Angular momentum in astrophysical discs and its impact on galaxy evolution

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Abstract

This thesis examines aspects of galaxy evolution from a theoretical perspective, aided by numerical experiments. The greatest focus is given to the fundamental role angular momentum plays in the formation of disc galaxies. Supporting this, details surrounding the cooling of hot gas, another vital component of galaxy formation, are investigated. To study the relevant astrophysical processes has involved development of semi-analytic models of galaxy formation, use of state-of-the-art cosmological, hydrodynamic simulations, and arguments from pure mathematics.

I first present the Semi-Analytic Galaxy Evolution (SAGE) model, developed by our research team at Swinburne, placing this in context of the historical development of semi-analytic models. I then extend this codebase to deliver a model with a much greater emphasis on the details of disc evolution in galaxies, culminating in DARK SAGE. With DARK SAGE, I explore the evolution of discs in annuli of fixed specific angular momentum, leading to a framework where disc evolution is described explicitly in terms of local angular momentum. Using this model, I investigate the origin of the net mass–spin relation of stellar discs observed in spiral galaxies. I show that the resolution of Toomre disc instabilities produces the observed correlation, as this process regulates the structure of discs, driving a greater fraction of mass into (pseudo)bulges for systems with lower specific angular momentum.

Next, I use the cosmological, hydrodynamic EAGLE simulations to learn about the nature of how gas cools onto galaxies and to test assumptions that go into semi-analytic models. I find that hot gas that cools has systematically higher specific angular momentum than that of the halo the gas resides in, where models often assume these to be equal. As this gas cools, most of its angular momentum is lost to its surroundings. In addition, the rotation axis of the cooling gas is initially better aligned with the cold gas already in the galaxy than with the rest of the hot gas. I show that gas settles with a surface density profile that is approximately exponential with a central cusp. The structure of these discs are in agreement with predictions made from DARK SAGE.

Further, I use the cosmological, hydrodynamic MassiveBlack-II simulation in conjunction with high-resolution zoom re-simulations of individual galaxies to address the topic of aperture bias in measuring the integrated properties of galaxies from simulations. I show that many of the techniques used in the literature are inherently flawed and often exclude a significant fraction of a galaxy’s mass. By finding a commonality in the cumulative baryonic mass profiles of simulated galaxies, I introduce a new aperture definition that minimises any aperture bias.

Finally, I extend the subject of angular momentum in discs down to the scale of black-hole accretion discs. Using purely analytic arguments from general relativity, I calculate the contribution that emission and scattering of photons in idealised scenarios could have in removing the
specific angular momentum necessary for accretion to take place. I find these processes can only be efficient in the inner tens of Schwarzschild radii of accretion discs around fast-spinning black holes.
Acknowledgements

It is mandatory that my first acknowledgements are of my supervisors. But I don’t just include them because they’re mandatory. In carefully considering where I wanted to do my PhD, I realised what mattered most was not the prestige of an institute, or even what the specific topic was. For me, what mattered was that I found a person I knew I would enjoy working with and who I thought would help me excel. That person was Darren Croton. Darren, your wisdom may only be finite, but there’s no limit to your support, encouragement, and positivity. Thank you for giving me the freedom to pursue the things that interested me, the ability to travel to so many eye-opening conferences, and thank you for steering me on track when things started to not go so well. I hope there are many more years to come in our professional relationship.

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meaningful relationships with people here (you know who you are) and I hope those friendships continue to last. Being around all of you has made what had the potential to be quite stressful mostly fun instead.

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Declaration

The work presented in this thesis has been carried out in the Centre for Astrophysics & Supercomputing at Swinburne University of Technology, and in the Institute of Astronomy and Kavli Institute for Cosmology Cambridge at the University of Cambridge, between 2013 and 2016. This thesis contains no material that has been accepted for the award of any other degree or diploma. To the best of my knowledge, this thesis contains no material previously published or written by another author, except where due reference is made in the text of the thesis.

The content of the chapters listed below has appeared or is due to appear in refereed journals. Minor alterations have been made to the published papers in order to maintain argument continuity and consistency of spelling and style. As most of these publications involved coauthors, the term ‘we’ (our) is used in the relevant chapters in place of ‘my coauthors and I’ (my). All text and figures in the works that I led were written and produced by me. The contents of Chapter 2 were co-led by me and my primary supervisor, Darren Croton, for which I have produced all the figures and written (or altered) most of text presented here (without making the presentation inconsistent with the paper).

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## Contents

Abstract i
Acknowledgements iii
Declaration v
Contents vii
List of Figures xi
List of Tables xv

1 The Universe 1
1.1 Cosmology 4
1.2 Structure formation 6
  1.2.1 Under ΛCDM 7
  1.2.2 Cosmological simulations 10
  1.2.3 Challenges and alternatives to cold dark matter 11
1.3 Astrophysics of galaxies 12
1.4 Simulated universes with galaxies 16
  1.4.1 Semi-analytic models 17
  1.4.2 Hydrodynamic simulations 18
1.5 Outline of this thesis 20
  1.5.1 A note on notation 22

2 Semi-Analytic Galaxy Evolution 25
2.1 N-body simulations 26
  2.1.1 The Millennium simulation 27
  2.1.2 The Bolshoi simulation 27
  2.1.3 The GiggleZ simulation suite 29
  2.1.4 Halo merger tree structure and required properties 29
2.2 An overview of SAGE 30
  2.2.1 Model calibration 31
2.3 Gas infall in haloes 34
2.4 Reionization 35
2.5 The hot gas halo 36
## Contents

2.6 Cold gas, star formation, and metal enrichment ........................................ 38
2.7 Supernova feedback and the galactic fountain ............................................ 43
  2.7.1 Reincorporation of ejected gas .......................................................... 44
2.8 Supermassive black holes and their feedback .............................................. 46
  2.8.1 The radio mode ..................................................................................... 46
  2.8.2 The quasar mode .................................................................................. 49
2.9 Galaxy mergers and intracluster stars ......................................................... 52
2.10 Disc instabilities ......................................................................................... 55
2.11 Starbursts ................................................................................................... 55
2.12 Discussion and summary ............................................................................. 56

3 Building galaxies with disc structure through angular momentum .......... 59
3.1 Physics and design of DARK SAGE ............................................................ 60
  3.1.1 Baryonic reservoirs and disc structure .................................................. 63
  3.1.2 Cooling of hot gas and formation of the interstellar medium ............... 64
  3.1.3 Passive star formation from molecular gas ....................................... 66
  3.1.4 Supernova feedback .............................................................. 68
  3.1.5 Gas precession .............................................................. 69
  3.1.6 Disc instabilities .............................................................. 70
  3.1.7 Stripping and accretion of subhaloes ............................................. 72
  3.1.8 Galaxy mergers .............................................................. 73
  3.1.9 Black holes and active galactic nuclei ......................................... 75
3.2 Galaxy structure .......................................................................................... 75
  3.2.1 Discs and pseudobulges ...................................................................... 77
3.3 Mass–spin relation of spirals ...................................................................... 81
  3.3.1 Stellar discs in the local Universe .................................................. 81
  3.3.2 Evolution of stellar discs .............................................................. 84
  3.3.3 Total stellar content .............................................................. 86
3.4 Outlook and conclusions ............................................................................. 89

4 Halo gas cooling in EAGLE .......................................................................... 93
4.1 The EAGLE simulations and data ............................................................... 95
  4.1.1 Overview of the simulations ............................................................. 95
  4.1.2 Halo finding ...................................................................................... 96
  4.1.3 Galaxy sample selection .............................................................. 97
4.2 Before cooling: hot gas in haloes ............................................................... 98
## Contents

4.2.1 Temperature profiles of hot gas ........................................... 98  
4.2.2 Density profiles of hot gas .............................................. 103  
4.2.3 Metallicity profiles of hot gas .......................................... 105  
4.2.4 Cooling-time profiles .................................................. 106  

4.3 Angular momentum of cooling gas ......................................... 110  
4.3.1 Conservation of angular momentum during cooling ...................... 110  
4.3.2 Relative orientation and magnitude of specific angular momentum 113  

4.4 After cooling: cold gas in galaxies ....................................... 116  
4.4.1 Radial surface density profiles of gas discs ............................ 116  
4.4.2 Angular-momentum structure of gas discs ............................. 119  

4.5 Summary .............................................................................. 122  

5 Where do galaxies end? .............................................................. 125  
5.1 The simulations ................................................................. 127  
5.1.1 The M12 simulations ...................................................... 127  
5.1.2 MassiveBlack-II ............................................................ 128  

5.2 Techniques for measuring galaxy properties ............................. 128  
5.2.1 Aperture choices in the literature ........................................ 129  
5.2.2 Employed subhalo finders ............................................... 132  
5.2.3 The ‘baryonic-mass profile’ (BaryMP) technique ....................... 133  
5.2.4 Employed aperture techniques ......................................... 135  

5.3 Results ................................................................................ 137  
5.3.1 Relative measurements for Milky Way-mass systems ............... 139  
5.3.2 Relative measurements for a broad galaxy population .............. 144  
5.3.3 Galaxy scaling relations ................................................... 146  

5.4 Discussion ........................................................................... 151  
5.4.1 Sizes of the simulated galaxies ......................................... 151  
5.4.2 Which method should one choose? ..................................... 151  

5.5 Conclusion ............................................................................. 152  

6 Angular momentum in black-hole accretion discs ......................... 155  
6.1 Mathematical formalisms and background ................................ 156  
6.2 Radiating and scattering away angular momentum .................... 157  
6.2.1 Pure, relatively isotropic emission ..................................... 158  
6.2.2 Relatively isotropic emission with internal angular-momentum transport 159  
6.2.3 Scattering with internal angular-momentum transport ............ 161
6.3 Discussion of the thin-disc picture ........................................ 163
6.4 Chapter recap .............................................................. 164

7 Thesis abridged ................................................................ 165
7.1 A good recipe includes SAGE ............................................ 166
7.2 The DARK SAGE semi-analytic model ................................. 166
7.3 How to get cool in the heat .................................................. 167
7.4 Integrated properties of simulated galaxies ......................... 168
7.5 Liberating angular momentum from accretion discs ............. 169
7.6 Upcoming research: phase 2 for the SAGE models ............... 170

Bibliography ........................................................................ 175

A Cosmological derivations .................................................... 199
A.1 Curvature ................................................................. 199
A.2 Derivation of the Friedmann Equation .............................. 201
A.3 Cosmological redshift .................................................. 204
A.4 Manipulation of the Friedmann Equation .......................... 205
A.5 Cosmic time ................................................................. 207

B Resolution study of SAGE ................................................ 209
B.1 Mass resolution ............................................................ 209
B.2 Temporal resolution ...................................................... 209

C Further details of DARK SAGE .......................................... 213
C.1 Model calibration .......................................................... 213
C.2 Rotation curves ............................................................ 217
  C.2.1 Dark matter .......................................................... 218
  C.2.2 Hot gas ................................................................. 219
C.2.3 Bulges and spheroids .................................................. 220
C.2.4 Intracluster stars ...................................................... 220

D Model-equivalent cooling rates from EAGLE ..................... 221

E The BaryMP aperture ......................................................... 225
E.1 The algorithm ................................................................. 225
E.2 The effect of the \( \epsilon \) parameter ..................................... 226
List of Figures

1.1 Relation between redshift and time ........................................... 6
1.2 Growth of large-scale structure in a simulated universe .................. 9
1.3 Simple depiction of galaxy evolution ........................................... 13

2.1 Stellar mass function at \( z = 0 \) for SAGE galaxies ....................... 32
2.2 Baryon breakdown in SAGE haloes at \( z = 0 \) ................................. 35
2.3 Effect of reionization on the SAGE stellar mass function ................. 36
2.4 Universal star formation history from SAGE ............................... 39
2.5 Kennicutt–Schmidt relation for SAGE galaxies at \( z = 0 \) ............... 40
2.6 Baryonic Tully–Fisher relation for SAGE galaxies ....................... 41
2.7 Mass–metallicity relation for SAGE galaxies ............................... 42
2.8 Stellar mass–specific star formation rate relation of star-forming galaxies in SAGE versus observations ........................................ 45
2.9 Luminosity–temperature relation of hot gas haloes in SAGE ............ 50
2.10 Black hole–bulge mass relation for SAGE galaxies ..................... 52
2.11 Quiescent fraction of galaxies in SAGE ..................................... 54

3.1 Overview of the DARK SAGE semi-analytic model ....................... 61
3.2 Stellar mass function of DARK SAGE galaxies at \( z = 0 \) .................. 63
3.3 Offset between gas and stellar discs in disc-dominated DARK SAGE galaxies .................................................. 70
3.4 Surface density profiles of Milky Way-like DARK SAGE galaxies compared with observations ................................................. 76
3.5 Normalised surface density profiles of DARK SAGE discs as a function of specific angular momentum ........................................... 78
3.6 Stellar mass functions of DARK SAGE and SAGE broken into disc-dominated and bulge-dominated galaxies ............................... 79
3.7 Size–mass relation of DARK SAGE disc galaxies compared with observations .................................................. 80
3.8 Mass–spin relation of stellar discs in spiral galaxies for DARK SAGE, SAGE, and observations .................................................. 82
3.9 How instabilities shape the mass–spin relation of galaxies in DARK SAGE .................................................. 83
3.10 Mass–spin relation of DARK SAGE galaxies when pseudobulges are dissociated from discs .................................................. 84
3.11 Evolution of the mass–spin sequence for stellar discs of DARK SAGE galaxies with and without the effect of disc instabilities .......... 85
3.12 Spin evolution of stellar discs as a function of their mass at \( z = 0 \) ....... 86
List of Figures

3.13 Mass–spin relation for all stars in DARK SAGE spiral galaxies compared to a simple model ........................................ 88
3.14 Ratios of stellar mass and specific angular momentum to that of the halo in DARK SAGE ........................................ 89
4.1 Ratio of a halo dynamical time to the snapshot separation in EAGLE .................. 98
4.2 Mass distribution of EAGLE haloes actively cooling ................................. 99
4.3 Temperature profiles of hot and cooling gas in EAGLE haloes ....................... 100
4.4 Temperature profiles of EAGLE haloes from alternate-feedback simulations ... 102
4.5 Density profiles of hot and cooling gas in EAGLE haloes ............................ 104
4.6 Evolution of the c_B parameter in EAGLE halo density profiles .................. 105
4.7 Metal fraction profiles of hot gas in EAGLE haloes ................................... 106
4.8 Cooling-time profiles of hot haloes in EAGLE ........................................... 109
4.9 Angular momentum lost by gas during cooling in EAGLE .......................... 111
4.10 Distributions of the ratio of specific angular momentum of cooling gas to that of the entire halo for EAGLE systems ...................... 114
4.11 Angular separations between rotation axes of gas particle groups during cooling .......................... 115
4.12 Angular separation between spin directions of hot and cooling gas as a function of radius ........................................ 116
4.13 Radial surface density profiles for cold and cooled gas in EAGLE galaxies ...... 117
4.14 Trends relating to the scale length of freshly cooled gas in EAGLE galaxies ..... 120
4.15 Surface density profiles as a function of specific angular momentum for cold and cooled gas in EAGLE galaxies ................................. 121
5.1 Example cumulative baryonic mass profiles of simulated galaxies with analytic fits 133
5.2 Example cumulative baryonic mass profiles of simulated galaxies, indicating where the gradient of the profiles becomes constant .......................... 134
5.3 Images of a high-resolution simulated galaxy at z = 2.15 with example apertures used for measuring integrated properties, seen before and after a subhalo finder is applied ........................................ 137
5.4 Images of a high-resolution simulated galaxy at z = 0 with example apertures used for measuring integrated properties ................................. 138
5.5 Imaged galaxy from the MassiveBlack-II simulation at z = 0.0625 with example apertures used for measuring integrated properties ................................. 138
5.6 Distribution and evolution of various aperture radii used for measuring the integrated properties of simulated Milky Way-like galaxies ................................. 140
### List of Figures

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.7</td>
<td>Distribution and evolution of stellar masses of simulated Milky Way-like galaxies measured within various apertures</td>
<td>141</td>
</tr>
<tr>
<td>5.8</td>
<td>Distribution and evolution of gas masses of simulated Milky Way-like galaxies measured within various apertures</td>
<td>142</td>
</tr>
<tr>
<td>5.9</td>
<td>Distribution and evolution of star formation rates of simulated Milky Way-like galaxies measured within various apertures</td>
<td>143</td>
</tr>
<tr>
<td>5.10</td>
<td>Evolution of gas accretion and ejection rates of Milky Way-like simulated galaxies measured within various apertures</td>
<td>144</td>
</tr>
<tr>
<td>5.11</td>
<td>Distribution of aperture size and integrated properties of MassiveBlack-II galaxies measured within those apertures at $z = 0.0625$</td>
<td>145</td>
</tr>
<tr>
<td>5.12</td>
<td>Relation between stellar mass and star formation rate of simulated galaxies determined from different aperture techniques</td>
<td>147</td>
</tr>
<tr>
<td>5.13</td>
<td>Kennicutt–Schmidt relation of simulated galaxies determined from different aperture techniques</td>
<td>150</td>
</tr>
<tr>
<td>6.1</td>
<td>Relative loss of specific angular momentum from a black-hole accretion disc to photons, as a function of radius</td>
<td>159</td>
</tr>
<tr>
<td>6.2</td>
<td>Relative loss of energy from a black-hole accretion disc to photons, as a function of radius</td>
<td>161</td>
</tr>
<tr>
<td>7.1</td>
<td>H I fraction of DARK SAGE satellite galaxies, in various environments, as a function of stellar mass</td>
<td>172</td>
</tr>
<tr>
<td>7.2</td>
<td>H I fraction of DARK SAGE satellite galaxies, in various environments, as a function of specific star formation rate</td>
<td>173</td>
</tr>
<tr>
<td>B.1</td>
<td>Stellar mass function variation in SAGE with mass resolution</td>
<td>210</td>
</tr>
<tr>
<td>B.2</td>
<td>Stellar mass function variation in SAGE with time resolution</td>
<td>211</td>
</tr>
<tr>
<td>C.1</td>
<td>H I mass function of DARK SAGE galaxies at $z = 0$</td>
<td>214</td>
</tr>
<tr>
<td>C.2</td>
<td>H$_2$ mass function of DARK SAGE galaxies at $z = 0$</td>
<td>215</td>
</tr>
<tr>
<td>C.3</td>
<td>H I mass fraction as a function of stellar mass for DARK SAGE galaxies at $z = 0$</td>
<td>215</td>
</tr>
<tr>
<td>C.4</td>
<td>Black hole–bulge mass relation for DARK SAGE galaxies at $z = 0$</td>
<td>216</td>
</tr>
<tr>
<td>C.5</td>
<td>Baryonic Tully–Fisher relation for DARK SAGE galaxies at $z = 0$</td>
<td>216</td>
</tr>
<tr>
<td>C.6</td>
<td>Mass–metallicity relation for DARK SAGE galaxies at $z = 0$</td>
<td>217</td>
</tr>
<tr>
<td>C.7</td>
<td>Universal star formation history from DARK SAGE</td>
<td>218</td>
</tr>
</tbody>
</table>
List of Figures

D.1 Model-equivalent cooling rates of EAGLE haloes using complete gas profiles versus analytic profiles .................................................. 222

E.1 Examples of how the BaryMP radius is calculated ........................................ 226
E.2 Average variation in results from the BaryMP technique for different values of the \( \epsilon \) parameter ..................................................... 227
E.3 Scatter in BaryMP radius and galaxy mass for varying \( \epsilon \) ......................... 228
List of Tables

2.1 $N$-body simulations used in conjunction with the SAGE .......................... 28
2.2 Parameter values used in the public release of SAGE ................................. 33
3.1 Fiducial parameter values for DARK SAGE ............................................. 62
5.1 Techniques used for measuring the integrated properties of simulated galaxies in
the literature ................................................................................................. 130
5.2 Aperture techniques used in the comparison of galaxy integrated properties .... 135
5.3 Summary of values and distributions of integrated properties of simulated galaxies
measured from various aperture techniques ............................................... 148
“You cannot believe in astronomical observations before they are confirmed by theory.”

A. E. Eddington, as quoted by S. Chandrasekhar (1974).
At the dawn of the twentieth century, our wider understanding of gravity and motion was still founded on the landmark laws introduced by Newton (1687). But just a few short years into that century, a deeper theory for the laws of motion was presented in the form of Special Relativity (Einstein 1905). A decade later, Einstein extended his work, presented as the General Theory of Relativity (Einstein 1915, 1916; Einstein et al. 1920), in which “Space tells matter how to move; Matter tells space how to curve” (Misner et al. 1973). Concepts such as distance, time, and mass were no longer understood to be absolute. Mathematically, this theory is expressed in terms of Einstein’s Field Equations (Einstein 1916, 1917),

\[ R_{\mu\nu} - g_{\mu\nu} \left( \frac{R}{2} - \Lambda \right) = \frac{8\pi G}{c^4} T_{\mu\nu}, \tag{1.1} \]

which relate gravity, the curvature of spacetime, and a ‘cosmological constant’ on the left-hand side to energy, momentum, and natural constants (the speed light and Newton’s gravitational constant) on the right-hand side (the explicit definitions of each term are defined where relevant throughout this thesis). In 100 years of physics research, this remains our best theory of gravity, with discoveries such as gravitational redshift (Pound & Rebka 1959; Pound & Snider 1964, 1965), gravitational delays in light travel time (Shapiro 1964; Shapiro et al. 1968), gravitational lensing (Walsh et al. 1979), and gravitational waves (Hulse & Taylor 1975; Abbott et al. 2016a) all supporting this description of the Universe.

While the world was beginning to digest General Relativity, velocity measurements of ‘nebulae’ gave rise to the first observational evidence that massive astrophysical bodies exist outside our Milky Way Galaxy (e.g. Slipher 1917), which naturally brought into question the scale of the Universe (Shapley & Curtis 1921). It soon became clear that, not only were these bodies extragalactic, but they came in a variety of morphologies (Hubble 1922, 1926). Furthermore, the distances of these systems were found to be correlated with their recessional velocities (Lemaître
1927; Hubble 1929), suggesting that the Universe was not only large, but that it was expanding. To add to that, before the century had finished, the expansion rate of the Universe was found to be accelerating (Riess et al. 1998; Perlmutter et al. 1999a). The distant objects from which we had learned all this¹ are modernly referred to as ‘galaxies’, a term derived from the Greek for ‘milky’. Indeed these galaxies are, in many respects, akin to our Milky Way.

If the Universe has always been expanding, then at some point in the past, it must have had an infinitesimal physical volume. There must, therefore, have elapsed a fixed amount of time since the Universe existed on a quantifiable scale. The discovery of the cosmic microwave background (Penzias & Wilson 1965) is widely regarded as confirmation that the Universe has a finite age. This radiation originated from the moment baryonic² matter and electromagnetic-force-carrying photons decoupled, around 400,000 years into the Universe’s existence. The distance travelled by these photons to reach us defines the surface of last scattering, which encompasses the entire observable Universe.

Concurrently in the last century came evidence that most of the matter in the Universe is non-baryonic and not luminous. The gravitational influence of this ‘dark matter’ was first observed in the motion of galaxies in the Coma cluster (Zwicky 1933, 1937). Soon after, it was also observed in the rotation curve of the Andromeda galaxy (Babcock 1939), which later lead to the same finding in other galactic discs (Freeman 1970; Rubin et al. 1980; Bosma 1981b). Famously, the Bullet Cluster (in fact, a merger of two clusters) was found to have clearly different distributions for its observed baryonic matter (from galaxies and X-ray-emitting hot gas) and total matter from gravitational lensing (Clowe et al. 2004). X-ray observations of dynamically relaxed clusters also indicate the majority of their mass to be in the form of dark matter, assuming the emitting gas to be in hydrostatic equilibrium (Vikhlinin et al. 2006).

The culmination of these remarkable discoveries has resulted in our standard cosmological model, named ‘ΛCDM’. Here, Λ is the cosmological constant present in Eq. 1.1, associated with the acceleration of the expansion of the Universe, which more generally is dubbed ‘dark energy’ (Huterer & Turner 1999; Perlmutter et al. 1999b). CDM stands for Cold Dark Matter, highlighting that dark matter must have a low thermal speed in order for gravitational collapse to occur across all astrophysically relevant scales, such that structure in the Universe is built hierarchically (see Section 1.2). Our best measurements to date suggest that, under ΛCDM, we are living in a Universe that is 13.8 billion years old, whose energy content is 69% dark energy, 26% dark matter, and 5% baryonic matter (with uncertainties smaller than the figure of significance given here –

¹In fact, supernovae observed in other galaxies lead to the discovery of accelerated expansion.
²In the proper particle physics sense, a ‘baryon’ is a particle composed of three quarks. For the purposes of astrophysicists, ‘baryonic matter’ or ‘baryons’ refer to all regular forms of matter. This includes, for example, electrons (and other mesons), despite these being composed of one quark and one anti-quark. The majority of regular-matter mass in the Universe (protons, neutrons) is genuinely baryonic though.
see Table 4 of Planck Collaboration 2016b). 75% of baryonic matter was initially in the form of hydrogen, and 25% in the form of helium (with a small fraction, \(<1\%\), of lithium – see Fields et al. 2014; Planck Collaboration 2016b). In $10^2$ years we have learned a lot about $\sim 10^{10}$ years of cosmic history, but naturally, there is much left to learn. For example, what dark matter and dark energy physically are is an ongoing area of research (e.g. Abbasi et al. 2010; Parkinson et al. 2012; Abbott et al. 2016b; Urquijo 2016).

While there is a wealth of information on what baryonic matter physically is, its role in astrophysics is vastly more complex to describe than dark energy or dark matter, owing to the multitude of ways it self-interacts (i.e. not just gravitationally, but electromagnetically, and through the strong and weak nuclear forces). While even early observations of galaxies last century gave us tremendous insight into the underlying nature of the Universe, understanding how those galaxies came to be the way they are requires a full consideration of complex baryonic physics. With advancements in instrumentation in the latter parts of the 1900’s came the rise of observational galaxy surveys (Tonry & Davis 1979; York et al. 2000; Colless et al. 2001). By studying the photometric and spectroscopic properties of large samples of galaxies statistically, the community has been able to place strict constraints on how galaxies must have formed and evolved. In the wake of these data, galaxy evolution has become one of the largest areas of research in astrophysics.

Our understanding of the Universe on large scales gives a framework to which any theory of galaxy evolution must adhere. But, to develop a sound scientific theory, one requires more than pure observation. For most fields of science, hypotheses can be tested through controlled experiments in a laboratory. Unfortunately, astronomers cannot construct other universes or galaxies in a laboratory, nor have we the technology to meaningfully affect the one Universe or Galaxy we are a part of. We must, therefore, place a strong emphasis on simulations to develop a theory of galaxy evolution. Coinciding with our enlightenment about galaxies and amassing of data, we entered the Computing Age. This has given us the ability to perform simulations of the Universe in enough detail to meaningfully interpret our data. Serendipitously, the physical scales over which modern integral-field unit surveys can resolve galaxies are comparable to the resolution of the most detailed cosmological simulations of volume $(100 \text{ Mpc})^3$ (cf. Sánchez 2015; Schaye et al. 2015).

This thesis contributes towards the field of galaxy evolution. My research focusses on developing semi-analytic models of galaxy evolution and interpreting state-of-the-art cosmological simulations. Using these tools, I highlight the fundamental importance angular momentum has in forming galactic discs and directing the evolution of galaxies. With this thesis, I address several scientific questions:

1. What drives the observed relation between the stellar mass and specific angular momentum of galaxies? To answer this, I have developed a model of disc galaxy evolution that functions
on local scales and has an explicit dependence on specific angular momentum.

2. How do galaxies accrete their angular momentum? I use cosmological, hydrodynamic simulations to assess their predictions and compare these against analytic models common in the literature.

3. How should the properties of simulated galaxies be best quantified? This is inherently tied to how the edge of a galaxy should be defined, for which there is a lack of conformity in the literature. I motivate and test a method that functions self-consistently for all simulated data.

4. How does material accreting onto a black hole lose its angular momentum? For this, I use general relativistic arguments to assess what could be lost through electromagnetic interactions in idealised scenarios.

The remaining sections of this chapter focus on providing an introduction to the foundational material on which my research is conducted.

1.1 Cosmology

The philosophical roots of cosmology can be traced to the Mediocrity Principle. In essence, this idea states that any phenomenon should be assumed normal, or not unusual, before it is assumed special or extravagant. An extension of this idea is that our location in the Universe is not special and that we are not privileged observers of the Universe. This is more commonly referred to as the Copernican Principle, named after Nicolaus Copernicus in honour of being the first to come forward with a scientifically plausible theory of the Solar System where the Earth was not placed at the centre (Copernicus 1543). By this thinking, we should expect the Universe to be roughly the same everywhere, and look approximately the same in all directions. In other words, averaged over large scales, the Universe should be homogeneous and isotropic. These are the pillars of the Cosmological Principle upon which our understanding of the Universe is founded. This is entirely supported by observations of the cosmic microwave background, which is found to only have small anisotropies that are consistent with Gaussianity (Planck Collaboration 2016a).

Homogeneity and isotropy are well-defined mathematical concepts which can be implemented in a description of the Universe by restricting the metric $g_{\mu \nu}$. An exact solution to the Einstein Field Equations that describes a homogeneous, isotropic Universe, with the freedom to be expanding (or contracting) and curved, can be found in the Friedmann–Lemaître–Robertson–Walker metric (Friedmann 1922, 1924, 1999; Lemaître 1927, 1931; Robertson 1935, 1936a,b; Walker
1.1. Cosmology

A detailed mathematical consideration of this metric is provided in Appendix A. This solution to Eq. 1.1 provides the mathematics that underlies the $\Lambda$CDM cosmological model, which is assumed throughout this thesis.

In an expanding Universe, modulo the peculiar motions from gravitational interactions on cosmologically small scales, more distant objects recede from us at faster rates. This means that not only does light from them take longer to arrive, but that light also undergoes a greater Doppler shift (redshift). As such, there is a one-to-one(-to-one) relation between the cosmological redshift of an observed distant object (e.g. a galaxy), $z$, the cosmic time at which the light we receive was emitted, $t$, and the relative size of the Universe at that time, $a$. As derived in Appendix A, one finds

$$a = \frac{1}{1 + z}, \quad (1.2a)$$

$$t(z) = \frac{1}{H_0} \int_z^\infty \frac{dz'}{(1 + z') \sqrt{\Omega_R (1 + z')^4 + \Omega_M (1 + z')^3 + \Omega_k (1 + z')^2 + \Omega_\Lambda}}. \quad (1.2b)$$

Here, $H_0$ is the Hubble constant, which describes the relative rate of expansion at the present epoch, which has the dimension of inverse time. The equivalent dimensionless Hubble parameter is

$$h \equiv \frac{H_0}{100 \text{ km s}^{-1} \text{ Mpc}^{-1}}. \quad (1.3)$$

$\Omega_R$, $\Omega_M$, $\Omega_k$, and $\Omega_\Lambda$ are the relative background energy densities of radiation, matter, curvature, and dark energy at the present epoch, respectively, which must all sum to 1. More precisely, these are ratios of the respective absolute densities to the critical density of the Universe,

$$\rho_{\text{crit}}(z) = \frac{3H_0^2}{8\pi G} \left( \Omega_R (1 + z)^4 + \Omega_M (1 + z)^3 + \Omega_\Lambda \right), \quad (1.4)$$

which is the average density of a universe without curvature that will asymptotically expand forever (see Appendix A.4). The Universe is currently in a state where radiation contributes negligibly to the overall energy content ($\Omega_R \approx 2 \times 10^{-5}$, Padmanabhan 2003) and observations are consistent with there being no curvature (de Bernardis et al. 2000; Planck Collaboration 2016b). In general, it is taken that $\Omega_k = \Omega_R = 0$ and the mean density of the Universe is $\bar{\rho} = \rho_{\text{crit}}$. Following common convention, throughout this thesis, epochs of the Universe will tend to be referred to by their associated cosmological redshift. Fig. 1.1 graphs how this relates with time, i.e. through Eq. 1.2b.

It is worth noting that other cosmological models for our Universe have not been ruled out. These often extend or generalise Eq. 1.1, affecting the functionality of gravity (e.g. Buchdahl 1970; Bekenstein 2004; Grande et al. 2010). Some of these models are motivated to explain the Universe without imposing the necessity for dark matter or dark energy. However, so far none of them has described the Universe more accurately than $\Lambda$CDM, and those that are equally as
predictive are more complex (which is undesirable under Occam’s razor). The focus of this thesis is thus given to $\Lambda$CDM.

1.2 Structure formation

Although homogeneous once smoothed over large scales, the Universe does have structure (on smaller scales). Our collective understanding is that quantum fluctuations gave rise to tiny perturbations in the initial density field of the Universe, and after a short period of inflationary growth, these were expanded to macroscopic scales (Guth 1981; Albrecht & Steinhardt 1982; Linde 1982). While inflation was wrapped up within $\sim 10^{-32}$ s, come the surface of last scattering at $\sim 10^{13}$ s, the fractional temperature fluctuations of the Universe were only of order $\sim 10^{-4}$ (as has been observed from the cosmic microwave background; Smoot et al. 1992; Planck Collaboration 2016a). Areas that are overdense (i.e. have a density greater than the Universe’s average) will naturally have a stronger gravitational attraction than those that are underdense. Even with an initially weak density contrast, over time, the overdense regions will attract more matter and continue to grow.

Without the gravitational collapse of structure, there would not be galaxies. One can directly see that galaxies are embedded in structure that spans larger scales by measuring their positions and redshifts (distances), and reconstructing a map. One of the earliest presentations of this came from the CfA redshift survey,\(^3\) which showed that galaxies are connected in a ‘cosmic web’ (Davis et al. 1982). This has since been shown in greater detail, such as by the 2-degree Field Galaxy

\(^3\)Named after the Harvard-Smithsonian Centre for Astrophysics.
1.2. Structure formation

Redshift Survey (Peacock et al. 2001). Of course, most of the underlying gravitating matter in the cosmic web is not visible. The most detailed images thus come from simulations (see Section 1.2.2 and Fig. 1.2).

1.2.1 Under $\Lambda$CDM

In the prevailing cosmological model, the nature of gravitational collapse is driven by cold dark matter (see Peebles 1982; Blumenthal et al. 1984). This means dark-matter particles slowed to a non-relativistic thermal speed in the very early Universe (within the first second – see Bringmann & Hofmann 2007). If the thermal motion of dark-matter particles is sufficient to move a distance greater than the scale of a perturbation before it has time to gravitationally collapse, then perturbations of that scale and below will be washed away. In the early stages of the Universe, the density field was dominated by radiation, for which $\rho_R \propto a^{-4}$. As the density of matter goes as $\rho_M \propto a^{-3}$ instead, the Universe transitioned to a period of matter domination at $z \approx 3000$. Around this point, gravitational collapse became an efficient process. The average distance travelled by a dark-matter particle up to this point sets its ‘free-streaming scale’, which marks the characteristic scale for gravitational collapse. The free-streaming scale of dark matter depends on when the particles became non-relativistic, which depends on the particle’s mass. *Cold* dark matter is defined such that its free-streaming scale is below the scale of galaxies. One of the first particle candidates to be hypothesised for cold dark matter (however now considered a candidate for warm dark matter – see Section 1.2.3) was a gravitino$^4$ of mass $\sim 1$ keV (Bond et al. 1982; Blumenthal et al. 1982), which is around the minimum mass required for the size of a dwarf galaxy to exceed the free-streaming scale. The favoured particle now is a neutralino$^4$ of mass $\sim 100$ GeV (Goldberg 1983; Ellis et al. 1984; for further details, see the review by Feng 2010), which corresponds to a free-streaming scale around the size of the Solar System, in which the mass of contained dark matter would be comparable to a terrestrial planet (Diemand et al. 2005; Green et al. 2005).

Gravitational collapse can be understood through the early work of Jeans (1902). Given a system of particles with a certain velocity dispersion and density, the Jeans scale describes the minimum scale necessary for that system to gravitationally collapse. Equivalently, the Jeans mass describes the mass enclosed within a sphere of radius the Jeans length. From linear perturbation theory, one can find an effective Jeans mass for dark matter:

$$M_{\text{Jeans}}(t) \propto \frac{\rho_M(t) \sigma_{\text{DM}}^2}{\rho^{3/2}(t)},$$  

(1.5)

$^4$Particle names ending in ‘-ino’ are hypothetical particles adhering to supersymmetry, for which every boson has a partner fermion with identical quantum numbers (other than spin, which differs by 1/2). For example, the gravitino is the supersymmetric partner of the graviton (the force-carrier for gravity). The neutralino is a mixture of supersymmetric particles and is charge-free.
where $\sigma_{\text{DM}}$ is the velocity dispersion (thermal speed) of dark matter, $\rho_M$ if the density of all matter, and $\bar{\rho}$ is the average density of the Universe (see above). If dark matter becomes non-relativistic and settles into the Hubble flow while the Universe is still radiation-dominated,\(^5\) then $\bar{\rho} \propto a^{-4}$ and $\sigma_{\text{DM}} \propto a^{-1}$. Hence, over this period, the Jeans scale remains constant, and is of the order the free-streaming scale. Once the Universe becomes matter-dominated, the Jeans scale begins to decrease, but perturbations at smaller scales have already been destroyed (see fig. 1 of Schneider et al. 2013). The first structures to collapse are therefore of the order of the free-streaming scale.

Overdense regions of the Universe are the formation sites of galaxies (and galaxy groups and clusters). Classically, the matter that surrounds a galaxy has been referred to as a ‘halo’. As such, collapsed overdensities, dominated by dark matter, are generally referred to as ‘haloes’. The mutual gravitational attraction of haloes leads to mergers, where smaller haloes combine to form larger haloes. In this way, the structure of the Universe is built hierarchically. This not only allows incredibly large haloes of mass $>10^{15} \, M_\odot$ to form (see, e.g., Sifón et al. 2013; Bellstedt et al. 2016), but has important consequences for how the galaxies in those merging haloes evolve (see Sections 2.9 and 3.1.8). Mergers are naturally more prominent in the most dense environments. In the lead up to a merger, the smaller halo will become embedded within the larger one, and thus is labelled a ‘subhalo’. Each subhalo can form its own galaxy. As a result, the largest haloes in the Universe, found at the intersections of filaments in the cosmic web (see Fig. 1.2), are host to galaxy clusters. Whether a visible galaxy will form in a subhalo, and how that galaxy evolves, is then dependent on a raft of factors (see Sections 2.9 and 3.1.7).

As overdensities continue to grow and form larger haloes, they will eventually reach a state of virial equilibrium. This relaxed state is found when the internal kinetic energy of a halo is twice the magnitude of its gravitational potential energy. It can be shown analytically that a uniform, overdense sphere, in a matter-dominated universe without curvature, will collapse to the point of virialization when its average density reaches $18\pi^2 \simeq 178$ times the background density. Cole & Lacey (1996) have shown that, for haloes in $N$-body simulations (see Section 1.2.2), the radius at which the average internal density is $178\rho_{\text{crit}}$ well describes the boundary between the virialized region and the region of collapse. In truth, the precise value depends on the chosen cosmology. For roundedness, the virial radius of a halo, $R_{\text{vir}}$, is generally regarded as that for which the average internal density is $200\rho_{\text{crit}}$. The virial mass, $M_{\text{vir}}$, is then the mass enclosed within this radius, and the virial velocity, $V_{\text{vir}}$, is the circular orbital velocity at this radius. These definitions are used throughout this thesis.

The concentration of matter leads to the generation of a tidal field. This results in haloes

\(^5\)During radiation domination, $H^2 \propto (1 + z)^4 \rightarrow H \propto a^{-2} \rightarrow \dot{a} \propto a^{-1}$. Given $\sigma_{\text{DM}} \propto \dot{a}$, then $\sigma_{\text{DM}} \propto a^{-1}$. See Appendix A.4 for further mathematical details.
1.2. Structure formation

Figure 1.2: Growth of large-scale structure in a simulated universe using the RAMSES code
(Teyssier 2002), with initial conditions generated using MPGRAFIC (Prunet & Pichon 2013). This
assumes a ΛCDM cosmology with $\Omega_M = 0.3$, $\Omega_\Lambda = 0.7$, and $h = 0.7$. The simulation box is
$50h^{-1}$ comoving Mpc in each dimension with periodic boundaries, and contains $128^3$ particles.
Each panel is a projection of the entire box, contrasting *comoving* column densities from $10^{-0.5}$
to $10^{3.5} \ M_\odot \ \text{pc}^{-2}$. 
being subject to torques, thereby acquiring angular momentum (Hoyle 1951; Peebles 1969). To describe tidal torques analytically is non-trivial and requires consideration of structure growth in the non-linear regime. With a simplified model, Peebles (1969) described the angular momentum of haloes through the dimensionless spin parameter

$$\lambda \equiv \frac{J|E|^{1/2}}{GM^{5/2}},$$

(1.6)

where $J$ is the total angular momentum of the halo, $E$ is its total (kinetic + potential) energy, and $M$ its total rest mass. Peebles (1969) suggested haloes should have $\lambda \simeq 0.08$. By running numerical simulations with cold dark matter, Barnes & Efstathiou (1987) established that the spin of haloes could be described by an almost log-normal distribution of $\lambda$, centred very near to Peebles’ value. The baryons in haloes are subject to the same tidal torques as the dark matter, which ultimately is how galaxies obtain their angular momentum. Much of this thesis, especially Chapters 3 and 4, focusses on the importance of angular momentum in galaxies and its consequences for how they evolve.

1.2.2 Cosmological simulations

While the large-scale structure of the Universe is visible from the manner in which galaxies are distributed, to completely visualise the cosmic web, one needs to run simulations. A cosmological $N$-body simulation works by discretising a universe into a finite number of particles, in a box that is of finite size but has periodic boundaries. The particles are first distributed uniformly throughout the box, then their positions are perturbed, thereby seeding overdensities, which are based on observations of the cosmic microwave background. The positions of these particles then evolve under the influence of gravity, allowing one to directly see the formation of the cosmic web.

Solving for the gravitational force imparted on each particle $i$ from each particle $j$,

$$\vec{F}_{i,j} = -\vec{F}_{j,i} = -\frac{Gm_i m_j}{R_{i,j}^3} \vec{R}_{i,j},$$

(1.7)

is too computationally expensive for cosmological simulations, given the large number of particles necessary (in modern simulations, $\gtrsim 10$ billion particles are used, e.g. Klypin et al. 2016). This is usually combated by one of two ways. One method is to employ a Tree algorithm, where particles are grouped, such that Eq. 1.7 is solved for the common centre of mass of each group (Appel 1985; Barnes & Hut 1986). This reduces the number of calculations necessary from order $N^2$ to order $N \log N$ (where $N$ is the number of particles). Alternatively, there is the Particle-Mesh (PM)
method, which discretises the simulation volume into cells and calculates the gravitational field through a Fourier transform (Eastwood & Hockney 1974; Hohl 1978). For neighbouring particles, Eq. 1.7 can still be solved directly to increase accuracy on small scales (the Particle-Particle-Particle-Mesh method or P³M; Hockney & Eastwood 1981). The grid size can also be refined in particle-busy regions with this method (Couchman 1991). The Tree and PM methods can further be combined for maximal efficiency, with the former dealing with calculations on smaller scales, and the latter on larger scales (Xu 1995).

To provide an example, Fig. 1.2 shows the growth of large-scale structure from a cosmological N-body simulation. This was run with RAMSES (Teyssier 2002), a publicly available adaptive-mesh code. Here, the filaments and voids of the cosmic web are seen to form. The N-body simulations used in this thesis for science purposes (see Section 2.1) include larger volumes and are of higher resolution than this example.

1.2.3 Challenges and alternatives to cold dark matter

While ΛCDM is the simplest model (with 6 parameters) that can simultaneously explain the large-scale structure of the Universe observed from galaxies and the structure of anisotropies in the cosmic microwave background, there remains debate over areas of potential conflict with some observations. Soon after the formulation of the cosmological model, it was noted that the number density of subhaloes produced by $z = 0$ in ΛCDM N-body simulations vastly exceeded the number density of observed satellite galaxies (Moore et al. 1999; Klypin et al. 1999). This ‘missing satellites problem’ was based on simple arguments that the relative mass of baryons contained in subhaloes should be equivalent to that of the cosmic baryon fraction, and that most of these baryons would be bright, i.e. in the form of stars. This, however, did not take into account any feedback associated with baryonic physics (see Section 1.3), which can prevent baryons from turning into stars, which has since been shown can solve the problem (Zolotov et al. 2012; Brooks et al. 2013).

An affiliated potential issue surrounding ΛCDM, dubbed ‘too big to fail’, is that the most massive subhaloes in simulated Milky Way-mass haloes are too dense to house satellite galaxies as bright as what is found around the Milky Way (Boylan-Kolchin et al. 2011). Again, these simulations excluded any consideration of baryonic physics. Fully hydrodynamic ΛCDM simulations (see Section 1.4.2) produce subhaloes with density profiles less steep than the pure N-body case, and can produce the right number of satellite galaxies about the Milky Way with the correct brightness (Wetzel et al. 2016).

By Occam’s razor, baryonic physics is the most favourable solution to the ‘missing satellites problem’ and ‘too big to fail’. However, the implementation of feedback physics in simulations is not perfect, and there may well be unexplored issues that arise as our simulation methods be-
come more sophisticated. An alternative solution to these challenges may lie in dark matter not being ‘cold’, but instead being ‘warm’. If the dark-matter particle is lighter, it will carry a higher thermal velocity, and hence have a longer free-streaming scale (Section 1.2.1). This will suppress perturbations from growing at larger scales. Candidates for warm dark matter include the sterile neutrino or gravitino (see Viel et al. 2005). These have masses around \( \sim 10^3 \) eV and would return a free-streaming scale of the order of a dwarf galaxy. The formation of small haloes (which would later become subhaloes) would hence be suppressed, leaving Milky Way-mass galaxies with fewer satellites. All larger structure would still be formed hierarchically. Simulations involving warm dark matter are ongoing and have had success demonstrating these results (e.g. Lovell et al. 2014), but there are problems surrounding the implementation of a sizeable free-streaming scale, where the discretisation of particles still leads to the formation of small subhaloes (see Schneider et al. 2013). Although structure below the free-streaming scale can form through dissipative fragmentation.

While the existence of warm dark matter is yet to be ruled out, ultra-relativistic dark matter can be ruled out, as this would eliminate hierarchical growth from too-large scales. In the absence of compelling evidence for warm dark matter being favourable, \( \Lambda \)CDM will be assumed throughout the rest of this thesis.

### 1.3 Astrophysics of Galaxies

This section provides a brief overview of the key physical processes involved in galaxy evolution. As part of this thesis is devoted to models of galaxy evolution, these processes will be described in more detail, with accompanying mathematics, in Chapters 2 and 3. Fig. 1.3 illustrates how baryons move between different parts of the halo and galaxy according to these processes at the simplest level.

Much of the collective wisdom on how galaxies first formed was outlined by Rees & Ostriker (1977) and White & Rees (1978). In the initial stages of structure formation in the Universe, baryonic matter, in gaseous form, follows dark matter in its gravitational collapse. Gas thus either collapses directly onto haloes or accumulates in filaments (cf. Fig. 1.2). In the latter case, gas is attracted through filaments towards haloes. Once in a sizeable potential well, gas can lose its energy more efficiently than dark matter through electromagnetic interactions. As gas cools, it begins to condense at the centre of the halo. Tidal torques (see Section 1.2.1) dictate that all haloes and their gas will carry some net angular momentum, which should, to first order, be conserved during cooling (see Peebles 1969; Efstathiou & Jones 1979; Barnes & Efstathiou 1987). This then leads to the formation of a rotating, flattened disc (e.g. Fall & Efstathiou 1980), which is the least energetic configuration for a structure with angular momentum. For haloes of moderate size, gas
Figure 1.3: A simple depiction of how galaxies evolve, indicating rough scales on the left-hand side (applicable at low redshift) and baryonic components on the right. Gas from filaments falls into the halo and can accrete directly onto (or form) a galaxy through cold streams. Hot gas in the halo cools onto the galaxy as well. Cold gas in the disc forms stars, which in turns leads to stellar feedback, which returns gas to the interstellar medium and can heat gas out of the disc. Accretion of hot and cold gas onto a central black hole leads to ‘active galactic nucleus’ (AGN) feedback. This visualises how processes are typically treated in a semi-analytic model.
is able to cool rapidly to form a disc galaxy. For those that are virialized and of a critical mass, gas can become shock heated upon infall, setting up a quasi-static hot-gas halo, where cooling flows are restricted. However, the merging of haloes and subhaloes leads to further accumulation of cold gas. As massive haloes cannibalise many smaller haloes, massive galaxies usually have some supply of cold gas maintained.

Once a gas disc has formed, fragmentation occurs, leading to the formation of gas clouds on scales up to hundreds of parsecs. The increased density and shielding from energetic photons of these clouds allows hydrogen to form molecules (e.g. Hollenbach et al. 1971; Glover et al. 2010). Further fragmentation occurs within these clouds (à la Jeans 1902), where each fragment gravitationally collapses to form a star. Much of the gravitational energy lost during collapse is transformed by dissociating molecules, ionizing much of the gas, and increasing its thermal energy (see Larson 1969). Collapse halts at the point hydrostatic equilibrium is reached, i.e. where the pressure induced from fusion balances the star’s self-gravity. The temperature and density in the inner regions of stars are sufficiently high for nuclear fusion to occur. This produces heavier elements, and, in doing so, releases the potential energy of the nucleons. After travelling through radiative and conductive zones (the exact make-up of which depends on the mass of the star), the energy from fusion is transferred to the surface of the star, where it is primarily released in the form of thermal (electromagnetic) radiation. As the surface temperatures of stars are typically thousands of Kelvin, a significant portion of its radiation is in the optical spectrum. Gratefully, this makes the Universe bright.

The life-time of a star is finite; at some point, the fuel for conducting fusion will be consumed. While massive stars have more fuel to burn, their significantly higher densities mean they are able to fuse the available hydrogen in their cores much faster. In the latter stages of their lives, they are able to not just conduct fusion in their cores, but in a series of shells (with heavier elements being fused nearer the centre). Ultimately, their life-times are shorter than low-mass stars (many of the important aspects of stellar evolution are covered in the early review by Iben 1967). Without a heat source to provide pressure to counteract gravity, the core of the star will undergo gravitational collapse. Much of the energy lost by the collapsing core is transferred to the outer regions of the star, which can cause the stellar envelope to expand by orders of magnitude and can lead to energetic outflows. These outflows return gas to the interstellar medium and enrich it with metals. The strength of the outflow and the type of remnant left at the end of the star’s life depends on the initial mass of the star. If the star’s core is relatively small, electron degeneracy pressure (caused by electrons being unable to occupy the same quantum state from Pauli’s exclusion principle – Pauli 1925; Stoner 1929) will balance gravity, leaving a white dwarf (typically composed of helium, In the true chemical sense, ‘metals’ make up a fixed subset of the periodic table of elements. In the astrophysics community, a ‘metal’ is regarded as any element that is not hydrogen or helium.)
oxygen, and/or carbon) at the centre of a planetary nebula. If the remnant exceeds a threshold mass of \(\sim 1.4 \, \text{M}_\odot\) (Chandrasekhar 1931),\(^8\) it will collapse into a neutron star, supported by neutron degeneracy pressure. If the remnant further exceeds a mass of \(\sim 2 \, \text{M}_\odot\) (cf. Bombaci 1996; Kiziltan et al. 2013), then gravity will overcome neutron degeneracy pressure, and so it will collapse into a black hole (Oppenheimer & Volkoff 1939; Tolman 1939). The formation of neutron stars and black holes are typically coupled with a supernova, where the outer layers of the original star are energetically driven away from the collapsar. Stellar winds and supernovae heat the interstellar medium and are capable of ejecting gas back out into the halo. Even ‘protostellar outflows’ can occur during the initial stages of a star’s formation (see the review by Frank et al. 2014). In this sense, stars are a source of feedback, which ensures star formation is a self-regulated process (for recent discussion and simulation advancement on this topic, see Padoan et al. 2014; Federrath 2015, respectively).

Beyond the stellar-mass black holes produced during a supernova, most galaxies have been measured to host a massive (or ‘supermassive’) black hole at their centre. For the Milky Way, this can be seen directly from the motions of stars at the Galactic Centre, which show Keplerian orbits about a point mass roughly four million times the mass of the Sun (Schödel et al. 2002; Ghez et al. 2008). Many galaxies are known to have much larger black holes; as early as \(z \sim 6\), supermassive black holes with masses \(>10^{10} \, \text{M}_\odot\) have been detected (e.g. Wu et al. 2015). How these black holes formed is still an open question; one potential solution is runaway collisions in very early-formed star clusters (see Katz et al. 2015). Regardless, the accretion of gas onto black holes has important consequences for galaxy evolution. For gas to accrete onto a black hole, it must lose most of its angular momentum and energy. That energy can result in outflows in the form of relativistic collimated jets, which heat the surrounding medium (Lynden-Bell 1969). This leads to a self-regulating feedback process, where the cooling of gas onto the galaxy is suppressed, which reduces the fuel for star formation and further accretion. When two galaxies merge, their central black holes will be attracted to the bottom of the potential well and will themselves merge.

Historically, galaxies have been classed by their stellar morphology, as observed in the optical spectrum (Hubble 1926). It is well documented that the stellar morphologies of galaxies range from being rotationally supported, flattened discs, to dispersion-supported, elliptical objects, with everything in between, and combinations thereof. Stellar discs are generally understood to form out of a gas disc, and can grow through the accretion of smaller galaxies. Ellipsoids are understood to form either through gravitational instabilities in discs driving low-angular-momentum material

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\(^8\)Chandrasekhar’s original value was 0.91 \(\text{M}_\odot\), but this has been revised on many occasions (see reviews by Ostriker 1971; Hartle 1978; Iben & Renzini 1984). A white dwarf that accretes mass and reaches the Chandrasekhar limit will collapse into a neutron star and is understood to generate a Type-Ia supernova (see the review by Hillebrandt & Niemeyer 2000).
Chapter 1. The Universe

into the centres of galaxies (Kormendy & Kennicutt 2004), or through major galaxy mergers, which can destroy discs, and upset any pre-existing kinematic structure, as evidenced by elliptical galaxies being particularly common in dense environments (Dressler 1980).

