Fishnet metamaterial with double negative refractive index in blue region of visible spectrum

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ABSTRACT

By optimizing the shape and scale of perforations as well as the depth of different layers, we obtained a metal-dielectric-metal double fishnet metamaterial with an unbroken 38-nm bandwidth of negative refractive index in visible spectrum, spanning from 459 to 497 nm. Moreover, the real parts of permittivity and permeability of this metamaterial are simultaneously negative from 460 to 478 nm in wavelength, reaching a frequency band as high as to the blue region — a territory that has never been explored before in visible spectrum.

Keywords: Fishnet metamaterial, double negative refractive index, shape optimization

1. INTRODUCTION

The seminal concept of Negative Refraction Index (NRI) proposed by Veselago in 1968\textsuperscript{1} did not attract a great amount of attention until it was experimentally validated in 2000.\textsuperscript{2} Since then, extensive research efforts have been devoted to this emerging area termed as "metamaterials" or "left-handed materials" and a variety of exceptional electromagnetic properties have been demonstrated and are being exploited for a broad range of applications, such as near field focusing,\textsuperscript{3} invisibility clock\textsuperscript{4} and perfect optical absorption.\textsuperscript{5} NRI denotes the real part of the index of refraction \( n' < 0 \), which can be guaranteed as long as the real (') and imaginary (") parts of the magnetic permeability \( \mu \) and electrical permittivity \( \epsilon \) satisfy \( \mu' \epsilon'' + \mu'' \epsilon' < 0 \).\textsuperscript{6} Such a relationship suggests that NRI can be obtained by either single-negative or double-negative (\( \mu' \) and \( \epsilon' \) are simultaneously negative) metamaterial.\textsuperscript{7} Double-negative Refractive Index (DNRI) is particularly useful as it avoids high-loss of properties.\textsuperscript{8} NRI could be achieved by the coincidence of magnetic and electric resonances respectively offered by Separated Ring Resonator (SRR) and straight line in the same microwave frequency regime.\textsuperscript{9} Such an overlapped regime is typically too low in frequency regime to be utilized for most optical applications. As a matter of fact it is highly desirable to render NRI in higher frequency regime, especially in the visible spectrum.

As the resonant frequencies of SRR and straight line are limited to the microwave regime, metal-dielectric-metal double fishnet structures perforated with an array of holes were devised to render refractive index negative in optical regime in recent years.\textsuperscript{11, 12} These structures sparked widespread interest because electromagnetic resonances in such metallodielectric composites can be tuned across a band of frequencies by altering the shapes of the holes, which are generally fabrication-friendly for mass production at nanoscale. Rectangular, square and circular-shaped holes were investigated with regard to the Figure of Merit (FOM=\(|n'/n''|\)) and it was found that FOM could differ by a factor of 2.5 times with different holes.\textsuperscript{13} In the original report for the fishnet metamaterials perforated with nearly square holes,\textsuperscript{10, 12} NRI occurs circa 780 nm. Through adopting 7-layer (4 metal and 3 dielectric layers) fishnet structures perforated by square holes, Garcia-Meca et al. observed wider NRI bandwidths spanning over 620-713 nm and 693-806 nm for two different fishnet metamaterials, which differed in the periodicity and size of the perforations.\textsuperscript{5} As the visible spectrum ranges from 390 to 780 nm, the maximum continuous NRI bandwidth of such multilayered structures in the visible range is 97 nm. If a multilayered fishnet structure perforated by crescent-shaped dielectric cylinders, rather than by rectangle holes as reported in,\textsuperscript{14} extends infinitely along the direction of incident light,\textsuperscript{15} it has three bands of NRI spanning over 628-645, 750-1005 and 1790-2000 nm, respectively.

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2. METHODOLOGY

Despite significant progress in ushering NRI to the frequency of red regime, NRI in higher frequency bands covering yellow and blue regimes is yet to be realized. Now that deliberately tuning the geometrical parameters of the perforations can make electric and magnetic resonances spectrally overlap, it is promising to obtain a higher frequency band in this way. However, such an approach has not been sufficiently explored to date because of the significant challenges in shape optimization for electromagnetic problems. As Gielis' superformula\textsuperscript{16} gives an extremely simple yet flexible expression of a class of complex shapes through very few parameters, it inspires us to take advantage of the gradient-free evolutionary algorithm\textsuperscript{17} to seek optimal shape of the perforations for NRI in higher frequency regime.

