A combined measurement of cosmic growth and expansion from clusters of galaxies, the CMB and galaxy clustering

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ABSTRACT
Combining galaxy cluster data from the ROSAT All-Sky Survey and the Chandra X-ray Observatory, cosmic microwave background (CMB) data from the Wilkinson Microwave Anisotropy Probe, and galaxy clustering data from the WiggleZ Dark Energy Survey, the 6-degree Field Galaxy Survey and the Sloan Digital Sky Survey III, we test for consistency the cosmic growth of structure predicted by General Relativity (GR) and the cosmic expansion history predicted by the cosmological constant plus cold dark matter paradigm (ΛCDM). The combination of these three independent, well-studied measurements of the evolution of the mean energy density and its fluctuations is able to break strong degeneracies between model parameters. We model the key properties of cosmic growth with the normalization of the matter power spectrum, σ8, and the cosmic growth index, γ, and those of cosmic expansion with the mean matter density, Ωm, the Hubble constant, H0, and a kinematical parameter equivalent to that for the dark energy equation of state, w. For a spatially flat geometry, w = −1, and allowing for systematic uncertainties, we obtain σ8 = 0.785 ± 0.019 and γ = 0.570±0.064−0.063 (at the 68.3 per cent confidence level). Allowing both w and γ to vary we find w = −0.950±0.069−0.070 and γ = 0.533 ± 0.080. To further tighten the constraints on the expansion parameters, we also include supernova, Cepheid variable and baryon acoustic oscillation data. For w = −1, we have γ = 0.616 ± 0.061. For our most general model with a free w, we measure Ωm = 0.278±0.012−0.011, H0 = 70.0 ± 1.3 km s−1 Mpc−1 and w = −0.987±0.054−0.053 for the expansion parameters, and σ8 = 0.789 ± 0.019 and γ = 0.604 ± 0.078 for the growth parameters. These results are in excellent agreement with GR+ΛCDM (γ ≃ 0.55; w = −1) and represent the tightest and most robust simultaneous constraint on cosmic growth and expansion to date.

Key words: cosmological parameters – cosmology: observations – dark energy – large-scale structure of Universe – X-rays; galaxies: clusters.

1 INTRODUCTION
The unexpected measurement from Type Ia supernova (SNIa) data of late-time cosmic acceleration by Riess et al. (1998) and Perlmutter et al. (1999) initiated a series of theoretical and observational efforts to unveil the nature of its underlying cause. However, to this day it is still unclear whether the origin of this phenomenon is due to a new energy component, spurious cosmological assumptions or modifications of gravity at large scales. A number of theoretical approaches and observational probes have been developed to investigate these different possibilities (for recent reviews see Copeland, Sami & Tsujikawa 2006;Frieman, Turner & Huterer 2008; Allen, Evrard & Mantz 2011; Clifton et al. 2012; Weinberg et al. 2012). Current data on the energy content, geometry, and expansion and growth histories of the Universe do not show any deviation from...
the standard cosmological paradigm, ΛCDM (Allen et al. 2008; Mantz et al. 2008, 2010a; Vikhlinin et al. 2009; Percival et al. 2010; Blake et al. 2011a,b, 2012; Conley et al. 2011; Hinshaw et al. 2012; Reid et al. 2012; Suzuki et al. 2012). However, the cosmological constant model suffers from well-known, serious theoretical problems that present-day dark energy models have not been able to improve upon. For modified gravity models, various approaches have been developed:1(parametrized frameworks (Hu & Sawicki 2007; Amin, Wagoner & Blandford 2008; Bertschinger & Zakim 2008), consistency tests of General Relativity (GR) (Linder 2005; Linder & Cahn 2007; Zhang et al. 2007; Acquaviva et al. 2008; Di Porto & Amendola 2008; Nesseris & Perivolaropoulos 2008) and alternative theories of gravity (Dvali, Gabadadze & Porrati 2000; Arkani-Hamed et al. 2004; Carroll et al. 2004; Nicolis, Rattazzi & Trincherini 2009; de Rham, Gabadadze & Tolley 2011). Recent works have used a variety of experiments and data sets to constrain gravity properties and models and found no significant deviations from GR (see e.g. Rapetti et al. 2009, 2010; Schmidt et al. 2009; Daniel et al. 2010; Giammanasio et al. 2010; Reyes et al. 2010; Zhao et al. 2010, 2012; Hojjati, Pogosian & Zhao 2011; Wojtak, Hansen & Hjorth 2011; Basilakos & Pouri 2012; Hudson & Turnbull 2012; Lombriser et al. 2012; Samushia et al. 2013; Simpson et al. 2013). To further test the overall standard paradigm, GR + ΛCDM, it is crucial to use data sets able to robustly constrain the key properties of the model, and to combine complementary data sets to break the degeneracies between the model parameters.

Rapetti et al. (2010, hereafter R10) tested GR using ROSAT All-Sky Survey (RASS) and Chandra X-ray Observatory (CXO) data of cluster abundance and scaling relations (Mantz et al. 2010a,b, hereafter M10a,b). R10 obtained strong constraints on GR, even when marginalizing over conservative systematic and astrophysical modelling uncertainties in the evolution of the cluster X-ray luminosity–mass relation. When combining the cluster growth data with measurements of the anisotropies in the cosmic microwave background (CMB) from the Wilkinson Microwave Anisotropy Probe (WMAP; Spergel et al. 2003, 2007; Dunkley et al. 2009; Komatsu et al. 2009, 2011; Bennett et al. 2012; Hinshaw et al. 2012, and companion papers), they highlighted a large degeneracy between γ and σ8, which limited the constraints on each parameter individually. Here we include complementary data that break this degeneracy. In particular, we use measurements of the growth rate and the Hubble parameter from joint redshift space distortions (RSD) and Alcock–Paczynski (AP) effect constraints (Blake et al. 2011c, hereafter B11) from the WiggleZ Dark Energy Survey (WiggleZ; Drinkwater et al. 2010). We also use a low-redshift RSD constraint (Beutler et al. 2012) from the final data release (DR3) of the 6-degree Field Galaxy Survey (6dFGS; Jones et al. 2009) and an RSD and AP constraint from the latest data release (DR9) of the Sloan Digital Sky Survey (SDSS) III Baryon Oscillation Spectroscopic Survey (BOSS; Reid et al. 2012).

In addition to our primary data sets, and to tighten the constraints on the expansion parameters, we also present results including data from the Union II SNIa sample (Suzuki et al. 2012), baryon acoustic oscillation (BAO) measurements from a combined analysis (Percival et al. 2010) of 2-degree Field Galaxy Redshift Survey (2dFGRS; Colless et al. 2003) and SDSS-Ⅱ DR7 (Abazajian et al. 2009) data as well as from a recent analysis (Reid et al. 2012) of SDSS-Ⅲ BOSS data at a higher redshift, and H0 measurements from the Supernovae and H0 for the Equation of State programme (SH0ES; Riess et al. 2011).

We find that by combining cluster, CMB and galaxy data, we are able to break the key degeneracies between γ and σ8 and obtain tight and robust constraints on cosmic expansion and growth. We model the expansion primarily with Ωm, H0, and w and the growth with σ8 and γ. We find that, individually, the CMB and galaxy data have large degeneracies in the growth plane but that, crucially, these degeneracies are nearly orthogonal. The individual and combined constraints from cluster, CMB and galaxy data are consistent with one another, making this a very robust measurement, and in good agreement with GR and ΛCDM. While individually clusters provide the tightest constraints on the growth plane, the combination of clusters, the CMB and galaxies provides significantly improved constraints and arguably the most robust measurement of cosmic structure growth to date.