Of course, gas is just as much a central component of a galaxy as stars are. The majority of gas in galaxies is in the form of neutral atomic hydrogen, \( \text{H}^\text{I} \). Without excitation, hydrogen atoms are not inherently bright. They do, however, passively emit at a wavelength of 21.1 cm, caused by the flip of the spin of the electron in the ground state of the atom. In addition, the molecular gas in galaxies, the majority of which is in the form of \( \text{H}_2 \), can be traced from the radio emission of carbon monoxide (see Young & Scoville 1991; Obreschkow & Rawlings 2009). Early observations with radio telescopes showed that gas discs reside in galaxies of a variety of stellar morphologies, that they are more extensive than stellar discs, and that they often exhibit warps (Bosma 1981a). Most gas studies of galaxies have been restricted to the local Universe, as the 21-cm signal is relatively weak; at present, the most distant galaxy seen from \( \text{H}^\text{I} \) emission is at \( z = 0.376 \) (Fernández et al. 2016), whereas the most distant galaxy (candidate) observed in the optical is at \( z = 11.1 \) (Oesch et al. 2016, based on a combination of grism spectroscopy and photometry). We can also learn about the hot gas environment around galaxy groups and clusters from X-ray emission, primarily generated by Bremsstrahlung. Again, this is far less advanced than optical studies of galaxies, as the weakness of emission from low-mass haloes (comparable to the Milky Way and smaller) makes it difficult to detect (but see, e.g., Anderson & Bergman 2011). Our understanding of gas in galaxies and haloes must therefore rely more on models and simulations (e.g. Lagos et al. 2015b; Sokołowska et al. 2016).

1.4 Simulated Universes with Galaxies

Cosmological \( N \)-body simulations provide a powerful way to simulate the growth of structure in the Universe. By only considering gravity as the means of particles interacting, these are representative of a universe composed entirely of dark matter, and hence, by themselves, miss all the baryonic physics associated with galaxy evolution. The question then is, what kind of galaxies should the haloes formed in these simulations house?

One way to populate haloes from a simulation is through statistically connecting them to observed galaxies. A Halo Occupation Distribution model prescribes more massive haloes to host a larger number of galaxies, with a minimum halo mass required for hosting one galaxy (Benson et al. 2000; Peacock & Smith 2000; Seljak 2000). The specific assignment of galaxies to haloes is calibrated to match the observed clustering of galaxies. An alternative and similar method to populate individual subhaloes with galaxies is the Subhalo Abundance Matching technique (Conroy et al. 2006). For a fixed redshift, one ranks the maximum circular velocities (or masses) of all
subhaloes at their time of infall, then ranks observed galaxies by their luminosity (or stellar mass), and populates each subhalo with the galaxy of the same rank. While Halo Occupation Distribution and Subhalo Abundance Matching can be used for the construction of mock galaxy catalogues, they offer only limited physical insight as to how galaxies evolve.

Beyond the above, two modes of approach have been prevalent in the literature for simulating the physical evolution of galaxies in a cosmological framework: semi-analytic models and hydrodynamic simulations. Semi-analytic models use the properties and histories of haloes from an $N$-body simulation, thereby evolving the galaxies as a post-processing step. Hydrodynamic simulations solve the equations of fluid mechanics directly in the simulation itself and include radiative cooling and relevant feedback physics. Both methods have their advantages, and each are described below. Further details can be found in the recent reviews by Benson (2010) and Somerville & Davé (2015).

### 1.4.1 Semi-analytic models

Semi-analytic models of galaxy evolution take advantage of the relative computational efficiency of $N$-body simulations by adding the bound baryons to a simulation as a post-processing step. By using information about the gravitationally bound haloes, such as their mass, size, spin, substructure, and merger history, the properties of galaxies hosted within these structures can be inferred through differential equations that describe the relevant physics macroscopically. The runtime of such a model is orders of magnitude less than the $N$-body simulation itself. Hence, after that initial investment, one can explore large regions of the parameter space that underly the baryonic physics by running the model many times.

In the original implementation of the semi-analytic method (White & Frenk 1991), haloes were drawn statistically from the Press–Schechter formalism for gravitational collapse (Press & Schechter 1974; later extended by Bond et al. 1991; Bower 2001), and the analytic baryonic physics was based on the milestone theory of White & Rees (1978). Kauffmann et al. (1993) introduced the use of merger trees for a model, where individual systems were connected across time within Press–Schechter theory. This formalism only provides a statistical take on what the masses of the progenitors of a halo are, however, which induces uncertainty in its history. Merger trees produced from an $N$-body simulation, where individual objects are directly tracked, do not suffer this issue, and are therefore preferable. These were first introduced into the semi-analytic framework by Kauffmann et al. (1999). Now, it is standard practice for semi-analytic models to be coupled to an $N$-body simulation (e.g. Hatton et al. 2003; Bower et al. 2006; Croton et al. 2006; Benson 2012; Henriques et al. 2015). For a more in-depth look at the history of semi-analytic models, see Baugh (2006).
Chapter 1. The Universe

The simplified picture of galaxy evolution in Fig. 1.3 shows how galaxy evolution processes are treated in a semi-analytic model. Haloes are initialised to carry a mass of hot gas that is determined by the cosmic baryon fraction. As haloes grow in their dark-matter content, so too do they accumulate fresh gas. The amount of gas that cools onto a galaxy at each time-step will depend on the total mass and metallicity of the gas in the halo. If a halo is sufficiently small, gas can fall onto the galaxy rapidly through cold streams (Rees & Ostriker 1977). The cooled gas will carry an analytic surface density profile, with a scale radius determined from the specific angular momentum of the halo. Subsequently, the mass of stars that form in the cold gas disc will depend on the mass and size of the gas disc. Every episode of star formation will heat gas out of the disc and increase the fractional contribution of metals to the gas mass. Black holes can accrete gas through the disc or directly from the halo, which in turn leads to ‘active galactic nucleus’ (AGN) feedback, which suppresses the cooling rate of gas, and can heat gas out of the cold disc. Each of these processes, and how they are handled in a semi-analytic model, are discussed in more detail in Chapter 2, where the recently published SAGE model (Croton et al. 2016) is presented.

While coarse in their description of astrophysics, semi-analytic models can be powerful tools to assess the relative importance of certain galaxy evolution processes. For example, the inclusion of AGN feedback in semi-analytic models highlighted the impact this could have on the stellar mass function, suppressing the most massive galaxies from forming new stars (Croton et al. 2006). Furthermore, their ability to work with large simulation volumes makes them ideal for the production of mock galaxy datasets (e.g. Blaizot et al. 2005; Overzier et al. 2013; Bernyk et al. 2016).

1.4.2 Hydrodynamic simulations

In addition to evolving discretised dark matter through gravity, in a hydrodynamic simulation, discretised baryonic matter is directly evolved through gravity and fluid mechanics, and is subject to radiative cooling and feedback processes from galaxies. This requires relating the density of gas, calculable in the simulation, to temperature and pressure through an equation of state. Stars and black holes are created on the fly and can directly impact nearby gas through outflows and supernovae. A plethora of techniques for implementing gas dynamics, star formation, and feedback in simulations exists, for which an overview is provided here. For the specific case of the EAGLE simulations, see Section 4.1.

One method for including fluid dynamics in an $N$-body simulation is the Lagrangian ‘smoothed-particle hydrodynamics’ (SPH) scheme (see the review by Price 2012a). Although originally developed for stellar simulations (Gingold & Monaghan 1977; Lucy 1977), SPH simulations of galaxy formation begin with two particle species. One is the dark-matter particle, which is treated
the same as a particle in a regular $N$-body simulation. The other is the gas particle, which is not only influenced by gravity, but is also subject to pressure forces, and hence is collisional. To determine the strength of hydrodynamical effects, a density, temperature, and pressure is calculated for each particle by smoothing over its neighbours. Gas particles are subject to radiative cooling, which requires some description of their chemical composition. Gas particles can generate or be transformed into star particles during a simulation (see below), where they are no longer subject to collisions. Given that the formation scenario of supermassive black holes is unknown, black-hole particles are generated manually, typically once a halo reaches a threshold mass (e.g. Springel et al. 2005b). The GADGET code (Springel et al. 2001a; Springel 2005) is popularly used for SPH simulations of galaxy formation (and is used in simulations analysed in this thesis), but other SPH codes are common too, e.g. GASOLINE (Wadsley et al. 2004) and GIZMO (Hopkins 2015).

Another means of including hydrodynamics in an astrophysical simulation is with an Eulerian, grid-based approach. The simulation box is broken into cells, where each cell has its own density, pressure, and temperature, which varies with time. This allows the hydrodynamic equations to be solved precisely at the boundaries between cells. Particles are still used for dark matter, stars, and black holes in these codes (see, e.g., the sink particle techniques for stars and black holes by Federrath et al. 2010; Dubois et al. 2012, respectively). It is usual to employ an Adaptive Mesh Refinement (AMR) scheme (Berger & Colella 1989), where dense cells above a threshold are broken into smaller cells, allowing more resolution to be placed in the areas of higher density, as is naturally done in SPH. Many AMR codes are widely used for cosmological simulations in the literature, e.g. ART (Kravtsov et al. 1997), RAMSES (Teyssier 2002), and ENZO (Norman et al. 2007).

There are pros and cons to each of AMR and SPH. For example, the Lagrangian approach of SPH is better suited to angular-momentum conservation and the handling of bulk flows than AMR, but SPH can have issues with discontinuities, which can arise from shocks (see Price 2012b). Recently, a hybrid Lagrangian-Eulerian method, based on a moving mesh, has become a primary contender for simulating galaxies (Springel 2010). The hydrodynamic simulations presented in this thesis, however, are an advanced form of SPH (with the exception of the Martig et al. 2012 simulations, which use a sticky-particle method – Section 5.1.1).

The most detailed, cosmologically representative, hydrodynamic simulations to date, that have been run to $z = 0$, reach a baryonic mass resolution of $\sim 10^6 \, M_\odot$ and spatial resolution of $\sim 700$ pc (Vogelsberger et al. 2014; Schaye et al. 2015). Even simulations that focus on individual galaxies
clusters or galaxies only achieve resolutions of $\sim 10^3 \, M_\odot$ (e.g. Powell et al. 2011; Brook et al. 2012a; Governato et al. 2012), which is comparable to the scale of giant molecular clouds, where stars are formed. This means that the process of star formation is not resolved. As such, ‘sub-grid’ models are employed for processes like star formation, where a gas particle has a probability of producing a star particle should its local density be sufficient to expect a non-zero star formation rate (based on Schmidt 1959). A single star particle represents a population of stars, from which stellar feedback can impart energy (and/or momentum) on neighbouring gas particles and raise their metal fractions (e.g. Springel & Hernquist 2003). Similarly, the accretion of gas onto black holes is not resolved and requires a sub-grid prescription. Feedback from AGN is also applied by directly affecting the temperature and/or velocity of gas particles near the black hole (e.g. Di Matteo et al. 2005; Taylor & Kobayashi 2014). The nature of sub-grid physics in hydrodynamic simulations means they are essentially semi-analytic at the level of the particle, rather than at the level of the galaxy.

While vastly more detailed than semi-analytic models, hydrodynamic simulations are considerably more computationally expensive. In fact, the addition of hydrodynamics comes at the cost of approximately two orders of magnitude in particle number; for example, the main run of the hydrodynamic EAGLE simulations (Schaye et al. 2015) used $2 \times 1504^3 \simeq 6.8 \times 10^9$ particles, but required approximately the same number of floating point operations as the main run of the pure $N$-body Dark Sky simulations (Skillman et al. 2014) with $1024^3 \simeq 1.1 \times 10^{12}$ particles. Semi-analytic models and hydrodynamic simulations are thus complementary methods for simulating galaxies; the best option will depend on the science question one wishes to explore. Both are used in this thesis.

1.5 Outline of this thesis

The overarching themes of this thesis are to (i) converge on a self-consistent model of galaxy evolution that considers key physical processes and can be related to the properties of observed galaxies; (ii) examine the fundamental importance of angular momentum in astrophysical discs, including how galactic discs obtain their angular momentum, how angular momentum is structured, and what consequences arise for the properties of galaxies as a result of the accretion and redistribution of angular momentum in discs across a range of scales; and (iii) present new methods and tools for simulating the evolution of galaxies and interpreting the results of those simulations.

The purpose of each chapter is as follows.

Chapter 2 of this thesis presents the new semi-analytic model of galaxy evolution developed by the theory research group at Swinburne, known as SAGE, for which I have been a primary
1.5. Outline of this thesis

contributor. In outlining SAGE, Chapter 2 describes the overall methodology and framework of semi-analytic modelling (beyond Section 1.4.1). With a single parameter set, the model is performed on three \( N \)-body simulations of various volume and resolution, providing galaxies that are self-consistent amongst the simulations and consistent with observations. Effects of resolution are highlighted in Appendix B. SAGE is used as a reference framework throughout this thesis, from which new methods in semi-analytic models are developed, and to which hydrodynamic simulations are compared.

In Chapter 3 (and Appendix C), I heavily modify the SAGE codebase in order to include the one-dimensional structure of galactic discs, culminating in the new model DARK SAGE. This treats physical processes on local scales, as opposed to global scales like a classical semi-analytic model, and explicitly considers galaxy evolution as a function of specific angular momentum. Using this model, I investigate the cause of the connection between the net mass and specific angular momentum (spin) of stellar discs. The results show that gravitational instabilities in discs, à la Toomre (1964), regulate the mass–spin sequence by driving low-angular-momentum material into a dispersion-supported bulge, with stars of moderate angular momentum naturally forming (i.e. without being prescribed) a rotationally supported ‘pseudobulge’.

Semi-analytic models and hydrodynamic simulations are bridged in Chapter 4, with an extension in Appendix D. Here, I use the EAGLE simulations (Schaye et al. 2015) as a numerical experiment to investigate the process of gas cooling and the accretion of angular momentum onto galaxies. I compare the results against the manner in which these processes are typically prescribed in semi-analytic models, with an emphasis on the methods employed in SAGE and DARK SAGE. The results from EAGLE suggest the specific angular momentum of gas that cools onto galaxies is decoupled from and is systematically higher than the specific angular momentum of the halo. This gas has less specific angular momentum by the time it reaches the disc, but is still greater than that of the halo. This result has important consequences for the handling of the angular momentum of galaxies in semi-analytic models.

Chapter 5 discusses an often overlooked question in handling data produced by hydrodynamic simulations: how should the integrated properties of simulated galaxies be best defined? I collate and compare the non-conformal methods employed in the literature, and go on to motivate a new method by which the edge of a galaxy can be consistently applied to all simulations, regardless of resolution or the details of the sub-grid physics. Using a range of hydrodynamic simulations, I test...
the effectiveness of this technique on a variety of galaxy properties and scaling relations. Further
details of this method are provided in Appendix E.

Chapter 6 moves from the scale of galactic discs down to the scale of black-hole accretion
discs. I calculate the fraction of specific angular momentum that can be removed from accretion
discs due to the emission and scattering of photons. This is done purely analytically in a general
relativistic framework under a Kerr (1963) metric. I show that inverse Compton scattering be-
comes a viable means of eliminating specific angular momentum in the final stages of accretion
in an idealised scenario for fast-spinning black holes. Other mechanisms that induce viscosity are
required to reach this stage, however.

The results of this thesis are summarised in further detail in Chapter 7. I also demonstrate
some ongoing work relating to SAGE and DARK SAGE, which extends work from this thesis and
is in preparation for journal submission.

1.5.1 A note on notation

In compiling this thesis, efforts have been made to normalise mathematical notation across chap-
ters. For example, upper-case $R$ is used for three-dimensional radial distances ($R^2 = x^2 + y^2 + z^2$),
and lower-case $r$ is used for two-dimensional radial distances ($r^2 = x^2 + y^2$, where the $z$-direction
is always parallel to the relevant rotation axis). Case is not important for likes of velocity ($V$ or
$v$) or mass ($M$ or $m$), however. Because the chapters were originally written as separate papers,
subtleties in terminology or emphasis may vary from chapter to chapter, which may result in subtle
changes in notation. For example, $m_{\text{cold}}$ in Chapter 2 and $m_{\text{gas}}$ in Chapter 5 essentially describe
the same concept (cold gas mass), although the precise definition of ‘cold’ is not identical. Some
symbols have multiple meanings; e.g., $\sigma$ may mean a velocity dispersion or standard deviation,
but these different uses will be physically separated. Much of the mathematics in Chapter 6 and
Appendix A, which deal with general relativity, is independent of the other chapters.

This thesis also employs a philosophy of avoiding the use of acronyms where appropriate.
Nevertheless, there are a few acronyms/abbreviations used. These are generally defined throughout
the thesis, but a quick list is also provided here:

- AGN = Active Galactic Nucleus
- AJ = the Astronomical Journal
- AMR = Adaptive Mesh Refinement
- ApJ = the AstroPhysical Journal

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1.5. Outline of this thesis

- ApJS = the AstroPhysical Journal Supplement series
- ARA&A = Annual Review of Astronomy & Astrophysics
- A&A = Astronomy & Astrophysics
- BaryMP = BARYonic-Mass Profile
- CDM = Cold Dark Matter
- C06 = Croton et al. (2006)
- EAGLE = Evolution and Assembly of GaLaxies and their Environments
- FB = FeedBack
- GAR = Gas Accretion Rate
- GER = Gas Ejection Rate
- GRMHD = General Relativistic MagnetoHydroDynamics
- GM = Gas Mass
- ID = IDentification number
- MB-II = MassiveBlack-II
- MR = Medium Resolution
- PASA = Publications of the Astronomical Society of Australia
- PASJ = Publications of the Astronomical Society of Japan
- PhRv = PPhysical ReView
- PhRvL = PPhysical ReView Letters
- PM = Particle-Mesh
- RecalHR = RECALibrated and High Resolution
- RPS = Ram-Pressure Stripping
- SAGE = Semi-Analytic Galaxy Evolution
- SFR = Star Formation Rate
- SLH = Softened Lagrangian Hydrodynamics
- SM = Stellar Mass
- SN = SuperNova
- SPH = Smoothed-Particle Hydrodynamics
- sSFR = Specific Star Formation Rate
Developing a complete theory of galaxy evolution is a formidable task. Without the ability to construct real universes in a laboratory, we are left to test ideas through conducting supercomputer simulations and comparing their results against what we observe. Arguably the most thorough way of doing this currently is through cosmological, hydrodynamic simulations (e.g. Carlberg et al. 1990; Dubois et al. 2014b; Vogelsberger et al. 2014; Khandai et al. 2015; Schaye et al. 2015), where the physics of baryons and the dark sector are self-consistently considered (Section 1.4.2). However, one can successfully reproduce the large-scale structure and formation sites of galaxies inside haloes with pure $N$-body simulations (e.g. Davis et al. 1985; Springel et al. 2005a; Kim et al. 2011; Klypin et al. 2011; Skillman et al. 2014; Poole et al. 2015), into which galaxies can be later added in post-processing (Section 1.4.1). By foregoing a simulation of costly hydrodynamic processes, simulators can invest their computational resources in increasing the number of particles. Structures on both smaller and larger scales are then resolved through improved particle mass resolution and by simulating larger volumes, respectively. Larger volumes also lead to less biased sampling and more haloes overall, which in turn allow for better statistical significance.

In this chapter, we present our new semi-analytic model, SAGE (Semi-Analytic Galaxy Evolution), which updates the work of Croton et al. (2006, hereafter C06). The new model revamps many prescriptions for the treatment of baryons, including the suppression of cooling within haloes from AGN feedback (active galactic nuclei), reincorporation of ejected gas, and the stripping of gas from satellite systems. While semi-analytic models have historically been designed and calibrated for a single $N$-body simulation, SAGE is designed to be run on any simulation, so long as the merger trees are provided in an appropriate format. We show SAGE’s performance on three cosmological $N$-body simulations; namely the Millennium (Springel et al. 2005a), Bolshoi (Klypin et al. 2011), and GiggleZ (Poole et al. 2015) simulations, all of which follow the standard ΛCDM
cosmological paradigm. That said, there is nothing to stop SAGE being run on simulations that explore alternate gravity models, or different dark matter candidates, for example.

The purpose of this chapter is two-fold. The first aim is to describe an updated model of galaxy evolution, detailing the important physical processes and how they interplay with each other. The second goal is to explore the semi-analytic methodology itself in detail. This will provide an important context for the chapters to follow, which extend the semi-analytic framework (Chapter 3), and compare prescriptions against the results of hydrodynamic simulations (Chapter 4).

Along with those from other semi-analytic models, SAGE galaxy catalogues built on various N-body simulations are available through the Theoretical Astrophysical Observatory (Bernyk et al. 2016). The codebase of SAGE is also publicly available, allowing the community to build further models, or modify those described here. The repository includes an IPYTHON notebook for conducting simple analysis with SAGE output, specifically showing how we produced some of the figures presented in this chapter.

The sections of this chapter are laid out in the following manner. The N-body simulations used as input for SAGE are summarised in Section 2.1. An overview of SAGE is provided in Section 2.2, covering which components of the model have been upgraded from that of C06. Sections 2.3-2.11 then describe the physics of the model in more detail, covering: gas infall in haloes (2.3); the role of reionization (2.4); cooling of gas from the hot halo (2.5); the consideration of cold gas, star formation, and metal enrichment in galactic discs (2.6); the role of supernova feedback (2.7); the growth of black holes and their associated AGN feedback (2.8); dealing with mergers and intracluster stars (2.9); disc instabilities (2.10); and starbursts (2.11). Finally, some discussion and concluding remarks are offered in Section 2.12. Throughout this chapter, we present all results assuming $h = 0.73$, based on the cosmology of the Millennium simulation, the primary simulation used for calibrating the model. Where relevant, we also use a Chabrier (2003) initial mass function to produce stellar masses.

## 2.1 N-BODY SIMULATIONS

In this chapter, we present the performance of SAGE on three cosmological simulations. Each simulation not only varies in terms of its cosmological parameters, having used the best available measurements at their various times of being run, but also in terms of the codes and pipelines used to generate the final data products. Even for simulations with identical initial conditions and cosmological parameters, the use of different codes for either running or post-processing cosmo-
2.1. **N-body simulations**

Logical simulations can lead to non-trivial differences in their results. Knebe et al. (2011, 2013b) assessed how and why the choice of (sub)halo finder can affect results, while complimentary studies investigated how the choice of merger tree code can change the derived structure formation histories and the consequences this has for semi-analytic models (Srisawat et al. 2013; Lee et al. 2014, respectively). Understanding these non-trivial differences is key to understanding the theoretical and numerical uncertainties associated with semi-analytic models, although we do not attempt to provide a detailed analysis of such effects here. We describe each simulation below and summarise their properties in Table 2.1.

2.1.1 **The Millennium simulation**

The Millennium simulation (Springel et al. 2005a) significantly upped the ante in cosmological simulations, boasting unrivalled detail for its time. It has since been the focus of a plethora of scientific studies.\(^4\) Ten years on from its completion, the simulation remains a benchmark, and continues to be used for science. Of the simulations used in this chapter, it remains the most balanced in terms of (cosmologically representative) size and resolution (cf. Table 2.1).

Millennium was run using the popular GADGET-2 code (Springel 2005). The cosmological parameters followed those from a combined analysis of WMAP1 (Wilkinson Microwave Anisotropy Probe, first year) data (Spergel et al. 2003) and the 2-degree Field Galaxy Redshift Survey (Colless et al. 2001). Arguably, the biggest weakness of Millennium is its dated cosmological parameters, which now differ significantly from the best-fitting values which are more precisely measured (for the latest observational results, see Planck Collaboration 2016b).

Structure and subhaloes were identified in the Millennium simulation with the SUBFIND algorithm (Springel et al. 2001b). Parent haloes are found through a friends-of-friends procedure, while subhaloes are defined as having at least 20 gravitationally bound particles by the halo finder. The merger trees which feed SAGE were constructed with the L-HALOTREE code (described in the supplementary information of Springel et al. 2005a).

2.1.2 **The Bolshoi simulation**

Bolshoi (Klypin et al. 2011) was run using the adaptive-mesh-refinement code ART (Adaptive Refinement Tree, Kravtsov et al. 1997). The chosen cosmological parameters were very close to those of the WMAP7 data (Jarosik et al. 2011), while maintaining consistency with WMAP5 (Dunkley et al. 2009; also see Komatsu et al. 2009). When compared with WMAP1, the data from these WMAP releases describe a universe with a greater average matter density, that presently expands more slowly, with smaller mass fluctuations. While smaller in box size, Bolshoi complements the

<table>
<thead>
<tr>
<th>Simulation</th>
<th>$N_{\text{part}}$</th>
<th>$M_{\text{part}} h$</th>
<th>$l_{\text{box}} h$</th>
<th>$\Omega_M$</th>
<th>$\sigma_8$</th>
<th>Code</th>
<th>Subhalo finder</th>
<th>Tree constructor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Millennium</td>
<td>2160$^3$</td>
<td>$8.60 \times 10^8$ (M$_\odot$)</td>
<td>500</td>
<td>0.250</td>
<td>0.900</td>
<td>GADGET-2</td>
<td>SUBFIND</td>
<td>L-HALOTREE</td>
</tr>
<tr>
<td>Bolshoi</td>
<td>2048$^3$</td>
<td>$1.35 \times 10^8$ (M$_\odot$)</td>
<td>250</td>
<td>0.270</td>
<td>0.820</td>
<td>ART</td>
<td>ROCKSTAR</td>
<td>CONSISTENT-TREES</td>
</tr>
<tr>
<td>GiggleZ-MR</td>
<td>520$^3$</td>
<td>$9.50 \times 10^8$ (M$_\odot$)</td>
<td>125</td>
<td>0.273</td>
<td>0.812</td>
<td>GADGET-2</td>
<td>SUBFIND</td>
<td>G. B. Poole et al. (in preparation)</td>
</tr>
</tbody>
</table>

Table 2.1: Details of the $N$-body simulations used in the analysis of SAGE in this chapter. The columns provide particle number, $N_{\text{part}}$; particle mass, $M_{\text{part}}$; periodic box length, $l_{\text{box}}$, in comoving units; contribution of matter to the average universal energy density at the present epoch, $\Omega_M$ (for which the equivalent for dark energy is $\Omega_\Lambda = 1 - \Omega_M$); the redshift-zero extrapolation of the root-mean-square linear mass fluctuation within a sphere of radius $8h^{-1}$ Mpc, $\sigma_8$; the code the simulation was run with; the code used to identify subhaloes; and the code used to build the merger trees.
results from Millennium due to its higher mass resolution, allowing us to probe the low-mass end of the mass function in more detail.

Subhaloes in Bolshoi were identified with the ROCKSTAR algorithm (Behroozi et al. 2013a),\(^5\) which builds a hierarchy of friends-of-friends subgroups and utilises one temporal and six phase-space quantities to determine which particles constitute those subhaloes. Merger trees were subsequently constructed using CONSISTENT-TREES (Behroozi et al. 2013b).\(^6\)

### 2.1.3 The GiggleZ simulation suite

The Gigaparsec WiggleZ simulation suite (GiggleZ, Poole et al. 2015) was run as a theoretical counterpart to the WiggleZ Dark Energy Survey (Drinkwater et al. 2010). Each simulation was performed using GADGET-2, with cosmological parameters based on data from WMAP5, baryonic acoustic oscillations, and supernovae (Komatsu et al. 2009). While the main box of GiggleZ boasts shear size \((1h^{-1} \text{ comoving Gpc on a side})\), due to its lower mass resolution we instead assess the complimentary GiggleZ-MR simulation for this chapter. GiggleZ-MR was run in a smaller box of side length \(125h^{-1} \text{ comoving Mpc}\) but with a particle mass much closer to Millennium.

GiggleZ subhaloes were identified with SUBFIND. Trees were built following the method in Poole et al. (in preparation). This approach repairs pathological defects in merger trees introduced by the halo-finding process (e.g. over linking, or the disappearance of haloes during pericentric passages) through a process of forward and backward matching which scans both ways over multiple snapshots.

### 2.1.4 Halo merger tree structure and required properties

SAGE is modular enough that it should run on any halo merger tree that is structured in a supported format and contains a minimum set of information per halo. The initial public release of SAGE requires halo measurements for (i) \(M_{\text{vir}}\), the halo virial mass; (ii) the number of particles in the (sub)halo; (iii) \(V_{\text{max}}\), a fit to the maximum circular velocity of the halo; and (iv) the cartesian position, (v) velocity, and (vi) spin vector of each halo.

In terms of tree structure, SAGE assumes the halo trees are organised in depth-first order, as described in the supplementary information of Springel et al. (2005a, see, in particular, their fig. 5). This is the same output format used for the trees of the Millennium simulation (Section 2.1.1), which were constructed with the GADGET-2/L-HALOTREE codebase. L-HALOTREE produces additional output properties required by SAGE that enable the membership and history of each halo:

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\(^6\)https://bitbucket.org/pbehroozi/consistent-trees
to be determined: ‘FirstProgenitor’, ‘NextProgenitor’, ‘FirstHaloInFOFgroup’, and ‘NextHaloIn-FOFgroup’. Given the modular nature of SAGE, other tree formats, such as those produced by \textsc{rockstar/consistent-trees}, may be supported in the future. In lieu of this, a tree converter has been made publicly available.\footnote{https://github.com/manodeep/ConvertCTrees}

## 2.2 AN OVERVIEW OF SAGE

The new SAGE model of galaxy formation is an updated version of that described in C06. One of the most important aspects of this update is that the code has been cleaned, significantly optimised, generalised to run on a wide variety of simulations, and ready to be used by the wider community. Key changes to the physics that distinguish SAGE from the C06 model are summarised below. A more detailed description of each model component is then given in the subsequent sections, where we also explore their consequences on the galaxy population and its evolution.

- **Gas cooling and AGN heating**: Cooling and heating of halo gas is now more directly coupled in the new SAGE model. We introduce a heating radius from radio mode AGN feedback, interior to which gas will never cool. This radius can only move outward, thus retaining the memory of previous heating episodes. The cooling rate is then calculated using only the hot gas between the cooling and heating radii.

- **Quasar mode feedback**: We have added feedback from the quasar mode, the dominant growth channel of the black hole, triggered by mergers or disc instabilities. This feedback is most effective at removing disc (and sometimes halo) gas at high redshift in gas rich galaxies.

- **Ejected gas reincorporation**: Gas ejected from the halo is now reincorporated according to the dynamical time of the halo modulated by $V_{\text{vir}}/V_{\text{crit}}$, with $V_{\text{crit}}$ a parameter set at the galaxy group halo mass scale. Previously, reincorporation was dependent on the dynamical time alone.

- **Satellite galaxies**: Hot gas is no longer instantaneously removed from the satellite/subhalo system upon infall, but stripped in proportion to the subhalo dark matter mass stripping rate. Satellites are treated in the same way as central galaxies for the longest time possible (e.g. we allow cooling in subhaloes).

- **Mergers and intracluster stars**: At the moment of infall, an expected satellite merger time is calculated. The satellite is then tracked until its baryon-to-subhalo mass falls below a critical
threshold (taken at ∼ 1). At this point the current survival time is compared to the expected merger time. If the subhalo has survived longer than expected we say it is more resistant to disruption and the satellite is merged with the central in the usual way. Otherwise the satellite is disrupted and its stars are added to a new ‘intracluster’ mass component. As a consequence, SAGE no longer produces satellite galaxies lacking a subhalo, the so-called orphan population.

2.2.1 Model calibration

A summary of the primary SAGE parameters, along with the choices that define our fiducial galaxy model, are given in Table 3.1 for quick reference. These parameters were manually selected to simultaneously perform well across all three simulation sets, with a slightly higher emphasis on Millennium. All figures and results in this chapter are calculated using a model with these parameter choices. Note that, due to the order-of-magnitude higher resolution of Bolshoi, we found it necessary to lower its baryon fraction parameter (only) from 0.17 to 0.13 to obtain comparable results to Millennium and GiggleZ-MR (see Section 2.3).

The primary constraint for our parameter choices was the ability to reproduce the \( z = 0 \) stellar mass function: that is, the number density of galaxies as a function of their stellar mass, \( \Phi \). We show in Fig. 2.1 that SAGE produces an excellent stellar mass function for each simulation with our fiducial parameter set, tightly matching the observational uncertainty range presented by Baldry et al. (2008). To conservatively account for the different simulation resolution limits, a minimum stellar mass equal to the median of galaxies in Millennium (sub)haloes made of 50 particles is adopted. We also compare the SAGE mass functions to the C06 model, but note that C06 used the luminosity function as its primary constraint instead.

We further used a set of secondary constraints for setting our fiducial parameter values. These constraints include the star formation rate density history (Fig. 2.4), the Baryonic Tully–Fisher relation (Fig. 2.6), the mass–metallicity relation of galaxies (Fig. 2.7), and the black hole–bulge mass relation (Fig. 2.10). We detail which parameters were allowed to be varied in the model calibration in order to meet these constraints in Table 3.1. All of the free parameters and several of the fixed parameters have different values to C06.

For the remainder of the chapter, we walk through the different baryonic reservoirs in the model and the physics that describe how mass and energy move between them.
Figure 2.1: Stellar mass functions at $z = 0$ for SAGE galaxies for each of the $N$-body simulations (given in the legend) compared to observational data from Baldry et al. (2008) and the Croton et al. (2006) model, taking $h = 0.73$. 
### Table 2.2: Fiducial \textsc{sage} parameters used throughout this chapter, and also compared to those from Croton et al. (2006). The fifth column indicates whether the value was kept fixed or used in the calibration. The last column indicates the section where the parameter is discussed. The cosmic baryon fraction used for Millennium and GiggleZ-MR is 0.17, while 0.13 was used for Bolshoi. All other parameters are common across all three simulations.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
<th>C06 value</th>
<th>Fixed</th>
<th>Section(s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_{\text{b}}$</td>
<td>(Cosmic) baryon fraction</td>
<td>0.17, 0.13</td>
<td>0.17</td>
<td>No</td>
<td>2.3, 2.4</td>
</tr>
<tr>
<td>$z_0$</td>
<td>Redshift when H II regions overlap</td>
<td>8.0</td>
<td>8.0</td>
<td>Yes</td>
<td>2.4</td>
</tr>
<tr>
<td>$z_r$</td>
<td>Redshift when the intergalactic medium is fully reionized</td>
<td>7.0</td>
<td>7.0</td>
<td>Yes</td>
<td>2.4</td>
</tr>
<tr>
<td>$\alpha_{\text{SF}}$</td>
<td>Star formation efficiency</td>
<td>0.05</td>
<td>0.07</td>
<td>No</td>
<td>2.6</td>
</tr>
<tr>
<td>$Y$</td>
<td>Yield of metals from new stars</td>
<td>0.025</td>
<td>0.03</td>
<td>No</td>
<td>2.6</td>
</tr>
<tr>
<td>$R$</td>
<td>Instantaneous recycling fraction</td>
<td>0.43</td>
<td>0.30</td>
<td>Yes</td>
<td>2.6, 2.7</td>
</tr>
<tr>
<td>$\epsilon_{\text{disc}}$</td>
<td>Mass-loading factor due to supernovae</td>
<td>3.0</td>
<td>3.5</td>
<td>No</td>
<td>2.7</td>
</tr>
<tr>
<td>$\epsilon_{\text{halo}}$</td>
<td>Efficiency of supernovae to unbind gas from the hot halo</td>
<td>0.3</td>
<td>0.35</td>
<td>No</td>
<td>2.7</td>
</tr>
<tr>
<td>$k_{\text{reinc}}$</td>
<td>Sets velocity scale for gas reincorporation</td>
<td>0.15</td>
<td>N/A</td>
<td>Yes</td>
<td>2.7</td>
</tr>
<tr>
<td>$\kappa_R$</td>
<td>Radio mode feedback efficiency</td>
<td>0.08</td>
<td>N/A</td>
<td>No</td>
<td>2.8.1</td>
</tr>
<tr>
<td>$\kappa_Q$</td>
<td>Quasar mode feedback efficiency</td>
<td>0.005</td>
<td>N/A</td>
<td>No</td>
<td>2.8.2</td>
</tr>
<tr>
<td>$f_{\text{BH}}$</td>
<td>Rate of black hole growth during quasar mode</td>
<td>0.015</td>
<td>0.03</td>
<td>No</td>
<td>2.8.2</td>
</tr>
<tr>
<td>$f_{\text{friction}}$</td>
<td>Threshold subhalo-to-baryonic mass for satellite disruption or merging</td>
<td>1.0</td>
<td>N/A</td>
<td>Yes</td>
<td>2.9</td>
</tr>
<tr>
<td>$f_{\text{major}}$</td>
<td>Threshold mass ratio for merger to be major</td>
<td>0.3</td>
<td>0.3</td>
<td>Yes</td>
<td>2.9</td>
</tr>
</tbody>
</table>
2.3 Gas infall in haloes

Perturbations in the primordial density field lead to gravitational instability that drives mass to continually collapse into ‘halo’-like structures. This process is dominated by dark matter, being the main mass component of the Universe, with the baryons tending to follow rather than lead the dynamical evolution. In a cosmological $N$-body simulation, such halo growth is typically well measured by a halo finder. Hence, the total baryonic content of each halo, and how it changes with time, can be inferred.

In the C06 model, as in SAGE, the total baryonic content of each dark matter halo at (nearly) all times is maintained at the universal fraction, $f_b$. This requires that for each simulation time-step, the baryons in each halo grow by $f_b M_{\text{vir}} - m_b$, where $m_b$ is the total mass of baryons present in the previous time-step. This mass difference is added to the hot gas reservoir of the system and assumed to be pristine. If halo mass were to decrease over a time-step, then ejected gas (see Section 2.7) or, secondarily, hot gas is removed from the system (cold gas and stars, located deep in the potential well of the halo, are always unaffected here). However, in low-mass haloes and at high redshift, little to none of the baryons are expected to be hot (Birnboim & Dekel 2003; Kereš et al. 2005); a decrease in halo mass can then lead to a temporary increase in the baryon fraction, i.e. above universal.

However unlike C06, SAGE considers the baryon fraction to be a free parameter during model calibration, which sets the baseline level of baryons within the virial radius of each halo. For the Millennium and GiggleZ-MR simulations, the original value of 0.17 provides the baryonic mass required to produce a well-calibrated model. However we find that the Bolshoi simulation, with its order-of-magnitude higher mass resolution, needs a lower baseline fraction of baryons (assuming the same remaining model parameters), $f_b = 0.13$, to obtain comparable results.

Investigating this further, it is interesting to see how the baryon fraction and simulation resolution limit together influence which haloes get gas at each epoch and when this gas turns into stars. Since haloes are identified earlier in Bolshoi, and given the power-law steepness of the halo mass function, a considerable fraction of baryons accumulate in low-mass haloes but are delayed in their opportunity to turn into stars by the reionization prescription (see Section 2.4). Our choice of a lower baryon fraction compensates for this accumulation by lessening the significance of such baryons once they are later able to contribute to galaxy evolution. A simple back-of-the-envelope calculation reveals that a model with $f_b = 0.17$ run on Millennium results in approximately the same baryonic mass per unit volume as a model with $f_b = 0.13$ on Bolshoi.

In Fig. 2.2 we show the baryon fraction contained within Millennium haloes of a given mass at $z = 0$, broken down by mass component: galaxy stars, ‘intracluster’ stars, hot gas, cold gas, and ejected gas (these will each be defined in the subsequent Sections). One can see the dominance of
2.4 Reionization

At high redshift, the baryonic content of low-mass haloes is most likely suppressed as a result of photoionization heating of the intergalactic medium by strong feedback from the first stars. This heating acts to lower the concentration of baryons in shallow potentials (Efstathiou 1992). Observationally, this can be seen in the low abundance of local dwarf galaxies relative to the prediction that results from the $\Lambda$CDM halo mass function (Section 1.2.3).

Gnedin (2000) showed using high-resolution SLH-P$^3$M (softened Lagrangian hydrodynamics) simulations that the effect of photoionization heating can be modelled by defining a filtering mass, $M_F$, below which the baryonic fraction in the halo is reduced relative to the universal value:

$$ f_{b,\text{halo}}(z, M_{\text{vir}}) = \frac{f_{b,\text{cosmic}}}{(1 + 0.26 M_F(z)/M_{\text{vir}})^3} . $$

Notice that the filtering mass is a function of time. In their simulations, the change was sharpest around the epoch of reionization.

Kravtsov et al. (2004) fitted an analytic model to approximate the behaviour of the Gnedin (2000) filtering mass. They defined two parameters that characterise this transition: $z_0$, which marks the redshift where the first $\text{H} \, \text{II}$ regions overlap, and $z_r$, which marks the time when the intergalactic medium is fully reionized. The best fit values to the Gnedin simulation are $z_0 = 8$

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure2.2.png}
\caption{Fraction of baryonic content to total mass for Millennium haloes at $z = 0$ from SAGE, binned by virial mass. Each curve represents the mean fraction of the labelled component.}
\end{figure}
Chapter 2. Semi-Analytic Galaxy Evolution

Figure 2.3: The effect of reionization in SAGE on the $z = 0$ stellar mass function for Bolshoi. Because of the simulation’s relatively high resolution, reionization is needed to get the low-mass end of the mass function right, while also affecting the high-mass end.

and $z_r = 7$. We adopt these in our model, leaving it identical to that used in C06. See appendix B of Kravtsov et al. (2004) for the expression of $M_F(z)$.

In Fig. 2.3, we show how reionization can have an important and positive effect on both the low-mass and high-mass ends of the stellar mass function at $z = 0$. We show this for Bolshoi, using all galaxies more massive than the median stellar mass in (sub)haloes made of 50 particles, which keeps us above the simulation resolution limit. Because reionization suppresses gas infall (and therefore star formation) at high redshift, satellite galaxies at $z = 0$ have less stars when reionization is turned on. Instead this gas cools at later epochs, being brought into more massive haloes by the satellites, and contributes directly to the higher-mass central galaxy. As such, these systems form more stars, and the high-mass end of the mass function kicks out. Higher-resolution simulations have more subhaloes, especially at high redshift, in which more gas can cool, and the strength of this effect is resolution-dependent (hence the effect is stronger for Bolshoi).

2.5 THE HOT GAS HALO

The physics of infalling gas is complicated and a detailed accounting is beyond the scope of the semi-analytic methodology. Its evolution can be approximated at a level of accuracy consistent with the rest of the model by assuming that new gas added is shock heated to the virial temperature of the system as it loses its gravitational potential energy. This results in the formation of a quasistatic hot halo of baryons. However, at early times and in low-mass systems, the shock heating may be unstable, or the shock not form at all. In this case, the gas rapidly radiates away any gained energy and instead is channelled to the centre as a cold stream, the so-called ‘cold
2.5. The hot gas halo

accretion’ or ‘rapid cooling’ mode. See Section 3.2.1 of C06 for further discussion.

In SAGE, we follow the hot-halo model described in C06. That model, based on the original
work of White & Frenk (1991), assumes that halo gas initially settles into an isothermal density
profile at the virial temperature of the system,

\[ T_{\text{vir}} = \frac{1}{2} \frac{\bar{\mu} m_p}{k} V_{\text{vir}}^2 = 1.436 \times 10^6 \left( \frac{V_{\text{vir}}}{200 \text{ km s}^{-1}} \right)^2 \text{K}, \]

(2.2)

where \( V_{\text{vir}} \) is the halo virial velocity and \( \bar{\mu} m_p \) is the mean particle mass, with \( \bar{\mu} = 0.59 \) nominally, and \( k \) is the Boltzmann constant. The density profile of the gas is thus

\[ \rho_g(R) = \frac{m_{\text{hot}}}{4\pi R_{\text{vir}}^2 R^2}, \]

(2.3)

where \( m_{\text{hot}} \) is the total hot gas mass in the halo which extends out to the virial radius \( R_{\text{vir}} \) (as defined in Section 1.2.1).

To calculate the rate at which gas cools out of this distribution, the similarity solutions of
Bertschinger (1989) are used. We define the cooling radius, \( R_{\text{cool}} \), as the radius at which the gas
cooling time is equal to the dynamical time of the system, \( t_{\text{cool}} = R_{\text{vir}}/V_{\text{vir}} \) (although other time-
scales can be used, for example the age of the system – see Lu et al. 2011). Bertschinger (1989)
showed that the cooling mass flux across this radius is proportional to the mass deposition rate at
the centre of the halo, with the proportionality constant close to one.

For a parcel of gas with local density \( \rho_g(R) \) and temperature \( T_{\text{vir}} \), the cooling time can be
approximated by the ratio of its specific thermal energy to the cooling rate per unit volume,

\[ t_{\text{cool}} = \frac{3}{2} \frac{\bar{\mu} m_p k T_{\text{vir}}}{\rho_g(R) \Lambda(T_{\text{vir}}, Z)}, \]

(2.4)

Here \( \Lambda(T, Z) \) is the cooling function (Sutherland & Dopita 1993), dependent on both the gas
temperature and metallicity, \( Z \) (metal production is described below in Section 2.6).

Combining Eqs. 2.3 and 2.4, and assuming \( t_{\text{cool}} \) above, allows us to find \( R_{\text{cool}} \) for each system.

We then solve the continuity equation of mass deposition,

\[ \dot{m}_{\text{cool}} = 4\pi \rho_g(R_{\text{cool}}) R_{\text{cool}}^2 \dot{R}_{\text{cool}}, \]

(2.5)

to determine the instantaneous cooling rate as

\[ \dot{m}_{\text{cool}} = \frac{1}{2} \left( \frac{R_{\text{cool}}}{R_{\text{vir}}} \right) \left( \frac{m_{\text{hot}}}{t_{\text{cool}}} \right). \]

(2.6)

Eq. 2.6 is valid as long as \( R_{\text{cool}} < R_{\text{vir}} \), which marks when the system is in the hot-halo regime,
as discussed above. Otherwise, infalling gas is in the cold-accretion regime, and we assume it deposits at the centre of the halo on a free-fall timescale, $R_{\text{vir}}/V_{\text{vir}}$. In the presence of radio mode AGN heating, this cooling rate is modified with the addition of an inner heating radius. We define our new hybrid cooling/heating model further in Section 2.8.1.

2.6 COLD GAS, STAR FORMATION, AND METAL ENRICHMENT

Cooling gas that settles in the centre of a halo is assumed to conserve angular momentum and spin up to form a rotationally supported disc. It is from this disc of cold gas that stars form. In the SAGE model, star formation proceeds as in C06. A more sophisticated accounting of the formation of new stars, including the tracking of atomic and molecular hydrogen to fuel star formation, will be introduced in Chapter 3.

In studying 15 local, star-forming galaxies, Kennicutt (1989) showed that their Hα surface brightnesses, which proxy star formation rate surface densities, scale with local cold-gas column density, but only once that column density is above a threshold. Below the threshold, gas is not dense enough to be molecular and hence does not form stars. Based on the Kennicutt (1989) data, and assuming a Toomre (1964) instability model with constant gas velocity dispersion, Kauffmann (1996) provided an analytic relation for the critical local gas surface density for star formation as a function of radius. As shown in C06, after assuming the gas is evenly distributed across the disc, one can convert this into a threshold gas mass, for which we find a value of

$$m_{\text{crit}} = 3.8 \times 10^9 \left( \frac{V_{\text{vir}}}{200 \text{ km s}^{-1}} \right) \left( \frac{r_{\text{disc}}}{10 \text{ kpc}} \right) \text{M}_\odot.$$  

(2.7)

Here, the disc radius is defined as

$$r_{\text{disc}} = \frac{3}{\sqrt{2}} \lambda R_{\text{vir}},$$

(2.8)

which is thrice the disc scale length proposed by Mo et al. (1998), based on the properties of the Milky Way (van den Bergh 2000). The spin parameter of the dark halo, $\lambda$ (Eq. 1.6), is calculable directly from the $N$-body simulation (Bullock et al. 2001).

A star formation rate can now be calculated from a Kennicutt–Schmidt-type relation (Kennicutt 1998):

$$\dot{m}_* = \alpha_{\text{SF}} \left( m_{\text{cold}} - m_{\text{crit}} \right) / t_{\text{dyn, disc}},$$

(2.9)

where $m_{\text{cold}}$ is the total mass of cold gas and $\alpha_{\text{SF}}$ is the star formation efficiency. In other words, a fraction $\alpha_{\text{SF}}$ of gas above the threshold is converted into stars in a disc dynamical time $t_{\text{dyn, disc}} = r_{\text{disc}}/V_{\text{vir}}$.

Assuming an exponential disc profile simply results in a minor adjustment to the value of $m_{\text{crit}}$. 
2.6. Cold gas, star formation, and metal enrichment

Many more physically detailed star formation laws exist in the literature beyond what we have prescribed for SAGE (e.g. Krumholz et al. 2012; Federrath 2013; Salim et al. 2015). The critical gas surface density associated with star formation (measured by Kennicutt 1989) is now known to really signify that star formation is more truly correlated with H$_2$ surface density (see, e.g., Schruba et al. 2011). In the spirit of keeping SAGE as modular and customisable as possible, we have opted for the simplest prescription that works well. In Chapter 3, we present a much more detailed star formation law within a more complex semi-analytic model.

This model of star formation, combined with the feedback processes described in the below sections, produces a galaxy population whose combined star formation rate density evolution is consistent with the observed Universe, as shown in Fig. 2.4. The supporting observational data were compiled by Somerville et al. (2001), which we used as a constraint for SAGE. Observational constraints on star formation rate density are non-trivial to obtain, naturally leading to large uncertainties (for discussion, see Somerville et al. 2001; Springel & Hernquist 2003). For example, in SAGE the time of peak star formation shows dependence on the N-body simulation: $z \sim 2$ for Bolshoi, $z \sim 2.5$ for GiggleZ, and $z \sim 3$ for Millennium, yet they all agree with the broadness of the observational data. Millennium exhibits a systematically higher star formation rate than the other simulations for $z \gtrsim 2$, which can be attributed to its larger $\sigma_8$ value. Despite a largely identical star formation law to C06, variations in other parts of the model in SAGE (e.g. AGN feedback) affect the star formation histories of galaxies noticeably. As seen in Fig. 2.4, the C06 model predicted star formation rates that were too high on average at high redshift.

While our star formation law is loosely based on the Kennicutt–Schmidt relation between

![Figure 2.4: Average star formation rate density history produced by SAGE for each cosmological simulation compared to observations and the Croton et al. (2006) model. Observational data were originally compiled and corrected by Somerville et al. (2001, see their table A2 for a complete list of references).](image)

Many more physically detailed star formation laws exist in the literature beyond what we have prescribed for SAGE (e.g. Krumholz et al. 2012; Federrath 2013; Salim et al. 2015). The critical gas surface density associated with star formation (measured by Kennicutt 1989) is now known to really signify that star formation is more truly correlated with H$_2$ surface density (see, e.g., Schruba et al. 2011). In the spirit of keeping SAGE as modular and customisable as possible, we have opted for the simplest prescription that works well. In Chapter 3, we present a much more detailed star formation law within a more complex semi-analytic model.

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While our star formation law is loosely based on the Kennicutt–Schmidt relation between
average gas surface density, $\Sigma_{\text{gas}}$, and average star formation rate surface density, $\Sigma_{\text{SFR}}$, our power law index does not match the nominal value of 1.4 by construction. As shown in Fig. 2.5, SAGE still produces an appropriate level of scatter for this relation, which can be attributed to the fact that output star formation rates are averaged over the previous time-step for a snapshot, whereas gas masses are instantaneous. Indeed, observationally, star formation rates are typically representative of some recent historical average, the time-scale of which depends on the chosen method of measurement.

A well-known constraint on the properties of nearby disc galaxies is the correlation between luminosity (a proxy for stellar mass) and rotation velocity (Tully & Fisher 1977). An even tighter correlation is found if one considers the contribution of cold gas to the mass of the systems: the so-called Baryonic Tully–Fisher relation (McGaugh et al. 2000). For each of the $N$-body simulations, SAGE successfully produces Sb and Sc Hubble-type galaxies (proxied by a bulge-to-total ratio cut between 0.1 and 0.5) that closely match this relation, as shown in Fig. 2.6. The observational backdrop shows the fitted relation from Stark et al. (2009) with their random uncertainties; i.e. we plot $\log_{10} \left( \left( m_* + m_{\text{cold}} \right)/M_\odot \right) = \left( 3.94 \pm 0.07 \right) \log_{10}(V_{\text{max}}/\text{km s}^{-1}) + \left( 1.79 \pm 0.26 \right) + 2 \log_{10}(0.75/0.73)$, where the last term accounts for the assumed value of $h$. We note that we have approximated the flat rotation velocity (as used by Stark et al. 2009) of SAGE galaxies by their halo’s maximum value, $V_{\text{max}}$. We also note that the contours for Bolshoi show more galaxies at low $V_{\text{max}}$ on account of the simulation’s higher mass resolution.

There is a suggestion from Fig. 2.6 that SAGE galaxies are systematically too massive for
2.6. Cold gas, star formation, and metal enrichment

Figure 2.6: Baryonic Tully–Fisher relation for SAGE galaxies at $z = 0$. 2500 representative galaxies (randomly sampled within the axes) with bulge-to-total ratios between 0.1 and 0.5 are plotted from Millennium, while contours encapsulating 68% of galaxies of the same cut are shown for the other $N$-body simulations. The maximum Keplerian velocity for the subhaloes, $V_{\text{max}}$, represents the rotational velocity of galaxies. Appropriately, only central galaxies were considered. The thick strip represents the scatter in the observed relation (Stark et al. 2009).

As stellar populations evolve, they enrich the interstellar medium with elements heavier than hydrogen and helium. Such metal enrichment is modelled in SAGE by assuming a yield $Y$ of metals are returned for each solar mass of stars formed. We deposit these metals into the cold disc of the galaxy, but more complicated models may wish to explore direct metal enrichment of the hot halo gas and beyond (e.g. Shattow et al. 2015). Furthermore, a fraction $R$ of the mass of newly formed stars is recycled immediately back to the cold gas disc: the so-called ‘instantaneous recycling approximation’ (see Cole et al. 2000). Our treatment of metals and recycling are identical to

Figure 2.7: Stellar mass – gas metallicity relationship for $z = 0$ central galaxies in SAGE. 2500 representative galaxies are plotted in the axes from Millennium, while vertical lines cover the 16th–84th percentile range for bins of stellar mass for the other N-body simulations. The shaded region compares the 16th–84th percentile range from observational data published by Tremonti et al. (2004), binned identically to the Bolshoi and GiggleZ-MR data.

that described in C06 (and De Lucia et al. 2004 before), aside from differences in the assumed $Y$ and $R$ parameters, which are guided by a change to the Chabrier (2003) initial mass function.

