MECHANICAL PROPERTIES AND ENERGY ABSORPTION OF ALUMINIUM FOAM AND SANDWICH PANELS

Weihong HOU

B. Eng., M. Eng., M. Sc.

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Faculty of Engineering and Industrial Sciences
Swinburne University of Technology

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ABSTRACT

Aluminium foams and aluminium sandwich structures with metal foam cores are novel materials and structures that have a great application potential in the automobile, marine, aircraft and building industries. They have the capability of absorbing considerable impact energy by large plastic deformation under quasi-static or dynamic loading, making them ideal structure protectors and energy absorbers. The microstructures of metal foams give them the ability to undergo large plastic deformation at nearly constant stress, thus absorbing a large amount of kinetic energy before collapsing to a more stable configuration or fracture. Studies on the mechanical properties of aluminium foam were presented by previous studies on the tensile, compressive loadings. The shear behaviour of aluminium foam is critical when it works as the core of sandwich structures. However, there has been very limited literature reporting on or addressing this aspect. Recent research on the performance of sandwich structures, especially on those made of various fibre-reinforced plastic laminates with polymer foam cores, has been focused on their behaviours under quasi-static loading and impact at a wide range of velocities. Work on the structural deformation and energy absorption of circular aluminium sandwich structures with metal foam cores is still limited.

In this research, experimental investigations were conducted on the shearing strength and energy absorption of aluminium foams. Variation of the ultimate shear stress against geometrical dimension and relative density as well as impact velocity is discussed and empirical formulae are obtained for the ultimate shear stress and shear energy in terms of the relative density. Finite element method (FEM) simulation corresponding to the shearing experiment has been undertaken using commercial software LS-DYNA 970 to analyse the energy dissipation during the shearing process.

Further experiments were carried out to study the structural performance of circular aluminium sandwich structures with metal foam cores under quasi-static and low-velocity indentation loading. Deformations of specimens in the tests were observed and analysed; quantitative results from different loading conditions were recorded and discussed; different failure modes were proposed and failure maps were constructed according to the geometrical configurations of the panels.

A finite element simulation has been performed to validate an analytical model of sandwich panels under indentation loading with large deflection. Comparative studies have been conducted for the analytical solutions of monolithic plates and circular sandwich panels with the
identical mass per unit area, but with different face-core thickness ratios. The analytical model has been proposed for the elastic and post-yield behaviours of the circular sandwich panels.

Experiments were also conducted on circular metallic sandwich panels with aluminium foam cores to investigate the energy absorption and plastic deformation of the structures under ballistic impact loading. A parametric study was carried out to examine the effect of the face sheets, the foam core, the shapes of projectiles and the impact velocities on the performance and behaviour of the sandwich structures. An empirical equation was suggested to approximately estimate the energy enhancement during dynamic perforation.

Finally, LS-DYNA 970 was used to construct numerical models of sandwich structures to simulate the deformation and energy absorption of sandwich structure under ballistic impact loading. The progress of damage and deformation was illustrated. The energy dissipation of the sandwich structure was analysed. The simulated perforation energy absorption by the sandwich structure will increase when the impact velocity increases, which agrees with the findings from previous impact experiments.
DECLARATION

I declare that this thesis represents my own work, except where due acknowledgement is made, and that it has not been previously included in a thesis, dissertation or report submitted to this university or to any other institution for a degree, diploma or other qualification.

Signed __________________________

Weihong HOU
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Chapter 2

$E^*$ Young’s modulus of the metal foam
$G^*$ Shear modulus of the metal foam
$\rho^*$ Density of the metal foam
$E_s$ Young’s modulus of the solid cell edge material
$G_s$ Shear modulus of the solid cell edge material
$\rho_s$ Density of the solid cell edge material
$\sigma^{pl}_*$ Plastic collapse strength of metal foam
$\sigma_{ys}$ Yield strength of the solid cell edge material
$\tau^{pl}_*$ Shear strength of the metal foam

Chapter 3

$\gamma$ Essential energy
$\nu_p$ Plastic Poisson’s ratio
$\nu$ Velocity
$W_{max}$ Final displacement
$T$ Loading duration
$\hat{\sigma}$ Equivalent stress
$\sigma_y$ Yield strength of the metal foam
$\sigma_{VM}$ von Mises stress
$\sigma_m$ Mean stress
$\sigma_p$ Plateau stress
$\sigma_c$ Compressive stress
$\sigma_t$ Tensile stress
$\varepsilon$ Equivalent strain
$\varepsilon_D$ Densification strain
\( \rho^*, \rho_f \) Density of aluminium foam

\( \rho_{fo} \) Density of original metal material

**Chapter 4**

\( E_f \) Young’s modulus of the skin sheets

\( \sigma_y \) Yielding stress of the skin sheets

\( \hat{\rho} \) Relative density of foam core

**Chapter 6**

\( a \) Radius of the punch

\( E_c \) Young’s modulus of the core material

\( E_f \) Young’s modulus of the face sheet material

\( G_f \) Shear modulus of the face sheet material

\( h \) Thickness of solid plate

\( h_f \) Face sheet thickness of sandwich plate

\( H_c \) Thickness of sandwich core

\( P_o \) Limit load for the monolithic circular plate

\( R \) Radius of sandwich plate

\( T \) Loading duration

\( w_m \) Central-point deflection of the panel

\( w_f \) Deflection at failure

\( W \) Plastic energy dissipation per unit area

\( \sigma_f \) Yield stress of face sheet

\( \nu_f \) Poisson’s ratio of face sheet

\( \rho_f \) Density of face sheet

\( \sigma_c \) Yield stress of core material

\( \rho_c \) Density of core material

\( \epsilon_{r}, \epsilon_{\theta} \) Components of tensile strain of the middle plane

\( \kappa_{r}, \kappa_{\theta} \) Principal curvatures of the middle plane.
\bar{\rho} = \rho_c / \rho_f \quad \text{Relative density of sandwich core}

\sigma_e \quad \text{von Mises stress}

\sigma_m \quad \text{Mean stress}

\varepsilon_F \quad \text{The ductility strain of face sheet material}

**Chapter 7**

\rho \quad \text{Mass density}

E \quad \text{Young’s modulus}

\nu \quad \text{Poisson’s ratio}

\sigma_s \quad \text{Tensile strength}

\sigma_{pl} \quad \text{Plateau stress}

\sigma \quad \text{Dynamic crushing stress for shock front theory}

\sigma_o \quad \text{Quasi-static yield strength}

\sigma_p \quad \text{Plateau stress}

\sigma_{yd} \quad \text{Dynamic flow stress}

\alpha, \alpha_2, \gamma, \beta \quad \text{Material parameters for Deshpande-Feck model}

V, V_i \quad \text{Impact velocity}

L \quad \text{Side length of a square sandwich panel}

\varepsilon^* \quad \text{Effective plastic strain}

\varepsilon_f \quad \text{Tensile failure strain}

\varepsilon_D \quad \text{Densification strain}

h_f \quad \text{Skin thickness}

H_c \quad \text{Core thickness}

\bar{\rho} \quad \text{Relative density of foam core}

V_b \quad \text{Ballistic limit}

E_p \quad \text{Perforation energy}

V_i \quad \text{Impact velocity of the projectile}

V_r \quad \text{Rear velocity of the projectile}

m_p \quad \text{Mass of the projectile}

E_d \quad \text{Dynamic perforation energy}
\( E_s \) Quasi-static perforation energy
\( \Phi \) Dynamic enhancement factor
\( r_p \) Radius of the projectile
\( A, B, C, n \) Material parameters of Johnson-Cook model

**Appendix A**

\( a \) Radius of the punch
\( C \) Function of the central deflection
\( D_{eq} \) Equivalent flexural rigidity
\( D \) Dissipation energy
\( E \) External work
\( E_c \) Young’s modulus of the core material
\( E_f \) Young’s modulus of the face sheet material
\( G_f \) Shear modulus of the face sheet material
\( Me \) Mass per unit area of the sandwich
\( h \) The thickness of solid plate
\( h_f \) Face sheet thickness of sandwich plate
\( H_c \) Thickness of sandwich core
\( M \) Bending moment
\( N \) Longitudinal force
\( M_0 \) Plastic values of bending moment
\( N_0 \) Plastic values of longitudinal force
\( p \) Load intensity
\( P_s \) Plastic limit load of simply supported sandwich plate
\( P_c \) Plastic limit load of clamped sandwich plate
\( P_o \) Limit load for the monolithic circular plate
\( R \) Radius of sandwich plate
\( T \) Loading duration
\( u \) Radial displacement of the middle plane
\( w_{cm} \) Central-point deflection of the panel
$w_{\text{max}}$ Final deflection of the panel

$w$ Deflection of the middle plane

$w_F$ Deflection at failure

$W$ Plastic energy dissipation per unit area

$\beta_1, \beta_2, \beta_3$ Parameters of the Prager-Onat plastic condition

$\sigma_f$ Yield stress of face sheet

$\nu_f$ Poisson’s ratio of face sheet

$\rho_f$ Density of face sheet

$\sigma_c$ Yield stress of core material

$\nu_c$ Poisson’s ratio of core material

$\rho_c$ Density of core material

$\varepsilon_r, \varepsilon_\theta$ Components of tensile strain of the middle plane

$\kappa_r, \kappa_\theta$ Principal curvatures of the middle plane.

$p = \rho_c / \rho_f$ Relative density of sandwich core

$\sigma_e$ von Mises stress

$\sigma_m$ Mean stress

$\varepsilon_F$ Ductility strain of face sheet material
CHAPTER ONE

INTRODUCTION

1.1 Motivation

Today, various transportation tools play a large role in society both domestically and internationally. The number of vehicles running on the roads, ships sailing on the water and aircraft flying in the sky is rapidly increasing. Not only does technology contribute to the increasing quantity of vehicles, it also allows for properties such as greater speed and larger mass in vehicles such as trucks and aircrafts [1]. These vehicles cause more serious damage to people and the environment when they are involved in accidents (Figure 1-1). A crash involving rapidly moving vehicles occurring in an instant will result in a large impact force, causing a huge acceleration to the structures and occupants of the vehicles. This impact acceleration pulse is loaded and unloaded in a very short duration at a high speed.

Hence the research and development of energy-absorbing structures and materials that dissipate kinetic energy during intense dynamic loading, have received increasing attention, especially for the automobile and military industries.

The design and analysis of energy-absorption structures are very different from conventional structural design and analysis. The energy absorbers have to sustain intense impact loads, so their deformation and failure involve large geometry changes, strain-hardening effects, strain-rate effects and various interactions between different failure modes such as bending and stretching. For these reasons, aluminium alloy as well as other ductile materials attracts an increasing number of applications and has become one of the most widely used materials. Aluminium foam together with fibre-reinforced laminates and polymer foams is commonly used, especially when the weight of the structures is a key consideration.
1.2 Aluminium Foam and Aluminium Sandwich Structures

Aluminium foams have a combination of properties that make them attractive in a number of engineering applications. Aluminium foams have the capacity to undergo considerably large deformation at a relatively constant stress. Figure 1-2 shows a schematic stress-strain curve for aluminium foam under uniaxial compressive loading. In the first stage, the foam responds with initial elasticity; then a nearly constant stress plateau forms and the stress plateau continues up to large strain and finally the stress increases sharply into densification stage. The energy absorption is equal to the area under the stress-strain curve, which is the integration of the curve against the horizontal axis; the longer the plateau zone, the more energy can be absorbed by the material. Aluminium foams with a long plateau zone are good energy absorbers under various loading conditions.
Aluminium sandwich structures with metal foam cores have good mechanical properties and are relatively light in weight. Recently developed processing techniques allow the manufacture of panels of complex shapes with integrally bonded faces at relatively low cost.

A schematic sketch of the assembly of an aluminium sandwich plate with a metal foam core is shown in Figure 1-3. Their metallic face-sheets absorb the kinetic energy by global bending and stretching at large deflections; at the same time, the plastic deformation of the foam cores bears the localised indentation and shearing damage during impact loading. The metal face-sheets of aluminium sandwich structures provide relatively high strength and the foam core allows significant energy absorption without generating damaging peak stresses. These properties make them ideal structure protections and energy absorption devices. Therefore a wide range of potential applications have been found for sandwich structures in industries such as aerospace, marine and automobile, where there is high possibility of various impact damages such as a bird flying toward an airplane, a ship hitting an iceberg and cars colliding with each other or running into buildings. Hence it is important to study the mechanical properties and energy-absorption characteristics in order to use these materials for optimum performance in real-life applications.
1.3 The Scope of this Study

The aim of this study is to investigate the mechanical strength, structural response and energy-absorption properties of sandwich structures with aluminium foam cores under various loading conditions, from static, low-velocity impact to ballistic impact. As a preliminary study, the shearing behaviour of aluminium foam is also investigated.

The research is carried out in four stages. In the first stage, the shear property of the aluminium foam is investigated by experiments; the ultimate shear stress against geometrical dimension and relative density is discussed and empirical formulae are obtained for the ultimate shear stress and shear energy in terms of the relative density; finite element simulation is applied to further study the damage procedure and energy dissipation during the shearing procedure.

In stage 2, further experiments are carried out to study the deformation and behaviour of circular aluminium sandwich plates with aluminium foam cores under quasi-static and low-velocity indentation loadings; quantitative data are recorded and analysed; the failure modes and parameters on the properties are analysed.

In stage 3, a numerical analysis is conducted to simulate the elastic and post-yield behaviours of the circular sandwich panels; the large deflection behaviour is compared with an analytical model by a yield criterion for sandwich constructions taking account
of core strength; comparative studies are conducted for monolithic plates and circular sandwich panels with the identical mass per unit area, but different face–core thickness ratio.

More experiments are then conducted to investigate the ballistic response of the circular sandwich structures. Parametric studies are carried out to identify the influences of several key parameters on the structural response. An empirical equation is suggested to approximately estimate the energy enhancement during dynamic perforation. Finally the FE method is used to simulate the deformation and energy dissipation of the sandwich panels under ballistic loading.

1.4 Organisation of the Thesis

The thesis is organised as follows:

Chapter 1: Introduction. Chapter 2 presents a literature review on the research of aluminium foams and sandwich structures with aluminium foam cores.

In Chapter 3 quasi-static experimental investigation is reported on the shearing behaviour of the aluminium foam core. Shearing strength and energy absorption properties of this foam are analysed. Failure progress analysis and finite element simulation are also presented.

The experiments on sandwich structures with aluminium foam cores under quasi-static and low-velocity impact are reported in Chapters 4 and 5 together with failure modes and parameter analysis.

Chapter 6 presents an FE model to simulate the load-carrying capacity of sandwich structures with large deflections under static loading.
The experimental and numerical investigation of the ballistic response of the sandwich structures with aluminium foam cores is presented in Chapter 7.

The findings in this research are summarised in Chapter 8 where future work is also suggested.
CHAPTER TWO

LITERATURE REVIEW

2.1 Introduction

The following review will focus on research on the mechanical properties of metal foam materials and studies for the structural behaviours of sandwich structures. The review is organised in the following sequence: section 2.2 on aluminium foam, section 2.3 on sandwich structures and a summary is presented in section 2.4.

2.2 Aluminium Foams

Many literature studies have been undertaken on the mechanical properties of metal foams. A broad survey of the understanding of the mechanical behaviours of a wide range of cellular solids is provided by Gibson and Ashby [2-4]. Others have carried out experiments to investigate the behaviours of metallic foams under different loading conditions, particularly the properties of metal foams under impact loading.

2.2.1 Mechanical properties of aluminium foams - theoretical analysis

Two types of metal foams, closed-cell and open-cell foams, can be made by different manufacturing methods. A closed-cell foam is shown in Figure 2-1a with faces separating the voids of each cell; an open-cell foam is shown in Figure 2-1b with truss edges connecting the cells. The cell size of most aluminium foams is in the range of 2 to 10mm. The relative density of foams is the most important feature affecting their mechanical properties; it is defined as the ratio of the density of the foam to that of the solid metal, \( \rho^* / \rho_s \), which is typically within the range of 0.03–0.2 [4].
It is difficult to model metal foams precisely due to their complex cell geometry. Here, we review the simple dimensional arguments method used to model the mechanisms of deformation and failure of ideal foams [2]. The results give the dependence of the properties on relative density and on cell wall property but not on cell geometry; the constants relating to cell geometry are determined by fitting the equations to experimental data or data from finite element simulation [5].