2 COSMOLOGICAL MODEL

We adopt a purely phenomenological model to conveniently test the consistency of current observations with both the cosmic expansion history and the cosmic growth history predicted by ΛCDM + GR.

Our model assumes neither the existence of a new component, dark energy, nor a modification of GR. Instead, the parameters of the model represent departures from key kinematical and dynamical features of ΛCDM + GR. Deviations from such benchmarks would indicate disagreement of the observed evolution of the background and density perturbations with the standard cosmological paradigm.2

2.1 Cosmic expansion history

We model the expansion history using the evolution parameter $E(a) \equiv H(a)/H_0$, where $H(a)$ is the Hubble parameter as a function of the scale factor $a$ and $H_0$ its present-day value. We parametrize $E(a)$ as follows:

$$E(a) = \left[\Omega_m a^{-3} + (1 - \Omega_m) a^{-3(1+w)}\right]^{1/2}.$$  

$\Omega_m$ is the present, mean matter density in units of the critical density of the Universe and $w$ a kinematical parameter inspired by the dark energy equation of state. Since for our test we do not assume any particular scenario for cosmic acceleration, such as dark energy, we use $w$ only to conveniently fit expansion history data, matching ΛCDM for $w = -1$. Below, we present results for two expansion models, $w = -1$ (ΛCDM) and $w$ constant ($w$CDM). For both cases, we assume a spatially flat geometry (i.e. the curvature energy density $\Omega_k = 0$).3

1 Similar approaches can also be used to study clustering dark energy models.

2 Note that different physical scenarios can cause similar departures from this paradigm. For example, specific models of clustering or interacting dark energy and of modified gravity might provide similar deviations from the density perturbations of ΛCDM + GR.

3 Using cluster, CMB and SNIa data, Rapetti et al. (2009) found a negligible correlation between $\Omega_k$ and $\gamma$. They also showed that the constraints on $\gamma$ were not significantly weaker when including $\Omega_k$ as a free parameter. Note also that if $\Omega_k$ were included as a free parameter, an extension of equation (2) proposed by Gong, Ishak, & Wang (2009) would fit better the predictions from GR.
2.2 Cosmic growth and cluster abundance

We model the growth history at late times by parametrizing the linear growth rate of density perturbations on large scales, $f(a)$, as a power law of the evolving mean matter density, $\Omega_m(a) = \Omega_m a^{-3} E(a)^{-2}$, such as (Peebles 1980; Wang & Steinhardt 1998; Linder 2005)

$$f(a) \equiv \frac{d \ln \delta}{d \ln a} = \Omega_m(a)^\gamma,$$  \hspace{1cm} (2)

where $\gamma$ is the growth index, for which we recover GR when $\gamma \simeq 0.55$. $\delta \equiv \delta \rho_m / \rho_m$ is the ratio of the comoving matter density fluctuations, $\delta \rho_m$, with respect to the cosmic mean, $\rho_m$. While at early times we assume GR, for $z < z_1$ we obtain $\delta(z)$ from equation (2) using as an initial condition $\delta(z_1)$ calculated within GR. Normalizing $\delta(z)$ to $\delta(z_1)$, we obtain the growth factor, $D(z) \equiv \delta(z)/\delta(z_1)$. Here we use $z_1 = 30$, which is well within the dark-matter-dominated era, when $f(a) \sim 1$ for both the $\gamma$-model (equation 2) and GR. We then calculate the matter power spectrum of such fluctuations for a given wavenumber, $k$, as

$$P(k, z) \propto k^{3-\gamma} T^2(k, z_1) D(z)^2,$$  \hspace{1cm} (3)

where $T(k, z_1)$ is the matter transfer function of GR in the synchronous gauge at redshift $z_1$ and $n_s$ the primordial scalar spectral index.

The variance of the linearly evolved density field, smoothed by a spherical top-hat window function of comoving radius $R$ enclosing mass $M = 4\pi R^3/3$, is

$$\sigma^2(M, z) = \frac{1}{2\pi^2} \int_0^\infty k^2 P(k, z) |W_M(k)|^2 dk,$$  \hspace{1cm} (4)

where $W_M(k)$ is the Fourier transform of the window function. From this expression, $\sigma_M^2$ is defined as the $z = 0$ variance in the density field at scales of $8 h^{-1} \text{ Mpc}$, where $\sigma_M$ is widely used as a parameter for the normalization of the matter power spectrum.

Here we use $\sigma(M, z)$ to calculate the abundance of dark matter haloes as a function of mass and redshift

$$n(M, z) = \int_0^M \frac{dM'}{M'} \frac{d \ln \sigma^{-1}}{dM'} D(z),$$  \hspace{1cm} (5)

where $\mathcal{F}(\sigma, z)$ is a convenient fitting formula obtained from large $N$-body simulations of dark matter particles (Tinker et al. 2008).

$$\mathcal{F}(\sigma, z) = A \left( \frac{\sigma}{\sigma_0} \right)^{-a} + 1 \ e^{-c/\sigma^2}.$$  \hspace{1cm} (6)

The parameters of this formula have a generic redshift dependence of the form $x(z) = x_0(1 + z)^m$, with $x$ representing $A, a, b$ or $c$. The values for each $x_0$ and $m$, are given in Tinker et al. (2008). As in M10a, we introduce an additional parameter, $\epsilon$, to account for residual systematic uncertainties in the evolution of $\mathcal{F}(\sigma, z)$ due to non-$\Lambda$CDM scenarios. Remarkably, $\mathcal{F}(\sigma, z)$ encapsulates the non-linear cosmic growth history and appears to be almost universal for a wide range of cosmologies (see R10 for more details).

We marginalize over the uncertainties in the parameters of $\mathcal{F}(\sigma, z)$, accounting for their covariance and for additional systematic uncertainties due to e.g. the presence of baryons following the method described in M10a. Note, though, that as shown in M10a the uncertainties in $\mathcal{F}(\sigma, z)$ are subdominant in the analysis. R10 also verified that $\epsilon$ is essentially uncorrelated with $\gamma$.

2.3 Integrated Sachs–Wolfe effect

In our CMB analysis we include the constraint on $\gamma$ from the integrated Sachs–Wolfe (ISW) effect of the CMB using the method and assumptions described by Rapetti et al. (2009) and R10. In brief, the low multipoles of the CMB are sensitive to the growth of cosmic structure due to the effect of the time-varying gravitational potentials of large-scale structures on the CMB photons crossing them. We calculate the contribution of these photons to the temperature anisotropy power spectrum as (Weller & Lewis 2003)

$$\Delta_{\text{ISW}}^T(k) = 2 \int dt \ e^{-\tau(t')} \phi(t') \ j(t(t' - t)),$$  \hspace{1cm} (7)

where $t$ is the conformal time and $t_0$ its present-day value, $\tau$ the optical depth to reionization, $j(x)$ the spherical Bessel function for the multipole $l$, and $\phi(t)$ the conformal time variation of the gravitational potential. Taking the derivative of the Poisson equation with respect to $t$, we calculate the latter quantity for the $\gamma$-model\(^6\) as $\phi(t) = 4\pi G a^{-2} k^{-2} H \delta \rho_m [1 - \Omega_m(a)^\gamma]$, where $H$ is the conformal Hubble parameter. Since the ISW effect is only relevant for $z < 2$, as an initial condition to solve this equation we match $\Delta_{\text{ISW}}^T(k)$ to that of GR at $z_1 = 2$.\(^7\)

Note, however, that the constraining power on $\gamma$ from the ISW effect is small compared to that of the cluster data (Rapetti et al. 2009). For the current analysis, the primary relevance of the CMB is its ability to tightly constrain the combination of growth parameters $\sigma_s$ and $\gamma$ (see Section 5).