Through the measurement of optical nebular emission lines supplemented by galaxy photometry, Lequeux et al. (1979) were among the first to show evidence for a correlation between total mass and gas metallicity for galaxies. Furthering this, Tremonti et al. (2004, and see references therein) showed there is a near quadratic relationship in logarithmic space for the oxygen abundance in galaxies, calculated based on the model of Charlot & Longhetti (2001), as a function of their stellar mass. In Fig. 2.7, we show that the redshift-zero population of SAGE galaxies is representative of this observed relation for each N-body simulation by comparing against the 16th–84th percentile range of Tremonti et al. (2004, cf. their table 3), adjusting for a conversion from a Kroupa (2001) to Chabrier (2003) initial mass function: $m_{*},\text{Chabrier} = (5/6)m_{*},\text{Kroupa}$. Our choice for the value of the yield is almost entirely driven by this constraint.

The scatter seen in Fig. 2.7 for the simulations is larger than the observational data on average (up to ~75%), although Bolshoi in particular shows strikingly good agreement. The scatter for Millennium and GiggleZ-MR are almost identical across the full mass range, which can be seen in more detail by the individual Millennium points. We note that there appears to be a slight downturn in the metallicities of galaxies at high masses, seen most clearly from Bolshoi (but present for the other simulations too). We suspect this is related to the high-mass galaxies acquiring low-metallicity cold gas from mergers at late times, which also leads to high specific star formation rates (see Section 2.9). This is an area of improvement for SAGE, as it appears inconsistent with
2.7 Supernova feedback and the galactic fountain

For each episode of new star formation, very massive stars have lifetimes much shorter than the typical time resolution of the simulations on which SAGE is built. Such stars end as supernovae. Supernovae play an important role in the life-cycle of a galaxy, injecting metals (as discussed above), mass and energy into the surrounding interstellar medium. Our modelling of supernova feedback follows that used in C06.

First, we assume that supernova winds remove cold gas from the disc, which in turn acts to suppresses star formation. The rate at which disc gas is driven from the galaxy, $\dot{m}_{\text{reheated}}$, is proportional to the rate at which new stars are forming:

$$\dot{m}_{\text{reheated}} = \epsilon_{\text{disc}} \dot{m}_*. \tag{2.10}$$

The proportionality constant, $\epsilon_{\text{disc}}$, is typically referred to as a mass-loading factor.

More generally, the energy released by supernovae during the star formation episode can be approximated as

$$\dot{E}_{\text{SN}} = \frac{1}{2} \epsilon_{\text{halo}} \dot{m}_* V_{SN}^2, \tag{2.11}$$

where $\frac{1}{2} V_{SN}^2$ is the mean energy in supernova ejecta per unit mass of stars formed, $V_{SN} = 630 \text{ km s}^{-1}$ taking the commonly accepted value, and $\epsilon_{\text{halo}}$ quantifies the efficiency with which this energy is able to reheat disc gas. If this reheated disc gas were moved to the hot halo without changing its specific energy, the total thermal energy of the hot gas would change by

$$\dot{E}_{\text{hot}} = \frac{1}{2} \dot{m}_{\text{reheated}} V_{\text{vir}}^2. \tag{2.12}$$

Eqs. 2.11 and 2.12 allow us to determine the excess energy in the hot gas after reheating: $\dot{E}_{\text{excess}} = \dot{E}_{\text{SN}} - \dot{E}_{\text{hot}}$. When $\dot{E}_{\text{excess}} < 0$, supernovae have failed to transfer enough energy with the reheated disc gas to unbind any hot halo gas. However $\dot{E}_{\text{excess}} > 0$ signals that there is enough energy in the system to unbind some of the hot halo and maintain virial balance. Ejected hot gas is accounted for in the model by placing it in an external reservoir:

$$\dot{m}_{\text{ejected}} = \frac{\dot{E}_{\text{excess}}}{E_{\text{hot}}} m_{\text{hot}} = \left( \epsilon_{\text{halo}} \frac{V_{SN}^2}{V_{\text{vir}}^2} - \epsilon_{\text{disc}} \right) \dot{m}_*. \tag{2.13}$$

Here, $E_{\text{hot}} = \frac{1}{2} m_{\text{hot}} V_{\text{vir}}^2$ is the total thermal energy of the hot gas. No gas is ejected when the right-hand side of Eq. 2.13 is less than zero; this signals that $\dot{E}_{\text{excess}} < 0$, as mentioned above.
Eq. 2.13 has many desirable properties. In low-mass haloes with shallow potentials, the feedback can be very destructive, with the possibility that the entire disc and halo gas be expelled from the system if the feedback (i.e. star formation) is strong enough. Conversely, in haloes with virial velocities above $200\, km\, s^{-1}$, the potential well is sufficiently deep that no amount of feedback will remove hot gas. Such haloes develop and maintain very stable hot atmospheres throughout their lives (barring mergers or other cataclysmic events).

2.7.1 Reincorporation of ejected gas

In a dynamically evolving universe, gas that is ejected may not stay ejected forever. In SAGE, we adopt a modified version of that used in C06 to determine ejected-gas reincorporation. Previously, C06 assumed that a fixed fraction of the ejected material returned to the hot halo over a halo dynamical time, and that this was true for all haloes. As addressed by Mutch et al. (2013a), this model was poorly constrained, where alterations to the reincorporation parameter could significantly alter the number of low-mass systems. In fitting SAGE for multiple simulations, we found that a better match to the data can be obtained when we allow the reincorporation rate to increase for the more massive haloes, and limit it to zero for the very lowest-mass haloes. We do this by assuming the mass of ejected gas reincorporated per time-step is

$$\dot{m}_{\text{reinc}} = \left( \frac{V_{\text{vir}}}{V_{\text{crit}}} - 1 \right) \frac{m_{\text{ejected}}}{t_{\text{dyn}}}, \quad (2.14)$$

where $t_{\text{dyn}} = R_{\text{vir}}/V_{\text{vir}}$, and assuming $V_{\text{vir}} > V_{\text{crit}}$ ($\dot{m}_{\text{reinc}} = 0$ otherwise). Here, $V_{\text{crit}} = k_{\text{reinc}}V_{\text{esc}}$, where $V_{\text{esc}} = V_{SN}/\sqrt{2}$ is the critical halo virial velocity, above which the supernova wind velocity is sufficient for the gas to escape, and $k_{\text{reinc}}$ parameterises how efficient this idealised process actually is (cf. Table 3.1). In effect, $V_{\text{crit}}$ sets the velocity scale below which no gas can reincorporate back into a hot halo. Conversely, far above this scale, ejected gas reincorporates in a fraction of the dynamical time, meaning it quickly becomes available for future cooling and hence star formation.

It has been suggested by Henriques et al. (2013) that using a virial-mass dependence rather than virial-velocity one for the reincorporation rate could be conducive to reproducing observations across multiple epochs, in particular the galaxy stellar mass function. Our initial exploration of this variation has not been quite as positive however (nor was it for White et al. 2015), and hence we do not adopt it here. In truth, this is an area for improvement for both SAGE and other popular models.

Note that the ejected gas component of our model need not be physically removed from the system. It merely marks gas that is unable to cool into the disc to form stars for a period of time.
2.7. Supernova feedback and the galactic fountain

Such evacuated gas and metals may still play a part in a galaxy’s evolution, simply one at a later epoch in its history.

Using halo mass accretion histories produced by extended Press–Schechter theory, Firmani et al. (2010) used a self-consistent model of galaxy evolution, which included the galactic fountain, and examined its effect on the stellar mass–specific star formation rate relation of disc galaxies. By comparing the model with and without the reincorporation of ejected gas from feedback with observations, they found the model could not reproduce the ‘down-sizing’ effect in either case, where the observations show higher mass galaxies should have lower specific star formation rates. We find SAGE suffers the same issue, which is shown by Fig. 2.8 for Millennium, both with and without reincorporation. Here, we have only included star-forming galaxies with sSFR $> 10^{-11} \text{yr}^{-1}$, and we show the 16th–84th percentile range for the models and observations. The observational data are Sloan Digital Sky Survey (SDSS; York et al. 2000) galaxies, specifically from Data Release 7 (Abazajian et al. 2009). Stellar masses and star formation rates come from the MPA-JHU catalogue, where star formation rates are based on Brinchmann et al. (2004). We only include galaxies at $z < 0.05$. Our only difference to the results of Firmani et al. (2010) is that the inclusion of reincorporation in SAGE does not lead to a significant systematic increase in star formation rates at fixed stellar mass at $z = 0$. It should be noted, however, that SAGE would need to be recalibrated after dropping reincorporation to meet our usual constraints, e.g. the stellar mass function.

\[ \log_{10}(\text{sSFR [yr}^{-1}]) \]

\[ \log_{10}(m^* [M_{\odot}]) \]

\[ \text{Observations} \]

\[ \text{Millennium with reincorporation} \]

\[ \text{Millennium without reincorporation} \]

\[ \text{Observations} \]

Figure 2.8: Stellar mass–specific star formation rate relation of star-forming galaxies (sSFR $> 10^{-11} \text{yr}^{-1}$) in SAGE at $z = 0$ versus observations from SDSS. The vertical bars and shaded regions cover the 16th–84th percentile range in each case.
2.8 Supermassive Black Holes and Their Feedback

Feedback from active supermassive black holes is now known to play a critical role regulating the life-cycle of many types of galaxies. C06 distilled a number of popular ideas about the interplay of active galactic nuclei (AGN) and galaxies into a simple picture of how this feedback shapes the properties of the low-redshift galaxy population (see also Bower et al. 2006; Cattaneo et al. 2006). Kitzbichler & White (2007) extended their comparison to higher redshift, specifically looking at galaxy masses, luminosities and number counts. Other authors have since adopted similar models and developed the basic framework to further explore how AGN evolve (e.g. Marulli et al. 2008; Somerville et al. 2008b; Guo et al. 2011).

C06 broke AGN into two general classes, loosely dubbed the ‘quasar mode’ and ‘radio mode’. These two classes can be distinguished by their triggering mechanism, lifetime, and accretion rate. We maintain this two-mode approach in SAGE but have updated both, for which we describe each in turn below.

We note that we do not consider the AGN luminosity function in the calibration or results of the model. This would require additional levels of modeling beyond the scope of this chapter. We do, however, plan to include more complex AGN physics to explore more observables in later developments of the model.

2.8.1 The radio mode

Radio mode feedback was introduced into semi-analytic models to solve the cooling flow problem, where the over-accretion of cooling gas onto the central galaxy led to galaxy properties that were inconsistent with observations (too massive, too blue, too discy). The first implementations were simple and framed either phenomenologically, which attempted to infer a black hole accretion rate (and hence feedback) based on the local black hole and gas properties (C06), or in terms of a sharp cutoff in cooling when the halo or galaxy evolved past a chosen critical state (Bower et al. 2006), e.g. a mass threshold.

The model we employ in SAGE is an enhancement of the Bondi–Hoyle accretion model described in section 5.2 of C06, which is also used by Somerville et al. (2008b). In this model, hot gas accretes onto the central black hole at a rate approximated using the Bondi-Hoyle formula (Bondi 1952):

$$\dot{m}_{\text{Bondi}} = 2.5\pi G^2 \frac{m_{BH}^2 \rho_0}{c_s^3}. \quad (2.15)$$

Here, the sound speed, $c_s$, is approximated by the virial velocity of the parent halo, $V_{\text{vir}}$, while $\rho_0$ is the density of hot gas around the black hole. To find $\rho_0$, C06 equated the sound travel time across a shell of diameter twice the Bondi radius ($R_{\text{Bondi}} \equiv 2Gm_{BH}/c_s^2$) to the local cooling time.
In a departure from C06, we insert a ‘radio mode efficiency’ parameter, $\kappa_R$, to the right-hand side of Eq. 2.16. While we have added this term by hand, it allows us to counteract the approximations used in the derivation of Eq. 2.16 and to modulate the strength of black hole accretion (and subsequently radio mode feedback) within SAGE. As such, the ‘best’ value need not be 1. In the present work, we employ a default value of 0.08 as obtained from our calibration procedure (see Section 2.2.1 and below).

The accretion rate given by Eq. 2.16 enables us to estimate the luminosity of the black hole in the radio mode, $L_{BH,R} = \eta\dot{m}_{BH,R}c^2$, where $\eta = 0.1$ is the standard\textsuperscript{10} efficiency with which inertial mass is liberated upon approaching the event horizon, and $c$ the speed of light. We assume this luminosity acts as a source of heating that offsets the energy losses from the cooling gas. If enough energy from the central AGN is injected into the cooling flow, it can be turned off entirely, leading to longer-term quenching, which was the focus of the work by C06. If we define the specific energy of gas in the hot halo as $\frac{1}{2}V_{\text{vir}}^2$, and heating as adding this amount of energy per unit mass to offset cooling, then the heating rate from radio mode feedback can be written as

$$\dot{m}_{\text{heat}} = \frac{L_{BH,R}}{\frac{1}{2}V_{\text{vir}}^2}. \tag{2.17}$$

\textbf{A more self-consistent treatment of the cooling–heating cycle}

In C06, the heating rate given by Eq. 2.17 was subtracted off the cooling rate given by Eq. 2.6 to determine an effective AGN-adjusted cooling rate. However, despite its ability to reproduce many key local galaxy properties, there remained important limitations. Significantly, in the C06 model (and many similar radio mode models), cooling and heating are decoupled, meaning the latter only offsets the former after both have been independently calculated. In the real Universe, one would expect episodes of AGN activity to have a lasting (or, at least, extended) effect on the surrounding gas, modifying its temperature and density profile in such a way that would alter the later cooling.

\textsuperscript{10}`Standard’ in terms of what has been adopted in the literature (e.g. Di Matteo et al. 2005; Sijacki et al. 2007; Booth & Schaye 2009; Dubois et al. 2014a,b; Henriques et al. 2015; Schaye et al. 2015), a value which falls between the efficiency expected for a non-spinning and maximally spinning black hole (Bardeen et al. 1972; also see fig. 1 of Maio et al. 2013). Many authors cite Shakura & Sunyaev (1973) as the justification for $\eta = 0.1$, despite their using $\eta = 0.06$. Recent hydrodynamic works have started to use $\eta = 0.2$ instead (e.g. Hirschmann et al. 2014; Sijacki et al. 2015), motivated by observations of more-luminous AGN (Davis et al. 2011; Yu et al. 2002, respectively).
A coupled cooling–heating model is clearly a desirable refinement if one wants to make realistic predictions for e.g. the X-ray halo cooling luminosity or AGN jet power. Unfortunately, there is no natural way to achieve such cooling–heating coupling within the paradigm of current semi-analytic models.

To achieve a coarsely equivalent behaviour, we implement a simple idea in SAGE to create an updated AGN model. We assume that cooling gas is heated by radio mode feedback out to a particular radius, called $R_{\text{heat}}$, interior to which the gas retains the memory of its past heating. This gas will never cool thereafter. To determine $R_{\text{heat}}$ we find the radius at which the energy deposited into the gas due to the radio mode equals the energy the halo gas interior to $R_{\text{heat}}$ would lose if it were to cool onto the galaxy disc from Eq. 2.6 alone. This is given by

$$R_{\text{heat}} = \frac{\dot{E}_{\text{heat}}}{\dot{E}_{\text{cool}}} R_{\text{cool}},$$

(2.18)

where

$$\dot{E}_{\text{heat}} = \frac{1}{2} \dot{m}_{\text{heat}} V_{\text{vir}}^2$$

and

$$\dot{E}_{\text{cool}} = \frac{1}{2} \dot{m}_{\text{cool}} V_{\text{vir}}^2,$$

(2.19)

and $\dot{m}_{\text{cool}}$ and $\dot{m}_{\text{heat}}$ are determined from Eqs. 2.6 and 2.17 above. The cooling rate given in Section 2.5 is then modified by the presence of this heating radius. The new cooling rate, in the presence of current and past radio mode episodes, now becomes

$$\dot{m}_{\text{cool}}' = \left(1 - \frac{R_{\text{heat}}}{R_{\text{cool}}} \right) \dot{m}_{\text{cool}}.$$

(2.20)

This model assumes that only the gas between $R_{\text{heat}}$ and $R_{\text{cool}}$ can cool, and when $R_{\text{heat}} > R_{\text{cool}}$, cooling is effectively quenched (i.e. $\dot{m}_{\text{cool}}' = 0$). To retain the memory of past heating episodes, the heating radius is never allowed to move inwards, only outwards.

This new model behaves almost identically to that used in C06, but with the added benefit that a more realistic cooling rate can be extracted from the cooling–heating cycle rather than simply an upper limit. To demonstrate this point, we show the cooling rates predicted by SAGE (i.e. Eq. 2.20) against those from the C06 model (i.e. Eq. 2.6 less Eq. 2.17) for the same haloes in Fig. 2.9 at $z = 0$. The cooling rates are shown against halo virial temperature (Eq. 2.2). For comparison, also plotted are various X-ray observations of hot gas haloes surrounding galaxy clusters and groups [Ponman et al. 1996 (P96); Peres et al. 1998 (P98); Anderson et al. 2015 (A15); Bharadwaj et al. 2015 (B15), as marked].\(^{11}\) Note that P98 measure the cooling luminosity (X-ray luminosity inside the cooling radius), whereas P96 and B15 measure the bolometric luminosity.

\(^{11}\)We have used a subset of the observational data in the literature to cover the plotted range of virial temperatures. Many other data exist (see, e.g., fig. 7 of Anderson et al. 2015 and references therein) and they are all consistent with the conclusions we draw here.
(X-ray luminosity inside $R_{500}$), for which the cooling luminosity of such systems will be lower. Furthermore, the observations statistically favour the brightest systems at any given mass scale, while the model draws more broadly from the wider galaxy and halo population. The A15 data are stacked observations of X-ray emission around ‘locally brightest galaxies’. These data should, in principle, be more representative of an average halo (suffering optical selection bias rather than X-ray selection bias), and therefore more directly comparable to SAGE. We have plotted the maximum of their measurement and bootstrapping errors for each binned point.

The upper-limit nature of the C06 model is highlighted in Fig. 2.9 by the tight band of points between $10^{0.4} < T_{\text{vir}}/\text{keV} < 10^{0.6}$ relative to the observations. In this model, the radio feedback required to offset the excessive cooling demanded $\kappa_{\text{R}} = 1$, with correspondingly higher absolute heating rates. With less cooling to counteract, our nominal value for $\kappa_{\text{R}}$ has been reduced to 0.08 for SAGE. In other words, our required heating rates are significantly lower than before, the results of which can be more meaningfully compared with observations.

In truth, if an AGN has been off for an extended period of time one might expect the hot gas in the halo to return to its previous state and cool at closer to the maximal rate. Ideally, $R_{\text{heat}}$ would shrink during this time, which is most likely why our cooling rates lie below the observations. In this sense, our prevention of $R_{\text{heat}}$ moving back inwards places a lower limit on the cooling rates of haloes. The addition of $R_{\text{heat}}$ in SAGE is still a step in the right direction though, and goes some way to include often missed effects, such as the observed entropy floors in many cluster hot gas profiles. We leave a more thorough treatment of the cooling–heating cycle for a future study.

### 2.8.2 The quasar mode

In most simulations of galaxy formation, quasars are triggered by mergers or from some form of instability in the disc. The key requirement is to funnel gas into the galactic centre on a very short time-scale, which results in high black hole accretion rates and rapid black hole growth. Hence, the quasar mode is the dominant mode through which a black hole gets its mass.

Galaxy mergers are described below in Section 2.9. Of relevance here, to model the effect of mergers on black hole growth we follow the work of Kauffmann & Haehnelt (2000), as well as the enhancements in C06, and assume that mergers trigger black hole growth according to the phenomenological relation

$$\Delta m_{\text{BH},Q} = \frac{f'_{\text{BH}} m_{\text{cold}}}{1 + (280 \text{ km s}^{-1}/V_{\text{vir}})^2} ,$$

where

$$f'_{\text{BH}} = f_{\text{BH}} (m_{\text{sat}}/m_{\text{central}})$$
Figure 2.9: Cooling luminosity of hot gas in haloes as a function of virial temperature at $z = 0$. Here we compare what the Croton et al. (2006) model for cooling would have returned against SAGE for Millennium haloes. 2500 randomly selected points are plotted within the axes for both the SAGE and Croton et al. (2006) cooling methods. Squares and starred points with error bars compare observational data of galaxy clusters from Peres et al. (1998), measured with the High Resolution Imager (HRI) and Position Sensitive Proportional Counter (PSPC) at ROSAT, respectively. Triangular points with errors show observational data of galaxy groups (Ponman et al. 1996; Bharadwaj et al. 2015). Stacked X-ray observations surrounding locally brightest galaxies from Anderson et al. (2015) are given by the horizontal marks with errors.
2.8. Supermassive black holes and their feedback

is an accretion efficiency parameter with constant $f_{\text{BH}}$, which controls the fraction of cold gas accreted by a black hole and is modulated by the satellite-to-central galaxy merger mass ratio.

Disc instabilities can similarly lead to rapid black hole growth. In Section 2.10 we describe our instability implementation, which is similar to that used in C06. For the sake of instability-driven accretion here, we modify Eq. 2.21 by taking $f'_{\text{BH}} = f_{\text{BH}}$ and substitute the mass of unstable cold gas for $m_{\text{cold}}$.

Although C06 used the quasar mode to grow black holes, they did not include quasar feedback when accounting for the evolution of the surrounding baryons. In SAGE, quasar mode feedback is included. This mode has little effect on the local galaxy population but can have a significant impact on early universe galaxy formation.

In the absence of a detailed understanding of how quasar accretion and feedback operates (but see Lynden-Bell 1969; Novikov & Thorne 1973; Costa et al. 2014), we adopt a simple phenomenological model that is consistent with the quasar mode narrative. When a merger or disc instability occurs and the black hole has undergone some form of rapid accretion, we assume a quasar wind follows with luminosity $L_{\text{BH,Q}} = \eta m_{\text{BH,Q}} c^2$, where $\eta = 0.1$ as before. This is used to calculate the total energy contained in the quasar wind,

$$E_{\text{BH,Q}} = \kappa_Q \frac{1}{2} \eta \Delta m_{\text{BH,Q}} c^2 ,$$  \hspace{1cm} (2.23)

where $\kappa_Q$ parameterises the efficiency with which the wind influences the surrounding gas as it escapes the galaxy and halo. Next, we calculate the total thermal energies in both the cold disc gas and hot halo gas:

$$E_{\text{cold}} = \frac{1}{2} m_{\text{cold}} V_{\text{vir}}^2 \hspace{0.5cm} \text{and} \hspace{0.5cm} E_{\text{hot}} = \frac{1}{2} m_{\text{hot}} V_{\text{vir}}^2 .$$  \hspace{1cm} (2.24)

Simply put, if the total energy in the quasar wind (Eq. 2.23) exceeds the total energy in the cold disc gas we blow out the cold gas and associated metals to the ejected gas reservoir. If the quasar energy is greater than the combined total energy in the cold gas and hot halo gas, the quasar wind instead ejects both the cold and hot gas (and metals) from the halo. This is an ‘all or nothing’ approach that is ripe for development.

Black hole population

While the radio mode regulates cooling in SAGE, the quasar mode is the dominant channel for black hole growth. In Fig. 2.10, we show that our treatment of black hole evolution is in general agreement with the observed black hole–bulge mass relation. We compare SAGE galaxies to the observed sample published in Scott et al. (2013), which considers Sersic and core-Sersic galaxies (the latter with typically more-massive bulges) separately (also see Graham & Scott 2015). Once
Figure 2.10: Black hole–bulge mass relation for SAGE galaxies. 2500 representative galaxies are plotted for Millennium within the axes, while contours encapsulating 68 percent of systems containing black holes and bulges are shown for the other N-body simulations. Error bars compare observational data from Scott et al. (2013), where cyan represents Sersic galaxies, and purple represents core-Sersic.

again, observational statistics favour large bulge masses, while numbers are naturally greater for low-bulge-mass systems from the theory perspective. The region of overlap spans over 1 dex in width though, and shows clear agreement.

We note that we do not consider either the quasar or radio AGN luminosity functions in the current work. This would require additional layers of modelling that are beyond the scope of this thesis. We do, however, plan to include more complex AGN physics in later developments of SAGE that will allow us to explore more observables.

2.9 Galaxy Mergers and Intracluster Stars

The new SAGE model treats satellite galaxies somewhat differently to the previous C06 model. In C06, satellites had their hot halo instantly stripped upon infall, and their orbits were followed using the host subhalo position until the subhalo dark matter mass stripped below the resolution limit of the simulation. Upon losing the subhalo, an analytic merger time was calculated assuming the dynamical friction model of Binney & Tremaine (1987) and the properties of the subhalo in the time-step before it was lost:

\[
\tau_{\text{friction}} = 1.17 \frac{V_{\text{vir}}}{Gm_{\text{sat}}} \ln \Lambda .
\]
2.9. **Galaxy mergers and intracluster stars**

Here, $m_{\text{sat}}$ is the total mass of the subhalo/satellite system (dark matter plus baryonic), and the Coulomb logarithm $\ln \Lambda$ is approximated by $\ln(1 + M_{\text{vir}}/m_{\text{sat}})$. The ‘orphan’ (i.e. subhalo-less) galaxy was then allowed to survive until this clock ran out, after which it was merged with the central galaxy.

A few important consequences resulted from this satellite galaxy model. In particular, it was realised that the colours of satellite galaxies were too red, as noted by many authors (C06; Weinmann et al. 2006; Guo et al. 2011). This was because the instantaneous stripping of hot gas was imposing an artificial quenching mechanism on satellites, leading to premature suppression of star formation (also see Font et al. 2008). Furthermore, in the C06 model, all satellites were assumed to merge with the central galaxy once their dynamical friction clock reached zero, whereas we know from observations that many satellites are shredded to pieces well before merging, and instead become part of an intragroup or intracluster stellar halo. While scatter in a mass-dependent dynamical friction formula is inevitable, hydrodynamic simulations have since suggested there is a better generic fit than Eq. 2.25 (Jiang et al. 2008, 2010).

In SAGE, we allow for the evolution of the satellite population by treating them more like central galaxies. Hot-halo stripping now happens in proportion to the dark matter subhalo stripping, rather than instantaneously. Any hot gas present in the subhalo is allowed to cool onto the satellite in the usual way. Upon infall, a merger time is calculated for the satellite, using the same dynamical friction formula as above (Eq. 2.25). We take this as the average merger time expected for systems of similar properties. We then follow the satellite with time and measure the ratio of subhalo-to-baryonic mass. When this ratio falls below a critical threshold, $f_{\text{friction}}$ (typically 1: cf. Table 3.1), we compare its current survival time with the average time determined at infall. If the subhalo has survived longer than average, then we say that the subhalo/satellite system was more bound than average and merge it with the central in the standard way. On the other hand, if the subhalo/satellite mass ratio has fallen below the threshold sooner than average, then we argue that the system was instead loosely bound and more susceptible to disruption. In this case, we add the satellite stars to a new intracluster stars component, and any remaining gas goes to the parent hot halo. This omission of orphan galaxies is one of SAGE’s notable points of difference to other semi-analytic models currently in the literature.

Once the occurrence of a galaxy–galaxy merger has been identified, we check the satellite-to-central baryonic mass ratio. If the ratio is above a threshold $f_{\text{major}}$ – in SAGE set at a default of 0.3 – we say the merger is ‘major’. In a major merger the discs of both galaxies are destroyed and all stars are combined to form a spheroid. Otherwise the merger is ‘minor’, and only the satellite stars are added to the central galaxy bulge. Furthermore, any cold gas present in either system can lead to a starburst, as described below in Section 2.11.
SAGE still overproduces the fraction of quiescent satellite galaxies, a problem shared with other semi-analytic models (e.g. Guo et al. 2011). In fact, this is true for galaxies at lower masses ($m_\ast \lesssim 10^{10} \, \text{M}_\odot$) in general. In Fig. 2.11, we show the fraction of quiescent\textsuperscript{12} galaxies from each $N$-body simulation, determined as those with specific star formation rates $< 10^{-11} \, \text{yr}^{-1}$, as a function of stellar mass. To compare to real galaxies, we calculate a quiescent fraction (by the same definition as the model galaxies) from the same SDSS sample used for Fig. 2.8 (see Section 2.7). We bin these data in stellar mass of width 0.1 dex and display Poisson errors for the quiescent fraction. Also compared against these data is the quiescent fraction for C06 galaxies. While SAGE has improved the quiescent fraction at masses around $10^{10} \, \text{M}_\odot$, the problem at lower masses, i.e. in satellites, persists (as does an overproduction of star-forming galaxies at high masses). We note that because Millennium was our primary simulation for constraining the model, and GiggleZ-MR is of similar resolution and used the same halo finder, these simulations produce more promising results than Bolshoi. Due to a number of effects, including Bolshoi being more affected by reionization, baryons cool in Bolshoi haloes at a later time, therefore systematically raising the specific star formation rates of galaxies at $z = 0$ (also seen in Fig. 2.4).

\textsuperscript{12}It is worth noting that a red fraction, defined by a cut in colour (typically $g - r$), is not equivalent to a quiescent fraction, defined by a cut in specific star formation rate.

\textbf{Figure 2.11:} Fraction of quiescent (i.e. low specific star formation rate) satellites at $z = 0$ in SAGE as a function of stellar mass. Compared are observed galaxies at $z < 0.05$ from SDSS along with the Croton et al. (2006) model. Too many quiescent galaxies are consistently seen in the models for $m_\ast \lesssim 10^{10} \, \text{M}_\odot$. 

\[ \log_{10}(m_\ast \, \text{[M}_\odot]) \]

\[ \begin{align*}
0.0 & \quad 0.2 \\
0.4 & \quad 0.6 \\
0.8 & \quad 1.0
\end{align*} \]

\[ \begin{align*}
0.0 & \quad 0.2 \\
0.4 & \quad 0.6 \\
0.8 & \quad 1.0
\end{align*} \]
clustering of the galaxy population through the halo occupation distribution model (see Berlind & Weinberg 2002), which also lacks an orphan population. An orphan-less semi-analytic model is significantly more modular and transportable between differing simulations, which is a desirable property in the SAGE codebase.

2.10 DISC INSTABILITIES

Galaxies also transform through instabilities that occur within a galaxy disc itself. Again, we follow C06: after each episode of star formation, we determine a stability criterion (Mo et al. 1998) using the disc mass, $m_{\text{disc}}$; radius, $r_{\text{disc}}$; and circular velocity, $V_c$, approximated by the maximum circular velocity of the halo. For the disc to be stable the following inequality must be met:

$$\frac{V_c}{(Gm_{\text{disc}}/r_{\text{disc}})^{1/2}} \geq 1 .$$

(2.26)

When the left side of Eq. 2.26 is less than unity, we transfer (in proportion) enough stellar and cold gas mass to the bulge to return the disc to stability. Unstable cold gas can both grow the central black hole (as described in Section 2.8.2) and lead to the formation of new stars in a starburst (as described in the next Section).

2.11 STARBURSTS

Starbursts mark the rapid formation of new stars, triggered as a result of a specific event. This is opposed to the more typical quiescent star formation that goes on in the galactic disc and described in Section 2.6. In SAGE, both galaxy mergers and disc instabilities act as triggers for starbursts.

**Mergers:** When a galaxy merger has been identified, the resulting starburst occurs in proportion to the total sum of the cold gas in the two merging galaxies. SAGE treats starbursts using the implementation of Somerville et al. (2001), where the fraction of cold gas converted to stars is given by

$$e_{\text{burst}} = \beta_{\text{burst}}(m_{\text{sat}}/m_{\text{central}})^{\alpha_{\text{burst}}} .$$

(2.27)

The two parameters above are fixed at $\alpha_{\text{burst}} = 0.7$ and $\beta_{\text{burst}} = 0.56$, which provides a good fit to the numerical results of Cox et al. (2004) and also Mihos & Hernquist (1994, 1996) for merger mass ratios ranging from 1:10 to 1:1. For minor mergers, the new stars are added to the galactic bulge and the stability of the disc is subsequently checked. As mentioned in Section 2.9, for major mergers, all stars go to the spheroid, including those
newly formed.

**Instabilities:** For instability-driven starbursts, $e_{\text{burst}}$ is taken as the fraction of cold disc gas that is unstable (Section 2.10), minus any gas that is accreted onto the central black hole (Section 2.8.2). All newly formed stars from a disc instability burst are then added to the bulge (adding them to the disc would simply leave the disc unstable).

Our starburst implementation for mergers is in contrast to the recent semi-analytic work of Padilla et al. (2014); rather than applying Eq. 2.27, those authors instead check for an instability to drive a starburst after merging, by virtue of evolving the size of discs rather than assuming Eq. 2.8.

### 2.12 Discussion and Summary

This chapter has presented our new, publicly available semi-analytic model of galaxy formation and evolution, SAGE. The model is based on that of Croton et al. (2006), but has updated many of the transfer processes between baryonic reservoirs, including gas infall, cooling, heating, and reincorporation. It further includes previously omitted quasar mode feedback for supermassive black holes and an intracluster star component for central galaxies. In addition, SAGE has a unique take on satellite galaxies, whereby they are assumed to be disrupted or merge when they lose their dark matter subhalo, rather than surviving as orphans.

The codebase that describes SAGE is modular and well suited to run on the halo merger trees of any cosmologically representative $N$-body simulation. In this chapter we have shown the performance of SAGE on each of the Millennium, Bolshoi, and GiggleZ simulations using a common set of default parameters. With these, SAGE can successfully reproduce many observational properties of the local galaxy population simultaneously. Our calibrations include the redshift-zero stellar mass function, baryonic Tully–Fisher relation, stellar mass–gas metallicity relationship, black hole–bulge mass relation, and the average star formation rate density history. We find only minor differences between simulations in the predicted scaling relations. This insensitivity to halo finding and merger tree construction reflects the robust nature of the included physical prescriptions.

However, current models of the galaxy population should not be considered complete. The physics that governs galaxy evolution can always be examined at a higher level of complexity, and in the current era of survey science, our instruments continue to produce increasingly vast and rich data sets. Within these data, the properties of galaxies are being measured in increasingly refined detail. To stay current, SAGE is ripe for further development in a number of key areas:
2.12. Discussion and summary

- More detailed modelling of gas outside of and within the halo and galaxy, including radio predictions for the large-scale distribution of atomic hydrogen, new baryonic reservoirs such as the circumgalactic medium and warm intergalactic medium, and the different phases of gas in the galactic disc, specifically its neutral and molecular hydrogen content.

- Improved modelling of the various stellar growth channels of discs and bulges in galaxies, their size and evolution, and how this modifies their broader predicted observed properties.

- An improved understanding of gas and star formation in satellite galaxies; in particular, the formation and abundance of low mass galaxies at high redshift, and the discrepancies found by many models with the quiescent fraction of satellites at low redshift.

- Expanding the model to produce predictions for the AGN population, such as radio jet luminosities and sizes, and the abundance of radio AGN as a function of host galaxy mass and redshift.

To explore one of these, the next chapter focuses on heavily modifying the SAGE codebase to include the structural evolution of discs, wherein galaxy evolution processes are calculated on local scales.

Orthogonal to its use to explore questions of science, the philosophy behind SAGE is one of transparency and reproducibility of scientific results. By making SAGE an open and community project, we are hoping to (a) widen the accessibility of such models to more astronomers, especially students; (b) enable wider development of the science modules, which will hopefully be useful to astronomers with more specific interests; and (c) increase the scrutiny of how such models are produced, their uncertainty and limitations, and their use in the literature. In this sense, SAGE is following a similar path to that already established by scientific codes such as GADGET-2 (Springel 2005) and GALACTICUS13 (Benson 2012), as well as many others (see the Astrophysics Source Code Library14 for further examples).

The default catalogues produced by SAGE and used here are publicly available for download at Swinburne University’s Theoretical Astrophysical Observatory (TAO, Bernyk et al. 2016). With TAO, users can additionally add a wide range of observational filters to produce apparent and absolute magnitudes, build custom light cones to mimic popular surveys, and create custom images of a mock galaxy population. These datasets form a solid basis for comparisons with survey data, although users may wish to locally re-run them using SAGE and tweak the parameters to further refine the model. When new simulations of significance become publicly available, and new models are built on top of them, we expect to use TAO to distribute these as well. For example,

---

13https://sites.google.com/site/galacticusmodel/
14http://ascl.net/
we are currently in the process of making SAGE data produced with the *MultiDark* simulations (Klypin et al. 2016) available.

Having an array of simulations and models on hand, and being able to generate new ones as needed, will enable astronomers to explore the theoretical uncertainty between different mock datasets. This is especially important for models which claim to follow a similar underlying physical narrative, but with different technical implementation. Expanding the pallet of theoretical predictions available to observers will add valuable context when using such models to compare with and interpret observational results.
Building galaxies with disc structure through angular momentum

The contents of this chapter have been published in A. R. H. Stevens et al., 2016a, MNRAS, 461, 859.

Efforts to develop a comprehensive theory of galaxy formation and evolution have gained significant momentum over the last few decades. Cosmological \( N \)-body simulations have not only shown us how the large-scale structure of the Universe forms, but also how overdensities give rise to haloes, the formation sites of galaxies. Under the generally accepted \( \Lambda \)CDM paradigm, these haloes merge throughout time, building the Universe hierarchically. In 30 years of running these simulations, the community has improved the level of detail, i.e. the number of particles, by over seven orders of magnitude (cf. Davis et al. 1985; Skillman et al. 2014). Yet we are still striving for a complete theoretical picture of galaxy evolution in this framework.

One piece to this puzzle lies in our understanding of the angular momentum of galaxies. Specific angular momentum, otherwise referred to as ‘spin’, is arguably one of the most fundamental properties of a galaxy. Following Fall (1983), there has been a recent focus in the literature on how the spin of a galaxy is correlated with its mass and related to its morphology (e.g. Romanowsky & Fall 2012; Fall & Romanowsky 2013; Obreschkow & Glazebrook 2014; Genel et al. 2015; Teklu et al. 2015; Cortese et al. 2016; Lagos et al. 2016; Zavala et al. 2016). These findings suggest one should be able to describe galaxy evolution explicitly as a function of spin. Indeed, several efforts have been made in the past to do this (e.g. Dalcanton et al. 1997; Firmani & Avila-Reese 2000; van den Bosch 2001; Stringer & Benson 2007). To truly test this idea requires numerical, cosmological simulations in which galaxies are evolved with a full consideration of all relevant astrophysics.

With this chapter, we present DARK SAGE, a semi-analytic model with an updated approach to galactic-disc evolution, explicitly based on angular momentum. This has used SAGE as a base (Chapter 2), but has been vastly altered. Where SAGE only evolved the integrated properties
of galaxies like a classical semi-analytic model, DARK SAGE also evolves the one-dimensional structure of galactic discs. Given semi-analytic models have historically served as a theoretical counterpart to observational surveys, it is prevalent for models to begin including disc structure in the era of surveys which are returning structurally resolved details of galaxies in large numbers (e.g. Cappellari et al. 2011; Ho et al. 2011; Croom et al. 2012; Sánchez et al. 2012; Brodie et al. 2014; Bundy et al. 2015). Similar to the models of Stringer & Benson (2007) and Dutton & van den Bosch (2009), we discretise disc structure into bins of specific angular momentum. This deviates from semi-analytic models developed by Fu et al. (2010, 2013), who instead bin by radius, where now the angular momentum of material within discs is naturally conserved.

In fact, semi-analytic models have recently begun to make a concerted effort to evolve the angular momentum of galaxies, but these efforts have typically assumed, and not evolved, the angular-momentum structure (e.g. Lagos et al. 2009; Guo et al. 2011; Benson 2012; Padilla et al. 2014; Tonini et al. 2016). Other angular-momentum studies have been conducted in post-processing of the semi-analytic model itself, after the historical properties of the galaxies have been determined (e.g. Lagos et al. 2015a). DARK SAGE’s point of difference is that it self-consistently evolves disc structure and the integrated properties of galaxies, inclusive of angular momentum as a vector quantity.

This chapter is laid out as follows. Section 3.1 gives a complete description of the model itself, including the physics and prescriptions that have gone into it. Our main results are presented in the following two sections, where we use the model to make predictions about how galaxies evolve and their properties at $z = 0$, comparing these to observations. Section 3.2 focuses on the surface density profiles of galaxies, both as a function of radius and specific angular momentum. Section 3.3 looks at the integrated specific angular momentum of galaxies, how this scales with mass, and how this compares to observations and other predictions. Finally, we offer conclusions and discuss future prospects for the model in Section 3.4. Again, all results (and compared data) in this chapter assume (or have been altered to assume) a Chabrier (2003) stellar initial mass function and a Hubble constant with $h = 0.73$.

### 3.1 Physics and Design of DARK SAGE

DARK SAGE is based on the architecture of SAGE (Chapter 2) and its predecessor, Croton et al. (2006), but is vastly different in terms of how properties within galactic discs are evolved. In this section, we fully describe the new evolutionary prescriptions of the model, which are founded on angular momentum. Each of the processes outlined in Sections 3.1.2–3.1.9 affects the evolution of the angular-momentum structure of galactic discs. We provide a visual realisation of some of these processes in Fig. 3.1. The only aspect of the model that remains identical to SAGE in terms of
3.1. Physics and design of DARK SAGE

**Figure 3.1:** A simple pictorial overview of the DARK SAGE semi-analytic model.  
**Panel (a):** Galactic discs are broken into annuli, with constant density assumed within a given annulus. Galaxy evolution processes take place in these annuli (see Section 3.1.1).  
**Panel (b):** Gas discs cool with a spin vector parallel to that of the halo. Subsequent cooling episodes may occur when the halo and disc spin vectors have an angular offset, leading to a change in the disc’s vector’s direction (see Section 3.1.2).  
**Panel (c):** Star formation is an analogous process, where the spin of the gas disc is used for new star formation episodes, which can lead to offsets in the gas and stellar discs (see Section 3.1.3).  
**Panel (d):** The gas disc precesses about the stellar spin axis, maintaining coplanarity (see Section 3.1.5).  
**Panel (e):** Unstable disc annuli transfer some of their mass to adjacent annuli, conserving angular momentum in the process. For the innermost annulus, unstable stars are partially transferred to the bulge, while unstable gas feeds the black hole (see Section 3.1.6).  
**Panel (e):** Merging satellite galaxies have their stellar content shifted to the bulge of the central galaxy, while their gas goes to disc annuli of the appropriate specific angular momentum (see Section 3.1.8). Further processes are described throughout Section 3.1.

In developing DARK SAGE, the definitions of many model parameters have changed from SAGE. The parameters (old and new) for this chapter and their values are compiled in Table 3.1. Only 8 parameters were allowed to vary during calibration. For the sake of simplicity, the calibration and results of DARK SAGE for this chapter only use the Millennium simulation merger trees (see Section 2.1.1).

In addition to the observational constraints used for SAGE, a greater emphasis has been placed on constraining the gas properties of galaxies in DARK SAGE. The new constraints include the H\textsc{i} and H\textsc{2} mass functions (Zwaan et al. 2005; Keres et al. 2003, respectively), and the H\textsc{i}–stellar mass scaling relation of Brown et al. (2015). The stellar mass function remains the main constraint and is presented in Fig. 3.2. Further details and plots regarding the calibration process are provided in Appendix C.1. For all plots in this chapter (and Appendix C.1), including the mass functions, we only present results for systems whose masses of the relevant species are above the median for a (sub)halo with 50 particles (e.g. $\sim 10^{8.5}$ M\sun for stellar mass). Both SAGE and DARK SAGE can reliably find acceptable agreement with observational constraints above this limit, and hence this is also applied to the results of this chapter.
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
<th>Fixed</th>
<th>Section</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_b$</td>
<td>Cosmic baryon fraction</td>
<td>0.17</td>
<td>Yes</td>
<td>2.4</td>
</tr>
<tr>
<td>$\epsilon_{SF}$</td>
<td>Star formation efficiency from $\text{H}_2$ [$10^{-4} \text{ Myr}^{-1}$]</td>
<td>3.96</td>
<td>No</td>
<td>3.1.3</td>
</tr>
<tr>
<td>$\theta_{\text{thresh}}$</td>
<td>Threshold angle for stars and gas to be considered coplanar [degrees]</td>
<td>10.0</td>
<td>Yes</td>
<td>3.1.3</td>
</tr>
<tr>
<td>$Y$</td>
<td>Yield of metals from new stars</td>
<td>0.025</td>
<td>No</td>
<td>3.1.3</td>
</tr>
<tr>
<td>$\mathcal{R}$</td>
<td>Instantaneous recycling fraction</td>
<td>0.43</td>
<td>Yes</td>
<td>3.1.3</td>
</tr>
<tr>
<td>$\epsilon_{\text{disc}}$</td>
<td>Mass-loading factor due to supernovae at $\Sigma_{0,\text{gas}}$</td>
<td>6.0</td>
<td>No</td>
<td>3.1.4</td>
</tr>
<tr>
<td>$\Sigma_{0,\text{gas}}$</td>
<td>Surface density scaling for supernova reheating [$\text{M}_\odot \text{ pc}^{-2}$]</td>
<td>8.0</td>
<td>No</td>
<td>3.1.4</td>
</tr>
<tr>
<td>$\epsilon_{\text{halo}}$</td>
<td>Efficiency of supernovae to unbind gas from the hot halo</td>
<td>0.4</td>
<td>No</td>
<td>2.7</td>
</tr>
<tr>
<td>$\vartheta_t$</td>
<td>Precession angle of gas discs about stars in a dynamical time [degrees]</td>
<td>5.0</td>
<td>Yes</td>
<td>3.1.5</td>
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<tr>
<td>$f_{\text{move}}$</td>
<td>Fraction of unstable gas that moves to adjacent annuli</td>
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<td>No</td>
<td>3.1.6</td>
</tr>
<tr>
<td>$f_{\text{major}}$</td>
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<td>Yes</td>
<td>3.1.8</td>
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<tr>
<td>$f_{\text{BH}}$</td>
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<td>No</td>
<td>3.1.8</td>
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<tr>
<td>$\kappa_R$</td>
<td>Radio mode feedback efficiency</td>
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<td>No</td>
<td>2.8.1</td>
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<tr>
<td>$\kappa_Q$</td>
<td>Quasar mode feedback efficiency</td>
<td>0.005</td>
<td>Yes</td>
<td>2.8.2</td>
</tr>
</tbody>
</table>

Table 3.1: Fiducial parameter values for DARK SAGE used for all the results presented in this chapter (and Appendix C.1). The fourth column indicates whether the parameters were fixed prior to calibration or considered free during calibration. The fifth column gives the section in this thesis where each parameter is first defined or used mathematically.
3.1. Physics and design of DARK SAGE

3.1.1 Baryonic reservoirs and disc structure

Akin to SAGE and other popular semi-analytic models, every halo from the simulation DARK SAGE is run on is given singular reservoirs for hot gas, ejected gas (hot gas unavailable for cooling), a black hole mass, a merger-driven bulge, an instability-driven bulge, and intracluster stars. Subhaloes (where satellite galaxies live) include these reservoirs except for the ejected gas and intracluster stars. Discs are instead broken into 30 reservoirs, allowing the one-dimensional structure of discs to be directly evolved within the hierarchical framework. The processes by which these reservoirs grow and how baryons move between them will be described in the following subsections.

We draw inspiration from the Fu et al. (2010, 2013) models for breaking galactic discs into annuli, but make several important points of difference. First, instead of breaking discs into bins of radius, \( r \), we break discs into bins of specific angular momentum,

\[
j = r v_{\text{circ}}(r),
\]

assuming baryons in discs follow Keplerian orbits [see Appendix C.2 for how \( v_{\text{circ}}(r) \) is calculated]. This binning method, also used in the models of Stringer & Benson (2007) and Dutton & van den Bosch (2009), means we evolve galactic discs explicitly as a function of specific angular momentum. Because \( j \) increases monotonically with radius, each bin still represents a disc annulus. The primary advantage of binning this way is that we naturally conserve angular momentum within discs, something that does not happen for radial bins, as the velocity structure of galaxies is very dynamic. Thus, we do not need to manually account for radial flows of gas, e.g. as done...
by Fu et al. (2013).

To first order, one would expect the surface density of discs to fall exponentially with radius and for $j$ to increase linearly with radius (but see Section 3.2.1 and Appendix C.2). As such, we use fixed specific-angular-momentum bins for all galaxies, spaced equally in logarithmic space. More specifically, the outer edge of each bin is given by

$$j_i = j_1 \times f_{\text{step}}^{i-1}, \quad i = 1, 2, ..., 30$$

(with the inner edge of the first annulus at $j = 0$). Our choice of using 30 annuli per galaxy allows for sufficient resolution to evolve disc structure accurately and follows Fu et al. (2010), but truthfully is arbitrary (Fu et al. 2013 suggest as few as 12 annuli might be sufficient for producing galaxies with realistic properties). We find our results are converged with $j_1 = 1.0 h^{-1} \text{kpc km s}^{-1}$, so we adopt this value. $f_{\text{step}}$ is then set so the outermost annulus roughly reaches the virial radius of a Milky Way-like system. This gives us $f_{\text{step}} = 1.4$. For an image of an example disc, see Fig. 3.1a.

The second difference from the Fu et al. models is that we do not force disc stars and gas in the galaxy to be coplanar. Instead we track separate spin vectors for each disc component (similar to Lagos et al. 2015a), but ensure they do not persist with non-physical, large angular separations through a gas precession module (Section 3.1.5). Hereafter, we shall refer to these components as the gas disc and stellar disc.

### 3.1.2 Cooling of hot gas and formation of the interstellar medium

Galaxies initially form and grow from radiative cooling and condensation of hot gas in haloes (White & Rees 1978). We follow the prescription of SAGE for determining how much gas cools at each time interval in the model. Based on White & Frenk (1991), the hot gas is assumed to be a singular isothermal sphere at the halo’s virial temperature. The gas cooling rate then follows the similarity solutions of Bertschinger (1989), which is limited by the free-fall time-scale of the halo, and uses the cooling function tables from Sutherland & Dopita (1993), which depend on temperature and metallicity. For further details, see Section 2.5.

Over the time step, $\Delta t$, the amount of cooling gas, $\Delta m_{\text{cold}}$, must be spread out to form or add to a galactic disc. Following the disc formation scenario proposed by Fall & Efstathiou (1980), it was shown explicitly by Fall (1983) and Mo et al. (1998) that if the gas from an isothermal sphere cools into a disc with an exponential surface density profile,

$$\Sigma_{\text{cool}}(r) = \frac{\Delta m_{\text{hot} \rightarrow \text{cold}}}{2 \pi r_d^2} e^{-r/r_d},$$

(3.3)
the specific angular momentum of the gas will be conserved if the newly formed, centrifugally supported disc rotates with a constant circular velocity, \( v_{\text{circ}} = V_{\text{vir}} \), and has a scale radius of

\[
r_d = \frac{\lambda}{\sqrt{2}} R_{\text{vir}}.
\]

(3.4)

Here, \( \lambda \) is the spin parameter of the halo:

\[
\lambda = \frac{J|E|^{1/2}}{G M_{\text{vir}}^{5/2}} = \frac{j_{\text{halo}}}{\sqrt{2} V_{\text{vir}} R_{\text{vir}}}.
\]

(3.5)

(cf. Eq. 1.6), where \( J \) and \( E \) are the total angular momentum and energy within the virial radius, respectively (Peebles 1969; Bullock et al. 2001). Here, the net specific angular momentum of the cooling gas matches that of the halo, \( j_{\text{halo}} \).

The above cooling picture can be generalised by considering the distribution of cooling gas as a function of specific angular momentum:

\[
\Sigma_{\text{cool}}(j) = \frac{\Delta m_{\text{cold}}}{2 \pi r_d^2} e^{-j/r_d V_{\text{vir}}}.
\]

(3.6)

We apply Eq. 3.6 when determining how the cooling gas is distributed among the \( j \) bins in DARK SAGE, thereby ensuring angular momentum is conserved as the gas cools. By using this instead of Eq. 3.3, we no longer assume cooling gas to have a singular rotational velocity. Cooling gas of a given \( j \) is then assumed to localise itself with gas in the disc of the same \( j \).

The orientation of the cooling gas’s angular momentum is always assumed to be parallel to that of the halo at the time of cooling. As both the magnitude and direction of a halo’s spin is very dynamic over a Hubble time, this means subsequent cooling episodes likely occur at angles to one another. If a gas disc does not exist prior to a cooling episode, then a gas disc is initialised with a spin vector parallel to the halo. If instead a gas disc already exists, the angular-momentum vectors of the pre-existing and cooling gas discs are summed together to define the new orientation of the gas disc. Both the pre-existing and cooling gas discs are then projected onto this new plane, where any orthogonal component of angular momentum is assumed to dissipate, and the discs are summed together. Discs are therefore more compact than in SAGE. See Fig. 3.1b for a depiction of this process.

It is worth noting that neither the direction nor magnitude of a halo’s spin from a cosmological simulation is totally precise (see, e.g., Bullock et al. 2001; Bett et al. 2010). The relative uncertainty of \( j_{\text{halo}} \) is correlated with both the number of particles in the halo and the magnitude of \( j_{\text{halo}} \) itself (Contreras et al. in preparation). A careful qualitative analysis of these uncertainties is left to be presented by Contreras et al., but we do not expect this to significantly impact the results.
of semi-analytic models that are primarily concerned with galaxies in well-resolved haloes, where uncertainties in angular momentum are minimised.

3.1.3 Passive star formation from molecular gas

In a gas disc, fragmentation leads to the formation of giant molecular clouds, which in turn collapse to form stars. At each time-step, we allow stars to form in each annulus based on the local H$_2$ content. Specifically, the star formation rate surface density is

$$\Sigma_{\text{SFR}}(r) = \epsilon_{\text{SF}} \Sigma_{\text{H}_2}(r),$$  \hspace{1cm} (3.7)

where $\epsilon_{\text{SF}}$ is the star formation efficiency. This relation is supported observationally (e.g. from the inner parts of spiral galaxies from Leroy et al. 2008) and has proven successful in semi-analytic models previously (e.g. Fu et al. 2013).