For open-cell foams, the cell edges initially deform by bending, and Young’s modulus is calculated from a dimensional analysis of the edge bending deflection. The relative Young’s modulus $E^*$ and the plastic collapse stress $\sigma_{pl}^*$ of the foam have power low relationships to the relative density $\rho^*/\rho_s [2, 4]$: 

$$\frac{E^*}{E_s} = C_1 \left(\frac{\rho^*}{\rho_s}\right)^2$$  \hspace{1cm} (2-1) 

$$\frac{\sigma_{pl}^*}{\sigma_{sys}} = C_3 \left(\frac{\rho^*}{\rho_s}\right)^{3/2}$$  \hspace{1cm} (2-2)
where $E^*$ and $\rho^*$ are Young’s modulus and the density of the metal foam; $E_s$ and $\rho_s$ are Young’s modulus and density of the solid metal, $C_1$ and $C_3$ are constants that can be determined by experiments.

Analysis of closed-cell foams is more complicated. There is stretching of the planar cell faces in addition to bending of the cell edges when the foam is loaded; adding a linear density dependence to Eqs 2-1 and 2-2, we obtain Young’s modulus and plateau strength for closed-cell foams as:

$$\frac{E^*}{E_s} = C_1\left(\frac{\rho^*}{\rho_s}\right)^2 + C_1\left(\frac{\rho^*}{\rho_s}\right)$$  \hspace{1cm} (2-3)

$$\frac{\sigma_{pl}^*}{\sigma_{ys}} = C_3\left(\frac{\rho^*}{\rho_s}\right)^{3/2} + C_3\left(\frac{\rho^*}{\rho_s}\right)$$  \hspace{1cm} (2-4)

where the constant $C_1$ and $C_3$ are related to the cell geometry and determined by experiments.

In shear, the cell edges also respond by bending. For open-cell foams, the relative shear modulus is proportional to the square of the relative density [2, 4]:

$$\frac{G^*}{G_s} = C_2\left(\frac{\rho^*}{\rho_s}\right)^2$$  \hspace{1cm} (2-5)

where $G^*$ and $G_s$ are the shear modulus for the foam and solid metal, $C_2$ is a constant determined by experiments.

The shear strength can be obtained from the yield criterion. The Deshpande-Fleck yield surface [2, 4] suggests that the shear strength $\tau_{pl}^*$ is 0.69 times the uniaxial strength $\sigma_{pl}^*$:

$$\tau_{pl}^* = 0.69\sigma_{pl}^*$$  \hspace{1cm} (2-6)

or, for open-cell foams using Eq. 2-2,
\[ \frac{\tau^*_{pl}}{\tau_{sys}} = C_2 \left( \frac{\rho^*}{\rho_s} \right)^{3/2} \]  

(2-7)

and, for closed-cell foams using Eq. 2-4,

\[ \frac{\tau^*_{pl}}{\sigma_{sys}} = C_2 \left( \frac{\rho^*}{\rho_s} \right)^{3/2} + C_4 \left( \frac{\rho^*}{\rho_s} \right) \]  

(2-8)

where the constants \( C_2 \) and \( C_4 \) are related to the cell geometry and can be determined by experiments.

### 2.2.2 Mechanical properties of aluminium foams – experimental investigation

The mechanical properties have been widely investigated by experiments in several aspects. They will be discussed in this section in the following sequence: first, the quasi-static uniaxial experiments – compression and tension; second, the quasi-static experiments – shearing; then the multi-axial experiment; and finally the strain-rate effect and dynamic response.

- **Quasi-static uniaxial experiments – compression and tension**

Studies [6-8] focused on the compressive and tensile properties of aluminium foams. Results show that the compressive process of aluminium foam was characterised by three deformation stages: linear elastic stage, plastic stage and densification stage. They found that the strength and elastic modulus were strictly affected by the relative density of the foams.

Andrews et al. [6] compared the uniaxial compressive and tensile behaviour of five different brands of aluminium foams, closed-cell: Alcan, Alporas, Fraunhofer and open-cell: ERG with models of ideal tetrakaidecahedral cells for cellular solids [2]. The
relative Young’s modulus $E^*/E_s$ and the plateau strength $\sigma_{pl}^*/\sigma_{ys}$ of the open-cell foam relating to the relative density $\rho^*/\rho_s$ of the aluminium foams were well described by the model, while the closed-cell foams had moduli and strengths that fell well below the predicted values as shown in Figure 2-2. They concluded that the reduced values were the results of defects in the cellular microstructure, which caused the bending rather than stretching of the cell wall; two features in the cell wall, the curvature and corrugations, were measured and modelled and it was approved that they accounted for most of the reduction in properties in closed-cell foams.

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Figure 2-2. Relative Young’s modulus and relative compressive strength plotted against relative density for aluminium foams. The dashed and solid lines represent Eq. 2-1, 2-3 and Eq. 2-2, 2-4 for ideal open- and closed-cell foams, respectively. The dash-dot line represents the reduction in the strength of the ideal closed-cell foam resulting from the measured cell wall curvature [6].

Tensile experiments were conducted on two different relative densities of Alporas aluminium foams [7]. The microscopic yield stress, peak stress, Young’s modulus, Poisson’s ratio and the work-hardening exponent were determined; the local deformation behaviour was also investigated with a special surface-strain mapping method. The deformation characteristics under tension were different from those observed in aluminium foam under compression. No plateau-stress regime accompanied by plastic instability, like the crushing of cells, could be observed in tension.

Yu et al. [8] investigated the effect of cell diameters on the compressive property and energy absorption of the closed-cell aluminium foam of relative density of 0.16. It is found that the effect of cell diameter is obvious: with the increase of cell size, the peak stress, nominal Young’s modulus and energy absorption increase.
Quasi-static experiments – shearing

Von Hagen and Bleck [9] found a nearly linear increase in shear strength with increasing density for Alporas and Alcon aluminium foams while analytical models predict a non-linear power law dependence [2-4]. Saenz et al. [10] only reported a few data points for the shear modulus and the shear strength of melt-foamed aluminium. Rakow and Waas [11] used an in-plane shearing device shown in Figure 2-3 and conducted experiments with CYMAT foams. They concluded that the shear modulus was linearly proportional to the relative density, while the ultimate shear strength was shown to have a power law relationship with the relative density. They examined the size effect in shear deformation of CYMAT aluminium foams with optical techniques through which the average shear strain is calculated over sub-regions of the samples. They concluded that the strain in these sub-regions deviates significantly from the global applied strain for regions smaller than 18 mean cell diameters, indicating a critical specimen size below which certain bulk properties are no longer representative.

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Figure 2-3. The loading frame used in shear testing of aluminium foam. The arrow marks the direction of drawing in the manufacturing process. The boxes mark the boundaries of the sub-regions over which the shear strain is averaged [11].
• **Quasi-static multi-axial experiments**

In real-life applications, aluminium foams may be subject to multi-axial loads. Previous works on multi-axial behaviour for cellular solids were presented [12–15]. Another purpose of multi-axial experiments is to investigate the initial failure criteria of metal foams.

Experiments were carried out to determine the stress-versus-strain responses of Alporas and Duocel foams under proportional axisymmetric compressive loading; and to investigate the shape of the initial yield surface and its evolution under hydrostatic and uniaxial compressive loadings [12]. It was found that the hydrostatic yield strength was of similar magnitude to the uniaxial yield strength. The yield surfaces were of quadratic shape in the stress space of mean stress versus effective stress, and evolved without corner formation. The initial yield surfaces for three foams were plotted in Figure 2-4. Both the mean stress and effective stress values had been normalised by the uniaxial yield strength of the respective specimen.

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Figure 2-4. Initial yield surfaces of the low and high-density Alporas, and Duocel foams [12]
Miller [13] proposed a yield surface that could be fitted to the plastic flow properties of a broad class of cellular solids exhibiting plastic compressibility and different yield points in uniaxial tension and compression.

Gioux *et al.* [14] measured the failure responses of the Duocel (open-cell) and Alporas (closed-cell) aluminium foam using an experimental program designed for aluminium foams under biaxial and triaxial loadings. They compared the data with the three yield criteria for metallic foams. The first criterion was based on analysis of the failure mechanisms of an ideal foam [2]. It overestimated the measured foam yield surface: the discrepancy could be related to imperfections in the foam structure. The Miller yield criteria [13] could be derived from the mechanistic yield surface for ideal open-cell foams by accounting for the effect of cell wall curvature; the Deshpande-Fleck criteria [12] could be so derived with the exception of a linear term in the mean stress. Both the Miller and Deshpande-Fleck criteria gave a good description of the multiaxial failure of aluminium foams.

Ruan *et al.* [15] carried out three types of tests: uniaxial compression, uniaxial tension and triaxial compression, on CYMAT aluminium foams with five different relative densities at a normal axial strain rate of $10^{-4}$ s$^{-1}$. The initial failure surfaces were constructed. It has been found that the Gibson *et al.* [2] criterion overestimates experimental yield stresses. The experimental data are consistent with both the Miller [13] and Deshpande and Fleck [12] criteria when suitable plastic Poisson’s ratios are employed.

- **Dynamic response and strain rate effect**

Intensive studies [16–23] have been reported to investigate the strain rate effect on the dynamic behaviour of aluminium foams. However contradicting results and conclusions have been reported with different brand of aluminium foams.
Deshpande and Fleck [16] studied the strain rate sensitivity of two aluminium foams (Alulight closed-cell foam and Duocel open-cell foam). Their results showed that the plateau stress was almost insensitive to the strain rate, for a strain rate up to 5000 s\(^{-1}\). Deformation was localised in weak bands in the Alulight foam but it was spatially uniform for the Duocel foam, over the full range of strain rates 10\(^{-3}\) to 1200 s\(^{-1}\). They estimated the magnitude of the above two sources and concluded that they were negligible. Ruan et al. [17] carried out experiments on compressive behaviour of CYMAT aluminium foams with relative densities ranging from 5% to 20% at strain rates ranging from 10\(^{-3}\) to 10\(^{+1}\) s\(^{-1}\). It was found that the plateau stress is insensitive to the strain rate and was related to the relative density by a power law. The dynamic plateau stress at the highest strain rate was 1.05 times (5%) larger than that at the lowest strain rate. This indicates that the strain rate had little influence on the plateau stress or the plateau stress was not sensitive to the strain rate within the test range. Similar conclusions were reported by other authors [18, 19].

Paul and Ramamurty [21] conducted an experimental investigation into the strain rate sensitivity of Alporas closed-cell aluminium foam at room temperature and under a compression loading at a strain rate from 3.33\times10^{-5} to 1.6\times10^{-1} s\(^{-1}\). Within this range, experimental results showed that the plastic strength and the energy absorbed increase (by 31 and 52.5%, respectively) with an increasing strain rate. However, the plastic strength was found to increase bilinearly with the logarithm of the strain rate, whereas dense metals tend to show only a linear response. As was the case with dense metals, the strain rate sensitivity of the foam was not a constant value, but found to be dependent on the strain and incremental change in the strain rate. Investigations [22] indicated that strain rate strengthening occurs in closed-cell aluminium (Alporas) foams. The effect is more apparent for the higher density (15%) foam investigated.

Results obtained by Montanini [22] are diversified. The author studied the compressive behaviour of three different cellular aluminium alloys (M-PORE, CYMAT, SCHUNK), in a wide range of relative density, under both quasi-static and dynamic loading. Drop dart impact measurements were carried out at a maximum strain rate of 100 s\(^{-1}\) by means of an instrumented pendulum machine and compared with static compression
tests performed at a low strain rate \(2 \times 10^{-3} \text{s}^{-1}\). The experimental results showed that the specific energy absorption capacity of metal foam with similar density could be quite different: strain rate sensitivity could be considered negligible for the M-PORE foam, which had an open-cell morphology, while it was significant for the two closed-cell foams (CYMAT, SCHUNK) investigated, although some distinctions between the two materials could be pointed out, which may be attributed to differences in their microstructure, arising from the way the two foams have been manufactured (melt gas injection and powder metallurgy route). Impact tests showed that the dependence of the plateau stress on the strain rate could be considered negligible for M-PORE and CYMAT foams while it was quite remarkable for SCHUNK foams. Moreover, it was found that the peak stress of CYMAT foams had a quite large sensitivity on the loading rate.

Kumar et al. [23] experimentally investigated the deep indentation response of a closed-cell aluminium foam under different rates of penetration, ranging from \(1.5 \times 10^{-2}\) to \(5 \times 10^{3}\ \text{mm s}^{-1}\). Closed-cell aluminium foam (Alporas) with an average cell size of 4.5mm and a relative density of 8% was used in their study. Instrumented drop-tower tests were conducted on 50mm-thick panels at room temperature. Two types of punches were used: flat-end punch (FEP) and spherical-end punch (SEP). The experiments showed a gradual increase in the plastic collapse strength as well as the energy absorbed with increasing displacement rate. The energy absorbed per unit displacement volume gradually increased in the quasi-static regime and showed a significant change in slope at 10m/s velocity, possibly due to the shockwave effect becoming significant at those velocities. The first peak load increased with an increase in velocity of indentation whereas the plateau load in general resembled that of a quasi-static regime. The indentation process absorbs higher energy vs uniaxial compression for similar displacement rates, which involved an additional mechanism of tearing and shearing of the cells that were at the periphery. Cross-sectional views of the indented specimens showed that the deformation was confined only to the material directly beneath the indenter with very little lateral spread, a consequence of the near-zero plastic Poisson’s ratio of the foam. The computed densification strain in the case of FEP indentation was similar to that observed in uniaxial compression, but significantly lower than that observed in SEP indentation.
From previous studies, the obvious strain rate enhancement is observed for some materials and it is negligible for other materials. The reasons for this phenomenon may be due to the different manufacturing methods of materials, the air within closed-cell foam, the inertia effect and shockwave effect.

2.3 Sandwich Structures

Sandwich structures with metal foam cores are successful engineering structures in many applications. The manufacturing process of these structures has been improved recently [24–27], which makes them more cost effective. They have similar properties to sandwich structures with polymer foam cores, or laminates with PVC foam cores as well as sandwich with honeycomb cores. Therefore, this review also covers the literature on similar structures.

The following sections will discuss separately the studies on sandwich beams and sandwich plates.

2.3.1 Sandwich beams

- Sandwich beams under static bending

The quasi-static response of sandwich beams is usually tested by 3-point or 4-point bending; investigations generally focus on (1) structural failure modes and (2) peak loads for initial plastic collapse. The sandwich beams may have composite, for example, E-glass/Epoxy or Carbon/Epoxy [28–33] or metallic skins [34–48] and a polymeric/metallic foam core. It has been found that the collapse modes of sandwich beams are correlated to their physical and geometric properties (core relative density, core thickness, face thickness and strength, cellular morphologies, etc.) and boundary conditions.
Compared with the face-sheets made from elastic-brittle solids such as composite laminates, the ductile metallic faces allow large plastic deformation, that is, global bending and stretching, and thus can produce much a higher load and energy absorption.

Experiments were conducted by Fleck and co-workers [34–36] on sandwich beams with aluminium face-sheets and aluminium foam cores. They set up experiments on aluminium foam sandwich beams under three-point bending and four-point bending under two types of supporting conditions (simply supported or fully clamped). Three different initial collapse failure modes were observed: face yielding, core shear and local indentation.

