequalities:

\[
\lambda_{\min}(P) \| \xi \|^2 \leq V(\xi) \leq \lambda_{\max}(P) \| \xi \|^2
\]

\[
\left\| \frac{\partial V}{\partial \xi} \right\| = \| 2\xi^T P \| \leq 2\lambda_{\max}(P) \| \xi \|
\]

\[
\frac{\partial V}{\partial \xi} A\xi = \xi^T (PA + A^T P) \xi = -\xi^T Q \xi \leq -\lambda_{\min}(Q) \| \xi \|^2
\]

Taking the time derivative of \( V \), with the help of (4.10) - (4.13), yields

\[
\dot{V} = \frac{\partial V}{\partial \xi} A\xi + \frac{\partial V}{\partial \xi} \Delta(\xi)
\]

\[
\leq -\lambda_{\min}(Q) \| \xi \|^2 + 2\rho(t) \lambda_{\max}(P) \| \xi \|^2
\]

If we let \( \rho(t) < \bar{\rho} < \frac{\lambda_{\min}(Q)}{2\lambda_{\max}(P)} \), then

\[
\dot{V}(t) < 0
\]

which means the perturbed system (4.9) is exponentially stable at the origin in the sense of Lyapunov.

The control objective is to design a robust SMLC to drive the virtual sliding variable \( \sigma \) to converge to zero, and therefore ensure the stability of the system dynamics (4.9) in the sliding mode.
In the next subsection, the robust SMLC will be proposed with the capability of stabilizing both input-output system states and the internal dynamics.

### 4.2.3 Sliding Mode Learning Controller

The learning controller is proposed as the following form:

$$ u(t) = u(t - \tau) + \Delta u(t) \quad (4.15) $$

where $\tau$ is the time delay interval.

In (4.15), the adaptation term $\Delta u(t)$ is designed as:

$$ \Delta u(t) = \begin{cases} 
\frac{-1}{b\sigma} \left( a\hat{V}(t - \tau) + \beta V^\vartheta(t) \right) & \text{for } \sigma \neq 0 \\
0 & \text{for } \sigma = 0 
\end{cases} \quad (4.16) $$

where $\hat{V}(t)$ computed by (4.17) denotes the approximation of $V(t)$, $\hat{V}(t)$ is the first order derivative of the Lyapunov function candidate chosen to be $V(t) = 0.5\sigma(t)^2 [108]$:

$$ \hat{V}(t) = \frac{3V(t) - 4V(t - \tau) + V(t - 2\tau)}{2\tau} \quad (4.17) $$

In (4.16), $\alpha, \beta > 0$ are the control parameters to be determined later, and the parameter $\vartheta = \frac{q}{p}$, where $p, q$ are positive odd integers and $p > q$.

**Remark 4.3:** The SMLC algorithm (4.15) with the adaptation law (4.16), which was first initiated in [85], is designed in such a way that it is capable of driving the sliding variable to converge to zero in finite time under the presence of uncertainties and thus guarantees the stability of the closed-loop dynamics in the sense of Lyapunov (4.14). In particular, the term $\beta V^\vartheta(t)$ in (4.16) behaves like an inertia factor and, by properly choosing the values of $\beta$ and $\vartheta$, the convergence of the closed-loop system can be improved. The term $a\hat{V}(t - \tau)$ on the other hand plays the role of checking the most recent stability status of the closed-loop system, and updating the control signal.
accordingly to ensure that the system states can converge to zero in finite time. Most importantly, if the most recent information shows that the system is unstable, the term \( aV(t - \tau) \) is capable of modifying the control signal in the sense that the closed-loop system can be driven from an unstable domain to a stable domain. The significance of this point can be seen from the convergence analysis in the next section.

**Remark 4.4:** It is worth noting that in this chapter, the approximation of the first order derivative of the Lyapunov function candidate is calculated by (4.17) based on “the derivative approximation methods by finite differences” [108], which is actually the estimated differentiation of the Lyapunov function with second order error \( O(\tau^2) \). This computation brings with it a number of benefits. Firstly, compared with the approximation with the first order error \( O(\tau) \) used in [85-87], the estimation (4.17) results in better precision with the truncation error of \( O(\tau^2) \). Secondly, the convergence rate increases due to the increase in the order of the approximation.

Taking the first-order derivative of (4.8) with the help of (4.7.a) and (4.7.b) yields:

\[
\dot{s} = \dot{a}(s, \xi, \theta) + \dot{b}(s, \xi)u - L(\dot{E}\xi + \dot{F}s + \dot{\delta}(s, \xi)) \\
= \ddot{f}(s, \xi, \theta) + \ddot{b}(s, \xi)u
\]  

(4.18)

where \( \ddot{f}(s, \xi, \theta) = \dot{a}(s, \xi, \theta) - L(\dot{E}\xi + \dot{F}s + \dot{\delta}(s, \xi)) \).

The analysis on the convergence and the stability of the proposed SMLC will be shown in the next section.

### 4.3 Convergence Analysis

**Theorem 4.2:** Consider the nonminimum phase nonlinear system described in (4.3) and (4.4), assume the pair \((E,F)\) in (4) is controllable and \( \dot{b}(s, \xi) \) is nonsingular. If the sliding manifold (4.8) and the proposed controller (4.15) are used, then the system states will be driven to reach the sliding surface \( \sigma = 0 \) and be retained on it thereafter. The closed-loop dynamics will then converge to zero asymptotically on the sliding surface.

**Proof:** Considering a Lyapunov candidate function
\[ V(t) = \frac{1}{2} \sigma(t)^2 \] (4.19)

The time derivative of \( V(t) \), upon using (4.15), (4.16) and (4.18), is expressed as:
\[
\dot{V}(t) = \sigma(t)\dot{\sigma}(t)
= \sigma(t)\left(\hat{f}(s, \xi, \theta) + \bar{b}u(t)\right)
= \sigma(t)\left[\hat{f}(s, \xi, \theta) + \bar{b}u(t - \tau)\right] + \sigma(t)\bar{b}\Delta u(t)
= \dot{V}(t, t - \tau) - \alpha\dot{V}(t - \tau) - \beta V^\theta(t) \tag{4.20}
\]

where \( \dot{V}(t, t - \tau) = \sigma(t)\left[\hat{f}(s, \xi, \theta) + \bar{b}u(t - \tau)\right] \).

Adding the term \( \dot{V}(t - \tau) - \dot{V}(t - \tau) \) to (4.20) yields
\[
\dot{V}(t) = \dot{V}(t, t - \tau) - \dot{V}(t - \tau) + \dot{V}(t - \tau) - \alpha\dot{V}(t - \tau) - \beta V^\theta(t)
\leq \left|\dot{V}(t, t - \tau) - \dot{V}(t - \tau)\right| + \dot{V}(t - \tau) - \alpha\dot{V}(t - \tau) - \beta V^\theta(t) \tag{4.21}
\]

Consider the continuity property of the Lyapnov function, the Lipschitz-like condition (3.11) can be used in this circumstance.

Substituting (3.11) into (4.21) yields
\[
\dot{V}(t) < \frac{1}{M}\left|\dot{V}(t - \tau)\right| + \dot{V}(t - \tau) - \alpha\dot{V}(t - \tau) - \beta V^\theta(t) \tag{4.22}
\]

- **For the case that** \( \dot{V}(t - \tau), \dot{V}(t - \tau) > 0 \):

One can obtain from (4.22) that
\[
\dot{V}(t) < \dot{V}(t - \tau) + \left(\frac{1}{M} - \alpha\right)\left|\dot{V}(t - \tau)\right| - \beta V^\theta(t) \tag{4.23}
\]

If the control parameter \( \alpha \) is chosen such that
\[
\frac{1}{M} < \alpha < 1 - \frac{1}{M} - \gamma \quad (4.24)
\]

then, (4.23) becomes

\[
\dot{V}(t) < \dot{V}(t - \tau) - \frac{1}{M} - \alpha \left| \hat{V}(t - \tau) - \beta V^\theta(t) \right|
\]

\[
< \dot{V}(t - \tau) \quad (4.25)
\]

The inequality (4.25) indicates that the proposed SMLC (4.15) always makes the value of \(\dot{V}(t)\) decrease for \(\dot{V}(t - \tau) > 0\). Suppose that, at time \(t = t_0\), \(\dot{V}(t_0) = 0\). Then at the time \(t = t_0 + \tau\), (4.20) can be expressed as:

\[
\dot{V}(t_0 + \tau) = \dot{V}(t_0 + \tau, t_0) - \alpha \hat{V}(t_0) - \beta V^\theta(t_0 + \tau) \quad (4.26)
\]

In fact, \(\dot{V}(t_0 + \tau, t_0) - \alpha \hat{V}(t_0)\) is upper bounded and \(V^\theta(t_0 + \tau) \neq 0\), thus there exists a positive number \(\beta\) such that the following inequality holds:

\[
\left| \dot{V}(t_0 + \tau, t_0) - \alpha \hat{V}(t_0) \right| < \beta V^\theta(t_0 + \tau) \quad (4.27)
\]

thus, with the chosen parameter \(\beta\) which satisfies (4.27), (4.26) becomes

\[
\dot{V}(t_0 + \tau) < 0 \quad (4.28)
\]

The analysis from (4.23) to (4.28) emphasizes that the proposed learning controller (4.15) is capable of reducing \(\dot{V}(t)\) from the positive value to the negative one. In other terms, the closed-loop trajectory is always driven into the stable region.

- **For the case that \(\dot{V}(t - \tau), \hat{V}(t - \tau) < 0\):**

One can rewrite (4.22) as:
4.3 Convergence Analysis

\[
\dot{V}(t) < \dot{V}(t-\tau) + \left(\frac{1}{M} + \alpha\right) |\dot{V}(t-\tau)| - \beta V^\theta(t)
\]  
(4.29)

From (3.12), one can easily obtain

\[
\dot{V}(t-\tau) < \dot{V}(t-\tau) + \gamma |\dot{V}(t-\tau)| < (\gamma - 1) |\dot{V}(t-\tau)|
\]  
(4.30)

Substituting (4.30) into (4.29) leads to

\[
\dot{V}(t) < (\gamma - 1) |\dot{V}(t-\tau)| + \left(\frac{1}{M} + \alpha\right) |\dot{V}(t-\tau)| - \beta V^\theta(t)
\]
\[
< \left(\frac{1}{M} - 1 + \gamma + \alpha\right) |\dot{V}(t-\tau)| - \beta V^\theta(t)
\]  
(4.31)

With the chosen parameter \(\alpha\) which satisfies (4.24), (4.31) becomes

\[
\dot{V}(t) < -\left|\frac{1}{M} + \alpha + \gamma - 1\right| |\dot{V}(t-\tau)| - \beta V^\theta(t)
\]
\[
< -\beta V^\theta(t)
\]  
(4.32)

In summary, based on the mathematical analysis above, the stability criterion (4.32) is satisfied which guarantees the finite time convergence of the virtual sliding variable \(\sigma(t)\) [52-53, 109]. As shown in Theorem 4.1, the pole placement method can be used to design \(L\) such that the system dynamics (4.9) with vanishing perturbation is asymptotically stable, that is to say \(\xi = \left(\frac{Z}{\eta}\right) \rightarrow 0\) in the sliding mode. Moreover, at the same time, on the sliding mode surface \((\sigma = 0)\), \(s\) defined in (4.5) also vanishes since \(|s| \leq ||L||\|\xi\| \rightarrow 0\) as \(\xi \rightarrow 0\). This concludes that both input-output dynamics \(\zeta\) and internal dynamics \(\eta\) of the closed-loop system converge to zero asymptotically.

**Remark 4.5:** As shown in [52-53], it is worth noting that the proper selection of \(\theta\) in (4.16) can guarantee a finite- time convergence of the sliding variable in the reaching phase. More specifically, it is easy to show that the stability condition (4.32) will
guarantee that the sliding variable converges to zero in finite time for all bounded initial conditions [109]. It can be concluded from Theorem 4.1 and Theorem 4.1 that the closed-loop system exhibits a strong robustness against uncertain dynamics. Moreover, since the adaptive learning term (4.16) is used, there is no sign function in the controller input, and thus the so-called chattering is completely eliminated.

**Remark 4.6**: It is noted that the Lipschitz-like condition (3.11) has been employed in the convergence analysis and stability discussion above. Owing to this condition, the uncertainties are all embedded in the left-hand side of (3.11), and thus no information on the bounds of the uncertainties is required for the controller design. It appears, moreover, that the Lipschitz-like condition plays an essential role in the convergence of the closed-loop system, which guarantees that the learning controller, synthesizing the adaptive learning mechanism, is always adjusted to correct the motion of the closed-loop dynamics and drives the virtual sliding variable to converge to zero in finite time, and thus the closed-loop dynamics including both input-output dynamics and internal dynamics asymptotically converges to zero in the sliding mode.

**4.4 Simulation Results**

In this section, two examples are presented to illustrate the effectiveness of the proposed learning control scheme in comparison with the conventional SMC and the backstepping control.

**Example 1:**

In this illustrative example, let us consider the following uncertain nonlinear system [103]:

\[
\dot{x} = \begin{bmatrix} -x_3 \\ \sin(x_2) + x_3 \\ 2x_1^2 - (1 + \theta)x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u
\]

(4.33)

where the uncertainty \( \theta \) is set as \( \theta = 0.2 \sin(4t) \), \( x_0 = [0.3 \ 0.4 \ 0.1]^T \), \( y = [1 \ 0 \ 0]x \).
From the definition in Assumption 4.1 and simple calculations below, we have the relative degree of output channel \( r = 2 \).

\[
\begin{align*}
\zeta_1 &= h(x) = x_1 \\
\zeta_2 &= L_f h(x) = \frac{\partial h(x)}{\partial x} f(x) = -x_3 \\
L_g h(x) &= \frac{\partial h(x)}{\partial x} g(x) = 0 \\
L_g L_f h(x) &= \frac{\partial L_f h(x)}{\partial x} g(x) = -1 \neq 0 \\
L_f^2 h(x) &= \frac{\partial L_f h(x)}{\partial x} f(x) \\
&= -2x_1^2 + (1 + \theta)x_3 \\
\eta &= x_2
\end{align*}
\]

Thus, (4.33) can be described in internal dynamic and input-output equations as follows:

\[
\begin{align*}
\dot{\zeta}_1 &= \zeta_2 \\
\dot{\zeta}_2 &= -2\zeta_1^2 - \zeta_2 - \theta \zeta_2 - u \\
\dot{\eta} &= \sin(\eta) - \zeta_2 \\
y &= \zeta_1
\end{align*}
\] (4.34)

The approximately linearized model of the internal system can be obtained as

\[
\dot{\eta} = \eta - \zeta_2
\] (4.35)

It is obviously seen that the internal mode (4.35) is unstable. From (4.5), the sliding variable is defined as:

\[
s = \zeta_2 + 10\zeta_1
\] (4.36)

In the coordinate \((s, \xi)\), system dynamics in (4.34) and (4.35) can be re-written as
\[ \dot{\xi} = \begin{pmatrix} -10 & 0 \\ 10 & 1 \end{pmatrix} \xi + \begin{pmatrix} 1 \\ -1 \end{pmatrix} s \]  
\[ (4.37) \]

where \( \xi = \begin{pmatrix} z \\ \eta \end{pmatrix} = \begin{pmatrix} \zeta_1 \\ \eta \end{pmatrix} \).

From (4.8) and (4.9), one can design the sliding variable parameter \( L = [12, 17] \) to guarantee the system dynamics on sliding mode surface

\[ \dot{\xi} = \begin{pmatrix} 2 & 17 \\ -2 & -12 \end{pmatrix} \xi \]  
\[ (4.38) \]

have eigenvalues on the left half complex plane, thus guarantees the asymptotic stability of the system (4.38) in the sliding mode.

For comparison purpose, we first consider using the conventional SMC designed in [103]:

\[ u = -2\zeta_1^2 - 2\zeta_2 - 8\eta + K\text{sign}(\sigma) + M\sigma \]  
\[ (4.39) \]

where the upper bound of uncertainty \( K \geq |\theta\zeta_2| \) is supposed to be known and \( M > 0 \).

The simulation results of the virtual sliding variable, input-output dynamics, internal state, and control input have been shown in Figure 4.1.a – Figure 4.1.d, respectively. It is seen that even though the closed-loop dynamics are stabilized, the conventional SMC exhibits the chattering phenomenon and the controller design still requires the information about the upper bound of the uncertainty.

We now turn to adopt the proposed SMLC for the system (4.33), which is designed as in (4.15) and (4.16) with the control parameters \( \alpha = 0.05, \beta = 0.02, \vartheta = 5/7 \), the simulation results of the virtual sliding variable, input-output dynamics, internal state, and control signal are shown in Figure 4.2.a – Figure 4.2.d, respectively. It is clearly seen that the proposed control scheme with the recursive-learning algorithm exhibits an excellent performance with both chattering-free characteristic and strong robustness with respect to uncertainty. Also, it is clearly seen that the convergence rate of the internal state is faster and smoother than that of the conventional SMC shown in the
previous case. Moreover, with the help of the Lipschitz-like condition in which all the system uncertainties have been embedded, the design of the proposed SMLC no longer requires prior knowledge of the bounds of the uncertainties.

![Virtual sliding variable](image1.png)

**Figure 4.1.a.** Virtual sliding variable $\sigma$ (Conventional SMC).

![Input-output states](image2.png)

**Figure 4.1.b.** Input-output states $x_1$ & $x_3$ (Conventional SMC).
Figure 4.1.c. Internal state $x_2$ (Conventional SMC).

Figure 4.1.d. Control input (Conventional SMC).

Figure 4.2.a. Virtual sliding variable $\sigma$ (Proposed SMLC).
4.4 Simulation Results

Figure 4.2.b. Input-output states $x_1$ & $x_3$ (Proposed SMLC).

Figure 4.2.c. Internal state $x_2$ (Proposed SMLC).