To form stars with this model, the H$_2$ content in each bin must first be calculated. We follow Blitz & Rosolowsky (2004) and assume that the mid-plane pressure of the gas determines the ratio of molecular to atomic hydrogen:

$$R_{\text{H}_2}(r) \equiv \frac{\Sigma_{\text{H}_2}(r)}{\Sigma_{\text{H}_1}(r)} = \left(\frac{P(r)}{P_0}\right)^\chi,$$  \hspace{1cm} (3.8)

with $P_0 = 5.93 \times 10^{-13} \, h^2 \, \text{Pa}$ and $\chi = 0.92$ (cf. Blitz & Rosolowsky 2006). Following Elmegreen (1989), the mid-plane pressure is calculated as

$$P(r) = \frac{\pi}{2} G \Sigma_{\text{gas}}(r) \left[\Sigma_{\text{gas}}(r) + \frac{\sigma_{\text{gas}}}{\sigma_s(r)} \Sigma_s(r)\right],$$  \hspace{1cm} (3.9)

where $\sigma$ is the velocity dispersion for the subscripted component. Eq. 3.9 is only directly applied if the gas and stellar discs are sufficiently aligned. If their angular separation exceeds some threshold, $\theta_{\text{thresh}}$ (nominally $10^\circ$), then the expression simplifies to

$$P(r) = \frac{\pi}{2} G \Sigma_{\text{gas}}^2(r).$$  \hspace{1cm} (3.10)

We assume $\sigma_{\text{gas}} = 11 \, \text{km s}^{-1}$ for all galaxies at all radii (Leroy et al. 2008). In truth, the H$\text{I}$ velocity dispersions of observed spiral galaxies are anti-correlated with radius, but a value of 11 km s$^{-1}$ is typical at an optical radius and is generally within a factor of 2 across the entire disc (Tamburro et al. 2009; also see Zheng et al. 2013).
For the stellar velocity dispersion profile, we follow the relation identified by Bottema (1993),
\[ \sigma_s(r) = \frac{1}{2} V_{\text{vir}} e^{-r/r_d}, \]  
(3.11)
where we use \(r_d\) from Eq. 3.4. An exponentially decreasing \(\sigma_s(r)\) profile is supported by \(N\)-body simulations of stellar discs (Khoperskov et al. 2003) and more recent observations of spiral galaxies (Martinsson et al. 2013).

Disc gas is composed of hydrogen, helium, and metals. We also assume some of the gas is in a warm, ionized phase, unavailable for star formation (following Obreschkow et al. 2009, based on Milky Way observations from Reynolds 2004). As such, we calculate the true fraction of \(H_2\) as
\[ f_{H_2}(r) \equiv \frac{\Sigma_{H_2}(r)}{\Sigma_{\text{gas}}(r)} = \frac{f_{\text{He}} f_{\text{warm}}}{R_{H_2}(r) + 1} [1 - Z_{\text{gas}}(r)], \]  
(3.12)
with \(f_{\text{He}} = 0.75\) and \(f_{\text{warm}} = 1.3^{-1}\) (as used by Fu et al. 2010),\(^1\) where
\[ Z_{\text{gas}}(r) \equiv \frac{\Sigma_{Z,\text{gas}}(r)}{\Sigma_{\text{gas}}(r)}, \]  
(3.13)
is the metallicity of the gas in the disc.

Every episode of star formation is assumed to form a simple stellar population. High-mass stars burn bright and die fast, returning gas and newly formed metals to the interstellar medium through stellar winds and supernovae (feedback is described in Section 3.1.4). We use the instantaneous recycling approximation (following Cole et al. 2000) and immediately return a fraction \(R\) of the gas used to form stars in a given annulus back to the gas disc. We use \(R = 0.43\), based on a Chabrier (2003) initial mass function; in reality, it may take \(\sim 2\) Gyr for 43% of the stellar mass of a population to be returned to the interstellar medium, but most of this is expelled quickly (see, e.g., Leitner & Kravtsov 2011). The net production of newborn stars is hence described by
\[ \Delta \Sigma_{\text{gas} \rightarrow *}(r) = (1 - R) \Sigma_{\text{SFR}}(r) \Delta t. \]  
(3.14)
The returned gas is also assumed to be enriched with new metals, given by a fixed yield, \(Y\) (as in De Lucia et al. 2004). The net change in gaseous metals for a given annulus after an episode of star formation is thus
\[ \Delta \Sigma_{Z,\text{gas}}(r) = \{Y[1 - Z_{\text{gas}}(r)] - [1 - R]Z_{\text{gas}}(r)\} \Sigma_{\text{SFR}}(r) \Delta t, \]  
(3.15)
where the factor of \([1 - Z(r)]\) next to \(Y\) accounts for the fact that metals cannot produce more
\(^1\)While \(\Sigma_{\text{gas}}\) represents ‘cold’ gas, this still includes the warm, ionized gas in the disc.
metals.

Beyond passive star formation from H$_2$, DARK SAGE includes two channels for starbursts (see Sections 3.1.6 and 3.1.8). These events follow the same recycling, enrichment, and feedback regimes.

Stars are always born in the plane of the gas disc. If no stellar disc exists prior to a star formation episode, the stellar disc orientation is initialised as equal to the gas disc’s. If a stellar disc already exists, then the pre-existing and newborn stellar discs are combined in the same manner as gas discs during cooling; the angular-momentum vectors of each disc are summed to define the new stellar disc plane, then both the pre-existing and newborn stellar discs are projected onto this plane, where any orthogonal angular momentum is dissipated (refer again to Fig. 3.1b).

We have tested calculating H$_2$ content with a metallicity-dependent prescription based on McKee & Krumholz (2010). Using this requires minor recalibration of the model, after which our results do not qualitatively change. We have opted to exclude results from this prescription for the sake of simplicity.

3.1.4 Supernova feedback

As the deaths of high-mass stars in the form of Type-II supernovae return mass and metals to the intergalactic medium, so too do they release energy capable of reheating the gas and removing it from the galaxy. In regions of high density, the energy produced by supernovae is dispersed over a higher mass of gas, thereby heating that gas less and allowing it to cool back faster (while maintaining its angular momentum). If the time-scales for this are short, then the net amount of heated gas from supernovae should depend on local gas density. To account for this, we follow the supernova feedback model of Fu et al. (2010) where the local heating rate surface density of gas is

\[
\Sigma_{\text{reheated}}(r) = \frac{\Delta \Sigma_{\text{gas} \rightarrow \text{hot}}(r)}{\Delta t} = \epsilon_{\text{disc}} \frac{\Sigma_{0, \text{gas}}}{\Sigma_{\text{gas}}(r)} \Sigma_{\text{SFR}}(r),
\]  

(3.16)

where \( \Sigma_{0, \text{gas}} \) is a reference surface density and \( \epsilon_{\text{disc}} \) is the mass-loading factor (see Table 3.1). Despite \( \Sigma_{\text{gas}}(r) \) appearing in the denominator of Eq. 3.16, our prescription still preferentially removes low-angular-momentum gas from the disc, as there is simply more of it. This is consistent with results of hydrodynamic simulations (Brook et al. 2011, 2012a).

In annuli where \( \Sigma_{\text{gas}} \ll \Sigma_{0, \text{gas}} \), Eq. 3.16 can shut off star formation entirely. To prevent this, we only use the instantaneous recycling approximation (Section 3.1.3) and apply supernova feedback if a star formation event in an annulus would produce \( > 100 h^{-1} M_\odot \) of new stars. Provided this limit is well below the simulation mass resolution, our results are impervious to its precise value, as stars that form in these annuli contribute little to a galaxy’s stellar mass overall.
3.1. Physics and design of DARK SAGE

The energy released by supernovae may be sufficient to not only heat gas out of the disc, but to eject gas from the halo entirely. To determine both the ejection and reincorporation rate of gas, we follow the standard SAGE prescription (Section 2.7.1).

3.1.5 Gas precession

While it is nice to not force the gas and stellar discs to be coplanar, it is not physically reasonable (i.e. not observed) to have a significant number of galaxies where these discs have a large angular offset. Tohline et al. (1982) discuss that a gas disc embedded in an axisymmetric potential with an initial angular offset will precess to become coaxial or counter-axial with the potential. Hydrodynamic simulations have also shown that a gas disc rotating at an angle to stars should settle to be coaxial or counter-axial within a few galaxy dynamical times if left undisturbed, where the angular-momentum component orthogonal to this axis is lost to other parts of the halo (van de Voort et al. 2015).

Following this narrative, DARK SAGE includes a precession prescription for the gas disc. To ensure a reasonable level of alignment between gas and stars, we assume the potential axis of symmetry to be coaxial with the stars’ rotational axis. Where stellar mass is primarily in a disc or instability-driven bulge, this is coaxial with the stellar disc (see Fig. 3.1c). Where most stellar mass lies in a merger-driven bulge (i.e. early-type galaxies), this is coaxial with the bulge’s spin (see Section 3.1.8).

We formulate a simple precession rate prescription, which is inversely proportional to a galaxy’s dynamical time (inspired by van de Voort et al. 2015), determined as a mass-weighted average of the annuli’s dynamical times,

$$\frac{\Delta \theta_{\text{gas disc}}}{\Delta t} = \vartheta_t \left[ \frac{1}{m_{\text{cold}}} \sum_{i=1}^{30} m_i \frac{\bar{r}_i}{v_{\text{circ}}(\bar{r}_i)} \right]^{-1},$$

where \( \bar{r}_i \) is the average radius of the mass in the \( i^{\text{th}} \) annulus and \( m_i = \Sigma_{\text{gas}}(r_i) \pi (r_i^2 - r_{i-1}^2) \).

Our precession parameter is fiducially set at \( \vartheta_t = 5^\circ \) per dynamical time. The precession angle, \( \Delta \theta_{\text{gas disc}} \), is limited to the angle necessary for gas and stars to become co- or counter-aligned (and hence is always < 90°).

Fig. 4.11 shows the importance this module has in the angular offset between stellar and gas discs at redshift zero. Without it, the number density of disc-dominated galaxies with a particular angular offset declines as a shallow exponential with the offset angle. With the module, most gas and stellar discs are coplanar by \( z = 0 \). As expected from the model’s design, a small portion (7.25%) of these coplanar systems are counter-aligned. We do not currently know of any published offsets between axes of rotation for gas and stars are observed in elliptical galaxies though (e.g. Davis et al. 2011).
Chapter 3. Building galaxies with disc structure through angular momentum

Figure 3.3: Number density of disc-dominated DARK SAGE galaxies (bulge-to-total ratio < 0.5) as a function of the angular offset between their gas and stellar discs at $z = 0$. The gas precession module (Section 3.1.5) is necessary to obtain galaxies with realistic offsets, where gas and stars are almost always coplanar.

The spin direction of a halo changes frequently. This means the spin direction of the gas disc keeps changing as more cooling happens (Section 3.1.2). Without precession, the stellar disc will always lag the gas disc when it comes to updating its orientation (new star formation episodes bring the discs more in line – Section 3.1.3). That lag means it is more likely to have a non-zero (but small) angular offset between the gas and stellar discs, without precession, which is seen by the peak in the dashed distribution in Fig. 4.11. Ultimately, the location of this peak is controlled by the mean rate of change of the haloes’ spin directions, which, in this case, comes from Millennium.

3.1.6 Disc instabilities

Following cooling episodes (Section 3.1.2) and mergers (Section 3.1.8), we check the level of stability of each annulus. We calculate a Toomre $Q$ parameter for both the gas and stars for each annulus, where

$$Q_s(r) = \frac{\kappa(r) \sigma_s(r)}{3.36 G \Sigma_s(r)}$$ (3.18)

(Toomre 1964) and

$$Q_{\text{gas}}(r) = \frac{\kappa(r) c_s}{\pi G \Sigma_{\text{gas}}(r)}$$ (3.19)
3.1. Physics and design of DARK SAGE

(Binney & Tremaine 1987), with epicyclic frequency

\[ \kappa(r) \equiv \sqrt{\frac{2v_{\text{circ}}(r)}{r^2}} \frac{dj}{dr} \tag{3.20} \]

(Pringle & King 2007). For the speed of sound, we approximate \( c_s \approx \sigma_{\text{gas}} = 11 \text{ km s}^{-1} \). \( \sigma_s(r) \) is given by Eq. 3.11. We start by assessing the stability of the outermost annulus of the gas disc and incrementally shift inward. We then do the same for the stellar disc.

If the misalignment between the gas and stellar discs exceeds \( \theta_{\text{thresh}} \), we consider gas and stellar instabilities independently. In that case, should either \( Q_s \) or \( Q_{\text{gas}} \) be < 1 for an annulus, the unstable mass is calculated as that required to be removed to bring the relevant \( Q \) up to 1.

If the gas and stellar discs are aligned, then stars will affect the stability of gas and vice versa. We hence calculate a combined \( Q_{\text{tot}} \) for each annulus, following Romeo & Wiegert (2011), where

\[
Q_{\text{tot}}^{-1}(r) = \begin{cases} 
Q_s^{-1}(r) + W(r) Q_{\text{gas}}^{-1}(r), & Q_{\text{gas}}(r) < Q_s(r) \\
W(r) Q_{\text{gas}}^{-1}(r) + Q_s^{-1}(r), & Q_{\text{gas}}(r) \geq Q_s(r) 
\end{cases} \tag{3.21a}
\]

\[
W(r) \equiv \frac{2 \sigma_{\text{gas}} \sigma_s(r)}{\sigma_{\text{gas}}^2 + \sigma_s^2(r)}. \tag{3.21b}
\]

If \( Q_{\text{tot}} < 1 \) for an annulus, an instability occurs. In that case, stars and/or gas must be shifted out of that annulus such that \( Q_{\text{tot}} \) is raised to 1. To determine how much of the unstable mass is in the form of stars and gas, we first calculate a value of \( Q_{\text{stable}} \), where if each of \( Q_{\text{gas}} \) and \( Q_s \) were equal to this value, \( Q_{\text{tot}} \) would equal 1:

\[
Q_{\text{stable}}(r) = 1 + W(r). \tag{3.22}
\]

If \( Q_{\text{gas}} > Q_{\text{stable}} \), all the unstable mass is in the form of stars. If \( Q_s > Q_{\text{stable}} \), all the unstable mass is in the form of gas. If both are < \( Q_{\text{stable}} \), first \( Q_{\text{gas}} \) is raised to \( Q_{\text{stable}} \), then \( Q_s \) is raised as necessary.

Gaseous instabilities can either be resolved by internal motion of gas or by rapid gravitational collapse of the gas to form stars. Once an unstable gas mass has been calculated, we transfer a parameterised fraction, \( f_{\text{move}} \), of the unstable gas to the adjacent annuli. The proportion of mass moved outward and inward ensures angular momentum is conserved. For the innermost annulus, the gas that would have moved to a lower-\( j \) bin instead feeds the black hole and results in quasar mode feedback (see Section 3.1.9). The remaining gas is consumed in a starburst and associated supernova feedback. These stars are added to the disc in the same way as an ordinary star formation episode. After the entire gas disc has been dealt with, we recalculate \( Q_s \), then deal with the stellar
This method ensures the discs find stability, where it is not necessary to directly transfer the instability-burst stars to the bulge, as done in SAGE for example (although, those stars might immediately be seen as unstable and migrate inward anyway).

Unstable stars are simply transferred to the adjacent annuli, again conserving angular momentum. Should unstable stars exist in the innermost annulus, the portion of unstable stars that lose angular momentum is assumed to lose all of it, and those stars are transferred to the instability-driven bulge. By construction, the instability-driven bulge has no angular momentum (note, this definition differs to the model of Tonini et al. 2016). Fig. 3.1d displays an example case of this process.

Our method of dispersing mass between adjacent annuli of fixed $j$ to resolve gravitational instabilities is complementary to disc models with radial annuli that explicitly impose that annuli torque their neighbours, with the value of that torque dependent on local $Q$ (e.g. Forbes et al. 2012, 2014). The net intended effect is the same.

Whenever stars or gas shift to an adjacent annulus, they take with them metals in proportion to the metallicity of their original annulus. This has consequences for the metallicity gradients of galaxies. This is a subject we intend to look into in a future paper.

An instability is typically regarded as either a global (averaged over the entire disc) or a small-scale, local process. Semi-analytic models, by their design, cannot treat truly local instabilities. While instabilities do not physically happen in annuli, as calculated in DARK SAGE, our model does act like a global prescription would. Because most unstable mass is transferred inward, if annulus $i$ is unstable, the likelihood that annulus $i - 1$ will be unstable is raised. Because we check annulus $i - 1$ next, this then cascades all the way to the centre of the galaxy, which has the same external appearance as a global instability.

### 3.1.7 Stripping and accretion of subhaloes

In addition to stripping of hot gas (Section 2.9), satellite galaxies are subject to ram-pressure stripping of their cold gas, caused by their relative motion to the intergalactic or intracluster medium. Gunn & Gott (1972) proposed that if ram pressure exceeds the gravitational restoring force per unit area of the galaxy then it will successfully strip the gas. At each time-step, for each annulus of each satellite, we check if

$$\rho_{\text{hot, cen}}(R_{\text{sat}}) \frac{v_{\text{sat}}^2}{2} \geq 2\pi G \Sigma_{\text{gas}}(r) [\Sigma_{\text{gas}}(r) + \Sigma^*(r)],$$

where $\rho_{\text{hot, cen}}$ refers to that of the central galaxy (see Eq. 2.3), $R_{\text{sat}}$ is the distance between the central galaxy and satellite, and $v_{\text{sat}}$ is the velocity of the satellite relative to the central. If this
3.1. Physics and design of DARK SAGE

criterion is fulfilled, the gas in that annulus is transferred to the hot reservoir of the main halo (associated with the central galaxy). $\Sigma_*(r)$ is removed from Eq. 3.23 if the gas and stellar discs are not aligned within $\theta_{\text{thresh}}$. Using a combined hydrodynamics and semi-analytic approach, Tecce et al. (2010) show that approximating an isothermal distribution for hot gas in an NFW halo tends to over-predict the strength of ram pressure, especially at higher redshift, but for $z \lesssim 0.5$, this should be accurate within a factor of 2 (see their fig. 3). We also note that Eq. 3.23 does not consider the orientation of the satellite galaxy.

3.1.8 Galaxy mergers

In line with the standard semi-analytic model narrative, when a galaxy–galaxy merger occurs in the model, we first check the baryonic-mass ratio of the merging galaxies (hot gas and intracluster stars are excluded). If this is above $f_{\text{major}}$ (nominally 0.3), then we class this as a major merger. Otherwise, fittingly, the merger is classed as minor. We describe how each type of merger is dealt with below. Both mergers are dealt with in a similar way to SAGE, but with extra details regarding discs.

Major mergers

We opt for a simple model of combining gas discs in major mergers. We sum the angular-momentum vectors of the gas discs of the two systems (as measured relative to their own respective centres of mass), then project both discs onto this axis and sum them together. All annuli are then subject to a merger starburst (Section 3.1.8). Some of this gas also directly feeds the black hole and leads to quasar mode feedback. We follow the phenomenological relation used in SAGE (from Kauffmann & Haehnelt 2000), but modify it so it applies to each annulus individually. The growth of the black hole from this channel is hence

$$\Delta m_{\text{cold} \rightarrow BH} = f_{\text{BH}} \left[ 1 + \left( \frac{280 \, \text{km s}^{-1}}{V_{\text{vir}}} \right)^2 \right]^{-1} \sum_{i=1}^{30} (m_{i,\text{cen}} + m_{i,\text{sat}}) \min \left( \frac{m_{i,\text{sat}}}{m_{i,\text{cen}}}, \frac{m_{i,\text{cen}}}{m_{i,\text{sat}}} \right),$$

(3.24)

where the $m_i$ values are the contributed gas masses to each annulus from each of the satellite and central. Because the $j$ distribution of discs, and hence collisional gas in mergers, is weighted toward the low end, the black hole preferentially accretes low-$j$ gas.

The stellar discs and instability-driven bulges of both galaxies are destroyed during a major merger, where all stellar mass forms a merger-driven bulge. The bulge is assigned a spin direction whose axis is parallel to the orbital axis of the merging galaxies at the last resolved moment before merging. This axis is only used for gas precession (Section 3.1.5) in elliptical galaxies in the
Chapter 3. Building galaxies with disc structure through angular momentum

model, and thus is not important for our results, which focus on spiral galaxies.

**Minor mergers**

For minor mergers, we assume the larger (central) galaxy maintains the integrity of its structure. If the baryons of the merging satellite are to be added to the disc of the central galaxy, they must be assigned some specific angular momentum relative to the central’s centre of mass. First, we measure the specific angular momentum of the satellite itself, relative to the central, \( \vec{j}_{\text{sat}} = \vec{R}_{\text{sat}} \times \vec{v}_{\text{sat}} \), at the last snapshot it is resolved before merging.\(^3\) We then project this vector onto the central’s gas rotation axis. We then convolve this \( j \) with a top-hat of width \( 2R_{\text{sat,projected}}V_{\text{vir,sat}} \), accounting for the spread of \( j_{\text{sat}} \) due to its rotation, and deposit the satellite’s gas in the central’s gas annuli of corresponding \( j \) (see Fig. 3.1e). Each of these annuli are then subject to a merger starburst (Section 3.1.8) and *direct* feeding of the black hole (Eq. 3.24). If the satellite’s orbit is retrograde as seen from the central’s gas disc, we consider twice the gas mass deposited in an annulus, less that consumed in the starburst, to be unstable (fed into the instability prescription above).

To be consistent with SAGE, and for simplicity, we deposit merging satellites’ stars in the merger-driven bulge of the central. We find that half of all minor mergers are retrograde relative to the central’s disc. Therefore, angular momentum for stars in minor mergers should be conserved on average. The direct feeding of the black hole from the gas throughout the disc also removes some of the angular momentum we artificially introduce from retrograde satellites. Our consideration of angular momentum during mergers is a first-order approximation only, and clearly has room for improvement.

**Merger starbursts**

Whenever gas originating from two galaxies collides in an annulus, a merger starburst occurs. To determine how much gas is consumed in the starburst, we again maintain the prescription in SAGE (from Somerville et al. 2001) but apply it to the annuli individually:

\[
\Delta m_{i,\text{burst}} = -\beta_{\text{burst}} (m_{i,\text{sat}} + m_{i,\text{cen}}) \left[ \min \left( \frac{m_{i,\text{sat}}}{m_{i,\text{cen}}}, \frac{m_{i,\text{cen}}}{m_{i,\text{sat}}} \right) \right]^{\alpha_{\text{burst}}},
\]

with \( \alpha_{\text{burst}} = 0.7 \) and \( \beta_{\text{burst}} = 0.56 \) (see Mihos & Hernquist 1994, 1996; Cox et al. 2004). All stars generated by merger starbursts are added to the merger-driven bulge.

\(^3\)In reality, dynamical friction could reduce the specific angular momentum of a merging satellite between the snapshot of last resolution and the actual time of merging. In this sense, our calculation of \( j_{\text{sat}} \) is more of an upper limit.
3.2. Galaxy structure

3.1.9 Black holes and active galactic nuclei

We maintain a two-mode model for black-hole growth and AGN feedback, with a ‘radio mode’ and ‘quasar mode’ (Section 2.8). The radio mode is implemented identically to SAGE. Here, we slightly modify the quasar feedback prescription. In SAGE, quasar feedback either heated all the cold gas in a galaxy or none of it. With DARK SAGE, we check if the energy is sufficient to heat the gas of each annulus, beginning from the centre and working outward until all the energy is used. If there is still energy left to eject the hot gas, this will occur as well.

3.2 Galaxy structure

The results we first present are the surface density profiles of Milky Way-like galaxies from DARK SAGE at $z = 0$. Comparing these profiles to well-studied observed galaxies serves as a test for whether the model can produce galaxies with realistic spatial structure. This is an important check of the model methodology. More explicitly, this legitimises using specific angular momentum as an independent variable of galaxy evolution. In order to make precise predictions for the net spin of galaxies and how this scales with other properties, one must integrate over the galaxies’ angular-momentum structure.

Because annuli have edges of fixed specific angular momentum, the corresponding disc radius changes from galaxy to galaxy, based on the given galaxy’s rotation curve. First, we interpolate the radial surface density profiles of the DARK SAGE galaxies for each species of stars, H\textsubscript{1}, and H\textsubscript{2} onto a common radial grid. Then we plot these in Fig. 3.4, showing the 16\textsuperscript{th}–84\textsuperscript{th} percentile range across the bins, as well as the mean and median. With cuts to virial velocity, morphology, stellar and gas mass, these are compared against analogue, observed galaxies from Leroy et al. (2008). To the stellar profiles, we add the contribution from the bulge of each galaxy by projecting their assumed profiles (see Appendix C.2) to two dimensions.

The predicted surface density profiles of DARK SAGE galaxies are broadly in agreement with observations, at a level just as good, if not better, than previous semi-analytic attempts to build disc structure (e.g. Fu et al. 2010, 2013). Because the result of Fig. 3.4 in no way affected the parameter values of the model, we can be relatively confident about the structure-dependent predictions the model makes.

We note there appears to be an excess in the central stellar surface density of our galaxies compared to observations (top panel of Fig. 3.4). This contribution is from the disc and not a bulge component (but see Section 3.2.1). This feature has shown up consistently in models of disc evolution, especially those based on angular momentum (e.g. Dalcanton et al. 1997; van den Bosch 2001; Bullock et al. 2001; Stringer & Benson 2007). This excess could be explained by the
Chapter 3. Building galaxies with disc structure through angular momentum

Figure 3.4: Face-on surface density profiles for galaxies produced by DARK SAGE at $z = 0$. The three panels display the stellar, H$_1$, and H$_2$ profiles, from top to bottom. The gas profiles are built solely from that in discs, while the stellar profiles include the bulge mass contribution (note that $\Sigma_*$ refers only to the disc everywhere else in this chapter). We include galaxies with $175 \leq V_{\text{vir}}/\text{km s}^{-1} \leq 235$, $m_* > 10^{10}h^{-1} \text{M}_\odot$, $m_{\text{cold}} > 10^{9.2}h^{-1} \text{M}_\odot$, and bulge-to-total ratios < 0.5. The mean, median, and 16th–84th percentile range of the profiles of the DARK SAGE galaxies within these cuts are presented. In support, profiles of observed galaxies with similar properties are shown with errors (Leroy et al. 2008), specifically NGC 628, 3184, 3351, 3521, 3627, 5055, 5194, and 6946.
model lacking consideration of radial dispersion support in the disc. While the assumption that discs are entirely rotationally supported is accurate at larger radii, a comparable level of dispersion support should exist towards the centres of discs, which breaks down the simplicity of building a rotation curve and applying Eq. 3.1 to convert from $j$ to $r$. As noted by Bower et al. (2006), the assumption that discs shrink to be entirely rotationally supported can make them too small and almost completely self-gravitating, which is consistent with our results. The mass contributions from the overdense regions of the discs are insufficient to notably affect their integrated properties. Our primary results of this chapter, concerned with angular momentum, are hence not directly affected by this.

The extra central stellar content leads to an artificial rise in the central gas pressure (through Eq. 3.9), which decreases the fraction of gas in an H\textsc{i} state. As such, the DARK SAGE galaxies also show a deficit in H\textsc{i} surface density in their centres (middle panel of Fig. 3.4). This has a minimal effect on their integrated H\textsc{i} content.

### 3.2.1 Discs and pseudobulges

Although gas cools initially into a disc with an exponential dependence on $j$ in the model, come redshift zero, discs profiles are not exponential in the inner regions. This is due to a culmination of processes: full rotation curve modelling leads to gas (and stars) settling into a more complex structure, various star formation channels convert gas to stars in different regions at different rates, feedback from star formation affects each annulus differently, instabilities drive material inward, etc. We demonstrate this by plotting the surface density profiles of discs alone for stars and all cold gas as a function of $j$ in Fig. 3.5. The profiles for each galaxy have been normalised, such that they can be fairly compared with one another, and such that one would expect an exponential profile in the case that Eq. 3.6 were the only expression driving disc structure (which, obviously, is not the case). An equivalent plot with $r/r_d$ on the $x$-axis looks nearly identical. We have fitted exponentials to the medians of the outer parts of the profiles and over-plotted Eq. 3.6. Because stars form primarily from low-$j$ gas (see Section 3.1.3), it is unsurprising that these exponentials are consistent for gas but inconsistent for stars.

Few galaxies have been studied in the local Universe in sufficient detail to directly measure $\Sigma_{\text{gas}}(r)$. While many spirals have had their H\textsc{i} structure observed, H\textsc{2} data are required as well for a complete picture. Bigiel & Blitz (2012) derived cumulative surface density profiles for H\textsc{i} and H\textsc{2} of 33 local spirals, promoting a ‘universal’ profile from their results. The normalised profiles of their galaxies (see their figs. 2 and 3) are consistent with an exponential gas disc with a central cusp. Our results are, therefore, at least qualitatively consistent with observations, but more data are required to test this thoroughly.
Chapter 3. Building galaxies with disc structure through angular momentum

Figure 3.5: Face-on surface density profiles of discs alone for stars (top) and cold gas (bottom) as a function of specific angular momentum. Surface densities for each galaxy have been normalised by their mass and presumed scale radius (Eq. 3.4), while \( j \) has been normalised in each case by the same radius and the virial velocity of the halo. We compare exponential profiles in each panel, fitted to the median curves over the range \( j/r_d V_{\text{vir}} > 0.9 \). The dot-dashed line in the bottom panel represents Eq. 3.6. The same DARK SAGE galaxy population is presented in Fig. 3.4. See text in Section 3.2 for a discussion.

Observationally, cusps at the centres of otherwise-exponential stellar surface density profiles are often associated with a bulge component. In our model, while we have accounted for a purely pressure-supported instability-driven bulge, we have no explicit consideration of pseudobulges. Characteristically, pseudobulges have a steeper density profile than an exponential disc (with Sérsic indices \( \lesssim 2 \)) but are rotationally supported like discs (Kormendy 1993; Kormendy & Kennicutt 2004; Fisher & Drory 2008). These can form through disc instabilities driving material towards the centre of galaxies and heating them, resulting in a rising thickness at lower radii (see Toomre 1966; Kormendy & Kennicutt 2004). The cusp seen at low \( j \) in our disc profiles matches this description, suggesting galaxies in DARK SAGE naturally form pseudobulges.

Fisher & Drory (2008) have empirically shown that pseudobulges, on average, have half-mass radii approximately equal to \( 0.2 r_d \). If we consider, in post-processing, for galaxies that have formed an instability-driven bulge, all the stellar disc mass within \( 0.2 r_d \) to be a part of a bulge instead of the disc, we naturally find different bulge-to-total ratios for our galaxies. To demonstrate this, we have compared the results of DARK SAGE with and without this redistribution of mass to observations of the bulge-dominated and disc-dominated stellar mass functions from the Galaxy And Mass Assembly (GAMA) survey (Moffett et al. 2016). This is presented in Fig. 3.6. We find overall better agreement with the data with this simplistic pseudobulge consideration (dashed
3.2. Galaxy structure

Figure 3.6: Stellar mass functions considering bulge-dominated (bulge-to-total ratio $> 0.5$) and disc-dominated ($\leq 0.5$) systems separately. Solid curves are the direct output of DARK SAGE at $z = 0$, whereas the dashed curves assume that stellar disc mass inside $0.2r_d$ counts as pseudobulge mass for systems that have developed an instability-driven bulge. Compared is the result from the public version of SAGE and observations from Moffett et al. (2016, based on Hubble type).

Another way of comparing the stellar structure of DARK SAGE galaxies against observations is to look at their sizes through some standard measure. In Fig. 3.7, we present the half-mass radii of stellar discs in DARK SAGE disc galaxies as a function of their mass at $z = 0$. These are compared to half-light radii for the complete profiles of S(B)0- and S(B)a-type galaxies from the GAMA survey, as a function of their stellar mass (Lange et al. 2016). Of the various size–mass relations presented by Lange et al. (2016), this is the most relevant, as it was determined with a single-component fit for the most disc-dominated systems. The cyan shaded region in Fig. 3.7 is the best-fitting power law to their data, including the quoted uncertainties in the fit. While the compared values are not identically defined, the size of the observed galaxies are remarkably similar to those produced by DARK SAGE. For $m_{*,\text{disc}} \gtrsim 10^{8.9} M_\odot$, the top panel of Fig. 3.7 suggests DARK SAGE galaxies may be slightly too small. This reflects the findings of Figs. 3.4 and 3.5 that too much stellar mass pools at the centre of the discs due to instabilities and a lack of modelled pressure support. By eliminating the galaxies that have suffered instabilities in their recent history (i.e. those with a non-zero instability-driven bulge mass), we find the model almost perfectly matches the observational data, as seen in the bottom panel of Fig. 3.7.
Chapter 3. Building galaxies with disc structure through angular momentum

Figure 3.7: Relation between mass and half-mass radius of stellar discs in DARK SAGE at $z = 0$. Top panel: Includes all DARK SAGE galaxies with bulge-to-total ratios $< 0.3$. Bottom panel: Added restriction for DARK SAGE galaxies of having no instability-driven bulge. Both panels give 4000 representative galaxies as crosses, with the mean, median, and 16th–84th percentile range given by the solid curves, dashed curves, and blue shaded regions, respectively. The cyan shaded region compares the single-component half-light radius and stellar mass of observed S(B)0- and S(B)a-type galaxies, using the best-fitting power law from Lange et al. (2016).
3.3 Mass–spin relation of spirals

Several observational studies of galaxies in the local Universe have suggested there is a strong correlation between the stellar mass of a galaxy and its specific angular momentum (e.g. Fall 1983; Romanowsky & Fall 2012; Obreschkow & Glazebrook 2014). These mass–spin sequences evidently vary for galaxies of different morphology.

DARK SAGE evolves the angular-momentum structure of discs, but makes no direct predictions for the angular momentum of spheroids. In the interest of comparing against larger datasets, we focus on results concerning the stellar specific angular momentum of spiral galaxies. We intend to address the total baryonic specific angular momentum of DARK SAGE galaxies in future work.

3.3.1 Stellar discs in the local Universe

With Fig. 3.8, we display the predicted stellar mass–spin relation at $z = 0$ for discs of spiral galaxies from DARK SAGE, i.e. showing integrated mass, $m_{*,\text{disc}}$, against net specific angular momentum,

$$j_{*,\text{disc}} \equiv \frac{\sum_{i=1}^{30} m_{*i} j_{i}}{\sum_{i=1}^{30} m_{*i}} = \frac{\sum_{i=1}^{30} m_{*i} (j_{i} + j_{i-1})}{2 m_{*,\text{disc}}}.$$  \hspace{1cm} (3.26)

We use galaxies with bulge-to-total ratios less than 0.3 as ‘spiral galaxies’. Compared in Fig. 3.8 are the equivalent results from the public version of SAGE (Chapter 2) and observations of spiral galaxies (Fall & Romanowsky 2013; Obreschkow & Glazebrook 2014).

In SAGE, $j$ is not an evolved quantity; instead, it is assumed that $j_{*,\text{disc}} = j_{\text{halo}}$. While $j_{\text{halo}}$ is set entirely by the simulation and not the semi-analytic model, the corresponding $m_{*,\text{disc}}$ comes from the galaxy evolution physics. This is indirectly and loosely constrained by the stellar mass function. The lack of two-dimensional freedom limits SAGE’s ability to examine the cause of the mass–spin relation of discs. With its complete evolution of angular-momentum structure, DARK SAGE is better equipped.

That said, both models overlap the observational data, where the data seem to lie in between the two models (Fig. 3.8). It is unsurprising that the DARK SAGE sequence has systematically lower $j$ than SAGE; while gaseous discs carry the same spin as the halo upon birth, subsequent cooling episodes, gas precession, and mergers in DARK SAGE all lead to a reduction in the specific angular momentum of the disc. In addition, stars are preferentially born from low-$j$ gas. The DARK SAGE sequence is also notably tighter than SAGE.

The observational data we have compared against have been measured with two techniques. Many of the Fall & Romanowsky (2013) data calculated $j_{*,\text{disc}}$ analytically by assuming stellar discs followed exponential surface density profiles with a constant rotation velocity. These were originally published in Romanowsky & Fall (2012), but the masses were updated in Fall & Ro-
Chapter 3. Building galaxies with disc structure through angular momentum

Figure 3.8: Integrated stellar disc mass versus net specific angular momentum of stellar discs from DARK SAGE galaxies with bulge-to-total ratios < 0.3. 4000 randomly sampled galaxies are shown by the black crosses in the axes, with the mean and inner 68% of galaxies in 0.1-dex bins also presented. Compared is the equivalent result from SAGE, which assumes the specific angular momentum of the disc and halo are the same. Supporting observational data from spiral galaxies are presented from Fall & Romanowsky (2013) and Obreschkow & Glazebrook (2014). Their uncertainties in $j_{\star,\text{disc}}$ are approximately the size of the data points, while the mass uncertainties are dominated by systematics at the level of 0.5 dex (see section 3 of Romanowsky & Fall 2012).

manowsky (2013), using a variable mass-to-light ratio. The Obreschkow & Glazebrook (2014) specific angular momenta were calculated by properly integrating the resolved angular momentum profiles, but their masses assumed a constant mass-to-light ratio. These authors find the uncertainty introduced by calculating $j_{\star,\text{disc}}$ with the method of Romanowsky & Fall (2012) is correlated with the value of $j_{\star,\text{disc}}$. Six galaxies exist in both samples.

Earlier in this project, we hypothesised that the instability prescription could drive the mass–spin relation of discs. This is because the prescription, in essence, places an upper limit on the amount of mass allowed in an annulus. As instabilities drive most of the unstable mass inward, low-angular-momentum mass from the disc is transferred to the bulge. This then approximately translates into a lower limit on the net specific angular momentum in a stellar disc of a given integrated mass.

The top panel of Fig. 3.9 presents the mass–spin relation for DARK SAGE stellar discs if all consideration of disc instabilities is removed from the model (without changing parameter values). With instabilities switched off, the low-$j$ part of the axes becomes occupied. In addition, the average $j_{\star,\text{disc}}$ is lowered (e.g. at $m_{\star,\text{disc}} = 10^{11} M_\odot$, it is lower by $\sim$0.4 dex). The scatter (standard deviation) also increases from $\sim$0.19 to $\sim$0.22 dex. There still exists a trend between mass and spin, but without instabilities, the model would simply not be able to match the data.
Although the above suggests instabilities drive the mass–spin relation of discs, some galaxies may have had stable discs since their formation. We can identify those galaxies in \textsc{Dark Sage} as having no instability-driven bulge (if these galaxies have had major mergers, their discs will have been stable since the last major merger). We can consider this group of galaxies somewhat as a control sample to test whether stable galaxies still obey the observed mass–spin relation. These galaxies are shown in the bottom panel of Fig. 3.9 and match the observational data almost perfectly. Indeed, if we use the halo identification numbers of these galaxies and rerun the model without instabilities, these systems still match the observations. Therefore, the galaxy evolution physics considered in \textsc{Dark Sage} (disregarding instabilities) naturally explains the mass–spin relation of spiral galaxies with stable discs. Toomre instabilities then act to regulate the specific angular momentum in the discs of the remaining galaxies to naturally bring them in line with observations.

As discussed in Section 3.2.1, the inner disc material of \textsc{Dark Sage} galaxies may actually represent a pseudobulge. In principle then, the \textsc{Dark Sage} discs should have lower masses and
higher specific angular momenta than that presented in Fig. 3.8. Once again, to approximate this to first order, we shift all stellar mass within $0.2r_d$ to the bulge in post-processing for the galaxies with non-zero instability-driven bulge masses. We then resample the spiral galaxies and plot the mass and specific angular momentum of their stellar discs in Fig. 3.10. With this adjustment, we now find DARK SAGE spiral galaxies in general to be in remarkable agreement with observations.

Pseudobulges are thought to form through disc instabilities, so, naturally, galaxies without an instability-driven bulge should be without a pseudobulge. Hence the results of the mass–spin relation after adjusting for the pseudobulges of general DARK SAGE spiral galaxies (Fig. 3.10) agree quite well with the spiral galaxies without instability-driven bulges (bottom panel of Fig. 3.9).

### 3.3.2 Evolution of stellar discs

We now turn our attention to the redshift evolution of the mass–spin relation for stellar discs of spiral galaxies. In the top panel of Fig. 3.11, we present the sequence from DArk SAGE at select redshifts, where we have selected galaxies based on their morphology at that redshift. We find there is a trend for stellar discs of fixed mass to have lower specific angular momentum at earlier epochs. The trend is relatively weak however, where the mean of the sequence only increases by $\sim 0.4$ dex from $z = 4.8$ to 0 at the high-mass end. This compares to a standard deviation about each sequence of $\sim 0.15$ dex. Qualitatively, the same trend is seen in the EAGLE hydrodynamic simulations (Lagos et al. 2016).

In the bottom panel of Fig. 3.11, we present the same mass–spin sequence evolution, but now without instabilities in the model. Not only does this alter the sequence for the local Universe,
but the dependence with time is opposite to the full model. Without the instability channel of star formation, stellar discs take longer to build up their mass. Also, without the ability to regulate where in the disc this stellar mass ends up, these discs are typically low-mass with high specific angular momentum at higher redshift. As time progresses, the low-$j$ stars, that otherwise would have been transferred to the bulge, loiter in the disc.

Even without instabilities, $j_{s,\text{disc}}$ of a given galaxy does not typically decrease with time. To illustrate this point, we show the evolution of $j_{s,\text{disc}}$ for the most-massive progenitors of $z = 0$ spirals in DARK SAGE in Fig. 3.12, given as a function of their final mass. These results include instabilities in the model, but the same qualitative result is found without them. Here we see that stellar discs, on average, clearly begin with lower specific angular momentum than they end up with.

As shown by Fig. 3.12, as we approach higher redshift, the mean spin of the local-spiral progenitors begins to lose its dependence on final mass. By design, gas discs are initially set to have the same spin as their halo. The spin of haloes follows a fairly strict log-normal distribution

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure3.11.png}
\caption{Evolution of the mass–spin sequence for stellar discs of DARK SAGE galaxies with bulge-to-total ratios < 0.3. The shaded region around each curve covers the 16th–84th percentile range for that redshift. \textit{Top panel:} Fiducial model. \textit{Bottom panel:} Instability prescription excluded from the model.}
\end{figure}
which is only weakly correlated with mass (Barnes & Efstathiou 1987; Bullock et al. 2001; Knebe & Power 2008). Further, because the gas discs have the same spin at birth on average, the first episode of star formation in these discs will mean the same is true for stellar discs. As such, the initial spin of a stellar disc should not depend on its final mass. Note that larger objects in the local Universe tend to have formed (resolved structures in the simulation) at earlier epochs. One thus has to go to relatively higher redshift to find the progenitors of local massive spirals at a state of equivalently low spin.

### 3.3.3 Total stellar content

Given a halo with a known mass profile and spin parameter (Eq. 3.5), one can calculate the halo’s specific angular momentum. Based on this idea, Romanowsky & Fall (2012) proposed a toy model for producing the mass–spin relation of galaxies, considering their total stellar content. Those authors approximated the mass profiles of haloes as NFW (Navarro et al. 1996, 1997) profiles with a fixed concentration, $c = 9.7$ (defined in Appendix C.2). Following this same process, but using the cosmology of the Millennium simulation and adjusting for the difference in definition of virial mass, which are implicitly assumed, this gives

$$
\frac{J_{\text{halo}}}{\text{kpc km s}^{-1}} \simeq 5.73 \times 10^4 \lambda \left( \frac{M_{\text{vir}}}{10^{12} M_\odot} \right)^{2/3}
$$

(cf. equation 14 of Romanowsky & Fall 2012). The authors then related these quantities to their stellar equivalents assuming the stellar specific angular momentum of a galaxy to be some fixed
fraction of the halo, \(^4\)
\[ f_j \equiv \frac{j_s}{j_{\text{halo}}}, \]  
(3.28)

and using the stellar–halo mass relation of Dutton et al. (2010, their equation 3). Under our implicit assumptions, this gives
\[ f_s(m_*) \equiv \frac{m_*}{f_b M_{\text{vir}}} \simeq 0.28 \sqrt{\frac{m_*/M_0}{1 + m_*/M_0}}, \]  
(3.29)

where \( M_0 = 10^{10.4} h^{-2} M_\odot \). Combining all of this leads to an expression for estimating \( j_* \) from \( \lambda \) and \( m_* \) alone:
\[ \frac{j_*}{\text{kpc km s}^{-1}} \simeq 4.02 \times 10^4 \frac{\lambda f_j}{f_s^{2/3}(m_*)} \left( \frac{m_*}{10^{11} M_\odot} \right)^{2/3}. \]  
(3.30)

We compare this theory to the direct predictions of DARK SAGE. In Fig. 3.13, we again plot the mass–spin relation for spiral galaxies, but now include stars both in the disc and bulge. We remind the reader that DARK SAGE currently makes no direct predictions for the magnitude of angular momentum present in a bulge. As such, we assume bulge stars to carry zero angular momentum.\(^5\) That is,
\[ j_* \equiv \sum_{i=1}^{30} m_{x_i} \left( j_s + j_{i-1} \right) \]  
(3.31)

This is a reasonable approximation, as, based on our morphology cut, bulge stars should contribute very little angular momentum compared to those in the disc (see section 2.3 of Obreschkow & Glazebrook 2014).

To produce values of \( j_* \) from the toy model of Romanowsky & Fall (2012), we take the same sample of DARK SAGE galaxies, and plug their values of \( \lambda \) and \( m_* \) into Eq. 3.30. In the bottom panel of Fig. 3.13, we plot the derived mass–spin relation, assuming \( f_j = 0.16 \), a value we treated freely to find the best visual agreement. The mean relation is similar to the true DARK SAGE one, but the toy model does induce a larger amount of scatter (standard deviations of \( \sim 0.18 \) dex versus \( \sim 0.25 \) dex). As a point of interest, had we instead presented DARK SAGE results without the instability prescription here, the scatter would have been consistent.

Of course, the quantities \( f_j \) and \( f_* \) are also predicted directly by DARK SAGE. The first two panels of Fig. 3.14 present these values for the same galaxies as shown in Fig. 3.13. Both quantities, especially \( f_j \), show a large amount of scatter. For reference, the dashed lines also

\(^4\)In Romanowsky & Fall (2012), \( f_j \) is discussed as the fraction of specific angular momentum lost by stars to other parts of the halo over time. Given that Eq. 3.27 assumes \( z = 0 \), in addition to the fact that masses and spins of haloes are dynamic quantities, their description is not to be taken literally.

\(^5\)With the exception of potential pseudobulges, which are incorporated in the disc profiles (Section 3.2.1).
Figure 3.13: Top panel: As for Fig. 3.8 but now calculating mass and specific angular momentum considering all stars in DARK SAGE spiral galaxies. Bottom panel: Reproducing the top figure using the same stellar masses, but calculating $j_*$ from the theory of Romanowsky & Fall (2012), our Eq. 3.30, assuming $f_j = 0.16$. We include the mean trends for values of $j_*$ in each case on both plots for comparison.
3.4 Outlook and conclusions

We have presented our new semi-analytic model of galaxy evolution, DARK SAGE. With this model, we have broken discs into annuli of fixed specific angular momentum, while evolving the angular momentum vectors of both the stellar and gas components of discs. This level of detail in the angular-momentum evolution of galaxies is a new step for semi-analytic models, and allows
processes such as star formation, supernova feedback, metal enrichment, ram-pressure stripping of cold gas in satellites, and disc instabilities to be performed locally. Galaxies are therefore evolved as an explicit function of specific angular momentum, on local scales, self-consistently, in a cosmological framework.

In this chapter, we have focussed on predictions of DARK SAGE for two scientific areas of interest. First, the model is capable of producing realistic surface density profiles for disc galaxies in the local Universe for each of stars, H\textsc{i}, and H\textsc{ii}, as presented in Section 3.2. Interestingly, the profiles of the discs themselves are not exponential in the centres, but rather form a cusp, likely indicating the formation of a pseudobulge. A first-order consideration of the mass of these pseudobulges brings the model in better agreement with observed morphological stellar mass functions.

Second, we have shown DARK SAGE produces a clear sequence for the net specific angular momentum and mass of stellar discs in spiral galaxies, presented in Section 3.3. This is in very good agreement with observations, especially after accounting for possible pseudobulges. We found that a Toomre instability criterion plays an important role in reducing the scatter of this sequence and for improving its agreement with observations. The minority of high-mass discs that suffer no instabilities since their last major merger naturally agree precisely with observations as well. We also found the instability prescription determines the direction in which the mass–spin sequence evolves with redshift (left to right versus bottom to top). The main progenitors of individual spiral galaxies at $z = 0$ typically grow in specific angular momentum over their lifetime, though, regardless of instabilities. Finally, excluding angular momentum from a merger-driven bulge, our model predicts the ratio of the stellar specific angular momentum of a galaxy to its halo is 0.4 on average, with a standard deviation of 0.29.

DARK SAGE is a model well suited for a number of different scientific investigations. For example, we intend to focus on the metallicity gradients of nearby spiral galaxies in a future study. Predictions for the model concerning H\textsc{i} scaling relations with respect to galaxy environment are to be presented in an upcoming paper as well (Stevens & Brown, in preparation). In another work, we will focus on the different evolutionary tracks galaxies take in the SAGE and DARK SAGE models by making object-to-object comparisons for the same $N$-body simulation haloes.

Plenty of room for further detail in models like DARK SAGE remains. For example, we only evolve the structure of discs in one dimension, thereby assuming axial symmetry. The semi-analytic method does not lend itself well to detailed multi-dimensional information about galaxies, but some additions could be made. Where we currently track a single angular-momentum vector for each gas and stellar disc, one could, in principle, track an angular-momentum vector for each annulus, which would allow for the study of disc warps. We also enforce all baryons in the disc to have positive specific angular momentum with respect to the net rotation axis, but hydrodynamic
3.4. Outlook and conclusions

Simulations have shown a fair portion should have retrograde motion (e.g. Chen et al. 2003). Bins for negative values of $j$ could be included to account for this. Furthermore, DARK SAGE has no explicit consideration of bars, which could affect the internal disc structure of galaxies and how instabilities are dealt with. The model could also include a more explicit consideration of radial migration. Currently, radial migration happens naturally through binning discs by $j$ (Section 3.1.1) and resolving disc instabilities (Section 3.1.6), but one may favour explicitly modelling diffusion, for example.

While the physics of this model is founded on widely used theory, aspects of this model can, and should, be tested. Specifically, the prescriptions for how gas settles into discs during cooling episodes and mergers could be compared with hydrodynamic simulations. It has already been shown, for example, that the net specific angular momentum of dark matter and hot gas in haloes differs both in direction and magnitude (van den Bosch et al. 2002, 2003; Chen et al. 2003). The process of how this gas cools is vital to the evolution of angular-momentum structure in galaxies. This is precisely the motivation for the next chapter.
Comparing analytic models of halo gas cooling with EAGLE

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Fundamentally, galaxies must form from the cooling and condensation of gas residing in overdense regions of the Universe (e.g. White & Rees 1978). The manner in which the gas is accreted over cosmic time is vital to the structure of a galaxy and how it evolves. This is especially true for late-type galaxies, where discs hold the majority of a galaxy’s baryonic mass, and when considering the high-redshift Universe, prior to mergers dominating the growth of the most massive galaxies.

Many of the semi-analytic models in the literature (e.g. Cole et al. 2000; Hatton et al. 2003; Croton et al. 2006, 2016; Lagos et al. 2008; Somerville et al. 2008b; Guo et al. 2011; Benson 2012) have been, in part, based on the disc formation scenario developed by Fall & Efstathiou (1980) and Mo et al. (1998). In these models, it is assumed that halo gas has a uniform temperature and carries the same total specific angular momentum as the halo (variants have adopted a statistical take on spin direction – see Padilla et al. 2014; Lagos et al. 2015a). Two regimes of gas accretion onto galaxies are typically employed. For the ‘hot mode’, infalling gas is first shock heated to the virial temperature, then must cool to reach the galaxy. In the ‘cold mode’, the cooling time-scale is small relative to the free-fall time-scale, and hence the former no longer sets the time-scale for accretion (see Birnboim & Dekel 2003; Kereš et al. 2005; Benson & Bower 2011). When the gas condenses onto a disc, it is assumed to conserve its specific angular momentum, and to settle with a surface density that decreases exponentially with radius. Often rotation curves of discs are approximated as completely flat. A variation of describing cooling profiles as an exponential function of specific angular momentum was also introduced in Chapter 3. While these assumptions all carry physical merit, like anything, they would ideally be subject to independent testing.

With this study, we are motivated to investigate the basic assumptions of the aforementioned
models with respect to the hot mode of accretion. That is, how is hot gas distributed in haloes, what is the nature and distribution of gas as it cools onto a galaxy, and does this gas conserve its specific angular momentum? These are not only important science questions in and of themselves, but answers to these have a direct impact on galaxy evolution model development. To address these topics with modern observations would be an unsurmountable challenge. In this sense, we cannot truly test the model assumptions. We can, however, look to numerical experiments with more detailed physics for insight, namely hydrodynamic, cosmological simulations. This allows us to see how the widely adopted theoretical description of gas cooling on global scales (Fall & Efstathiou 1980; White & Frenk 1991) compares to predictions from modelling galaxy evolution physics on local ($\lesssim$ kpc) scales. Hydrodynamic simulations also carry the advantage that we can immediately relate any results regarding gas cooling to the properties of dark-matter haloes.

We use the EAGLE simulations (Evolution and Assembly of GaLaxies and their Environments; Schaye et al. 2015; Crain et al. 2015) to study the gas particles in haloes that cool from a hot state down onto galaxies. We assess how these particles are distributed in physical space and in terms of specific angular momentum, both prior to and after cooling episodes. We compare this against the overall hot and cold gas profiles in haloes, investigating how the process of cooling leads to evolution in these structures. We further determine whether the angular momentum of these particles is conserved as they cool, both on an individual basis and as a collective system. EAGLE provides an ideal testbed to address these questions, as there is growing evidence the simulation produces a realistic galaxy population in terms of mass (Furlong et al. 2015a), size (Furlong et al. 2015b), specific angular momentum (Lagos et al. 2016), and gas content (Lagos et al. 2015b; Bahé et al. 2016).

Recently, Guo et al. (2016) compared the galaxy populations of two semi-analytic models, run on the dark-matter-only halo merger trees of EAGLE, against the main hydrodynamic EAGLE simulation. Those authors found consistency in the evolution of the stellar mass function and the specific star formation rates of galaxies, but noted clear differences when galaxies were broken into passive and star-forming. However, their study did not address whether the physical description and approximations of galaxy evolution processes in the semi-analytic models were supported by hydrodynamic simulations. Our study contributes by focussing on the physics of the models, rather than testing whether the end result, i.e. the galaxy properties, are in agreement with each other or observations.

This chapter is structured as follows. In Section 4.1, we provide details on the design of the EAGLE simulations and the data products that we have used (the cosmology of the simulation detailed there is assumed throughout the results of this chapter). In Section 4.2, we examine the state of hot gas in haloes, comparing this to the particles which are about to cool. In Section 4.3, we
4.1. The EAGLE simulations and data

discuss the direction and magnitude, with respect to the halo, of the specific angular momentum of gas as it cools, and whether this is conserved. Once those particles have cooled onto a disc, we study their distribution, and that of the disc in full, in Section 4.4. The results of this chapter are then summarised in Section 4.5.