Tagarielli and Fleck [36] focused on the effect of clamped boundary conditions on the flexural behaviours at finite deflections of sandwich beams comprising an aluminium face and aluminium foam core (Alporas). Analytical models were proposed and three dominant collapse mechanisms were identified as face yield, core shear and indentation, as shown in Figure 2-5. Specimens were tested on three-point bending with two different bounding conditions for comparison. The load-versus-deflection response of the beams may be subdivided into three phrases, elastic phase, transition phase and membrane phase, as indicated in Figure 2-6. Boundary conditions played an important role in the shape and strength and failure modes of the sandwich beams. Simply supported beams underwent continued plastic collapse at nearly constant load; eventually, the transverse deflection became sufficiently large that the structure failed by fracture of the face-sheets or core. In contrast, clamped beams underwent membrane stretching of the face-sheets beyond initial yield, and this gave rise to a hardening macroscopic response. Initial plastic collapse of clamped sandwich beams occurred by face yield, core shear, or indentation at small transverse deflections. Subsequent transverse deflection, however, involves tensile stretching of the faces and core. The stress distribution within the beam evolved from that associated with the initial collapse load to that of pure membrane action, with the membrane solution achieved when the deflection was about equal to the thickness of the beam. Thereafter, the beam deformed in a membrane mode, and yielded axially until the face-sheets tore when the axial
plastic strain attained the material ductility. Equilibrium considerations gave an expression for the load \( F \) versus deflection \( u \) in the membrane phase as

\[
F(u) = \frac{8t\sigma_f}{l^2 - a}
\]  

(2-9)

where \( b \) is the width of the beam, \( t \) is face thickness, \( a \) is the width of a flat-ended punch. It is assumed that the deflection \( u \) is small compared with the span \( l \), and that the net axial force in the faces is much greater than that in the core.

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Figure 2-5. (1) Initial collapse of face yielding, (2) Initial collapse by core shear, (3) Initial collapse by indentation [36]

Chen et al. [39] investigated the plastic collapse of sandwich beams experimentally and theoretically. Sandwich beams were manufactured with an Alporas’ aluminium alloy foam core, and cold-rolled aluminium face-sheets in the half-hard condition. Collapse mechanism maps, with the geometrical parameters of the beam as axes, were generated using limit load formulae. Minimum weight designs were obtained as a function of an appropriate non-dimensional structural load index \( \bar{F} \), which is defined as, in terms of collapse load \( F \), beam width \( b \), length \( l \) and the yield strength of the face skin \( \sigma_{fy} \),

\[
\bar{F} \equiv F / bl\sigma_{fy}
\]

For this purpose, it was instructive to plot the contours of weight and load on the collapse mechanism map.

McCormack et al. [37] had also constructed a failure map that illustrates the dominant failure mode for practical sandwich beam design with a metallic foam core. The results of the analysis were compared with experiments on sandwich beams in three-point bending. The peak loads and the failure modes were described well by the analysis.
Research had also been carried out into other aspects of sandwich beams under bending. Styles [44] focused on the effect of core thickness on the flexural behaviour of aluminium foam sandwich structures.

Kesler and Gibson [45] studied the size effect in metallic foam core sandwich beams. Sandwich beams with aluminium foam cores (Alporas) were tested in three-point bending to characterise the effect of the beam depth on the limit load. Beams of constant ratio of core thickness to span length \((c/L, t/L)\), but different absolute values of core thickness were tested; the measured limit loads were compared to analytical values. In most cases, discrepancies between the measured and calculated failure loads were less than 12%. The discrepancy between the measured and calculated failure loads increased to over 20% if the size effect in the core shear strength was negligible.

- **Sandwich beams under impact loading**

Experiments were also carried out to study the difference of the failure modes and energy absorption of sandwich beams under impact loadings compared with quasi-static loadings [40, 49–53].
Mines et al. [49, 50] investigated the behaviours of polymer composite sandwich beams under static and impact loadings. It was shown that a preferential failure mode of sandwich beams for energy absorption was the top skin compressive failure followed by a stable core crushing and then the back face tensile failure. This kind of failure mode occurred mainly in a sandwich beam with a high relative density core. They suggested that the back face bending failure could be modelled by an elastic-plastic analysis that employs the ‘bending hinge’ concept in the dynamic plastic response of metallic structures. Based on this concept, Li et al. [51] developed an analytical model to predict the behaviour of a simply supported composite sandwich beam subjected to a mass impact at the mid-span at small deflections. The dynamic response corresponding to different intensities was characterised by the three regimes: elastic response regime, core-crushing failure regime and final failure regime.

Yu et al. [52, 53] set up experiments to study the response and failure of dynamically loaded sandwich beams with identical aluminium alloy face-sheets and an aluminium-foam core under quasi-static and dynamic loading. It was found that the energy-absorbing capacity of beams loaded dynamically was higher than that for quasi-static loading due to large local indentation and damage. They found that the failure mode and the load history predicted by a modified Gibson’s model agreed well with the quasi-static experimental data. The failure modes and crash processes of beams under impact loading were similar to those under quasi-static loading when the impact velocity was lower than 5m/s, but the dynamic force-time curves exhibited great oscillation.

Crupi and Montanini [40] conducted three-point bending tests with aluminium foam sandwiches (Schunk and Alulight) with identical nominal dimensions under static and impact loading. As far as impact tests are concerned, no significant strain rate sensitivity at low-impact speeds has been found for both Schunk and Alulight panels. This result is consistent with that obtained by Yu et al. [52] on open-cell foam sandwiches under three-point bending loads.
2.3.2 Sandwich plates

Although studies on aluminium sandwich beams were often reported in literature, the studies on aluminium foam sandwich panels are limited. In this section, we will present the studies covering other sandwich panels that have similar properties, such as composite laminates, FML (fibre-metal-laminate)-reinforced, FRP (fibre-reinforced plastic) laminates sandwich structures and aluminium sandwich structures with honeycomb cores under static and impact loading.

- **Sandwich plates under quasi-static and low-velocity impact**

In general, performances of sandwich plates under quasi-static and low-velocity impact properties are assessed using two approaches: (1) indentation by a concentrated force and (2) compressed by a uniformly distributed pressure. Studies were presented on the influence of several parameters on the properties of sandwich panels: skin thickness, core thickness and density, impact velocity, the geometry of loading indenters as well as boundary conditions.

The indentation tests on sandwich structures can be performed by servo-hydraulic material-testing machines (for quasi-static tests) or a drop hammer (for low-velocity impact tests), which can produce impacts with the velocity up to 15m/s. This type of study is mainly concerned with the structural response, load-carrying capacity, failure pattern and energy absorption of the sandwich panels due to indentation damages. Figure 2-6 shows the damage evolution of two typical sandwich plates [54]. It is found that the low-velocity impact failure modes are similar to static indentation failure modes, but the energy absorption in the impact cases is slightly higher [55].

Compared with a large amount of research work carried out in experimental [54–62] and numerical studies [56, 57, 64–66], far fewer investigations have been available on the analytical solutions for sandwich structures due to the complex interaction between
the face-sheets and cores during deformation and failure. There are two widely used models, the spring-mass model and energy-balance model. In the spring-mass model, the sandwich panel is modelled as a discrete dynamic system with equivalent masses and spring/dashpots. The load-deflection response is calculated by a combination of bending, shear, membrane and contact springs. In the energy-balance model, the kinetic energy of the projectile is equated to the sum of energies due to contact, bending, shear and membrane deformations, and then the maximum deflection and force can be estimated.

Hoo Fatt and co-workers [67–71] analytically studied the response of sandwich panels under different loading conditions and supports. A spring-mass model was used [67, 68] to predict the low-velocity impact effect on composite sandwich panels rigidly supported, two-sided clamped, simply supported and four-sided clamped. The analysis related to local indentation and structural global deformations, and the strain rate effect was considered as well. Another analytical model was developed to calculate the impact force and velocity for several damage initiation modes: tensile and shear fracture of the top face-sheet, core shear failure and tensile failure of the back face. The energy-balance model is proposed in [72, 73].

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<table>
<thead>
<tr>
<th>Coremat panel</th>
<th>Aerolam panel</th>
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Figure 2-6. Sketches of the cross-section and surfaces of two typical sandwich panels at different stages of static loading [54]
• **Sandwich plates under high-velocity impact/ballistic impact**

The ballistic impact is defined as the impact that is able to perforate the entire structure completely. Impact tests can be carried out by drop hammer or gas gun with different impact mass. Investigations focus on several aspects, such as the dimensions and materials of the sandwich samples, impact velocity and the geometry and shape of projectiles. To analyse the impact response of the sandwich structures, two important parameters, the ballistic limit \( V_b \) and perforation energy \( E_p \) are used to evaluate the perforation-resistance behaviour and energy absorption of the structures.

- **Experimental investigations**

Experimental studies have been conducted to investigate the high-velocity response of sandwich structures [74–81]. Damage behaviour of sandwich structures in perforation is complex and depends on the material and structural configuration of the sandwich panels as well as the shapes of indenters.

Wen and co-workers [74, 75] presented a study on the indentation, penetration and perforation of composite laminates and sandwich panels under quasi-static and projectile loadings. Composite laminates and sandwich panels were used in this study for penetration and perforation tests. Three types of indenters were designed: they were flat-faced, hemispherical-ended and conical-nosed. Tests were conducted under quasi-static, drop-weight and ballistic impact conditions with impact velocity up to 305m/s.

The fracture patterns as well as the load-displacement characteristics of the sandwich panels subject to low-velocity impact could be expected to be similar to those observed in corresponding quasi-static cases. But higher energy was required to perforate a sandwich panel dynamically than quasi-statically. This change was attributed to such factors as inertia and material strain-rate sensitivity.
Villanueva and Cantwell [76] studied the high-impact response of composite and novel FML-reinforced sandwich structures. The structures exhibited a number of energy-absorbing mechanisms such as fibre-matrix delamination, longitudinal splitting and fibre fracture in the composite skins and indentation, progressive collapse and densification in the aluminium foam core. Tests showed that the unidirectional FML-reinforced aluminium foam sandwich structures offered specific perforation energies approximately 23% higher than their plain composite counterparts with similar volume fractions.

Kepler [77, 78] reported the impact penetration of sandwich panels at different velocities. The sandwich panels he used consisted of FRP (Fibre Reinforced Polymer) with PVC foam core. Penetration was also done in quasi-static conditions, the total energy absorption was measured, and the damage patterns described in terms of different geometry shapes of impact projectiles as well as different impact velocities. The individual energy contributions were estimated from simple physical models. It was shown that the most important contributions were from membrane-state fibre stretching, core compression and friction between core material and impactor. Lesser contributions were from delamination, core fracture, and debonding between core and back face-sheet.

Skvortsov et al. [86] considered ballistic impact and penetration in sandwich panels at projectile velocity above the ballistic limit. An analytical model was developed to solve the partition of the energy absorption, allowing for quantitative estimation of the energy fraction consumed via the panel elastic response and the one consumed through irreversible damage. Different variants of load histories and distributions had been analysed, and only a weak dependence of the ratio of the plastic response energy $E_{\text{pan}}$ and damage energy $E_{\text{dam}}$ on the loading pattern had been found. It had been observed that the energy partition $E_{\text{pan}} / E_{\text{dam}}$ changed by not more than 20% within the entire range of considered load histories for absorbed energy. Experimental measurements of the damage energy were performed for simply supported and backed panels. Comparison of the experimental and theoretical results indicated the validity of the developed model.
Bull and Hallstrom [80] investigated the response of sandwich structures subjected to impact velocities of virtually 0m/s and approximately 1000m/s. A series of 1x1m$^2$ sandwich panels was manufactured using vacuum-infusion. The higher velocity exceeded both the longitudinal and the transverse wave propagation velocity of the core material in the sandwich panels. It was found that the size and shape of the debonding was governed by the bending properties of the face and fracture toughness of the core material as well as impactor energy and velocity. The face-core debonding propagated in the core material in the quasi-statically impacted panels. In the high velocity impacted panels, however, the debonding seemed to propagate partially in the core and partially in the face-sheet.

In some cases, the energy required for penetration under impact loading is significantly higher than in the static case. The effect of perforation velocity on this energy enhancement phenomenon has been studied extensively.

Goldsmith et al. [81] conducted an experimental study of the ballistic limits of aluminium honeycomb sandwich panels with aluminium alloy sheets using projectiles in the form of spheres, 60° cylindric-conical bullets and blunt-nosed cylinders. It was found that the ballistic limit for each size and shape of penetrator exhibited a nearly linear relation with the areal density of the target. However, the smaller projectiles evidenced a greater slope than the larger sizes. The variation of terminal velocity as a function of initial velocity beyond the ballistic limit followed the standard pattern of a short, concave-downward curve. The perforation resistance of the target as dominant was followed by a linear region where the piercing process represented a small fraction of the total initial kinetic energy of the striker.

Mines et al. [82] tested the low-velocity perforation performance of two sandwich constructions, namely woven glass vinyl ester skins with Coremat core and woven glass epoxy pre-preg skins with honeycomb core. Results showed that the density of the core influences the progression of failure and that higher impact velocity increased the energy absorption of the panels. The core crush dominated overall energy absorption.
and the increase of perforation energy from static to dynamic loading could be due to a change of the deformation geometry as well as the material strain rate effect.

Roach et al. [83, 84] studied the relation of energy absorption and damage of sandwich panels with GR laminate skins and laminate skins with a closed-cell PVC core perforated by a flat-ended solid cylindrical indenter in both quasi-static and impact-loading conditions. It was found that the ratio of dynamic penetration energy to static penetration energy raised rapidly initially with velocity, plateauiing at about 100m/s. More energy was required to penetrate laminates with a core foundation. There existed a linear correlation between the impact energy and the area of damage, at all velocities, and this correlation was most accurate for the foam-backed laminates.

Zhao et al. [85] used an instrumented Hopkinson pressure bar as a perforator and at the same time a measuring device. The facility aims to get a high-quality piercing force record during the entire perforation process, which is a weak point of common free-flying projectile-target testing systems. Sandwich samples were made of CYMAT foam cores and aluminium alloy sheets as top and bottom skins. This new testing arrangement allows for the measurement of piercing force-displacement curves under quasi-static and impact loadings of sandwich specimens. Compared with quasi-static top skin peak loads (the maximum load before the perforation of top skins) obtained under the same geometric and clamping conditions, a significant enhancement under impact loading (25%) of the top skin peak load was found. However, the used foam core and skin sheet were known and have been confirmed to be barely rate-sensitive by separate tests on foam cores as well as on the skin sheets. A possible explanation of these results might be the following: the foam core under the perforation was locally more compressed under impact loading because of the inertia effect. As the used foam cores have quite an important strain-hardening behaviour, the strength of the foam cores before the failure of the top skin is higher than that under static loading, which leads to an increase of the top skin peak load under impact loading. Tests on a uniformly precompressed sandwich sample indeed exhibit higher top skin peak loads, which also supports this aforementioned concept.
Analytical models

A few analytical models have been developed to predict the ballistic limit and energy absorption of the sandwich structures during perforation.

A theoretical model was proposed by Hoo Fatt and co-workers [70–72] for the perforation of sandwich panels with E-glass/polyester composite skins and an aluminium honeycomb core. The complete process of perforation was split into three stages: initiation of the top face-sheet failure, penetration of the top face-sheet and perforation of the bottom face-sheet (Figure 2-7).

The three-stage perforation model was considered in the dynamic analysis and the conservation of energy was combined with the equations of motion to obtain the ballistic limit.

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(a)     (b)     (c)

Figure 2-7. Three stages of perforation, (a) Stage 1 – perforation of top face-sheet, (b) Stage 2 – perforation of honeycomb core, (c) Stage 3 – perforation of bottom face-sheet [70]

Wen and co-workers [74, 75] performed penetration tests on the FRP laminates and two FRP-skinned sandwich panels. Simple analyses using spring models have been developed to describe the initial failure of the top skin of the sandwich panels load quasi-statically by different indenters. Formulae for the loads corresponding to the three principal failure modes, that is, indentation, core shear and global bending, and the
transition equations between these modes have also been given in the quasi-static case. Based on the experimental data, an empirical equation was derived to predict the ballistic limit and perforation energy of fibre-reinforced plastic laminates and sandwich panels with such skins and with foam cores subjected to quasi-static and impact loading by these indenters.