Figure 4.2.d. Control input (Proposed SMLC).
Example 2:

In this example we consider a SISO nonlinear system described by [95]:

\[
\dot{x} = \begin{bmatrix}
-x_1 + x_2 \\
-3x_2 + x_1^3 + \theta \\
x_1 - 2x_3 \\
-x_4 + x_3^2
\end{bmatrix} + \begin{bmatrix}
0 \\
2 + \sin^2(x_4) \\
0 \\
0
\end{bmatrix} u 
\] (4.40)

where \( \theta = 0.3 \sin(3t) \), \( x_0 = [10 \; 5 \; 5 \; 1]^T \), and \( y = [1 \; 0 \; -3 \; 0]^T x \).

Similarly, one can obtain the relative degree of the system (4.40) as \( r = 2 \).

\[
\begin{align*}
\zeta_1 &= h(x) = x_1 - 3x_3 \\
\zeta_2 &= L_f h(x) = \frac{\partial h(x)}{\partial x} f(x) = -4x_1 + x_2 + 6x_3 \\
L_g h(x) &= \frac{\partial h(x)}{\partial x} g(x) = 0 \\
L_g L_f h(x) &= \frac{\partial L_f h(x)}{\partial x} g(x) = 2 + \sin^2(x_4) \neq 0 \\
L_f^2 h(x) &= \frac{\partial L_f h(x)}{\partial x} f(x) \\
&= 10x_1 - 7x_2 + x_1^3 - 12x_3 + \theta \\
\eta_1 &= x_3 \\
\eta_2 &= x_4 
\end{align*}
\]

then (4.40) can be described in internal dynamics and input-output equations below

\[
\begin{align*}
\dot{\zeta}_1 &= \zeta_2 \\
\dot{\zeta}_2 &= -18\zeta_1 - 7\zeta_2 - 24\eta_1 + (\zeta_1 + 3\eta_1)^3 \\
&\quad + (2 + \sin^2(\eta_2)) u \\
\dot{\eta} &= \begin{bmatrix} \dot{\eta}_1 \\ \dot{\eta}_2 \end{bmatrix} = \begin{bmatrix} \eta_1 + \zeta_1 \\ \eta_1^2 - \eta_2 \end{bmatrix} \\
y &= \zeta_1 
\end{align*}
\] (4.41)

The approximately linearized model of the internal system can be obtained as
\[ \dot{\eta} = \begin{pmatrix} 1 & 0 \\ 2 & -1 \end{pmatrix} \eta + \begin{pmatrix} 1 \\ 0 \end{pmatrix} \zeta \]  

(4.42)

It is obviously seen that the internal mode (4.42) is not completely stable.

For comparison purpose, we first employ the concept of a backstepping technique designed as in [95]:

\[ u = -\frac{a(x)}{b(x)} + \frac{1}{b(x)} \left[ -(c_1 \rho_1 + c_2 \rho_2) - (k_1 + 1)(\zeta_1 + \zeta_2 + \eta_1) \right] \]  

(4.43)

where

\[ \rho_1 = \zeta_1 + (k_1 + 1)\eta_1 \]
\[ \rho_2 = \zeta_2 + (k_1 + 1)(\zeta_1 + \eta_1) \]
\[ [k_1 \ c_1 \ c_2] = [5 \ 5 \ 5] \]

Figure 4.3.a – Figure 4.3.c show the resulting responses of the input-output states, internal dynamics and control signal, respectively.

Next, the SMLC proposed in (4.15) and (4.16) is applied to the system (4.40) with the selected control parameters = \([-1 \ 24 \ 0]\), \([\alpha, \beta] = [0.014, 0.011]\), and \(\theta = \frac{5}{7}\). The results of the virtual sliding variable, input-output states, internal dynamics and control input are shown in Figure 4.4.a – Figure 4.4.d, respectively. It can be seen that, by using the SMLC, the internal dynamics have been completely stabilized to fulfil the stability of the closed-loop system. It is seen that, compared with the backstepping control, the proposed SMLC scheme achieves a better performance in terms of stabilizing both input-output and zero dynamics. The closed-loop dynamics with the proposed SMLC asymptotically converge to zero at a faster rate, and also the control signal is smoothen out. Moreover, the design of SMLC is much more simplified without any constraints on the system uncertainties.
Figure 4.3.a. Input-output states $x_1$ & $x_2$ (Backstepping).

Figure 4.3.b. Internal states $x_3$ & $x_4$ (Backstepping).

Figure 4.3.c. Control signal (Backstepping).
Figure 4.4.a. Virtual sliding variable (Proposed SMLC).

Figure 4.4.b. Input-output states $x_1 \& x_2$ (Proposed SMLC).

Figure 4.4.c. Internal states $x_3 \& x_4$ (Proposed SMLC).
In this work, the novel sliding mode learning control with merits of both chattering-free attributes and asymptotic convergence has been successfully developed for a class of nonminimum phase nonlinear systems. It is worth noting that the developed control is capable of learning from the history of system dynamics and driving the virtual sliding variable to converge to the sliding surface and remain on it thereafter. The closed-loop dynamics can then be asymptotically stabilized in the sliding mode. The simulation results have illustrated the superior performance of the new sliding mode control technique. The proposed control scheme exhibits a strong robustness with respect to uncertainty and the constraint about the prerequisite knowledge of the bounds of uncertain dynamics required in many existing sliding mode control schemes has been lifted. The extensive work to control of an MIMO nonlinear system and its applications is under the authors’ investigation.

Figure 4.4.d. Control signal (Proposed SMLC).

4.5 Conclusion
Chapter 5

Sliding Mode Learning Based Congestion Control for DiffServ Networks

In this chapter, a robust sliding mode learning control (SMLC) scheme is developed for congestion control problem in differentiated services (DiffServ) networks. The network is modelled by a nonlinear fluid flow model corresponding to two classes of traffic, namely, the premium traffic and the ordinary traffic. The proposed congestion controller does take into account the associated physical network resource limits and is studied analytically to guarantee the stability of the closed-loop system with a strong robustness against unknown and time-varying delays. Numerical results are presented to illustrate the effectiveness and capabilities of the proposed congestion control strategy.

5.1 Introduction

It is widely agreed that the solving of the congestion control problem is of paramount importance in communication networks due to the soaring needs for speed, size, load, and connectivity of progressively integrated services. This has necessitated the design and utilization of innovative network architectures by incorporating more effective congestion control algorithms in addition to the standard Transport Control Protocol (TCP) technologies [110]. Unlike integrated service (IntServ) which is a flow-based mechanism, DiffServ is a class-based networking architecture for classifying and managing network traffic and providing quality of service (QoS). DiffServ is often used to provide low-latency to critical network traffic such as voice and streaming media [111-112].

In general, there are three significant traffic services in DiffServ networks: the premium, the ordinary, and the best-effort. Premium service is designed for applications with stringent delay and loss requirements on a per packet basis that can specify upper
bounds on their traffic needs and required QoS. The ordinary traffic is intended for applications that have relaxed delay requirements and allow their rates into the network to be controlled. This kind of traffic can use any leftover capacity that is not used by the premium traffic. Finally, the best-effort service has no delay or loss requirements. It opportunistically capitalizes on any instantaneously leftover capacity that is not used by both premium and ordinary traffic services. There is thus no control expected for the best-effort service, for this reason, the best-effort service is not considered in this chapter.

An “ideal” congestion control must be able to simultaneously satisfy the QoS specifications of the aggregate traffics in addition to congestion avoidance. A large body of work has been carried out regarding diverse congestion control techniques [113-120]. Specifically, in [113, 115, 120], adaptive nonlinear congestion controllers have been proposed in an attempt to effectively alleviate the effect of unknown and time-varying delays on the performance of network service. In fact, modelling and analyzing the performance metrics like network throughput, queuing delay, and packet loss rate in a formal, quantitative, and analytical manner is not an easy task, because their effects on the congestion control problem are typically nonlinear in nature. Hence, the congestion control problem may become unmanageable unless effective, robust, and decentralized methods are developed. The development of such effective congestion control algorithms require integration of advanced networking and control techniques.

Amongst many control techniques for dynamical systems with uncertainties, or network systems in this case, sliding mode control (SMC) is widely accepted as an efficient control method for uncertain dynamical systems thanks to its robustness against unknown dynamics [8, 13]. Many researchers have made good use of the merits of SMC in congestion control problems [114, 117-118]. However, the conventional SMC brings with it several setbacks such as a chattering phenomenon due to discontinuous motions of switching control signals and the information of the bounds required in the controller design. These drawbacks have greatly refrained SMC from being applied to many practical circumstances. Having said all that, the SMLC recently proposed by [84] and later successfully applied to various disciplines [86-88] has beautifully addressed the aforementioned issues and demonstrated a plausible alternative of robust control technique for networking control systems.
5.2 Problem Formulation

In this chapter, the SMLC technique, that has been recently developed to effectively serve a large class of uncertain dynamic systems, is further adopted to tackle the congestion control problems in a networking system. First of all, a nonlinear fluid flow model is used to model the congestion control problem in DiffServ networks subject to unknown and time-varying delays. Next, SMLC is designed to address the queue regulation of premium and ordinary buffers. It is seen that the SMLC is able to drive the system dynamics to converge to the desired states. The proposed controller exhibits an excellent performance of both robustness with respect to uncertainties and chattering-free characteristics. More importantly, by using the Lipschitz-like condition [84-88], the information of the bounds of uncertainties is no longer required for the controller design.

The remaining part of this chapter is organized as follows: in Section 5.2, the congestion control problem associated with DiffServ networks and the proposed SMLC scheme are first formulated. Next, the stability of the closed-loop dynamics is analyzed in detail in Section 5.3. In Section 5.4, simulation results are presented to validate the effectiveness of the proposed control technique. Lastly, Section 5.5 draws a conclusion and future work on this trend.

5.2 Problem Formulation

This section is concerned with the problem formulation associated with the fundamental of congestion control of a DiffServ network and the design of the proposed SMLC for such a system.

5.2.1 Congestion Control for DiffServ Networks

Following the fluid flow model [111], a validated nonlinear DiffServ network dynamics can be expressed as:

\[
\dot{x}(t) = -\frac{x(t)}{x(t) + 1}C(t) + \lambda(t)
\]  

(5.1)

where \(x(t)\) is the queue length, \(C(t)\) represents the link capacity and is chosen as the control input for a premium buffer, a nonlinear function \(\lambda(t)\) denotes the average incoming traffic rate and is taken as the control input for an ordinary buffer.
In (5.1), we have assumed that the sources of data are persistent and ignored the latency of incoming traffic. However, in fact, the input and output of traffic is shifted in time. Therefore, (5.1) can be reformulated as

\[
\dot{x}_i(t) = -\frac{x_i(t)}{x_i(t) + 1} C_i(t) + \lambda_i(t - \tau_i)
\] 

(5.2)

where \(\tau_i\) is the time delay, the index \(i = (p, r)\) where \(p\) and \(r\) indicates premium and ordinary buffer dynamics respectively throughout this chapter.

Following the leader-follower approach [113-116], the proposed control schematic diagram is sketched out in Figure 5.1. In (5.2), \(x_p(t), x_r(t)\) are the state variables. For the premium service, the control signal is link capacity \(C_p(t)\) while the data incoming rate \(\lambda_p(t)\) can be treated as the disturbance of the system. The control objective is to make the system queue length \(x_p(t)\) trace the desired reference queue length \(x_p^{ref}(t)\) through accommodating link capacity. For the ordinary buffer, the link capacity \(C_r(t)\) is the leftover capacity calculated from \(C_r(t) = C_{server}(t) - C_p(t)\), which is uncontrolled. The control signal of ordinary service is \(\lambda_r(t)\), whose control objective is to ensure that the queue length \(x_r(t)\) closely tracks the desired reference queue length \(x_r^{ref}(t)\) by adjusting the arriving rate of data \(\lambda_r(t)\).

Figure 5.1. Proposed control scheme for DiffServ traffic.
Remark 5.1: It is worth noting that certain network physical constraints should be formally identified and specified. The router is embedded with a buffer and output link capacity limit. Furthermore, the transmitter can only support a maximum transmission rate of $\lambda_{max}$. Thus, the queue, the capacity, and the instantaneous traffic transmission rate should satisfy the following constraints

$$
0 \leq x_i(t), x_i^{ref}(t) \leq x_{buffer}
$$

$$
0 \leq C_i(t) \leq C_{server}
$$

$$
0 \leq \lambda_i(t) \leq \lambda_{max}
$$

5.2.2 Sliding Mode Learning Controller Design

Firstly, a sliding variable is defined as follows:

$$
\sigma_i(t) = x_i(t) - x_i^{ref}(t)
$$

(5.4)

Taking the first-order derivative of (5.4) yields:

$$
\dot{\sigma}_i(t) = \dot{x}_i(t) - \dot{x}_i^{ref}(t)
$$

$$
= -\frac{x_i(t)}{x_i(t) + 1} C_i(t) + \lambda_i(t - \tau_i) - \dot{x}_i^{ref}(t)
$$

$$
= -\frac{x_i(t)}{x_i(t) + 1} C_i(t) + \lambda_i(t) - \tau_i \dot{\lambda}_i - \dot{x}_i^{ref}(t)
$$

(5.5)

where the delayed signal $\lambda_i(t - \tau_i)$ is approximated by its first order as $\lambda_i(t - \tau_i) = \lambda_i(t) - \tau_i \dot{\lambda}_i$ with $\tau_i$ be an unknown delay coefficient.

The control objective here is to design a robust sliding mode learning controller to drive the sliding variable to converge to zero, and therefore ensure the stability of the closed-loop dynamics in the sense of Lyapunov.

The SMLC is proposed for the primary traffic as
\[ C_p(t) = C_p(t - \tau) + \Delta C_p(t) \]  \hspace{1cm} (5.6)

where the learning term \( \Delta C_p(t) \) is defined as:

\[
\Delta C_p(t) = \begin{cases} 
    \frac{x_p(t) + 1}{\sigma_p(t)x_p(t)} \left( \alpha_p \hat{V}_p(t - \tau) + \beta_p \left| \hat{V}_p(t - \tau) \right| \right) & \text{for } \sigma_p(t) \neq 0 \\
    0 & \text{for } \sigma_p(t) = 0
\end{cases}
\]  \hspace{1cm} (5.7)

and for the ordinary traffic as:

\[ \lambda_r(t) = \lambda_r(t - \tau) + \Delta \lambda_r(t) \]  \hspace{1cm} (5.8)

where the adaptive learning term \( \Delta \lambda_r(t) \) is defined as:

\[
\Delta \lambda_r(t) = \begin{cases} 
    -\frac{1}{\sigma_r(t)} \left( \alpha_r \hat{V}_r(t - \tau) + \beta_r \left| \hat{V}_r(t - \tau) \right| \right) & \text{for } \sigma_r(t) \neq 0 \\
    0 & \text{for } \sigma_r(t) = 0
\end{cases}
\]  \hspace{1cm} (5.9)

It is noted from (5.6) - (5.9) that \( \tau \) is the time delay interval of the controller, \( \hat{V}_i(t - \tau) \) is the approximation of \( V_i(t - \tau) \), \( \hat{V}_i(t) \) is the first order derivative of the Lyapunov function candidate \( V_i(t) = 0.5\sigma_i(t)^2 \), and \( \hat{V}_i(t - \tau) \) is defined as:

\[ \hat{V}_i(t - \tau) = \frac{V_i(t) - V_i(t - \tau)}{\tau} \]  \hspace{1cm} (5.10)

and \( \alpha_i, \beta_i > 0 \) are control parameters which will be determined later.

**Remark 5.2**: The minimal value of the time delay \( \tau \) is equal to the sampling period. If \( \tau \) is sufficiently small, it is reasonable to assume that

\[ \text{sign} \left( \hat{V}_i(t - \tau) \right) = \text{sign} \left( \hat{V}_i(t - \tau) \right) \]  \hspace{1cm} (5.11)

\[ \left| \hat{V}_i(t - \tau) - \hat{V}_i(t - \tau) \right| < \gamma_i \left| \hat{V}_i(t - \tau) \right| \]  \hspace{1cm} (5.12)
for $\dot{V}_i(t - \tau) \neq 0, \ddot{V}_i(t - \tau) \neq 0$ and $0 < \gamma_i < 1$.

The next section is dedicated to the convergence and stability analysis of the proposed learning control scheme.

### 5.3 Stability Analysis

**Theorem 5.1**: Considering the DiffServ network traffic (5.2), if the sliding variable (5.4) and the proposed controller (5.6) and (5.8) are respectively used for premium and ordinary traffic, the sliding variable will be driven to reach the sliding surface $\sigma_i = 0$ and be retained on it thereafter, the system queue length can accordingly track the desired reference queue length asymptotically.