4.1 The EAGLE simulations and data

4.1.1 Overview of the simulations

The EAGLE simulations are a suite of state-of-the-art hydrodynamic, cosmological simulations, first presented by Schaye et al. (2015). They were run using a significantly modified version of GADGET3, an N-body Tree-PM smoothed-particle hydrodynamics (SPH) code. Various modifications of GADGET have been developed by the community. The most recent official public release was GADGET-2 (Springel et al. 2005b). The simulations assumed a ΛCDM cosmology with parameters based on the 2013 Planck data release: \( \Omega_M = 0.307, \Omega_\Lambda = 0.693, \Omega_b = 0.048, h = 0.6777, \sigma_8 = 0.8288, \) and \( n_s = 0.9611 \) (Planck Collaboration 2014). The main ‘Reference’ simulation used a dark-matter particle mass of \( 9.70 \times 10^6 \) M\(_\odot\), an initial gas particle mass of \( 1.81 \times 10^6 \) M\(_\odot\), and a Plummer-equivalent gravitational softening scale of 2.66 comoving kpc for \( z > 2.8 \) and 700 physical pc otherwise. The simulations implemented a modified version of SPH, dubbed ANARCHY (Dalla Vecchia in preparation; but see appendix A of Schaye et al. 2015), along with the time-step limiter of Durier & Dalla Vecchia (2012). Schaller et al. (2015b) presented a comparison of the simulations with the old SPH version, and found that the modifications were important for active galactic nucleus (AGN) activity and star formation rates of large galaxies, but, by and large, did not have a big impact on stellar masses and sizes.

The simulations include a set of sub-grid models that describe physical processes below the resolution limit. These are described in full in Schaye et al. (2015). Briefly, these include the following.

- Radiative cooling of gas particles is calculated by following 11 elements (Wiersma et al. 2009a), which are assumed to be in ionization equilibrium. Gas is exposed to the cosmic microwave background, and after \( z = 11.5 \), a uniform ionizing background is included (Haardt & Madau 2001). Self-shielding and local stellar radiation are ignored.

- Star formation is based on the local pressure of gas (Schaye & Dalla Vecchia 2008) and is consistent with the Kennicutt-Schmidt relation (Kennicutt 1998). This is only triggered above a threshold local density, which has a metallicity-dependent value (Schaye 2004). Star particles are each considered to be single stellar populations that follow the initial
Chapter 4. Halo gas cooling in EAGLE

mass function of Chabrier (2003) and the stellar evolution models described by Wiersma et al. (2009b). Stellar feedback from supernovae is implemented with the stochastic thermal model of Dalla Vecchia & Schaye (2012).

- Black-hole particles of mass $10^5 h^{-1} M_\odot$ are seeded in haloes once they reach a mass of $10^{10} h^{-1} M_\odot$ (Springel et al. 2005b). These accrete gas from neighbouring particles, with consideration of angular momentum as in Rosas-Guevara et al. (2015), which leads to AGN feedback where nearby gas particles are stochastically heated (Schaye et al. 2015).

The free parameters of the sub-grid physics in EAGLE were calibrated to best reproduce a small number of key observables of galaxies in the local ($z \sim 0$) Universe: the stellar mass function (Li & White 2009; Baldry et al. 2012), the stellar size–mass relation of Shen et al. (2003), and the black hole–stellar mass relation of McConnell & Ma (2013). See Crain et al. (2015) for further details.

In this chapter, in addition to the Reference simulation, we also use runs with modified feedback physics but equal resolution. These include runs with stellar feedback half the Reference strength (WeakFB), stellar feedback with twice the Reference strength (StrongFB), and an absence of active galactic nucleus feedback (NoAGN). We also use the high-resolution run of EAGLE (with 8-fold superior mass resolution), for which the strength of feedback was recalibrated to meet the same observational constraints as mentioned above (RecalHR). The alternate-feedback and high-resolution runs were performed in 25-Mpc boxes. As we want to fairly compare these to Reference haloes, we use the 25-Mpc Reference simulation for this chapter as well. Further details on these simulations can also be found in Crain et al. (2015).

4.1.2 Halo finding

To identify structures in EAGLE, SUBFIND (Springel et al. 2001b; Dolag et al. 2009) was run on the output of the simulations. For the purposes of the code, a ‘halo’ is a collection of particles found with a friends-of-friends algorithm. A ‘subhalo’ is a set of particles in a halo which is enclosed by isodensity contours that traverse saddle points, where only particles gravitationally bound to the substructure are included. A subhalo encompasses a galaxy and its immediate surroundings. All haloes have a minimum of one subhalo (haloes with more than one subhalo have satellite galaxies). Neither haloes nor subhaloes are inherently restricted to their virial radii (we specify when only particles within $R_{\text{vir}}$ are considered). Note that all results in this chapter concerning ‘haloes’ only consider the primary subhalo.
4.1.3 Galaxy sample selection

For the purposes of this chapter, we are interested in studying haloes undergoing cooling episodes. To identify a relevant sample of systems in EAGLE, first we select central subhaloes (most massive in their parent halo) with total gas masses $> 10^{9.5} \, M_\odot$, then we compare the state of gas particles of the same IDs at temporally adjacent snapshots within each subhalo. More specifically, we take each relevant subhalo at snapshot number $s$ and tabulate the IDs of all particles we consider ‘hot’. We then find the same particles at snapshot $s + 1$ and determine which of these are now ‘cold’ and within the same system. Those that have transitioned from hot to cold, or from hot to a star particle, from snapshot $s$ to $s + 1$ are labelled ‘cooled’ particles. Equivalently, we refer to these as ‘cooling’ particles at snapshot $s$. If the number of cooling particles exceeds a threshold value (64, see below), then we deem a resolved cooling episode to have happened and include that subhalo in our sample. We perform this same process for all snapshot pairs.

Note that in order to have a clean sample of central galaxies to compare against how semi-analytic models treat centrals, we exclude galaxies from our analysis that are satellites at either snapshot (that is, we only consider the primary subhalo of each halo). We further exclude galaxies that undergo a major merger between snapshots. We define a major merger as one where the smaller galaxy has a stellar mass at least 30% that of the central. These would have only constituted a small fraction of our sample. As such, had we included them, our results would not have noticeably changed. When selecting haloes from the alternate-feedback and high-resolution runs, we add a requirement that only haloes also present in the Reference sample can be included.

Throughout this chapter, we define ‘cold’ particles as either having non-zero star formation rates or having both temperature $T < 10^{4.5} \, K$ and hydrogen number density $n_H > 0.01 \, \text{cm}^{-3}$, which should capture the interstellar medium of most galaxies. Conversely, ‘hot’ gas particles have zero star formation rates and $T > 10^{5} \, K$.

A cooling episode should occur over a time-span comparable to the dynamical time of a halo, which is directly tied to the Hubble flow at the given redshift:

$$ t_{\text{dyn}} = \frac{R_{\text{vir}}}{V_{\text{vir}}} = [10 \, H(z)]^{-1} . \quad (4.1) $$

As we want to capture the before and after states of a cooling episode, snapshots separated by a dynamical time would be well suited for our study. By no coincidence, the temporal separation between snapshots in EAGLE is always within 40% of a dynamical time (with the exception of the first two snapshots in the simulation, which we do not use). This is illustrated by Fig. 4.1.

The precise threshold imposed for the minimum number of cooling particles is, at some level, arbitrary. A threshold is required to avoid numerical noise and to ensure a meaningful mass cools.
Chapter 4. Halo gas cooling in EAGLE

We find a round value of 64 particles returns a sizeable number of haloes (an average of 48 per snapshot interval in the 25-Mpc box), each with sizeable cooling rates (\( \geq 0.2 \, \text{M}_\odot \, \text{yr}^{-1} \)). As will be shown, this threshold leads to results that are consistent with the RecalHR simulation when we select the same haloes. We are, however, restricted to studying haloes at \( z < 4 \). The average temporal separation between snapshots in this range is 681 Myr.

We present the masses of our sample of Reference EAGLE haloes in Fig. 4.2. We break these into three redshift bins, each with 6 snapshot pairs, which we use throughout the chapter. The normalised histograms show the virial masses of the haloes, as well as the gas and stellar masses of all particles in each respective main subhalo inside an aperture of \( R_{\text{vir}} \). Note that the gas masses are for all gas, i.e. hot + cold + everything in between.

4.2 BEFORE COOLING: HOT GAS IN HALOES

4.2.1 Temperature profiles of hot gas

In the absence of cold streams, before finding itself in a galaxy, gas will find itself in a hot state, sitting in a halo. Models of galaxy evolution often operate under the assumption that hot gas in haloes, and therefore hot gas that will cool, carries a uniform temperature that is equal to the virial temperature. To compare against this, we examine the temperature profiles of haloes in our EAGLE sample, each normalised by its virial temperature. The virial temperature is given by Eq. 2.2.

We present temperature profiles of hot gas from our sample of EAGLE haloes from the Reference and RecalHR runs in the top panels of Fig. 4.3. The profiles were built by measuring the mean temperature of hot gas particles within spherical shells. We use (a maximum of) 100 shells
4.2. Before cooling: hot gas in haloes

Figure 4.2: Normalised histograms of masses of Reference EAGLE systems used in this chapter, according to the selection criteria given in Section 4.1.3. The solid distributions give the haloes’ virial masses. The dashed and dot-dashed distributions give the gas and stellar masses, respectively, of all particles in the main subhalo within $R_{\text{vir}}$. Each panel covers a different redshift range, as labelled, with the same number of snapshots.
Figure 4.3: Temperature profiles of non-star-forming, hot ($T > 10^5$ K) gas in haloes in the Reference and RecalHR runs of EAGLE. The top row of panels presents the temperature profiles for all hot gas in the haloes, while the bottom panels only include hot gas particles that are known to cool onto the galaxy by the following snapshot. The temperature in both these cases is normalised by the virial temperature of each halo. Columns of panels are for different redshift ranges. The median (solid and dashed curves) and inner 68% (shaded regions of corresponding colour) of profiles are shown in each panel for the two simulations. Vertical dashed lines are given at $R_{\text{vir}}$ to indicate the cut-off for what is typically regarded as part of a halo. While the hot-gas temperature rises in the centre, the gas that cools continues to get cooler.

for each halo, where each shell encompasses the same number of particles for a given halo, unless that width is below the spatial resolution (softening scale) of the simulation. We show the evolution of these profiles by grouping systems into three redshift bins, where each bin includes the same number of snapshots. We display profiles for the median and $16^{\text{th}}$–$84^{\text{th}}$ percentile range of our samples in Fig. 4.3. It is immediately clear that the hot gas is not truly isothermal, but rather temperature tends to decrease with radius. At higher redshift, the gas is within a factor of $\sim 2$ of being isothermal and only approaches $T_{\text{vir}}$ towards the centre. As time evolves, the centres become hotter and the temperature gradient steepens.

The bottom panels in Fig. 4.3 consider only the hot gas that will cool onto the galaxy before the subsequent snapshot, which we dub ‘cooling’ particles. As these particles only constitute a small fraction of the hot gas, we reduce the number of spherical shells to 20 in each halo for measuring these profiles. $T_{\text{cooling}}(R)$ has a clear gradient from high redshift onward. This shows that gas that settles onto a galaxy from a larger distance tends to already be cooler than the inner gas. Physically, this makes sense, as gas at large radii is at lower density (see below) and hence has less opportunity to lose its energy through collisions over the same time. It is not surprising that the average temperature of cooling gas is consistently below the hot gas in general; the hotter
4.2. Before cooling: hot gas in haloes

particles would require longer to cool. Perhaps what is more interesting is that as the centres of haloes heat overall, the temperature of the cooling particles decreases. Feedback processes lead to energy being transferred out of the galaxy, raising the mean temperature at the centre of the hot halo. This reheated gas takes time to mix with its surroundings and/or cool back onto the galaxy. Meanwhile, the gas that was already in the process of cooling continues to cool, and is evidently oblivious to the heating of its surroundings. Small differences are seen between the Reference and RecalHR runs (which we come back to briefly in Section 4.3.2), but they are broadly consistent with each other.

To better quantify the effect of feedback, we have produced the same temperature profiles for the alternate-feedback runs of EAGLE. We present these for the lowest redshift bin in Fig. 4.4, comparing the median profile of the Reference simulation to the median, 16th and 84th percentiles of the WeakFB, StrongFB and NoAGN runs. At higher redshift, the profiles of the alternate-feedback runs more closely matched those of the Reference run (as they have the same initial conditions). The differences seen at low redshift are hence directly the result of feedback associated with galaxy evolution. Note that the number of systems meeting our selection criteria in each run varies as a result of the differing feedback. These numbers are given in the legend of Fig. 4.4.

Let us first examine the role of stellar feedback. Stellar feedback is implemented thermally in EAGLE. This raises the temperature of gas particles in the centre of the haloes, where the exploding stars are. These hot particles rise and can drive shocks that heat their neighbours. In our sample, the masses of the haloes are large enough that supernova energy is insufficient to eject gas from galaxies out of the halo entirely. The result instead is that the gradient of the temperature profile becomes generally steeper. Both the $T_{\text{cooling}}(R)$ and $T_{\text{hot}}(R)$ profiles for the WeakFB run lie below the Reference curves, as one would expect from having less heating. For the StrongFB run, the rise of $T_{\text{hot}}(R)$ is even more stark than the Reference run, yet the median $T_{\text{cooling}}(R)$ profile matches the Reference simulation. This supports the argument raised above that while stellar feedback affects the mean temperature of halo gas, it does not immediately impact the gas already in the process of cooling.

Let us now consider the role of AGN. Perhaps counter-intuitively, switching AGN off leads to an evolution in $T_{\text{hot}}(R)$ that looks similar to the StrongFB case. While AGN feedback does heat the central gas particles in a halo, the net effect is less a case of the temperature simply rising at the centre, and more a case of AGNs regulating the temperature of the gas throughout the haloes. Strong outflows from AGNs advect hot particles to larger distances from the galaxy than stellar-driven outflows would, thereby spreading the heat out and reducing the gradient. The difference between the NoAGN run and the others is that $T_{\text{cooling}}(R)$ has also increased throughout the halo. This shows that an AGN suppresses the ability of hot gas to cool, rather than directly heating gas,
Figure 4.4: Temperature profiles of hot gas in haloes for various alternate-feedback runs of EAGLE, only considering systems at $z < 0.6$. The top panel includes all hot gas in the profiles, while the bottom panel only includes that which is known to cool before the next time-step. The thick, opaque curves give the median profiles of the different EAGLE runs. The thin, transparent lines give the 16th and 84th percentiles for the alternate-feedback runs. The bracketed numbers in the legend indicate the number of haloes in the sample of each run. Both stellar and AGN feedback affect the temperature profiles of hot haloes, but the gas in the process of cooling is not affected identically.
4.2. Before cooling: hot gas in haloes

as now, without AGN, hotter gas is seen to cool more efficiently. This is precisely how radio mode AGN feedback is considered in semi-analytic models (e.g. Bower et al. 2006;Croton et al. 2006); that is, the heating rate of a radio AGN is subtracted directly off the cooling rate of a halo to calculate its net cooling rate (see Appendix D). Although, note that EAGLE does not distinguish between radio and quasar modes of AGN, instead opting for a single model of AGN feedback which is more in line with the quasar mode (see Schaye et al. 2015). Even if the net effect of AGN feedback in EAGLE is to make the cooling process less efficient, one would expect the properties of AGN-driven winds in EAGLE to differ if radio mode feedback were implemented directly in the simulation (see Dubois et al. 2012).

4.2.2 Density profiles of hot gas

We now turn our attention to the density profiles of hot gas in haloes. Using the same spherical shells as used for the temperature profiles, we measure the density within each shell of our EAGLE haloes. We normalise the density profile of each hot halo by $m_{\text{hot}}/R_{\text{vir}}^3$. We then make the same measurements for the cooling gas, instead normalised by $m_{\text{cooling}}/R_{\text{vir}}^3$. This allows us to directly compare the density profiles of both the hot and cooling gas, regardless of the halo size or total hot/cooling mass. In Fig. 4.5, we present these for the same 3 redshift bins as before. The Reference and RecalHR runs produce consistent profiles (as do the alternate-feedback runs, but these are not included for clarity).

A clear evolution is seen in the hot gas density profiles, where the gradient at the centre becomes shallower with time. The cooling gas density profile evolves less noticeably, however. This is consistent with our findings for the temperature evolution. That is, while the hot gas at the centre becomes hotter and less dense (relative to the outer parts of the same halo) from supernovae and stellar winds, the gas that actually cools is almost unaffected by the feedback.

Some models of galaxy formation assume hot gas follows the density profile of a singular isothermal sphere (e.g. Croton et al. 2006, 2016; Lagos et al. 2008; Somerville et al. 2008b; Lee & Yi 2013; Stevens et al. 2016a), where $\rho_{\text{hot}}(R)$ is given by Eq. 2.3. This relation is over-plotted in each relevant panel in Fig. 4.5. This profile is based primarily on simplicity, where there is an absence of cited, explicit physical motivation in the models. An alternative, motivated by both theory and observations of galaxy groups and clusters, is a $\beta$ profile (see the review by Mulchaey 2000). This has been incorporated in the GALFORM family of models (e.g. Cole et al. 2000;
Figure 4.5: As for Fig. 4.3 but instead showing density profiles of hot and cooling gas, normalised by the average density within $R_{\text{vir}}$. These are compared against a singular isothermal sphere profile in all panels, and against a $\beta$ profile for all hot gas, which is fitted to median density profile of the Reference simulation in each case. The numbers in the bottom left of those panels give the value of $c_\beta$ for the fit (see Eq. 4.3). A fitted $\beta$ profile describes the hot gas as a whole reasonably well, yet the gas that cools is roughly consistent with a singular isothermal sphere.

Benson et al. 2003; Font et al. 2008). The $\beta$ profile prescribes the gas density as

$$
\rho_{\text{hot}}(R) = \rho_0 \left[ 1 + \left( \frac{R}{R_c} \right)^2 \right]^{-3\beta/2},
$$

where $\rho_0$ is the central density and $R_c$ is a core radius. Under the assumption that all haloes follow the same value of $\beta$ and that $R_c = c_\beta R_{\text{vir}}$, where $c_\beta$ is a constant (e.g., as assumed by Benson et al. 2003), we can compare our EAGLE profiles to the $\beta$ profile. Taking $\beta = 2/3$, the expression becomes

$$
\rho_{\text{hot}}(R) = \frac{m_{\text{hot}}}{4\pi c_\beta^2 R_{\text{vir}}^3} \left[ 1 - c_\beta \tan^{-1} \left( \frac{1}{c_\beta} \right) \right]^{-1} \left[ 1 + \left( \frac{R}{c_\beta R_{\text{vir}}} \right)^2 \right]^{-1}.
$$

Using the median density profiles of the Reference simulation in Fig. 4.5, we have fitted for $c_\beta$ and over-plotted for comparison.

Fig. 4.5 clearly shows that singular isothermal spheres are not representative of the distribution of hot gas in our haloes. While the $\rho_{\text{cooling}}(R)$ profiles are closer to being exponential on average than anything else, curiously, they do encompass the singular isothermal sphere profile within their 68% confidence range, at least for $z > 0.6$. A singular isothermal sphere describes the cooling gas density profile as well as a best-fitting $\beta$ profile would (i.e. the best-fitting $c_\beta$ is large). Despite
4.2. Before cooling: hot gas in haloes

Figure 4.6: Best-fitting concentration parameter of $\beta$ profile fits to the hot-gas density profiles of haloes in our EAGLE sample at each snapshot, assuming $\beta = 2/3$, for various runs as given in the legend. Each curve gives the median relation from each simulation, with the exception of the perfectly smooth curve, which is a fit to the Reference simulation median (Eq. 4.4), which happens to describe many of the other runs well too. The shaded region covers the 16th–84th percentile range for the Reference simulation.

not actually having isothermal haloes, the manner in which gas cools onto galaxies in EAGLE is consistent with randomly drawing particles out of a singular isothermal sphere, which we have found is true for the alternate-feedback EAGLE runs as well (not shown here).

A $\beta$ profile captures the total hot-gas density reasonably well out to $\sim 0.8 R_{\text{vir}}$, when $c_\beta$ is free, as seen in the top panels of Fig. 4.5. By allowing this, we find the best-fitting $c_\beta$ decreases with time. To quantify this, we have fitted $c_\beta$ to each hot-gas density profile of our sample haloes for each snapshot individually. We have done this for the Reference, RecalHR, and alternate-feedback EAGLE runs, and present the evolution of $c_\beta$ for these in Fig. 4.6. We include the median relation for all runs, and include the scatter for the Reference simulation. We find that weak stellar feedback can affect the value of $c_\beta$ at $z \lesssim 1$, but otherwise the fit is fairly robust to feedback changes. In addition, we find the median $c_\beta(z)$ curve for the Reference simulation is fitted almost perfectly by a simple analytic expression:

$$c_\beta(z) \simeq 0.21e^{-1.5z} - 0.036z + 0.27.$$  \hspace{1cm} (4.4)

This least-squares fit (which weights each snapshot equally) is included in Fig. 4.6. We note that for all $z < 4$, the best-fitting $c_\beta$ is above the values assumed in previous incarnations of GALFORM: 0.07 (Benson et al. 2003) and 0.1 (Font et al. 2008).

4.2.3 Metallicity profiles of hot gas

Another common approximation made in analytic models is that the metallicity of hot gas is uniform throughout the halo. In reality, we would expect hot gas at the centre of haloes to be more metal-rich than at the outskirts, as the stars at the bottom of the potential well are what
Chapter 4. Halo gas cooling in EAGLE

4.2.4 Cooling-time profiles

The primary purpose of semi-analytic models including prescriptions for the hot-gas density profile, temperature, and metallicity is to determine a cooling rate at each time-step. It is, therefore, of interest what consequences will arise for the cooling rates from the differences in the profiles of the EAGLE haloes versus analytic approximations. To investigate this, we must first address the typical process for calculating cooling rates in semi-analytic models.

The majority of semi-analytic models (e.g. Cole et al. 2000; Hatton et al. 2003; Cora 2006; Croton et al. 2006, 2016; Somerville et al. 2008b; Guo et al. 2011; Lee & Yi 2013) calculate cooling rates using some variation of the method presented in White & Frenk (1991), which is
as follows (but see, e.g., Monaco et al. 2007 for a more detailed treatment). Given a spherically
symmetric distribution of hot gas, the ‘cooling time’ for hot gas at a given radius is first defined as
\[ t_{\text{cool}}(R) \equiv \frac{3}{2} \frac{T_{\text{hot}}(R)}{\rho_{\text{hot}}(R)} \frac{\bar{\mu} m_p k_B}{\Lambda(T_{\text{hot}}(R), Z_{\text{hot}}(R))}, \]
where \( \Lambda(T, Z) \) is the cooling function, commonly drawn from the tables of Sutherland & Dopita
(1993), dependent on the temperature and metallicity of the gas (cf. Eq. 2.4). One then defines the
‘cooling radius’, \( R_{\text{cool}} \), as the radius at which the cooling time equals a relevant time-scale. For
the purposes of this chapter, we have chosen to equate this time-scale to the dynamical time, in
line with the sAGE family of models (see Section 2.5), but this could have been informed by the
free-fall time-scale instead, as in GALFORM, for example. The general argument then is that the
cooling mass that crosses \( R_{\text{cool}} \) is approximately equal to the cold-gas mass deposition rate onto
the galaxy (Bertschinger 1989). From this continuity law, one can calculate the rate at which gas
cools onto the galaxy as
\[ \dot{m}_{\text{cool, model}} = 4\pi \rho_{\text{hot}}(R_{\text{cool}}) R_{\text{cool}}^2 \left( \frac{\mu_{\text{coal}}}{dR} \right)_{R \rightarrow R_{\text{cool}}}^{-1}. \]
In the case where \( R_{\text{cool}} > R_{\text{vir}} \), haloes are assumed to be in a cold-accretion regime in a semi-
analytic model, where the cooling rate is then taken as the ratio of the hot mass to the dynamical
(or free-fall) time. We remind the reader that our EAGLE haloes are selected to be in the hot mode
of accretion.

Our intent here is not to compare the true cooling rates of EAGLE haloes to those inferred
by Eq. 4.6. To make that comparison would require a complete deconstruction of how feedback
influences cooling in both EAGLE and in semi-analytic models. This is non-trivial, as the way
in which feedback and cooling are handled in semi-analytic models is not directly comparable to
what goes on in a hydrodynamic simulation, and there is plenty of variation amongst models as
well (see Lu et al. 2011). A model can have degeneracies when it comes to cooling and heating
rates, where the free parameters governing these are only really constrained in a relative sense,
such that the net growth of (sometimes just the stellar content of) galaxies is representative of
the observed Universe. This is why many semi-analytic–hydrodynamic comparison projects have
excluded feedback (and sometimes even star formation) from the simulations entirely (Benson et
al. 2001; Yoshida et al. 2002; Helly et al. 2003; Cattaneo et al. 2007; Viola et al. 2008; Saro et al.
2010). What we aim to address here is how the cooling-time profiles, through Eq. 4.5, vary when
we include the detail of the density, temperature, and metallicity profiles we have available to us
from EAGLE. In Appendix D, we go one step further, and calculate how the \( t_{\text{cool}}(R) \) profiles of
our EAGLE haloes would translate into an effective semi-analytic cooling rate.
It is normal for the purposes of a semi-analytic model to simplify Eq. 4.5 by taking $T_{\text{vir}}$ as the temperature for all hot gas in the halo, and assuming all hot gas to have the same metallicity. This then leaves $t_{\text{cool}}(R)$ for a given halo dependent on the assumed density profile. We address the impact of the density profile on $t_{\text{cool}}(R)$ in Fig. 4.8. The long-dashed and dot-dashed curves give the median $t_{\text{cool}}(R)$ profiles after using analytic profiles of a singular isothermal sphere and $\beta$ profiles with $c_\beta = 0.1$ and $c_\beta(z)$ from Eq. 4.4, respectively. These profiles differ little, where the $\beta$ fit only makes a notable difference for $R \lesssim 0.3R_{\text{vir}}$ at low redshift, where it is generally true that $t_{\text{cool}} < t_{\text{dyn}}$. As a result, the cooling radii calculated from these density profiles are all consistent. This is shown by the horizontal lines in Fig. 4.8, which cover the $16^{th}$–$84^{th}$ percentile range for $R_{\text{cool}}/R_{\text{vir}}$ in each case, with the intersecting vertical marks giving the medians.

As a direct comparison to the analytic profiles, we calculate $t_{\text{cool}}(R)$ using the actual density profiles from each Reference EAGLE halo, while maintaining the use of $T_{\text{hot}}(R) = T_{\text{vir}}$ and $Z_{\text{hot}}(R) = \bar{Z}_{\text{hot}}$. The median profile (dotted curves in Fig. 4.8) is consistent with the analytic cases in the outer parts of the halo, but diverges for $R \lesssim 0.4R_{\text{vir}}$ for all $z < 4$ (the scatter in all these cases is consistent, but this is not shown for clarity). As a result, the cooling radii are systematically lower than their analytic counterparts. This highlights the fact that, even when using a density profile that fits the general population by construction, it is difficult to recover the true cooling radii of haloes from a hydrodynamic simulation.

Next, we consider the role of the temperature profiles of the EAGLE haloes on $t_{\text{cool}}(R)$. We again solve Eq. 4.5 with $Z_{\text{hot}}(R) = \bar{Z}_{\text{hot}}$, but use the actual profiles of the haloes for $T_{\text{hot}}(R)$ and $\rho_{\text{hot}}(R)$. The median $t_{\text{cool}}(R)$ profile in this case is given by the short-dashed curves in Fig. 4.8. Comparing this to the dotted curves, we see the profile flattens, which leads to a much broader range in cooling radii for the haloes. Even though a typical EAGLE halo will only have a radial variation in its temperature around a factor of 3 (cf. Fig. 4.3), this temperature structure can significantly impact the $t_{\text{cool}}(R)$ profile one infers for a halo, and therefore would impact the cooling rate one would determine in a semi-analytic model.

As a final step, we include the metallicity profiles of the EAGLE haloes and recalculate $t_{\text{cool}}(R)$. We include the median relation with the solid curves, and the $16^{th}$–$84^{th}$ percentile range with the shaded region in each panel of Fig. 4.8. The addition of the $Z_{\text{hot}}(R)$ profile restores sensible cooling radii values that are consistent with the haloes being in the hot mode of accretion. For completeness then, a model of halo gas that includes temperature structure should also include metallicity structure for the sake of calculating cooling radii and rates. For $z > 0.6$, these cooling radii are in moderate agreement with the purely analytic profiles. The $t_{\text{cool}}(R)$ profiles flatten at low redshift, which leads to a broad range in cooling radii, which become systematically less than the cases of the analytic profiles.
4.2. Before cooling: hot gas in haloes

Figure 4.8: Cooling time of hot gas in our EAGLE halo sample, normalised by their respective dynamical times, as a function of radius, normalised by their virial radii. We present median profiles calculated from the EAGLE haloes through Eq. 4.5 in cases where we consider (i) only the density profiles of the halo with \( T_{\text{hot}}(R) = T_{\text{vir}} \) and \( Z_{\text{hot}}(R) = \bar{Z}_{\text{hot}} \) (dotted curves), (ii) both the density and temperature profiles with \( Z_{\text{hot}}(R) = \bar{Z}_{\text{hot}} \) (short-dashed curves), and (iii) the density, temperature, and metallicity profiles of the haloes (solid curves). In the latter case, we show the 16\(^{\text{th}}\)–84\(^{\text{th}}\) percentile range with the shaded region. Included for comparison are the median cooling-time profiles when assuming the density profile follows (i) a singular isothermal sphere, (ii) a \( \beta \) profile with constant \( c_\beta = 0.1 \), and (iii) a \( \beta \) profile with \( c_\beta \) calculated from Eq. 4.4, which take \( T_{\text{hot}}(R) = T_{\text{vir}} \) and \( Z_{\text{hot}}(R) = \bar{Z}_{\text{hot}} \) in all 3 cases (see the legend in the right-hand panel). The horizontal lines cover the inner 68\% of \( R_{\text{cool}}/R_{\text{vir}} \) values in each case, with matching linestyles. The vertical marks through these give the median cooling radii. Complete information on the density, temperature, and/or metallicity profiles of hot haloes leads to a greater range of cooling profiles than analytic approximations would give, which can impact cooling radii, thereby affecting the cooling rates one would calculate in a semi-analytic model.
4.3 ANGULAR MOMENTUM OF COOLING GAS

4.3.1 Conservation of angular momentum during cooling

An important aspect of most models of gas cooling is the assumption that the angular momentum of the gas is conserved. Of course, cooling gas can exchange angular momentum with many other parts of the halo in principle, especially through collisions with gas particles not involved in cooling (including those already cold). Early attempts at forming spiral galaxies with cosmological, hydrodynamic simulations were plagued by an ‘angular-momentum catastrophe’, where gas cooled too quickly and lost too much angular momentum (see, e.g., Katz & Gunn 1991; Navarro & Benz 1991; Navarro & Steinmetz 1997, 2000). Solutions to this problem were found in better resolution (Governato et al. 2004) and more efficient subgrid feedback (e.g. Brook et al. 2011, 2012a). We are now in a position to use a simulation like EAGLE to make predictions about angular-momentum conservation of cooling gas, or lack thereof, which is informative for analytic models of gas cooling.

We measure the specific angular momentum, $j$, of cooling and cooled particles (i.e. immediately before and after cooling) in our EAGLE haloes along their cooling axes on an individual basis and for the summed quantity of all cooling particles involved in a single episode. The relative change in $j$ for each case is presented in the upper and middle panels of Fig. 4.9, respectively. We bin our systems by redshift, as in the previous sections, and show distributions for the relative change in angular momentum for each of these bins. Because resolution is known to play a role in $j$ losses, we present both the Reference and RecalHR simulations in the left and right panels of Fig. 4.9, respectively.

It is clear from the top panels of Fig. 4.9 that individual particles do not conserve angular momentum while cooling. Comparing this to the middle panels, those particles appear to be exchanging some of their angular momentum between one another, as the net change in $j$ for the collection of particles provides a narrower distribution with a peak closer to zero. In both cases, the means of the distributions are negative. Therefore, on average, angular momentum is lost by gas as it cools onto EAGLE galaxies. There is no clear evolution in the distributions for either the upper or middle panels of Fig. 4.9. For the Reference simulation; for the highest- to lowest-redshift bins, the average net loss of $j$ during a cooling episode is approximately 55, 64, and 57 percent, respectively. The highest- and lowest-redshift histograms are also consistent according to the Kolmogorov–Smirnov test. We find that at higher resolution (the RecalHR run), $j$ losses during cooling are reduced at higher redshift, on average. We suggest that, by itself, this is not enough to claim any generic correlation between $\Delta j/j_i$ and $z$, especially as the results between simulations agree at low redshift. We come back to this point in Section 4.3.2.
4.3. Angular momentum of cooling gas

Figure 4.9: Relative change in the specific angular momentum component parallel to the rotation axis of cooling particles during cooling episodes, compared to the initial state. The left-hand panels include haloes from our Reference EAGLE sample, while the right-hand panels include those from the RecalHR run. The top panels consider particles individually. The middle and bottom panels consider the change in net specific angular momentum of all cooling particles in a single episode, i.e. on a halo-by-halo basis. Where the top two rows of panels bin by redshift, the bottom row of panels bins by the $\lambda_R$ value of the central galaxy for all $z < 1.7$. The short vertical lines give the mean of each distribution for respective linestyles. If angular momentum were always conserved along the cooling axis, the distributions would be $\delta$ functions matching the long, vertical, dashed line. Instead, we see angular-momentum losses in the cooling gas. We perform Kolmogorov–Smirnov tests to compare the consistency of the halo distributions between $4 > z > 1.7$ and $z < 0.6$, and between the distributions for $\lambda_R \leq 0.25$ and $\lambda_R > 0.75$. The $D$-value printed in the panels gives the maximum vertical separation of the cumulative probability distributions, while the $p$-value is the probability that the means of the distributions would be as separated as they are under the pretext that they come from the same underlying population. A little less angular momentum appears to be lost when gas cools onto a rotationally supported galaxy.
Galactic discs in EAGLE are known to be realistic in their size (Furlong et al. 2015b; Schaye et al. 2015; Lange et al. 2016). Yet, the gas that cools loses a fraction of angular momentum that is consistent with the large percentages reported when the ‘angular-momentum catastrophe’ produced simulated discs that were too concentrated (cf. Katz & Gunn 1991). While, indeed, too much absolute angular momentum was lost during cooling in early hydrodynamic, cosmological simulations, evidently the correct amount of specific angular momentum was lost, based on our results.

The frequency of particle interactions and collisions will determine the potential for angular-momentum loss of cooling particles. We hypothesise that in a rotationally supported system, where there is less random motion, collisions might be fewer, and thus less angular momentum might be lost. To test this, we first quantify the relative level of rotation and dispersion support in our galaxies using the $\lambda_R$ parameter, introduced by Emsellem et al. (2007). Galaxies with $\lambda_R \sim 0$ are predominantly dispersion-supported, whereas those with $\lambda_R \sim 1$ are predominantly rotationally supported. For our EAGLE galaxies, we measure the property as

$$\lambda_R = \frac{\sum m_\ast j_{z,\ast}}{\sum m_\ast \sqrt{j_{z,\ast}^2 + r_{z,\ast}^2 v^2_{z,\ast}}} ,$$

where the sums are over all star particles associated with the main subhalo within the ‘BaryMP’ galactic radius defined in Section 5.2.3 (where the cumulative mass profile of the stars and cold gas reaches a constant gradient), and $j_z$ and $v_z$ are the specific angular momentum and velocity components along the galaxy’s rotation axis, respectively, as measured in the galaxy’s centre-of-momentum frame.\(^1\) In the bottom panels of Fig. 4.9, we again show the relative net loss of specific angular momentum during cooling events, but now bin galaxies by $\lambda_R$, including all systems for $z < 1.7$. By excluding those in the range $1.7 < z < 4$, we eliminate the population of high-$z$ systems from RecalHR that we have shown lose less $j$ during cooling. We would have otherwise had biased results, as the average $\lambda_R$ of the galaxies is lower at higher $z$. For the Reference and RecalHR simulations, we find a weak, but statistically significant, tendency for $j$ losses to be reduced for high-$\lambda_R$ systems. The difference in the means is less for RecalHR than the Reference simulation, but it is still in favour of our hypothesis. Our results suggest any reduction in $j$ losses during cooling caused by stronger rotation in the central galaxy is, at most, a few tens of percent.

\(^1\)Note that this calculation for $\lambda_R$ does not include the intricacies required to compare against observations as in Naab et al. (2014), as we simply require a relative measure of this quantity for internally comparing EAGLE galaxies.
4.3. Angular momentum of cooling gas

4.3.2 Relative orientation and magnitude of specific angular momentum

As already discussed, galaxy formation models typically assume that the net specific angular momentum of hot gas about to cool, \( \vec{j}_{\text{cooling}} \), is equivalent to that of all the hot gas in the halo, \( \vec{j}_{\text{hot}} \), and to that of the entire halo itself, \( \vec{j}_{\text{halo}} \), both in terms of magnitude and direction. Without the ability to directly measure the motion of dark matter in haloes to independently measure a halo’s spin, it is impossible to determine if \( \vec{j}_{\text{cooling}} \) and \( \vec{j}_{\text{halo}} \) are the same empirically with observational methods. Simulations are the only current means of addressing this in any capacity. Previous studies of cosmological, hydrodynamic simulations have shown that gas and dark matter in haloes tend to have different and offset specific angular momenta (e.g. van den Bosch et al. 2002, 2003; Chen et al. 2003; Sharma & Steinmetz 2005; Sharma et al. 2012). Attention has not been given specifically to the cooling particles before, however.

Magnitude

We directly measure the ratio \( j_{\text{cooling}}/j_{\text{halo}} \) from our EAGLE haloes, and present distributions for this ratio for bins of redshift in the top panels of Fig. 4.10. We see that cooling gas typically has more specific angular momentum than the halo. This result is contrasting (but not opposing) to the idea that the specific angular momentum of the stellar content of galaxies is typically lower than that of their haloes, as suggested by both observations (Romanowsky & Fall 2012) and models (Fig. 3.14). As discussed in Section 4.3.1, some of this angular momentum is lost during the cooling process. As seen in the top panels of Fig. 4.10, there is no clear evolution in \( j_{\text{cooling}}/j_{\text{halo}} \) for our EAGLE haloes. We do, however, find the mean ratio to be lower for RecalHR at higher redshift, which appears to be statistically significant, based on a Kolmogorov–Smirnov test (see the top right panel of Fig. 4.10). This is balanced by the fact that less angular momentum is lost during the cooling process for these specific haloes, as shown in Section 4.3.1. If we compare these findings with the lower left panel of Fig. 4.3, we see that the cooling gas in these haloes is lower at the ‘beginning’ of the cooling episode for the RecalHR run. The evidence suggests the population of RecalHR haloes in our sample for \( 4 > z > 1.7 \) effectively had a head start in cooling over the same haloes in the Reference simulation, and thus were measured at a moment when the gas was already cooler and had already lost some of its specific angular momentum. This would imply the angular momentum measurements in the Reference and RecalHR runs are entirely consistent.

We have already shown tentative evidence that gas cooling onto a rotationally supported galaxy in EAGLE loses less specific angular momentum than that cooling onto a dispersion-supported galaxy (Fig. 4.9). In the bottom panels of Fig. 4.10 we also break \( j_{\text{cooling}}/j_{\text{halo}} \) into bins of \( \lambda_R \). There is a population of low-\( \lambda_R \) galaxies, in both the Reference and RecalHR simulations, that
Figure 4.10: Probability distributions for the ratio of the magnitudes of net specific angular momentum of cooling gas to that of the halo, drawn from our sample of Reference and RecalHR EAGLE systems. The top panels cover all systems in each redshift bin, while the bottom panels cover all $z < 1.7$ and bins by $\lambda_R$ of the central galaxy. The dashed, vertical line indicates where $j_{\text{cooling}} = j_{\text{halo}}$, which is often assumed in galaxy formation models. The short vertical lines mark the means of the distributions. We perform Kolmogorov–Smirnov tests on the consistency of the distributions in each panel, and note the cases where there is potential inconsistency (see caption of Fig. 4.9). Cooling gas carries more specific angular momentum than the halo on almost all occasions.

show high values for $j_{\text{cooling}}/j_{\text{halo}}$, but are not abnormal in any other respect we can find. We find nearly all the excess specific angular momentum is lost during cooling for these systems. Modulo those few systems, the distributions for $j_{\text{cooling}}/j_{\text{halo}}$ for various $\lambda_R$ are entirely consistent with each other. In other words, gas should not be aware of what type of galaxy it will cool onto at the moment it begins to cool, which appears to indeed be the case for EAGLE.

Orientation

The direction of the angular momentum of the cooling gas is also important for how the new material will alter the disc. In the top panels of Fig. 4.11 we show the angular offsets of the spin vectors of hot gas about to cool and hot gas in general for our EAGLE haloes. Regardless of redshift, these are typically offset by tens of degrees. Interestingly, the particles about to cool show better alignment with the cold disc than with the hot gas from whence they came (cf. top and middle panels of Fig. 4.11). Because the cooling gas’s rotation axis is consistently offset from the hot gas as a whole, an offset is built between the rotation axes of the cold gas in the galaxy and its hot halo; this is shown by the bottom panels of Fig. 4.11. Again, we find minimal difference between the Reference (left panels) and RecalHR (right panels) runs, suggesting this is indeed a
4.3. Angular momentum of cooling gas

Reference

\[ 4 > z > 1.7 \]

\[ 1.7 > z > 0.6 \]

\[ z < 0.6 \]

\[ \cos^{-1} |\hat{j}_{\text{cooling}} \cdot \hat{j}_{\text{hot}}| \]

\[ \cos^{-1} |\hat{j}_{\text{cooled}} \cdot \hat{j}_{\text{cold}}| \]

\[ \cos^{-1} |\hat{j}_{\text{cold}} \cdot \hat{j}_{\text{hot}}| \]

Figure 4.11: Angular separations between rotation axes of gas particle groups during cooling. The top panels give the difference between all the hot particles and those about to cool. The middle panels are for all cold gas and the particles that have just cooled. The bottom panels are for all cold gas and all hot gas. Gas that cools is better aligned with gas that is already cold than the rest of the hot gas in the halo.

physical effect.

Using the EAGLE simulations, Zavala et al. (2016) showed that the spin direction of a galaxy is more aligned with that of the inner parts of its halo \( ( < 0.1 R_{\text{vir}} ) \) than that of the entire halo. Also, because most cooling occurs in the inner part of the hot halo (Fig. 4.5), one might expect the inner hot halo’s rotation axis to be better aligned with the cooling gas. In light of these findings, we check whether the rotation axis offset between cooling and hot gas varies when only hot gas internal to a given radius is included. This is presented in Fig. 4.12. Contrary to expectation, the inner hot gas is typically less aligned with the cooling gas. Any evolution in these profiles is minimal for our sample of EAGLE haloes, as seen by the similarity of the curves in Fig. 4.12. The Reference (upper panel) and RecalHR (lower panel) simulations again produce the same result, so we can trust that this is not predominantly an effect of resolution.

The relative orientations and magnitudes of various particle types in haloes in general is an interesting topic which we are investigating in detail in an accompanying paper (Contreras et al. in preparation). The sample of cooling systems used here is consistent with the differences in direction and magnitude of \( j \) between baryons and dark matter in all EAGLE haloes to be presented there. That paper will address uncertainties in angular momentum as a function of the number of particles used; in general \( \geq 100 \) particles is required for trustworthy measurements of individual
Chapter 4. Halo gas cooling in EAGLE

4.4 AFTER COOLING: COLD GAS IN GALAXIES

4.4.1 Radial surface density profiles of gas discs

In the picture of disc formation proposed by Fall & Efstathiou (1980), gas cools and collapses to form an exponential disc while conserving angular momentum:

$$\Sigma_{cooled}(r) = \Sigma_0 e^{-r/r_d} \approx \frac{m_{cooled}}{2\pi r_d^2} e^{-r/r_d},$$

(4.8)

where $r_d \ll R_{\text{vir}}$ is the scale radius of the disc. The exponentiality of discs is a typical assumption of galaxy formation models, where some even maintain that a disc must always be exponential (e.g. Cole et al. 2000; Hatton et al. 2003; Croton et al. 2006, 2016; Somerville et al. 2008a; Guo et al. 2011). But only relatively recently have resolved observations of the H I and CO (H_2) distribution in local spiral galaxies been made (e.g. Walter et al. 2008; Leroy et al. 2009), allowing us to actually see what they are like. The 33 galaxies analysed by Bigiel & Blitz (2012) suggest that while an exponential profile broadly describes the average galaxy at intermediate radii, there is plenty of deviation from exponentials, and a strong suggestion of cusps existing at the centres of these discs. With EAGLE, we can not only check the exponentiality of gas discs, but also compare directly to Eq. 4.8.

Figure 4.12: Angular separation between spin directions of hot gas in haloes within a given radius and all cooling gas within in the same haloes. Cooling gas is poorly aligned with hot gas on all scales, regardless of redshift or resolution.

haloes. We do not find any significant changes to our conclusions here (which are concerned with the population) if we exclude haloes with <100 particles though.
4.4. After cooling: cold gas in galaxies

Figure 4.13: Surface density profiles for cold gas particles in our sample of EAGLE galaxies. As for previous figures, we present 3 redshift ranges for the Reference and RecalHR simulations, presenting their median profiles (solid and short-dashed curves, respectively) and 16th–84th percentile ranges (shaded regions). Radial distances have been normalised for each galaxy by their fitted exponential scale radius, \( r_{d,\text{fit}} \). The top row of panels considers only the recently cooled particles, whereas the bottom panels consider all cold particles. All surface densities have been normalised by the amount of cooled/cold gas considered and the square of the scale radius. The long-dashed line is a precise exponential (Eq. 4.8). The dot-dashed curves give the median profile for the rotation-dominated galaxies (\( \lambda_R > 0.75 \)) for each subsample of the Reference haloes. With the addition of a central cusp, an exponential describes the profile of recently cooled gas well, but only works as well for cold discs as a whole at low redshift.

We present surface density profiles for recently cooled and all cold gas of our sample of EAGLE haloes in Fig. 4.13 (top and bottom panels, respectively). To build these profiles, we find the rotation axis of the cold (cooled) particles to determine the plane of the galaxy (freshly cooled gas), and measure the surface density in annuli in this plane. Consistent with our measurements of the hot gas profiles, the widths of the annuli are adapted to contain the same number of particles, while respecting the softening scale. 100 annuli are used for the cold gas, 20 for the cooled gas. We then fit an exponential to each profile and normalise the annuli’s radii by the fitted scale length. By normalising the surface densities as well, we are able to see how well an exponential describes the entire galaxy population as a whole. Note that \( r_{d,\text{fit}} \) is a different quantity for the \( \Sigma_{\text{cooled}}(r) \) and \( \Sigma_{\text{cold}}(r) \) profiles in Fig. 4.13.

The top panels of Fig. 4.13 show that \( \Sigma_{\text{cooled}}(r) \) for EAGLE galaxies is nicely centred on Eq. 4.8, with a scatter increasing at lower redshift (the 68% confidence range is 0.53, 0.62, and 0.79 dex tall on average for the highest- to lowest-redshift bins, respectively). The profiles include a cusp, seen at \( r \lesssim 0.6r_d \) for the highest-redshift bin, with the prominence of this feature growing

\[ \text{For our intents and purposes, an annulus is a cylindrical shell that cannot exceed the virial sphere in height.} \]
modestly down to $z = 0$.

Of course, the galaxies in our sample need not be all rotationally supported, and hence may not have well-behaving, classical discs. To this point, we over-plot the median surface density profiles for cold and recently cooled gas for the rotation-dominated galaxies with $\lambda_R > 0.75$ in Fig. 4.13 with dot-dashed curves. By selecting galaxies we expect to be more disc-like, we find no difference in the normalised surface density profiles for cold or recently cooled gas. While the recently cooled gas in these galaxies lost less $j$ during the cooling process, this has not translated into a difference in $\Sigma_{\text{cooled}}(r)$.

In a semi-analytic model, one must put in what the scale length of the cooling disc is. Fall (1983) and Mo et al. (1998) have shown that by assuming the gas cools from a singular isothermal sphere, that $j_{\text{halo}} = j_{\text{cooling}} = j_{\text{cooled}}$, and the rotation curve of this gas is completely flat, where $v_{\text{rot}} = V_{\text{vir}}$, the scale radius can be related to the spin of the halo:

$$r_{d,\text{model}} \equiv \frac{\lambda}{\sqrt{2}}R_{\text{vir}} = \frac{j_{\text{halo}}}{2V_{\text{vir}}}$$

(cf. Eqs. 3.4 & 3.5). This model also assumes $\lambda$ measured from a dark-matter-only simulation is equivalent to considering all matter in a halo in reality (which might not be true – Schaller et al. 2015a, for example, have shown the masses of haloes in the EAGLE simulations are systematically higher when run without baryonic physics). Eq. 4.9 has formed the basis of determining (initial) disc sizes in many semi-analytic models (e.g. Hatton et al. 2003; Croton et al. 2006; Fu et al. 2010), but more complex algorithms are also popular (e.g. Cole et al. 2000). We showed in Section 4.3 that the assumptions about the angular momentum of the cooling gas required for Eq. 4.9 to be accurate do not agree with EAGLE, so we would expect some difference between the model scale length, $r_{d,\text{model}}$, and the fitted scale length, $r_{d,\text{fit}}$. After measuring net specific angular momentum of each halo in our EAGLE sample as

$$j_{\text{halo}} = \frac{\sum_p m_p \vec{r}_p \times \vec{v}_p}{\sum_p m_p}$$

(4.10)

(where subscript $p$ is for particles of all types within $R_{\text{vir}}$), we can quantify how discrepant $r_{d,\text{model}}$ is from $r_{d,\text{fit}}$.

In Fig. 4.14a, we present normalised histograms for the ratio of $r_{d,\text{fit}}/r_{d,\text{model}}$ for both the Reference and RecalHR haloes, for all $z < 4$. Both simulations find an almost log-normal distribution for this ratio, where it is almost always true that $r_{d,\text{fit}} > r_{d,\text{model}}$. The idea that $r_d \propto j_{\text{halo}}/V_{\text{vir}}$ (Eq. 4.9) came from the assumptions that $j_{\text{cooled}} = j_{\text{halo}}$ and that the cooled gas all rotated with velocity $V_{\text{vir}}$. In practice, we should really expect $r_d \propto j_{\text{cooled}}/v_{\text{rot}}$, where $v_{\text{rot}}$ is mass-weighted
mean tangential velocity of the cooled gas. This then gives

\[
\frac{r_{d,\text{fit}}}{r_{d,\text{model}}} \approx \frac{\dot{j}_{\text{cooled}} V_{\text{vir}}}{\dot{j}_{\text{halo}} v_{\text{rot}}} .
\]  

(4.11)

In Fig. 4.14b, we compare the left- and right-hand side of Eq. 4.11 for the Reference and RecalHR simulation haloes, displaying the median and 16th–84th percentile range for each. Note that we have binned data along the y-axis of this plot (and Fig. 4.14d, but all other plots in this chapter bin along the x-axis). Both simulations corroborate Eq. 4.11, with the median trend for the RecalHR simulation (with \( \gtrsim 512 \) cooled particles per halo) nearly matching it perfectly. In addition to an exponential not describing every individual cooling profile, the scatter in Fig. 4.14b can be attributed to the cooling particles having an angular-momentum structure (see Section 4.4.2) where the specific angular momentum of individual particles does not have a precise monotonic relationship with either position or rotational velocity.

It is useful to relate \( r_{d,\text{fit}}/r_{d,\text{model}} \) to global properties of the halo, as, if there is a strong correlation with one, the prescription for calculating \( r_d \) in a semi-analytic model could be easily modified to better match the results of EAGLE. We find that the halo property that best correlates with \( r_{d,\text{fit}}/r_{d,\text{model}} \) is the spin parameter, \( \lambda \). This is shown in Fig. 4.14d. By performing a least-squares fit for \( \log_{10}(r_{d,\text{fit}}/r_{d,\text{model}}) \) as a function of \( \log_{10} \lambda \) for the RecalHR simulation, we obtain the relation

\[
\log_{10} \left( \frac{r_{d,\text{fit}}}{r_{d,\text{model}}} \right) = -0.77 \log_{10} \lambda - (0.52 \pm 0.18) ,
\]  

(4.12)

where the uncertainty is the standard deviation of the points about the fit. Combining this result and Eq. 4.9, we suggest a modified model for the scale radius of cooling gas in a halo of given size and spin:

\[
\log_{10} \left( \frac{r_d}{R_{\text{vir}}} \right) = 0.23 \log_{10} \lambda - 0.67(\pm0.18) .
\]  

(4.13)

In Fig. 4.14e, we show that our EAGLE haloes are representative in terms of their distribution of spins, which should be roughly log-normal (cf. Barnes & Efstathiou 1987; Bullock et al. 2001).

### 4.4.2 Angular-momentum structure of gas discs

There is a building trend for models of disc evolution to calculate local processes in annuli of specific angular momentum, rather than radius (e.g. Stringer & Benson 2007; Dutton & van den Bosch 2009; Stevens et al. 2016a). These have been partially motivated by the fact that galaxies have dynamic and non-uniform velocity structures. One can rewrite Eq. 4.9 as an exponential function of specific angular momentum, thereby relaxing the requirement that cooling gas should
Figure 4.14: Trends relating to the ratio of the fitted exponential scale length of the surface density profiles of recently cooled gas in EAGLE galaxies to a typical assumed scale length in semi-analytic models (Eq. 4.9). Panel (a): Normalised histograms for this ratio for each of the Reference and RecalHR simulations. Panel (b): Relation between this ratio and a combination of the specific angular momentum of the cooled gas, that of the halo, the virial velocity of the halo, and the rotational velocity of the cooled gas. These quantities are expected to be nearly equal (Eq. 4.11, as given by the long-dashed line). The solid and short-dashed curves give the median for the Reference and RecalHR simulations respectively, with the 16th–84th percentile range for each shown by the shaded regions (for data binned along the y-axis). Panel (c): Normalised histograms for the y-axis value of panel (b). Panel (d): Relation between the scale radius ratio and the spin parameter of the halo. In addition to the median and 16th–84th percentile range (for data binned along the y-axis), we include a least-squares fit for the RecalHR simulation, including the standard deviation for points about this fit, where $\log_{10} \lambda$ has been taken as the independent variable. Panel (e): Normalised histograms for the spin parameter of the haloes. Only in the highest-spin haloes do the specific angular momenta of the cooled gas and halo become similar enough for the scale radius to resemble the model value.
4.4. After cooling: cold gas in galaxies

Figure 4.15: As for Fig. 4.13, but now presenting normalised surface density profiles for recently cooled (top panels) and all cold (bottom panels) gas as a function of specific angular momentum. If these profiles were generally well described by an exponential, we should see a tight relation about the long-dashed line in each panel. Instead, we find an exponential as a function of specific angular momentum is a less accurate description of the profiles than a function of radius (cf. Fig. 4.13).

settle with a constant rotational velocity:

\[ \Sigma_{\text{cooled}}(j) = \Sigma_0(j_d)e^{-j/j_d} \]  

(cf. Eq. 3.6). Here, we investigate the angular-momentum structure of the cooled and cold gas profiles of our EAGLE galaxies and determine whether Eq. 4.14 can approximate them.