2.4 The AP effect and RSD

The AP test is a geometrical means of probing the cosmological model by a comparison of the observed tangential and radial dimensions of objects which are assumed to be isotropic in the correct choice of model. It can be applied to the two-point statistics of galaxy clustering if the RSD, the principal additional source of anisotropy, can be successfully modelled (Ballinger, Peacock & Heavens 1996; Matsuura & Suto 1996; Matsubara 2000; Seo & Eisenstein 2003; Simpson & Peacock 2010). By equating radial and tangential physical scales, the AP test determines the observable $F(z) = (1 + z) D_s(z) H(z)/c$, where $D_s(z)$ is the physical angular diameter distance and $c$ is the speed of light.

In the model fit for $F(z)$, the normalized growth rate, $f_r(z)$, is determined simultaneously. Here $f_r(z)$ is again the logarithmic rate of change of the growth factor at redshift $z$ (see equation 2) and $\sigma_r(z) = |D(z)/D(0)| \sigma_s$. In B11, RSD were modelled using the fitting formulæ provided by Jennings, Baugh & Pascoli (2011) to determine the density–velocity and velocity–velocity power spectra, marginalizing over a linear bias factor. Tests were performed to

\(^4\)Many models of modified gravity predict a growth index that varies with time and length scale, $\gamma(a, k)$. Note again, though, that here we do not use this parameter as a diagnostic of the true theory of gravity, but rather as a consistency test for GR.

\(^5\)For current results, this value is a good approximation to be used as a GR reference. At higher accuracy, though, the growth index of GR has relatively small redshift and background parameter dependences (see e.g. Polarski & Gannouji 2008).

\(^6\)Since here we are testing GR, we assume no contributions to $\phi$ from the anisotropic stress and energy flux of the Weyl tensor (Challinor & Lasenby 1999).

\(^7\)For our analysis, the difference from calculating $\delta(z)$ using $z_i$ equal to $2$ or $30$ is negligible since both redshifts are well within the dark-matter-dominated era, when $f(a)$ tends to $1$ for any $\gamma$. 

We model the growth history at late times by parametrizing the linear growth rate of density perturbations on large scales, $f(a)$, as a power law of the evolving mean matter density, $\Omega_m(a) = \Omega_m a^{-3} E(a)^{-2}$, such as (Peebles 1980; Wang & Steinhardt 1998; Linder 2005)
ensure that the results were not very sensitive to the model used for the non-linear RSD, the real-space power spectrum or the range of scales fitted ($k_{\text{max}} < 0.2 \, h \, \text{Mpc}^{-1}$, for the measurements used here).

For a low-redshift survey such as 6dFGS, the AP distortion is negligible (since distances in $h^{-1}$ Mpc are approximately independent of the assumed cosmological model). For 6dFGS, the growth rate measurement of Beutler et al. (2012) was obtained by again assuming the model of Jennings et al. (2011) to describe non-linear RSD.

For the BOSS measurements of the RSD and AP effect, the modelling of the matter density and velocity fields was performed following the approach of Reid & White (2011). The latter uses perturbation theory to calculate the non-linear redshift space clustering of haloes in the quasi-linear regime and the halo model framework to describe the galaxy–halo relation. This model was tested against a large set of galaxy catalogues from N-body simulations and only fitted over those scales where the quasi-linear velocity field was thought to dominate the signal and the small-scale random velocities could be simply modelled and marginalized over.

For all the RSD and AP effect measurements employed in the paper (see Section 4.2), the parameters used to fit the 2D galaxy power spectrum and galaxy correlation function data have negligible covariance with the parameters in the current analysis. Also, the linear model as well as the non-linear corrections assumed in those analyses lie within the GR+$\Lambda$CDM paradigm tested here.8

3 PHYSICS OF THE OBSERVABLES

In this section, we describe the physical mechanisms behind the principal degeneracies between our most relevant growth and expansion parameters, for each of our primary observations.

3.1 CMB anisotropies

From the normalization and tilt of the CMB temperature anisotropy power spectrum, we can primarily constrain the scalar amplitude and spectral index of primordial fluctuations; from the position of its first peak, the mean energy density of curvature and dark energy; and from the amplitudes of the second and third peaks, those of dark and baryonic matter. These measurements provide strong constraints on the content of the background energy density and its linear density fluctuations at high redshift. For a given value of the growth index, $\gamma$, these translate into tight constraints on the amplitude of the matter power spectrum today, $\sigma_8$. A model with faster perturbation growth, i.e. with a small $\gamma$, implies large fluctuations today, i.e. large $\sigma_8$, and vice versa. This provides a large, negative correlation between $\sigma_8$ and $\gamma$ (see Fig. 1). At low redshift, the ISW effect of the CMB data (see Section 2.3) constrains $\gamma$, which is otherwise unconstrained by this data set.

3.2 Distribution of galaxies

From measurements of the anisotropic clustering of galaxies, we use constraints on the product $f(z)\sigma_8(z)$ and on the quantity $F(z)$, where the latter are purely expansion history constraints, i.e. on $\Omega_m(z)$.

Both of these constraints, from RSD and AP effect measurements, respectively, are required to measure $\gamma = \ln(f(z)/\Omega_m(z)$ from galaxy data alone. The current uncertainty on the linear galaxy bias, $b(z)$, limits the ability to measure $\sigma_8$ from the normalization of the galaxy power spectrum, which scales with $\sigma_8(z)b(z)$, and to measure $\sigma_8$ using RSD constraints on $f(z)/b(z)$, as previously commonly used. As proposed by Song & Percival (2009), here we use instead RSD constraints on $f(z)\sigma_8(z)$, which are independent of $b(z)$, and obtain a positive correlation between $\gamma$ and $\sigma_8$ (see Fig. 1) for a $\Lambda$CDM expansion model and data within a relatively low-$z$ range, where $f(z)$ increases towards 1. The faster the perturbations grow (small $\gamma$), the smaller the present-day perturbation amplitude, $\sigma_8$, needs to be to provide the same amount of anisotropy in the distribution of galaxies at redshift $z$. At higher $z$, where $f(z) \sim 1$ and $f(z)\sigma_8(z) \sim \sigma_8(z)$, the correlation between $\gamma$ and $\sigma_8$ becomes negative (see Section 3.3). Adding high-$z$ data from future missions will then help to break the large degeneracy of the current data between $\gamma$ and $\sigma_8$.