Skvortsov et al. [86] developed a mathematical model to describe the penetration in sandwich panels for intermediate and high-range incident velocities, where it was assumed that the fraction energy can be divided into two parts: the first part was the energy absorbed due to the damage of sandwich composites, that is, matrix cracking, core crushing, delaminating and fibre failure; the second part was the energy absorption due to in panel elastic response (deformation and motion). Based on the similar energy partition, Vemurrugan et al. [87] proposed another simple analytical model to predict the impact response of a sandwich structure, including the ballistic limit.

- **Numerical simulations**

Compared to the high cost and the long time delay to design and carry out experiments to obtain test results, the computer simulation can be a cost-effective aid to design and analyse sandwich plates. The finite element method offers the possibility to evaluate and optimise sandwich structures in the preliminary design stage and to determine the influence of each design parameter on the structural behaviour under certain conditions. Finite element analysis can be used to compare a proven construction with an alternate construction, to determine where the difference occurs and to predict distribution of internal stress and strain that is difficult to measure experimentally. The finite element analysis can also be employed to understand how sandwich structures fail and to identify critical parameters.

The mostly common commercial software used to simulate the impact loading of sandwich structures is LS-DYNA [57, 64, 89–71] and ABAQUS [72, 73, 92, 93].
Three basic formulations are generally used for impact/dynamic mechanics problems involving large deformation and solid/shell interactions: (1) the Lagrangian Solution, (2) the Eulerian Solution, and (3) the Arbitrary Lagrangian-Eulerian (ALE).

The Lagrangian solution is well suited to moderately large strain problems where mesh distortion and element entanglement are not a significant problem. In this system, the mesh deforms with the material being modelled so that there is no material flow between elements. The advantage of the Lagrangian approach is that the free surface of the material is automatically captured by the mesh. The mesh and free surface can handle moving boundary and multiple materials naturally. The main disadvantage of the Lagrangian approach is that problems develop in physical situations that involve highly deformed surfaces.

In a Eulerian-based formulation, the mesh is stationary and the material flows through the mesh. The Eulerian approach is originated in the fluid dynamics field and is best suited to very large deformation flow problems. The greatest disadvantage of the Eulerian approach is that a fine mesh is required to capture the material response, making the method computationally very expensive.

The Arbitrary Lagrangian-Eulerian (ALE) approach is a compromise between the above two methods. In its most basic sense, the ALE method defines that the mesh motion is independent of the motion of the material being analysed. The greatest advantage of the ALE method is that it allows smoothing of a distorted mesh without performing a complete remesh. This smoothing allows the free surface of the material to be followed automatically without encountering the distortional errors of the Lagrangian approach.
For the modelling of materials of monolithic metal face-sheets of sandwich structures, Johnson-Cook’s material model is a widely used constitutive relation, which describes plasticity in metals under strain, strain rate and temperature conditions [94].

A large number of computational models have been developed for composite laminates, where the strain rate effect and failure criteria were defined [95].

Periodic cellular solids can be either modelled as 3-D solid elements or as 2-D shell elements. The former approach is to model the sandwich core, which requires the material properties for the particular core to be determined through mechanical testing. The second approach involves modelling each core cell with shell elements, so the final model is an accurate and detailed representation of the real geometry. The disadvantage of this approach is that it would be very time-consuming and would not permit an exploration of trends. On the other hand, modelling the core as a solid is more practical where the effective properties of the solid mimic those of the cellular core. A detailed review of constitutive models for cellular materials applicable to structural impact and shock analyses has been presented by Hanssen et al. [98]. The models have different formulations for the yield surface, hardening rule and plastic flow rule, while fracture is not accounted for in any of them.

2.4 Summary

Metal foams have attracted many studies to investigate their mechanical behaviour under various loading conditions. Theoretical modelling and experimental studies have been carried out to investigate the tension, compression properties, strain rate effect and failure criteria.

When aluminium foams are used as cores of sandwich structures, the shear behaviour of the aluminium foam is critical to the overall performance of the structure. Hence an understanding of the strength and energy absorption of foams under shearing is important. However, the shear resistance of metal foams and its associated energy
absorption mechanism is less documented. Future work needs to be done to further investigate the shearing properties with different relative densities and their capacity of energy absorption especially the dynamic shearing response of aluminium foam.

Previous literature covered experimental/analytical research on the collapse modes and mechanical response of aluminium sandwich beams as well as sandwich panels of FRL/FML/PVC foam cores. Most focused on static or low-velocity with only small deflections. High-velocity impact or ballistic impact studies are mainly on sandwich structures with composite face-sheets.

Very little research has been done on the high-impact response of sandwich panels of aluminium skin with aluminium foam core structures.

This thesis reports on research carried out on the quasi-static, low-velocity and ballistic penetration of sandwich panels with aluminium foams as core material. Experiments were made to study the static and dynamic shearing behaviour of aluminium foam as a premium study. A large number of experiments were also conducted to study the energy dissipation by projectiles of various geometries at different impact velocities. The effect of skin material, core material and their dimensions have been thoroughly investigated. Different models of deformation and failure modes were studied and identified. Finite element analysis was carried out to simulate the details of a penetration process and the energy dissipation was analysed.
CHAPTER THREE

SHEAR STRENGTH AND ENERGY ABSORPTION OF ALUMINIUM FOAM

3.1 Introduction

The objective of the chapter is to report our experimental work on the static and dynamic shearing properties of CYMAT aluminium foam. An experimental device was designed to ensure that foam specimens undergo steady shear failure along the small clearance (2mm) between the indenter and the clamps during the test. Variation of the ultimate shear stress against geometrical dimension and relative density is discussed and empirical formulae are obtained for the ultimate shear stress and essential shear energy in terms of the relative density. Dynamic shear tests were carried out using a drop hammer tower to investigate the strain rate effect on aluminium foams under dynamic loading. A finite element simulation was also carried out to analyse the shearing process and the energy dissipation during the shear loading procedure.

3.2 Experimental Set-up and Specimens

3.2.1 Specimens

Continuously cast and closed-cell aluminium foams from CYMAT Aluminium Corporation (Canada) were used in this study. Specimens with different nominal relative densities (5%, 10%, 12%, 15%, 17% and 20%, as marked by the manufacturer) were cut into beams of rectangular cross-section with different widths from 25mm to 200mm (Figure 3-1). The specimens were numbered by nine-digit numbers indicating relative density (2), width (3), thickness (2) and repeating times (2). Letters were added for some special cases such as G for glue effect cases and SD for dynamic cases. CYMAT aluminium foams were produced by melt route method, in which the
aluminium is melted, and then gas injected into the melt. SiC particles were used to stabilise the foam until cooling was complete. Foams manufactured by melt route method have a high degree of irregularity and imperfections within the cellular microstructure. Such irregularity results in the scattering of measured mechanical properties, reported by previous studies [6, 11, 100 and 101], was also evident in the current experiments.

To avoid the effect of edge constraint and cell size, the specimens were made larger than the critical value – eight cell size, as suggested by Andrews *et al* [100, 101]; each specimen contained at least 10 cells in the width direction. The thicknesses of the samples are as received: there are two different values: 25mm and 50mm. The relative density of the samples is calculated by dividing the actual weight of each specimen by its measured volume and then normalised by the density of the corresponding solid of the foam material, which is 2760kg/m$^3$ as provided by the manufacturer. The actual relative density of the samples deviated several percentage points from the nominal value given by the manufacturer. Hence careful measurement of each specimen is necessary to understand the scattering of the mechanical properties of the aluminium foam. Figure 3-1 shows an aluminium specimen before the test.

### 3.2.2 Experimental set-up

- **Static shear loading**

Experiments were carried out using an MTS universal testing machine. An experimental apparatus was designed to fix the specimens (Figure 3-2). The aluminium foam beam was fully clamped at the both ends and the load was applied via two rectangular plates (size: 260mm x 96mm x 20mm) clamping (by four screws at four corners, refer Figure 3-3) the central portion of the beam. The clearance (gap) Δ between the plates and the clamp was set at 2mm for the test specimens. This arrangement enabled the specimen to fail by shear along the clamping edge. The loading rate was fixed at 0.02mm/sec (1.2mm/min) for all the tests to obtain a quasi-static shearing loading. Tests were
stopped when the total crosshead displacements reached the whole thickness of the beams and by then the specimens had fully fractured by shear. Displacement and load curves were collected automatically by a computer connected to the MTS machine.

Figure 3-1 A typical rectangular-shaped specimen of aluminium foam (1707502 with relative density: 17%, width: 75mm and thickness: 25mm) for shearing tests

Figure 3-2 Photograph of the shearing experimental set-up with MTS
Figures 3-3 Sketches of experimental apparatus for shearing test

- **Low-velocity impact shear loading**

A drop hammer tower was used to carry out low-velocity shearing impact tests on aluminium foam beams. The same fixture used in static experiments was used in dynamic loading case (Figure 3-4). Samples were cut into rectangular shapes of 100mm $\times$ 200mm with two different thicknesses. The drop weight was set at 15kg with different heights up to 2.9m, which resulted in an initial impact velocity up to 7.58m/s. Initial impact velocity was measured by a laser velocimeter, which was installed on the drop hammer tower right above the fixture. A laser displacement transducer and a load washer were used to record simultaneously the time-displacement curves and time-load curves respectively. The load-displacement curves were obtained by combining the two curves.
3.3 Experimental Results and Observation

3.3.1 Quasi-static shear loading

The dimensions of specimens and testing results of the quasi-static test are listed in Table 3-1. Figure 3-5 shows photographs of a typical aluminium beam (171502501 with relative density: 17%, width: 150mm and thickness: 25mm) after a quasi-static shearing test. It is observed that the sample was sheared through the two clearance gaps (shearing planes as shown in Figure 3-7) along the clamped edges.

It is observed that the shearing fracture focused on the shearing plane as shown in the photographs in Figure 3-5; a few small side fractures also appeared on the top surface on some of the samples in the close region of the shearing plane. This shows a small degree of non-shearing loading at the edges near the gap. It is not reasonable to assume that the pure shearing deformation is transferred to every point of every cell member; points within the cellular microstructure experience non-uniform states of tension and compression rather than shear [3, 11]. We conducted tests on two identical samples to
investigate the effect of gluing the rectangular sample to the clamping edges, which helps to reducing the small fractures on the top surface near the gap during the shearing process. G171252501 and G171252502 were tested. The former was glued to the clamping plates using 3M Epoxy adhesive SW2216, cooling for 36 hours at room temperature, and the latter was set up as clamped as usual. Figure 3-6 shows the load and displacement curves of the two samples. There is only a slight difference between the two different clamping methods. The glued sample has a peak load, which appeared earlier than the one without glue. Except for that, there is no significant influence of the glue in regard to shearing strength and overall energy absorption. Thus the side fractures are minor and can be negligible.

A typical load displacement curve is shown in Figure 3-7. The photographs of the specimen 171252502 at different stages during the shearing process are also indicated by the corresponding numbers in the curve.

Load-displacement curves of aluminium foams of relative densities, 5%, 10%, 12%, 15%, 17% and 20%, have been shown in Figure 3-8 – Figure 3-13. Generally speaking, the curves can be divided into four phases indicated on the top of the curves. In Phase 1, the load increases sharply at the beginning of the test; the load-displacement curve is almost linear with a large slope until the force reaches a peak. After the peak load is reached, the load drops dramatically to only 20–50% of the peak load (Phase 2) and through-thickness cracks are visible along the clearance. Two typical trends were observed for Phase 3 for specimens with different relative densities. Taking specimens with a relative density of 12% and 17% as examples, the load increases very slowly to form a secondary peak in Phase 3 as shown in Figure 3-10; while the load decreases or forms a plateau in Figure 3-12; this is attributed to the cracked cell walls clinging to each other. Finally, the load drops further until the specimen is completely separated and the load is nearly zero (Phase 4).
Figure 3-5. Photographs of a typical specimen of aluminium foam (171502501 with relative density: 17%, width: 150mm and thickness: 25mm) after quasi-static shear test
Table 3-1. Dimensions of aluminium foam specimens and test results

<table>
<thead>
<tr>
<th>Specimen Number</th>
<th>Length $L$ [mm]</th>
<th>Width $B$ [mm]</th>
<th>Thickness $t$ [mm]</th>
<th>Weight [g]</th>
<th>Relative Density [%]</th>
<th>Shear Strength [MPa]</th>
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Similar observations were made with other specimens, load-displacement curves of aluminium foams of relative densities, 5%, 10%, 15% and 20%. There are two different thickness groups, 25mm and 40mm, for specimens with a nominal relative density of 5%. From the load-displacement curves, two specimens, 052002501 and 052004001, were observed having much higher secondary peak loads compared with their own primary peak loads. This may be due to the fact that the cracked cell walls from the shear damage accumulated and tangled together creating a very high-resistance force at the third phase of shearing process. The thicker samples generally show higher peak loads. It is noted the peak loads of the specimens with a nominal 15% relative density scattered to a very high degree. This may be due to the irregularity and imperfections of the microstructure of the aluminium foam.
Figure 3-7. Typical response curve and photographs for aluminium foam under quasi-static shear loading. The relative density is 17.2% and width is 126.3 mm. The loading stages of successive photographs are indicated on the curve.
Figure 3-8. Load-displacement curves for aluminium foams (nominal relative density 5%)

Figure 3-9. Load-displacement curves for aluminium foams (nominal relative density 10%)
Figure 3-10. Load-displacement curves for aluminium foams (nominal relative density 12%)

Figure 3-11. Load-displacement curves for aluminium foams (nominal relative density 15%)
Figure 3-12. Load-displacement curves for aluminium foams (nominal relative density 17%)

Figure 3-13. Load-displacement curves for aluminium foams (nominal relative density 20%)
3.3.2 Low-velocity impact shear loading

The effect of dynamic loading on the shearing property of aluminium foams is important when they work as sandwich cores, in which case the foam cores bear most of the localised shear deformation. This type of structures being used in real-life environments are always at risk of various dynamic impacts.

Low-velocity shearing tests were carried out to investigate the effect of dynamic shear loading using the drop hammer facility. The dimensions of the specimens and test results are listed in Table 3-2.

Figure 3-14 shows a pair of load-displacement curves for two nominally identical specimens under static and dynamic shear loadings. It is observed that the peak load of the specimen SD151002501 under dynamic shear loading is 13.24KN, which is slightly higher than the one under quasi-static loading 151002502, which is 10.34KN. The primary peak load occurred later for the dynamic case than the static one. The secondary peak, which appeared on the quasi-static curve, disappeared on the dynamic loading curve. Similar phenomena were observed for other specimens.
Table 3-2. Dimensions of aluminium foam specimens under dynamic shear tests and their test results

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<tr>
<th>Specimen number</th>
<th>Length $L$ [mm]</th>
<th>Width $B$ [mm]</th>
<th>Thickness $t$ [mm]</th>
<th>Weight [g]</th>
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<th>Initial impact velocity (m/s)</th>
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3.4 Analysis and Discussion

The effect of sample size, relative density and dynamic impact on the peak load, ultimate shear strength and energy absorption of aluminium foams are discussed in the following sections.

3.4.1 Effect of sample size

It is observed from the quasi-static load-displacement curves (Figure 3-8 – Figure 3-13), that the displacement corresponding to the peak load increases with the increasing width of the samples. Figure 3-15 shows a nearly linear relationship between the peak load displacement and the width of specimens of uniform thickness of 25mm and nominal relative density of 5%, 12%, 15% and 17% respectively. The peak load displacements normalised by the width of the samples are plotted in Figure 3-16. It is observed that
specimens with the same nominal relative density have similar values of normalised peak load displacement with the exception of small size samples. The specimen of relative density of 17% and width of 25mm and the one with relative density of 12% and width of 50mm have significantly higher values. This is attributed to the small width of these two samples. The average cell size is about 3.5mm for relative density of 17% and 5.5mm for 12%; the samples with 17% relative density and 25mm width have approximately seven cell sizes and the sample with 12% relative density and 50mm width has nine cell sizes in width direction respectively. This observation is in agreement with the conclusion that the fundamental length scale for the bulk shear properties of CYMAT aluminium foams is approximately 18 cell sizes [11].