We measure the mean \( \langle j \rangle \) of particles in each annulus for which we have measured surface density values for our EAGLE galaxies. This gives us \( \Sigma_{\text{cooled}}(j) \) and \( \Sigma_{\text{cold}}(j) \) profiles, to which we fit Eq. 4.14, then normalise these profiles based on the fitted \( j_d \). These are presented in Fig. 4.15. At intermediate to low redshifts, an exponential for \( \Sigma_{\text{cooled}}(j) \) describes the profiles reasonably well, but this is not the case at high redshift. For \( 1.7 > z > 0.6 \), there is less of a featured cusp for \( \Sigma_{\text{cooled}}(j) \) than for \( \Sigma_{\text{cooled}}(r) \). This could be explained by the rotational velocities (and hence specific angular momenta) decreasing rapidly at the centres of discs, where pressure support becomes comparable to rotational support. In general, we find the scatter in the profiles is reduced when considering surface density as a function of radius instead (cf. Figs. 4.13 and 4.15). We find these conclusions extend to the overall cold gas profile, and they apply to the general galaxy population (in our sample) as well as the most rotationally supported systems (\( \lambda_R > 0.75 \), which should be the most discy – cf. solid and dot-dashed curves in Fig. 4.15).

While our findings suggest one would be better off using Eq. 4.8 rather than Eq. 4.14 in a
model of galaxy formation, it has been shown in Chapter 3 that using this expression for cooling in a semi-analytic model with disc structure can successfully reproduce the properties of local galaxies, including the relation between total mass and specific angular momentum of stellar discs. Of course, disc instabilities and feedback regulate the structure of a galaxy (both as a function of specific angular momentum and radius), so the manner in which gas cools does not provide a complete picture by itself. An assessment of the importance of details surrounding cooling versus internal regulatory processes in accurately describing galaxy evolution with respect to observations would serve as a useful follow-up study to this work.

4.5 Summary

We have studied the hot mode of accretion of gas onto galaxies in the EAGLE simulations. As detailed in Section 4.1.3, we selected systems where a sufficient number of gas particles cooled between snapshots in order to learn about the state of haloes undergoing cooling and the galaxies they host. Our findings presented in this chapter can be summarised as follows.

- The temperature of hot gas in haloes is within a factor of 2 of the virial temperature across all radii (Section 4.2.1). The gradient of the temperature profiles steepens with time, due in part to stellar feedback. As centres become hotter, they become less dense, leading to shallower density gradients and the formation of a core in the hot gas (Section 4.2.2). The hot-gas density profiles are well approximated by a $\beta$ profile, which can be parameterised as a simple function of redshift (Eq. 4.4).

- Using the precise density, temperature, and metallicity profiles of hot gas in the EAGLE haloes leads to a range of cooling radii that is wider than one would obtain assuming a singular isothermal sphere or $\beta$ profile for a halo with the same total mass content, net metallicity, and virial temperature. At low redshift, these cooling radii are systematically smaller as well, which would translate into lower cooling rates under the model of White & Frenk (1991).

- The temperature and density profiles of cooling gas remain nearly unchanged with time, irrespective of the underlying hot gas (Section 4.2.1). Outflows from AGN feedback appear to help regulate the former. While perhaps most alike an exponential, the density profiles of cooling gas are consistent with a singular isothermal sphere (Section 4.2.2). So long as models only require cooling gas to look like it is coming from an isothermal sphere, this seems to be a reasonable approximation.
4.5. Summary

- Over the course of cooling, gas loses approximately 60% of its specific angular momentum, on average (Section 4.3.1). This value is comparable to losses quoted during reports of the ‘angular momentum catastrophe’, yet the galaxies in EAGLE are far from catastrophic. We find a weak tendency for gas cooling onto rotationally supported galaxies to lose a lesser fraction of its specific angular momentum.

- Gas in the process of cooling typically begins with specific angular momentum several times greater than that of the halo and is offset from the rest of the hot gas by tens of degrees (Section 4.3.2). Despite this, freshly cooled gas is typically well aligned with pre-existing gas discs. Interestingly, the inner hot gas of the halo is even less well aligned with the cooling gas, despite the cooling gas predominantly originating from the inner halo (cf. Sections 4.2.2 and 4.3.2).

- Recently cooled gas is well approximated by an exponential surface density profile as a function of radius (Section 4.4.1). In general, this is more precise than an exponential as a function of specific angular momentum (Section 4.4.2). Because the cooled gas tends to have higher specific angular momentum than that of the halo, the best-fitting scale radius for these surface density profiles is typically larger than expected from the standard model of disc formation (Fall & Efstathiou 1980; Mo et al. 1998). This radius is still strongly tied to the spin parameter of a halo, for which we provide a new expression (Eq. 4.13).

Our results suggest some of the assumptions surrounding cooling of hot gas in (semi-)analytic galaxy formation models should be revised. While the parameters of models are often tuned to match the properties of galaxies in the local Universe, by altering the prescriptions surrounding cooling, the star formation histories and higher-redshift properties of these galaxies will change. This could, therefore, be fruitful in the quest for developing a model of galaxy evolution that can simultaneously explain the high- and low-redshift Universe.
Where do galaxies end?


From humble beginnings (Carlberg et al. 1990; Katz et al. 1992; Evrard et al. 1994), hydrodynamic supercomputer simulations of the formation and evolution of galaxies and structure in the Universe have grown to be highly sophisticated, both in terms of their modelled physics and technical specifications (e.g. Crain et al. 2009, 2015; Sijacki et al. 2012). While the current state of the art makes such simulations an excellent tool for studying galaxy evolution and cosmology, interpreting the vast volumes of particle data output to conduct science presents many notable challenges.

To investigate the gross evolution of galaxies, it is informative to study their integrated properties. Many surveys have been conducted with this as a central purpose, e.g. SDSS (York et al. 2000) and 6dF (Jones et al. 2004). While there are established means of using simulations to predict observables (e.g. Jonsson 2006), ultimately, observations attempt to measure quantities which should be ‘directly’ measurable from simulations.

In order to measure the integrated properties of a hydrodynamic-simulation galaxy, one needs a way of defining which particles/cells actually belong to the galaxy. This sounds like a rather trivial task, but it is not. There is no solitary ‘right’ way to define, for instance, ‘the size of the galaxy’ or, rather, the boundary between the galaxy and the rest of the (sub)halo, nor is there to identify satellites or substructure that should not (yet) be considered part of the galaxy of interest. For cosmological simulations that include many objects, one must also first face the task of structure location and halo definition (see Knebe et al. 2013b). Each decision carries an inevitable level of arbitrariness, partly driven by an incomplete definition of what a real galaxy is (see Forbes & Kroupa 2011, who propose a vote on the definition).

An array of techniques for defining galaxies in hydrodynamic simulations has been used in the literature. Some rely primarily on the results of subhalo finders (e.g. Kereš et al. 2005, 2009a, 2012; Saro et al. 2010; Governato et al. 2012; Neistein et al. 2012; Sales et al. 2012; Haas et al.
Chapter 5. Where do galaxies end?

2013; Moster et al. 2014; Munshi et al. 2013), which invoke overdensity, friends-of-friends (FoF), and/or gravitational binding criteria. Usually, this is coupled with restrictions on gas properties. It has also often been assumed that a galaxy will fall within some prescribed spherical aperture: examples include the radius at which the modelled surface brightness reaches some threshold (e.g. Brook et al. 2012a; Martig et al. 2012), an aperture of a fixed size in physical or comoving coordinates (e.g. Martig et al. 2009; Kereš et al. 2012, respectively), or with a radius set to a fixed fraction of the virial radius (e.g. Hirschmann et al. 2012; Sales et al. 2012; Scannapieco et al. 2012; Benítez-Llambay et al. 2013; Marinacci et al. 2014; Roškar et al. 2014).

We aim to address the arbitrary choices involved in determining what constitutes a simulated galaxy and to compare results of integrated properties produced from an assortment of techniques, many of which have been used in publications. The goal is to provide a discussion on the motivations behind each technique, while addressing details of their implementation and determining which properties are most sensitive to the choice of technique, thereby realising the level of uncertainty associated with these measurements. This tangentially builds on previous suggestions by Guedes et al. (2011) and Munshi et al. (2013) that stellar mass can vary by ∼20% from summing the mass of all star particles within a simulated halo versus undertaking mock observations (also see Obreja et al. 2014, who compare direct measurements with spectral energy distribution fits for particles within the same optically motivated radius). Further, we aim to address the influence the techniques have on galaxy scaling relations. We cover two different, but complementary, simulation regimes, considering both a suite of high-resolution zoom re-simulations of individual haloes (Martig et al. 2012, hereafter referred to as the M12 simulations), and subhaloes identified in the aptly massive cosmological simulation MassiveBlack-II (Khandai et al. 2015).

In this chapter, Section 5.1 describes the simulations we have used to perform our analysis. Section 5.2 outlines, in more detail, how galaxy particles are typically identified in hydrodynamic simulations. Here, we cover specific techniques used in the literature, describing those we have used for our analysis, while outlining a new technique of our own. We also describe how well those techniques succeed at capturing galaxies. We provide comparative results of integrated property measurements from our analysis in Section 5.3. We offer a brief discussion on what to make of the results in Section 5.4. Section 5.5 concludes the chapter with a summary.

Except where the expansion factor \( a = (1 + z)^{-1} \) is explicitly written, all distances quoted in this chapter are in physical coordinates, not comoving. Because \( h \) is a dimensionless constant and not a unit (see Croton 2013), we dissociate explicit factors of \( h \) from relevant units, in favour of it either being attached to the number in question instead or having its value substituted, whichever is more exact (but note that many of the papers cited in this chapter adopt different values of \( h \)).
5.1. THE SIMULATIONS

5.1.1 The M12 simulations

The M12 simulations are a set of 33 high-resolution zoom re-simulations run by Martig et al. (2012), which each focus on a central, Milky Way-size galaxy and its surrounding satellites. Here, merger and diffuse-accretion histories were first extracted for each object of interest from a cosmological, dark-matter-only simulation. These histories were then re-simulated at higher resolution, where diffuse accretion was modelled assuming gas follows dark matter in an amount given by the cosmic baryon fraction. Merging haloes were tracked in the simulation and replaced with higher-resolution haloes housing galaxies (made of a stellar disc and bulge, and a gas disc) upon their entrance of one of the re-simulated volumes.

The parent simulation was run using the code RAMSES (Teyssier 2002) in a periodic box of length $20h^{-1}\text{a}\text{Mpc}$, with $512^3$ particles of mass $6.9 \times 10^6 \text{M}_\odot$, assuming ΛCDM parameters $\Omega_M = 0.3$, $\Omega_\Lambda = 0.7$, $h = 0.7$, and $\sigma_8 = 0.9$.

The re-simulations were performed using a Particle-Mesh code, where gas dynamics was modelled with a sticky-particle scheme. Star formation followed a Schmidt (1959) law, and supernova feedback and stellar mass loss were included (but not active galactic nucleus feedback). Particle masses are $3 \times 10^5 \text{M}_\odot$ for dark matter and $1.5 \times 10^4 \text{M}_\odot$ for gas and stars.\(^1\) Details on the zoom technique are outlined in Martig et al. (2009). Each re-simulated box had a length of 800 kpc and spatial resolution of 150 pc. The re-simulations started at $z = 5$ and ran to $z = 0$. For $z \lesssim 2$, the systems are sufficiently evolved such that measurements of the galaxies’ properties are trustworthy.

We used a sample of 16 re-simulated galaxies, providing a fair representation of the diverse histories these simulations showcase spiral galaxies to have. At $z = 0$, the sample covers halo masses from 2.7 to $20 \times 10^{11} \text{M}_\odot$ and bulge-to-total ratios between 0.02 and 0.53. We analysed snapshots at 375-Myr intervals.

Gas particle densities were recovered using a refinable mesh, where particles within each cell were assigned the average density of that cell. Temperatures were subsequently calculated using a polytropic equation of state (Teyssier et al. 2010):

\[
\frac{T}{10^4 \text{K}} = \begin{cases} 
\left(\frac{n_H}{0.3 \text{ cm}^{-3}}\right)^{-1/2}, & n_H > 0.3 \text{ cm}^{-3} \\
1, & n_H \in [10^{-3}, 0.3] \text{ cm}^{-3} \\
400 \left(\frac{10^3 n_H}{\text{cm}^{-3}}\right)^{2/3}, & n_H < 10^{-3} \text{ cm}^{-3}
\end{cases}
\]

\(^1\)True for star particles formed during the simulations. Star particles existent from the initial conditions have mass $7.5 \times 10^4 \text{M}_\odot$. 

where $n_H$ is the number density of hydrogen atoms, which we approximated gas to be entirely composed of. Given the piecewise nature of Eq. 5.1, throughout this chapter, all gas particles with $T \leq 10^4$ K are considered ‘cold’ for these simulations, with the remainder considered ‘hot’. Note that this differs from the definitions used in Chapter 4.

5.1.2 MassiveBlack-II

MassiveBlack-II (Khandai et al. 2015) is a cosmological, hydrodynamic simulation run with $2 \times 1792^3 \simeq 11.5$ billion particles in a $100 h^{-1}$ Mpc periodic box, using the TreePM-SPH code P-GADGET (a modification of GADGET-2; Springel 2005). The cosmology of the simulation followed $\Lambda$CDM with $\Omega_M = 0.275$, $\Omega_{\Lambda} = 0.725$, $h = 0.702$, and $\sigma_8 = 0.8$. Gas and dark matter particles have initial$^2$ masses equal to $3.16 \times 10^6$ and $1.57 \times 10^7$ M$_{\odot}$, respectively, while gravitational softening occurred at $1.85 h^{-1}$a kpc. MassiveBlack-II ran to $z = 0$ and was a follow-up simulation to MassiveBlack (Di Matteo et al. 2012) which ran to $z = 4.75$ using 8 times as many particles, at lower mass resolution, in a volume 151 times larger. Star formation and stellar feedback followed Springel & Hernquist (2003). A full consideration of black holes and active galactic nucleus feedback was also included (following Di Matteo et al. 2005; Springel et al. 2005b; Di Matteo et al. 2008).

We analyse one snapshot of MassiveBlack-II at $z = 0.0625$, measuring results from subhaloes whose gas (hot + cold) and stellar masses are each between $10^8$ and $10^{12}$ M$_{\odot}$. This ensured that each subhalo assessed was well resolved (by at least 63 gas and star particles each). This gave a sample of 224,585 galaxies.

To maintain consistent definitions of ‘cold’ and ‘hot’ throughout this work, densities of gas particles in MassiveBlack-II were calculated via the same means as in the M12 simulations, where temperatures were subsequently calculated with Eq. 5.1.$^3$

5.2 Techniques for measuring galaxy properties

As galaxies occupy (sub)haloes, particles belonging to a galaxy should be a subset of the baryons belonging to a (sub)halo. Baryons in (sub)haloes can be broken into three populations: those in the galaxy of interest, those in other galaxies within the same (sub)halo (i.e. satellites), and those diffusely occupying the rest of the (sub)halo. As has been done in the literature, this breakdown can be achieved with an automated, generally applicable method involving three steps:

$^2$Stars particles are created with mass $1.58 \times 10^6$ M$_{\odot}$, meaning a gas particle’s mass is halved upon creating a star particle.

$^3$A proper treatment of hydrogen number density could affect the number of hot and cold particles for MassiveBlack-II here, where the difference would be negligible for the M12 simulations.
5.2. Techniques for measuring galaxy properties

1. Use of a subhalo finder. Subhalo finders not only locate and provide the (sub)haloes that galaxies of interest occupy, but are also proficient at identifying satellite systems to be stripped.

2. Temperature and density restrictions on gas, as gas within a galaxy should be relatively dense and cool.

3. A cut with a spherical aperture. This sets a boundary between the galaxy and the rest of the (sub)halo, classifying which of the remaining star and cold gas particles are part of the galaxy. Spherical apertures are both simple in their implementation and a fair shape to broadly apply to all galaxies.

There are many widely used, well-established subhalo-finding codes available (for a comparative study on popular codes’ abilities to locate galaxies, see Knebe et al. 2013a), while the cold/hot temperature boundary for gas is often motivated by how temperature was treated within the simulation of interest. The choice of spherical aperture is hence the most contentious step. This ultimately determines where the galaxies end: a choice that has shown great variation throughout the literature. Indeed, the primary goal of this chapter is to quantify the differences in integrated properties returned from these aperture choices. We stress the value the above methodology has in being, in theory, blindly applicable; this is mandatory for analysing large simulation datasets efficiently.

5.2.1 Aperture choices in the literature

We present a summary of the techniques for defining particles/cells associated with simulated galaxies in the literature in Table 5.1. Many of these cases include only one or two of the ideal steps listed above. For instance, while most of the examples we list include the application of an aperture, it has also been popular to not apply one at all. We note techniques beyond what we have listed exist in the literature as well (e.g. Doménech-Moral et al. 2012; Few et al. 2012, who decompose the galaxy into disc and bulge particles). The aperture choices listed in Table 5.1 can be broken into categories of an optical limit \( R_{25} \), fixed aperture (comoving or physically fixed), and fraction of the virial radius. We discuss the potential motivations and short falls of each in turn below.

Optical limits describe where a galaxy would appear to end against a sky background, and hence are appropriate for more-direct comparisons to observations. However, they do not aim

---

4The work presented in this chapter was published prior to the release of the EAGLE and Illustris simulations. As such, these do not feature in Table 5.1. For reference, EAGLE adopts a fixed aperture of 30 physical kpc (Schaye et al. 2015) and Illustris’s aperture is 2\( R_{1/2} \) (twice the stellar half-mass radius; Vogelsberger et al. 2014).
Table 5.1: Summary of techniques used for measuring integrated properties of galaxies published in the literature. ‘Type’ represents whether the study was of a periodic-box (P) or zoom (Z) simulation. The third column provides the best resolution of baryonic mass used in each reference. Column four lists any subhalo finders or mock observing codes applied [with references a(Gelb & Bertschinger 1994; Katz et al. 1996); b(Jonsson 2006); c(Fioc & Rocca-Volmerange 1997); d(Domínguez-Tenreiro et al. 2014)]. An aperture radius of $R_{25}$ is the radius at which the modelled surface brightness reaches 25 mag arcsec$^{-2}$ in the listed band. $R_P$ denotes the Petrosian radius (Blanton et al. 2001). The sixth and seventh columns provide the maximum allowable temperature and minimum allowable density, respectively, for gas particles. $\bar{\rho}_{\text{bary}}$ is the mean baryonic density. Column eight lists the properties measured in the papers, including stellar mass (SM), gas mass (GM), star formation rate (SFR), and gas accretion rate (GAR). Properties with a star were measured via a different method to the one in this table.

<table>
<thead>
<tr>
<th>Reference</th>
<th>Type</th>
<th>Best mass res.</th>
<th>Code</th>
<th>Aperture radius</th>
<th>$T_{\text{max.}}$</th>
<th>$n_H$ or $\rho$</th>
<th>Properties</th>
</tr>
</thead>
<tbody>
<tr>
<td>Haas et al. (2013)</td>
<td>P</td>
<td>$1.850.0h^{-1}$</td>
<td>FoF</td>
<td>-</td>
<td>-</td>
<td>0.1 cm$^{-3}$</td>
<td>SM, GM</td>
</tr>
<tr>
<td>Saro et al. (2010)</td>
<td>Z</td>
<td>170,000.0$h^{-1}$</td>
<td>SUBFIND</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>SM, SFR</td>
</tr>
<tr>
<td>Sales et al. (2012)</td>
<td>Z</td>
<td>1,400.0$h^{-1}$</td>
<td>SUBFIND</td>
<td>$0.15R_{\text{vir}}$</td>
<td>-</td>
<td>-</td>
<td>SM, GM</td>
</tr>
<tr>
<td>Moster et al. (2014)</td>
<td>Z</td>
<td>$\sim 100.0$</td>
<td>SUBFIND</td>
<td>10.0</td>
<td>0.1 cm$^{-3}$</td>
<td>SM, GM</td>
<td></td>
</tr>
<tr>
<td>Neistein et al. (2012)</td>
<td>P</td>
<td>86,600.0$h^{-1}$</td>
<td>SUBFIND</td>
<td>30$h^{-1}$ kpc</td>
<td>3.0</td>
<td>2000$\bar{\rho}_{\text{bary}}$</td>
<td>SM, GM, SFR</td>
</tr>
<tr>
<td>Kereš et al. (2012)</td>
<td>P</td>
<td>740.0$h^{-1}$</td>
<td>SUBFIND</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>SM, GM</td>
</tr>
<tr>
<td>Munshi et al. (2013)</td>
<td>Z</td>
<td>0.4</td>
<td>AHF</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>SM, GM</td>
</tr>
<tr>
<td>Kereš et al. (2005, 2009a)</td>
<td>P</td>
<td>1,700.0</td>
<td>SKID$^a$</td>
<td>-</td>
<td>3.0</td>
<td>1000$\bar{\rho}_{\text{bary}}$</td>
<td>SM, GM, SFR, GAR</td>
</tr>
<tr>
<td>Guedes et al. (2011)</td>
<td>Z</td>
<td>20.0</td>
<td>SUNRISE$^b$</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>SM</td>
</tr>
<tr>
<td>Martig et al. (2012)</td>
<td>Z</td>
<td>15.0</td>
<td>PEGASE$^c$</td>
<td>$R_{25}$ g-band</td>
<td>-</td>
<td>-</td>
<td>SM, GM, SFR, GAR$^*$</td>
</tr>
<tr>
<td>Brook et al. (2012a)</td>
<td>Z</td>
<td>4.8</td>
<td>SUNRISE</td>
<td>$R_{25}$ i-band</td>
<td>-</td>
<td>-</td>
<td>SM, GM$^*$</td>
</tr>
<tr>
<td>Obreja et al. (2014)</td>
<td>Z</td>
<td>75.0</td>
<td>GRASIL-3D$^d$</td>
<td>$2R_P$</td>
<td>-</td>
<td>-</td>
<td>SM, SFR</td>
</tr>
<tr>
<td>Roškar et al. (2014)</td>
<td>Z</td>
<td>89.0</td>
<td></td>
<td>-</td>
<td>0.1$R_{\text{vir}}$</td>
<td>-</td>
<td>SM, SFR$^*$</td>
</tr>
<tr>
<td>Marinacci et al. (2014)</td>
<td>Z</td>
<td>230.0</td>
<td></td>
<td>-</td>
<td>0.1$R_{\text{vir}}$</td>
<td>-</td>
<td>SM, GM, SFR</td>
</tr>
<tr>
<td>Dekel et al. (2013)</td>
<td>Z</td>
<td>10.0</td>
<td></td>
<td>-</td>
<td>0.1$R_{\text{vir}}$</td>
<td>1.5</td>
<td>GM, SFR, GAR</td>
</tr>
<tr>
<td>Scannapieco et al. (2012)</td>
<td>Z</td>
<td>200.0</td>
<td></td>
<td>-</td>
<td>0.1$R_{\text{vir}}$</td>
<td>10.0</td>
<td>SM, GM, SFR</td>
</tr>
<tr>
<td>Hirschmann et al. (2012)</td>
<td>Z</td>
<td>4,200.0$h^{-1}$</td>
<td></td>
<td>-</td>
<td>0.1$R_{\text{vir}}$</td>
<td>log $T &lt; 0.3 \log \rho + 3.2$</td>
<td>SM, GM, SFR</td>
</tr>
<tr>
<td>Benítez-Llambay et al. (2013)</td>
<td>Z</td>
<td>60.5</td>
<td></td>
<td>-</td>
<td>0.15$R_{\text{vir}}$</td>
<td>-</td>
<td>SM, SFR</td>
</tr>
<tr>
<td>Martig et al. (2009)</td>
<td>Z</td>
<td>21.0</td>
<td></td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>SM, GM, SFR</td>
</tr>
</tbody>
</table>
to probe where a galaxy actually ends. While integrated properties derived from this technique certainly have value, they will not necessarily be the ‘true’ integrated properties of a given galaxy, which are of interest here.

For a small sample of galaxies of similar shapes and masses at some epoch, it might be reasonable that each galaxy ends at an approximately equivalent radius. However, using an aperture of fixed comoving size to define the end of galaxies does not work on a general basis, simply as galaxies come in a range of sizes. This is at least preferable to an aperture of fixed physical size though, as it considers the average growth of galaxies with time.

Using a fraction of the virial radius is the most common aperture method in the literature. For this to be appropriately motivated, there would need to be a strong correlation between the size of galaxies and their parent (sub)haloes.\(^5\) A study by Kravtsov (2013), which compared the cumulative number density of observed galaxies in the local Universe as a function of their size to the same distribution of simulated dark matter haloes, found a trend between a galaxy’s stellar half-mass radius, \(R_{\frac{1}{2}}\), and its virial radius. Specifically, the author showed

\[
4.5 < \frac{R_{\frac{1}{2}}}{R_{\text{vir}}} \times 10^3 < 45 . \tag{5.2}
\]

This relationship is consistent with the model of Mo et al. (1998), who suggested the radii of discs should be proportional to \(\lambda R_{\text{vir}}\), where \(\lambda\) is the dimensionless spin parameter of the galaxy’s parent halo. The scatter one expects from the probability distribution of \(\lambda\) strongly correlates to the allowable range in Eq. 5.2 (Kravtsov 2013).

This result has been interpreted by others (e.g. Stringer et al. 2014) to mean there is a tight correlation between the size of a galaxy and its parent (sub)halo. We find several points to be troubling about this conclusion:

1. The relation only considers the stellar component of any galaxy. Galaxies also consist of gas. The distribution of gas should be included in the interpretation of where a galaxy ends.
2. There is a spread of an order of magnitude for the correlation. To use it to infer the size of a galaxy hence carries a large uncertainty.
3. A correlation for a half-mass radius only infers a correlation to a full-mass radius if the baryonic density profiles of all galaxies are equivalent, which they are not (see below).

The majority of examples in Table 5.1 study a limited number of integrated properties, some of which are not measured with equivalent techniques. We emphasise that, especially if one wishes to

\(^5\)None of the literature examples we present describes any motivation behind using a precise fraction of \(R_{\text{vir}}\), so we are left to speculate.
measure multiple properties that are derivatives of time (e.g. star formation rate, gas ejection rate, gas accretion rate, star death rate), a solitary technique that is self-consistent in a mass-conserving sense should be used to measure all properties.

5.2.2 Employed subhalo finders

We employed AHF\(^6\) (Gill et al. 2004; Knollmann & Knebe 2009) to identify substructure in the M12 simulations. AHF operates by first finding density peaks in a simulation using a refinable-mesh scheme, then places spherical apertures around those regions based on an input overdensity criterion (typically 200\(\rho_{\text{crit}}\)). Finally, the program performs an unbinding procedure on the particles, where only particles with kinetic energy (kinetic + thermal for gas) less than their potential energy relative to the overdensity are considered bound to the structure. We ensured the refinement process of AHF could only go down to a cell length equal to the gravitational softening of the M12 simulations (150 pc). AHF offers the choice of a critical velocity to determine particles as unbound; we chose this to be exactly equal to the escape velocity calculated within the program. In accordance with the previously published techniques, all subhaloes have been stripped from the AHF data presented.

Subhaloes in MassiveBlack-II were found with SUBFIND (Springel et al. 2001b). SUBFIND identifies subhaloes within FoF groups by enclosing areas with isodensity contours that traverse saddle points, then groups particles whose energies are sufficiently low to be gravitationally bound (the same final step as AHF).

While, to first order, AHF and SUBFIND achieve the same goal, their methods of identifying substructure are not identical. Comparison studies of these and other subhalo finders (Knebe et al. 2011, 2013a; Onions et al. 2012) have shown their results to be in good agreement, however. As such, the impact of using them on the different simulations is small when compared to the much larger variations between the aperture techniques (see below).

The primary difference between the codes is that AHF uses spherical apertures to define the extent of subhaloes (for our results, this is \(R_{\text{vir}}\)). There is no change to our results from MassiveBlack-II if an aperture cut at \(R_{\text{vir}}\) is additionally made to the SUBFIND particles, however. This is because the properties we assess are only concerned with particles that naturally exist toward the centre of subhaloes; for all results involving use of a subhalo finder, integrated gas masses and transfer rates only consider cold gas. For properties concerning dark matter and hot gas, this additional aperture cut would have some effect.

\(^6\)AMIGA Halo Finder. Downloadable at http://popia.ft.uam.es/AMIGA/
5.2. Techniques for measuring galaxy properties

5.2.3 The ‘baryonic-mass profile’ (BaryMP) technique

We propose a new technique for determining the size of the spherical aperture that defines the end of a galaxy. Ideally, the end of a galaxy should be directly informed by the way in which baryons are distributed in the (sub)halo of interest. Moreover, we see no reason to bias the size on the stellar distribution over the cold gas distribution or vice versa. As such, we use the cumulative one-dimensional baryonic (stars + cold gas) mass profiles of each respective (sub)halo (without substructure) to infer their aperture sizes.

From both the M12 and MassiveBlack-II galaxies, we discovered the baryonic mass profiles for all the simulated (sub)haloes follow a common form. This can be analytically approximated as

\[
\frac{m_{\text{bary}}(< R)}{m_{\text{bary}}(< R_{\text{vir}})} = \left(1 - e^{-bR/R_{\text{vir}}}ight) \left(\xi \frac{R}{R_{\text{vir}}} + k\right),
\]

where \( b \) is defined to be positive, and \( 0 \leq \xi < 1 \) (see below and Appendix E). Because the left side of Eq. 5.3 must be 1 when \( R = R_{\text{vir}} \), \( b \) and \( \xi \) are the only 2 parameters, where

\[
k = (1 - e^{-b})^{-1} - \xi.
\]

We provide examples of baryonic mass profiles from the simulations placed against their analytic counterparts with best-fitting parameters in Fig. 5.1. For our sample of MassiveBlack-II galaxies, the best-fitting \( b \) parameters produce a smooth, approximately Gaussian distribution, with mean and standard deviation values of 14.2 and 3.8, respectively. 75% of these systems also have \( \xi < 0.1 \); the parameter is more important for the other 25% of cases.
The \( \frac{\xi R}{R_{\text{vir}}} + k \) term in Eq. 5.3 describes a straight-line asymptote that the function approaches as \( R \) increases. For large \( R \) then, the cumulative baryonic mass effectively goes linearly with \( R \) (with gradient \( \xi \)); in other words, baryonic density goes as \( R^{-2} \). This follows the relation for an isothermal sphere and also holds at the scale radius for dark matter haloes in virial equilibrium (Navarro et al. 1996). We define the diffuse halo component to occupy the region of the actual profiles where the gradient is roughly constant (for a profile void of substructure or abnormalities, this would equate to the asymptotic region of the analytic approximation). By extension, all (cold) baryons internal to this radius constitute the galaxy.

With the above motivation, we have devised an algorithm to determine the radius where the gradient of the baryonic mass profiles of (sub)haloes is sufficiently close to constant, \( R_{\text{BaryMP}} \). This is done via iterative straight-line fits to the outer parts of the profiles (Eq. 5.3 is not explicitly used). We provide a thorough description and code for the BaryMP algorithm in Appendix E. We have subsequently employed this aperture technique for measuring the integrated properties of simulated galaxies.

Fig. 5.2 shows where the BaryMP aperture cuts are taken for example profiles from MassiveBlack-II and two epochs of the M12 simulations. We note that, for these examples, the aperture radii returned are various fractions of their respective virial radii. At \( z \sim 0 \), we typically find \( 0.2 \lesssim R_{\text{BaryMP}} / R_{\text{vir}} \lesssim 0.4 \), with only one occasion from M12 in agreement with an aperture of \( 0.15 R_{\text{vir}} \). At higher redshift for M12, aperture radii consistently exceed \( 0.3 R_{\text{vir}} \).
5.2. Techniques for measuring galaxy properties

<table>
<thead>
<tr>
<th>Aperture radius</th>
<th>M12</th>
<th>MB-II</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1R_{vir}</td>
<td>D</td>
<td>S</td>
</tr>
<tr>
<td>0.15R_{vir}</td>
<td>D, S</td>
<td>S</td>
</tr>
<tr>
<td>R_{i25}</td>
<td>D</td>
<td>S</td>
</tr>
<tr>
<td>30h^{-1},a kpc</td>
<td>S</td>
<td>S</td>
</tr>
<tr>
<td>BaryMP</td>
<td>S</td>
<td>S</td>
</tr>
</tbody>
</table>

Table 5.2: Aperture techniques applied for our analysis, noting for each simulation set (M12 and MassiveBlack-II) which have been used as direct aperture techniques (denoted by D) and which have been used in combination with a subhalo finder and gas temperature restriction (denoted by S). R_{i25} is the radius at which the surface brightness profile reaches 25 mag arcsec^{-2} in the i-band.

5.2.4 Employed aperture techniques

In this chapter, we closely mimic many of the techniques presented in Table 5.1 (namely from Brook et al. 2012a; Governato et al. 2012; Kereš et al. 2012; Sales et al. 2012; Benítez-Llambay et al. 2013; Munshi et al. 2013; Roškar et al. 2014; Marinacci et al. 2014). The primary aim of this is to indicate how the relevant publications’ results should be compared with each other and our BaryMP technique. We summarise the aperture techniques we apply in Table 5.2. We note that to keep the number of test techniques limited and to assess the most ideal cases, wherever subhalo finders are used, hot gas is also stripped.

We refer to techniques that only included an aperture cut from the full simulation data as ‘direct aperture techniques’. These involve no attempt to remove satellites that fall in the aperture, nor is there a requirement to separate gas by temperature or density. Direct aperture techniques were employed in some of the literature examples presented in Table 5.1 (Benítez-Llambay et al. 2013; Marinacci et al. 2014; Roškar et al. 2014).

To study galaxies in a cosmological simulation like MassiveBlack-II, a subhalo finder is needed to locate galaxies initially, regardless of whether there is intent to apply direct aperture techniques. In addition to the fact that no direct aperture techniques were performed on periodic-box simulations listed in Table 5.1, we have only applied them to the M12 simulations. Table 5.2 lists the instances where direct apertures have been applied.

For the direct aperture techniques we employed that use a fraction of the virial radius, R_{vir} values were calculated considering all particles, again irrespective of whether they were part of any substructure. R_{vir} values were recalculated without the substructure when subhalo finders were used. Typically, the difference in these values is negligible.

For an optical radius, we find R_{25} values in the i-band (hereafter R_{i25}). To calculate these radii, we used the simple stellar population evolution model of Maraston (2005)\footnote{Downloadable at http://www-astro.physics.ox.ac.uk/~maraston/Claudia%27s\Stellar\Population\Models.html} with a Kroupa
(2001) initial mass function to produce spectral energy distributions for each star particle. We subsequently applied the $i$-band filter and integrated the spectra to produce $i$-band luminosities for each particle. Solar metallicity was assumed for the M12 star particles, while metallicity was tracked in MassiveBlack-II. Surface brightness profiles used to evaluate $R_{i25}$ were built using the AB magnitude system irrespective of redshift, i.e. assuming galaxies were being viewed face-on (defined below) at a near enough distance where angular-diameter and luminosity distances are negligibly different.\footnote{In this regime, (apparent) surface brightness has no dependence on distance. This can also be thought of as building ‘absolute’ surface brightness profiles, treating high-redshift systems fairly.} No additional dust modelling was included.

Figs. 5.3 and 5.4 present example images of an M12 simulation at two epochs with apertures from Table 5.2 overdrawn. Fig. 5.5 similarly presents an example from MassiveBlack-II. All three of these show ‘face-on’ images, i.e. where the net angular momentum vector of the substructure-stripped star particles within $R_{\text{vir}}$ of the (sub)halo points out of the page toward the reader. Fig. 5.3 also highlights the potentially significant effect of removing substructure identified by AHF.

Fig. 5.3 presents an example where, by eye, there is a distinguishable end to the stellar component of the galaxy, while there appears to be no end to the cold gas, which fills the entire halo. With temperature restrictions rendered unhelpful to resolve this (and, by extension, density restrictions, due to the polytropic equation of state), there is no distinction between galaxy and halo without the use of an aperture. Arguably, many of the apertures presented do a reasonable job of encompassing the stellar component of the galaxy, but it is not obvious from imaging alone which best encompasses the cold gas. As such, the aperture technique applied needs to be solidly motivated.

Figs. 5.4 and 5.5 show how several of the aperture techniques can exclude a significant portion of the galaxy at lower redshift (as quantified in Section 5.3.1). Specifically, $0.1R_{\text{vir}}$ and $0.15R_{\text{vir}}$ consistently underestimate the extent of galaxies. Despite being the most observationally motivated technique, $R_{i25}$ also frequently excludes parts of the galaxy. While Figs. 5.3 and 5.4 provide examples where a cut at $30h^{-1}a$ kpc seems reasonable, Fig. 5.5 exemplifies that even for systems of comparable present-epoch mass, the technique is not generally applicable. Fig. 5.4 also shows an occasion where it appears satellites have remained after removing the identified substructure. As we discuss in Appendix E and is exemplified in Fig. 5.4, the BaryMP radius successfully cuts inside the largest (star-dominated) satellite, as designed.

Conversely to Fig. 5.3, the cold gas in Fig. 5.5 shows a fairly distinctive cut-off by eye, whereas the stars are dispersed more continuously throughout the subhalo. Because BaryMP considers cold gas and stars equivalently, the technique is motivated to fairly compute an aperture radius in both instances, despite their differences.
Figure 5.3: Illustrations of one of the M12 simulations at $z = 2.15$. Left panels display stars, with black pixels indicating a column density $\leq 10^{-1} \, M_{\odot} \, \text{pc}^{-2}$ and white pixels $\geq 10^{3.5} \, M_{\odot} \, \text{pc}^{-2}$. Right panels display gas from $10^{-1}$ to $10^2 \, M_{\odot} \, \text{pc}^{-2}$. The width, height, and depth of each frame is equal to $2R_{\text{vir}}$, with $R_{\text{vir}}$ calculated after stripping substructure and hot gas. Top panels display all particles within the image frame. Bottom panels only show particles associated with the main halo according to AHF (substructure and hot gas stripped). Apertures have been overdrawn in the following colour code (in order of smallest to largest): magenta = $0.1R_{\text{vir}}$, green = $R_{\text{r25}}$, red = $0.15R_{\text{vir}}$, blue = BaryMP, white = $30h^{-1}$ a kpc.

5.3 Results

We present results in a relative context, where measurements from each technique are normalised to the respective integrated properties of their full parent (sub)halo (i.e. where no aperture has been used, while substructure and hot gas has been stripped). We normalise to this technique not only
Chapter 5. Where do galaxies end?

**Figure 5.4:** The same M12 simulation as in Fig. 5.3 but at $z = 0$, with stars on the left and gas the right. All AHF-identified substructure and hot gas has been stripped from these images (despite what appear to be satellite systems in the frame). Each frame has width, height, and depth of $R_{\text{vir}}$. We exclude images prior to stripping substructure and hot gas, as, visually, there is little difference for this case. Colour code for apertures is: magenta = $0.1R_{\text{vir}}$, red = $0.15R_{\text{vir}}$, green = $R_{\text{i}25}$, white = $30h^{-1}\alpha$ kpc, blue = BaryMP. Intensity scale matches Fig. 5.3.

**Figure 5.5:** Stars (left) and cold gas (right) of an example subhalo from MassiveBlack-II at $z = 0.0625$ with integrated masses from each particle species comparable to those in the M12 simulations. Frame width, height, and depth are each $R_{\text{vir}}$. Intensity scale is identical to Figs. 5.3 and 5.4. Colour code for apertures is: green = $R_{\text{i}25}$, magenta = $0.1R_{\text{vir}}$, red = $0.15R_{\text{vir}}$, white = $30h^{-1}\alpha$ kpc, blue = BaryMP.
Results

5.3. Results

because it has been popular in the literature to assume the properties of galaxies and their parent (sub)haloes are equivalent (cf. Table 5.1), but also as it shows what each aperture technique says about the baryons surrounding the galaxy. For relatively isolated systems (e.g. the M12 galaxies), the baryons that occupy the outer regions of (sub)haloes represent streams of material and star clusters stripped from former satellites. For systems in dense environments, these particles are also representative of the observed intracluster light (see, e.g., Section 2.9; Gonzalez et al. 2005).

5.3.1 Relative measurements for Milky Way-mass systems

We first address Milky Way-mass systems by focusing on measurements from the M12 simulations with a supporting subsample of the MassiveBlack-II galaxies. This latter subsample includes subhaloes with stellar and gas masses each in the same range as the M12 simulations: $m_\ast \in [3, 25] \times 10^{10} \, M_\odot$, $m_{\text{gas}} \in [0.85, 8.5] \times 10^{10} \, M_\odot$ (hot + cold). In total, this provided 282 supporting subhaloes.

Figs. 5.6–5.9 present the primary results of this subsection. Each of these figures contains three panels. The (a) panels show the average of the relative measurements from the full suite of M12 simulations for each technique at each snapshot. The (b) panels show the probability distribution functions for the M12 simulations for each technique for the snapshot nearest $z = 0.0625$. These distributions are generated using a Gaussian kernel density estimator, with a bandwidth obtained from Scott’s rule (Scott 1992). The (c) panels provide equivalently generated distributions from the MassiveBlack-II Milky Way-mass subsample. Although some distributions may appear to continue outside the plotted $x$-axis range, it physically makes no sense for radii to be lower than 0, nor does it for any of the other properties to be outside $[0, 1]$. As such, each distribution is normalised to have an area of 1 within the physically meaningful boundaries, and is effectively cut at these boundaries. Direct aperture techniques are represented by dashed curves while those that included a subhalo finder (and excluded hot gas) are given by solid curves. Colours are associated with aperture techniques as defined in the legends. Each of these figures is analysed in turn in the following subsidiary subsections.

We summarise results from this subsection and Section 5.3.2 in Table 5.3. There, we provide the means and standard deviations for each of the datasets used to generate the distributions presented in Figs. 5.6–5.11.
Chapter 5. Where do galaxies end?

Figure 5.6: Aperture radii for each technique normalised to the systems’ virial radii. Panel (a): Average of the M12 simulation sample at each redshift. Panel (b): Probability distribution function generated from a Gaussian kernel density estimator for the normalised radii of the M12 galaxies at the snapshot nearest to \( z = 0.0625 \). Panel (c): Equivalent distribution for MassiveBlack-II subhaloes at \( z = 0.0625 \) for a subsample of 282 galaxies with comparable gas and stellar masses to the M12 galaxies. Dashed lines represent the direct aperture techniques, while solid lines indicate the additional use of a subhalo finder and omission of hot gas. Colours correspond to aperture techniques as follows: magenta = \( 0.1 R_{\text{vir}} \), red = \( 0.15 R_{\text{vir}} \), green = \( R_{i25} \), black = \( 30h^{-1}a \) kpc, blue = BaryMP. See Section 5.3.1.

Aperture radii

To put the measurements of the integrated properties in context, the aperture radii from each technique are first presented in Fig. 5.6. All measurements are normalised to \( R_{\text{vir}} \), calculated using only the particles identified by the relevant subhalo finder.

From Fig. 5.6a, it is evident that, for the M12 simulations, the average \( R_{i25} \) radius (green line) is practically equivalent to \( 0.15 R_{\text{vir}} \) (red lines), regardless of redshift. Fig. 5.6b shows that the data producing the \( R_{i25}/R_{\text{vir}} \) distribution come with \( \sigma = 0.03 \) at low redshift, though. The \( 0.15 R_{\text{vir}} \) technique hence provides a rough approximation for the optical limit of these simulated galaxies. The same can not quite be said for the MassiveBlack-II systems however; while they carry the same variance for the \( R_{i25}/R_{\text{vir}} \) distribution, the mean is smaller (cf. Fig. 5.6c, Table 5.3).

From the blue distributions in Figs. 5.6b and 5.6c, we see again that BaryMP radii can cover a wide range of fractions of (sub)haloes’ virial radii. Over all examined redshifts for the M12 galaxies, the mean of the \( R_{\text{BaryMP}}/R_{\text{vir}} \) distribution does vary, but remains approximately between 0.3 and 0.4 (dropping below 0.3 at \( z = 0 \), consistent with the discussion in Section 5.2.3; see Fig. 5.6a).

Comparison of Figs. 5.6b and 5.6c shows that the average M12-analogue in MassiveBlack-II returns radii at lower fractions of \( R_{\text{vir}} \) for the fixed (comoving) aperture technique, while returning higher fractions for BaryMP. The former point suggests the M12 galaxies occupy smaller haloes

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\footnote{With the exception of gas mass for direct aperture techniques, as they count hot gas particles. As such, those distributions were not renormalised. In practice, this has negligible impact on our results.}
5.3. Results

Figure 5.7: Integrated stellar mass for each aperture technique normalised to the value for the full substructure-stripped (sub)halo. Panel (a): Average values for the M12 systems as a function of redshift. Panel (b): Probability distribution functions for the M12 galaxies for the snapshot closest to \( z = 0.0625 \). Panel (c): Equivalent distributions for the Milky Way-mass MassiveBlack-II systems. Differences between techniques are of order tens of percent, and can be as large as a factor of 2. Further details in Section 5.3.1. Plotting conventions maintained from Fig. 5.6.

on average, while the latter indicates that the MassiveBlack-II galaxies are less concentrated than the M12 ones. The latter point is also supported by \( R_{25} \) values being larger for M12 systems. We note that while the distribution for BaryMP in Fig. 5.6b appears to reach radii of zero, there are no instances where this actually happens; this is merely a result of the extrapolative nature of kernel density estimation.

Stellar mass

Stellar mass measurements are presented in Fig. 5.7. Over all redshifts, the 0.15\( R_{\text{vir}} \) and \( R_{25} \) apertures, on average, predict stellar masses \( \sim \)10% less than that of the full halo for the M12 simulations. The 0.1\( R_{\text{vir}} \) technique naturally shows a larger average difference, nearing 20% at \( z = 0 \) and 2. The MassiveBlack-II systems return lower average stellar masses for 0.1\( R_{\text{vir}} \), 0.15\( R_{\text{vir}} \), and 30\( h^{-1} \)a kpc than those of M12, consistent with the M12 galaxies being more concentrated (and in smaller haloes). The shapes of the distributions for these techniques are almost identical for the different simulations, though (cf. Figs. 5.7b and 5.7c).

Comparing the two most popular techniques in the literature (0.1\( R_{\text{vir}} \) and no aperture; cf. Table 5.1), an average MassiveBlack-II galaxy shows a difference in stellar mass of 32%. At the lower extreme of the relevant distribution (magenta, Fig. 5.7c), the potential for a factor-of-two difference in stellar mass is seen.

For the M12 simulations, the measured stellar mass from BaryMP is never more than a few percent less than the full halo. The larger sample of MassiveBlack-II galaxies indicates there is still the potential to find up to a 30-per-cent difference with the application of BaryMP versus the full subhalo, however (lower extreme, blue distribution, Fig. 5.7c).
Chapter 5. Where do galaxies end?

Figure 5.8: As for Fig. 5.7 but displaying the integrated gas mass instead. For techniques that used a subhalo finder, only cold gas particles contributed, while direct aperture techniques included hot gas. Large differences are evident from the various apertures, with the potential to exclude cold gas entirely. More analysis in Section 5.3.1.

Gas mass

The radial density gradient of gas in galaxies is typically noticeably shallower than it is for stars, especially towards their centres. As such, there is a greater variation in measured gas mass from each technique, as shown in Fig. 5.8. Comparing the application of 0.1Rvir and no aperture (the two most popular and yet most contrasting techniques), for the average M12 simulation, there is factor of $\sim 3$ difference in the measured gas mass at high redshift, coming closer to 2 at low redshift. At $z \sim 0$, the greatest individual difference for the M12 galaxies is a factor of 5, while 23% of MassiveBlack-II galaxies show differences at least this large (see magenta distributions in Figs. 5.8b and 5.8c, respectively).

As for other properties, the BaryMP gas masses do not deviate excessively from the full M12 halo values (blue distribution, Fig. 5.8b). The MassiveBlack-II systems show a much wider range of BaryMP gas masses, however, with cases beyond a factor-of-two difference to the full subhaloes. Fig. 5.8a also shows the aperture to play a larger role at higher redshift.

Star formation rate

The integrated star formation rate measurements for the M12 galaxies, displayed in Fig. 5.9, were calculated by summing the mass of all star particles identified by a given technique that were also identified as being gas particles in the galaxy in the previous snapshot, and dividing this value by the time-step. In truth, this means each value is the average star formation rate over the 375 Myr prior. At high redshift, most of the star formation occurs in a small, central region, meaning most

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10 This is a natural consequence of stars forming from gas and only doing so in the densest regions. Visually, this can be readily seen from Fig. 5.3 (but also from Figs. 5.4 and 5.5), noting the difference in intensity scale given in the caption. This has also been empirically shown to be the case with H1 + H2 observations (e.g., see appendix F of Leroy et al. 2008).
techniques return similar values. With the formation of discs, the star-forming region extends, with $>20\%$ of star formation within the halo occurring beyond $0.1R_{\text{vir}}$ for the average M12 system. For individual systems, values are shown to vary by up to and above $60\%$ for different techniques for the M12 galaxies (magenta distribution, Fig. 5.9b).

Following Springel & Hernquist (2003), each gas particle in MassiveBlack-II has a tracked star formation rate. Integrated values for each galaxy were calculated by summing these. In truth, this provides the forecasted star formation rate over the next simulation time-step, rather than the rate at which stars actually formed in the end. Regardless of the differing definition to the M12 galaxies, each technique (with perhaps the exception of BaryMP) shows similar distributions for the two simulation sets. For the majority of these MassiveBlack-II galaxies, star formation is confined to a more central region, leading to star formation rate measurements showing less susceptibility to aperture technique than gas mass.

In some cases for MassiveBlack-II, however, the choice of aperture can exclude the star-forming region entirely. We note that if a system had no star formation within the entire subhalo, that system did not contribute to any of the distributions presented in Fig. 5.9c.

*Gas accretion and ejection*

Two properties that show an even greater dependence on measurement technique are the gas accretion and ejection rates. We present relative measurements for the M12 galaxies in Fig. 5.10, where the respective rates were determined by tracking which gas particles were considered part of the galaxy in temporally adjacent snapshots. Once again, only cold gas particles were counted for the techniques that used AHF. No separation between steady accretion and mergers was made. The order of which techniques give the largest differences remains similar to the other properties at high redshift, but is reversed at low redshift for these plots. This is consistent with the notion that,
at early epochs, haloes accumulate cold gas rapidly from their surroundings, where much of this gas penetrates to the central galaxy in cold streams. Shortly after, as the haloes’ masses increase, the majority of gas is shock-heated and sits in the halo (Rees & Ostriker 1977). At lower redshift, less gas shifts through the boundary of the virial sphere, while the gas inside it continues to cool onto the galaxy, and feedback mechanisms cause outflows which repopulate the halo gas content. As a rule of thumb, at low redshift, the closer to the centre of the galaxy one defines a surface, the faster the gas will be measured to pass through that surface in both directions. Differences of up to and above an order of magnitude in gas transfer rates emphasise the importance of how the end of galaxies is defined. The technique-to-technique difference varies strongly from galaxy to galaxy (see Table 5.3). A study of the gas accretion and ejection rates in MassiveBlack-II systems fell beyond the scope of the project.

5.3.2 Relative measurements for a broad galaxy population

We present results for the full MassiveBlack-II sample with four plots in Fig. 5.11. All comments in this subsection are made with respect to this figure.

The range of radii relative to the virial radius for the BaryMP technique for this large sample of simulated galaxies is consistent with the Milky Way-mass sample, indicating that the technique is unbiased toward subhalo baryonic mass (cf. blue distributions in Figs. 5.6c and 5.11a). The hugely variant size of subhaloes in this sample leads to the fixed aperture returning a very wide distribution when normalised to the virial radius (black histogram, Fig. 5.11a), with the aperture larger than \( R_{\text{vir}} \) itself on some occasions, highlighting the inappropriateness of broadly applying such a technique. While \( R_{25} \) radii are comparable to \( 0.1R_{\text{vir}} \) for the Milky Way-mass systems, the optical limit for the general MassiveBlack-II galaxy is much smaller (cf. green distributions in
5.3. Results

Figure 5.11: Integrated properties of a general galaxy population (covering 4 orders of magnitude in stellar and gas mass) relative to the properties of their parent subhaloes: (a) aperture radius, (b) stellar mass, (c) gas mass, (d) star formation rate. Each panel is similar to the (b) panels of Figs. 5.6–5.9 but covers the full MassiveBlack-II sample ($2.2 \times 10^5$ galaxies) and shows the actual binned values. With the exception of stellar mass, overall, this broader galaxy population shows lower aperture–dependence on integrated-property measurements (analysis in Section 5.3.2). Note that the y-axis in panel (d) is zoomed in significantly further than the others.

Figs. 5.6c and 5.11a).

Stellar mass measurements, presented in Fig. 5.11b, show a relatively strong dependence on technique, with $0.1R_{\text{vir}}$ averaging 25% less mass than the full subhalo, with the tail of the magenta distribution maintaining a potential for differences of 50%. $R_{25}$ stellar masses show even more extreme cases, where almost all the star particles in the subhalo can be excluded by the aperture. The distributions from $0.1R_{\text{vir}}$ and $0.15R_{\text{vir}}$ exhibit similarities to the Milky Way-mass sample, but with higher averages (cf. Figs. 5.7c and 5.11b). Given that the small, low-mass systems vastly outnumber the high-mass ones, the largeness of an aperture of radius $30h^{-1}$ a kpc leads to stellar masses barely smaller than the full subhalo, on average. Meanwhile, BaryMP typically returns stellar masses 4% lower than the full subhalo, but can be 20% lower in the extreme cases.