3.3 Cluster abundance and masses

For clusters, we have direct constraints on $\sigma_8(z)$ and $\Omega_m(z)$ from abundance, mass calibration and gas mass fraction data (see Sections 2.2 and 4.1). $\sigma_8(z)$ measurements provide us with constraints not only on $\sigma_8(z) = 0$, from the local cluster mass function, but also on the growth rate $f(z) = -(1+z)\ln\sigma_8(z)/dz$. Together, the constraints on $\sigma_8(z)$ and $\Omega_m(z)$ constrain $\gamma$.

The evolution of $\sigma_8(z) = \sigma_8(z)e^{-\gamma(z)}$ depends on $\gamma$, $\Omega_m$ and $w$ as follows:

$$g(z) = \int_0^z (1+z')^{-1} \left[ p(z') - 1 \right]^{-\gamma} p(z')^{-1} dz'$$

$$= \left( 3w^\gamma \right)^{-1} \left[ \lambda(z) - \lambda(0) \right],$$

where $\lambda(z) = \left[ p(z) - 1 \right]^{-1} p(z')^{-1} p(z')^{-1} 2F_1[1, 1; 1 + \gamma; p(z')], 2F_1$ is a hypergeometric function, $p(z) = p_0(1+z)^{-3w}$ and $p_0 = \Omega_m/(\Omega_m - 1)$. In practice, a negative degeneracy between $\sigma_8$ and $\gamma$ exists due to the limited precision of cluster mass estimates, but it is notably smaller than those described above (see Fig. 1). Within the precision of the data, indistinguishable cluster count evolution can be produced by models with e.g. $\sigma_8$ of 0.8 and a growth rate consistent with GR, or with a slightly larger present-day amplitude and faster growth (smaller $\gamma$), for which $\sigma_8(z)$ decreases with $z$ a bit more steeply.

For the $\gamma+w$CDM model, the dependence of $\sigma_8(z)$ on the product $w \gamma$ implies a negative correlation on the $w, \gamma$ plane (see Fig. 2). Within the precision of the data, a fast expansion history (small $w$) can be mimicked by a slow growth history (large $\gamma$), and vice versa.

4 DATA ANALYSIS

4.1 Galaxy cluster data

For clusters we use two experiments: growth of structure (M10a,b) and gas mass fraction ($f_{\text{gas}}$; Allen et al. 2008).11

8 Beutler et al. (2012) calculated the uncertainties in $F(z)$ and showed that they are unimportant.

9 The non-linear modelling from Jennings et al. (2011) used in the WiggleZ and 6dFGS analyses also encompasses a range of quintessence dark energy models.

10 In the same way as for the AP effect, the addition of the BAO constraints on $\Omega_m(z)$ improves significantly the measurement of $\gamma$ for the combination gal+$\Lambda$CDM (see the right-hand panel of Fig. 1).

11 Note that the cluster growth analysis employs the $f_{\text{gas}}$ analysis to calibrate the masses for the scaling relations of Section 4.1.1 using gas mass as a proxy for total mass (see details in M10a).
Following the methods developed by M10a,b for the cluster growth analysis, we self-consistently and simultaneously combine X-ray survey and follow-up observations to obtain the best constraints possible while accounting fully for selection biases. We employ the survey data to determine cluster abundances and the follow-up data to calibrate cluster masses from three observables: luminosity, temperature and X-ray emitting gas mass. For the survey data we employ three wide-area cluster samples drawn from RASS: the Bright Cluster Sample in the northern sky (BCS; \( z < 0.3 \) and \( F_X(0.1-2.4 \text{keV}) > 4.4 \times 10^{-12} \text{erg s}^{-1} \text{cm}^{-2} \)), the ROSAT-Einstein X-ray survey in the southern sky (REFLEX; \( z < 0.3 \) and \( F_X > 3.0 \times 10^{-12} \text{erg s}^{-1} \text{cm}^{-2} \)) and the Bright Massive Cluster Survey with \( \sim 55 \) per cent sky coverage (Bright MACS; \( 0.3 < z < 0.5 \) and \( F_X > 2 \times 10^{-12} \text{erg s}^{-1} \text{cm}^{-2} \)). To keep systematic uncertainties to a minimum and maintain a trivial constant scaling between X-ray gas mass and total mass, for all three samples we impose a lower luminosity cut of \( 2.5 \times 10^{44} \text{erg s}^{-1} \) (0.1–2.4 keV) leaving a total of 78 clusters from BCS, 126 clusters from REFLEX and 34 clusters from Bright MACS. In total, we use 238 clusters. For 94 of these clusters, we employ follow-up observations from XCO or pointed observations from ROSAT (M10b; distributed along the same redshift range of the survey data \( 0 < z < 0.5 \) to constrain simultaneously the luminosity–mass (\( L-M \)) and temperature–mass (\( T-M \)) relations using the model from M10b (see a brief description in Section 4.1.1).

For the \( f_{\text{gas}} \) analysis, we use the methods and data set of Allen et al. (2008) for 42 massive, hot \((kT \sim 5 \text{keV})\), dynamically relaxed, X-ray luminous galaxy clusters spanning the redshift range \( 0.5 < z < 1.1 \).

### 4.1.1 Scaling relation model

We model the \( L-M \) scaling relation as (M10b)

\[
\langle \ell(m) \rangle = \beta_{\ell m} t + \beta_{\ell m}^m m + \beta_{\ell m}^\sigma \log_{10}(1 + z) ,
\]

with a log-normal intrinsic scatter at a given mass

\[
\sigma_{\ell m}(z) = \sigma_{\ell m}(1 + \sigma_{\ell m}^\sigma z) ,
\]

where \( \ell \equiv \log_{10}[L_{500}(z)/10^{44} \text{erg s}^{-1}] \) and \( m \equiv \log_{10}[M_{500}(z)/10^{14} \text{M}_\odot] \). The subscript 500 refers to quantities measured within radius \( r_{500} \), at which the mean, enclosed density is 500 times the critical density of the Universe at redshift \( z \). We model the \( T-M \) scaling relation \( \langle t(m) \rangle \), where \( t \equiv \log_{10}(kT_{500}/\text{keV}) \), and its scatter \( \sigma_{\ell m}(z) \) (using the same equations (9) and (10)) but with the parameters \( \beta_{\ell m}^t, \beta_{\ell m}^{tm}, \beta_{\ell m}^\sigma, \sigma_{\ell m} \) and \( \sigma_{\ell m}^\sigma \) instead of those with index \( \ell \). When \( \beta_{\ell m}^t = 0 \) and \( \beta_{\ell m}^{tm} = 0 \), we have ‘self-similar’ evolution of the \( L-M \) and \( T-M \) relations, respectively (Kaiser 1986; Bryan & Norman 1998).\(^{12} \) \( \sigma_{\ell m}^t = 0 \) and \( \sigma_{\ell m}^\sigma = 0 \) correspond to scaling relations with non-evolving scatter.

M10b showed that current data do not require departures from self-similar evolution and constant scatter. R10 demonstrated that \( \gamma \) correlates weakly with departures from self-similarity and constant scatter in the \( L-M \) relation and negligibly for those in the \( T-M \) relation. Here we therefore assume self-similar evolution and constant scatter for both relations (\( \beta_{\ell m}^t = \beta_{\ell m}^{tm} = \beta_{\ell m}^\sigma = \sigma_{\ell m} = 0 \)).

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\(^{12} \) Self-similar evolution is entirely determined by the \( E(z) \) factors in the definitions of \( \ell, t \) and \( m \).