**Proof**: Considering a Lyapunov candidate function

$$V_i(t) = \frac{1}{2} \sigma_i(t)^2 \tag{5.13}$$

The time derivative of $V_i(t)$, upon using (5.5), can be expressed as:

$$\dot{V}_i(t) = \sigma_i(t) \dot{\sigma}_i(t)$$

$$= \sigma_i(t) \left( - \frac{x_i(t)}{x_i(t) + 1} C_i(t) + \lambda_i(t) - \tau_i \dot{\lambda}_i - \dot{x}_i^{ref}(t) \right) \tag{5.14}$$

Substituting (5.6)-(5.9) into (5.14) leads to

$$\dot{V}_p(t) = \sigma_p(t) \left( - \frac{x_p(t)}{x_p(t) + 1} C_p(t - \tau) + \lambda_p(t) - \tau_p \dot{\lambda}_p - \dot{x}_p^{ref}(t) \right)$$

$$- \sigma_p(t) \frac{x_p(t)}{x_p(t) + 1} \Delta C_p(t)$$

$$= \dot{V}_p(t, t - \tau) - \alpha_p \dot{\dot{V}}_p(t - \tau) - \beta_p \left| \ddot{V}_p(t - \tau) \right| \tag{5.15}$$

and
\[ \dot{V}_r(t) = \sigma_r(t) \left( - \frac{x_r(t)}{x_r(t) + 1} C_r(t) + \lambda_r(t - \tau) - \tau_r \delta_p - \dot{x}_r^{ref}(t) \right) - \sigma_r(t) \Delta \lambda_r(t) \]

= \dot{V}_r(t, t - \tau) - \alpha_r \dot{V}_r(t - \tau) - \beta_r \left| \dot{V}_r(t - \tau) \right| \quad (5.16)

where

\[ \dot{V}_p(t, t - \tau) = \sigma_p(t) \left( - \frac{x_p(t)}{x_p(t) + 1} C_p(t - \tau) + \lambda_p(t) - \tau_p \delta_p - \dot{x}_p^{ref}(t) \right) \quad (5.17) \]

\[ \dot{V}_r(t, t - \tau) = \sigma_r(t) \left( - \frac{x_r(t)}{x_r(t) + 1} C_r(t) + \lambda_r(t - \tau) - \tau_r \delta_r - \dot{x}_r^{ref}(t) \right) \quad (5.18) \]

For the sake of a convergence proof in the later part, (5.15) and (5.16) can be generalized as follows

\[ \dot{V}_i(t) = \dot{V}_i(t, t - \tau) - \alpha_i \dot{V}_i(t - \tau) - \beta_i \left| \dot{V}_i(t - \tau) \right| \quad (5.19) \]

**Remark 5.3:** Considering the continuity of both \( \dot{V}_i(t) \) and \( \dot{V}_i(t, t - \tau) \), as the time delay \( \tau \) is sufficiently small, there exists a positive number \( M_i \gg 1 \) such that the following inequality is always satisfied [84-88]:

\[ \left| \dot{V}_i(t, t - \tau) - \dot{V}_i(t - \tau) \right| < \frac{1}{M_i} \left| \dot{V}_i(t - \tau) \right| \quad (5.20) \]

for \( \dot{V}_i(t, t - \tau) \neq 0, \dot{V}_i(t - \tau) \neq 0 \), and \( \dot{V}_i(t - \tau) \neq 0 \).

Adding the term \( \dot{V}_i(t - \tau) - \dot{V}_i(t - \tau) \) to (5.19) yields

\[ \dot{V}_i(t) = \dot{V}_i(t, t - \tau) - \dot{V}_i(t - \tau) + \dot{V}_i(t - \tau) - \alpha_i \dot{V}_i(t - \tau) - \beta_i \left| \dot{V}_i(t - \tau) \right| \]

\[ \leq \left| \dot{V}_i(t, t - \tau) - \dot{V}_i(t - \tau) \right| + \dot{V}_i(t - \tau) - \alpha_i \dot{V}_i(t - \tau) - \beta_i \left| \dot{V}_i(t - \tau) \right| \quad (5.21) \]

Substituting (20) into (21) yields
\[ \dot{V}_i(t) < \frac{1}{M_i} |\dot{V}_i(t - \tau)| + \dot{V}_i(t - \tau) - \alpha_i \ddot{V}_i(t - \tau) - \beta_i |\ddot{V}_i(t - \tau)| \] (5.22)

- **For the case that** \( \dot{V}_i(t - \tau) > 0\):

One can obtain from (5.22) that

\[ \dot{V}_i(t) < \dot{V}_i(t - \tau) + \left( \frac{1}{M_i} - \alpha_i \right) |\dot{V}_i(t - \tau)| - \beta_i |\ddot{V}_i(t - \tau)| \] (5.23)

If the control parameter \( \alpha_i \) is chosen such that

\[ \frac{1}{M_i} < \alpha_i < 1 - \frac{1}{M_i} - \gamma_i \] (5.24)

then, (5.23) becomes

\[ \dot{V}_i(t) < \dot{V}_i(t - \tau) \] (5.25)

The inequality (5.25) indicates that the proposed controller (5.6) - (5.9) always makes the value of \( \dot{V}_i(t) \) smaller than \( \dot{V}_i(t - \tau) \). Hence, \( \dot{V}_i(t) \) always decreases for \( \dot{V}_i(t - \tau) > 0 \). Suppose that, at time \( t = t_0 \), \( \dot{V}_i(t_0) = 0 \). Then at the time \( t = t_0 + \tau \), (19) can be expressed as:

\[ \dot{V}_i(t_0 + \tau) = \dot{V}_i(t_0 + \tau, t_0) - \alpha_i \ddot{V}_i(t_0) - \beta_i |\ddot{V}_i(t_0)| \] (5.26)

In fact, \( \dot{V}_i(t_0 + \tau, t_0) - \alpha_i \ddot{V}_i(t_0) \) is upper bounded and \( \dot{V}_i(t_0) \neq 0 \), thus there exists a positive number \( \beta_i \) such that the following inequality holds:

\[ |\dot{V}_i(t_0 + \tau, t_0) - \alpha_i \ddot{V}_i(t_0)| < \beta_i |\ddot{V}_i(t_0)| \] (5.27)

thus, with the chosen parameter \( \beta_i \) which satisfies (5.27), (5.26) becomes
\[ \dot{V}_i(t_0 + \tau) < 0 \]  \hspace{1cm} (5.28)

The analysis from (5.23) to (5.28) emphasizes that the proposed learning controller is capable of reducing \( \dot{V}_i(t) \) from the positive value to the negative one. In other terms, the closed-loop trajectory is always driven into the stable region, in which the virtual sliding variable is guaranteed to reach and be retained on the sliding surface; the system dynamics then asymptotically converge to zero in the sliding mode.

- **For the case that** \( \dot{V}_i(t - \tau) < 0 \):

One can rewrite (5.22) as:

\[ \dot{V}_i(t) < \frac{1}{M_i} \left| \hat{V}_i(t - \tau) \right| + \dot{V}_i(t - \tau) - \alpha_i \hat{V}_i(t - \tau) - \beta_i \left| \hat{V}_i(t - \tau) \right| \]  \hspace{1cm} (5.29)

By using (5.12) in (5.29), one can obtain

\[ \dot{V}_i(t) < \frac{1}{M_i} \left| \hat{V}_i(t - \tau) \right| + \dot{V}_i(t - \tau) + \gamma_i \left| \dot{V}_i(t - \tau) \right| - \alpha_i \hat{V}_i(t - \tau) - \beta_i \left| \hat{V}_i(t - \tau) \right| \]

\[ < \left( \frac{1}{M_i} - 1 + \gamma_i + \alpha_i \right) \left| \dot{V}_i(t - \tau) \right| - \beta_i \left| \hat{V}_i(t - \tau) \right| \]  \hspace{1cm} (5.30)

With the chosen parameter \( \alpha_i \) which satisfies (5.24), (5.30) becomes

\[ \dot{V}_i(t) < 0 \]  \hspace{1cm} (5.31)

In summary, the stability criterion \( \dot{V}_i(t) < 0 \) is satisfied which ensures the asymptotic convergence of the sliding variable \( \sigma_i(t) \), and thus guarantees the stability of the closed-loop dynamics on the sliding mode surface.

**Remark 5.4:** It is noted that the inequality (5.20) is termed the Lipschitz-like condition, which mathematically states that the difference between the current value of the gradient of the Lyapunov function and its most recent value is very small as the time
delay $\tau$ is sufficiently small. Owing to this condition, the uncertainties are all embedded in the left-hand side of (5.20), and thus no information on the bounds of the uncertainties is required for the controller design. Moreover, as the learning algorithm is adopted, there is no sign function involved in the controller. Hence, the chattering phenomenon is successfully eliminated.

5.4 Simulation Results

In this section, the simulation results for the premium and ordinary traffic will be shown separately in comparison with the conventional SMC and second-order sliding mode controller (SOSMC).

The router parameters are chosen as: $C_{server} = 220000, \lambda_{max} = 200000, \tau_p = 0.02s, \tau_r = 0.06s$. The traffic incoming rate of the premium buffer is set to be a square waveform shown in Figure 5.2.

For comparison purpose, we first consider using the conventional SMC as in [121]

$$
C_p(t) = \frac{-p(t) + \dot{x}^r_p(t)}{-x_p(t)} + 25000 \tanh(\sigma_p) + 1
$$

$$
\dot{C}_p(t) = -C_p(t) \left( \dot{p}(t) - C_p(t) \frac{\dot{x}_p(t)}{x_p(t) + 1} - x_p^r(t) - D_{2p} + \gamma_p \sigma_p + \rho_p \sigma_p \right) + \epsilon_p \text{sign}(\sigma_p)
$$

Secondly, we employ the SOSMC designed in [114]:

$$
\dot{C}_r(t) = \frac{\dot{x}_r(t)}{(x_r(t) + 1)^2} + \dot{C}_r(t) - \frac{x_r(t)}{x_r(t) + 1} + \dot{x}_r^r(t) + C_r(t) + D_{2r} - \gamma_r \sigma_r - \rho_r \sigma_r
$$

$$
\dot{C}_p(t) = \frac{\dot{x}_p(t)}{(x_p(t) + 1)^2} + \dot{C}_p(t) - \frac{x_p(t)}{x_p(t) + 1} - x_p^r(t) - D_{2p} + \gamma_p \sigma_p + \rho_p \sigma_p
$$

$$
\dot{C}_r(t) = \frac{\dot{x}_r(t)}{(x_r(t) + 1)^2} + \dot{C}_r(t) - \frac{x_r(t)}{x_r(t) + 1} + \dot{x}_r^r(t) + C_r(t) + D_{2r} - \gamma_r \sigma_r - \rho_r \sigma_r
$$

$$
\dot{C}_p(t) = \frac{\dot{x}_r(t)}{(x_r(t) + 1)^2} + \dot{C}_r(t) - \frac{x_r(t)}{x_r(t) + 1} - x_r^r(t) - D_{2r} + \gamma_r \sigma_r + \rho_r \sigma_r
$$

$$
\dot{C}_r(t) = \frac{\dot{x}_r(t)}{(x_r(t) + 1)^2} + \dot{C}_r(t) - \frac{x_r(t)}{x_r(t) + 1} + \dot{x}_r^r(t) + C_r(t) + D_{2r} - \gamma_r \sigma_r - \rho_r \sigma_r
$$
where $d_p = 0.05s, d_r = 0.1s, D_{2p} = D_{2r} = 10, \gamma_l = 80, \varepsilon_l = 150, \rho_l = 6250$.

Figure 5.3.a – Figure 5.3.d show the simulation results of the buffer length and control input for the premium and ordinary traffic, respectively, using the conventional SMC (5.32) and (5.33). Also, the performance of the SOSMC (5.34) and (5.35) is shown in Figure 5.4.a – Figure 5.4.d, respectively. It is seen that the SOSMC performs better than the conventional SMC in terms of having shorter settling time and chattering-free control inputs.

We now turn to the proposed SMLC designed as in (5.6)-(5.9) with the parameters $\alpha_p = 0.987, \beta_p = 0.985$ and $\alpha_r = 0.978, \beta_r = 0.972$. The simulation results of the buffer length and control input for the premium and ordinary traffic have been shown in Figure 5.5.a – Figure 5.5.d, respectively. The proposed control scheme exhibits an excellent performance with both chatter-free characteristic and strong robustness with respect to the disturbance and the time-varying latency. Firstly, it is clearly seen that the stability convergence rate of the tracking error between the system dynamics and the desired reference dynamics in the proposed SMLC scheme is faster than that of the conventional SMC and SOSMC shown in Figure 5.3.a – Figure 5.3.d and Figure 5.4.a – Figure 5.4.d, respectively. Secondly, the simulation results have shown that the network using SOSMC exhibits more overshoot and oscillation than that using the proposed SMLC. The proposed control scheme not only significantly reduces the overshoot but also shortens the settling time. As a result, the SMLC technique can effectively compensate for data packet losses and make better use of the link capacity. Moreover, the design of the SMLC controller does not require any prior knowledge of the bounds of the uncertainties normally required in conventional SMC approaches. The results have confirmed the superior performance offered by the proposed SMLC technique.

**Remark 5.5:** In this chapter, instead of using sinusoidal signals as seen in [114], we have used all step signals for numerical illustration. Especially, the incoming rate of the premium traffic, which acts as the disturbance to the system, is set at a high frequency as to reflect the most rigorous networking conditions. As a consequence, there may exist some spikes in terms of control effort on the occasions of resonant influences between sudden changes to the reference signals and the disturbance. However, unlike mechanical systems whose control input rates are essentially required to be smooth and bounded in order not to destroy an actuator and mechanism, in networking, rate control...
is merely a numerical adaptation and thus such control signals can be adopted in real networking systems. Therefore, the merits of the proposed SMLC and its application to the control of networking systems can be justified.

![Figure 5.2. Incoming rate of premium traffic.](image)

![Figure 5.3.a. Buffer length of premium traffic (Conventional SMC).](image)
Figure 5.3.b. Control signal of premium traffic (Conventional SMC).

Figure 5.3.c. Buffer length of ordinary traffic (Conventional SMC).

Figure 5.3.d. Control signal of ordinary traffic (Conventional SMC).
5.4 Simulation Results

Figure 5.4.a. Buffer length of premium traffic (SOSMC).

Figure 5.4.b. Control signal of premium traffic (SOSMC).

Figure 5.4.c. Buffer length of ordinary traffic (SOSMC).
Figure 5.4.d. Control signal of ordinary traffic (SOSMC).

Figure 5.5.a. Buffer length of premium traffic (SMLC).

Figure 5.5.b. Control signal of premium traffic (SMLC).
5.5 Conclusion

This chapter is concerned with the SMLC design to address the congestion control problem in DiffServ networks. The superiority of the proposed congestion controller guarantees both robust stability and the chatter-free property of the closed-loop system under rigorous networking conditions. The simulation results demonstrate that in both premium and ordinary services transient response and oscillatory behavior have been greatly improved by utilizing the proposed SMLC when compared to the other available control approaches in the literature, which result in better link utilization, lower packet
loss and smaller queue fluctuation. Future research will involve investigation of our proposed control methodology for large scale networks.
Chapter 6

Robust Sliding Mode Based Learning Control for Steer-by-Wire Systems in Modern Vehicles

In this chapter, a robust sliding mode learning control (SMLC) scheme is developed for steer-by-wire (SbW) systems. It is shown that an SbW system with uncertain system parameters and unknown external disturbance from the interactions between the tires and the variable road surface can be modelled as a second-order system. A sliding mode learning controller can then be designed to drive both the sliding variable and the tracking error between the steered front-wheel angle and the hand-wheel reference angle to asymptotically converge to zero. The proposed SMLC scheme exhibits many advantages over the existing schemes, including: (i) no information about vehicle parameter uncertainties and self-aligning torque variations is required for controller design; and (ii) the control algorithm is capable of efficiently adjusting the closed-loop response based on the most recent history of the closed-loop stability and ensuring a robust steering performance. Both simulations and experiments are presented to show the excellent steering performance and the effectiveness of the proposed learning control methodology.

6.1 Introduction

STEER-BY-WIRE (SbW) systems have been considered to serve the next generation of road vehicles for improving steering performance, enhancing vehicles’ manoeuvrability, and also providing drivers with better comfort and proactive safety. The distinct characteristics of SbW systems are as follows: (i) The mechanical linkage used to connect the hand wheel with the steered front wheels in conventional steering systems is removed, (ii) an ac or dc motor is adopted to steer the front-wheels
so that the steered front-wheel angles closely track the hand-wheel reference angle, and
(iii) another motor is coupled with the hand-wheel shaft to provide a driver with a
feeling of the interactions between the front tires and the road surface.

To date, the SbW control systems have been intensively studied in the automotive
industry [122-131]. The bottleneck for the design of high-quality SbW control systems
is how the effects of uncertain vehicle dynamics and highly nonlinear self-aligning
torque variations, due to different road conditions, on the steering performance can be
eliminated. In [132–135], a number of control methods have been proposed for SbW
systems with the strategies that the controllers are designed based on the estimated road
surface conditions and chassis sideslip angle. In fact, a good steering performance by
using these schemes for SbW systems can be achieved only when accurate estimates of
the road surface conditions and the chassis sideslip angle can be obtained. In [136-140],
a few adaptive control techniques for SbW systems have been developed for improving
the steering performance. However, how the system states, uncertain parameters, and
unknown lateral forces can be accurately online estimated under varying road
environments to ensure a robust steering performance is still an open issue.

Recently, sliding mode control (SMC) has been employed in SbW control systems
in [141-147]. It has been seen that the SbW systems equipped with the SMC are able to
eliminate the effects of SbW system uncertainties and unknown complex road
conditions on the steering performance by using the upper and lower bound information
of uncertainties. Although the steering performance with the SMC strategy is better than
the performances of other existing control techniques, how chattering in control signals
should be eliminated and how the constraints on the system uncertainties could be eased
without degenerating the robustness and convergence performance need to be further
studied.

In this chapter, we will develop a new sliding mode learning control (SMLC)
scheme for SbW systems based on [84-85] to improve the steering performance against
the uncertain system dynamics and the unknown road environment. It is worth noting
that the proposed SMLC algorithm in this chapter is based on the concept of the
“Lipschitz-like condition” recently proposed in [84-85]. The Lipschitz-like condition
describes the continuity of the gradient of the Lyapunov functions for a large class of
systems. It states that a continuous-time system is a Lipschitz-like system if the
difference between the current value of the gradient of a Lyapunov function and its most
recent value is very small as the sampling period is sufficiently small. As shown in [84-85], the controller designs based on the Lipschitz-like condition no longer require prior information about system uncertainties since the uncertain system dynamics are all embedded in the Lipschitz-like condition. It will be shown from the theoretical discussions and experimental results in this chapter that, unlike the conventional SMC, the proposed SMLC is continuous in the state space and no chatter occurs in the closed-loop system. In addition, the analysis on the convergence and stability of the closed-loop SbW system equipped with the proposed SMLC algorithm will show that the new learning control is capable of driving the closed-loop dynamics from an unstable domain to a stable domain and ensuring that both the sliding variable and the tracking error can asymptotically converge to zero.