Conversely, with the exception of $R_{25}$, gas mass shows relatively little variation with technique (Fig. 5.11c) for this broader sample. This suggests that the cold gas in the average galaxy is more tightly concentrated than in Milky Way-mass systems. Still, 30% of galaxies for $0.1R_{\text{vir}}$ and 14.5% for $0.15R_{\text{vir}}$ give gas masses $\geq$ 20% less than when no aperture is used, while half the subhaloes have at least 37% of their cold gas excluded by the $R_{25}$ aperture. There is also a
Chapter 5. Where do galaxies end?

noticeable population of galaxies with gas masses of zero for all the techniques. In these cases, the gas in the subhalo is offset from the stellar centre. Processes such as ram-pressure stripping can be held responsible for this.

The star formation rate distributions are effectively scaled-down versions of cold gas mass (Fig. 5.11d); only the very dense gas can form stars. This property is scarcely subject to technique for galaxies in general, with, for example, only 4\% of the sample returning values \geq 5\% less than the \textsc{subfind} results for the 0.1\( R_{\text{vir}} \) aperture. There remains a small population of systems with measured star formation rates of zero from several techniques, corresponding to the subhaloes where the (star-forming) gas is off-centre. We note again that systems where the entire subhalo had no star formation were omitted from this analysis (i.e. there is star formation occurring within the subhaloes but outside the apertures for those examples).

5.3.3 Galaxy scaling relations

Observationally, galaxy properties are known to follow certain scaling relations. Often such relations can provide tests for how well a simulation has reproduced the real Universe. We find it informative to check whether a simulation’s ability to match these relations depends on the choice of technique for measuring the relevant galaxy properties.

Stellar mass–star formation rate relation

A redshift-dependent power-law relation has been observed between stellar mass and star formation rate (e.g. Daddi et al. 2007; Elbaz et al. 2007; Noeske et al. 2007). Studying blue galaxies at low redshift, with a linear fit in log-log space, Elbaz et al. (2007) showed

\[
\frac{\text{SFR}}{M_\odot \text{yr}^{-1}} = 8.7^{+7.4}_{-3.7} \left( \frac{m_\ast}{10^{11} M_\odot} \right)^{0.77}, \quad 0.015 < z < 0.1 .
\]  

(5.5)

We compare stellar masses and star formation rates for each aperture technique listed in Table 5.2 (with subhalo finders applied where listed) with Fig. 5.12, where contours represent the \textit{MassiveBlack-II} systems and dots the M12 galaxies. The two M12 snapshots in the quoted redshift range of Eq. 5.5 were considered simultaneously.

For each technique, the M12 simulations show good agreement with the range suggested by Eq. 5.5, with root-mean-squared (RMS) scatter values <0.27 dex from the centre of the yellow strip. The best-fitting slopes from these data alone vary little from aperture to aperture, but do differ from the Elbaz et al. (2007) value, with values between 1.04 and 1.09.

For the \textit{MassiveBlack-II} galaxies, we only considered those with \( m_\ast > 5 \times 10^8 M_\odot \), matching the mass range used in the Elbaz et al. (2007) fit. Fig. 5.12 shows a clear offset between the
Figure 5.12: Stellar mass–star formation rate relation for each aperture technique applied with a subhalo finder where possible (see Table 5.2). The distributions of MassiveBlack-II galaxies are shown with contours, which cover 25% and 68% of the galaxies. Individual dots represent the M12 galaxies for each of the two snapshots that fall in the redshift range of Eq. 5.5, while the equation itself is shown by the yellow shaded region. Aperture techniques follow the colour-coding convention of the previous figures. Displayed straight-line fits to each dataset show a lack of dependence on aperture technique to reproduce this scaling relation.
Table 5.3: Summary of the datum samples that produced the distributions presented in Figs. 5.6–5.11, i.e. for the \( z = 0.0625 \) or nearest available snapshots. Mean, \( \mu \), and standard deviation, \( \sigma \), values are provided for each technique applied to each simulated-galaxy sample for each property. Each aperture has been applied with a subhalo finder, except in the cases of \( 0.15R_{\text{vir}} \) and \( R_{i25} \) for the M12 systems (as per Table 5.2). Non-applicable entries are filled with horizontal lines. These values can be used to quantify, with uncertainties, how different techniques compare at measuring galaxy properties. We note that the \( \mu \) and \( \sigma \) values for the distributions presented in Figs. 5.6–5.9 vary slightly (<10%) to the values given here, which use the actual data.

<table>
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<th>Property</th>
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<th>( 0.15R_{\text{vir}} )</th>
<th>( R_{i25} )</th>
<th>( 30h^{-1}) a kpc</th>
<th>BaryMP</th>
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<td></td>
<td></td>
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<td>( \sigma )</td>
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observed relation and the MassiveBlack-II galaxies, which exists for all techniques, with an RMS deviation of order 1.5 dex. The best-fitting slopes match the M12 systems almost identically. For both simulation sets then, there is no distinct technique that shows superior agreement for this scaling relation.

The relation derived by Elbaz et al. (2007) was done with blue galaxies, defined by colour index $U - g < 1.45$. However, an equivalent cut for the MassiveBlack-II galaxies does not produce a conducive comparison, as at low redshift, they are all red; using theoretical spectra for each MassiveBlack-II subhalo (for how these were derived, see Khandai et al. 2015), we found only $\sim$2% of galaxies to have $U - g < 1.45$. The fact that all the galaxies are red is consistent with their low (specific and standard) star formation rates. Hydrodynamic simulations have been known to produce galaxies with low star formation rates and high stellar masses at low redshift; for example, see Kereš et al. (2009b), who show that by normalising simulated galaxy masses to match the observed stellar mass function, one can also match the observed stellar mass-specific star formation rate relation (both their simulation and MassiveBlack-II produce too many low-mass and high-mass systems).

Kennicutt–Schmidt relation

The Kennicutt–Schmidt relation is a well-known scaling relation, where the average star formation rate surface density goes approximately as a power law of the average gas surface density, i.e. $\Sigma_{\text{SFR}} \propto \Sigma_{\text{gas}}^n$. Specifically, Kennicutt (1998) empirically determined the relationship

$$\frac{\Sigma_{\text{SFR}}}{M_{\odot} \, \text{yr}^{-1} \, \text{pc}^{-2}} = \frac{2.5 \pm 0.7}{10^{10}} \left( \frac{\Sigma_{\text{gas}}}{M_{\odot} \, \text{pc}^{-2}} \right)^{1.40 \pm 0.15} \ .$$

We note that Eq. 5.6 was derived taking galaxies to end at their optical limit. Nevertheless, we find it informative to assess whether using alternative definitions for the radial extent of a galaxy affects how well simulations match the observed relation or whether they produce an alternative equivalent relationship. We further note that, in reality, there are more physically motivated ways to relate gas surface density with star formation that return better correlations than Eq. 5.6 (see Section 3.1 of this thesis; Federrath 2013; Salim et al. 2015), but our test here is important for historical context.

We have combined measurements of star formation rate, gas mass, and aperture radius from each technique in Table 5.2 to produce $\Sigma_{\text{gas}}$ and $\Sigma_{\text{SFR}}$ values, as plotted in Fig. 5.13. For those techniques plotted, subhalo finders were included where possible (denoted in Table 5.2). The surface area in each case was taken to be $\pi R_{\text{aper}}^2$. We only considered cold gas to contribute toward $\Sigma_{\text{gas}}$, even for the direct aperture techniques. For clarity, we use dots to only show the $z = 0$ case.
Figure 5.13: Kennicutt–Schmidt relation for each aperture technique (with subhalo finders applied where listed in Table 5.2). Dots represent the M12 galaxies at $z = 0$. MassiveBlack-II galaxies are represented with contours, which include 25% and 68% of the galaxies. The orange shaded region follows Eq. 5.6. The best-fitting lines to each dataset (which included all snapshots of the M12 galaxies where $z < 2$) indicate technique-independence for falling within the shaded region, but that the derived slope can vary with the choice of technique.

For the M12 galaxies in Fig. 5.13, but data from all $z < 2$ snapshots were combined when fitting the straight-line relationships. Again, contours represent the MassiveBlack-II galaxies.

For the M12 galaxies, we found the best-fitting power-law slope, $n$, to be between 1.17 and 1.58 for the techniques listed in Table 5.2. The slope of the relation for the M12 galaxies varies negligibly for a given aperture applied with and without the additional use of AHF. The MassiveBlack-II systems instead give values of $n$ between 0.86 and 1.02. The M12 systems show a much smaller level of scatter, with RMS deviations $\lesssim 0.20$ dex, while the MassiveBlack-II systems instead show an RMS of order 0.5 dex for each technique. While the best-fitting slopes in many cases do not fall in the quoted range of Kennicutt (1998), we found the RMS for a forced fit of $n = 1.4$ to be scarcely different for when $n$ was fitted in each case ($< 10\%$ increase), with the M12 galaxies also showing general agreement with Eq. 5.6 for each technique.

Evident from Fig. 5.13, the average MassiveBlack-II galaxy has a low $\Sigma_{\text{gas}}$ compared to the M12 galaxies (and observed galaxies, for that matter). This is consistent with the galaxies’ low concentration. However, spreading gas and star formation over a larger area will not maintain agreement with the Kennicutt–Schmidt relation unless the overall star formation rate decreases too (as $n > 1$). As discussed in Section 5.3.3, these galaxies do have low star formation rates. As such, their offset from the observed Kennicutt–Schmidt relation is small (and less than from the observed stellar mass–star formation rate relation).
5.4 Discussion

5.4.1 Sizes of the simulated galaxies

To claim our results impact the post-processing of hydrodynamic simulations in general, one should address whether our sample of simulated galaxies is representative. If the ‘full stellar mass’ is calculated using all the star particles within a (sub)halo (no substructure), then, for stellar half-mass radii of the M12 galaxies, we find $0.021 < R_{1/2}/R_{\text{vir}} < 0.041$ at $z = 0$. These values fall within the observed range of Kravtsov (2013, Eq. 5.2 in this thesis). While this means the sizes of the galaxies are physically reasonable (for more on the sizes of the galaxies, see Martig et al. 2012), it does suggest our sample is biased by lower-concentration galaxies compared to an observed sample. Consistent with the notion that the MassiveBlack-II galaxies are even less concentrated, $R_{1/2}/R_{\text{vir}}$ values for MassiveBlack-II are larger than M12 on average, with most larger than observed galaxies.

It should be noted that the differences in measured properties from the techniques that invoke an aperture of a fixed fraction of the virial radius versus the other techniques would be less extreme in a sample of more-concentrated galaxies. However, if a given technique fails to capture a simulated galaxy in the less-concentrated instances, it fails to be generally applicable. After all, if a simulation produces all low-concentration galaxies, the properties of those galaxies will likely not be discarded, as they still have scientific merit. Their low concentration may not be noted in the published results either. The fact that the concentration of simulated galaxies changes for different redshifts, formation histories, and the design of the simulation itself, reinforces the notion that a fixed fraction of the virial radius can not define the general extent of galaxies. Ultimately, the M12 and MassiveBlack-II galaxies provide as fair a tool as any to compare the aperture techniques.

5.4.2 Which method should one choose?

As outlined in Section 5.2, we suggest that if one wants to study the evolution of a galaxy in a simulated (sub)halo, particles that are attached to any form of substructure, or that are diffusely spread throughout the (sub)halo, should not contribute. Even for single-halo simulations, we therefore encourage using a subhalo finder as a first step, and subsequently the exclusion of hot, low-density gas. We further encourage an aperture technique, not only to discern galaxies from the rest of their parent (sub)haloes, but also because no subhalo finder is perfect at locating all substructure to be removed (as seen in Fig. 5.4).

It is clear some of the aperture techniques assessed in this chapter failed to encompass a reasonable fraction of the galaxy. As already discussed, $0.1R_{\text{vir}}$ and $0.15R_{\text{vir}}$ poorly define the edge of a galaxy, due to the variation in the galaxy-to-virial radius fraction for (sub)haloes. Despite
these techniques’ popularity, we advise against them.

While a technique such as \( R_{25} \) is fair for comparison to some observations, the additional level of stellar-population modelling required and the technique’s common trait of cutting out a legitimate portion of the galaxy make it arguably less than ideal for an equally footed technique across all simulations. Because stellar mass and brightness track each other to first order, a technique that invokes an absolute cut-off in stellar surface density would prove to similar effect, with simpler implementation, and would not force observational constraints which impede measuring a galaxy’s true integrated properties.

We suggest the most sensible approach for a technique to measure any and all galaxy properties self-consistently is one that includes an aperture whose radius is directly determined by the baryonic content within each respective (sub)halo. While the suggestion of Figs. 5.7, 5.8, and 5.11 is that the additional application of such an aperture, BaryMP, would alter values of gas and stellar mass by only a few percent on average, there are a sufficient number of instances where a greater difference occurs for this to have a significant effect. Gas accretion and ejection rates also show a large difference with the application of the aperture versus none. BaryMP comes with an additional advantage of being unaffected by straggling satellites not identified by the subhalo finder (see Appendix E).

It is totally within reason that the scope of one’s study might not mean that a highly detailed analysis on which particles are part of the main galaxy of a (sub)halo is necessary. If, for example, one desired to measure just the integrated stellar mass of a Milky Way-like simulated galaxy, then any technique used in this chapter would be appropriate to return an answer approximately with a 20% uncertainty. In such a case, to minimise computation and effort, a direct aperture technique would be sensible.

5.5 Conclusion

When studying the gross properties of galaxies from hydrodynamic simulations, the first step is to determine which particles/cells should contribute to those properties. Throughout the published literature, a number of different techniques has been used for this step. Using a subsample of the high-resolution simulations of Martig et al. (2012) and a selection of galaxies from the MassiveBlack-II simulation, the integrated properties of galaxies were measured using a range of different techniques, including several from the literature. These techniques include the use of a subhalo finder, temperature constraint for gas, and/or spherical apertures, the latter defining where galaxies end.

Comparison of the two most popular techniques in the literature [an aperture of \( 0.1 R_{\text{vir}} \) versus the entire (sub)halo with substructure removed] shows differences in the average Milky Way-mass
system of order 30% for stellar mass, a factor of 3 for gas mass, and 40% for star formation rate. Gas accretion and ejection rates are the most susceptible properties to the choice of technique, with variations of an order of magnitude not unlikely. The choice of technique is hence key to the interpretation of simulation results. Our Table 5.3 further details these differences and the standard deviation of the distributions that go alongside them.

For a more general population of galaxies (stellar and gas mass each between \(10^8\) and \(10^{12}\) \(M_\odot\)), gas mass and especially star formation rate measurements exhibit less susceptibility to technique, assuming only cold gas is considered, but can still show significant differences for systems where the gaseous and stellar centres of mass are offset. Stellar mass, on the other hand, shows a similar level of variation to the Milky Way-mass population, with the exception of the observationally motivated \(R_{25}\) aperture.

Among the techniques we compared, in attempt to separate a galaxy from its diffuse baryonic surroundings in a physically motivated manner, we defined a new aperture technique, whose radius is determined from the cumulative (cold) baryonic mass profiles of (sub)haloes. While, on average, each integrated property measurement was only a few percent less than that of the full (sub)halo, the distributive nature of these differences highlights the importance of applying such an aperture to obtain universally fair values. The average gas mass and star formation rate measurements were also significantly less than the full (sub)halo for the Milky Way-mass MassiveBlack-II systems (cf. Table 5.3).

While the choice of technique for the measurement for individual integrated properties is important, we have shown that scaling relations do not rely heavily on technique to match observations. One can, however, derive a difference in the slope of the Kennicutt-Schmidt relation of order 25% using a single simulation set based on the various measurement techniques we have assessed.

For the sake of clarity, we encourage all authors to be explicit in how they have defined galaxies within their simulations, but more importantly to discuss and justify their choice of technique. Our analysis sheds light on the uncertainties associated with comparing results of simulations that use different techniques to measure galaxy properties. While there may not be a perfect technique, we have shown that apertures defined by a constant fraction of the virial radius do not succeed on a general basis, and better alternatives exist.

While the question ‘Where do galaxies end?’ is difficult to answer, it is an important question to ask if we are to have a common understanding of the properties of galaxies, both from simulation and observation perspectives, which ultimately impacts our understanding of the underlying astrophysics that generates these properties.
Angular momentum in black-hole accretion discs

It has long been widely accepted that active galactic nuclei are powered by gravitationally liberated energy from accretion discs around massive black holes (Lynden-Bell 1969). While the study of accretion is an interesting prospect in itself, the feedback it entails is significantly consequential for the evolution of galaxies as well (e.g. Di Matteo et al. 2005). To truly gauge the effect of feedback requires highly detailed cosmological, hydrodynamic simulations that self-consistently track the growth of black holes and the emission from their accretion discs. However, even the most state-of-the-art simulations presently (e.g. Vogelsberger et al. 2014; Schaye et al. 2015) are unable to resolve accretion discs, and must use subresolution models to describe their physics (e.g. Springel et al. 2005b; Booth & Schaye 2009), in a similar vein to semi-analytic models (e.g. Croton et al. 2006; Benson 2012). An analytic understanding of the functioning of accretion discs is hence key for this cause.

In the simple picture of a thin accretion disc, particles quasi-statically shrink on equatorial, circular orbits, where pressure forces are assumed negligible, until they reach the orbit of lowest energy, after which they are assumed to be captured by the black hole (Lynden-Bell 1969; Bardeen 1970). In doing so, those particles must be liberated of their angular momentum. Either angular momentum is lost through the disc out to higher radii, or it is emitted vertically and removed from the disc entirely. The latter occurs naturally through thermal emission of the disc and through scattering of photons if the disc is subject to irradiation from an external source. It is of interest then to assess the contribution of angular-momentum liberation that photons provide in order to better understand the process of accretion itself. This chapter aims to calculate exactly this, using purely general relativistic arguments.

In Section 6.1 of this chapter, relevant mathematical formulae for studying accretion discs are outlined. General relativistic calculations are performed based on these formulae in Section 6.2,
where limiting cases for the liberation of angular momentum via photons, as well as from the standard Novikov & Thorne (1973) disc, are considered. The results are placed in context of the approximations surrounding the thin-disc picture in Section 6.3. Concluding remarks are provided in Section 6.4.

6.1 Mathematical formalisms and background

The unique metric for spacetime around a (non-charged) rotating source mass (e.g. a black hole) was first discovered by Kerr (1963), usually now written in Boyer–Lindquist coordinates (Boyer & Lindquist 1967, note that the $r$ coordinate has a different meaning to the use of $r$ as a two-dimensional radial distance as in the other chapters of this thesis), for which the invariant interval is

$$
-c^2 dt^2 = \left(1 - \frac{R_s r}{\rho}\right) c^2 dt^2 + \frac{\rho}{\Delta} dr^2 + \rho d\theta^2 + \left(r^2 + \alpha^2 + \frac{R_s r \alpha^2 \sin^2(\theta)}{\rho}\right) \sin^2(\theta) d\phi^2 - \frac{2R_s r \alpha^2 \sin^2(\theta)}{\rho} c dt d\phi; \quad (6.1a)
$$

$$
\rho \equiv r^2 + \alpha^2 \cos^2(\theta), \quad \Delta \equiv r^2 - R_s r + \alpha^2, \quad (6.1b)
$$

where $c$ is the speed of light, $R_s \equiv 2GM/c^2$ is the Schwarzschild radius, and $\alpha \equiv J/Mc$ is the spin parameter (specific angular momentum) of the source of mass $M$ and angular momentum $J$. The metric is stationary, axisymmetric, and, as Carter (1968) showed (but see also Misner et al. 1973, §33.5), exhibits four constants of motion. Two of these constants are the azimuthal and time components of covariant four-momentum. Taking the limit $r \to \infty$, one finds these to be relativistic analogues of energy and azimuthal angular momentum, usually referred to as the energy and angular momentum ‘at infinity’. Often the ‘at infinity’ is dropped for brevity, and the usual symbols for these quantities are used, i.e.

$$
E \equiv -p_t, \quad L_z \equiv p_\phi. \quad (6.2)
$$

When discussing the emission or transport of energy or angular momentum in accretion discs (or Kerr geometry in general), these are the quantities that are meant.

Throughout the rest of this chapter, most quantities will be expressed in a dimensionless form, represented by a bar placed on the quantity of interest. For quantities with dimensions of distance, this means normalising to half the Schwarzschild radius, e.g. $\bar{r} \equiv 2r/R_s, \bar{\alpha} \equiv 2\alpha/R_s$, in line with

\footnote{In fact, those authors showed this for the more general, charge-inclusive Kerr–Newman metric (Newman et al. 1965).}
6.2. Radiating and scattering away angular momentum

By analysing equations of motion for particles in a Kerr spacetime, Bardeen et al. (1972) obtained expressions for the energy and specific angular momentum for circular (i.e. $p^r = 0$), equatorial (i.e. $\theta = \pi/2$ and $p^\theta = 0$), Keplerian (i.e. gravity is entirely centrifugally balanced) orbits:

$$\bar{E} \equiv \frac{E}{mc^2} = \frac{\bar{r}^{3/2} - 2\bar{r}^{1/2} \pm \bar{\alpha}}{\bar{r}^{3/4} (\bar{r}^{3/2} - 3\bar{r}^{1/2} \pm 2\bar{\alpha})^{1/2}}, \quad (6.3a)$$

$$\bar{L}_z \equiv \frac{L_z}{mcR_s} = \frac{\pm \bar{r}^2 - 2\bar{\alpha}\bar{r}^{1/2} \pm \bar{\alpha}^2}{2\bar{r}^{3/4} (\bar{r}^{3/2} - 3\bar{r}^{1/2} \pm 2\bar{\alpha})^{1/2}}, \quad (6.3b)$$

where upper signs are for prograde orbits and lower signs retrograde. Note that $m$ represents the rest mass of a particle (at infinity), not its inertial mass. By checking the derivatives of these quantities,

$$\frac{d\bar{E}}{d\bar{r}} = \frac{8\bar{\alpha}\bar{r}^{1/2} - 3\bar{\alpha}^2 + \bar{r}(\bar{r} - 6)}{W}, \quad (6.4a)$$

$$\frac{d\bar{L}_z}{d\bar{r}} = \frac{1}{W^{-1}} \left\{ \bar{\alpha}^2 \bar{r}^{1/2} \left( 4 - \frac{3}{2}\bar{r} \right) + \bar{r}^{5/2} \left( \frac{1}{2} \bar{r}^2 - 3 \right) - 3 \left[ \frac{1}{2} \bar{\alpha}^3 - \bar{\alpha}\bar{r} \left( \frac{3}{2} \bar{r} - 1 \right) \right] \right\}; \quad (6.4b)$$

$$W \equiv 2\bar{r}^{7/4} \left[ 2\bar{\alpha} + \bar{r}^{1/2} (\bar{r} - 3) \right]^{3/2}, \quad (6.4c)$$

one finds these two relations share a common minimum, referred to in the literature as the innermost stable circular orbit (ISCO), where $\bar{r}_{\text{ISCO}}$ is given by equation 2.21 of Bardeen et al. (1972).

Under the picture where particles transit between infinitesimally adjacent orbits in the process of accretion (until they reach the ISCO), the above equations provide the starting point for calculating how much specific angular momentum can be lost from photon emission and/or scattering. Hereafter, the use of $E$ and $L_z$ (with or without bars, but without further subscripts) refers to the orbiting states for which these equations (Eqs. 6.3–6.4) apply.

### 6.2 Radiating and scattering away angular momentum

In each of the following subsections, the relative specific-angular-momentum loss to photons in an accretion disc, $-\frac{d\bar{L}_z,\gamma}{d\bar{L}_z}$ (where subscript $\gamma$ is for photons), as a function of radius, is calculated for a different idealised situation, where each builds on the last. Each result is plotted in Fig. 6.1 for $r \geq r_{\text{ISCO}}$, allowing for comparisons between each individual case. The models considered here are all of thin, relativistic accretion discs, whereby the mathematics of Section 6.1 is applicable.

---

2In their notation, $r_{\text{ms}}/M$ and $a/M$ are equivalent to $\bar{r}_{\text{ISCO}}$ and $\bar{\alpha}$ here, respectively.
6.2.1 Pure, relatively isotropic emission

Before considering consequences for other forms of energy transport, the energy carried away by a photon supplied by a particle moving to an adjacent lower-energy circular orbit should, at most, be the energy difference between the orbits. This can be written in terms of differentials as

\[ dE_\gamma = -dE. \]  

If this energy is lost primarily through radiation, one expects each photon to be emitted with statistical isotropy from the frame of the emitting particle. For a non-rotating disc, this would make the average direction of emission from each face of the disc vertical (i.e. initially completely in the \( \pm \theta \)-direction). For a rotating disc perceived from an external frame (i.e. one static with the Boyer–Lindquist coordinates), this can then be modelled by stating that photons are emitted in the \( \phi-\theta \) ‘plane’ with a three-velocity in the \( \phi \)-direction equivalent to that of the disc, naturally a function of radius. This (angular) velocity is found as \( V_\phi = \frac{p_\phi}{p_t} \).

Recognising \( p_t = -g_{tt} E + g_{t\phi} L_z \) and \( p_\phi = -g^{\phi\phi} E + g^{\phi\phi} L_z \) (cf. Eqs. 6.1 & 6.2), obtaining the contravariant metric components by taking the matrix inverse of Eq. 6.1, and expanding and simplifying with Eq. 6.3, one concludes consistently with Bardeen et al. (1972) that

\[ \frac{p_\phi}{p_t} = \frac{\pm (2R_s)^{1/2} c}{2r^{3/2} \pm (2R_s)^{1/2} \alpha}. \]  

Let us write the (contravariant) four-momentum components of an emitted photon as \( dp^\mu \). One can then simultaneously solve

\[ g_{tt} dp^t + g_{t\phi} dp^\phi = -dE_\gamma \]  

and Eq. 6.6 (for the latter, the left-hand side now reads \( dp^\phi / dp^t \)) to obtain explicit functions for \( dp^t \) and \( dp^\phi \). Further calculating \( dp_\phi = dL_{z,\gamma} \), one obtains

\[ \frac{d\tilde{L}_{z,\gamma}}{dE_\gamma} = \frac{\tilde{L}_z}{E} = \frac{\pm \tilde{r}^2 - 2\tilde{\alpha} \tilde{r}^{1/2} \pm \tilde{\alpha}^2}{2 \left( \tilde{r}^{3/2} - 2\tilde{r}^{1/2} \pm \tilde{\alpha} \right)}, \]

consistent with the report of Lynden-Bell (1986, section 3.10). Now through Eq. 6.5, combined with use of Eq. 6.4, one finds an explicit form of \( -d\tilde{L}_{z,\gamma}/d\tilde{L}_z \), as presented by the dot-dashed lines in Fig. 6.1. For non-spinning black holes, the analytic relation simplifies to

\[ \frac{d\tilde{L}_{z,\gamma}(\tilde{\alpha} = 0)}{d\tilde{L}_z} = \frac{1}{\tilde{r} - 2}, \]

which further reduces to the Newtonian case presented by Johnson (2011) for \( r \gg R_s \).
6.2. Radiating and scattering away angular momentum

Figure 6.1: (Specific) angular momentum lost to photons relative to the (specific-)angular-momentum gap between adjacent, equatorial, circular orbits in a relativistic accretion disc as a function of radius. Thick curves apply for an accretion disc around a non-spinning black hole, while thin curves are for a hole spinning with \( \bar{\alpha} = 0.998 \) (the maximum of Thorne 1974). The dot-dashed curves assume photons are emitted with energy equal to the difference of the orbits and are angled to the accretion disc plane such that the \( \phi \)-velocity of the photons matches that of the disc itself (see Section 6.2.1). The solid curves follow the solution of Novikov & Thorne (1973), which include effects of internal torques (outlined further in Section 6.2.2). The dashed curves also account for internal torques, and show the upper limit of scattering, where the momentum imparted on photons is parallel to the \( \phi \)-direction (Section 6.2.3).

Under this picture, one finds that specific-angular-momentum removal by photons is important beyond the percent level out to \(~50R_s\). For non-spinning holes, as particles approach the ISCO, the radiative efficiency of angular momentum approaches 25\%, while for maximally spinning black holes, the efficiency approaches 87\%. Already from this analysis, it is clear, and perhaps unsurprising, that radiation is insufficient by itself to liberate an accretion disc of its necessary specific angular momentum.

6.2.2 Relatively isotropic emission with internal angular-momentum transport

The previous subsection considered a limiting role of photons without any additional form of angular-momentum transport. Because photons are unable to remove all the necessary angular momentum if they remove all the necessary energy, some other mechanism must remove angular momentum, which consequently must alter the energy liberated by photons too.

The standard model of thin accretion discs proposed by Shakura & Sunyaev (1973), and extended to be relativistic by Novikov & Thorne (1973), considers angular momentum to be transported radially through internal torques (generated by magnetically induced ‘viscosity’ – see Sec-
tion 6.3), in addition to removal from photons, in an accretion disc whose structure is completely stable (i.e. not a function of time). If \( \frac{d\bar{L}_z}{dr} \) represents the radial net removal of specific angular momentum from internal torques during a particle transition to an infinitesimally adjacent orbit, then it must hold that

\[
\frac{d\bar{L}_z}{dr} + \frac{d\bar{L}_{z,\text{int}}}{dr} + \frac{d\bar{L}_{z,\gamma}}{dr} = 0.
\]

With some small rearranging, the solution derived purely from the continuity equations of rest mass, angular momentum, and energy by Novikov & Thorne (1973) provides

\[
\frac{d\bar{L}_{z,\text{int}}}{dr} = -\frac{1}{2} \frac{d}{dr} \left( \sqrt{\bar{r}} \mathcal{Q} \right),
\]

\[
\frac{d\bar{L}_{z,\gamma}}{dr} = -\frac{3}{2} \frac{\bar{L}_z}{\bar{r}^2} \mathcal{B} \sqrt{\mathcal{\ell}},
\]

where \( \mathcal{Q} \), \( \mathcal{B} \), and \( \mathcal{\ell} \) are dimensionless quantities that approach unity for increasing \( r \): the reader is referred to equation 5.4.1 of Novikov & Thorne (1973) for their formal definitions.3

As for the previous subsection, \(-\frac{d\bar{L}_{z,\gamma}}{d\bar{L}_z}\) is presented for the Novikov & Thorne (1973) model in Fig. 6.1 as solid lines. For non-spinning black holes, the liberation of angular momentum from photons at low radii is noticeably less effective than the previous limiting case. Faster spinning holes reach a peak efficiency of radiative angular-momentum removal of nearly 40%.

Naively, one may have expected the solid lines in Fig. 6.1 to lie underneath the dot-dashed lines (i.e. the result of Section 6.2.1), because a new mode of angular-momentum transport has been introduced since Section 6.2.1. However, as noted by Shakura & Sunyaev (1973), internal angular-momentum transport provides a net energy source for \( r \gg r_{\text{ISCO}} \), meaning more angular momentum must be liberated by photons, specifically by a factor of 3. This is reconcilable by considering an energy outflow balance equation (i.e. an energy version of Eq. 6.10),

\[
\frac{d\bar{E}}{dr} + \frac{d\bar{E}_{\text{int}}}{dr} + \frac{d\bar{E}_\gamma}{dr} = 0.
\]

Using a combination of Eq. 6.4a, 6.8, & 6.11b, one can determine \(-\frac{d\bar{E}_\gamma}{d\bar{E}}\) to show that indeed this asymptotes to a value of 3. This is plotted in Fig. 6.2 (solid curves). When interpreting Fig. 6.2, one should appreciate that the absolute energy gap between adjacent orbits tends to zero as \( \bar{r} \to \infty \), and that thin discs have higher densities toward their centres.

3Page & Thorne (1974) note a sign error for equation 5.4.1h. Those authors also provide an analytically integrated expression for \( \mathcal{Q} \), but using that provides different results and does not satisfy Eq. 6.10, whereas numerically integrating the equations of Novikov & Thorne (1973) does.
6.2. Radiating and scattering away angular momentum

6.2.3 Scattering with internal angular-momentum transport

If an accretion disc is irradiated by an external source, incoming photons can be absorbed or scattered, causing particles in the disc to lose angular momentum, à la the Poynting–Robertson effect (Poynting 1904; Robertson 1937; Burns et al. 1979). In the absorption case, where the motive absorbing particles re-emit the radiation, the analysis of Section 6.2.2 remains sound. However, it could also transpire that charged particles in the disc anisotropically transfer their energy to the incoming photons via inverse Compton scattering.

An external irradiative source is observationally motivated by X-ray reflection spectra generated by an accretion disc’s corona, which provide a means for measuring black holes’ spins (for a review, see Reynolds 2014). At very high redshift, the cosmic background radiation could also provide an irradiative source (e.g. Fukue & Umemura 1994; Mineshige et al. 1998), although far more modest in temperature. For fast-spinning holes, a notable portion of emitted radiation from accretion discs is expected to fall back on the discs as well (Cunningham 1976). So long as the Thomson regime is applicable in the scattering particle’s reference frame, the scattered photon can have its energy significantly multiplied and be beamed in the direction of the scatterer’s motion as perceived by an external observer (see section 7.1 of Rybicki & Lightman 1979). While the
precise direction of the photon’s change of momentum would depend on its initial energy relative to the scattering particle, one can consider the extreme upper limit where this is the $\phi$-direction, as shown immediately below.

Let us now write the four-momentum components of that imparted on the scattered photon as $dp^\mu$ [it is very important to note that this is the change in photon’s momentum from scattering; when considering radiation, this was the momentum of the (average) photon itself, as it did not exist prior (i.e. it had no initial momentum)]. Because photons have no rest mass, it must hold that $g_{\mu\nu} dp^\mu dp^\nu = 0$, where the $g_{\mu\nu}$ terms are the metric components obtainable from Eq. 6.1, hence

$$g_{tt}(dp^t)^2 + 2g_{t\phi} dp^t dp^\phi + g_{\phi\phi}(dp^\phi)^2 = 0.$$  \hspace{1cm} (6.13)

Simultaneously solving Eqs. 6.7 & 6.13, recognising $d\bar{L}_{z,\gamma} = dp^\phi = g_{t\phi} dp^t + g_{\phi\phi} dp^\phi$ in this maximal limit (cf. Eq. 6.2), and using Eq. 6.5, one obtains

$$\frac{d\bar{L}_{z,\gamma}}{d\bar{E}_\gamma} = \frac{\pm \bar{r} \sqrt{\bar{\alpha}^2 + \bar{r}^2 - 2\bar{r} - 2\bar{\alpha}}}{2\bar{r} - 4}.$$  \hspace{1cm} (6.14)

One can consider taking the same energy lost to photons from Section 6.2.2 but instead using it to kick (scatter) photons in the $\phi$-direction. The relative increase in specific-angular-momentum loss can then be found by taking the ratio of Eq. 6.14 to Eq. 6.8. However, by increasing $d\bar{L}_{z,\gamma}/d\bar{r}$, it must be true that $d\bar{L}_{z,\text{int}}/d\bar{r}$ decreases, in order to satisfy conservation of angular momentum (Eq. 6.10). If $d\bar{L}_{z,\text{int}}/d\bar{r}$ decreases, then so must $d\bar{E}_\text{int}/d\bar{r}$ by an amount found by taking the ratio of Eq. 6.10 to Eq. 6.12 after making the internal-torque terms the arguments for each:

$$\frac{d\bar{L}_{z,\text{int}}}{d\bar{E}_\text{int}} = \frac{\frac{d\bar{L}_z}{d\bar{r}} + (\frac{d\bar{L}_{z,\gamma}}{d\bar{r}})_{6.2.2}}{\frac{d\bar{E}_\gamma}{d\bar{r}} + (\frac{d\bar{E}_{\gamma}}{d\bar{r}})_{6.2.2}},$$  \hspace{1cm} (6.15)

where subscript 6.2.2 implies the quantities as determined from Section 6.2.2. Consequently $d\bar{E}_{\gamma}/d\bar{r}$ must increase from energy conservation (Eq. 6.12), and therefore $d\bar{L}_{z,\gamma}/d\bar{r}$ must be higher than initially calculated. One can iteratively work through these calculations until finding a converged result.\(^4\)

Using the above method, one can calculate the maximum $-d\bar{L}_{z,\gamma}/d\bar{L}_z$ for photon scattering with the effects of internal transport included. This is shown by the dashed lines in Fig. 6.1. Consistent with the above results, under this maximal regime, accretion discs can remain efficient above the percent level for $-d\bar{L}_{z,\gamma}/d\bar{L}_z$ beyond $10^4 R_s$.

As was the case for the standard Novikov & Thorne (1973) disc, $-d\bar{E}_{\gamma}/d\bar{E}$ asymptotically approaches a value of 3 for increasing radii, but does so more slowly. It also exceeds a value of 1

\(^4\)One can update the terms with subscript 6.2.2 in the iterations, but the end result is the same.
6.3. Discussion of the thin-disc picture

at the same radius, as displayed by the dashed curves in Fig. 6.2.

It should again be stressed that the calculations in this subsection are an upper limit. In truth, one should expect the relevant curve on Fig. 6.1 to lie between the solid and dashed ones presented, with a bias towards the former.

6.3 Discussion of the thin-disc picture

The inclusion of internal angular-momentum transport in Sections 6.2.2 and 6.2.3 says nothing of the source of the internal torques. Magnetic viscosity is the favourite source, whereas molecular viscosity would be vastly insufficient (Lynden-Bell 1969; Shakura & Sunyaev 1973; Lin et al. 2013). A full consideration of the effects of magnetic fields on accretion discs would require relaxation of the assumption that particles follow Keplerian orbits in the accretion disc. Radiative viscosity is another possible source of internal torques, but this demands non-Keplerian orbits (Loeb & Laor 1992). Furthermore, assuming Keplerian orbits neglects the possibility of advection-dominated accretion flows (Narayan & Yi 1994). In this sense, the models in Section 6.2 are limited.

One may expect the inner part of accretion discs to rotate in the black hole’s equatorial plane, even if they were initially tilted, due to Lense–Thirring precession (Bardeen & Petterson 1975; King et al. 2005). While warping or even breaking of discs has been demonstrated with post-Newtonian hydrodynamic simulations, this will not occur if the alignment timescale vastly exceeds the accretion timescale (Nealon et al. 2015). Recent general relativistic magnetohydrodynamic (GRMHD) simulations of initially tilted discs suggest this effect is indeed too weak to cause alignment, although the effect is seen more strongly in retrograde discs (Morales Teixeira et al. 2014; Zhuravlev et al. 2014). GRMHD simulations have shown a ‘magneto-spin’ alignment to occur for magnetically saturated discs (McKinney et al. 2013) to account for this, but the Keplerian-orbit picture becomes even less physical in this regime.

Even assuming a thin Keplerian disc, the models presented in Section 6.2 neglect ejection of matter as a means of liberating specific angular momentum. A mechanism for ejection that would also raise the specific angular momentum of the ejected material (and therefore lower that of the material that remains in the disc) would be needed. Shakura & Sunyaev (1973) hypothesised that a disc accreting at super-Eddington rates could cause vertical ejection of matter, but there is little motivation for the ejecta to steal extra specific angular momentum on their way out. Meanwhile, the standard picture of jet formation has the ejecta steal energy from the black hole itself rather than the accretion disc (Blandford & Znajek 1977). Matter ejection is likely negligible for the consideration of this work then.

An implicit assumption in the calculations of Section 6.2 was that discs are sufficiently ex-
tended. Section 6.2.3 suggested a disc extending beyond a few $\times 10^4 R_s$ would be valid for the consideration of angular-momentum loss through scattering. Where thermal emission is more important, calculations in Sections 6.2.1 and 6.2.2 suggest a disc extending a few $\times 10^2 R_s$ would enter the regime where radiation is negligible for angular-momentum loss, and hence the picture within would be valid (cf. Fig. 6.1). Within their uncertainty, observations of microlensed quasars are consistent with accretion discs extending to order $10^2 R_s$ (e.g. Jiménez-Vicente et al. 2012, 2014), while attempts to confirm further extension would be difficult due to the expected relatively low brightness at large radii.

6.4 CHAPTER RECAP

As material accretes onto massive black holes, there must be a process by which specific angular momentum is removed from the system. As accretion discs are known to be bright sources of radiation, the emission of photons provides one channel for this angular-momentum loss. If a disc is irradiated, inverse Compton scattering provides another channel. This chapter has provided calculations of the contribution of angular-momentum liberation through photons in thin, relativistic accretion discs. In addition to situations where transport by internal torques was included, such that the usual conservation laws of physics were satisfied, these calculations included limiting situations where photon emission was responsible for all the necessary energy removal and where photon scattering was angled to remove maximal angular momentum.

On scales up to the order of a hundred Schwarzschild radii, photons remove a small ($>1$) percentage of angular momentum in accretion discs that emit in an expected fashion, but beyond this contribute negligibly. At the absolute most, discs subject to strong irradiation are potentially capable of scattering away angular momentum as efficiently out two orders of magnitude farther from the black hole. The contribution in both cases becomes stronger near the horizon of a fast rotating black hole, especially in the latter case ($\approx 60\%$, cf. thin dashed line, Fig. 6.1). By and large though, angular momentum is transported far more efficiently through the disc internally, rather than liberated from the disc electromagnetically.
Through gravitational attraction, perturbations in the initial density field of the Universe lead to the formation of large-scale structure, wherein haloes that house galaxies are connected through filaments. These haloes tidally induce torques on each other, providing each one with a non-zero angular momentum. As gas cools in haloes, this angular momentum is carried thorough (although not necessarily all of it – see Section 4.3.1), leading to the formation of a rotating disc. The specific angular momentum of this disc determines its size and density, and is inherently tied to the efficiency with which that disc can produce stars. The manner in which angular momentum is distributed further determines the level of stability of a disc, which, if unstable, can lead to a cuspy density profile. Further, gas losing angular momentum at the centres of galactic discs can form an (much smaller) accretion disc around a central supermassive black hole. The efficiency with which angular momentum is lost and accretion occurs determines the level of active galactic nucleus feedback that can halt the cooling process and shut down star formation. The angular momentum of astrophysical discs is thus fundamental to the evolution of galaxies.

In this thesis, a range of analytic and numerical techniques have been used to further the field of galaxy evolution. Two new semi-analytic models of galaxy formation have been presented: one that operates in a classical fashion of dealing with the integrated properties of galaxies, and one that co-evolves the angular-momentum structure of galaxies and deals with disc annuli. In addition to a suite of high-resolution zoom re-simulations of galaxies, the evolution and properties of galaxies in two state-of-the-art hydrodynamic, cosmological simulations have also been studied. In combining these tools with purely analytic considerations of black-hole accretion, this thesis has addressed (i) how galactic discs obtain angular momentum through the cooling of gas, and how this relates to the halo; (ii) how the angular momentum of stellar discs co-evolves with mass, which is strongly regulated by disc instabilities; (iii) how the integrated properties of simulated galaxies can be self-consistently quantified; and (iv) a potential mechanism for removing specific angular momentum from the inner parts of a black hole accretion disc.
Chapter 7. Thesis abridged

The main chapters of this thesis have been presented in five papers, either published in academic journals (Stevens et al. 2014, 2016a; Stevens 2015; Croton et al. 2016) or currently under peer review (Stevens et al. 2016b). Each of these is summarised in the sections below.

7.1 A GOOD RECIPE INCLUDES SAGE

Chapter 2 described a new publicly available codebase for modelling galaxy formation in a cosmological context, the ‘Semi-Analytic Galaxy Evolution’ model, or SAGE for short. SAGE is a significant update to the popular model of Croton et al. (2006) and has been rebuilt to be modular and customisable. The model runs on any N-body simulation whose merger trees are organised in a supported format and contain a minimum set of basic halo properties. The updated physics of the model include: gas accretion, ejection due to feedback, and reincorporation via the galactic fountain; a new gas cooling–radio mode AGN heating cycle; AGN feedback in the quasar mode; a new treatment of gas in satellite galaxies; and galaxy mergers, disruption, and the build-up of intracluster stars.

In this work, the baryonic prescriptions of SAGE were described, for which the free parameters were calibrated by running the model on three N-body simulations: Millennium, Bolshoi, and GiggleZ (Springel et al. 2005a; Klypin et al. 2011; Poole et al. 2015, respectively). With a single parameter set, the model can produce populations of galaxies at \( z = 0 \) that are self-consistent across the simulations, and consistent with observations, in terms of the stellar mass function, Baryonic Tully–Fisher relation, stellar mass–gas metallicity relation, black hole–bulge relation, and star formation rate density history.

There are several challenges that remain for semi-analytic models like SAGE. For example, matching the cooling rates of haloes to observations has proven difficult. In the Croton et al. (2006) model, these were overestimated by \( \sim 1.5 \) dex, whereas with SAGE, after refining the implementation of radio mode feedback, the cooling rates are now underestimated by the same factor (Fig. 2.9). In addition, the model overproduces the number of quiescent galaxies at low stellar mass, and underproduces them at high stellar mass (Fig. 2.11). These are areas which are intended to be addressed with further development of the model. Updates to the public version will be readily available through GitHub. This work has been published in ApJS (Croton et al. 2016).

7.2 THE DARK SAGE SEMI-ANALYTIC MODEL

Classically, semi-analytic models have been designed to tackle galaxy evolution by describing processes in terms of integrated properties. The motivations for this were two-fold. First, simple models are more digestible for humans, faster to run for computers, and can describe key processes
of galaxy evolution to an appreciable degree of accuracy (see Dekel et al. 2013; Mutch et al. 2013b). Second, galaxy surveys with which semi-analytic models were originally compared (e.g. York et al. 2000; Colless et al. 2001; Jones et al. 2004) only recovered the integrated properties of large numbers of galaxies. Now, integral-field unit surveys (e.g. Croom et al. 2012; Sánchez et al. 2012; Bundy et al. 2015) provide structural properties of galaxies in droves, demanding equivalent details of galaxies be considered in models.

Chapter 3 of this thesis presented the new semi-analytic model of galaxy evolution, DARK SAGE, a heavily modified version of the publicly available SAGE code. The model is designed for detailed evolution of galactic discs. Discs are evolved in a series of annuli with fixed specific angular momentum, which leads to predictions for the radial and angular-momentum structure of galaxies. Most physical processes, including all channels of star formation and associated feedback, are performed in these annuli. Discs are initialised with a specific angular momentum equivalent to that of the halo, but evolve in orientation, spin, and size, as fresh gas is supplied from different directions at each time-step, based on the instantaneous properties of the halo. The surface density profiles of both gas and stars in DARK SAGE galaxies are in good agreement with observations (Fig. 3.4). The discs also naturally build a pseudobulge (Fig. 3.5), and once these are considered, the morphologies of galaxies are found to agree well with observations over all resolved mass scales ($m_* > 10^{8.5} M_\odot$; Fig. 3.6).

The main results of Chapter 3 were focussed on predictions relating to the net mass–specific angular momentum relation of stellar discs. The model produces a distinct sequence between these properties in remarkable agreement with recent observational literature (Fig. 3.10). It was found that Toomre disc instabilities play a crucial role in shaping this sequence, where they regulate both the mass and spin of discs. Without instabilities, high-mass discs would be systematically deficient in specific angular momentum by a factor of $\sim2.5$, with increased scatter (Fig. 3.9). Typically, in the model, a galaxy’s stellar specific angular momentum is around 40% its halo’s (Fig. 3.14). Instabilities also appear to drive the direction in which the mass–spin sequence of spiral galaxy discs evolves. With them, galaxies of fixed mass have higher specific angular momentum at later epochs (Fig. 3.11). This work has been published in MNRAS (Stevens et al. 2016a).

7.3 HOW TO GET COOL IN THE HEAT

It can be observationally difficult, or even impossible, to test some of the physical descriptions of galaxy evolution that go into models. For example, when gas cools onto a galaxy, it is generally assumed to carry the same specific angular momentum as the halo. However, the dark matter that makes up haloes is not visible, and thus this assumption cannot be empirically confirmed. The best one can do to motivate physical descriptions of galaxy evolutionary processes like this is to
compare predictions of hydrodynamic simulations with coarser models.

Chapter 4 of this thesis focussed on using the hydrodynamic, cosmological EAGLE simulations to investigate how hot gas in haloes condenses to form and grow galaxies. Actively cooling haloes were selected from the simulations, and the temperature, distribution, and metallicity of their hot, cold, and transitioning ‘cooling’ gas were studied. These findings were placed in the context of semi-analytic models, assessing typical assumptions such as angular-momentum conservation of cooling gas and the exponentiality of freshly cooled discs. To be included in the sample, 64 particles in the central subhalo were needed to cool across a time-step. This criterion led to a sample of Milky Way-like haloes.

The hot-gas density profiles of the EAGLE haloes were found to form a progressively stronger core over time (Fig. 4.5), the nature of which can be captured by a $\beta$ profile that has a simple dependence on redshift (Eq. 4.4). In contrast, the hot gas that actually cools is broadly consistent with a singular isothermal sphere. Both a $\beta$ profile and singular isothermal sphere are commonly adopted to describe the hot gas in haloes in semi-analytic models, where it is further assumed that gas carries a uniform temperature and metallicity (see Section 4.2). However, once the complete profiles for the density, temperature, and metallicity of the EAGLE haloes are considered, the cooling-time profiles are noticeably changed compared to the analytic approximations (Fig. 4.8), suggesting cooling rates in models based on White & Frenk (1991) may be overestimated at low redshift (Appendix D).

A primary result of Chapter 4 was that cooling gas carries a few times the specific angular momentum of the halo and is offset in spin direction from the rest of the hot gas. The gas loses $\sim$60% of its specific angular momentum during the cooling process (Fig. 4.9), generally remaining greater than that of the halo, and is better aligned with the cold gas already in the disc than anything else (Fig. 4.11). Angular-momentum losses were found to be slightly larger when cooling took place onto dispersion-supported galaxies (i.e. with a low $\lambda_R$ parameter).

Finally, for EAGLE, an exponential surface density profile for gas arriving on a disc remains a reasonable approximation, but a cusp is always present, and disc scale radii are larger than predicted by a vanilla Fall & Efstathiou (1980) model. These scale radii are still closely correlated with the halo spin parameter, for which Eq. 4.13 provides an updated prescription that could easily be folded into galaxy formation models. This work is currently under peer review with MNRAS (Stevens et al. 2016b).

### 7.4 Integrated Properties of Simulated Galaxies

An often overlooked question in handling the output of hydrodynamic simulations of galaxy formation is ‘How should the integrated properties of galaxies be best quantified?’ Beyond identify-
ing particles gravitationally bound to (sub)structure, one usually defines an aperture, within which the properties of particles can be summed to give the net property of a galaxy. There has, however, been a lack of consistency in the literature for how to define this aperture, which is problematic for comparing results between simulations and with observations.

Using the suite of high-resolution zoom re-simulations of individual haloes by Martig et al. (2012), and the large-scale simulation MassiveBlack-II, in Chapter 5, a variety of techniques used in the literature were applied to measure the integrated properties of galaxies in the simulations. For the average Milky Way-mass system, the two most popular techniques in the literature (which use an aperture of $0.1 R_{\text{vir}}$ and no aperture at all) were found to return differences of order 30% for stellar mass, a factor of 3 for gas mass, 40% for star formation rate, and factors of several for gas accretion and ejection rates. Individual cases showed variations greater than this, with the severity dependent on the concentration of a given system. The average differences in integrated properties for a more general galaxy population ($m_\ast > 10^8 \, M_\odot$) were not found to be as striking, but are still significant for stellar and gas mass, especially for optical-limit apertures. Technique choice was not found to greatly affect simulated galaxies from lying within the scatter of observed scaling relations, such as the stellar mass–star formation rate relation, but it did alter the derived best-fit slope for the Kennicutt–Schmidt relation.

To help alleviate inconsistencies, a new aperture technique was introduced in Section 5.2.3 (and is detailed further in Appendix E), which defines the end of a galaxy to be where the gradient of the cumulative baryonic mass profile becomes constant. This method worked consistently for the high-resolution simulations and MassiveBlack-II (cf. Figs. 5.4 & 5.5), highlighting that it is not dependent on the simulation having a particular resolution, nor does it require the sub-grid physics of the simulation to produce galaxies of a certain concentration. This work has been published in MNRAS (Stevens et al. 2014).

## 7.5 Liberating angular momentum from accretion discs

A key component of explaining the array of galaxies observed in the Universe is the feedback of active galactic nuclei, each powered by a massive black hole’s accretion disc. For accretion to occur, angular momentum must be lost by that which is accreted. Electromagnetic radiation must offer some respite in this regard, the contribution for which was quantified in Chapter 6, using solely general relativity, with a Kerr (1963) metric, under the thin-disc regime. Calculations were first performed for extremised situations where photons are entirely responsible for energy removal in the disc. These were then extend to include internal angular-momentum transport, relating them to the standard relativistic accretion disc outlined by Novikov & Thorne (1973). While there is potential for the contribution of angular-momentum removal from photons to be
≥1% out to \( \sim 10^4 \) Schwarzschild radii if the disc is irradiated and maximally liberated of angular momentum through inverse Compton scattering, it is more likely of order \( 10^2 \) Schwarzschild radii if thermal emission from the disc itself is stronger (Fig. 6.1). The effect of radiation/scattering is stronger near the horizons of fast-spinning black holes, but, ultimately, other mechanisms must drive angular-momentum liberation/transport in accretion discs. This work has been published in PASA (Stevens 2015).

### 7.6 Upcoming Research: Phase 2 for the SAGE Models

While much of the work in this thesis has contributed towards the development of semi-analytic models, it has been pointed out on several occasions that there remains much room for improvement (e.g. Sections 2.8, 2.9, 3.4, and Chapter 4). It has further been raised that there are a variety of models in the literature, which each have their own set of prescriptions for galaxy evolution processes. Finding common ground amongst the models and realising their strengths and weaknesses is an ongoing process being undertaken by the Mocking Astrophysics collaboration, for which I am a member. A first study by Knebe et al. (2015) compared the out-of-the-box performance of 14 galaxy formation models, including SAGE, as run on the same \( N \)-body simulation. Their stellar mass functions varied by up to a factor of 3. Many of the differences in the galaxy populations discussed are likely the result of the models having not been retuned for that specific simulation. Furthermore, modellers would need to conform to a common set of observational constraints before their predictions could be placed on an equal footing, which has not yet been the case. I am coauthor on two papers currently in preparation (lead authors A. Knebe and R. Asquith) which experiment with the same group of galaxy formation models as in Knebe et al. (2015) that have now undergone a common calibration procedure. These works not only compare the models’ abilities to conform to a set of constraints, but also (still) show diversity in predictions of galaxy properties, especially at higher redshift. A complete dissection of the cause of this diversity will be informative for learning about how to best mathematically describe the most important galaxy evolution processes.