### 4.2 Galaxy clustering data

For WiggleZ, a series of growth and expansion analyses have recently been released, and here we build on one in particular: the joint analysis of the AP effect and growth of structure presented by B11, which contains four redshift bins of width \( \Delta z = 0.2 \), spanning the redshift range \( 0.1 < z < 0.9 \). The WiggleZ survey at the Australian Astronomical Observatory was designed to extend the study of large-scale structure over large cosmic volumes to higher redshifts \( z > 0.5 \), complementing SDSS observations at lower redshifts. The survey, which began in 2006 August, completed observations in 2011 January and has obtained of the order of 200 000 redshifts for UV-bright emission-line galaxies covering of the order of 1000 square degrees of equatorial sky.

For the WiggleZ analysis we fit our cosmological models to the joint measurements of RSD and AP distortion presented by B11. For this, we use the constraints obtained by B11 as a bivariate Gaussian likelihood for \( f \sigma_8(z) \) and \( F(z) \), including the large correlations between them. From B11, we have four bins with effective redshifts \( z = (0.22, 0.41, 0.60, 0.78) \) and \( f \sigma_8(z) = (0.53 \pm 0.14, 0.40 \pm 0.13, 0.37 \pm 0.08, 0.49 \pm 0.12) \) and \( F(z) = (0.28 \pm 0.04, 0.44 \pm 0.07, 0.68 \pm 0.06, 0.97 \pm 0.12) \) and correlation coefficients \( r = (0.83, 0.94, 0.89, 0.84) \).

For the 6dFGS analysis we use the growth rate of structure measurement obtained by Beutler et al. (2012). The 6dFGS is a combined redshift and peculiar velocity survey covering nearly the entire southern sky with the exception of a 10° band along the Galactic plane. Observed galaxies were selected from the 2MASS Extended Source Catalog (Jarrett et al. 2000) and the redshifts were obtained with the 6-degree Field multibreif instrument at the UK Schmidt Telescope between 2001 and 2006. The final 6dFGS sample contains about 125 000 galaxies in five bands distributed over ~17 000 square degrees with a mean redshift of \( z = 0.052 \).

For the analysis of the RSD from 6dFGS data, we use the constraints obtained by Beutler et al. (2012) as a Gaussian likelihood for \( f \sigma_8(z) = 0.423 \pm 0.055 \) at an effective redshift \( z = 0.067 \).

The analysis of the SDSS-III BOSS results from Reid et al. (2012) is based on the high-z sample CMASS, which consists of 264 283 galaxies in the redshift range 0.43 < \( z < 0.7 \) over 3275 square degrees. As part of SDSS-III (Eisenstein et al. 2011), BOSS has imaged the South Galactic sky for an additional 3100 square degrees over SDSS-II. This has increased the total sky coverage of SDSS imaging to 14 055 square degrees. As its primary goal, BOSS targets for spectroscopy luminous galaxies selected from the SDSS imaging. Within BOSS, CMASS is a roughly volume-limited sample of massive, luminous galaxies (for more detail, see e.g. Masters et al. 2011) tracing a cosmological volume at a high enough density to enable powerful statistical studies of large-scale structure.

For the analysis of the growth rate and AP effect measurements of CMASS BOSS, we use a bivariate Gaussian likelihood for \( f \sigma_8(z) = 0.43 \pm 0.07 \) and \( F(z) = 0.68 \pm 0.04 \) with a correlation coefficient \( r = 0.87 \) at an effective redshift \( z = 0.57 \) (Reid et al. 2012).\(^{13} \) Note that this redshift is similar to that of the third redshift bin of the WiggleZ analysis, \( z = 0.6 \). Due to the small overlap and the uncorrelated shot noise between the two surveys, their covariance should be minimal. Importantly, the results obtained by the two

\(^{13} \) For the results in Section 5 that include these and the distance-scale constraints from the BAO on the CMASS BOSS data, we extend this likelihood to account for the correlations between these three measurements as discussed in Section 4.4.
independent experiments, which target very different galaxy types, and require very specific studies of their non-linear properties and modelling uncertainties, are consistent.

4.3 CMB data

For the CMB analysis, we use the data and likelihood code\textsuperscript{14} from \textit{WMAP}.\textsuperscript{15} For the analyses including CMB data, we also fit for the mean physical baryon and dark matter densities, $\Omega_b h^2$ and $\Omega_c h^2$, the optical depth to reionization, $\tau$, the logarithm of the adiabatic scalar amplitude, $\ln(A_s)$, which is related to $\sigma_8$, and the adiabatic scalar spectral index, $n_s$. For these analyses, instead of $H_0$ we fit $\theta$, the (approximate) ratio of the sound horizon at last scattering to the angular diameter distance, which is less correlated with other parameters than $H_0$ (Kosowsky, Milosavljevic, & Jimenez 2002). We also marginalize over the amplitude of the Sunyaev–Zel’dovich effect from galaxy clusters, $0 < A_{SZ} < 2$ (Spergel et al. 2007).

4.4 Additional data sets

We also present results including constraints from the Union II SNIa data set of Suzuki et al. (2012), the SH0ES programme of Riess et al. (2011), and the BAO analyses of Percival et al. (2010), at two intermediate redshifts, and Reid et al. (2012), at a higher redshift.

The SNIa data set consists of a compilation of 580 SNIa from a variety of sources. For the likelihood analysis of these data, we use the cosmomc\textsuperscript{16} module of Suzuki et al. (2012), including their systematic uncertainties: 1 for CMB (see Section 4.3), 7 for additional data sets as those in Section 4.4. We also use a modified version of the code \textsc{camb}\textsuperscript{21} (Lewis, Challinor & Lasenby 2000) that includes $\gamma$ in the analysis of the ISW effect of the CMB data (Rapetti et al. 2009).

For our most general model, we simultaneously fit a total of 34 parameters. From these, 8 are cosmological parameters and 26 are used to model astrophysical variables and marginalize over systematic uncertainties: 1 for CMB (see Section 4.3), 7 for $f_{\text{gas}}$ (see details in Allen et al. 2008) and 18 for cluster growth data (see Sections 2.2 and 4.1.1, and M10a,b for full details).

For analyses without CMB data, we fix $n_s$ to 0.95 since, for such analyses, $n_s$ is degenerate with $\sigma_8$ (see M10a). For these analyses, we also use Gaussian priors on $H_0$ from the SH0ES programme (Riess et al. 2011), and $\Omega_b h^2 = 0.0213 \pm 0.0010$ from big bang nucleosynthesis (BBNS) studies (Pettini et al. 2008).