The remainder of this chapter is organized as follows. In Section 6.2, the modelling of SbW systems and the learning structure of the proposed SMLC are formulated. In Section 6.3, the analysis on the asymptotic error convergence and stability of the closed-loop SbW systems is presented in detail. Numerical simulations and experimental results are presented in Section 6.4 and 6.5, respectively, to illustrate the advantages of the proposed control scheme. Lastly, Section 6.6 gives a conclusion and discusses further work.

### 6.2 Problem Formulation

This section presents some underlying preliminaries associated with dynamics of SbW systems, AC motor control, and design of the proposed SMLC scheme.

#### 6.2.1 Dynamics of SbW Systems

A working scheme of a SbW system can be sketched out as in Figure 6.1, where $\theta_h$, $\delta_f$, and $\delta_{sm}$ are respectively rotational angles of hand wheel, front wheels, and steering motor shaft, $\tau_h$, $\tau_{hm}$, and $\tau_{sm}$ are respectively torques generated by driver, hand-wheel motor, and steering motor, and $\tau_e$ is the self-aligning torque.
It is shown that the mechanical linkage used to connect the hand wheel with the steered front wheels in conventional steering systems has been substituted by two motors, i.e., the steering motor and the hand-wheel motor. The role of the steering motor is to steer the front wheels and ensure that the front-wheel angle can closely track the hand-wheel angle.

On the other hand, the hand-wheel motor provides a driver with a reaction torque from the interactions between the vehicle tires and road surface. For simplicity, we assume that the backlash between the rack and pinion gear teeth is zero. Thus, the following relationships hold [132, 146]:

\[
\frac{\delta_f}{\delta_{sm}} = \frac{\delta_f}{\delta_{sm}} = \frac{\delta_f}{\delta_{sm}} = \frac{1}{\sigma r} = \frac{\tau_{12}}{\tau_5}
\]

(6.1)

where \(\tau_{12}\) is the torque exerted on the steering motor shaft by the front wheels, \(\tau_s\) is the torque transmitted to the steering arms of the front wheels by the steering motor through the rack and pinion gearbox, \(\sigma\) is the rack and pinion system’s gear ratio, and \(r\) is a scale factor accounting for the conversion from the linear motion of the rack to the rotation of the front wheels.
The rotation of the steering motor shaft is described by the following dynamical equation [146, 148]:

\[ J_{sm} \ddot{\delta}_{sm} + B_{sm} \dot{\delta}_{sm} + \tau_{12} = \tau_{sm} \] (6.2)

where \( J_{sm} \) and \( B_{sm} \) are the moment of inertia and the viscous friction of the steering motor, respectively.

Also, the rotation of the steered front wheels about their vertical axes crossing the wheel centers can be described by [135]:

\[ J_{sw} \ddot{\delta}_{f} + B_{sw} \dot{\delta}_{f} + \tau_{e} + \tau_{F} = \tau_{s} \] (6.3)

where \( J_{sw} \) and \( B_{sw} \) are the moment of inertia and the viscous friction of the steering front-wheels, respectively, \( \tau_{F} \) is the Coulomb friction defined as

\[ \tau_{F} = F_{s} \text{sign} (\dot{\delta}_{f}) \] (6.4)

with \( F_{s} \) the Coulomb friction constant.

Using (6.1) and (6.2), we can express \( \tau_{s} \) as

\[ \tau_{s} = k (\tau_{sm} - J_{sm} \ddot{\delta}_{sm} - B_{sm} \dot{\delta}_{sm}) \] (6.5)

where \( k = \sigma r \).

Substituting (6.5) into (6.3) leads to

\[ J_{sw} \ddot{\delta}_{f} + B_{sw} \dot{\delta}_{f} + F_{s} \text{sign} (\dot{\delta}_{f}) + \tau_{e} = k (\tau_{sm} - J_{sm} \ddot{\delta}_{sm} - B_{sm} \dot{\delta}_{sm}) \] (6.6)

By eliminating \( \dot{\delta}_{sm} \) in (6.6) with the aid of (6.1), we obtain
Re-arranging (6.7), we have

\[ J_{eq} \dot{\delta}_f + B_{eq} \dot{\delta}_f + F_s \text{sign}(\dot{\delta}_f) + \tau_e = \tau_{eq} \]  \hspace{1cm} (6.8)

where \( J_{eq}, B_{eq}, \) and \( \tau_{eq} \) are respectively the moment of inertia, the viscous friction, and the drive torque of the equivalent system (6.8), and defined as

\[
J_{eq} = J_s + k^2 J_{sm} \\
B_{eq} = B_s + k^2 B_{sm} \\
\tau_{eq} = k \tau_{sm}.
\]  \hspace{1cm} (6.9)

**Remark 6.1**: It is seen from (6.8) that, although the SbW system contains a few components such as the steered front-wheels, the rack and pinion gearbox, and the steering motor, the integrated SbW system can be modelled by a second-order differential equation. Hence, it is possible to use some advanced control techniques to design controllers to ensure a robust steering performance of road vehicles [146].

**Remark 6.2**: It is known that the variable gear ratio steering (VGRS) has been widely used in many modern vehicles [131, 151-153]. Although the SbW system in this research has a fixed gear ratio, it will be seen from the later discussions that the proposed learning controller is capable of eliminating the impacts of the variable gear ratio on the steering performance since the gear ratio \( \sigma \) via the parameter \( k = \sigma r \) in (6.5)-(6.8) has been embedded in the parameters of the equivalent second-order model.

Figure 6.2 shows the overall control diagram of the SbW system with the SMLC, where \( DR_s \) and \( DR_h \) denote the steering and hand-wheel motors associated with their servo-drivers, respectively, receiving the torque references \( \tau^*_{sm} \) and \( \tau^*_{hm} \) from the SMLC and the proportional-derivative (PD) controller, respectively.
Figure 6.2. SbW system with the SMLC scheme.

Remark 6.3: In Figure 6.2, the hand-wheel motor provides the driver with the feeling of the interactions between the tires and road surface. Thus, a PD controller can be designed for the hand-wheel control system using the error signal between the steered front-wheel angle and the hand-wheel reference angle [130]. Additionally, in this work, the estimated self-aligning torque \( \hat{\tau}_e \) is fed back to the hand-wheel control loop in order to continuously provide the driver with the sensation of the reaction torque after the tracking error vanishes.

Remark 6.4: In this research, two permanent magnet synchronous motors (PMSM) are used as the steering motor and the hand-wheel motor, respectively. Compared with the control of DC motors based SbW systems [123, 126-128], the control of the AC motors based SbW systems by its nature exhibits the more complex features in terms of designing control architectures and handling system nonlinearities. A detailed study will be seen in the later sections.

6.2.2 Steering AC Motor Torque Perturbation

The model of the PMSM is described as [149-150]:

\[
\frac{d i_d}{dt} = \frac{1}{L_d} \left( v_d - R_s i_d + \omega_e L_q i_q \right) \tag{6.10}
\]
\[
\frac{di_q}{dt} = \frac{1}{L_q} \left( v_q - R_s i_d - \omega_e L_d i_d - \omega_e L_{md} i_d \right) \quad (6.11)
\]
\[
\tau_{sm} = \frac{3}{2} p L_{md} I_{fd} i_q + (L_d - L_q) i_d i_q \quad (6.12)
\]

where \( v_d \) and \( v_q \) are the \( d \) - and \( q \) - axis stator voltages, \( i_d \) and \( i_q \) are the \( d, q \) - axis stator currents, \( L_d \) and \( L_q \) are the \( d, q \) - axis inductances, \( R_s \) and \( \omega_e \) are the stator resistance and the rotor electrical speed, \( L_{md} \) is the \( d \) - axis mutual inductance, \( I_{fd} \) is the equivalent \( d \) - axis magnetizing current, and \( p \) is the number of pole pairs, respectively.

If \( i_d \) is adjusted to zero ( \( i_d \equiv 0 \) ), (6.12) becomes
\[
\tau_{sm} = \frac{3}{2} p \psi_{sm} i_d \quad (6.13)
\]

where \( \psi_{sm} = L_{md} I_{fd} \) is the \( d \)-axis flux linkage due to the permanent magnet.

By properly controlling \( i_d \) and \( i_q \) in the current control loop of the motor control system, one can generate the actuation torque required to drive the front-wheels to closely track the hand-wheel reference angle. In fact, however, there always exist disturbances and noises due to both the flux linkage perturbation and the current control error which might be, without loss of generality, expressed as follows:
\[
\psi_{sm} = \psi_{sm}^* + \Delta \psi_{sm} \quad (6.14)
\]
\[
i_q = i_q^* + \Delta i_q \quad (6.15)
\]

where \( \psi_{sm}^* \), \( \Delta \psi_{sm} \) denote the nominal value and the perturbation of the \( d \) - axis flux linkage, and \( i_q^*, \Delta i_q \) denote the reference and the control error of the \( q \) - axis current control loop, respectively.

Using (6.14) and (6.15), we re-write (6.13) as:
\[
\tau_{sm} = \tau_{sm}^* + \Delta \tau_{sm} \quad (6.16)
\]
where
\[
\Delta \tau_{sm} = \frac{3}{2} p \left( \psi_{dm}^* \Delta i_q + i_q^* \Delta \psi_{dm} + \Delta i_q \Delta \psi_{dm} \right) \tag{6.17}
\]
represents the lumped torque perturbation and
\[
\tau_{sm}^* = \frac{3}{2} p \psi_{am}^* i_q^* \tag{6.18}
\]
denotes the nominal torque reference signal for the steering motor provided by the SMLC.

Substituting (6.16) in (6.8) leads to
\[
J_{eq} \ddot{\delta}_f + B_{eq} \dot{\delta}_f + \tau_i = k \tau_{sm}^* \tag{6.19}
\]
where the lumped uncertainty \( \tau_i \) can be expressed as:
\[
\tau_i = \tau_e + F_e \text{sign}(\dot{\delta}_f) - k \Delta \tau_{sm} \tag{6.20}
\]

For further analysis, (6.19) can be expressed in the state space form as follows:
\[
\begin{align*}
\dot{z}_1 &= z_2 \\
\dot{z}_2 &= a(t) + bu(t)
\end{align*} \tag{6.21}
\]

where
\[
\begin{align*}
z &= [z_1 \ z_2]^T = [\delta_f \ \dot{\delta}_f]^T, \quad u(t) &= \tau_{sm}^*, \\
b &= \frac{k}{J_{eq}}, \quad a(t) = -\frac{1}{J_{eq}} \left( B_{eq} \dot{\delta}_f + \tau_i \right)
\end{align*} \tag{6.22}
\]

**Remark 6.5:** The lumped uncertainty \( \tau_i \) including the self-aligning torque, the Coulomb friction, and the steering motor torque perturbation is bounded but unknown. It will be
shown that the prior knowledge of the bounds of the lumped uncertainty $\tau_l$ is not required in the proposed controller design. Such an advantage makes the proposed control scheme exhibit a strong robustness against uncertainties.

### 6.2.3 Sliding Mode Learning Control

The tracking error is defined as:

$$
\varepsilon(t) = \delta_f(t) - \frac{1}{N_\theta} \theta_h(t) = \delta_f(t) - \theta_{hr}(t)
$$

(6.23)

where the parameter $N_\theta$ denotes the ratio between the hand-wheel angle $\theta_h$ and the steering angle $\delta_f$.

A sliding variable is then defined as:

$$
\sigma(t) = \dot{s}(t) + \lambda \varepsilon(t)
$$

(6.24)

where $\lambda > 0$.

Thus, the time derivative of (6.24) is expressed as:

$$
\dot{s}(t) = a(t) + bu(t) - \dot{\theta}_{hr}(t) + \lambda (\delta_f(t) - \dot{\theta}_{hr}(t)) = f(t) + bu(t)
$$

(6.25)

where

$$
f(t) = a(t) - \dot{\theta}_{hr}(t) + \lambda (\delta_f(t) - \dot{\theta}_{hr}(t))
$$

(6.26)

In this chapter, the sliding mode learning controller is proposed as follows:

$$
u(t) = u(t - \tau) - \Delta u(t)
$$

(6.27)

where the correction term $\Delta u(t)$ is defined as:
\[ \Delta u(t) = \begin{cases} \frac{1}{b s(t)} (\alpha \hat{\nu}(t-\tau) + \beta |\hat{\nu}(t-\tau)|) & \text{for } s(t) \neq 0 \\ 0 & \text{for } s(t) = 0 \end{cases} \quad (6.28) \]

\( \tau \) is the time delay, \( \hat{\nu}(t-\tau) \) is the estimate of the first-order derivative of the Lyapunov function candidate \( V(t-\tau) = 0.5 s(t-\tau)^2 \), defined as:

\[ \hat{\nu}(t-\tau) = \frac{V(t) - V(t-\tau)}{\tau} \quad (6.29) \]

and the control parameters \( \alpha \) and \( \beta \) to be determined.

In the next section, the asymptotic convergence and the stability analysis of the proposed SMLC are discussed in detail.

### 6.3 Convergence Analysis

**Theorem 6.1**: Consider the SbW system in (6.19), if the control input in (6.27) with the correction term in (6.28) is used, the tracking error \( \varepsilon(t) \) defined in (6.23) will asymptotically converge to zero.

**Proof**: Please refer to the proof in Theorem 3.1 of Chapter 3.

### 6.4 Numerical Simulations

In this chapter, we consider an electric vehicle with its bicycle model. The dynamics of the yaw motion of the vehicle are given by [136, 155]:

\[
\begin{bmatrix}
\dot{\beta} \\
\gamma
\end{bmatrix} =
\begin{bmatrix}
\frac{-C_f - C_r}{m V_{CG}} & \frac{C_r l_r - C_f l_f}{I_z} \\
\frac{C_r l_r - C_f l_f}{l_z V_{CG}} & -1 + \frac{m V_{CG}^2}{m V_{CG}^2 - C_f l_f^2 - C_r l_r^2}
\end{bmatrix}
\begin{bmatrix}
\beta \\
\gamma
\end{bmatrix}
+ \begin{bmatrix}
\frac{C_f}{C_f l_f} \\
\frac{C_r}{C_f l_f}
\end{bmatrix} \delta_f \quad (6.30)
\]

where the vehicle body slip angle \( \beta \) at the centre of gravity (CG) and the yaw rate \( \gamma \) are the state variables, respectively, the constant longitudinal velocity \( V_{CG} \) is about 10 m/s.
at the CG, the vehicle mass $m$ is 2000 kg, the moment of inertia $I_z$ of the vehicle about the CG is 1300 kg.m$^2$, the distances of the front-wheel and rear-wheel axles from the CG are $l_f = 1.2$ m and $l_r = 1.05$ m, respectively, and the front and rear cornering stiffness coefficients $C_f$ and $C_r$ are respectively chosen as:

$$C_f = C_r = \begin{cases} 4000 \text{(snowy road)} & \text{for } t \leq 15 \text{ s} \\ 12000 \text{(dry road)} & \text{for } t > 15 \text{ s} \end{cases} \quad (6.31)$$

Please note that the dynamics of the yaw motion of the vehicle in (6.30) are derived under the assumption that the tire slip angle is less than four degrees. Thus the nonlinear self-aligning torque $\tau_e$ in the linear region can be approximated as follows [155]:

$$\tau_e = -C_f(t_p + t_m) (\beta + \frac{\gamma I_f}{V_{CG}} - \delta_f) \quad (6.32)$$

where $t_p (= 0.023 \text{ m})$ and $t_m (= 0.016 \text{ m})$ are the pneumatic and mechanical trails, respectively.

The nominal parameters of the dynamical model of the steered front-wheels in (6.3) are chosen as $J_{sw} = 3.8 \text{ kg.m}^2$, $J_{sm} = 0.0035 \text{ kg.m}^2$, $B_{sw} = 10 \text{ Nms/rad}$, and $B_{sm} = 0.018 \text{ Nms/rad}$, the parameters $\sigma$ and $r$ in (1) are set as $\sigma = 3$ and $r = 6$, and the Coulomb friction constant in (6.4) is chosen as $F_s = 30 \text{ Nm}$. The hand-wheel reference angle is generated by the following function:

$$\theta_{hr}(t) = 0.4 \sin(0.7\pi t) \text{ rad} \quad (6.33)$$

The sampling period is chosen to be equal to the time delay $\Delta T = \tau = 0.001 \text{ s}$, and the initial value of the front-wheel angle $\delta_f(0) = -0.1 \text{ rad}$. To simulate the effects of the self-aligning torque variations due to different road environments on the steering performance, two different road conditions are considered as in (6.31), with a snowy road surface for the first 15 seconds and a dry road surface for the next 15 seconds, respectively.
6.4 Numerical Simulations

Figure 6.3. Transient responses of SMLC:
(a) sliding variable, (b) tracking performance.

First, the control parameters in (6.28) are chosen as $\alpha = 0.95$ and $\beta = 0.395$. The transient responses of the sliding variable and the tracking performance in the first 5 seconds have been shown in Figure 6.3.a and Figure 6.3.b, respectively. It is seen that the asymptotic convergences of both the sliding variable and the tracking error of the closed-loop SbW system with the proposed SMLC have been achieved.

Next, the effectiveness of the proposed SMLC is illustrated in comparison with the H-infinity control and the conventional SMC. The H-infinity controller for the SbW system is designed as follows [158]:

$$
\tau_{sm}^* = \frac{J_{eq}}{k} \left[ \frac{B_{eq}}{J_{eq}} \ddot{\delta}_f + \dot{\theta}_{hr} - c_1 \dot{\epsilon} - c_2 \epsilon + L \left[ \begin{array}{c} \epsilon \\ \dot{\epsilon} \end{array} \right] \right]
$$

(6.34)

where $L = -\begin{bmatrix} 0 \\ \phi^2 \end{bmatrix}P$, the control parameters $\phi, c_1$ and $c_2$ are set to 0.1, 1, and 150, respectively, and the matrix $P$ is given by

$$
P = \begin{bmatrix} 25.2224 & 0.0103 \\ 0.0103 & 0.1669 \end{bmatrix}
$$

(6.35)

The conventional sliding mode control with the boundary layer (BL-SMC) for the SbW system is given by [146-147]:

\[
\text{[Equation]} \]
\[
\tau_{sm}^* = -\frac{1}{k} \text{sat}(s) \left[ J_{eq} \left( \tilde{\theta}_{hr} + \lambda |\varepsilon| \right) + B_{eq} |\dot{\delta}_r| + \tau_i \right]
\]  
(6.36)

where \( \lambda = 12 \), the \( \text{sat}(s) \) function is defined as

\[
\text{sat}(s) = \begin{cases} 
\text{sign}(s) & \text{for } |s| \geq 0.8 \\
\frac{s}{0.8} & \text{for } |s| < 0.8 
\end{cases}
\]  
(6.37)

and the upper bounds of \( \tau_i, \tilde{\theta}_{hr}, J_{eq}, B_{eq} \) are chosen as \( \tau_i = 8 \text{ Nm}, \tilde{\theta}_{hr} = 10 \text{ rad/s}^2, \)
\( J_{eq} = 10 \text{ kg.m}^2, \) and \( B_{eq} = 30 \text{ Nms/rad}, \) respectively.