A separate project, which also involves the simultaneous use of multiple semi-analytic models, is the production of galaxy catalogues using the MultiDark simulations. Each of the SAGE (Croton et al. 2016), GALACTICUS (Benson 2012), and SAG (Gargiulo et al. 2015) models are being presented on these new simulations with \( 3840^3 \) particles each (see Klypin et al. 2016) that cover a range of volumes. The first results for the \((1h^{-1}\text{Gpc})^3\) volume concerning the clustering of galaxies are in preparation (led by A. Knebe and F. Prada). Having calibrated SAGE against the new simulations and produced the final galaxy catalogues, I will coauthor this paper. These mock data will be made publicly available through the Theoretical Astrophysical Observatory (Bernyk
Beyond what has been presented in Chapter 3, DARK SAGE has also undergone significant development. New options for the code surrounding how hot and cold gas are stripped from satellites have been added. For example, ram-pressure stripping of the cold gas is now only applied to satellites if the total baryonic mass (cold gas + stars) of the galaxy exceeds the hot gas mass of the subhalo. This provides a simple way of accounting for the requirement that hot gas around a satellite should be preferentially removed before the gas at the (local) minimum of the potential well is susceptible to stripping. A ram-pressure-stripping-like prescription is also available for the hot gas (à la Font et al. 2008; McCarthy et al. 2008), such that it is not necessarily stripped in proportion to the dark matter. With these tools, in collaboration with T. Brown, I am exploring a range of environmental processes that can affect the gas content of satellites. The H\text{\textsc{ i}} fractions of these galaxies are being studied as a function of stellar mass and specific star formation rates, for bins of parent halo mass. The predictions are being very closely compared with the recent observational data presented by Brown et al. (2016), who used stacking of H\text{\textsc{ i}} observations of satellites to find the mean trends of H\text{\textsc{ i}} fraction with stellar mass and specific star formation rate in various environments. Example figures of the tests being undertaken are provided in Figs. 7.1 & 7.2. These will be assessed in full in Stevens & Brown (in preparation).

Finally, Chapter 4 of this thesis offered suggestions as to how cooling could be better handled in semi-analytic models. An immediate next step will be to implement these suggestions, specifically Eq. 4.13, into DARK SAGE and assess the difference this makes to the high-redshift population of galaxies once the model has been recalibrated to $z = 0$. 

et al. 2016).
Figure 7.1: Mean relations of the H\textsubscript{I} fraction of DARK SAGE satellite galaxies, as a function of stellar mass, for various environments and various runs of the model at $z = 0$. These were produced using a subset of the merger trees of the Millennium simulation (Section 2.1.1). Dashed curves include all subhaloes with satellite galaxies, whereas the solid curves only include subhaloes that were composed of at least 100 particles at some point in their history (and hence are comfortably resolved). Squares are observational data from Brown et al. (2016). Colours are used to indicate the mass of the main halo that the satellite galaxies reside in: blue for $M_{\text{vir}} < 10^{12} \text{ M}_\odot$, green for $10^{12} \leq M_{\text{vir}}/\text{M}_\odot < 10^{13.5}$, and red for $10^{13.5} \leq M_{\text{vir}}/\text{M}_\odot < 10^{15}$. Each panel corresponds to a different run of the model (the same observational data are given in each). The top left panel is for the updated model (detailed only briefly in Section 7.6), using a metallicity-dependent prescription for calculating the fraction of gas in the form of H\textsubscript{I} and H\textsubscript{2} in each annulus (McKee & Krumholz 2010). The top middle panel uses the mid-plane pressure prescription described in Section 3.1.3. The remaining panels each alter one aspect of the model, as labelled (RPS = ram-pressure stripping, SN = supernovae, AGN = active galactic nuclei). These results will be presented in Stevens & Brown (in preparation).
7.6. Upcoming research: phase 2 for the SAGE models

Figure 7.2: As for Fig. 7.1 but now for H I fraction as a function of specific star formation rate.
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Cosmological derivations

In order to adhere to common convention and to provide clear mathematics in this Appendix, not all notation is consistent with the rest of the thesis. For example, what was previously written as $\Omega_M$ will now be $\Omega_{M_0}$, with $\Omega_M(t)$ being considered a function of time.

A.1 Curvature

Imagining curvature physically in more than two dimensions is, at best, difficult. One can imagine, however, that on two-dimensional curved surfaces (e.g. a 2-sphere), the sum of angles within a triangle is not necessarily $\pi$ as we expect in Euclidean space. This provides a probe for measuring curvature, and, indeed, is the part of the definition of Gaussian curvature,

$$K \equiv \frac{\alpha_1 + \alpha_2 + \alpha_3 - \pi}{A}, \quad (A.1)$$

where the $\alpha$-values are the angles in the triangle and $A$ is the area of the triangle. The radius of curvature, $a$, is then defined by the relationship

$$K = \frac{k}{a^2}, \quad (A.2)$$

where $k = -1, 0, +1$ to ensure $a$ is a positive quantity.\(^1\) A surface with uniform $K$ must therefore have uniform $a$. As a result, there exist exactly three types of surface for which curvature is constant, parameterised by $k$. These correspond to hyperbolic ($H^2$), flat ($R^2$) and spherical ($S^2$) space, respectively.

An interval for a two-dimensional surface with constant radius of curvature can be described in polar coordinates by

$$ds^2 = a^2(d\theta^2 + \sin^2(\theta) \, d\phi^2), \quad (A.3)$$

\(^1\text{Weinberg (1972, section 1.1) defines } K \text{ by Eq. A.2, with } a \text{ defined beforehand.}\)
with $\theta \in [0, \pi], \phi \in [0, 2\pi]$. For a spherical surface, one can define a dimensionless radial parameter, $r$;

$$r \equiv \sin(\theta) \Rightarrow dr = \cos(\theta) \ d\theta \Rightarrow d\theta^2 = \frac{dr^2}{\cos^2(\theta)} = \frac{dr^2}{1 - \sin^2(\theta)} = \frac{dr^2}{1 - r^2}.$$ 

An equivalent process can be done for a hyperbolic surface, where instead $r \equiv i \sin(\theta)$. For flat space, $a$ can be considered a scale radius, with $r$ a coordinate based on a percentage of $a$. All three cases can be written in a combined manner as

$$ds^2 = a^2 \left( \frac{dr^2}{1 - kr^2} + r^2 \ d\phi^2 \right).$$ \hspace{1cm} (A.4)

This can be extended to three-dimensional surfaces (hypersurfaces residing in either $\mathbb{R}^4$ or $\mathbb{R}^{3,1}$, written in Robertson–Walker coordinates):

$$ds^2 = a^2 \left( \frac{dr^2}{1 - kr^2} + r^2 (d\vartheta^2 + \sin^2(\vartheta) \ d\phi^2) \right), \quad k = \begin{cases} +1 & S^3 \cr 0 & \mathbb{R}^3 \cr -1 & H^3 \end{cases},$$ \hspace{1cm} (A.5)

$$0 \leq \vartheta \leq \pi, \ 0 \leq \varphi \leq 2\pi, \quad \begin{cases} 0 \leq r < \infty & \mathbb{R}^3, H^3 \cr 0 \leq r \leq 1 & \text{half of } S^3 \end{cases}.$$

The above allows the mathematical freedom to describe space in our Universe as a three-dimensional surface. As a result, following from the Cosmological Principle, the invariant interval for the Universe, be it flat or curved either way, may be written in terms of a Friedmann–Lemaître–Robertson–Walker metric:

$$ds^2 = -c^2 \ dt^2 + a^2(t) \left( \frac{dr^2}{1 - kr^2} + r^2 (d\vartheta^2 + \sin^2(\vartheta) \ d\phi^2) \right).$$ \hspace{1cm} (A.6)

Although it was a requirement for $a$ to be uniform anywhere on the surface, there is no restriction on its ability to change with time. This radius of curvature, $a$, hence describes the scale of the Universe at any time, $t$, and, as a result, is more commonly referred to as the scale parameter.
Eq. A.6 is equivalent to writing a metric in a \((t, r, \vartheta, \varphi)\) coordinate system\(^2\) with components

\[
g_{\mu\nu} = \begin{pmatrix}
-c^2 & 0 & 0 & 0 \\
0 & a^2(t) & 0 & 0 \\
0 & 0 & a^2(t) r^2 & 0 \\
0 & 0 & 0 & a^2(t) r^2 \sin^2(\vartheta)
\end{pmatrix}.
\] (A.7)

It is important to note that isotropy and homogeneity are only assumed on the largest scales. When dealing with scales such as the Solar System, Galaxy, or a supercluster, local spacetime curvature dominates the metric. In other words, Eq. A.7 displays the background or average metric of the Universe. Details of cosmic averages are not assessed in this document; for that, the interested reader is referred to Wiltshire (2007).

A.2 DERIVATION OF THE FRIEDMANN EQUATION

Assuming the Universe can be described as an ideal fluid, the components of the energy-momentum tensor in an arbitrary coordinate basis can be written as (derivation not shown here, but see Peebles 1993, pp. 259-265; Weinberg 1972, sections 2.10 and 3.4)

\[
T^{\mu\nu} = (\rho + \frac{P}{c^2}) u^\mu u^\nu + g^{\mu\nu} P,
\] (A.8)

where \(\rho\) is the mass-energy density (the energy density measured in units of mass density), \(P\) the fluid’s pressure, and \(u^\mu\) the four-velocity components.

Homogeneity and isotropy not only provide a nicely restricted metric, but restrict the energy-momentum tensor components in an orthonormal basis, \(x^\hat{\mu} = (ct, x^\hat{i}, x^\hat{2}, x^\hat{3})\), as well. The Cosmological Principle would require \(u^\hat{\mu} = (c, 0, 0, 0)\), with \(\rho\) and \(P\) free to be functions of time only. Using Eq. A.8 in an orthonormal basis, it then follows that \(T^{\hat{0}\hat{0}} = \rho(t) c^2\), \(T^{\hat{i}\hat{i}} = P(t)\) and \(T^{\hat{0}\hat{i}} = T^{\hat{i}\hat{0}} = 0\), where \(i = 1, 2, 3\). Recognising that \(T_{\hat{\mu}\hat{\nu}} = \eta_{\hat{\alpha}\hat{\beta}} \eta_{\hat{\beta}\hat{\delta}} T^{\hat{\alpha}\hat{\delta}}\), as \(g_{\hat{\mu}\hat{\nu}} = \eta_{\hat{\mu}\hat{\nu}}\), the energy-momentum tensor is found as

\[
T_{\hat{\mu}\hat{\nu}} = \begin{pmatrix}
\rho(t) c^2 & 0 & 0 & 0 \\
0 & P(t) & 0 & 0 \\
0 & 0 & P(t) & 0 \\
0 & 0 & 0 & P(t)
\end{pmatrix}.
\] (A.9)

\(^2\)It might be more familiar to see a \((ct, r, \vartheta, \varphi)\) coordinate system. In which case, \(g_{00}\) would be \(-1\) instead of \(-c^2\).
We can transform this into our \((t, r, \vartheta, \varphi)\) coordinate basis through
\[
T_{\mu\nu} = T_{\alpha\beta} \frac{\partial x^\alpha}{\partial x^\mu} \frac{\partial x^\beta}{\partial x^\nu} .
\] (A.10)

Making use of Eq. A.9 and the property \(<\partial_\mu, f dx^\nu> = f \delta^\nu_\mu\), where \(f\) is some scalar function, leads to the result
\[
T_{\mu\nu} = \begin{pmatrix}
\rho(t) c^4 & 0 & 0 & 0 \\
0 & \frac{a^2(t)}{1 - kr^2} P(t) & 0 & 0 \\
0 & 0 & a^2(t) r^2 P(t) & 0 \\
0 & 0 & 0 & a^2(t) r^2 \sin^2(\vartheta) P(t)
\end{pmatrix} .
\] (A.11)

Now that we have restricted and well defined metric and energy-momentum tensors, we can reduce the Einstein Field Equations. With Eq. A.11, we already have the right-hand side of Eq. 1.1. To obtain the left-hand side, the Ricci tensor components and Ricci scalar need to be calculated. The Ricci tensor can be written in terms of Christoffel symbols, \(\Gamma^\alpha_{\mu\nu}\), where
\[
R_{\alpha\beta} \equiv R^\lambda_{\alpha\lambda\beta} = \partial_\lambda \Gamma^\lambda_{\beta\alpha} + \Gamma^\lambda_{\lambda\sigma} \Gamma^\sigma_{\beta\alpha} - \partial_\beta \Gamma^\lambda_{\lambda\alpha} - \Gamma^\lambda_{\beta\sigma} \Gamma^\sigma_{\lambda\alpha} .
\] (A.12)

The Ricci scalar is defined as the trace of the Ricci tensor:
\[
R \equiv R^\lambda_\lambda = g^{\lambda\sigma} R_{\lambda\sigma} .
\] (A.13)

The Christoffel symbols are calculable using the metric and its derivatives:
\[
\Gamma^\rho_{\mu\nu} = \frac{1}{2} g^{\rho\lambda} (\partial_\mu g_{\nu\lambda} + \partial_\nu g_{\mu\lambda} - \partial_\lambda g_{\mu\nu}) .
\] (A.14)

Using the metric of Eq. A.7, after two or three lines of algebra per component, each \(\Gamma^\rho_{\mu\nu}\) can be found. The algebra will not be shown here, but it is vastly simplified by \(g_{\mu\nu}\) being diagonal, where, because of this, \(g^{\mu\nu} = 1/g_{\mu\nu}\). In total, there are 19 non-zero Christoffel symbols, 10 of which are different. Those non-zero quantities are
\[
\begin{align*}
\Gamma^0_{11} &= \frac{a(t) \dot{a}(t)}{c^2 (1 - kr^2)} \\
\Gamma^0_{22} &= \frac{a(t) \dot{a}(t)}{c^2} r^2 \\
\Gamma^0_{33} &= \frac{a(t) \dot{a}(t)}{c^2} r^2 \sin^2(\vartheta)
\end{align*}
\]
A.2. Derivation of the Friedmann Equation

\[ \Gamma^1_{01} = \Gamma^1_{10} = \Gamma^2_{02} = \Gamma^2_{20} = \Gamma^3_{03} = \Gamma^3_{30} = \frac{\dot{a}(t)}{a(t)} \]

\[ \Gamma^1_{11} = \frac{kr}{1 - kr^2} \]

\[ \Gamma^1_{22} = -r (1 - kr^2) \]

\[ \Gamma^1_{33} = -r \sin^2(\vartheta) (1 - kr^2) \]

\[ \Gamma^2_{12} = \Gamma^2_{21} = \Gamma^3_{13} = \Gamma^3_{31} = \frac{1}{r} \]

\[ \Gamma^2_{33} = -\sin(\vartheta) \cos(\vartheta) \]

\[ \Gamma^3_{23} = \Gamma^3_{32} = \cot(\vartheta). \] \hspace{1cm} (A.15)

The recognition of the components that are zero lead to a reduction in Eq. A.12, albeit longer to write:

\[ R_{\alpha\beta} = \partial_0 \Gamma_{\beta\alpha}^0 + \partial_1 \Gamma_{\beta\alpha}^1 + \partial_2 \Gamma_{\beta\alpha}^2 + 3 \Gamma_{10}^1 \Gamma_{\alpha\beta}^0 + \Gamma_{1\beta\alpha}^1 (\Gamma_{11}^1 + 2 \Gamma_{21}^2) + \Gamma_{32}^3 \Gamma_{\alpha\beta}^2 - \partial_\beta \Gamma_{\lambda\alpha}^\lambda - \Gamma_{\beta\sigma}^\lambda \Gamma_{\alpha\lambda}^\sigma, \] \hspace{1cm} (A.16)

with many of these terms being zero for a specific \( \alpha \) and \( \beta \). Substituting in the values from Eq. A.15 to Eq. A.16 produces the values of the Ricci tensor components. Ultimately, the aim is to substitute these values into the Einstein Field Equation. As such, because the right-hand side of Eq. 1.1 is only non-zero when \( \alpha = \beta \) (as shown by Eq. A.11), only the diagonal values of the Ricci tensor need to be calculated. Omitting further algebra, these results are

\[ R_{00} = -3 \frac{\ddot{a}(t)}{a(t)} \]

\[ R_{11} = \frac{2 \dot{a}^2(t) + 2kc^2 + a(t)\dot{a}(t)}{c^2 (1 - kr^2)} \]

\[ R_{22} = \frac{r^2}{c^2} \left( 2 \dot{a}^2(t) + 2kc^2 + a(t)\dot{a}(t) \right) \]

\[ R_{33} = \frac{r^2 \sin^2(\vartheta)}{c^2} \left( 2 \dot{a}^2(t) + 2kc^2 + a(t)\dot{a}(t) \right), \] \hspace{1cm} (A.17)

with the Ricci scalar found to be

\[ R = \frac{6}{a^2(t) c^2} \left( \dot{a}^2(t) + kc^2 + a(t)\dot{a}(t) \right). \] \hspace{1cm} (A.18)

The results of Eqs. A.15, A.17, & A.18 can also be found in Narlikar (1993, §4.1).\(^3\)

\(^3\)Note that some of the Christoffel symbols presented here carry an extra factor of \( c \) in their denominators compared
We have now calculated all the necessary values to obtain the Friedmann equation. This is performed by substituting values from Eqs. A.11, A.17, & A.18 into Eq. 1.1, with $\mu, \nu = 0, 0$. The Friedmann equation is

$$3 \left( \frac{\dot{a}^2(t)}{a^2(t)} + \frac{kc^2}{a^2(t)} \right) - \Lambda c^2 = 8\pi G \rho(t). \quad (A.19)$$

Equally, using all the above information, the Einstein Field Equations for $\mu, \nu = i, i$ ($i = 1, 2, 3$) reduce to

$$- \frac{\dot{a}^2(t)}{a^2(t)} - \frac{kc^2}{a^2(t)} - 2\frac{\ddot{a}(t)}{a(t)} + \Lambda c^2 = \frac{8\pi G}{c^2} P(t). \quad (A.20)$$

### A.3 Cosmological redshift

For an expanding universe, i.e. one where $\dot{a} \neq 0$, when we observe a comoving object, e.g. another galaxy, that object should appear redshifted. Generated by the Doppler effect with the exception that the object is moving with space rather than through space, this is the cosmological redshift. Unlike the scale parameter, redshift is a quantity we can directly measure. Thankfully, however, a relationship between the scale parameter and redshift can be derived via the following steps.

Consider a radially inward falling photon, i.e. where its path has $d\vartheta = d\varphi = 0$ and $dr/dt < 0$. Noting that photons travel on null intervals (where $ds^2 = 0$), using Eq. A.6, one finds for the photon that

$$-c^2 \, dt^2 + a^2(t) \frac{dr^2}{1 - kr^2} = 0 \Rightarrow c \frac{dt}{a(t)} = -\frac{dr}{\sqrt{1 - kr^2}}.$$

By integrating from the photon’s creation ($t = t_1, r = r(t_1) \equiv r_1$) to its observation ($t = t_0, r = 0$), we find

$$c \int_{t_1}^{t_0} \frac{dt}{a(t)} = \int_0^{r_1} \frac{dr}{\sqrt{1 - kr^2}} \equiv f_k(r_1). \quad (A.21)$$

Now consider another photon being emitted at some very small interval $\delta t_1$ after the previous one, observed at time $t_0 + \delta t_0$. Assuming the emitting object is comoving, it’s $r$-coordinate should not change with time, meaning $r(t_1 + \delta t_1) = r_1$. Integrating over this path in the same manner as before shows

$$f_k(r_1) = c \int_{t_1 + \delta t_1}^{t_0 + \delta t_0} \frac{dt}{a(t)} = c \left( -\int_{t_1}^{t_1 + \delta t_1} \frac{dt}{a(t)} + \int_{t_1}^{t_0} \frac{dt}{a(t)} + \int_{t_0}^{t_0 + \delta t_0} \frac{dt}{a(t)} \right).$$

to Narlikar (1993) because the zeroth coordinate was chosen to be $t$ in this document, rather than $ct$. This does not affect the values of the Riemann tensor or scalar.
A.4. Manipulation of the Friedmann Equation

\[ = f_k(r_1) + c \left( - \int_{t_1}^{t_1 + \delta t_1} \frac{dt}{a(t)} + \int_{t_0}^{t_0 + \delta t_0} \frac{dt}{a(t)} \right) \]

\[ \Rightarrow \int_{t_1}^{t_1 + \delta t_1} \frac{dt}{a(t)} = \int_{t_0}^{t_0 + \delta t_0} \frac{dt}{a(t)}. \]

Because \( \delta t_1 \) and \( \delta t_0 \) are very small, \( a \) will remain constant over the periods of integration, hence

\[ \frac{\delta t_1}{a(t_1)} = \frac{\delta t_0}{a(t_0)}. \]  

(A.22)

For a sustained emission, \( \delta t_1 \) could represent the period of emission (i.e. the time between the emission of wave crests). Because wavelength is proportional to period, this provides a relationship between the observed change in wavelength and the ratio of scale factors at the time of emission and observation:

\[ \frac{a(t_0)}{a(t_1)} = \frac{\lambda_{\text{obs}}}{\lambda_{\text{em}}}. \]  

(A.23)

Redshift, \( z \), is defined as

\[ z \equiv \frac{\lambda_{\text{obs}} - \lambda_{\text{em}}}{\lambda_{\text{em}}} = \frac{\lambda_{\text{obs}}}{\lambda_{\text{em}}} - 1. \]  

(A.24)

By generalising the time of emission to be any time, \( t \), we now have a relationship between cosmological redshift\(^4\) and the scale parameter of the Universe at the epoch we are observing:

\[ \frac{a(t)}{a_0} = \frac{1}{1 + z}, \]  

(A.25)

where \( a_0 \equiv a(t_0) \). From now on, all subscripts of zero represent the quantity measured at time \( t_0 \).

### A.4 Manipulation of the Friedmann Equation

To derive the relationship between cosmic time and cosmological redshift, it is useful to define new parameters to simplify the appearance of the Friedmann equation. These new parameters will be those standard to the cosmology literature. First, the expansion of space is often expressed in terms of the Hubble parameter,

\[ H(t) \equiv \frac{\dot{a}(t)}{a(t)}. \]  

(A.26)

\(^4\)By Eq. A.24, the quantity \( z \) is not restricted to mean cosmological redshift (i.e. the redshift due purely to the expansion of the Universe); in truth, it should be the net redshift (which is what we measure). Despite this, when discussing cosmology in this manner, \( z \) is conventionally used to mean cosmological redshift.
Currently, the energy density term in Eq. A.19, \( \rho(t) \), accounts for the contribution of matter and radiation (i.e. \( \rho(t) = \rho_M(t) + \rho_R(t) \)). By defining
\[
\rho_\Lambda \equiv \frac{\Lambda c^2}{8\pi G},
\]
we can absorb the cosmological constant component into \( \rho \). Note that \( \rho_\Lambda \) is constant. By also making use of Eq. A.26, one can rearrange the Friedmann equation to
\[
\frac{kc^2}{H^2(t) a^2(t)} = \frac{8\pi G \rho(t)}{3H^2(t)} - 1,
\]
where now \( \rho(t) = \rho_M(t) + \rho_R(t) + \rho_\Lambda \). With this, we can define a critical density, \( \rho_c(t) \), where the Universe would just manage to expand forever. This occurs when \( k = 0 \), and therefore when Eq. A.28 = 0, giving Eq. 1.4. As is about to be shown, it is often convenient to write densities in terms of a fraction of the critical density, where
\[
\Omega_M(t) \equiv \frac{\rho_M(t)}{\rho_c(t)}, \quad \Omega_R(t) \equiv \frac{\rho_R(t)}{\rho_c(t)}, \quad \Omega_\Lambda(t) \equiv \frac{\rho_\Lambda}{\rho_c(t)}.
\]
By further defining
\[
\Omega_k(t) \equiv -\frac{kc^2}{a^2(t) H^2(t)},
\]
one can rewrite the Friedmann equation to simply be
\[
\Omega_M(t) + \Omega_R(t) + \Omega_\Lambda(t) + \Omega_k(t) = 1.
\]

With the above variables now defined, we can proceed to derive a relationship between the Hubble parameter and cosmological redshift. To begin, we rewrite the Friedmann Equation once again:
\[
H^2(t) = \frac{8\pi G}{3} (\rho_R(t) + \rho_M(t)) - \frac{kc^2}{a^2(t)} + \frac{\Lambda c^2}{3}.
\]
Assuming the total mass of the Universe remains constant with time, it stands that \( \rho_M(t) \propto a^{-3}(t) \). Following from laws of thermodynamics, one also finds \( \rho_R(t) \propto a^{-4}(t) \) (not derived or discussed here, but see, e.g., Narlikar 1993, §4.2). Combining these with Eq. A.30 gives
\[
H^2(t) = \frac{8\pi G}{3} \left( \rho_R_0 \frac{a_0^4}{a^4(t)} + \rho_M_0 \frac{a_0^3}{a^3(t)} \right) + \Omega_k(t) H^2(t) + \frac{\Lambda c^2}{3}.
\]
Now invoking Eqs. A.27 & A.29 with the realisation that \( \Omega_k(t) \propto a^{-2}(t)H^{-2}(t) \) (Eq. A.30)
A.5. Cosmic time

provides

\[ H^2(t) = \Omega_R H_0^2 \frac{a_0^4}{a^4(t)} + \Omega_M H_0^2 \frac{a_0^3}{a^3(t)} + \Omega_k H_0^2 \frac{a_0^2}{a^2(t)} + \Omega_\Lambda H_0^2. \quad (A.34) \]

Substituting in Eq. A.25 then shows

\[ H^2(z) = H_0^2 \left( \Omega_R (1+z)^4 + \Omega_M (1+z)^3 + \Omega_k (1+z)^2 + \Omega_\Lambda \right). \quad (A.35) \]

Eq. A.35 is incredibly useful, as it is written in terms of parameters we directly measure (with the exception of \( \Omega_k \), which is limited to exactly three possible values).

A.5 COSMIC TIME

For comoving observers, \( dr = d\varphi = d\theta = 0 \). Hence \( -c^2 dr^2 = ds^2 = -c^2 dt^2 \), so the time comoving observers measure is a proper time, \( \tau \). Neglecting peculiar velocities and local gravitational effects, this means there exists a proper time which all parts of the Universe experience, which we call cosmic time. Since the introduction of the Friedmann–Lemaître–Robertson–Walker metric in Section A.1, \( t \) has represented cosmic time and will continue to do so now. What we wish to find is \( t(z) \), i.e. the age of the Universe when the light was emitted from an object we observe at cosmological redshift \( z \).

In classical physics, we define a velocity in terms of the differential of position with time:

\[ v = \frac{dx}{dt} \Rightarrow \int dt = \int f \frac{dx}{v}. \]

This can be used as an analogue for comoving objects, where we replace \( x \) with proper distance (in the radial direction), \( l_r(t) = a(t) f_k(r) \), and \( v \) with the Hubble flow ‘velocity’, \( l_r(t) H(t) \):

\[ t_L = \int_{a(t)}^{a_0} \frac{da}{a(t) H(t)}, \quad (A.36) \]

where \( t_L \) is the look-back time (A.K.A. the light-travel time for the observed photons). By differentiating Eq. A.25, we know

\[ da = \frac{-a_0}{(1+z)^2} dz, \quad (A.37) \]

\[ ^5 \text{Note that there is bad mathematical practice in this last step, where } H^2(t) \text{ has been relabelled as } H^2(z) \text{ when the functional dependence of } H \text{ on } t \text{ is clearly different to that on } z. \]
allowing us to present $t_L$ as a function of cosmological redshift:

$$t_L(z) = - \int_z^0 \frac{dz'}{H(z') (1 + z')}$$

$$= \frac{1}{H_0} \int_0^z \frac{dz'}{(1 + z') \sqrt{\Omega_R(1 + z')^4 + \Omega_M(1 + z')^3 + \Omega_k(1 + z')^2 + \Omega_\Lambda}}$$

(A.38)

(where $z'$ is just a dummy relabelling of redshift within the integrals). Cosmic time of an object can be found as the difference between the look-back time to the Big Bang and the look-back time to the object of interest:

$$t(z) = t_L(\infty) - t_L(z) = \frac{1}{H_0} \int_z^\infty \frac{dz'}{(1 + z') \sqrt{\Omega_R(1 + z')^4 + \Omega_M(1 + z')^3 + \Omega_k(1 + z')^2 + \Omega_\Lambda}}.$$  

(A.39)
Resolution study of SAGE

B.1 Mass resolution

One advantage of having a simulation suite like GiggleZ is that we can test how SAGE varies solely as a function of mass resolution, where all other parameters of the simulation are held the same. In Fig. B.1, we show the stellar mass functions produced by SAGE for each of the low-(LR), nominal- (NR), medium- (MR), and high-resolution (HR) runs of GiggleZ, with a particle mass of $6 \times 10^{10}$, $7.5 \times 10^9$, $9.5 \times 10^8$, and $1.2 \times 10^8h^{-1}M_\odot$, respectively. There are clear differences between each of these mass functions when we apply our fiducial parameter set. For example, both the low- and nominal-resolution runs vastly under-predict the number of systems leftward of the knee ($m_* \lesssim 10^{11}M_\odot$), while all except the medium-resolution run over-predict the number of high-mass systems. This is primarily due to the fixed parameter set adopted, chosen for simulations with resolution similar to that of GiggleZ-MR.

The above is not a short-coming of SAGE, per se. What characterises semi-analytic models is how the prescriptions link certain variables together, not what the absolute values of parameters are. The freedom or tune-ability of parameters highlights our inability to capture the true ‘microscopic’ nature of galaxy formation with these models. Parameter values therefore should change with mass resolution, as the scales for which the parameters are compensating are changing. This is true not only for semi-analytic models, but for sub-resolution models used in hydrodynamic simulations as well (for an extended and in-depth discussion on this topic, see Schaye et al. 2015, section 2).

B.2 Temporal resolution

While convergence with mass resolution is not necessary for a common parameter set, a semi-analytic model should ideally produce converged results with temporal resolution. The coarser the
steps in a merger tree, the less accurate merging times and the determination of minor versus major
mergers will be. Using the GALACTICUS model (Benson 2012), Benson et al. (2012) suggested
128 snapshots should produce convergence to the five-percent level for galaxy masses on average,
while individual systems could still show much greater variation. Millennium, for instance, uses
64 snapshots, while Bolshoi uses 181.

To test temporal convergence for SAGE, we built merger trees for GiggleZ-MR using four
different snapshot spacings, with average time intervals of 58, 117, 233, and 466 Myr. We ran
identical versions of the model for each and provide their stellar mass functions in Fig. B.2.

While we do not assess galaxies on an individual basis, our interest is in whether the statistical
sample of galaxies is representative. The mass function appears relatively well converged for the
temporal resolutions of the N-body simulations we have used. There is a leftward movement of
the knee as temporal resolution increases, but such minor changes are readily remedied with small
alterations to parameter values. We are hence satisfied that the z = 0 properties of SAGE do not
show a strong dependence on temporal resolution. We note though that temporal resolution is
valuable for studies concerning instantaneous time derivatives of galaxy properties, especially at
higher redshift.

Figure B.1: Stellar mass functions for the GiggleZ runs at various mass resolutions (for details,
see Poole et al. 2015) for our fiducial parameter set for SAGE.
Figure B.2: Stellar mass functions for the GiggleZ-MR merger trees with varying snapshot temporal resolution, produced with our fiducial parameter set for SAGE. $N_{\text{snaps}}$ indicates the number of snapshots used in the trees’ construction from $z = 30$ to 0.
Further details of DARK SAGE

The contents of this appendix have been published in Stevens et al. (2016a).

C.1 MODEL CALIBRATION

We have constrained the free parameters of DARK SAGE by hand to match a set of observables, focussed on the local galaxy population. Many of these constraints are identical to those used for SAGE. We used the smaller ‘Mini Millennium’ simulation, with a box length of $62.5h^{-1}$ Mpc (but otherwise matching the details in Section 2.1.1), for calibrating most of the parameters. The plots presented in this appendix were preferably made with the larger Millennium simulation though.

Our primary constraints include galaxy mass functions for each of stars (already presented in Fig. 3.2), H I, and H$_2$ at $z = 0$, in addition to the H I–stellar mass scaling relation. The mass function, $\Phi$, describes the number density of galaxies with a particular mass (in a particular baryonic species) per unit volume. Of these, only the stellar mass function was originally used to constrain SAGE. This was primarily because that model did not break cold gas into H I and H$_2$ (for a version of SAGE that did do this, see Wolz et al. 2016).

We constrain the H I mass function using observational data from Zwaan et al. (2005), who built their mass function from the H I Parkes All Sky Survey. This is shown in Fig. C.1. Each mass bin had a sufficient number of galaxies such that their published Poisson errors are smaller than the thickness of the line over the range plotted here. For completeness, we also compare the mass function measured equivalently from the Arecibo Legacy Fast ALFA survey (Martin et al. 2010), covering the published uncertainty range.

To constrain the H$_2$ mass function, we initially used data from Keres et al. (2003), as shown in Fig. C.2. Those authors measured the CO luminosity function from observations of local galaxies, where the derived H$_2$ mass function assumed a constant conversion factor between the CO and H$_2$ content. In the later stages of calibration, we found the model that best fit the other constraints fell
Appendix C. Further details of DARK SAGE

Figure C.1: H I mass function of DARK SAGE galaxies at $z = 0$ compared to observations (Zwaan et al. 2005; Martin et al. 2010).

in between the H$_2$ mass functions of Keres et al. (2003) and Obreschkow & Rawlings (2009). The latter authors used the same data but applied a variable conversion factor from CO to H$_2$.

Through stacking H I observations of galaxies which would otherwise be non-detections, Brown et al. (2015) found an average scaling relation between the H I mass fraction of galaxies and their stellar mass. We use this to constrain the H I mass fractions of DARK SAGE galaxies. We show these in Fig. C.3 and include the mean trend, binned similarly to the observational data. Observational errors from jackknifing are smaller than the width of the line. We note that the lowest mass bin, $10^9 \lesssim m_\ast/M_\odot \lesssim 10^{9.5}$, always showed a deficit in the H I fraction throughout our calibration process.

The remaining constraints were identical to what was done for SAGE. Fig. C.4 provides points and the sequence for the black hole–bulge mass relation of DARK SAGE galaxies. We compare against observational data with uncertainties from Scott et al. (2013), who split galaxies into Sérsic and core-Sérsic types. Fig. C.5 shows the Baryonic Tully–Fisher relation (maximum rotational velocity against the sum of stellar and cold gas mass) for relevant DARK SAGE galaxies, with the fitted relation to observational data from Stark et al. (2009) including their random uncertainties. Fig. C.6 displays the stellar mass–gas metallicity relation, by calculating $12 + \log_{10}(O/H) = 9 + \log_{10}(m_{Z,cold}/0.02 m_{cold})$. Both DARK SAGE and the observational data (Tremonti et al. 2004) display the 16$^{th}$–84$^{th}$ percentile range of metallicity from bins of approximately 0.1-dex width in stellar mass.

Finally, we present the comoving star formation rate density history, also known as the Madau–Lilly diagram (à la Lilly et al. 1996; Madau et al. 1996), in Fig. C.7. Observational data used for this constraint were compiled by Somerville et al. (2001), with a complete set of references in
**Figure C.2:** H$_2$ mass function of DARK SAGE galaxies at $z = 0$ compared to observations (Keres et al. 2003; Obreschkow & Rawlings 2009).

**Figure C.3:** H I mass fraction as a function of stellar mass at $z = 0$. We show 4000 representative galaxies from DARK SAGE as well as the mean trend to be compared against the observed relation of Brown et al. (2015).
Appendix C. Further details of Dark Sage

Figure C.4: Black hole–bulge mass relation at $z = 0$. We show 4000 representative galaxies from Dark Sage, and include the mean and 16th–84th percentile range of the relation. Observational data from Scott et al. (2013) are compared.

Figure C.5: Baryonic Tully–Fisher relation for Dark Sage galaxies at $z = 0$ with bulge-to-total ratios between 0.1 and 0.5. We show 4000 example galaxies in the axes and the mean rotation velocity for mass bins of width 0.1 dex. Compared is the observational trend published by Stark et al. (2009).
### C.2 Rotation curves

It is important for many parts of the model to convert between $j$ and $r$ for the disc annuli. From Eq. 3.1, this means $v_{\text{circ}}(r)$ must be known. At each time-step, we construct a rotation curve for each disc, where

$$v_{\text{circ}}(r) = \sqrt{\frac{GM(<r)}{r}}. \quad (C.1)$$

We then iteratively solve the equation

$$j = r v_{\text{circ}}(r) = \sqrt{G M(<r) r} \quad (C.2)$$

to calculate a radius for each annulus edge. The mass internal to radius $r$, $M(<r)$, is determined by summing the mass profiles of each of the components in the (sub)halo,

$$M(<r) = m_{\text{cold}}(<j) + m_{*,\text{disc}}(<j) + m_{\text{DM}}(<r) + m_{\text{hot}}(<r)$$
$$+ m_{m-\text{bulge}}(<r) + m_{i-\text{bulge}}(<r) + m_{\text{ICS}}(<r) + m_{\text{BH}}. \quad (C.3)$$
Figure C.7: Madau–Lilly diagram. Observational data originate from a variety of sources; we constrained DARK SAGE against those compiled by Somerville et al. (2001). Compared are data compiled by Madau & Dickinson (2014).

The mass internal to \( j \) in the disc and the black hole mass, \( m_{\text{BH}} \), is directly known, while we describe how each of the other components is modelled below.

The value of \( v_{\text{circ}}(r) \) is not allowed to exceed the \( V_{\text{max}} \) of the halo as recorded by the halo finder. This ensures rotation curves do not spike to excessively large values at low radius, which happens for disc-dominated systems without the restriction. This problem is associated with the excessive pooling of baryonic mass towards the centres of discs, shared by many models of disc evolution (see Stringer & Benson 2007, and references therein).

C.2.1 Dark matter

We use the NFW profile (Navarro et al. 1996, 1997) for dark-matter haloes, which gives a cumulative mass of

\[
m_{\text{DM}}(< r) = m_{\text{DM}} \left[ \ln \left( \frac{r + r_h}{r_h} \right) - \frac{r}{r + r_h} \right] \left[ \ln \left( \frac{R_{\text{vir}} + r_h}{r_h} \right) - \frac{R_{\text{vir}}}{R_{\text{vir}} + r_h} \right]^{-1},
\]

where \( r_h \) describes a scale radius, related to the concentration of the halo,

\[
c \equiv \frac{R_{\text{vir}}}{r_h}.
\]

Over cosmic time, as haloes accumulate mass and condense, their concentrations increase. Larger haloes also achieve higher concentrations in shorter time-scales. Using a series of \( N \)-body simulations which covered five orders of magnitude of halo mass, Dutton & Macciò (2014) showed there is a generic dependence of the concentration of dark-matter-only haloes on their mass and...
redshift:

\[
\log_{10}(c_{\text{DM}}) = a + b \log_{10}(M_{\text{vir}}h/10^{12}M_\odot),
\]

\[a = 0.520 + 0.385e^{-0.617z^{0.21}},\]  
\[b = -1.01 + 0.026z.\]

This expression is valid for \(z < 5\). For higher redshifts, we approximate the concentration of haloes to be as if it were redshift 5. This has a negligible effect on the evolution of galaxies in DARK SAGE, given the relatively few snapshots considered at \(z > 5\) compared to those at lower redshift.

It is now widely accepted that baryonic physics plays an important role in the distribution of dark matter in haloes, especially in their centres (see, e.g., Duffy et al. 2010; Di Cintio et al. 2014; Brook 2015; Schaller et al. 2015a). With hydrodynamic simulations, Di Cintio et al. (2014) showed that the effect of baryons on the concentration of dark-matter haloes can be captured by the proportion of the virial mass in the form of stars in the galaxy:

\[c = \left\{1 + 3 \times 10^{-5} e^{3.4[\log_{10}(m_*/M_{\text{vir}})+4.5]}\right\} c_{\text{DM}}.\]  

We substitute Eq. C.6 into Eq. C.7 to solve for the dark-matter halo concentration in DARK SAGE.

In truth, an NFW profile is insufficient to account for all the baryonic effects on the dark-matter profile. A more generic profile for the family of Jaffe (1983), Hernquist (1990), and NFW profiles (à la Merritt et al. 2006) would be more accurate, where Di Cintio et al. (2014) provide scalings for the parameters of these profiles. Solving Eq. C.2 becomes immensely more computationally expensive, however, and this level of detail is beyond that necessary for DARK SAGE.

### C.2.2 Hot gas

The cumulative mass profile of hot gas in DARK SAGE comes directly from the assumption that the gas is a singular isothermal sphere:

\[m_{\text{hot}}(<r) = m_{\text{hot}} \frac{r}{R_{\text{vir}}}.\]  

\[C.7\]
C.2.3 Bulges and spheroids

We consider merger-driven and instability-driven bulges each as independent components of a galaxy, but approximate both as having Hernquist (1990) profiles truncated at the virial radius,

\[
m_{\text{bulge}}(<r) = m_{\text{bulge}} \left[ \frac{r(R_{\text{vir}} + a_{\text{bulge}})}{R_{\text{vir}}(r + a_{\text{bulge}})} \right]^2 ,
\]

where the half-mass radius of the bulge component is \( (1 + \sqrt{2})a_{\text{bulge}} \), assuming \( a_{\text{bulge}} \ll R_{\text{vir}} \).

For the merger-driven bulge, we use the empirical average size–mass scaling from Sofue (2016),

\[
\log_{10} \left( \frac{a_{m-\text{bulge}}}{\text{kpc}} \right) = 1.13 \left[ \log_{10} \left( \frac{m_{m-\text{bulge}}}{\text{M}_\odot} \right) - 10.21 \right] .
\]

For the instability-driven bulge, we use the observed average size–mass scaling of pseudobulges from Fisher & Drory (2008),

\[
a_{i-\text{bulge}} = 0.2 r_d (1 + \sqrt{2})^{-1} ,
\]

where \( r_d \) is taken from Eq. 3.4.

This model for the bulge size is incredibly crude, but its precision is not an important factor for the goals of the model in studying discs. For a modified version of SAGE with a thorough evolution of bulge sizes, see Tonini et al. (2016).

C.2.4 Intracluster stars

Modelling the mass distribution of intracluster stars is only of moderate significance for brightest cluster galaxies. Gonzalez et al. (2005) found the scale radius for intracluster light correlates with that of the elliptical brightest cluster galaxy. We follow their simple scaling relation, assuming intracluster stars are distributed in a Hernquist (1990) sphere (Eq. C.9) with

\[
a_{\text{ICS}} = 13 a_{m-\text{bulge}} .
\]
Model-equivalent cooling rates from EAGLE

The contents of this appendix have appeared in Stevens et al. (2016b).

In an ideal world, one would know the unique density, temperature, and metallicity profile of hot gas in every halo processed through a semi-analytic model. By using the measured profiles from EAGLE, we can solve Eq. 4.6 numerically for each halo, thereby determining the ‘semi-analytic equivalent’ for the cooling rate with ideal information. This can be compared against a more realistic calculation for a semi-analytic model, where in addition to an analytic density profile, it is assumed that $T_{\text{hot}}(R) = T_{\text{vir}}$ and $Z_{\text{hot}}(R) = \bar{Z}_{\text{hot}}$ for each halo. The results of Section 4.2.4 suggest any of a singular isothermal sphere, a $\beta$ profile with $c_\beta \neq 0.1$, or a $\beta$ profile with $c_\beta(z)$ from Eq. 4.4 will work practically the same for calculating a cooling rate. In the left-hand panel of Fig. D.1 we compare the cooling rates calculated by Eq. 4.5 using the full EAGLE profile information against the case of an assumed singular isothermal sphere (we have checked that this is essentially the same for the case of a $\beta$ profile).

The spread in the true $R_{\text{cool}}$ values for the EAGLE haloes is greater than that of the analytic models, as seen in all panels of Fig. 4.8. This then gives rise to the notable spread in the relative cooling rates seen in the left-hand (and middle, see below) panel of Fig. D.1. In the lower redshift bins, the model $R_{\text{cool}}$ values are systematically too large, most clearly seen in the right-hand panel of Fig. 4.8. This then translates into systematically higher cooling rates, as seen by the evolution in the distributions of the left-hand (and middle) panel of Fig. D.1.

Note that the cooling rates calculated by Eq. 4.6 do not consider feedback (of any kind), and hence are gross cooling rates (as opposed to net cooling rates – see below). If feedback were to affect the density or temperature profiles of the hot gas in haloes significantly, then we might expect this feedback to still cause differences in the gross cooling rates. While we showed that feedback does alter the temperature at the centre of EAGLE haloes in Fig. 4.4, this is less than a factor of 2. Because the hot-gas density profiles also do not change significantly when the strength of
Appendix D. Model-equivalent cooling rates from EAGLE

Figure D.1: Normalised histograms for the relative analytic cooling rates calculated for model density profiles of hot gas versus those using the actual density, temperature, and metallicity profiles from our sample of Reference and NoAGN EAGLE haloes, as labelled. The left and middle panels compare gross cooling rates (Eq. 4.6), while the right-hand panel compares net cooling rates (Eq. D.2). If the analytic approximations were sufficient for calculating cooling rates, the distributions would be δ functions at the vertical dashed line.

feedback is varied, the gross cooling rates calculated by Eq. 4.6 should agree for the Reference and alternate-feedback simulations. To demonstrate one example, we compare \( \dot{m}_{cool} \) for the NoAGN simulation, as we did for the Reference simulation, in the middle panel of Fig. D.1. While there are small differences compared with the left-hand panel, the evolution of the distributions and their peaks are in agreement. Results for WeakFB and StrongFB are indeed similar, so we omit them for simplicity.

While stellar feedback is often considered independently of calculating cooling rates in semi-analytic models (but not always – see, e.g., Monaco et al. 2007), radio mode AGN feedback is generally implemented by directly suppressing the cooling rates calculated through Eq. 4.6 (see Bower et al. 2006; Croton et al. 2006). Galaxies hosting a larger supermassive black hole will typically have their cooling suppressed more. At later times, the haloes in our sample are bigger (see Fig. 4.2) and host heavier black holes. Because the suppressive heating from AGN need not be tied directly to the gross cooling rates of haloes, ratios of net cooling rates as calculated in semi-analytic models with different hot-gas radial profiles will not be the same as the ratio of gross cooling rates. They will, in fact, be systematically smaller for systems with an AGN, effectively counteracting the redshift evolution of the gross cooling rate ratios presented in the left (and central) panels of Fig D.1.

To demonstrate the impact AGN heating might have on cooling rates in semi-analytic model, we use an example model of AGN heating to calculate effective net cooling rates for the haloes. We calculate the heating rate based on energy released from Bondi–Hoyle accretion (Bondi 1952) of hot gas in the halo,

\[
\dot{m}_{heat, model} = \frac{15\pi}{16} \frac{(\mu m_p)^2}{\Lambda(T_{vir}, Z_{hot})} G c^2 \kappa \eta m_{BH}
\]
(as implemented similarly in Croton et al. 2006; Somerville et al. 2008b), where $m_{BH}$ is the black-hole mass, $\kappa$ is a model parameter used to control the strength of feedback, and $\eta$ is the efficiency with which the inertial mass of particles is released during accretion onto the black hole. Here, we assume typical values of $\kappa = \eta = 0.1$. The net cooling rate of gas in a halo can then be found as the difference between Eqs. 4.6 & D.1:

$$\dot{m}_{\text{net, model}} = \dot{m}_{\text{cool, model}} - \dot{m}_{\text{heat, model}}.$$  

(D.2)

We calculate $\dot{m}_{\text{net}}$ for our haloes in the Reference simulation, using the true and analytic (singular isothermal sphere and $\beta$) radial profiles. In the right-hand panel of Fig. D.1 we present the ratio of the model-equivalent net cooling rates using the EAGLE radial profiles (density, temperature, and metallicity) to the singular isothermal sphere profile. Not only is the evolution of the distributions minimised compared to the ratio of gross cooling rates, but the widths of the distributions are also greatly reduced. The latter would not be true if the central temperature and metallicity of the hot haloes were used in the Bondi–Hoyle model, however.

\footnote{94% of the (sub)haloes in our sample have more than one black-hole particle. In a semi-analytic model, galaxies typically are only allowed one black hole; when a merger brings in a new black hole, it is assumed to merge immediately with any pre-existing black hole. To mimic this, we sum the masses of all black holes in an EAGLE galaxy to calculate $\dot{m}_{\text{heat, model}}.$}
E.1 The Algorithm

We know the gradient of the baryonic mass profiles must be steeper than $\xi$ (Eq. 5.3) for $R \ll R_{\text{vir}}$, as baryons are more concentrated toward the centre of (sub)haloes (i.e. in galaxies). This places an upper limit on $\xi$ of 1. Also, because the profiles are cumulative, $\xi \geq 0$. As such, our algorithm begins by locating the point on each profile where the gradient is 1, following its initial climb. Because $\xi < 1$, this provides a lower limit for the aperture radius. The profile from this point onward is then extracted and a straight line fitted. Parts of the profile where the cumulative baryonic mass fraction exceeds a separation of $\epsilon$ from the fitted line are then removed and the line refitted. This is done continuously until a converged result is found. The lowest of the remaining points of the profile is then taken to be the radius of the aperture.

For the results presented in Chapter 5, we applied the algorithm with $\epsilon = 0.01$. A sufficiently small $\epsilon$ is needed to return an aperture radius above the lower limit, while a value too small would exclude the majority of the profile from the final fit. An appropriately chosen $\epsilon$ also comes with the advantage of dealing with late rises in profiles caused by satellites and other substructure (no subhalo finder is perfect at locating all satellites, as was the case on several occasions for these simulations, e.g. as presented in Fig. 5.4). The top profile in Fig. E.1 displays the success here with $\epsilon = 0.01$. Should any substructure lie beyond the lower limit of the aperture, the final radius will be found internal to that substructure, hence eliminating its contribution to the (sub)halo. Should sharp rises in the profiles occur internal to the lower limit, we determine the responsible substructure to be sufficiently close to / merged with the central galaxy to be considered the same system (also seen in the top profile of Fig. E.1). A PYTHON routine for calculating the BaryMP radius has been published in appendix A of Stevens et al. (2014).
Figure E.1: Illustration of the BaryMP algorithm for two example M12 haloes. Red, solid curves show the baryonic mass profiles of the haloes (with AHF-identified substructure and hot gas stripped). Black crosses mark where the profiles have a gradient of 1, while black, dashed lines show the initial linear fit from these points onward. After several iterations of removing parts of the profile separated from the linear fit by $\epsilon = 0.01$ and refitting the line, the final parts of the profile fitted are given in green. The aperture radius is taken at the lowest-radius point on the green parts of each profile, emphasised by the blue plus-filled circles. The effect of bumps in profiles caused by unidentified satellites are shown to be minimised by the algorithm. The final linear fits for these examples almost overlay the initial fits exactly (but are not shown).

E.2 The effect of the $\epsilon$ parameter

While we have qualitatively described that choosing $\epsilon = 0.01$ works reasonably well, there was no specific motivation behind this exact value. As such, we briefly assess the variation in BaryMP results induced from different choices of $\epsilon$. To do this, we recalculated the BaryMP radii for each snapshot of the M12 simulations where $z < 2$ for 198 uniformly spaced values of $\epsilon$ between $10^{-3}$ and $10^{-1}$, recording the (cold) baryonic content within each radius and the fraction of the profile (beyond the lower limit, where the gradient is 1) used in the final linear fit. We present the average of each of these values as a function of $\epsilon$ in Fig. E.2.

On average, for $\epsilon \gtrsim 0.1$, the linear fit is converged on its first attempt. That is, the entirety of the profile beyond the lower limit deviates by $<0.1$ in the $y$-direction from the initial fit. This places an obvious upper limit on an appropriate value of $\epsilon$. The BaryMP technique’s success lies in its ability to ignore both early parts of the profile with varying gradient and straggling satellites that cause bumps later in the profile. In other words, it works well if a fraction, but not too large of a fraction, of the profile is removed for the final linear fit. Fig. E.2 therefore suggests it would be appropriate if $0.01 \lesssim \epsilon \lesssim 0.04$ (those values corresponding to average profile fractions of 0.5 and 0.93, respectively). Within this range, the average $R_{\text{BaryMP}} / R_{\text{vir}}$ value clearly varies. Despite this, however, the variation in the average enclosed baryonic content is small.
E.2. The effect of the $\epsilon$ parameter

Figure E.2: Average variation in results from the BaryMP technique for the M12 simulations as a function of the profile-fit separation threshold, $\epsilon$. The green, dashed curve gives the fraction of the profile beyond the lower limit used in final linear fit. The blue, dot-dashed curve gives the average aperture radius (as a fraction of the virial radius) inferred from that fit. Note that these values have been normalised to fit within the axes ($\zeta = 2.745$). The black, solid curve indicates the (cold) baryonic content within those apertures. The red, vertical line shows our choice of $\epsilon = 0.01$.

With Fig. E.3, we specifically assess how the BaryMP aperture radius and enclosed baryonic content would change relative to our currently presented results if we picked a different $\epsilon$ between 0.01 and 0.04. A choice of $\epsilon = 0.04$ would decrease the average BaryMP radius by 24%, with individual cases potentially having their radii halved. These rarer cases occur where an especially large radius was found for $\epsilon = 0.01$ (a prominent bump in the blue curve in Fig. 5.11a and subtler bumps in Figs. 5.6b and 5.6c all show small populations of galaxies with $R_{\text{BaryMP}} \gtrsim 0.6R_{\text{vir}}$). These anomalies crop up at various values of $\epsilon$, but do not greatly affect the values of integrated properties. As shown by Fig. E.3, the average enclosed baryonic mass decreases by only 4% for $\epsilon = 0.04$, where decreases of more than 6% happen for less than 16% of the M12 simulation snapshots.

The noticeable dependence of $R_{\text{BaryMP}}$ on $\epsilon$ emphasises the difficulty of defining or locating where a galaxy ends. The lack of dependence on $\epsilon$ for the baryonic mass suggests our choice of $\epsilon$ should not affect our conclusions surrounding the integrated properties of galaxies, and shows that the BaryMP method provides a relatively consistent means of measuring them.
Appendix E. The BaryMP aperture

Figure E.3: BaryMP aperture radius and enclosed (cold) baryonic mass as a function of $\epsilon$, normalised to the choice of $\epsilon = 0.01$. The blue, dot-dashed curve gives the median aperture radius over all the M12 simulation snapshots for $z < 2$, with the blue shaded region covering the central 68% of the values. Similarly, the black curve and shaded region show the median and 68% spread for the baryonic mass, respectively.