\textbf{5 RESULTS}

5.1 Constraints on the $\gamma$ + aCDM model

The left-hand panel of Fig. 1 shows the joint constraints in the $\sigma_8$, $\gamma$ plane for the $\gamma$ + aCDM model. The green contours show the constraints obtained from the RSD and AP effect data from WiggleZ, 6dFGS and CMASS BOSS (hereafter referred to as galaxy/gal data); the blue contours those from the CMB data; and the red contours

\textsuperscript{14} \url{http://lambda.gsfc.nasa.gov/}
\textsuperscript{15} Here we use the five-year WMAP data (Dunkley et al. 2009; Komatsu et al. 2009, and companion papers). For various of our results we have incorporated the galaxy data by importance sampling Markov Chain Monte Carlo (MCMC) chains that had clusters and WMAP5 data. Note that, for aCDM and wCDM models, the constraints on $\Omega_b h^2$, $\Omega_c h^2$ (see comments on this parameter in Section 5.2), $w$, $H_0$ and $\sigma_8$ from WMAP7 data alone do not differ significantly from those of WMAP5 (a maximum of 15 per cent in the errors of these parameters much less in most cases), and even less when combined with additional data sets as those in Section 4.4. We therefore expect a relatively small impact on the results from using WMAP7 instead. The new WMAP9 data, which appeared in the last stage of the present work, have up to 36 per cent better errors than WMAP5 for those parameters, and promises then a somewhat larger impact on the results.
\textsuperscript{16} \url{http://supernova.lbl.gov/Union/}
\textsuperscript{17} To calculate $z_d$ we use the exact expression (see e.g. appendix B of Hamann et al. 2010) instead of the approximate fitting formula of Eisenstein & Hu (1998). Note, however, that since Percival et al. (2010) and Reid et al. (2012) used the latter formula to fit the data, our results of the sound horizon need to be appropriately rescaled (see again Hamann et al. 2010). Besides being more accurate, the exact calculation of $z_d$ is independent of the standard assumptions used to obtain the fitting formula, and therefore valid for other models. Interestingly, though, for the extended models used here we find no significant differences in the results obtained from using either calculation.
\textsuperscript{18} \url{http://cosmologist.info/cosmomc/}
\textsuperscript{19} \url{http://www.slac.stanford.edu/~drapetti/fgas_module/}
\textsuperscript{20} We use these and standard modules for the other data sets in Section 4.4 either to run new MCMC chains or to perform importance sampling on existing ones. For selected examples, we have explicitly checked that both methods provide the same results.
\textsuperscript{21} \url{http://camb.info/}
those from the cluster abundance and $f_{\text{gas}}$ data (hereafter referred to as cluster/cl data). Combining the cluster+CMB+galaxy data, we obtain the tight constraints shown by the gold contours.

As shown in the figure, individually, the CMB and galaxy data exhibit significant degeneracies in the $\sigma_8$, $\gamma$ plane, as expected (see Section 3). For the cluster data, the correlation between these two parameters is much weaker, enabling independent constraints on both parameters. Importantly, the constraints from the three independent experiments (which are affected by very different systematic uncertainties) are in excellent agreement. This agreement motivates us to combine the constraints, leading to the results shown in the inner gold contours. Combining the three data sets we obtain marginalized constraints on $\gamma = 0.570^{+0.063}_{-0.054}$ (in good agreement with GR) and $\sigma_8 = 0.785 \pm 0.019$ (see also Table 1).

If we also include SNIa, BAO and the SH0ES measurement of $H_0$, the constraints on the growth parameters are, as expected, almost the same (see Table 1) although interestingly we obtain a small 4 per cent improvement in the error in $\gamma$. For this combination we also obtain improved, tight constraints on the expansion parameters $\Omega_m = 0.277 \pm 0.011$ and $H_0 = 70.2 \pm 1.0$ km s$^{-1}$ Mpc$^{-1}$. It is worth noting that the addition of the BAO data alone provides almost the same improved constraints on the $\Omega_m$, $H_0$ plane as those from adding all three data sets (see Table 1).

The right-hand panel of Fig. 1 shows a zoom into the central regions of the constraints shown in the left-hand panel, together with the constraints for the combinations of cl+CMB data (purple contours), cl+gal data (magenta contours) and CMB+gal data (turquoise contours). The gold, tightest contours correspond again to the combination of the three data sets, cl+CMB+gal. Notably, the nearly orthogonal degeneracies of the CMB (blue contours) and galaxy (green contours) constraints allow their combination (turquoise contours) to provide tight marginalized constraints on the growth plane. The area enclosed by the 95.4 per cent confidence contour in the $\sigma_8$, $\gamma$ plane is only slightly more than one third larger for CMB+gal than for the three data sets combined.

As found by R10, for the cl+CMB data (purple contours) $\sigma_8$ and $\gamma$ are highly correlated, with a correlation coefficient $\rho = -0.85$. The addition of the galaxy data breaks this degeneracy. With respect to constraints obtained from cl+CMB, those for the cl+CMB+gal provide more than a factor of 5 reduction in the area enclosed by the 95.4 per cent confidence contour in the $\sigma_8$, $\gamma$ plane.

In the right-hand panel of the figure, we also show the constraints from the combination gal+BAO (pale green contours), for which both data sets come from the analysis of different properties of galaxy redshift surveys. Interestingly, even though the BAO data on their own provide only constraints on expansion parameters, those on $\Omega_m$ help in reducing the large degeneracies that the galaxy growth data have in the $\Omega_m$, $\gamma$ and $\Omega_m$, $\sigma_8$ planes, with correlation coefficients of $\rho = 0.83$ and 0.74 for each plane. Adding BAO to gal we obtain then a significant improvement in the constraints on the growth plane $\sigma_8$, $\gamma$.

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22 When combining gal+BAO we also obtain results that are more comparable to those of the clusters due to the degeneracies broken by this combination (see further details in the text).

23 For this paper, we quote marginalized mean values and central credible intervals. For the latter, equal fractions of the volume of the posterior lies on each side of the interval (see e.g. Hamann et al. 2007). Instead, in R10 results were presented using marginalized peak values and minimal credible intervals, for which the size of the interval is minimized. For approximately symmetric posteriors, such as those for our combined data, both choices provide similar results, although by construction those from the former tend to be slightly more conservative. For our individual data sets, for which the posteriors are less symmetric, we show full marginalized distributions in Figs 4 and 5.

24 Note that the results presented in R10 were for a combination of cluster+CMB+SNIa+BAO data. However, the constraints on the $\sigma_8$, $\gamma$ plane were primarily driven by the cluster+CMB data.
5.2 Constraints on the $\gamma + \rho$CDM model

The left-hand panel of Fig. 2 shows the joint constraints in the $w$, $\gamma$ plane for the $\gamma + \rho$CDM model. For the combination of our primary data sets, cl+CMB+gal, we obtain the gold contours. For these, we find marginalized constraints on $w = -0.95_{-0.06}^{+0.06}$ and $\gamma = 0.533 \pm 0.080$ at the 68.3 per cent confidence level. These results are simultaneously consistent with GR and $\Lambda$CDM. The platinum contours in this panel show the joint constraints on the $w$, $\gamma$ plane when we add SNIa+SH0ES+BAO to the cl+CMB+gal data.25 In this case, we find marginalized constraints of $w = -0.98_{-0.05}^{+0.05}$ and $\gamma = 0.604 \pm 0.078$. Again the results are consistent with GR+$\Lambda$CDM.

In the right-hand panel of the figure, the purple contours correspond to cl+CMB, the magenta contours to cl+gal, the turquoise contours to CMB+gal and the gold contours again to the combination of the three data sets. The horizontal dashed and vertical dot-dashed lines mark $\gamma = 0.55$ (GR) and $w = -1$ ($\Lambda$CDM), respectively.

Comparing the cl+CMB with the cl+CMB+gal results, we find 46 and 62 per cent improvements in the constraints on $\gamma$ and $\sigma_8$.