![Figure 6.4. Tracking errors among different control techniques.](image)

Figure 6.4 shows a comparison of the tracking errors between the steered front-wheel angle and the hand-wheel reference angle of the SbW system with the proposed SMLC (6.27), the H-infinity control (6.34), and BL-SMC (6.36), respectively. It is seen that the H-infinity controller is unable to handle a large variation of road surface conditions, the BL-SMC, however, has greatly improved the tracking performance and behaves with a strong robustness against the variation of the road conditions. Furthermore, the proposed SMLC has demonstrated the best tracking performance with the smallest tracking error.
6.5 Experimental Results

Figure 6.5 shows the SbW platform in the Mechatronics Laboratory at Swinburne University of Technology. Two AC motors (Mitsubishi HF-SP102) driven by the servo drivers (Mitsubishi MR-J3-100A) are used as the steering motor and the hand-wheel motor, respectively, in the SbW platform. A gearhead of 10:1 ratio is adopted to amplify the steering motor drive torque. Two angle sensors are installed to measure the hand-wheel and front-wheel angles, respectively.

The Advantech PCI 1716 multifunction card is interfaced to the desktop computer for real-time control with the Real-time Windows Target toolbox in Matlab/Simulink. The nominal parameters of the motors, the rack and pinion gearbox of the SbW platform are the same as the ones in the simulation section.

For a comparison of the steering performances with different control techniques, we use the signal in (6.33) as the reference signal for the steered front-wheels to follow. The sampling period is selected as equal to the time delay $\Delta T = \tau = 0.001\ s$.

It has been noted that the dynamical model of the yaw motion of the vehicle in the simulation section is derived in the linear region of the nonlinear self-aligning torque $\tau_e$. However, the proposed SMLC has no constraint on the operating region of the nonlinear self-aligning torque $\tau_e$. In order to test the robustness of the SbW system against the changes of the self-aligning torque, a voltage signal $V_{\tau_e}$ is input to the
steering motor to produce a nonlinear torque disturbance for modelling the self-aligning torque \( \tau_e \) [156-157]:

\[
V_{\tau_e} = \begin{cases} 
\eta_1 \tanh(2\delta_f) & \text{for } t \leq 15 \text{ s} \\
\eta_2 \tanh(\delta_f) & \text{for } t > 15 \text{ s}
\end{cases}
\]  

(6.38)

where \( \eta_1 = 1 \) and \( \eta_2 = 5.6 \) to ensure that the values of the self-aligning torque \( \tau_e \) in the first 15 seconds and in the second 15 seconds are different.

Figure 6.6.a – 6.6.c show the performance of the SbW system with the H-infinity controller (6.34). It is seen that, after the road surface condition changed \( t > 15 \text{ s} \), the H-infinity controller was unable to drive the front-wheels to follow the reference signal well.
Figure 6.6. H-infinity control of the SbW system: (a) Steering performance, (b) Tracking error, (c) Control input.

Figure 6.7.a–6.7c show the steering performance, tracking error, and control input, respectively, with the BL-SMC (6.36). It is seen that, although the steering performance with the BL-SMC is much better than the one with the H-infinity controller. The steering angle could not track the reference angle well after the road surface condition varied. This is because the pre-set value of the boundary layer parameter in (6.37) was not adjusted for $t > 15$ s, the steady state error was therefore largely increased.
Figure 6.7. BL-SMC of the SbW system: (a) Steering performance, (b) Tracking error, (c) Control input.

Figure 6.8.a–6.8.c show the experimental results of the steering performance, tracking error, and control input, respectively, with the proposed SLMC (6.27) where the control parameters are chosen as $\alpha = 0.5, \beta = 0.1$, respectively. It is seen that the steering performance has been significantly improved with a very small tracking error, compared with the one of the H-infinity control in Figure 6.6.a–6.6.c and the one of the BL-SMC in Figure 6.7.a–6.7.c, respectively. Such an excellent steering performance of the SMLC is largely due to the excellent learning capability of the SMLC that has ensured that the control gain can be effectively adjusted in time as the road surface condition changes.
Figure 6.8. SMLC of the SbW system: (a) Steering performance, (b) Tracking error, (c) Control input.
Now we consider the SMLC for the SbW system with the real reference signal generated by a driver with the hand-wheel in Figure 6.5. The corresponding control parameters and the experiment settings are the same as those in Figure 6.8.a–6.8.c. The hand-wheel model is described by the following equation:

\[ J_h \ddot{\theta}_h + B_h \dot{\theta}_h + \tau_{hm} = \tau_h \]  

(6.39)

where the moment of inertia and the viscous friction of the hand wheel are set as \( J_h = 0.08 \text{ kg.m}^2 \), \( B_h = 0.15 \text{ Nms/rad} \), respectively, and the parameter \( N_\theta \) in (6.23) is chosen as \( N_\theta = 12 \).

The PD controller is designed together with the estimated self-aligning torque \( \hat{\tau}_e \) as an input to drive the hand-wheel motor to generate a torque to model the interactions between the front tires and road surface:

\[ \tau^{*}_{hm} = k_p \varepsilon + k_d \dot{\varepsilon} + \frac{\hat{\tau}_e}{N_\theta} \]  

(6.40)

where the control parameters \( k_p = 15 \) and \( k_d = 2 \).
Figure 6.9. SMLC of the SbW system with a driver’s input: (a) Steering performance, (b) Tracking error, (c) Front-wheel control input, (d) Hand-wheel control input.
The estimated self-aligning torque \( \hat{\tau}_e \) is output by the hand-wheel motor with the corresponding voltage input given by:

\[
V_{\hat{\tau}_e} = \xi \tanh(\delta_f)
\]  

(6.41)

where the parameter \( \xi = 1.2 \).

Figure 6.9.a–6.9.d show the steering performance, tracking error, front-wheel control input, and hand-wheel control input, respectively. It is seen that the proposed SMLC scheme has exhibited an excellent steering performance with a strong robustness against the varying road conditions.

### 6.6 Conclusion

In this chapter, a new SLMC technique has been developed for SbW systems with uncertain dynamics and varying road conditions. It is seen that the developed control scheme is capable of learning the closed-loop dynamics from its history and then driving both the sliding variable and the tracking error to converge to zero asymptotically. Such a learning control process ensures that the strong robust steering performance can be achieved. Both the numerical and experimental results have further confirmed the excellent steering performance of the proposed learning control methodology. Further work on the sliding mode learning-based observers for diagnosis of SbW systems is under the authors’ investigation.
Chapter 7

Robust Sliding Mode Learning Control for Uncertain Discrete-Time MIMO Systems

A robust sliding mode based learning control scheme is newly developed for a class of uncertain discrete-time multi-input multi-output systems. In particular, a recursive-learning controller is designed to enforce the sliding variable vector to reach and remain on the intersection of the sliding surfaces, and the system dynamics are then guaranteed to asymptotically converge to zero on the pre-described sliding manifold with respect to uncertainty. The “Lipschitz-like condition” for sliding mode control systems, which presents an essential property of the continuity of uncertain systems, is further extended to the discrete-time case establishing in this chapter. The appealing attributes of this approach include: (i) knowledge of the bounds of the uncertainties is not required for the controller design, (ii) the closed-loop system exhibits a strong robustness against uncertain dynamics, and (iii) the control scheme enjoys the chattering-free characteristic. Simulation results are given to illustrate the effectiveness of the proposed control technique.

7.1 Introduction

In recent years, sliding mode control (SMC) as a powerful technique has been well investigated and successfully applied for many industrial applications in robust control of linear and nonlinear systems with uncertainties [7, 8, 11, 52, 159]. Especially, studies in discrete-time sliding mode control (DSMC) have been receiving intensive attention due to the widespread use of digital controllers recently [80, 107, 160-169]. Nowadays, with the vast growth of intelligent digital electronic devices, the sampling period is much more reduced that makes the performance of a discrete-time (DT) system
close to its continuous-time (CT) counterpart. However, high sampling frequencies may meanwhile cause some undesired system behaviour and performance deterioration. Therefore, SMC designs in a DT domain need to be further studied in both academia and industry.

In fact, there still exist some drawbacks in the conventional DSMC schemes for uncertain systems. One of the challenging issues is about the chattering problem due to the involvement of a sign function in control inputs, which has greatly restricted the applications of DSMC in many industrial circumstances. As a chattering control signal may excite some undesired high frequency modes, it degenerates system performance and may cause the instability of the closed-loop system. Though the boundary layer technique has been widely used to alleviate the chattering phenomenon [27, 170], the property of zero convergence is sacrificed, since in this case, there no longer exists the ideal sliding mode and the system dynamics are only guaranteed to stay inside the boundary layer. A more recent attempt to alleviate the chattering effect is to adopt the higher order SMC [44-45], which will however result in high computational complexity.

In addition, the conventional DSMC designs require prior knowledge of the bounds of uncertainties that might not be achievable in practice. Many researchers have proposed different techniques for handling the uncertainties. Among these, in [162-164], the multirate output feedback sliding mode control techniques have been introduced wherein the sensor output is sampled at a rate faster than the control input sampling rate. The algorithm would result in the system performance that is very close to that obtained by a continuous-time control algorithm. However, due to the fact that the control input and system output are sampled at different rates, the control scheme may lead to a high complexity of controller design, and also, high sampling rates in turn require much larger memories because of the high volume of data. Also, in [165] a robust output tracking control was proposed for uncertain systems via discrete-time integral sliding mode, yet the zero-error convergence is lost as one step-delayed disturbance value used to estimate the lumped uncertainties can only guarantee the stability in the vicinity of the sliding surface. These above limitations in the conventional DSMC have raised a need for the development of DSMC from the perspective of handling uncertainties and removing the chattering phenomenon in DSMC systems.

More recently, in [84] the sliding mode based learning control was proposed in an effort to effectively handle system uncertainties and external disturbances without
requiring prior knowledge of the bounds of uncertainties in controller designs. Shortly after, this initiative was further developed and applied successfully to different engineering disciplines supported by great simulation and experiment results [87, 107, 154]. This has enabled us to completely remove the requirement of the prior information of the bounds of uncertainties in controller designs, which is one of the main drawbacks remaining in many conventional SMC schemes. Inspired by these pioneering works, in this chapter we have further developed the learning control strategy to serve a large class of dynamic systems with uncertainties extensively studied in a discrete-time framework.

In particular, this chapter is dedicated to develop a robust sliding mode-based learning control for a class of linear discrete-time multi-input multi-output (MIMO) systems with uncertainties. It will be shown that the proposed controller, like the recursive-learning control algorithm [171], consists of a most recent control signal and a learning term. The learning term, based on the most recent stability status of the closed-loop system, is designed to search for the sliding manifold and adjust the stability and convergence of the closed-loop system. More importantly, if the closed-loop system is unstable, the learning term is able to correct the control signals so as to reduce the gradient value of the Lyapunov function from the positive to the negative, and thus drive the closed-loop trajectories to reach and stay on the sliding mode surface. As a result, the desired closed-loop dynamics with both the chattering-free characteristics and zero convergence are achieved. In this chapter, a novel Lipschitz-like condition for SMC systems is studied for discrete-time MIMO systems, which states that the difference between the current value of the gradient of the Lyapunov function and its most recent value is very small as long as the discrete-time system is sufficiently smooth [84, 87, 107, 154]. The merit of using the Lipschitz-like condition is that the uncertainties are able to be embedded in this condition. The knowledge of the bounds of the uncertainties is not, therefore, required for the design of the proposed control scheme. Furthermore, it is seen that the developed controller that effectively employs a recursive learning algorithm exhibits a chattering-free characteristic, meanwhile, it both guarantees the stability of the closed-loop system, and achieves an asymptotic zero convergence regardless of uncertainties.

The remainder of the chapter is organized as follows: In Section 7.2, the discretization of an uncertain CT MIMO system and the design of the robust discrete-time sliding mode-based learning control (DSMLC) are formulated. In Section 7.3, the
convergence analysis of the closed-loop system with the proposed learning control scheme is discussed in detail. Some important remarks are also highlighted in Section 7.2 and Section 7.3. In Section 7.4, numerical simulations are given to illustrate the effectiveness of the proposed control technique in comparison with conventional DSMC techniques. Finally, Section 7.5 draws a conclusion and future work.

### 7.2 Problem Formulation

This section is dedicated to give some preliminaries about the discretization of a MIMO system and the design of the proposed learning controller for the discretised system.

#### 7.2.1 Discretization of Continuous-Time MIMO Systems

Let us first consider a linear CT MIMO system with uncertainties as:

\[
\dot{x}(t) = (A + \Delta A(t))x(t) + (B + \Delta B(t))u(t) + f(t) \\
y(t) = Cx(t)
\]

where \(x(t) \in \mathbb{R}^n, u(t) \in \mathbb{R}^m, y(t) \in \mathbb{R}^p, f(t) \in \mathbb{R}^l\) are vectors of the system states, control inputs, output signals, and exogenous disturbances, respectively; \(A \in \mathbb{R}^{n \times n}, B \in \mathbb{R}^{n \times m}, \) and \(C \in \mathbb{R}^{p \times n}\) are constant matrices; the matrices \(\Delta A(t) \in \mathbb{R}^{n \times n}\) and \(\Delta B(t) \in \mathbb{R}^{n \times m}\) represent unknown parametric uncertainties.

Before we proceed further, the following definition and assumptions are made throughout the chapter.

**Definition 7.1** [172-173]: The magnitude of a variable \(\sigma\) is said to be \(O(T^n)\) if and only if

\[
\lim_{T \to 0} \frac{\sigma}{T^n} \neq 0, \quad \lim_{T \to 0} \frac{\sigma}{T^{n+1}} = 0
\]

where \(n\) is an integer and denote \(O(T^0) = O(1)\).

**Remark 7.1**: Associated with the above definition, provided that \(\sigma_1, \sigma_2 \in O(T^n), \sigma_3 \in O(T^{n+1})\) and the sampling period \(T\) is sufficiently small, the following approximations are always valid:
7.2 Problem Formulation

- \( \sigma_1 + \sigma_2 = O(T^n) + O(T^n) \approx O(T^n) \) for \( \sigma_1 + \sigma_2 \neq 0 \).
- \( M_0 \sigma_1 = O(1), O(T^n) \approx O(T^n) \), where \( M_0 \neq 0 \) is a finite number.
- \( \sigma_1 \gg \sigma_3 \) or \( \sigma_1 + \sigma_3 = O(T^n) + O(T^{n+1}) \approx O(T^n) \).
- If \( \sigma_1 \) is a vector or matrix, then \( \sigma_1 \in O(T^n) \) also implies \( \|\sigma_1\| \in O(T^n) \), which states that \( \sigma_1 \) lies in the small region, \( \|\sigma_1\| < \varepsilon \in O(T^n) \), where \( \|\cdot\| \) denotes the Euclidian norm, and \( \varepsilon > 0 \) is arbitrarily small as \( T \) is sufficiently small.

Assumption 7.1: The system \((A, B, C)\) is controllable and observable.

Assumption 7.2: The parametric uncertainties and the exogenous disturbances satisfy the so-called matching condition [171], that is to say, there exist matrices of appropriate dimension \( D_a, D_b \) and a vector \( d_f(t) \) such that

\[
\Delta A = BD_a, \Delta B = BD_b, f(t) = Bd_f(t)
\] (7.2)

Remark 7.2: In general, the design of SMC consists of two steps:

- Design a sliding manifold such that in the sliding mode, system response acts like the desired dynamics.
- Design the control law in order to ensure the sliding mode is reached and sustained thereafter.

It is noted that Assumption 7.1 and Assumption 7.2 are widely used in the first step of the SMC design where a sliding manifold is pre-described by the designer with desired sliding motion. In other words, the controllability of the system is to enable the arbitrary assignment of the closed-loop eigenvalues, while the matching condition is to guarantee the insensitivity and asymptotic zero convergence of the system during sliding mode with respect to uncertainties. This will be pointed out in Theorem 7.1. Even for a general system, a linear transformation can be found to change the coordinates such that the induced system is controllable and observable. Interested readers can explore this technique in many existing references [174-175]. For the sake of simplicity, this chapter is mainly focused on designing a sliding mode controller for an uncertain discrete-time MIMO system that satisfies the above assumptions.

Based on Assumption 7.2, let us denote \( f(t) = D_a x(t) + D_b u(t) + d_f(t) \) as the lumped uncertainty of the CT system.
Through zero-order-hold (ZOH), $u(t) = u[k]$ over the time interval $[kT, (k + 1)T)$, where $T > 0$ is the sampling period; the DT representation of the dynamic system (1) can be obtained as:

$$x[k + 1] = \phi x[k] + \Gamma u[k] + d[k]$$
$$y[k] = C x[k]$$

(7.3)

where $\phi = e^{AT}$, $\Gamma = \left( \int_0^T e^{A\lambda} d\lambda \right) B$, and the generalized uncertainty $d[k] = \int_0^T e^{A\lambda} B f_1((k + 1)T - \lambda)d\lambda$.