The influence of the thickness on the peak load and peak load displacement were rather scattered and no clear trend is evident. A similar finding was reported by [10].

![Figure 3-15 Peak load displacement for different sample widths](image)
3.4.2 Effect of relative density

- Shear strength

From the quasi-static load-displacement curve, shear stress is obtained by dividing the total load by the area under shear \(2Br\). Shear stress is plotted in Figure 3-17 – Figure 3-22 against the displacement normalised with thickness \(t\), for relative densities from 5% to 20%, respectively. Ultimate shear strength is defined as the maximum shear stress.

It is found that for the aluminium foam specimens with the same relative density, the values of ultimate shear stress are almost identical, although some of the values scattered due to the imperfections and irregularity of the microstructure of the melt-route aluminium foam. The values of the ultimate shear stress are listed in Table 3-1. It
is almost independent of the sizes of foams but varies with the relative density. For example, for a relative density of 12%, the average ultimate shear stress is 0.86 MPa; while for 17%, it is 3.37 MPa.

Figure 3-17. Shear stress versus normalised displacement (nominal relative density 5%)
Figure 3-18. Shear stress versus normalised displacement (nominal relative density 10%)

Figure 3-19. Shear stress versus normalised displacement (nominal relative density 12%)
Figure 3-20. Shear stress versus normalised displacement (nominal relative density 15%)

Figure 3-21. Shear stress versus normalised displacement (nominal relative density 17%)
Earlier work [17] by Ruan et al. shows that the yield behaviour of CYMAT aluminium foams agrees with Deshpande-Fleck yield surface; this suggests that the shear strength is 0.69 times the uniaxial strength:

\[ \tau_m^* = 0.69\sigma_{pl}^* \]  

(3-1)

where \( \sigma_{pl}^* \) is the uniaxial strength. Here the value of plastic Poisson’s ratio is taken as 0.3. Based on the uniaxial strength obtained from the compression experiments [17], the average shear strength according to Eq. 3-1 is calculated for each nominal relative density. Table 3-3 lists a comparison between the calculated shear strength to the average shear strength obtained from current experiment. For lower relative densities up to 15%, the present experimental data are in good agreement with the calculated values while for the higher relative densities 17% and 20%, the equation underestimates the experimental data. The calculated values for nominal relative densities of 17% and 20% are 2.68MPa and 2.90MPa, while the corresponding experimental values are 3.37MPa and 4.47MPa respectively.
Table 3-3. Comparison of normalised shear strength

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<th>Average shear strength from current experiment (MPa)</th>
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<tbody>
<tr>
<td>5%</td>
<td>0.42</td>
<td>0.29</td>
<td>0.28</td>
</tr>
<tr>
<td>10%</td>
<td>1.59</td>
<td>1.10</td>
<td>1.06</td>
</tr>
<tr>
<td>12%</td>
<td>1.26</td>
<td>0.87</td>
<td>0.86</td>
</tr>
<tr>
<td>15%</td>
<td>2.20</td>
<td>1.52</td>
<td>1.51</td>
</tr>
<tr>
<td>17%</td>
<td>3.89</td>
<td>2.68</td>
<td>3.37</td>
</tr>
<tr>
<td>20%</td>
<td>4.20</td>
<td>2.90</td>
<td>4.47</td>
</tr>
</tbody>
</table>

As suggested by Gibson [2], the shear strength, $\tau^*_{m}$, normalised by the yield strength of the aluminium cell edge, $\sigma_{ys}$, is given as follows, for open-cell metal foams:

$$\frac{\tau^*_m}{\sigma_{ys}} = 0.21 \left( \frac{\rho^*_m}{\rho_y} \right)^{3/2}$$  \hspace{1cm} (3-2)

while for closed-cell foams:

$$\frac{\tau^*_m}{\sigma_{ys}} = 0.23 \left( \frac{\rho^*_s}{\rho_y} \right)^2 + 0.30 \left( \frac{\rho^*_s}{\rho_y} \right)$$  \hspace{1cm} (3-3)

The data in Figure 3-23 from our experiments with CYMAT aluminium foams, which are closed-cell foams, lie well below the results given by Eq. 3-3. The empirical relationship for the ultimate shear stress, which best fits the experimental data, is obtained as:

$$\frac{\tau^*_m}{\sigma_{ys}} = 0.337 \left( \frac{\rho^*_s}{\rho_y} \right)^2 + 0.001 \left( \frac{\rho^*_s}{\rho_y} \right)$$  \hspace{1cm} (3-4)
This suggests that plastic bending of cell edges contributes significantly to the failure of the CYMAT aluminium foam and the contribution of cell wall tension and stretching is negligible; thus the behaviour of CYMAT foam is much like that of open-cell foams, approximately described by Eq. 3-2. The experimental data from Rakow and Waas [11] are also shown in this figure, and exhibit a similar trend to ours. Thus Eq. 3-4 can be simplified to:

$$\frac{\tau^*}{\sigma_{ys}} = 0.34 \left( \frac{\rho^*}{\rho_s} \right)^{2}$$

\[(3-5)\]

![Figure 3-23. Normalised shear strength versus relative density](image)

- **Essential energy**

The shear energy is calculated by integrating the load-displacement curve and, assuming little energy is dissipated in regions other than the two fracture planes, the essential energy is obtained by dividing the total energy by the fracture area. Figure 3-24
illustrates variation of essential energy with the relative density. The essential energy \( \gamma \) derived from this group of shearing tests can be approximated by the following empirical formula:

\[
\gamma = 550.8 \left( \frac{\rho}{\rho_s} \right)^{1.8} \text{(kJm}^{-2})
\]

(3-6)

The data for essential energy obtained from the indentation and bearing tests by Olurin et al [114] are also shown in Figure 3-24. It is noted that their data were for Alporas foams. It can clearly be seen that, with increasing relative density, the essential energy under shear from the present tests broadly exhibits the same trend as that from the indentation and bearing tests, but the present values are slightly higher.

3.4.3 Effect of dynamic impact

Rakow and Waas [11] also tested nominally identical CYMAT aluminium foam samples under two different strain rates, static \((3.65\text{E-5s}^{-1})\) and quasi-static \((0.17\text{s}^{-1})\). It
is found the samples under the higher strain rate demonstrated an increase in shear strength and specific energy absorption, which goes against strain rate sensitivity for uniaxial compression found by previous studies [17-19].

A comparison of shear strength of specimens under quasi-static and dynamic shear loading is shown in Figure 3-25. It is noted that there are no obvious enhancements of dynamic effects of CYMAT aluminium foams with the tested range in general, which is similar to the conclusion from previous compression tests [15] with the same materials. A high degree of scattering is observed from the test data, which may be due to the imperfection of the CYMAT foam materials, which are manufactured by the melt-route method [11]. Figure 3-26 shows the essential energy absorbed by the specimens under quasi-static and dynamic shear loading respectively. The energy absorbed by the specimens with dynamic shear loading is slightly higher than those by quasi-static shear loading.

![Figure 3-25. Comparison of the shear strength of specimens under quasi-static and dynamic shear loading](image-url)
3.5 Finite Element Simulation

3.5.1 Geometric modelling of aluminium foam under shear

Based on the experiments, corresponding Finite Element simulation was carried out using the non-liner software package – LS-DYNA, which is an ideal tool for simulating the mechanics of solids, fluids and their interactions. In the FE model, the quasi-static shear loads are applied on aluminium beams by a couple of plates. Due to the symmetry of the structure and loading conditions, only one half of the whole system is considered, as shown in Figure 3-27. The couple of plates and the clamping plates are modelled as rigid bodies of 2D shell elements; aluminium is modelled as 3D brick elements respectively. Figure 3-27 shows the mesh and boundary conditions. The mesh is generated so as to reflect the size in the experiment. It is assumed that the contact surfaces between the plates and the specimen do not allow tangential slip. The shear
load is applied on the plates, and its movement has a velocity field governed by the following relationship [99]:

$$v(t) = \frac{\pi}{(\pi - 2)} \frac{w_{\text{max}}}{T} (1 - \cos \left( \frac{\pi}{2T} t \right))$$

(3-7)

where $T$ is the loading duration, with $0 \leq t \leq T$. $w_{\text{max}}$ is the final displacement. Integrating Eq. 3-7 over time gives the maximum value of $w_{\text{max}}$. On the other hand, differentiating Eq. 3-7 with respect to time leads to the initial acceleration equalling zero. Using this velocity curve makes it easy to control on displacement increasement. Therefore, the expression can ensure that the displacement load increases gradually, which can avoid the dynamic effect in the simulation especially at the beginning.

Figure 3-27 Geometric model and meshing of the aluminium beam under shear loading
3.5.2 Material modelling of aluminium foam

Due to the crushable properties and relatively low shearing strength, deformation domain is localised around the shearing gap during the shearing process. In the FE model, the aluminium specimen was in the shape of rectangle as in the previous experiments, and was modelled using material model No.154 of LS-DYNA, i.e. *Mat_DESHPANDE_FLECK_FOAM*. The yield function of the aluminium foam is defined as [12]:

\[
\Phi = \tilde{\sigma} - \sigma_y
\]  
(3-8)

where the equivalent stress \(\tilde{\sigma}\) is given by:

\[
\tilde{\sigma}^2 = \frac{1}{1+\alpha/3} \left[ \sigma_{VM}^2 + \alpha^2 \sigma_m^2 \right]
\]  
(3-9)

Here the \(\sigma_{VM}\) is von Mises stress, \(\sigma_m\) is the mean stress and \(\alpha\) is a parameter defining the shape of the yield surface and is related to the plastic Poisson’s ratio by [15]:

\[
\alpha = 3 \left( \frac{0.5 - \nu}{1+\nu} \right)^{1/2}
\]  
(3-10)

The yield stress can be expressed as [98, 99]:

\[
\sigma_y = \sigma_p + \gamma \frac{\dot{\varepsilon}}{\varepsilon_D} + \alpha_2 \ln \left[ \frac{1}{1-(\dot{\varepsilon}/\varepsilon_D)^\beta} \right]
\]  
(3-11)

where \(\sigma_p\) is the plateau stress of aluminium foam and \(\alpha_2, \gamma, \) and \(\beta\) are hardening parameters of the foam; and \(\dot{\varepsilon}\) is the equivalent strain. The densification strain is defined as:

\[
\varepsilon_D = -\ln \left[ \frac{\rho_f}{\rho_{f_0}} \right]
\]  
(3-12)

here \(\rho_f\) is the density of the aluminium foam and \(\rho_{f_0}\) is the density of the original material.

Material parameters of CYMAT aluminium foam are shown in Table 3-4. Uniaxial compression curves (Figure 3-28) obtained from mechanical tests of previous studies by Ruan et al. [15, 17] were converted into true strain–stress and used to compute the
hardening parameters $\alpha_2$, $\gamma$ and $\beta$ by using the least-square method. The plateau stress $\sigma_p$ was taken as the average of the uniaxial compressive strength $\sigma_c$ and tensile strength $\sigma_t$, i.e. $\sigma_p = (\sigma_c + \sigma_t)/2$. Fracture strain $\varepsilon_f$ is set at 0.1.

![Nominal stress–strain curves from quasi-static uniaxial compression tests](image)

Table 3-4. Material parameters of CYMAT aluminium foam

<table>
<thead>
<tr>
<th>Nominal relative density ($\rho$)</th>
<th>Poisson's ratio ($\nu$)</th>
<th>Plateau stress ($\sigma_p$) (MPa)</th>
<th>$\alpha$ (MPa)</th>
<th>$\gamma$ (MPa)</th>
<th>$\sigma_z$ (MPa)</th>
<th>$\beta$</th>
<th>$\varepsilon_f$</th>
</tr>
</thead>
<tbody>
<tr>
<td>5%</td>
<td>0.17</td>
<td>0.59</td>
<td>1.59</td>
<td>0.61</td>
<td>163.81</td>
<td>3.83</td>
<td>2.99</td>
</tr>
<tr>
<td>10%</td>
<td>0.17</td>
<td>1.34</td>
<td>1.59</td>
<td>1.19</td>
<td>137.31</td>
<td>3.79</td>
<td>2.30</td>
</tr>
<tr>
<td>12%</td>
<td>0.23</td>
<td>2.45</td>
<td>1.41</td>
<td>1.28</td>
<td>123.64</td>
<td>3.58</td>
<td>2.12</td>
</tr>
<tr>
<td>15%</td>
<td>0.29</td>
<td>2.77</td>
<td>1.21</td>
<td>2.36</td>
<td>109.89</td>
<td>2.73</td>
<td>1.89</td>
</tr>
<tr>
<td>17%</td>
<td>0.30</td>
<td>4.27</td>
<td>1.18</td>
<td>3.27</td>
<td>19.19</td>
<td>2.64</td>
<td>1.77</td>
</tr>
<tr>
<td>20%</td>
<td>0.30</td>
<td>4.67</td>
<td>1.18</td>
<td>4.97</td>
<td>13.77</td>
<td>1.07</td>
<td>1.61</td>
</tr>
</tbody>
</table>
3.5.3 Numerical results

The experimental and simulated load displacement curves of aluminium foam with nominal relative density of 20% are shown in Figure 3-29. Good agreement has been achieved in terms of the peak load and energy absorption. The four stages of shearing deformation observed in static shearing experiments were also shown in FE simulation. They are as follows:

1. Phase 1, the shear force increases nearly linearly until the peak load;
2. Phase 2, the load reaches to the peak and the foam crashes, the shear force decreases dramatically to a much lower level;
3. Phase 3, the load forms a plateau stage as the shearing process continues;
4. Phase 4, the load decreases further until the foam beam thickness is sheared thoroughly.

The agreement between the FE model and the experiment approves the finite element model suitable for further analysis of shearing properties of aluminium foams.

A comparison of FE simulation and experimental energy-displacement curves of aluminium foam with a nominal relative density of 20% is also shown in Figure 3-30.
Figure 3-29. FE simulation and experimental load-displacement curves of aluminium foam with a nominal relative density of 20%.

Figure 3-30. FE simulation and experimental energy-displacement curves of aluminium foam with a nominal relative density of 20%.
Figure 3-31 shows the distribution of Mises stress during the process of shearing. Although the agreement for shearing is not perfect due to the frictions after fracture in the shearing gap it is not possible to be simulated in the FE model. The results from FE model agree fairly well with the experiments in regard to energy absorption, general trend and overall characteristics.

Figure 3-31 Finite element simulation of the process of static shearing (to be continued)
3.5.4 Analysis on energy dissipation

Three different sections of the foam beam (Figure 3-32) were defined as three different parts separately in the DYNA-FE model in order to export energy absorption with each section and analyse the energy dissipation under shear loading. The three parts are defined as the shearing gap ‘A’, the left portion of the foam ‘C’ between and the right portion of the foam beam ‘B’. The energy dissipation is shown as Figure 3-32. It is observed that the energy dissipated outside the shearing gap is relatively low compared to the fracture energy absorbed by the shearing gap. The internal energy absorbed by the shearing gap is 92KJ about 65% of the total energy absorbed by the foam beam. This analysis helps us to understand energy absorption characteristics of aluminium foam.
under shear loading especially, when they work as sandwich cores in which case the loading pattern is similar to our experiment settings.

Figure 3-32. Analysis of the energy dissipation of aluminium foam under static shearing loading

3.6 Summary

An experimental study of shearing properties of CYMAT aluminium foams is presented in this chapter. Analysis on the maximum shear load and total energy absorption of the foam beams has indicated a linear relationship with the beam width. Fair agreement has been observed between the ultimate shear stress from the shearing tests and that derived from uniaxial compression incorporating Deshpande-Fleck yield criterion. The present data agree with those obtained by other workers using different test methods. Both the shear strength and essential energy under shear have been given, empirically, as a
function of the relative density, using a power-law relationship. There is no obvious enhancement observed from our low-velocity impact shearing tests in the current velocity range. The conclusion is in agreement with the findings of previous study with the same material [17].