It is also worth noting that the improvement in the joint measurement of $w$ and $\gamma$ is larger than that for each individual parameter. We find more than a factor of 3 reduction in the area enclosed by the 95.4 per cent confidence contour of the joint $w$, $\gamma$ constraints. Note that the correlation between $w$ and $\gamma$ increases from $\rho = -0.47$, for cl+CMB, to $\rho = -0.66$, for cl+CMB+gal, which suggests that additional constraints on $w$ might also help improving those on $\gamma$. In fact, even though SNIa and SH0ES data provide direct additional constraints on only cosmic expansion parameters, for which we obtain e.g. a 27 per cent improvement on $w$ when adding them to cl+CMB+gal, the combined, marginalized constraints on $\gamma$ represent a small improvement of 4 per cent due to the correlation between $w$ and $\gamma$. For these data sets combined, cl+CMB+gal+SNIa+SH0ES, the correlation in the $w$, $\gamma$ plane is still of $\rho = -0.65$. Interestingly, the correlation between $\gamma$ and $\Omega_\Lambda h^2$ is also relatively large, $\rho = 0.72$,26 which indicates that e.g. the significant improvements in the constraints on this parameter

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25 From Table 1, note that the main additional constraint on this plane comes from the SNIa data. The SH0ES and BAO data, though, significantly help in constraining the combination of parameters $\Omega_m$ and $H_0$.
from the CMB measurements of the Planck satellite\(^\text{27}\) should help in constraining $\gamma$.

Fig. 3 shows constraints for the same model and subsets of the data for three different planes: the growth plane $\sigma_8$, $\gamma$ (left-hand panel) and the expansion planes $\Omega_m$, $w$ (middle panel) and $\Omega_m$, $H_0$ (right-hand panel) for the $\gamma + w$CDM model. The left-hand panel of this figure shows that the correlation between $\sigma_8$ and $\gamma$ reduces dramatically from cl+CMB (purple contours), $\rho = -0.56$, to cl+CMB+gal (gold contours), a negligible $\rho = 0.08$. The reduction in the area enclosed by the 95.4 confidence contour in this growth plane when adding gal to the cl+CMB data is substantial.

The platinum contours in the three panels correspond to the constraints obtained when adding SNIa+SH0ES+BAO to the cl+CMB+gal data. The improvement in the growth plane of the left-hand panel of the figure is small, while those in the expansion planes of the middle and right-hand panels are significant due to the degeneracy breaking power of the additional data in these planes. For this model, the combined constraints on $\Omega_m = 0.278^{+0.012}_{-0.011}$ and $H_0 = 70.0 \pm 1.3$ km s$^{-1}$ Mpc$^{-1}$ are again very tight.

6 DISCUSSION

6.1 Comparing results

For a $\Lambda$CDM expansion model, and combining galaxy and CMB data, recent studies have presented constraints on $\gamma$ that are similar to and in agreement with ours. For example, Hudson & Turnbull (2012) combined data from two peculiar velocity surveys at low redshifts (Davis et al. 2011; Turnbull et al. 2012) and RSD (but not

\(^{27}\) http://www.esa.int/Planck
AP effect) data from various galaxy surveys. Samushia et al. (2013) used primarily RSD, AP effect and BAO data from the CMASS BOSS results of Reid et al. (2012) together with RSD and AP effect data from other surveys. In their combined results, both studies include WiggleZ, 6dFGS and CMASS BOSS data, as we do here, in addition to other galaxy and expansion data sets. Both analyses use CMB data from WMAP7. The former study uses previous results from CMB and expansion data only as a prior, while the latter uses the full CMB likelihood.28 Neither of these analyses, however, uses the low multipoles of the CMB to constrain $\gamma$ with the ISW effect (see Section 6.1.1).

Note also that these studies include BAO constraints from Percival et al. (2010) and Reid et al. (2012), respectively. As discussed in Section 6.1.2, both BAO data sets (and especially the latter) prefer larger values for $\Omega_m$, which in combination with growth data implies a preference for larger values of $\gamma$. This, together with the fact that these works do not include the cluster data or the ISW effect constraints from the CMB data, which both prefer smaller values for $\gamma$, is consistent with their results on $\gamma$ being at the high end of ours in Table 1. Although all these results and those in Table 1 are consistent with GR ($\gamma \simeq 0.55$), the differences highlight the importance of studying each individual data set as well as their various combinations in detail before combining all of them. For upcoming, more statistically powerful data sets, this will also be increasingly important.

6.1.1 ISW effect

Even though the ISW effect has only a relatively small impact on the combined results, it is not negligible. Using our analysis, it is interesting to compare results including or not the ISW effect for $\gamma$. For the $\gamma + \Lambda$CDM model and the combination CMB+gal, we obtain $\gamma = 0.607^{+0.078}_{-0.080}$ without the ISW effect, which as expected (see Section 6.2) is slightly higher (3 per cent) than our default result (see Table 1) and weaker by 8 per cent.29 For cl+CMB, we obtain $\gamma = 0.43^{+0.152}_{-0.153}$ and $\sigma_8 = 0.842 \pm 0.057$, which are 20 and 16 per cent weaker than our default results (see also a similar comparison in Rapetti et al. 2009). For cl+CMB+gal, $\gamma = 0.585 \pm 0.067$ is only 6 per cent weaker than the corresponding result including the ISW effect for $\gamma$.

6.1.2 Adding the BAO data

Our results show (see both Table 1 and the left-hand panel of Fig. 4) that, compared with the cluster and CMB data, the combination gal+BAO prefers significantly larger values of $\Omega_m$, and therefore of $\gamma$ due to the covariances between $\Omega_m$ and $\gamma$, and $\Omega_m$ and $\sigma_8$ (see Section 5.1).30 Also, for any of the data set combinations in Table 1, the addition of the BAO data shifts the constraints on $\Omega_m$ and $\gamma$ to larger values. Adding BAO to all the other data sets combined and for the $\gamma + w$CDM model, we have increases of 9 and 12 per cent for each parameter.31 It is interesting to note, though, that

28 Note that it is important to include $\gamma$ in the full CMB analysis, in combination with the other experiments, to account for all the degeneracies of the CMB parameters with both expansion and growth parameters, such as e.g. that of $\gamma$ with $\Omega_m h^2$ or the CMB shift parameter. If these covariances are not included, one may obtain spuriously tight results.

29 For the same data but for the $\gamma + w$CDM model, we obtain $\gamma = 0.547^{+0.092}_{-0.093}$, higher by 4 per cent and weaker by 5 per cent, and $w = -0.914 \pm 0.073$, tighter by 10 per cent. For this model, the background expansion parameter $w$ modifies the ISW effect in an approximately opposite way to that of the density perturbation parameter $\gamma$. Therefore, by artificially ignoring $\gamma$ in the calculation of the ISW effect, tighter constraints on $w$ are obtained. This is comparable to what happens for a $w$-model of a dark energy fluid when its dark energy perturbations are erroneously not taken into account (Weller & Lewis 2003; Bean & Dore 2004; Rapetti et al. 2005).

30 For $\gamma + \Lambda$CDM, using the CMB data alone we have $\Omega_m = 0.260 \pm 0.030$. For gal+BAO, using only the BAO data set from Percival et al. (2010), we obtain $\Omega_m = 0.345 \pm 0.050$ and $\gamma = 0.719^{+0.245}_{-0.244}$, and using instead only the BOSS BAO data set, $\Omega_m = 0.417 \pm 0.073$ and $\gamma = 0.974^{+0.350}_{-0.363}$, which are clearly larger.