**Remark 7.3**: As can be seen from (7.1), both system parameter uncertainties and external disturbances have been taken into consideration. Following the commonly used method to model uncertain dynamics from (7.1) to (7.3), these uncertainties can then be generalized and represented by a lumped uncertainty $d[k]$ during the discretization process. Therefore, it is worth noting that, without loss of generality, the unmodeled dynamics considered in this chapter well represent a wide class of uncertainties existing in real circumstances.

**Remark 7.4**: If $(A, B, C)$ is controllable and observable, the system $(\phi, \Gamma, C)$ is also controllable and observable for almost all choices of $T$ [8].

**Remark 7.5**: In fact, satisfying the matching condition (7.2) for the CT system (7.1) does not necessarily guarantee to hold for its DT counterpart (7.3), since ZOH does not take place in the disturbance channels. In detail, for the smooth bounded disturbance $f_1(t)$, the generalized uncertainty $d[k]$ in (7.3) can be expressed as [172]:

$$d[k] = \Gamma f_1[k] + \frac{1}{2} \Gamma \vartheta_1[k]T + O(T^3)$$
$$\approx \Gamma \left( f_1[k] + \frac{1}{2} \vartheta_1[k]T \right)$$

(7.4)

where

$$\vartheta_1[k] = \vartheta_1(kT), \quad \vartheta_1(t) = \frac{d}{dt} f_1(t).$$
In other words, the magnitude of the unmatched part in the disturbance $d[k]$ is the order of $O(T^3)$, thus it is reasonable to assume that the uncertainty $d[k]$ remains matched as $T$ is sufficiently small.

### 7.2.2 Design of Sliding Manifold

First, the linear sliding variable vector is defined as:

$$s[k] = Gx[k]$$  \hspace{1cm} (7.5)

where $G \in \mathbb{R}^{m \times n}$ is the sliding mode parameter matrix designed in such a way that $\det(G\Gamma') \neq 0$ and the dynamics of $s[k]$ are stable in the sliding mode.

Substituting (7.3) into (7.5) yields

$$s[k + 1] = G\phi x[k] + Gd[k] + G\Gamma u[k]$$  \hspace{1cm} (7.6)

The sliding condition for the MIMO system is chosen as:

$$\Delta V[k] = V[k + 1] - V[k] < 0$$  \hspace{1cm} (7.7)

where $V[k] = \|s[k]\|$ is the candidate of the Lyapunov function.

**Remark 7.6:** It is clearly seen that the reaching law (7.7) guarantees $\|s[k + 1]\| < \|s[k]\|$, which is the necessary and sufficient condition to ensure the existence of the global sliding mode in the sense of Lyapunov. It implies that all the sliding variables would finally move toward the intersection of the sliding surfaces $s[k] = 0$, and the closed-loop dynamics can then exponentially converge to zero in the sliding mode.

The asymptotic stability of closed-loop dynamics in the sliding mode will be given in Theorem 1.
Theorem 7.1: Consider the system (7.3). If the sliding manifold (7.5) is employed, the closed-loop dynamics are completely invariant with respect to matched uncertainties and the asymptotic zero convergence of the system states is achieved.

Proof: The system dynamics on the sliding mode are derived by solving $s[k + 1] = 0$, which leads to

$$u[k] = -(G \Gamma)^{-1}G(\phi x[k] + d[k])$$

(7.8)

Thus, substituting (7.8) into (7.3) yields the dynamical equation of the closed-loop system in the sliding mode:

$$x[k + 1] = (\phi - \Gamma(G \Gamma)^{-1}G\phi)x[k] + (I - \Gamma(G \Gamma)^{-1}G)d[k]$$

(7.9)

where $I$ is the unity matrix.

If $d[k]$ is matched as in (7.4), Equation (7.9) becomes

$$x[k + 1] = (\phi - \Gamma(G \Gamma)^{-1}G\phi)x[k]$$

(7.10)

Equation (7.10) can be considered as a linear state feedback control, thus the method of matrix transformation and Ackermann’s pole assignment scheme, subject to Assumption 1, can be used to design the matrix $G$ such that the closed-loop dynamics are asymptotically stable in the sliding mode [17, 174].

Theorem 7.1 summarizes the behaviour of the closed-loop dynamics once they reached the sliding mode surface. To achieve the sliding mode, a novel DSMLC is proposed in the next subsection.

7.2.3 Design of Discrete-Time Sliding Mode Learning Controller

In this scheme, the DSMLC is proposed as the following:

$$u[k] = u[k - 1] - \Delta u[k]$$

(7.11)
with the learning term:

\[
\Delta u[k] = \begin{cases} 
(GF)^{-1}(\xi s[k] - \Omega s[k - 1]) & \text{for } s[k] \neq 0 \\
0 & \text{for } s[k] = 0 
\end{cases} 
\tag{7.12}
\]

where \( \xi = diag(\varepsilon_1, \varepsilon_2, \ldots, \varepsilon_m) \) and \( \Omega = diag(\omega_1, \omega_2, \ldots, \omega_m) \), with \( \varepsilon_i > 0, \omega_i > 0, i = 1, m \) are the control parameters that will be determined later.

**Remark 7.7:** Considering the Assumption 7.1 and Assumption 7.2, it is reasonable to assume the DT system (7.3) whose dynamics are sufficiently smooth such that the following inequality is always held:

\[
|\Delta V[k, k - 1] - \overline{\Delta V}[k - 1]| < \frac{1}{M} |\Delta V[k - 1]| 
\tag{7.13}
\]

for \( \overline{\Delta V}[k, k - 1] \neq 0, \Delta V[k - 1] \neq 0 \) and \( M \gg 1 \)

where

\[
\begin{align*}
\Delta V[k - 1] & = \|s[k]\| - \|s[k - 1]\| \\
\overline{\Delta V}[k - 1] & = \|s[k] - s[k - 1]\| \\
\overline{\Delta V}[k, k - 1] & = \|s[k, k - 1] - s[k]\| \\
s[k, k - 1] & = G\phi x[k] + Gd[k] + GFu[k - 1] 
\end{align*}
\tag{7.14}
\]

In (7.14), \( s[k, k - 1] \) explicitly denotes a function \( s[k] \) with the delayed input \( u[k - 1] \). It is noted that (7.13) is the so-called Lipschitz-like condition in a discrete-time domain [84], which describes the essential property of the smoothness of uncertain discrete-time systems. In other words, it intuitively shows that the change of the gradient of the sliding variable vector between two consecutive sampling instances is very small as long as the sampling period \( T \) is sufficiently small (see the validation given in Appendix B.1).

Furthermore, considering the smoothness of \( s[k] \), provided that the sampling period \( T \) is sufficiently small, it yields that the difference between \( \|s[k] - s[k - 1]\| \) and \( \|s[k]\| - \|s[k - 1]\| \)
\|s[k - 1]\| is very small, thus there always exists a positive number \(\delta, 0 < \delta \ll 1\) such that the following inequality holds (refer to the proof given in Appendix B.2):

\[
\overline{\Delta V}[k - 1] - \Delta V[k - 1] < \delta \|s[k]\| \quad (7.15)
\]

In the next section, the stability of the closed-loop system and its convergence analysis will be discussed in detail.

### 7.3 Convergence Analysis

**Theorem 7.2**: Consider the system model (7.3). If the proposed DSMLC (7.11) with the learning term (7.12) is used and the control parameters \(\xi, \Omega\) are designed such that

\[
\frac{1}{M} < \omega_{\text{max}} < 1 - \frac{1}{M} \quad (7.16.\text{a})
\]

\[
\max\{|1 - \varepsilon_i|\} < 1 - \omega_{\text{max}} - \frac{2}{M} - \delta \quad (7.16.\text{b})
\]

with \(M \gg 1\) and \(0 < \delta \ll 1\), where \(\omega_{\text{max}} = \max\{\omega_i\}\), for \(i = 1, m\), then the controller (7.11) will drive the system states to the intersection of the sliding surfaces, and guarantee the existence of the sliding mode.

**Proof**: Substituting (7.11)-(7.12) into (6) yields

\[
s[k + 1] = G\phi x[k] + Gd[k] + G\Gamma u[k - 1] - \xi s[k] + \Omega s[k - 1] \quad (7.17)
\]

With the help of (7.14), (7.17) can be re-written as:

\[
s[k + 1] = s[k, k - 1] - \xi s[k] + \Omega s[k - 1] \quad (7.18)
\]

Choosing the candidate of Lyapunov function for the DT MIMO systems as:

\[
V[k] = \|s[k]\| \quad (7.19)
\]
one can then obtain

$$\Delta V[k] = V[k + 1] - V[k] = \|s[k + 1]\| - \|s[k]\|$$  \hspace{1cm} (7.20)

Substituting (7.18) into (7.20) yields

$$\Delta V[k] = \|s[k, k - 1] - \xi s[k] + \Omega s[k - 1]\| - \|s[k]\|$$  \hspace{1cm} (7.21)

$$= \|s[k, k - 1] - s[k] + (I - \xi)s[k] + \Omega s[k - 1]\| - \|s[k]\|$$

$$\leq \|s[k, k - 1] - s[k]\| + \|I - \xi\|\|s[k]\| + \|\Omega\|\|s[k - 1]\| - \|s[k]\|$$

$$\Delta V[k, k - 1] + \|I - \xi\|\|s[k]\| + \|\Omega\|\|s[k - 1]\| - \|s[k]\|$$  \hspace{1cm} (7.22)

$$= \Delta V[k, k - 1] - \Delta V[k - 1] + \Delta V[k - 1] + \|I - \xi\|\|s[k]\|$$

$$\leq |\Delta V[k, k - 1] - \Delta V[k - 1]| + \Delta V[k - 1] + \|I - \xi\|\|s[k]\|$$

$$\Delta V[k] \leq |\Delta V[k, k - 1] - \Delta V[k - 1]| + \Delta V[k - 1]$$

$$+ (\max\{|1 - \varepsilon_i| - 1\}\|s[k]\| + \omega_{max}\|s[k - 1]\|)$$  \hspace{1cm} (7.23)

Substituting (7.13) into (7.23) leads to

$$\Delta V[k] < \frac{1}{M}|\Delta V[k - 1]| + \Delta V[k - 1]$$

$$- \Delta V[k - 1] + \Delta V[k - 1] + (\max\{|1 - \varepsilon_i| - 1\}\|s[k]\| + \omega_{max}\|s[k - 1]\|)$$  \hspace{1cm} (7.24)

With the help of (7.15), inequality (7.24) is rewritten as:
\[
\Delta V[k] < \frac{1}{M} |\Delta V[k-1]| + \delta \|s[k]\| + \Delta V[k-1] + (\max\{|1-\varepsilon_i|\} - 1)\|s[k]\|
+ \omega_{\max} \|s[k-1]\|
\]
\[
= \frac{1}{M} |\Delta V[k-1]| + (1 - \omega_{\max}) \Delta V[k-1] + (\max\{|1-\varepsilon_i|\} + \omega_{\max} - 1 + \delta)\|s[k]\|
\]
(7.25)

- **For the case that** \(\Delta V[k-1] > 0\)

From (7.25), we have

\[
\Delta V[k] < \left(\frac{1}{M} + 1 - \omega_{\max}\right) \Delta V[k-1]
+ (\max\{|1-\varepsilon_i|\} + \omega_{\max} - 1 + \delta)\|s[k]\|
\]
(7.26)

Considering the conditions (7.16.a) and (7.16.b), one can easily verify that

\[
0 < \frac{1}{M} + 1 - \omega_{\max} < 1
\]
(7.27)
\[
\max\{|1-\varepsilon_i|\} + \omega_{\max} - 1 + \delta < 0
\]
(7.28)

then (7.26) can be expressed as:

\[
\Delta V[k] < \Delta V[k-1]
\]
(7.29)

The inequality (7.29) indicates that the learning controller (7.11) always makes the value of \(\Delta V[k]\) smaller than \(\Delta V[k-1]\) when \(\Delta V[k-1] > 0\). Suppose that, at \(k = k_0\), \(\Delta V[k_0] = 0\) or \(\|s[k_0 + 1]\| = \|s[k_0]\|\). Then at \(k = k_0 + 1\), (7.23) can be expressed as:

\[
\Delta V[k_0 + 1] \leq \Delta V[k_0 + 1, k_0] + (\max\{|1-\varepsilon_i|\} - 1)\|s[k_0 + 1]\| + \omega_{\max} \|s[k_0]\|
\]
\[
= \Delta V[k_0 + 1, k_0] + (\max\{|1-\varepsilon_i|\} + \omega_{\max} - 1)\|s[k_0 + 1]\|
\]
(7.30)

Considering the fact that
\[ |\Delta V[k_0]| = |\|s[k_0 + 1]\| - \|s[k_0]\|| \leq \|s[k_0 + 1]\| + \|s[k_0]\| \]

thus, (7.13) can be expressed as:

\[ |\Delta \bar{V}[k_0 + 1, k_0] - \Delta \bar{V}[k_0]| < \frac{1}{M} (\|s[k_0 + 1]\| + \|s[k_0]\|) \quad (7.31) \]

Since \( \Delta V[k_0] = 0 \) or \( \|s[k_0 + 1]\| = \|s[k_0]\| \), (7.31) will become:

\[ \Delta \bar{V}[k_0 + 1, k_0] < \frac{2}{M} \|s[k_0 + 1]\| \quad (7.32) \]

for \( s[k_0 + 1] \neq 0 \) and \( s[k_0 + 1, k_0] \neq 0 \).

Using (7.32) in (7.30) yields:

\[
\Delta V[k_0 + 1] < \frac{2}{M} \|s[k_0 + 1]\| + (\max[|1 - \epsilon_i| + \omega_{max} - 1]) \|s[k_0 + 1]\|
\]

\[
= \left( \max[|1 - \epsilon_i| + \omega_{max} - 1 + \frac{2}{M}] \right) \|s[k_0 + 1]\|
\]

(7.33)

From (7.16.b), we have

\[
\max[|1 - \epsilon_i| + \omega_{max} - 1 + \frac{2}{M}] < 0
\]

then (7.33) becomes

\[ \Delta V[k_0 + 1] < 0 \quad (7.34) \]

The analysis from (7.26) to (7.34) implies that the proposed learning controller (7.11) is capable of reducing \( \Delta V[k] \) from a positive value to a negative one, or equivalently the closed-loop trajectories are always driven into a stable region, in which the sliding
variable vector is guaranteed to reach and be retained on the intersection of the sliding surfaces \( s[k] = 0 \).

- **For the case that** \( \Delta V[k-1] < 0 \)

The inequality (7.25) can be expressed as:

\[
\Delta V[k] < \left( \frac{1}{M} + \omega_{\text{max}} - 1 \right) \Delta V[k-1] + (\max\{|1 - \epsilon_i|\} + \omega_{\text{max}} - 1 + \delta)\|s[k]\|
\]

(7.35)

From conditions (7.16), one can easily verify that

\[
-1 < \frac{1}{M} + \omega_{\text{max}} - 1 < 0 \quad (7.36)
\]

\[
\max\{|1 - \epsilon_i|\} + \omega_{\text{max}} - 1 + \delta < 0 \quad (7.37)
\]

thus, (7.35) becomes:

\[
\Delta V[k] < 0
\]

(7.38)

In summary, the sufficient condition (7.7) for the existence of the global sliding mode \( s[k] = 0 \) is satisfied.

**Remark 7.8:** It has been shown from the discussions above that the proposed DSMLC (7.11), whose parameters are designed as in (7.16), is capable of driving the sliding variable vector to converge to zero, thus the closed-loop dynamics can asymptotically converge to zero in the sliding mode. It also ensures the stability of the closed-loop system in the sliding mode with a strong robustness with respect to the uncertain dynamics. Moreover, the proposed learning control scheme, based on the recursive algorithm, inherits the chattering-free characteristic.

**Remark 7.9:** It is worth noting that (7.13) is simply the discrete-time version of the Lipschitz-like condition proposed in [84, 87, 154], describing the important property of
the continuity of uncertain systems. The advantage of using the Lipschitz-like condition is that the concept about system uncertainties considered in many existing robust sliding mode control designs could be embedded in this condition, thus the knowledge of the bounds of uncertainties usually used in the conventional SMC schemes is not required for developing the DSMLC any longer. It is therefore believed that the proposed control technique exhibits more flexibility and applicability in terms of relaxing the constraint on uncertain dynamics in conventional approaches.

### 7.4 Illustrative Examples

To illustrate the effectiveness of the proposed control technique, the DSMLC is employed in a discrete-time MIMO system with unmodelled dynamics and external disturbances in comparison with some existing control schemes.

**Example 1:**

In this numerical demonstration, we first consider an aircraft model used in [176]. The state-space model of the aircraft is given by:

\[
\begin{bmatrix}
\dot{\phi} \\
\dot{\theta} \\
\dot{\delta}_e
\end{bmatrix} =
\begin{bmatrix}
-0.277 & 1 & -0.0002 \\
-17.1 & -0.178 & -12.2 \\
0 & 0 & -6.67
\end{bmatrix}
\begin{bmatrix}
\phi \\
q \\
\delta_e
\end{bmatrix} +
\begin{bmatrix}
0 \\
0 \\
6.67
\end{bmatrix} u
\]

\[
y = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \phi \\ q \\ \delta_e \end{bmatrix}
\]

where \(\phi, q, \delta_e\) are the attack angle, pitch rate and elevator angle, respectively, \(u\) is the command to the elevator and \(y\) is the measurement vector.