Figure 1(a) schematically illustrates the 3-layer double fishnet metamaterial whose top/bottom metal and middle dielectric layers are spread in the horizontal ($x - y$) plane infinitely. The structure is vertically ($z$) perforated by periodical holes whose shapes are defined by Gielis' superformula. On account of its negative permittivity and low loss at visible frequencies\textsuperscript{18} and sensitivity to the perforations shape,\textsuperscript{19} silver is selected as the constitute metal. Since the widely-used Drude model for frequency-dependent permittivity of silver might be invalid in the optical regime,\textsuperscript{18} Johnson-Christy data,\textsuperscript{20} specially measured for thin film metals, are adopted herein. Akin to the original work on the fishnet metamaterial,\textsuperscript{12} the dielectric layer is composed of MgF\textsubscript{2} with $n = 1.38$ and the structure is located on a 5-nm indium-tin-oxide (ITO with $n = 1.5$) layer to protect it from charging effects in a fabrication process such as electron-beam-writing. Considering the periodicity of the structure, a unit cell model (Figure 1(b)) is extracted for simulation in \textit{CST MicroWave Studio}, a commercial electromagnetic mode solver with finite integration technique.\textsuperscript{21} The incident radiation is applied on the external surface with the $y$-axis polarization (electric field $E$) and $z$-axis Poynting vector $k$. The transmittance and reflectance of the waves measured at two parallel surfaces above and below the fishnet structure are used to retrieve the effective permeability $\mu$ and effective permittivity $\epsilon$ by an algorithm considering the bianisotropy of the structure.\textsuperscript{22} The branch of the $n'$ is determined in terms of Kramers-Kronig relationship.\textsuperscript{23} A good agreement between our simulation results and the experiment-verified data reported in\textsuperscript{12} is obtained, from which the reliability of the effective parameters $\mu$, $\epsilon$ and $n$ in this paper is guaranteed.

We seek to express the hole in Figure 1(b) by using Gielis’ superformula here,\textsuperscript{16} which defines the shape in cylindrical coordinate as $x = r(\theta) \cos(\theta)$ and $y = r(\theta) \sin(\theta)$, where the ranges of azimuthal angle is $-\pi \leq \theta \leq \pi$. The radius of Gielis' shape is defined by:

$$r(\theta) = (|\cos(m\theta/4)/a|^{n_2} + |\sin(m\theta/4)/b|^{n_3})^{-1/n_1}$$

The unit cell is periodic with a repeat distance $p$ in the order of 300 nm to maintain consistency with the samples in.\textsuperscript{12} The size of this enclosed geometry is determined by factors $a$ and $b$ while its shape by the parameters $n_1$, $n_2$ and $n_3$. The number of rotational symmetries is governed by parameter $m$, which is an integer to denote
the number of vertices. As the odd \( m \) leads to unsymmetrical shape on which the periodical boundary condition cannot be applied on, a symmetrical shape by reflecting the shape in the first quadrant about the \( x \)-axis and \( y \)-axis is used in such a situation. To decrease the number of design parameters, \( n_2 = n_3 \) and \( a = b \) are imposed without sacrificing shape diversity. Following abovementioned prerequisites, a variety of complex shapes derived from Gielis’ superformula are illustrated in Figure 2(a) with different colours. It is noted that in addition to common shapes such as conventional square, circle and regular hexagon, several gear/star-like geometries can be obtained from Gielis’ superformula. Considering that the thicknesses of the metal \( (d_1 \text{ and } d_2) \) and dielectric \( (d_3) \) layers are of great significance to the NRI bandwidth, they are considered as the design variables. Thus the shape optimization becomes searching for a stationary point in the hyperspace defined by parameters \( (a, \ldots) \).
Figure 3. (a) Magnitude; and (b) phase of the simulated S parameters for the optimized structure; Retrieved (c) permittivity; (d) permeability; (e) impedance; and (f) refraction index.