A finite element simulation was also carried out to analyse the shearing process and the energy dissipation during the shear loading procedure. The results show good agreement with the experiment data. The shear energy absorbed by the shearing plain dominates the whole energy dissipation.
CHAPTER FOUR
CIRCULAR ALUMINIUM SANDWICH PANELS UNDER QUASI-STATIC LOADING

4.1 Introduction

Theoretical and experimental studies were previously carried out on aluminium sandwich beams to study their collapse and failure modes. Up to now, relatively few literature studies have been done on aluminium sandwich panels with metallic foam cores under indentation, despite a major application of sandwich structure to be used as large panels in automotive and aerospace industries. The mechanical properties of the sandwich panels under impact loading are very important. In this chapter, experiments were set up to test aluminium sandwich panels with metal foam cores under static loading using a hemi-spherical indenter. Face-sheets and foam cores with different thickness and relative density $\rho$ (the ratio of foam density to that of the cell walls) were considered. Collapse modes, strength and energy absorption mechanisms were observed.

4.2 Specimens and Experimental Set-up

4.2.1 Materials and specimens

Sandwich panels were made of aluminium skin sheets with aluminium foam cores. The aluminium skin sheets were of aluminium 5005H34. The sandwich cores were continuously cast closed-cell aluminium foam (trade name CYMAT, from Cymat Aluminium Corporation, Canada). An epoxy adhesive film was used to glue the aluminium alloy sheets onto the surfaces of the foam core. The face-sheets were well cleaned with solvents before being assembled to the core using a structural adhesive SA80 (SP Company, New Port, Isle of Wight, UK). Two steel plates were then used to clamp the assembled plates. The clamped plates were cured in an oven at 120°C for one
hour. Some assembled plates were cured for seven hours at 90°C to achieve better adhesive performance. Core foams of three different relative densities (5%, 12% and 20%) were used in order to observe their effect on the failure modes. Table 4-1 lists dimensions of the sandwich panels that have been tested in this study. Figure 4-1 shows photographs of two different types of circular samples for simply supported and fully fixed boundary conditions.

![Image](image_url)

**Figure 4-1. Specimens of static test.** (a) Specimen for simply supported test; (b) Specimen for fully fixed test

Standard tensile tests were conducted for each skin thickness (0.6mm or 3.0mm) to obtain their stress–strain curves. Young’s modulus was found to be $E_f = 66$GPa and the
yielding stress of the skin sheets is $\sigma_y = 134$ MPa shown in Figure 4-2. The shear properties of foams were studied and reported in detail in the previous chapter.

4.2.2 Experimental set-up

Specimens were cut into two types according to supporting conditions: circular and square panels. Circular panels were cut for simply supported boundary conditions and square-shaped specimens were for the fully fixed boundary conditions. They were placed on a specially designed frame that has a circular opening in the centre. The exposed circular area has a diameter of 300mm. Circular specimens with an average diameter of 320mm were simply supported as shown in Figure 4-3a. Square-shaped specimens with an average side length of 380mm were fully clamped as shown in Figure 4-3b. The purpose of this was to see the effect of boundary conditions on the failure modes of the sandwich panels.

Experiments were conducted on an MTS universal testing machine. An 80mm diameter hemispherical-ended indenter was used in all the tests. The speed of the indenter was set at 0.02mm/second (1.2mm/min). Load-displacement curves were recorded automatically by a computer connected to the MTS machine.

Figure 4-2. Uniaxial stress–strain curves for aluminium skin sheets
4.3 Experimental Results

Close observation was carried out on the failure modes of sandwich panels during quasi-static indentation tests. Broadly, four types of failure modes were discovered from our experiments as sketched in Figure 4-4:

- Failure mode I: Global bending, Figure 4-4I, was seen in simply supported specimens with skin thickness of 3.0mm (CS4 and CS5). Specimen CS3, which had a skin thickness of 0.6mm but with a high foam relative density of 20%, failed also by global bending.

- Failure mode II: For most fully fixed specimens and those simply supported with thin skin and low relative density, the most common failure mode was localised indentation with initial tearing of the top skin under the indenter, followed by further penetration resulting in perforation through the sandwich panels (mode II, Figure 4-4.II).

- Failure mode III: Localised indentation with global bending, Figure 4-4.III, occurred for simply supported specimens CS5 and CS1.
Table 4-1. Dimensions of the test specimens under quasi-static indentation tests

<table>
<thead>
<tr>
<th>Specimen</th>
<th>Nominal skin thickness(mm)</th>
<th>Nominal core thickness(mm)</th>
<th>Nominal core relative density(%)</th>
<th>Average diameter(mm)</th>
<th>Average side length(mm)</th>
<th>Support condition</th>
</tr>
</thead>
<tbody>
<tr>
<td>CS1</td>
<td>0.6</td>
<td>50</td>
<td>5</td>
<td>-</td>
<td>-</td>
<td>Simply Supported</td>
</tr>
<tr>
<td>CS2</td>
<td>0.6</td>
<td>25</td>
<td>12</td>
<td>320</td>
<td>-</td>
<td>Simply Supported</td>
</tr>
<tr>
<td>CS3</td>
<td>0.6</td>
<td>25</td>
<td>20</td>
<td>320</td>
<td>-</td>
<td>Simply Supported</td>
</tr>
<tr>
<td>CS4</td>
<td>3</td>
<td>25</td>
<td>12</td>
<td>320</td>
<td>-</td>
<td>Simply Supported</td>
</tr>
<tr>
<td>CS5</td>
<td>3</td>
<td>25</td>
<td>20</td>
<td>320</td>
<td>-</td>
<td>Simply Supported</td>
</tr>
<tr>
<td>CS6Top</td>
<td>3</td>
<td>25</td>
<td>12</td>
<td>320</td>
<td>-</td>
<td>Simply Supported</td>
</tr>
<tr>
<td>CS6BF</td>
<td>3</td>
<td>25</td>
<td>12</td>
<td>320</td>
<td>-</td>
<td>Simply Supported</td>
</tr>
<tr>
<td>SS1</td>
<td>0.6</td>
<td>25</td>
<td>12</td>
<td>-</td>
<td>380</td>
<td>Fully Fixed</td>
</tr>
<tr>
<td>SS2</td>
<td>0.6</td>
<td>25</td>
<td>20</td>
<td>-</td>
<td>380</td>
<td>Fully Fixed</td>
</tr>
<tr>
<td>SS3</td>
<td>3</td>
<td>25</td>
<td>12</td>
<td>-</td>
<td>380</td>
<td>Fully Fixed</td>
</tr>
<tr>
<td>SS4</td>
<td>3</td>
<td>25</td>
<td>20</td>
<td>-</td>
<td>380</td>
<td>Fully Fixed</td>
</tr>
</tbody>
</table>
Failure mode IV: Localised indentation with bending along the clamping edge, Figure 4-4.IV, was seen in the specimens that were fully fixed and with a thick skin face (3.0mm), medium and high relative density (12%, 20%).

Figure 4-5 shows the distribution of different failure modes against the specifications of sandwich specimens. Among failure modes II, III and IV, the top skins of specimens were seen with two different kinds of tearing, pedal tearing and circumference tearing. Pedal tearing started from the centre point right under the indenter forming several pedals and then continued cracking when the indenter penetrated further; circumference tearing began from the circumference of the indenter and cracking continued with a circular line around the head of the indenter to finally form a round disc. Top skins were seriously bent in global bending failure mode (I and III) accompanied with local wrinkling. The bottom skin of the specimens only had bending or crinkling. In modes II and IV, the bottom skins of the specimens were completely penetrated and in mode IV the bottom face-sheets with obvious plastic hinges along the clamping edge.

Foam core shearing and compression in the vicinity of the indenter were observed in all the specimens tested.

Figure 4-4. Failure modes of sandwich panels under static loading. I) Global bending, II) Localised indentation, III) Localised indentation with global bending, IV) Localised indentation with bending along the clamping edge
4.3.1 Simply supported specimens

For simply supported circular panels, failure mode is dominated by skin thickness. Figure 4-6 shows the failure modes vs. sandwich specimen configurations for simply supported specimens. Figure 4-7 shows the photographs of the top and bottom views of four circular specimens after testing.

For panels with 0.6mm skin thickness, deformation started with skin cracking in the vicinity under the indenter, then face skin cracking continued with core shearing until the face skin was penetrated by the indenter, followed by further core shearing and bottom skin bending and buckling.

Specimen CS1 had a failure mode III (Figure 4-7, a and b). The top skin was penetrated by the indenter, the bottom skin was damaged by bending and wrinkling; specimen CS2 also had a failure mode III but the bottom skin was torn with a large crack (Figure 4-7, c and d). Specimen CS3 had a failure mode I, both the top and bottom skin faces were bent without cracking, and this may be due to the high density of the foam core, which provided the sandwich structure with relatively greater strength.
For circular specimens with 3.0mm skin thickness (CS4 and CS5), failure was also dominated by skin bending and core shearing (failure modes I and III). Deformation started with delamination between the foam and skin, followed by face skin bending with core shearing. Then the face skin under the indenter started cracking. This face bending/cracking together with core shearing continued until the bottom skin also began to bend. Top skins had small cracks rather than being penetrated. The bottom skins only bent. Specimens CS4 and CS5 had failure mode I.

Figure 4-6. Failure modes vs. sandwich specimen specifications for simply supported specimens. The force-displacement curves (I: Global bending, III: Localised indentation with global bending)

It is observed that the higher density of the foam core slightly increased the overall strength and energy absorption of the sandwich panels with the same skin and core thickness as shown in Figures 4-8 and 4-9. It was also noticed that for specimens with the same skin thickness, the lower density foam core with higher thickness resulted in a similar plastic curve but with much lower yield strength. For specimen CS3 with the relative density \( \rho = 20\% \) and core thickness \( t = 25\text{mm} \), the curve appears to be irregular. This may be due to the fact that the skins delaminated at an early stage of the process, and then the failure was dominated by core shearing and bending of the skins; there was no perforation of the skins. Figure 4-10 (a) and (b) shows the early delamination and the specimen after test.
Figure 4-7. Photographs of the top and bottom view of four circular specimens after test (see Table 1 for their dimensions)
Figure 4-8. Force-displacement and energy-displacement curves for simply supported circular panels (skin thickness = 0.6mm, CS1: $\hat{\rho} = 5\%$, $t = 50\text{mm}$; CS2: $\hat{\rho} = 12\%$, $t = 25\text{mm}$; CS3: $\hat{\rho} = 20\%$, $t = 25\text{mm}$)
Figure 4-9. Force-displacement and energy-displacement curves for simply supported circular panels (skin thickness = 3.0mm, CS4: \( \hat{\rho} = 12\% \), \( t = 25mm \); CS5: \( \hat{\rho} = 20\% \), \( t = 25mm \); CS6BF: \( \hat{\rho} = 12\% \), \( t = 25mm \) with bottom skin and foam core)
4.3.2 Fully fixed specimens

For fully fixed specimens, different failure modes were also observed. Panels fully fixed had shown failure modes II and IV. Perforation occurred in all the specimens. That is, the indenter penetrated through the top skin, core and the bottom skin of each sandwich panel.

Initial face failure started soon after loading began. The top skin yielded causing a multi-angle cracking together with some crushing in the foam core. After this the load increased steadily, with further cracking of the face skin as well as crushing of the foam core, until the load directly acted on the bottom skin causing perforation and bending (for sandwich panels with thick face skins) and finally the bottom skin failed and was penetrated by the indenter.

Figure 4-11 shows the failure modes vs. sandwich specimen configurations for fully fixed specimens.

Specimens with 0.6mm skin thickness had failure mode II while panels with 3.0mm thickness had failure mode IV. Figure 4-12 shows photographs of specimen SS3 during and after testing.

Figure 4-10. Specimen CS3: $\hat{\rho} = 20\%$ and $t = 25\text{mm}$; (a) early delamination (b) specimen after test
The response of specimens and the effect of relative density of the foam core and the skin thickness are shown on the force-displacement curves in Figures 4-13 and 4-14. Figure 4-15 shows the energy absorption of the fully fixed specimens.

It is observed that higher relative density of the foam core has relatively increased both the strength and energy absorption of the sandwich panels. For example, specimens SS1 and SS2 have the same face thickness of 0.6mm and the same core thickness of 25mm; SS1 has a foam core of relative density $\rho$ of 12% while SS2 has a relative density $\rho$ of 20%. The difference of relative density has resulted in an increase of the peak load from 7.83KN to 13.77KN. A similar trend is also shown in Figure 4-14 with specimens SS3 and SS4. The strength and energy absorption are dominated by skin thickness as shown in Figure 4-15. More detailed discussions will be presented in the following section.

Figure 4-11. Failure modes vs. sandwich specimen configurations for fully fixed specimens (II: Localised indentation, IV: Localised indentation with bending along the clamping edge)
Figure 4-12. Photographs of specimens SS3: (a) face initial deformation, (b) perforation of the whole panel, (c) top view after test, (d) bottom view after test

Figure 4-13. Force-displacement curves for circular panels (skin thickness = 0.6mm, SS1: $\dot{\rho} = 12\%$, $t = 25mm$; SS2: $\dot{\rho} = 20\%$, $t = 25mm$)
Figure 4-14. Force-displacement curves for circular panels (skin thickness = 3.0mm, SS3: $\hat{\rho} = 12\%$, $t = 25mm$; SS4: $\hat{\rho} = 20\%$, $t = 25mm$)

Figure 4-15. Energy-displacement curves for circular panels
4.4 Analysis and Discussion

4.4.1 The effect of skin thickness (1) – simply supported

Comparing the force-displacement curves of two sandwich samples that were simply supported in Figure 4-16, we find that the skin thickness has a significant effect on the strength and energy absorption of the sandwich panels. With the same foam core, an increase in skin thickness dramatically increases the yield strength of the sandwich panel. For example, specimen CS2 with skin thickness of 0.6mm has yield strength of 6.3KN while specimen CS4 with skin thickness of 3.0mm has yield strength of 50KN, although both specimens have the same nominally foam core with relative density of 12% and thickness of 25mm.

![Figure 4-16. Effect of skin thickness for simply supported specimens](image)

4.4.2 The effect of skin thickness (2) – fully supported

Similar results were observed from fully fixed specimens (Figure 4-17). It is also clear that the skin thickness has a great effect on the overall strength of the panels. For specimens SS1 and SS3, both of which have the same relative density (12%) and core thickness (25mm), SS1 with a skin thickness of 0.6mm has yield strength of 7.5KN.
while SS3 with skin thickness of 3.0mm has yield strength of 60KN. Specimens SS2 and SS4 have the same foam core, with a relative density of 20% and a core thickness of 25mm; SS2 has a face thickness of 0.6mm and SS4 has a skin thickness of 3.0mm; the overall strength is 12KN for SS2, it increases to 78KN for SS4. Hence it can be concluded that skin thickness is the dominating factor in respect of the strength of sandwich structures.

![Figure 4-17. Effect of skin thickness for fully fixed specimens](image)

**4.4.3 The effect of the relative density of the foam core**

Close observations also found that foam core relative density contributes considerably to panels’ strength especially in the case of fully fixed boundary conditions. For specimens SS1 and SS2 with 0.6mm skin thickness, when the relative density increases from 12% to 20%, the ultimate load increases from 7.5KN to 13.5KN. For specimens SS3 and SS4, of which skin thickness is 3.0mm, the maximum load increases from 60KN to 79KN when the relative density increases from 12% to 20%. In the case of simply supported specimens, the increase of relative density does not increase the overall strength significantly but it has resulted in an obvious increase of the overall Young’s modulus of the structures, the specimens reaching the peak load in a shorter displacement.
4.4.4 The effect of support methods

The effect of support methods is shown in Figure 4-18. Specimens CS3 and SS2 have the same geometry, that is, thickness: 25mm, relative density: 20%, skin thickness: 0.6mm. CS3 was simply supported, SS2 was fully supported. It is seen that the fully fixed sample SS2 has been strengthened by the clamped edge. Specimens CS4 and SS3 also appear to have the same geometry (thickness: 25mm, relative density: 12%, skin thickness: 3.0mm, while CS4 was simply supported and SS3 was fully supported.