The parametric uncertainty and disturbance are respectively assumed to be

\[
D_a = 0.2[\sin(t) \cos(t) \quad \sin(3t) + \cos(t) \quad -1 + \sin(t)\cos(2t)]
\]

\[
D_b = 0.1 \sin(t)
\]

\[
d_f = 0.1 \sin(4t)
\]
With the sampling period $T = 0.005$ s, the resulting discrete-time system matrices (7.3) can be obtained accordingly as:

$$\phi = \begin{bmatrix}
0.9997 & 0.001 & 0 \\
-0.0171 & 0.9998 & -0.0122 \\
0 & 0 & 0.9934
\end{bmatrix},$$

$$\Gamma = \begin{bmatrix}
0 \\
0 \\
0.0066
\end{bmatrix}, C = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix}$$

For comparison purpose, we first employ the concept of the multirate output feedback controller as in [11, 12]. Following the design procedure in [163] and choosing the value of $N, q, \varepsilon$ as $N = 2, q = 2, \varepsilon = 1$, the multirate sliding mode controller can be obtained as

$$u[k] = [-139.1 \ 0.35 \ 139.2 \ 0.35]y[k] - 0.0007u[k - 1] - 1.05\text{sign}(s[k])$$

Figure 7.1.a – 7.1.c show the sliding variable, the output response and the control input, respectively. It is seen that the multirate SMC exhibits chattering phenomenon due to the sign function and it causes undesired high frequencies that deteriorate the system performance.

Figure 7.1.a. Sliding variable (Multirate SMC).
7.4 Illustrative Examples

Now, we turn to apply the proposed DSMLC scheme. The simulation results of sliding variable, output response, and control input are shown in Figure 7.2.a – Figure 7.2.c, respectively, where the control parameters of $\xi, \Omega$ in (7.14) are set as $\xi = \text{diag}(0.88, 0.88)$ and $\Omega = \text{diag}(0.8, 0.8)$, respectively. It is seen that the sliding variable is driven to converge to zero, the system dynamics then asymptotically converge to zero in the sliding mode. Also, the control signal is completely chattering-free. A superior
performance with strong robustness of the stability of the closed-loop system is achieved. Furthermore, the prior knowledge of the bounds of the uncertainties is not required in the design of the proposed DSMLC.

Figure 7.2.a. Sliding variable (DSMLC).

Figure 7.2.b. Output response (DSMLC).
Example 2:

Let us consider an MIMO system with matched uncertainty as given in [172]

\[
A = \begin{bmatrix}
1 & -2 & 3 \\
-4 & 5 & -6 \\
7 & -8 & 9
\end{bmatrix},
B = \begin{bmatrix}
1 & -2 \\
-3 & 4 \\
5 & 6
\end{bmatrix},
\]

\[
C = [-1 1 0], f(t) = \begin{bmatrix}
0.1 \sin(3\pi t) \\
0.1 \cos(3\pi t)
\end{bmatrix}
\]

and the sampling period \(T = 0.001\) s, the discrete-time system matrices can be obtained accordingly as:

\[
\phi = \begin{bmatrix}
1.001 & -0.002 & 0.003 \\
-0.004 & 1.005 & -0.006 \\
0.007 & -0.008 & 1.009
\end{bmatrix}, \Gamma = \begin{bmatrix}
0.001 & -0.002 \\
-0.003 & 0.004 \\
0.005 & 0.006
\end{bmatrix}
\]

The initial state vector is selected as \(x[0] = [3 \ 4 \ -2]^T\). The sliding mode parameter matrix \(G\) in (7.5) is chosen such that the poles of the sliding dynamics are \(p = [0 \ -5 \ 0]^T\) in continuous-time, or \(z = [0 \ 0.995 \ 0]^T\) in discrete-time [172]:

\[
G = \begin{bmatrix}
0.2621 & -0.3108 & -0.0385 \\
3.4268 & 2.4432 & 1.1787
\end{bmatrix}
\]
For the purpose of comparison, let us first consider using the quasi discrete-time sliding mode controller via output feedback (OF-SMC) as follows [176]:

\[
    u[k] = -\alpha (G\Gamma)^{-1} s[k] - (G\Gamma)^{-1} G (x[k] - \phi x[k - 1] - \Gamma u[k - 1])
\] (7.39)

where the control parameter \( \alpha = 0.03 \).

Now, we turn to adopt the proposed DSMLC scheme for the given system, where the control parameters \( \xi, \Omega \) in (7.16) are set as \( \xi, \Omega = \text{diag}(0.117, 0.117) \) and \( \text{diag}(0.115, 0.115) \), respectively. Figure 7.3 and Figure 7.4 respectively show the output response \( y = x_2 - x_1 \) and the control inputs \( u_1, u_2 \) obtained by applying the proposed DSMLC in comparison with those obtained by using the output feedback SMC (7.39). It is clearly seen that even though the two controllers have been able to handle the uncertainty, the proposed DSMLC outperforms the OF-SMC with both smaller settling time and significantly reduced overshoot.

![Figure 7.3. Output response (DSMLC vs OF-SMC).](image)
7.5 Conclusion

In this chapter, a robust DSMLC has been newly developed for a class of linear discrete-time MIMO systems with uncertainties. It has been shown that the closed-loop system exhibits strong robustness with respect to parameter uncertainties and external disturbances. The closed-loop stability is guaranteed with an asymptotic zero convergence. Furthermore, the control signals are chattering-free and the control design does not require prior knowledge of the bounds of uncertainties at all. The future work on the extension to more advanced control of uncertain nonlinear systems and nonminimum phase systems is currently under the authors’ investigation.

Figure 7.4. Control inputs (DSMLC vs OF-SMC).
Chapter 8
Discrete-Time Sliding Mode Learning Based Congestion Control for Connection-Oriented Communication Networks

A sliding mode learning controller is developed for connection-oriented communication networks. Firstly, the networks are modelled as discrete-time nth-order dynamic systems subject to time-varying delay, the problem of congestion control in communication networks is then addressed by adopting a novel sliding mode learning controller. It not only considers the setbacks of conventional sliding mode control problems, but also guarantees the closed-loop system stability with strong robustness against uncertainties. The chapter presents the plausible applications of a discrete-time sliding mode learning controller in communication networks and its great potential in years to come.

8.1 Introduction

Over the last decade, networking services and long-distance traffic intensity in telecommunication systems have evolved at an unprecedented pace. The ever increasing world-wide demand and the high level of complexity in networking communication systems have created the need for novel technological solutions to the existing concepts of flow control and resource allocation. Too often, the bandwidth demand is always growing faster than the physical channel capacity, to combat the congestion and meanwhile ensure high throughput in the system, it appears that a viable solution must engage the application of appropriate flow control mechanisms.
The overview of earlier congestion control schemes can be found in [177]. Since then, various researchers have proposed the use of different control techniques to regulate the data flow rate in communication networks [178-180]. On the other hand, sliding mode control is known to be an efficient and robust regulation technique [7-8]. To date, sliding mode controllers and sliding mode based congestion control ones have been intensively studied and successfully applied to control of communication networks [120, 181]. In this chapter, we introduce the newly developed sliding mode learning controller to address the congestion control of networking systems. The goal of this chapter is to contribute to the congestion control of networking systems using a robust and efficient sliding mode learning controller to achieve a desired control performance. As a consequence, the proposed control strategy exhibits a fast and robust flow regulation mechanism to fulfill the traffic requirements of users and the superior quality of service.

The remainder of the chapter is organized as follows: In Section 8.2, the state space model of the networking systems and the discrete-time sliding mode-based learning controller (DSMLC) are presented. Section 8.3 comprises a stability analysis of the closed-loop system and the discussion of the property of the proposed control methodology. A numerical simulation is given in Section 8.4 to illustrate the effectiveness of the proposed control technique. Finally, Section 8.5 draws a conclusion.

8.2 Problem Formulation

This section gives some preliminaries of network modelling of connection-oriented communication networks in accordance to the design of a DSMLC scheme.

8.2.1 Network Model

In this chapter, a single virtual circuit in a connection-oriented network is considered. The network model is illustrated in Figure 8.1. The source sends data packets at discrete-time instants with the amounts determined by the controller placed at a network node. The congestion control problem can be solved through an appropriate input rate adjustment. After forward propagation delay $T_f$, packets reach the bottleneck node and are served pertaining to the bandwidth availability at the output link. The remaining data accumulates in the buffer. The packet queue length in the buffer at time $kT$ is denoted as
8.2 Problem Formulation

\( y(k) \), and its demand value \( y_d \) are used to calculate the current amount of data to be sent by the source \( u(k) \). Once the control units appear at the end system, they are turned back to arrive at the origin, with backward propagation delay \( T_b \) after being processed by the congested node. Since management units are not subject to the queuing delays, round-trip time \( RTT = T_f + T_b = mT \), where \( m \) is a positive integer remaining constant for the duration of connection. Quantities \( T_f \) and \( T_b \) may differ, but their sum \( RTT \) remains unchanged.

The available bandwidth (the number of packets that may leave the bottleneck node at each \( kT \) instant) is modelled as an a priori unknown bounded function of time \( d(k) \), \( 0 \leq d(k) \leq d_{\text{max}} \), which accounts for possible nonlinear interactions between various virtual circuits in the network. If there are packets ready for transmission in the buffer then bandwidth consumed by the source \( h(k) \) (the number of packets actually leaves the node) will be equal to the available bandwidth. Otherwise, the output link is underutilized, and the exploited bandwidth matches the data arrival rate at the node. Thus, we may write:

\[
0 \leq h(k) \leq d(k) \leq d_{\text{max}}
\]  \hspace{1cm} (8.1)

The network can be represented in the state space:

\[
x(k + 1) = Ax(k) + bu(k) + vh(k) \\
y(k) = q^T x(k)
\]  \hspace{1cm} (8.2)
where \( x(k) = [x_1(k) \ x_2(k) \ \ldots \ x_n(k)]^T \) is the state vector with \( x_1(k) = y(k) \), the state matrix \( A \in \mathbb{R}^{n \times n} \) and the vectors \( b, v \) and \( q \in \mathbb{R}^{n \times 1} \) are respectively described as follows:

\[
A = \begin{bmatrix}
1 & 1 & 0 & \cdots & 0 \\
0 & 0 & 1 & \cdots & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
0 & 0 & 0 & \cdots & 1 \\
0 & 0 & 0 & \cdots & 0
\end{bmatrix}
\]

\[
b = \begin{bmatrix}
0 \\
0 \\
\vdots \\
0 \\
1
\end{bmatrix}, \quad v = \begin{bmatrix}
-1 \\
0 \\
\vdots \\
0 \\
0
\end{bmatrix}, \quad q = \begin{bmatrix}
1 \\
0 \\
\vdots \\
0 \\
0
\end{bmatrix}
\]

**8.2.2 Design of Discrete-Time Sliding Mode Learning Controller**

Firstly, the sliding variable is defined as follows:

\[
s(k) = c^T e(k) \tag{8.4}
\]

where \( c^T = [c_1 \ c_2 \ \cdots \ c_n] \) is the sliding mode parameter vector such that \( \det(c^T b) \neq 0 \). The closed-loop system error is denoted as \( e(k) = x_d - x(k) \), with the desired system state defined as \( x_d = [x_{d1} \ x_{d2} \ \cdots \ x_{dn}]^T = [y_d \ 0 \ \cdots \ 0]^T \).

**Remark 8.1:** The vector \( c \) is selected such that the closed-loop dynamics are stable on the sliding mode surface \( s(k + 1) = 0 \). It can be verified that the closed-loop system state matrix has the following form \( A - b(c^T b)^{-1} c^T A \). Thus, a pole placement method can be used to design the sliding mode parameter \( c^T \) such that all the eigenvalues are located inside the unit circle which guarantees the asymptotic stability of the closed-loop discrete-time system.

In this scheme, the DSMLC is proposed as the following:

\[
u(k) = u(k - 1) + \Delta u(k) \tag{8.5}
\]

with the learning term:
\[ \Delta u(k) = \begin{cases} 
(c^T b)^{-1}(\alpha s(k) - \beta s(k - 1)) & \text{for } s(k) \neq 0 \\
0 & \text{for } s(k) = 0 
\end{cases} \quad (8.6) \]

where \( \alpha, \beta > 0 \) are the control parameters that will be determined later.

### 8.3 Stability Analysis

**Theorem 8.1:** Consider the system model (8.2). If the proposed DSMLC (8.5) with the learning term (8.6) is used and the control parameters \( \alpha, \beta \) are designed such that

\[
\frac{1}{M} < \beta < 1 - \frac{1}{M} \\
|1 - \alpha| < 1 - \beta - \frac{2}{M} - \frac{1}{N} 
\]

where \( M, N \gg 1 \), then the system states will be driven to the sliding surface, and therefore guarantee the asymptotic convergence of the closed-loop dynamics on the sliding mode surface.

**Proof:** Substituting (8.2), (8.5) and (8.6) into (8.4) yields

\[
s(k + 1) = c^T(x_d - Ax(k) - vh(k)) - c^T bu(k - 1) - \alpha s(k) + \beta s(k - 1) \\
= s(k, k - 1) - \alpha s(k) + \beta s(k - 1) \]

where \( s(k, k - 1) = c^T(x_d - Ax(k) - vh(k)) - c^T bu(k - 1) \).

Choosing a Lyapunov function candidate for the DT systems as:

\[
V(k) = |s(k)| \quad (8.9) 
\]

one can then obtain
\[ \Delta V(k) = V(k + 1) - V(k) = |s(k + 1)| - |s(k)| \quad (8.10) \]

Substituting (8.8) into (8.10) yields

\[ \Delta V(k) = |s(k, k - 1) - \alpha s(k) + \beta s(k - 1)| - |s(k)| \leq |s(k, k - 1) - s(k)| + |1 - \alpha| |s(k)| + \beta |s(k - 1)| - |s(k)| \quad (8.11) \]

\[ \triangleq \overline{\Delta V}(k, k - 1) - \overline{\Delta V}(k - 1) + \overline{\Delta V}(k - 1) + |1 - \alpha| |s(k)| + \beta |s(k - 1)| - |s(k)| \quad (8.12) \]

where \( \overline{\Delta V}(k, k - 1) \triangleq |s(k, k - 1) - s(k)| \) and \( \overline{\Delta V}(k - 1) \triangleq |s(k) - s(k - 1)| \).

Considering the smoothness of the discrete-time system, it is shown that the following inequality is always held:

\[ |\overline{\Delta V}(k - 1) - \overline{\Delta V}(k - 1)| < \frac{1}{M} |\Delta V(k - 1)| \quad (8.13) \]

and

\[ |\overline{\Delta V}(k - 1) - \Delta V(k - 1)| < \frac{1}{N} |s(k)| \quad (8.14) \]

for \( M, N \gg 1 \).

Using (8.13) and (8.14) in (8.12) leads to

\[ \Delta V(k) < \frac{1}{M} |\Delta V(k - 1)| + (1 - \beta) \Delta V(k - 1) + \left( \beta - 1 + |1 - \alpha| + \frac{1}{N} \right) |s(k)| \quad (8.15) \]

- **For the case that \( \Delta V(k - 1) > 0 \)**

From (8.15), we have
\[ \Delta V(k) < \left( \frac{1}{M} + 1 - \beta \right) \Delta V(k - 1) + \left( \beta - 1 + |1 - \alpha| + \frac{1}{N} \right) |s(k)| \quad (8.16) \]

Consider the conditions (8.7), one can easily verify that

\[ 0 < \frac{1}{M} + 1 - \beta < 1 \quad (8.17) \]
\[ \beta - 1 + |1 - \alpha| + \frac{1}{N} < 0 \quad (8.18) \]

then (8.16) can be expressed as:

\[ \Delta V(k) < \Delta V(k - 1) \quad (8.19) \]

The inequality (8.19) indicates that the learning controller (8.5) always makes the value of \( \Delta V(k) \) smaller than \( \Delta V(k - 1) \) when \( \Delta V(k - 1) > 0 \). Suppose that, at \( k = k_0 \), \( \Delta V(k_0) = 0 \) or \(|s(k_0 + 1)| = |s(k_0)|\). Then at \( k = k_0 + 1 \), (8.12) can be expressed as:

\[ \Delta V(k_0 + 1) \leq \Delta \bar{V}(k_0 + 1, k_0) + (|1 - \alpha| + \beta - 1)|s(k_0 + 1)| \quad (8.20) \]

Considering the fact that \(|\Delta V(k_0)| \leq |s(k_0 + 1)| + |s(k_0)|\), thus (8.14) can be expressed as:

\[ |\Delta \bar{V}(k_0 + 1, k_0) - \Delta \bar{V}(k_0)| < \frac{1}{M} (|s(k_0 + 1)| + |s(k_0)|) \quad (8.21) \]

Since \( \Delta V(k_0) = 0 \) or \(|s(k_0 + 1)| = |s(k_0)|\), (8.21) will become:

\[ \Delta \bar{V}(k_0 + 1, k_0) < \frac{2}{M} |s(k_0 + 1)| \quad (8.22) \]

for \( s(k_0 + 1) \neq 0 \) and \( s(k_0 + 1, k_0) \neq 0 \).
Using (8.22) in (8.20) yields:

$$\Delta V(k_0 + 1) < \left( \beta + |1 - \alpha| - 1 + \frac{2}{M} \right) |s(k_0 + 1)|$$  \hspace{1cm} (8.23)

Considering (8.7.b), we have

$$\Delta V[k_0 + 1] < 0$$ \hspace{1cm} (8.24)

The analysis from (8.16) to (8.24) implies that the proposed learning controller (8.5) is capable of reducing $\Delta V(k)$ from a positive value to negative one, or equivalently the closed-loop trajectories are always driven into a stable region.

- **For the case that $\Delta V(k - 1) < 0$**

The inequality (8.15) can be expressed as:

$$\Delta V(k) < \left( \frac{1}{M} + \beta - 1 \right) |\Delta V(k - 1)| + \left( |1 - \alpha| + \beta - 1 + \frac{1}{N} \right) |s(k)|$$ \hspace{1cm} (8.25)

From conditions (8.7), one can easily verify that

$$\Delta V(k) < 0$$ \hspace{1cm} (8.26)

**Remark 8.2:** It has been shown from the discussions above that the proposed DSMLC (8.5), whose parameters are designed as in (8.6), is capable of driving the sliding variable vector to converge to zero, thus the closed-loop dynamics can asymptotically converge to zero in the sliding mode. It also ensures the stability of the closed-loop system in the sliding mode with a strong robustness with respect to the uncertain dynamics. Moreover, the proposed learning control scheme, based on the recursive algorithm, inherits the chattering-free characteristic.
8.4 Simulation Example

To verify the proposed control strategy, a simulation was performed using an MATLAB simulator. Firstly, the model of the network was constructed according to the formulation given in Section II. The system parameters are chosen as follows: discretization period $T = 1\,\text{ms}$, propagation delay $RTT = mT = 10\,\text{ms}$, maximum available bandwidth $d_{\text{max}} = 10$ packets, the demand queue length $y_d = 200$ packets. The controller parameters are set to $\alpha = 0.9, \beta = 0.2$, respectively. Figure 8.2 shows that the function $d$ experiences sudden changes of large amplitude, which reflects the most rigorous networking conditions.