$m, n_1, n_2, d_1, d_2, d_3$ so that the highest working frequency for NRI is achieved. Since the number of design variables to be determined is relatively small, we adopted the evolutional algorithm\textsuperscript{17} for the optimization. Such a non-gradient method is fairly simple but rather efficient, especially suitable for optical optimization problems in which the gradient of the cost function with respect to the design variables is sometimes too sensitive to be stabilized.\textsuperscript{24–27}

The evolutionary algorithm starts from a set of parent vectors randomly selected in the given parametric space. For each parent vector, a fitness (to be maximized) is obtained by solving Maxwell’s equations. Then an offspring is introduced by a mutation rule defined as the summation of the weighted difference between a pair of parent vectors and the third one. The offspring is required to mend by a cross-over process for improving the diversity. In this step the weighing factor and cross-over probability are set to be 0.5 and 1, respectively, for the following examples. The mutation and cross-over processes are repeated until the best performance is obtained.

The upper and lower bounds of design variables must be determined before the optimization and they should be wide enough to encompass the feasible values. Through a number of tests, we took the allowable values for $m$ ranging from 1 to 32 and the shape parameters $n_1$, and $n_2$ ranging from 0.01 to 200. The optimized fishnet structure is obtained from the optimization and its shape is shown in Figure 2(b) in which only the area enclosed by the black dished lines is used to obtain the symmetrical unit cell. The periodic pattern of this fishnet metamaterial is illustrated in Figure 2(c). In the simulation, the unit cell is not the black dashed-line box in Figure 2(c) but the one shown as in Figure 2(d), which is a mesh view in the $x$–$y$ plane with $z = 0$. The mesh size is as small as to 2 nm, leading to $150 \times 150$ elements in $x$–$y$ plane that can describe the complex

<table>
<thead>
<tr>
<th>Structure</th>
<th>$m$</th>
<th>$n_1$</th>
<th>$n_2$</th>
<th>$d_1$ (nm)</th>
<th>$d_2$ (nm)</th>
<th>$d_3$ (nm)</th>
<th>$\epsilon', \mu' &lt; 0$ (nm)</th>
<th>DNRI (nm)</th>
<th>$n' &lt; 0$ (nm)</th>
<th>NRI (nm)</th>
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<tbody>
<tr>
<td>Dolling$^\text{a}$\textsuperscript{12}</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>40</td>
<td>40</td>
<td>17</td>
<td>-</td>
<td>-</td>
<td>762-813</td>
<td>51</td>
</tr>
<tr>
<td>Optimized</td>
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<td>166.0472</td>
<td>31.4227</td>
<td>31.4227</td>
<td>47.3656</td>
<td>460-478</td>
<td>18</td>
<td>459-497</td>
<td>38</td>
</tr>
</tbody>
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Table 1. Gielis’ parameters and index features of the optimized structures in comparison with previous work.
geometry of the air parts within allowable error. Gielis’ parameters and index features of the optimized structure in comparison with Dolling’s structure in\textsuperscript{12} are listed in Table 1.

By applying periodic boundary conditions on the two sets of opposite surfaces parallel to $z$ axis, we obtained the magnitude (Figure 3(a)) and phase (Figure 3(b)) of the $S$ parameters for the optimized fishnet structure. The retrieved material parameters are presented in Figures 3(c)-(f). The dip in the phase of $S_{21}$ indicates the presence of a negative index band, confirmed by the retrieved index in Figure 3(f), where NRI ranges from 459 to 497 nm. The overlapping negative band of $\epsilon'$ and $\mu'$ in Figure 3(d) (cyan area) shows that this structure indeed has a DNRI from 460 to 478 nm, a bandwidth of 18 nm that is slightly narrower than the reported widest 19 nm DNRI metamaterial around 813 nm.\textsuperscript{28} However, this structure providers DNRI in the blue region of visible spectrum, which is firstly achieved in this higher frequency regime.

The effective refraction index $n'$ of the optimized fishnet metamaterial (shown as in 4(c)) is compared with that of Dolling’s structure (Figure 4(b)) in Figure 4(a). Note that the optimized structure exhibits NRI and DNRI in blue region—a territory that has never been explored before. Whilst NRI of Dolling’s structure occurs merely in the infrared range. It was reported that a 3-layer fishnet structure perforated with square holes can intrudes yellow region only with NRI.\textsuperscript{10} Since the optimized structure render index negative in shorter wavelength as low to 459 nm, it is of considerable potential for many applications, e.g. superlens and biosensors, which require higher working frequency for excitation to attain larger optical resolution.\textsuperscript{10}

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REFERENCES