Figure 4-18. The effect of support methods
4.5 Summary

In this work, sandwich panels made of aluminium skin with CYMAT foam cores were tested on an MTS universal testing machine under quasi-static loading using a hemispherical indenter. Four different types of failure modes were proposed. It was found that the skin thickness, the relative density and the thickness of the foam core have significant influence on the failure modes of sandwich panels. The supporting condition is another factor in the deformation and final failure of the panels.
CHAPTER FIVE

CIRCULAR ALUMINIUM SANDWICH PANELS UNDER LOW-VELOCITY IMPACT

5.1 Introduction

In this chapter, experiments were set up to test aluminium sandwich panels with metal foam cores under low-velocity impact loading using a hemispherical indenter. The tests were carried out on a drop hammer testing tower. Several parameters – face-sheets thickness, foam cores with different thickness and relative density $\rho$ (the ratio of foam density to that of the cell walls) – as well as boundary conditions were considered. Impact strength and the energy absorption mechanism were obtained and analysed. Comparison was also conducted with the corresponding static tests to analyse the effect of impact velocity.

5.2 Specimens and Experimental Set-up

5.2.1 Materials and specimens

Sandwich panels were made of aluminium skin sheets with aluminium foam cores using the same materials as in the quasi-static tests discussed in the previous chapter. Specimens were prepared using the same method as referred to in the previous chapter. Table 5-1 lists the dimensions and specifications of the sandwich panels tested in this experiment.

5.2.2 Experimental set-up

Specimens were also cut into two types according to supporting conditions: circular and square panels. They were placed on the same fixture used in our quasi-static tests. The exposed circular area has a diameter of 300mm. Circular specimens with an average
diameter of 320mm were simply supported as shown in Figure 5-1a. Square-shaped specimens with an average side length of 380mm were fully clamped as shown in Figure 5-1b.

Experiments were conducted on a drop hammer tower. An 80mm diameter hemispherical-ended indenter (which is the same as in the quasi-static tests) was used in all low-velocity impact tests. Figure 5-2 shows the photograph of the drop hammer tower. The drop hammer has a total height of 5m and with a drop weight up to 150kg. In all the tests, the drop height has been kept constant, which results in the impact velocity of the indenter around 6m/second (360m/min). Two different dropping weights have been applied, that is, 40kgs and 150kgs. Load-time curves were recorded by a KISTLER™ quartz load washer, and displacement-time curves were recorded by a laser displacement transducer simultaneously. Thus the load-displacement charts were obtained by combining the two curves. A high-speed camera was also used to record the impact process of two sandwich specimens.

5.3 Experimental Results

The failure patterns/modes of sandwich panels under low-velocity impact indentation were observed and grouped into four different types similar to the quasi-static tests as shown in Figure 5-3. Mode I, global bending (Figure 5-3.I) and Mode III, localised penetration of face-sheet and core with bottom sheet, were commonly seen in simply supported specimens, while failure (Figure 5-3.III) Mode II, localised penetration (Figure 5-3.II) and Mode VI, penetration with bending along the clamping (Figure 5-3.IV) occurred with the fully fixed samples.

It was also noticed that the deformation of specimens under low-velocity impact indentation was with a much higher intensity compared to quasi-static tests. The deflection of the top skin face and bottom face-sheets of the sandwich structures was increased with a much larger degree of bending. More serious shearing and compression of the foam core under the vicinity of the indenter were observed in all the tested specimens.
Figure 5-1. Two types of experimental set-ups for the sandwich panels under low-velocity indentation tests, (a) simply supported, (b) fully fixed

To observe the indentation process, a high-speed camera was used to record the progress of the penetration of two specimens under the same impact velocity. The high-speed camera was set at 5000fps with a resolution of 512×1024. For the first specimen, the high-speed camera was set at the top front of the fixture to record the top face-sheet under the indention as illustrated in Figure 5-4; for the second specimen, the high-speed camera was set at the bottom front of the fixture to record the exit of the indenter and the graduate damage of the bottom face-sheet as shown in Figure 5-5.
Figure 5-2. Photograph of the drop hammer tower used in low-velocity indentation tests
Table 5-1. Dimensions of the specimens under low-velocity indentation

<table>
<thead>
<tr>
<th>Specimen</th>
<th>Nominal skin thickness (mm)</th>
<th>Nominal core thickness (mm)</th>
<th>Nominal core relative density (%)</th>
<th>Average diameter (mm)</th>
<th>Average side length (mm)</th>
<th>Support condition</th>
</tr>
</thead>
<tbody>
<tr>
<td>DC060525</td>
<td>0.6</td>
<td>50</td>
<td>5</td>
<td>320</td>
<td>-</td>
<td>Simply Supported</td>
</tr>
<tr>
<td>DC061525</td>
<td>0.6</td>
<td>25</td>
<td>15</td>
<td>320</td>
<td>-</td>
<td>Simply Supported</td>
</tr>
<tr>
<td>DC062025</td>
<td>0.6</td>
<td>25</td>
<td>20</td>
<td>320</td>
<td>-</td>
<td>Simply Supported</td>
</tr>
<tr>
<td>DC201525</td>
<td>2.0</td>
<td>25</td>
<td>15</td>
<td>320</td>
<td>-</td>
<td>Simply Supported</td>
</tr>
<tr>
<td>DC202025</td>
<td>2.0</td>
<td>25</td>
<td>20</td>
<td>320</td>
<td>-</td>
<td>Simply Supported</td>
</tr>
<tr>
<td>DC300525</td>
<td>3.0</td>
<td>50</td>
<td>5</td>
<td>320</td>
<td>-</td>
<td>Simply Supported</td>
</tr>
<tr>
<td>DC301525</td>
<td>3.0</td>
<td>25</td>
<td>15</td>
<td>320</td>
<td>-</td>
<td>Simply Supported</td>
</tr>
<tr>
<td>DC302025</td>
<td>3.0</td>
<td>25</td>
<td>20</td>
<td>320</td>
<td>-</td>
<td>Simply Supported</td>
</tr>
<tr>
<td>DS060525</td>
<td>0.6</td>
<td>50</td>
<td>05</td>
<td>380</td>
<td>-</td>
<td>Fully Fixed</td>
</tr>
<tr>
<td>DS062025</td>
<td>0.6</td>
<td>25</td>
<td>20</td>
<td>380</td>
<td>-</td>
<td>Fully Fixed</td>
</tr>
<tr>
<td>DS200525</td>
<td>2.0</td>
<td>50</td>
<td>05</td>
<td>380</td>
<td>-</td>
<td>Fully Fixed</td>
</tr>
<tr>
<td>DS201525</td>
<td>2.0</td>
<td>25</td>
<td>15</td>
<td>380</td>
<td>-</td>
<td>Fully Fixed</td>
</tr>
<tr>
<td>DS202025</td>
<td>2.0</td>
<td>25</td>
<td>20</td>
<td>380</td>
<td>-</td>
<td>Fully Fixed</td>
</tr>
<tr>
<td>DS300525</td>
<td>3.0</td>
<td>50</td>
<td>05</td>
<td>380</td>
<td>-</td>
<td>Fully Fixed</td>
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<tr>
<td>DS301525</td>
<td>3.0</td>
<td>25</td>
<td>15</td>
<td>380</td>
<td>-</td>
<td>Fully Fixed</td>
</tr>
<tr>
<td>DS302025</td>
<td>3.0</td>
<td>25</td>
<td>20</td>
<td>380</td>
<td>-</td>
<td>Fully Fixed</td>
</tr>
</tbody>
</table>
Figure 5-3. Failure patterns of sandwich panels under low-velocity loading
Figure 5-4. Photographs show the top view of the progress of an aluminium sandwich panel under low-velocity indentation (the high-speed camera was set at 5000fps with a resolution of 512×1024)
Figure 5-5. Photographs show the bottom view of the progress of an aluminium sandwich panel under low-velocity indentation (the high-speed camera was set at 5000 fps with a resolution of 512 x 1024)
5.3.1 Simply supported specimens

The relationship between the failure modes and sandwich specimen configurations for simply supported specimens under low-velocity indentation tests is illustrated in Figure 5-6.

For panels with lower skin thickness (0.6mm) that were impacted with the drop weight of 40kgs, deformation of the top skin was localised in the vicinity of the indenter; the face skin cracked and was penetrated by the indenter; the foam core was also sheared and penetrated, followed by bottom skin bending and buckling. Mode I and Mode III were seen with these samples.

For circular specimens with higher skin thickness (2.0mm and 3.0mm) that were impacted with the drop weight of 150kgs, failure was dominated by top skin bending and core shearing (failure modes II and IV: top skins had small cracks rather than being penetrated with the bottom skins only bending). Figure 5-7 shows the force-displacement curves of the simply supported specimens under low-velocity impact loading.

Figure 5-6. Failure modes vs. sandwich specimen specifications for simply supported specimens (I: Global bending, III: Localised indentation with global bending)
5.3.2 Fully fixed specimens

Panels fully fixed had shown failure modes II and IV. Specimens with skin thickness of 0.6mm were impacted with the drop weight of 40kgs while specimens with a higher skin thickness were impacted with a heavier weight of 150kgs.

Figure 5-8 shows the failure modes vs. sandwich specimen configurations for fully fixed specimens under low-velocity impact loading. The force-displacement curves are illustrated in Figure 5-9.
Figure 5-8. Failure modes vs. sandwich specimen configurations for fully fixed specimens under low-velocity impact loading (II: Localised indentation, Localised indentation with bending along the clamping edge)

Figure 5-9. Force-displacement curves for fully fixed specimens under low-velocity impact loading
5.4 Analysis and Discussion

5.4.1 The effect of skin thickness

Comparing curves in Figure 5-10, we found that skin thickness has a significant effect on the strength and energy absorption of the sandwich panels. With the same foam core, an increase in skin thickness dramatically increases the yield strength of the sandwich panel. For example, specimen DC061525 with a skin thickness of 0.6mm has a yield strength of 16.3KN; specimen DC201525 with a skin thickness of 2.0mm has a yield strength of 27.8KN while specimen DC301525 with a skin thickness of 3.0mm has a yield strength of 43.2KN, although all the specimens have nominally the same foam core with relative density of 12% and core thickness of 25mm.

Similar results were observed from fully fixed specimens (Figure 5-11). For example, of this group of specimens, all have nominally identical foam cores with the same relative density of 5% and core thickness of 25mm. The yield strength of specimen DS060525 with a skin thickness of 0.6mm is 11.5KN; it is 33.7KN for DS200525 with a skin thickness of 2.0mm, and 39.7 for DS300525 with a skin thickness of 3.0mm.

Figure 5-10. Effect of skin thickness for simply supported specimens under low-velocity impact indentation
5.4.2 The effect of the relative density of the foam core

The relative density of the foam core was found to contribute considerably the sandwich panels’ strength for specimens under the simply supported and fully fixed boundary conditions. Figures 5-12 and 5-13 show two groups of specimens, which are of the same skin thickness. In Figure 5-12 the skin thickness of the simply supported specimens is 3.0mm. The yield strength of three specimens with different foam core relative densities of 5%, 15% and 20% is 33.1KN, 39.9KN and 42.2KN respectively. The fully fixed group of specimens shown in Figure 5-13 have the same skin thickness of 2.0mm; when the relative density increases from 5% and 15% to 20%, the yield strength is 33.14KN, 52.9KN and 70.6KN respectively. In the case of simply supported specimens, the increase of relative density does not increase the overall strength significantly but it results in an obvious increase of the overall Young’s modulus of the structures. The specimens reach the peak load in a shorter displacement. However, the yield strength of the fully fixed samples is significantly enhanced by the increase of core relative density.
5.4.3 The effect of boundary conditions

Figure 5-14 shows load-displacement curves of three couples of identical specimens under different boundary conditions. Each couple of specimens DC062025 and DS062025, DC201525 and DS201525, and DC202025 and DS202025, has the same geometry but was tested with different boundary conditions. It is obvious that the strength of the fully fixed specimens has been enhanced by the clamped edge. The fully fixed samples with thicker skin thickness of 2.0mm have much higher initial yield strength than their counterparts, which were simply supported. For example, the strength of DS202025 (fully fixed) is 70.6KN, more than double the strength, 33.1KN, of specimen DC202025 (simply supported). A comparison of the initial yield strength of this group of specimens is illustrated in Figure 5-15.

![Figure 5-14. Load-displacement curves under different boundary conditions.](image)

Figure 5-12. Effect of the foam core for simply supported specimens under low-velocity impact indentation
Figure 5-13. Effect of the foam core for fully fixed specimens under low-velocity impact indentation

Figure 5-14. The effect of boundary conditions: load-displacement curves
5.4.4 The effect of the impact velocity

Figure 5-16 shows the load-displacement curves of four couples of identical specimens under quasi-static loading or low-velocity indentation loading. It is noted that for the group of sandwich panels with thin skin thickness of 0.6mm, the load-displacement curves under quasi-static and low-velocity loading have a similar pattern, however, the dynamic curves are of much higher value; for the group of sandwich panels with thick skin face of 3.0mm, the pattern of the dynamically loaded specimens is different from the quasi-statically loaded ones. The dynamically loaded samples have two obvious peak loads with the second peak reaching up to the highest load. The quasi-statically loaded samples have an initial yield point with a limited load drop but soon the load picks up to reach a much higher peak load. The dynamically loaded samples also have a higher slope in the initial stage of the load-displacement curves.

Figure 5-17 shows a comparison of the initial yield strength of the specimens under different loading conditions. For the sandwich panels with thin skin faces of 0.6mm, the dynamic impact has dramatically enhanced the yield strength as seen with DC062025
and CS3, and DS062025 and SS2. For sandwich panels with thicker skin thickness DC302025 and CS5, and DS302025 and SS4, the initial yield strength is also increased significantly in the dynamically loaded samples compared to the quasi-statically loaded samples.

Figure 5-16. The effect of impact velocity: load–displacement curves
Figure 5-17. The effect of impact velocity: comparison of initial yield strength

### 5.5 Summary

Sandwich panels made of aluminium skin with CYMAT foam cores were tested on a drop hammer tower under a low-velocity indentation loading using a hemi-spherical indenter. Four types of failure modes similar to static loading tests were discovered. It was found that the impact velocity, the skin thickness, the relative density and the thickness of the foam core as well as boundary conditions have a significant influence on the strength and energy absorption of sandwich panels.
CHAPTER SIX

FINITE ELEMENT SIMULATION OF CIRCULAR ALUMINIUM SANDWICH PANELS AT LARGE DEFLECTION

6.1 Introduction

Based on a yield locus, analytical solutions shown in the appendix A are derived on the elastic stiffness, initial collapse strength, and post-yield response of simply supported and clamped circular sandwich panels. This analytical model can be used to assess the load-bearing capacities of the sandwich constructions, and to optimise the geometric parameters of the structures in different loading conditions.

In this chapter, a numerical simulation is designed to validate the analytical model for the elastic and post-yield behaviours of circular sandwich panels, which is presented in Appendix A. A parametric study is carried out to identify the effect of several key parameters, for example, boundary conditions, core/face thickness, punch radius, core density, and so on, on the load-deflection response. A comparison on the energy dissipating performance is conducted between sandwich plates and monolithic panels with an identical mass per unit area, but various face–core thickness ratios.

6.2 Numerical Simulation

FE model

Finite element analysis has been carried out to validate the analytical model. In the FE model, the quasi-static loads are applied on sandwich panels by a central punch. Due to the symmetry of the structure and loading conditions, only one quarter of the whole system is considered, as shown in Figure. 6-1. The flat punch is modelled with a rigid
body. It is assumed that the contact surfaces between the punch and the panel do not allow a tangential slip. The punch load is applied on the central zone with \( r = a \), and its movement has a velocity field governed by the same velocity curve used in the previous chapter 3.

The calculations are performed using the commercial finite elements code ABAQUS/explicit, which has the ability to handle contact conditions and the large deformation behaviour of the foam core. A mesh sensitivity study has been performed to ensure the convergence of the results.