The transmission rate generated by the proposed algorithm is illustrated in Figure 8.3, and the resultant queue length is shown in Figure 8.4. It is clearly seen that the rates calculated by the algorithm are always nonnegative and upper bounded. Moreover, the queue length does not increase beyond the demand value and never drops to zero. This means that the buffer capacity is not exceeded, and all of the available bandwidth is used for the data transfer. As a consequence, the maximum throughput in the network is achieved. Furthermore, the evolution of the sliding variable is shown in Figure 8.5, which was driven quickly to the sliding mode surface and remains on it thereafter.

![Figure 8.2. Available bandwidth.](image-url)
Figure 8.3. Transmission rate.

Figure 8.4. Packet queue length.

Figure 8.5. Sliding variable.
8.5 Conclusion

In this chapter, a sliding mode based learning controller has been newly developed for a single virtual circuit of a connection-oriented network. It has been shown that the closed-loop system exhibits strong robustness with respect to time-varying delay. The closed-loop stability is guaranteed with an asymptotic convergence. The future work on the extension to a multisource framework is currently under the authors’ investigation.
Chapter 9

Conclusion and Future Work

In this chapter, the major contributions of the proposed SMLC scheme as well as the key contributions to take away from this thesis are recapitulated prior to the proposal of some relevant work regarding future research directions.

9.1 Summary of Contributions

This thesis has been concerned with the study and development of a sliding mode learning controller design for a large class of dynamic systems with unmodeled dynamics, and also the implementations of the proposed SMLC in various practical applications. In particular, we focus on both theoretical development and practical implementation of the novel SMLC scheme to completely address the limitations of existing control techniques, i.e., the constraints on the bounds of uncertainties in conventional controller design have been freed. The proposed SMLC approach, meanwhile, still possesses the inherent benefits of SMC, including chattering-free characteristics and strong robustness with respect to uncertainty. To reiterate, the core contributions of the thesis can be highlighted as follows.

In Chapter 3, the concept of the SMLC scheme has been investigated for uncertain dynamic systems. The zero convergence and stability of the closed-loop system have been analysed and discussed in detail to boost the significance of the proposed control scheme. Simulation results have been illustrated to further confirm the merits of the proposed SMLC algorithm.

In Chapter 4 and Chapter 5, the SMLC technique has been further developed to stabilize nonobservable dynamics of nonminimum phase systems and successfully applied to the congestion control of DiffServ networks, respectively. The results achieved in comparison with other existing control methods have shown the effectiveness of the proposed control scheme.
Furthermore, in Chapter 6 the proposed SMLC has been successfully implemented in an SbW platform. It is seen that the SMLC exhibits superior performance over existing control techniques.

Finally, the work in Chapter 7 and Chapter 8 is dedicated to the development of the SMLC in a discrete-time framework. In particular, the DSMLC is newly developed for an MIMO system with uncertainties and then applied to the congestion control of communication networks.

9.2 Future Work

Following the current research trend, the work of the thesis also piques much more interest and motivation for several ideas that shape future research.

9.2.1 Time-Delayed Systems

Time delays often occur in many dynamic systems especially for network control systems. The existence of time delays in system states and control input often become the sources of deterioration of the performance of control systems and potentially cause instability of the network systems. In order to ensure the closed-loop systems have strong robustness, [176] and [177] proposed a sliding mode predictive controller based on a discrete-time system. However, the proposed control scheme cannot reduce the chattering of the sliding mode. In this way, the proposed sliding mode learning control is the best candidate for networked control systems with variable time delays.

9.2.2 Observations and Identifications

In this thesis, an estimate of the first order derivative of the Lyapunov function is always required in the controller design. In practice, this is often not possible, since the system states are not always available or are too expensive to measure [175]. In general, the observers for such systems are devised under assumption that only the outputs are available, but not their derivatives. Thus, the development of a system state observer is needed to overcome this drawback. Furthermore, many research works have shown that these observers are very helpful in various applications such as fault detection, system parameter estimation, and unknown input identification. For this reason, it is strongly
believed that the design of an observer based sliding mode learning control is a very promising field in modern control theory.

9.2.3 Real-World Applications

Last but not least, since the SMLC is much more practical and applicable, it is also important to disseminate the proposed SMLC technique to many other real-world applications so that more practical problems can be solved. In [87], the sliding mode learning control scheme has been successfully applied to a steer-by-wire platform. Therefore, it is expected that more practical work in terms of the development of SMLC systems or SMLC based ones will be conducted in the future to enhance the robustness and performance of large-scale dynamic systems in real-world applications such as robotic manipulators, electric power systems, and communication networks.
Appendix A

A.1 Proof of the Lipschitz-Like Condition in a Continuous-Time Domain Given in the Inequality (3.11)

Let us denote $\mathcal{F}(t)$ intuitively described as a function of the lumped perturbed dynamics $h(t)$ and the delayed control input $u(t - \tau)$. Also, $\dot{V}(t - \tau) = F(h(t - \tau), u(t - \tau))$.

Considering the fundamentals of the Lipschitz functions of both $V(t)$ and $\dot{V}(t)$, there exists a positive function $\rho \in \mathcal{K}_\infty$ (continuous and strictly increasing), such that for the time interval $[t - \tau, t]$, the following condition is held [22]

$$|F(h(t), u(t - \tau)) - F(h(t - \tau), u(t - \tau))| \leq \rho(|h(t) - h(t - \tau)|) \quad (A.1)$$

For the Lipschitz perturbed dynamics $h(t)$, there exists positive constant $L$ such that

$$|h(t) - h(t - \tau)| \leq \tau L \quad (A.2)$$

From (A.1) and (A.2), one can obtain

$$|\dot{V}(t, t - \tau) - \dot{V}(t - \tau)| \leq \rho(\tau L) \quad (A.3)$$

Provided $\tau$ is sufficiently small, the magnitude of $\rho(\tau L) \in O(\tau)$ is very small. Based on the continuity property of a dynamic system, one can assume that the difference $|\dot{V}(t, t - \tau) - \dot{V}(t - \tau)|$ remains within the boundary of $(1/M)|\dot{V}(t - \tau)|$ at least in between two consecutive sampling instants, where $M \gg 1$.

In other words, there always exists a constant $M \gg 1$ such that

$$|\dot{V}(t, t - \tau) - \dot{V}(t - \tau)| \leq \rho(\tau L) < \frac{1}{M} |\dot{V}(t - \tau)|, \quad \dot{V}(t - \tau) \neq 0 \quad (A.4)$$
A.2 Proof of the Condition (3.12)

Considering the Lyapunov function $V(t)$ differentiable up to n-th order of interest, we take the Taylor series expansion of $V(t)$ as:

$$V(t - \tau) = V(t) - \tau \dot{V}(t) + \frac{\tau^2}{2!} \ddot{V}(t) - \frac{\tau^3}{3!} \dddot{V}(t) + \cdots$$  \hspace{1cm} (A.5)

$$V(t - 2\tau) = V(t) - 2\tau \dot{V}(t) + \frac{(2\tau)^2}{2!} \ddot{V}(t) - \frac{(2\tau)^3}{3!} \dddot{V}(t) + \cdots$$  \hspace{1cm} (A.6)

The higher order approximation to the first derivative can be obtained by using more Taylor series and wisely weighting the various expansions in a sum. In particular, by taking $(A.6)-4*(A.5)$ and withdrawing $\dddot{V}(t)$ from the sum leads to

$$\dot{V}(t) = \frac{3V(t) - 4V(t - \tau) + V(t - 2\tau)}{2\tau} + \frac{\tau^2}{3} \dddot{V}(t) + \cdots$$  \hspace{1cm} (A.7)

Assuming that $V(t)$ and its derivatives $V^{(n)}(t)$ are Lipschitz and bounded, or there exists a constant $L$ such that $|\dddot{V}(t)| \leq L < \infty$, one can obtain the backward different approximation of the time derivative of the Lyapunov function as

$$\dot{V}(t) \leq \frac{3V(t) - 4V(t - \tau) + V(t - 2\tau)}{2\tau} + \frac{\tau^2}{3} L \approx \dddot{V}(t) + O(\tau^2)$$  \hspace{1cm} (A.8)

$$\left| \dot{V}(t) - \dddot{V}(t) \right| \approx O(\tau^2)$$  \hspace{1cm} (A.9)

where $O(\tau^2)$ denotes a truncation error term proportional to $\tau^2$.

Thus, as the time delay $\tau$ is sufficiently small, the left-hand side of (A.9) (the boundary layer of the approximation error) will be extremely diminutive. For this reason, there exists a positive constant $\gamma \ll 1$ such that the approximation error
\( \left| \hat{V}(t) - \hat{V}(t) \right| \in O(\tau^2) \) will be confined within the boundary layer of \( \gamma \left| \hat{V}(t) \right| \), that is to say

\[
\left| \hat{V}(t) - \hat{V}(t) \right| \approx O(\tau^2) < \gamma \left| \hat{V}(t) \right| \quad \text{for } \hat{V}(t) \neq 0 \tag{A.10}
\]

\section*{A.3 Verification of the Condition (3.13)}

The condition (A.10) can be expressed as

\[
\left| \hat{V}(t - \tau) - \hat{V}(t - \tau) \right| < \gamma \left| \hat{V}(t - \tau) \right| \tag{A.11}
\]

for \( \hat{V}(t - \tau) \neq 0, \hat{V}(t - \tau) \neq 0 \), and \( 0 < \gamma \ll 1 \).

Inequality (A.11) can be rewritten as

\[
-\gamma \left| \hat{V}(t - \tau) \right| < \hat{V}(t - \tau) - \hat{V}(t - \tau) < \gamma \left| \hat{V}(t - \tau) \right|
\]

\[
\hat{V}(t - \tau) - \gamma \left| \hat{V}(t - \tau) \right| < \hat{V}(t - \tau) < \hat{V}(t - \tau) + \gamma \left| \hat{V}(t - \tau) \right| \tag{A.12}
\]

- \textbf{If } \( \hat{V}(t - \tau) > 0 \)

One can obtain from (A.12) that

\[
0 < \hat{V}(t - \tau) < \hat{V}(t - \tau) + \gamma \left| \hat{V}(t - \tau) \right| \tag{A.13}
\]

Mathematically, we can easily conclude \( \hat{V}(t - \tau) > 0 \)

- \textbf{If, however, } \( \hat{V}(t - \tau) < 0 \)

One can obtain from (A.12) that

\[
\hat{V}(t - \tau) - \gamma \left| \hat{V}(t - \tau) \right| < \hat{V}(t - \tau) < 0 \tag{A.14}
\]

Mathematically, we can easily conclude \( \hat{V}(t - \tau) < 0 \).
In summary, we always have \( \text{sign} \left( \hat{V}(t - \tau) \right) = \text{sign} \left( \tilde{V}(t - \tau) \right) \).

### A.4 Verification of the Continuity of the Proposed SMLC \( u(t) \) Given in the Equation (3.6)

When \( s(t) \) asymptotically converges to zero as time \( t \to \infty \), both \( s(t) \) and \( s(t - \tau) \) are the infinitesimals of the same order. We can therefore have the following relationship between \( s(t) \) and \( s(t - \tau) \) as time \( t \to \infty \):

\[
\lim_{s \to 0} \frac{s(t)}{s(t - \tau)} = \rho \tag{A.15}
\]

where \( 0 < |\rho| \leq 1 \).

From (A.15), we have:

\[
\lim_{s \to 0} \Delta u = \lim_{s \to 0} \frac{1}{2b\tau s(t)} \left( \alpha \hat{V}(t - \tau) + \beta |\hat{V}(t - \tau)| \right)
= \lim_{s \to 0} \frac{1}{2b\tau s(t)} \left[ \alpha \left( s^2(t) - s^2(t - \tau) \right) + \frac{\beta}{2\tau} |s^2(t) - s^2(t - \tau)| \right]
= \lim_{s \to 0} \frac{1}{2b\tau s(t)} \left[ \lim_{s \to 0} \alpha \left( s^2(t) - s^2(t) \right) + \lim_{s \to 0} \beta \left| s^2(t) - \frac{s^2(t)}{\rho^2} \right| \right]
= \lim_{s \to 0} \frac{1}{2b\tau s(t)} \left[ \lim_{s \to 0} \alpha s^2(t) \left( 1 - \frac{1}{\rho^2} \right) + \lim_{s \to 0} \beta s^2(t) \left| 1 - \frac{1}{\rho^2} \right| \right]
= \lim_{s \to 0} \frac{as(t)}{2b\tau} \left( 1 - \frac{1}{\rho^2} \right) + \lim_{s \to 0} \frac{\beta s(t)}{2b\tau} \left| 1 - \frac{1}{\rho^2} \right| = 0 \tag{A.16}
\]

Thus, we conclude that the proposed SMLC (3.6) is continuous at every time instant in the time domain.
Appendix B

B.1 Validation of the Lipschitz-Like Condition in a Discrete-Time Domain Given in the Inequality (7.13)

Consider the left side of (7.13)

\[ |\Delta V[k, k - 1] - \Delta V[k - 1]| \leq ||s[k, k - 1] - s[k] - s[k] + s[k - 1]| | \quad (B.1) \]

Firstly, we have

\[ s[k, k - 1] - s[k] = G\phi x[k] + G\Gamma u[k - 1] + Gd[k] - G\phi x[k - 1] \]
\[ -G\Gamma u[k - 1] - Gd[k - 1] \]
\[ = G\phi (x[k] - x[k - 1]) + G(d[k] - d[k - 1]) \]

Thus, one can obtain

\[ s[k, k - 1] - s[k] - s[k] + s[k - 1] = G\phi (x[k] - x[k - 1]) + G(d[k] - d[k - 1]) \]
\[ -G(x[k] - x[k - 1]) \]
\[ = G(\phi - I)(x[k] - x[k - 1]) + G(d[k] - d[k - 1]) \] \quad (B.2)

Substituting (B.2) into (B.1) yields

\[ |\Delta V[k, k - 1] - \Delta V[k - 1]| \leq ||G(\phi - I)(x[k] - x[k - 1]) + G(d[k] - d[k - 1])|| \]
\[ \leq ||G(\phi - I)|| ||x[k] - x[k - 1]|| + ||G(d[k] - d[k - 1])|| \] \quad (B.3)

With the approximation

\[ \phi = e^{AT} = I + AT + \frac{A^2 T^2}{2!} + O(T^3) \]
\[ G(\phi - I) = GA^2T^2 \frac{1}{2!} + O(T^3) \approx O(T) \]  \hspace{1cm} (B.4)

Also, one can have
\[
\|x[k] - x[k-1]\| = \|(\phi - I)x[k-1] + \Gamma u[k-1] + d[k-1]\|
\leq \|(\phi - I)\|\|x[k-1]\| + \|\Gamma\|\|u[k-1]\| + \|d[k-1]\| \hspace{1cm} (B.5)
\]

Considering the fact that \(\|d[k-1]\| \in O(T), \|\Gamma\| \in O(T), d[k] - d[k-1] \in O(T^2)\), and with the assumption that both \(x[k]\) and \(u[k-1]\) are smooth and bounded \([172]\), one can obtain the following
\[
\|x[k] - x[k-1]\| \leq O(T). O(1) + O(T). O(1) + O(T) \approx O(T) \hspace{1cm} (B.6)
\]
\[
\|G(d[k] - d[k-1])\| \in O(T^2) \hspace{1cm} (B.7)
\]

Substituting (B.4), (B.6) and (B.7) into (B.3) leads to:
\[
|\Delta V[k,k-1] - \Delta \overline{V}[k-1]| \leq O(T). O(T) + O(T^2) \approx O(T^2) \hspace{1cm} (B.8)
\]

If the sampling period \(T\) is sufficiently small such that the magnitude \(O(T^2)\) is very small, the change \(|\Delta V[k,k-1] - \Delta \overline{V}[k-1]|\) is actually very small. Based on this smoothness property of the discrete-time system, one can assume that the change \(|\Delta V[k,k-1] - \Delta \overline{V}[k-1]|\) remains within the small boundary of \(\frac{1}{M}|\Delta V[k-1]|\), where \(M \gg 1\). In other words, there always exists \(M \gg 1\) such that
\[
|\Delta V[k,k-1] - \Delta \overline{V}[k-1]| < \frac{1}{M}|\Delta V[k-1]|
\]

### B.2 Validation of the Inequality (7.15)

Using (B.6), it is easy to verify that
\[
\Delta V[k - 1] = \|s[k] - s[k - 1]\| \leq \|G\|\|x[k] - x[k - 1]\| \approx O(T) \quad (B.9)
\]

Also,
\[
\Delta V[k - 1] = G(\|x[k]\| - \|x[k - 1]\|) \approx O(T) \quad (B.10)
\]

From (B.9) and (B.10), one can consider the left side of (7.15) as
\[
\Delta V[k - 1] - \Delta V[k - 1] \approx O(T) \quad (B.11)
\]

Similarly, if the sampling period \(T\) is sufficiently small such that the magnitude \(O(T)\) is very small, the difference between \(\Delta V[k - 1]\) and \(\Delta V[k - 1]\) is actually very small. Thus, one can assume that the difference \(\Delta V[k - 1] - \Delta V[k - 1]\) remains within the small boundary of \(\delta\|s[k]\|\), where \(\delta \ll 1\). In other words, there always exists \(\delta \ll 1\) such that:
\[
\Delta V[k - 1] - \Delta V[k - 1] < \delta\|s[k]\|
\]
Author’s Publications

Peer Reviewed Journal Papers


Conference Publications


Bibliography