31 Note that the shift between the gold solid and platinum solid-thin lines in Fig. 5 is mainly due to the addition of BAO.
using only the BOSS BAO data set, we obtain similar shifts of 7 and 11 per cent, although slightly weaker constraints on $\Omega_m$, and similar constraints on $\gamma$. Using instead only the BAO data set of Percival et al. (2010), we find about half of those increases, 5 per cent for both parameters, and also a bit weaker constraints on $\Omega_m$. The constraints on $\gamma$, though, are slightly tighter due to the reduction in the tension with the other data sets. The mild tension on $\Omega_m$ between the BAO and the other data sets translates in some cases into a smaller constraining power for $\gamma$ (and also for $w$) when combining them. Table 1 shows e.g. that for $\gamma+w$CDM, adding both BAO data sets to cl+CMB+gal provides somewhat weaker constraints on $\gamma$, and also that these are 13 per cent weaker than those for instead adding SNIa to cl+CMB+gal. In addition, using all the data sets combined except BAO, we obtain the tightest constraints on $\gamma$ for $\gamma+\Lambda$CDM, and on both $\gamma$ and $w$ for $\gamma+w$CDM. However, the increase in constraining power on these parameters is small compared with the decrease in constraining power on the other expansion parameters when not using BAO.

The BAO and SH0ES data also present a mild tension in the direction of the well-known degeneracy between $H_0$ and $\Omega_m$ (see e.g. Hinshaw et al. 2012). The addition of SH0ES to any of the data combinations in Table 1 that include our primary data sets shifts $H_0$ to larger values, and therefore $\Omega_m$ to smaller values through the correlation between these two parameters.

### 6.2 Constraining power

As discussed in Section 5.2, the combination CMB+gal provides tight constraints on the $\sigma_8$, $\gamma$ plane (see Fig. 1) due to the complementarity between the constraints from the individual data sets. However, the large degeneracies of the individual constraints make the combination prone to potential biases from systematic uncertainties. The left-hand panel of Fig. 4 shows that for galaxies alone (green long-dashed line) the marginalized pdf for $\gamma$ has a large tail towards values higher than that for GR, although interestingly the peak is close to the GR value (vertical dashed line).\footnote{Adding the expansion data set BAO to gal shortens this tail (see Fig. 1) and shifts the peak to a larger value (see Table 1).} On the other hand, for the CMB (blue dotted line) values larger than that for GR are significantly constrained by the data (due to the ISW effect), while lower values are largely unconstrained and degenerate with $\sigma_8$, which has an extended tail towards large values (see the right-hand panel of the figure).

From comparing the normalized pdf’s in the figure, it is worth noting that while the constraining power of the cluster data on $\gamma$ (left-hand panel) and $\sigma_8$ (right-hand panel) is notably better than that of the CMB or galaxy data, the combination cl+CMB+gal is much more powerful than the cluster data alone. Note also that the power of the current data for constraining $\sigma_8$ (right-hand panel) is considerably greater than for constraining $\gamma$ (left-hand panel).

#### 6.2.1 Full model: $\gamma+w$CDM

For our most general model, $\gamma+w$CDM, only the cluster data can alone constrain this model at a significant level. We obtain $w = -1.021^{+0.190}_{-0.187}$ and $\gamma = 0.507^{+0.256}_{-0.242}$ (see also Fig. 5).\footnote{For the combination gal+BAO (see Table 1), we obtain similar constraints on $w$ and $\gamma$, while those on $\Omega_m$ and $\sigma_8$ are notably weaker.} Since our other primary data sets do not have strong direct constraints on $\gamma$ (see Section 3), their constraining power depends critically on the complexity of the model used. For our extended model, we allow departures from the standard expansion and growth histories equally. Combining all our data sets, we obtain the tightest and most robust results to date on this model. The addition of SNIa, SH0ES and BAO data is particularly helpful for constraining the expansion parameters in this model. The right-hand panel of Fig. 5 shows that when we include these data sets (platinum solid thin line) the constraining power on $w$ clearly increases. The figure also shows the progression in the pdf’s of $\gamma$ (left-hand panel) and $w$ (right-hand

\[ \begin{align*}
& \gamma = 0.507^{+0.256}_{-0.242} \\
& w = -1.021^{+0.190}_{-0.187}
\end{align*} \]
panel) when adding one at a time the other primary data sets to the cluster data. Remarkably, for these combinations (as well as for the others of the primary data sets), we can measure at the same time $\gamma$ (cosmic growth) and $w$ (cosmic expansion) with similar precision.

7 CONCLUSIONS

We have combined cluster growth and $f_{\text{gas}}$ data from RASS and CXO, CMB data from WMAP, and RSD and AP effect data from WiggleZ, 6dFGS and CMASS BOSS to simultaneously constrain the evolution of cosmic structure and background expansion. To test for consistency with GR and ACDM, we have used convenient parameterizations: $\Omega_m$, $H_0$ and $w$ for the expansion history, and $\sigma_8$ and $\gamma$ for the growth history. We find that the combination of clusters+CMB+galaxies breaks key degeneracies in the growth plane, $\sigma_8$ versus $\gamma$, for the data sets individually. In combination, the data provide tight, robust constraints that are in excellent agreement with GR+ACDM.

Fixing $w = -1$, we obtain marginalized constraints on the growth parameters $\sigma_8 = 0.785 \pm 0.019$ and $\gamma = 0.570^{+0.064}_{-0.063}$. Including SNIa, SH0ES and BAO data, we obtain $\gamma = 0.616 \pm 0.061$. Allowing $w$ to vary, we have $\sigma_8 = 0.780 \pm 0.020$ and $\gamma = 0.533 \pm 0.080$ for the combination of clusters+CMB+galaxies. For this, we find a correlation between $w$ and $\gamma$ of $\rho = -0.66$. Including SNIa+SH0ES+BAO, we obtain $\Omega_m = 0.278^{+0.012}_{-0.011}$; $H_0 = 70.0 \pm 1.3 \text{ km s}^{-1} \text{ Mpc}^{-1}$ and $w = -0.987^{+0.053}_{-0.063}$ for the expansion parameters, and $\sigma_8 = 0.789 \pm 0.019$ and $\gamma = 0.604 \pm 0.078$ for the growth parameters.

Our results highlight the potential of combining forthcoming galaxy cluster data [from e.g. the South Pole Telescope (SPT), the Atacama Cosmology Telescope (ACT), XMM–Newton wide-area surveys, the Dark Energy Survey (DES), the Large Synoptic Survey Telescope (LSST), and the extended Röntgen Survey with an Imaging Telescope Array (eROSITA)], CMB data [from e.g. SPT, ACT and Planck] and galaxy data [from e.g. SDSS-III, the Subaru Measurement of Images and Redshifts (SuMIRe) project, BigBOSS, the Dark Energy Spectrometer (DESpec) and the Euclid mission] for constraining dark energy and modified gravity models.

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REFERENCES

Conley A. et al., 2011, ApJS, 192, 1
Eisenstein D. J. et al., 2011, AJ, 142, 72
Komatsu E. et al., 2011, ApJS, 192, 18
Linder E. V., 2005, Phys. Rev. D, 72, 043529