Figure 6-1. FE model of the circular sandwich panel

In the FE model, the material of the face-sheets (AL-2024-O aluminium alloy) \cite{112} is modelled as an elasto-plastic solid. Its mechanical properties are: Young’s modulus \( E_f = 73.1 \text{GPa} \); shear modulus \( G_f = 28 \text{GPa} \); Poisson’s ratio \( \nu_f = 0.33 \) and yield strength \( \sigma_f = 75.8 \text{GPa} \). The classical isotropic hardening flow theory based on von Mises yield criterion is employed. The annealed aluminium alloy face-sheets can sustain a strain of up to 20% without fracture.

For the foam with relative density \( \bar{\rho} = 20\% \), its Young’s modulus is \( E_c = 1.06 \text{GPa} \), its tensile yield strength is \( \sigma_c = 5.1 \text{MPa} \), its tensile ductility is 1.1%. In the simulation, the behaviour of the aluminium foam core is described using the constitutive law
proposed by Deshpande and Fleck [12], in which the yield surface is elliptical in the space of von Mises stress $\sigma_e$ and mean stress $\sigma_m$. This model has been integrated in ABAQUS as a user-defined material law (UMAT) by Chen and Fleck [113].

6.3 FE Simulation Results and Discussion

The analytical solution is validated against the numerical model, and a parametric study is carried out to identify the effect of several key parameters, for example, boundary conditions, core and face thickness, punch radius, core density, and so on, on the load-deflection response of the circular sandwich panels.

The deformation contour plots of both the simply supported and fully clamped circular sandwich panels are shown in Figures 6-2 and 6-3, respectively. The results indicate that the sandwich panels deform with a dent first developing at the centre, and this zone expands towards the outside of the plate. This is finally followed by a large global plastic bending and stretching. A direct comparison of the analytical and computational load-deflection curves is shown in Figure 6-4. In this case, circular sandwich panels have face-sheet thickness $h_f = 1mm$; core thickness $H_c = 5mm$; panel radius $R = 160mm$ and punch radius $a = 20mm$. The curves from the theoretical solutions agree well with the numerical predictions under both boundary conditions. The analytical predictions reveal the overall load-deflection response of circular sandwich panels. It should be emphasized that an elastic deformation phase is added to the analytical modelling just for comparison purposes. In the elastic bending phase, the bending rigidities from the analytical models are slightly smaller than the FE results. One possible reason is that the core shear is disregarded in the theoretical analyses. Increasing the core thickness would raise the shear rigidity of the core material, whose magnitude could even reach the same order as that of the bending rigidity. After the initial plastic collapse, the slopes of curves from the analytical predictions are very close to those from the numerical simulations. For a simply supported panel, when the initial collapse has been attained, plastic deformation continues at a nearly constant plateau load, until the deflection reaches the order of the sandwich panel thickness. After that,
the bending-dominated deformation mechanism turns to a stretching-dominated mode; and then with further increase of the load, tearing damage takes place at the central area of the back face, and the structure fails eventually. It can also be seen from the figure that a discrepancy between the numerical and analytical predictions takes place at the stage of large deflection. One explanation is that in the theoretical analysis the radial central displacement of the panel is assumed to be zero, which can significantly reduce the computational complexity. However, since the deflection $w$ is not continuous at the centre of the plate, the radial displacement at that position should be small, but may not be zero. The present analytical model may overestimate the influence of the membrane force, but it might be useful in practical engineering applications as a simple method to obtain approximate estimations of sandwich panel deflection.

Figure 6-2. A typical deformation process of a simply supported circular sandwich panel loaded by a flat punch in the transverse direction
Compared with the simply supported case, a better agreement is obtained for the fully clamped panel, especially at the stage of large deflection. Due to the constraint at the clamped edge, an evident membrane effect, together with the strain-hardening behaviour is observed. The load-deflection curve increases in an approximately linear manner. Subsequently, the structure continues its deforming until the radial plastic strain of the face-sheets reaches the failure strain of their material, which leads to tearing damage.

The effect of the core thickness for simply supported and fully clamped boundary conditions is compared in Figures 6-5(a) and (b) respectively. All the panels have identical material and geometrical parameters defined above, except for the core thickness $H_c$, which has three different values, 5mm, 8mm and 9mm. The results show a considerable influence of the core thickness on the load-deflection behaviour in both the loading conditions. For the simply supported panels, larger core thickness results in
a longer plateau after the initial yield point, and the deformation mode is similar to that
of the metallic foam. On the other hand, the fully clamped panels show an
approximately linear post-yield behaviour, which reflects the effect of the membrane
force.

Figure 6-4. Comparison of analytical and numerical load-deflection response of circular sandwich panels
for fully clamped and simply supported boundary conditions (dash line – FE prediction, solid line –
analytical prediction)

Figure 6-6 indicates the effect of skin thickness. In this case, the face thickness $h_f$ has
three different values: $1\text{mm}$, $1.5\text{mm}$, $2\text{mm}$. The comparison shows a good agreement at
the large deflection stage, but the analytical solutions over-predict the initial collapse
load in both the loading conditions. Increasing the face-sheet thickness, sandwich plates
tend to behave like monolithic structures.

The influence of the core strength is revealed in Figure 6-7. Increasing the yield strength
of core materials from 2.1MPa to 5.1MPa, no significant difference in the post-yield
load-displacement response has been seen. This indicates the face yielding is the main
mechanism after the initial collapse, especially at the stage of large deflection. The post-
yield behaviour is insensitive to the yield strength of the core, although the initial collapse of the sandwich structure is highly dependent on the core strength.

(a) Simply supported cases

(b) Fully clamped cases

Figure 6-5. The effect of core thickness on the load-deflection response of circular sandwich panels for the different boundary conditions (dash line — FE predictions, solid line – analytical predictions)
Figure 6-6. The effect of face-sheet thickness on the load-deflection response of circular sandwich panels for the different boundary conditions (dash line – FE predictions, solid line – analytical predictions)
Figure 6-7. The effect of core strength on the load-deflection response of circular sandwich panels for the different boundary conditions (dash line – FE predictions, solid line – analytical predictions)

The dependence of structural response on the radii of punches is examined in Figure 6-8. It is clearly shown that larger indenters produce higher limit loads and more evident strain-hardening effects.
(a) Simply supported cases

(b) Fully clamped cases

Figure 6-8. The effect of punch radius on the load-deflection response of circular sandwich panels for the different boundary conditions (dash line – FE predictions, solid line – analytical predictions)

6.4 Performance Comparison with a Solid Panel
In this section, an effort is made to identify the near-optimal configurations of circular sandwich panels with various face–core thickness ratios, and the results are compared with the performance of a solid panel with equal mass. In the comparative study, only fully clamped panels are considered, since they are more commonly used in practice.

We define a simple failure criterion based on an estimation of the strain in the face-sheets due to in-plane stretching, with the effect of plastic bending disregarded. For a face-sheet material with ductility $\varepsilon_F$, the deflection $w_F$ at failure is given by [36]

$$w_F = 2R\sqrt{\varepsilon_F/2} \quad (6-2)$$

Figure 6-9 illustrates the relationship between the normalised load $P/P_0$ and the normalised central deflection $w_m/R$ for sandwich plates with various configurations but identical mass per unit area, $M_c = 2h_f\rho_f + H_c\rho_c$ being the mass per unit area of the sandwich panel, and $P_0$ the limit load of a solid plate:

$$P_0 = \frac{1}{2}\pi\sigma_f h^2\alpha_0/\alpha_1 \quad (6-3)$$

For the comparison, the corresponding monolithic plates with the same mass per unit area are also included in the figure. The deformation mechanism of monolithic plates has been extensively studied and well understood. At the initiation of the deformation, bending is the key effect on the structural response. The upper sheet of the structure is in compression and the lower one is in tension. However, later the whole structure is dominated by stretching, until tearing damage occurs on the face. In both the cases, when normalised deflection is less than $w_m/R \approx 0.075$, sandwich plates can bear more loads than their solid counterparts; at larger deflections, the performance of monolithic constructions is superior. In particular, sandwich panels with smaller face–core thickness ratios yield a higher load-carrying capacity.

The total energy absorption $W$ is the area under the load-versus-deflection curve of the circular sandwich panels. Neglecting the energy absorption in the elastic-bending phase, the nondimensional $W$ is given as

$$W = \frac{W(w_m)}{\sigma_f M_c\rho_f} = \int_0^{w_m} Pdw_m/\sigma_f M_c\rho_f \quad (6-4)$$

\[116\]
For the sandwich constructions discussed above, the normalised plastic energy dissipation per unit area against normalised deflection is plotted in Figure 6-10. When the deflection satisfies $w_m/R \leq 0.12$, sandwich plates absorb more energy in both cases than the solid plates of equal mass. Likewise, at a certain deflection, the plates with a smaller face–core thickness ratio absorb more energy, but just slightly.

Figure 6-9. The load-deflection response of circular sandwich panels of equal mass with various face–core thickness ratios. The response of the solid panel having the same total mass is also plotted for comparison.
Figure 6-10. Plastic energy dissipation for circular sandwich panels of equal mass with various face–core thickness ratios. Plastic energy dissipation of the solid panel having the same total mass is also plotted for comparison.
6.5 Summary

A finite element simulation has been performed to validate an analytical solution. The analytical model assumes that global deformation is the main deformation mechanism and no local indentation takes place. The large deflection response is estimated by assuming a velocity field, which is defined according to the initial deformation pattern of flat panel and the boundary condition. The effect of membrane force on the load-bearing behaviour of circular sandwich plates (either simply supported or fully clamped) is studied in detail in this research. A parametric study is then carried out to examine the effect of boundary conditions, face and core thickness ratios and core strength on the structural response. Good agreement confirms that the overall deformation behaviour of circular sandwich plates is well captured by the analytical model. The membrane effect caused by the geometric change has a significant influence on the post-yield response, especially in the case of a fully clamped boundary. Finally, using the analytical model, a comparative study with monolithic plates on the energy absorption is given for sandwich plates with identical mass per unit area, but various face–core thickness ratios.
CHAPTER SEVEN
CIRCULAR ALUMINIUM SANDWICH PANELS UNDER BALLISTIC IMPACT

7.1 Introduction

This chapter reports on the large number of perforation tests that were conducted on the sandwich panels with aluminium foam cores and two identical aluminium face-sheets, which were subjected to quasi-static loading and impact at the velocity ranging from 70m/s to 250m/s. The experimental set-ups and procedures are presented in Section 7.2. In Section 7.3 the specimen perforation process is recorded and the failure/damage patterns are described in detail. Based on the impact and exit velocities measured in the tests, the effect of face thickness, core thickness and relative density and the projectile shapes on the ballistic limit and energy absorption are analysed in Section 7.4. In addition, an empirical equation is derived to describe the influence of impact velocity on the perforation energy. FE simulation focusing on the deformation and energy dissipation of the sandwich panel under ballistic impact loading is reported in Section 7.5–7.7.

7.2 Experiments

7.2.1 Specimens, projectiles and material properties

Specimens were cut into square shapes. Sandwich panels tested consisted of two aluminium face-sheets with identical thickness and an aluminium foam core. The face-sheets were made from the same material used in previous chapter 4 and 5, ie. Al-5005H34 and had four thicknesses: 0.6mm, 1.0mm, 1.5mm and 2.0mm, respectively. The CYMAT™ closed-cell aluminium foam cores used had five relative densities: 5%, 10%, 15%, 18% and 20%. The cores were machined into 120mm×120mm plates with
two different thicknesses (25mm and 50mm). The face-sheets were glued onto the surfaces of the foam core using an epoxy adhesive, and then cured in room temperature for 36 hours. Figure 7-1 schematically illustrates the preparation procedure of the sandwich panels. The stress-strain relationships of the face and foam cores were determined via standard tensile and compression tests, and the resultant curves are presented in Figures 7-2 and 7-3, respectively.

Figure 7-1. Preparation process of the specimens

Figure 7-2. Tensile stress-strain curves of the Al-5005H34 skins with different thicknesses
Figure 7-3. Uniaxial compression stress-strain curves of the CYMAT™ aluminium foam with different relative densities

The stainless steel projectiles used in the tests had a similar mass and three shapes: (1) flat-ended, (2) hemispherical-nosed, and (3) conical-nosed. The geometries and dimensions are shown in Figure 7-4.

The main material properties of face, core and projectile are summarised in Table 7-1.

Figure 7-4. Geometries and dimensions of the three types of projectiles: flat-ended, hemispherical-nosed and conical-nosed
Table 7-1. Material properties of the face-sheets and foam cores

<table>
<thead>
<tr>
<th>Component part</th>
<th>Material</th>
<th>Density ρ (kg/m³)</th>
<th>Young’s modulus E (GPa)</th>
<th>Poisson’s ratio ν</th>
<th>Tensile strength σ_v (MPa)</th>
<th>Plateau stress σ_pl (MPa)</th>
<th>Tensile failure strain ε_f</th>
<th>Densification strain ε_D</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.6mm face-sheet</td>
<td>Al-5005H34</td>
<td>2700</td>
<td>65.04</td>
<td>0.33</td>
<td>123.47</td>
<td>--</td>
<td>2.7%</td>
<td>--</td>
</tr>
<tr>
<td>1.0mm face-sheet</td>
<td>Al-5005H34</td>
<td>2700</td>
<td>62.57</td>
<td>0.33</td>
<td>130.51</td>
<td>--</td>
<td>8.9%</td>
<td>--</td>
</tr>
<tr>
<td>1.5mm face-sheet</td>
<td>Al-5005H34</td>
<td>2700</td>
<td>77.09</td>
<td>0.33</td>
<td>139.55</td>
<td>--</td>
<td>7.6%</td>
<td>--</td>
</tr>
<tr>
<td>2.0mm face-sheet</td>
<td>Al-5005H34</td>
<td>2700</td>
<td>66.69</td>
<td>0.33</td>
<td>129.43</td>
<td>--</td>
<td>7.8%</td>
<td>--</td>
</tr>
<tr>
<td>20% foam core</td>
<td>Al-Si(7-9%)-Mg(0.5-1%)</td>
<td>540</td>
<td>0.14</td>
<td>0.30</td>
<td>5.20</td>
<td>9.92</td>
<td>3.5%</td>
<td>0.60</td>
</tr>
<tr>
<td>18% foam core</td>
<td>Al-Si(7-9%)-Mg(0.5-1%)</td>
<td>486</td>
<td>0.05</td>
<td>0.30</td>
<td>4.79</td>
<td>2.82</td>
<td>3.7%</td>
<td>0.35</td>
</tr>
<tr>
<td>15% foam core</td>
<td>Al-Si(7-9%)-Mg(0.5-1%)</td>
<td>405</td>
<td>0.03</td>
<td>0.29</td>
<td>3.19</td>
<td>1.70</td>
<td>3.5%</td>
<td>0.42</td>
</tr>
<tr>
<td>10% foam core</td>
<td>Al-Si(7-9%)-Mg(0.5-1%)</td>
<td>270</td>
<td>0.07</td>
<td>0.17</td>
<td>1.70</td>
<td>1.59</td>
<td>0.9%</td>
<td>0.41</td>
</tr>
<tr>
<td>5% foam core</td>
<td>Al-Si(7-9%)-Mg(0.5-1%)</td>
<td>135</td>
<td>0.01</td>
<td>0.17</td>
<td>0.73</td>
<td>0.31</td>
<td>0.7%</td>
<td>0.39</td>
</tr>
</tbody>
</table>
7.2.2 Experiment set-up

- **Quasi-static perforation set-up**

A universal MTS system was used to perform the quasi-static perforation. Figure 7-5 shows the experimental set-up. Three indenters with an identical diameter of 7.5mm and three different shapes were used. Specimens were fully clamped at the edges using two steel frames, which had a circular opening (diameter D=100mm) in the centre. The indenter head speed was set at 0.02mm/s to penetrate sandwich structures. Force-displacement data were recorded automatically by a computer connected to the MTS machine.

![Figure 7-5. Experimental set-up of the quasi-static perforation](image)

- **Impact perforation set-up**

Impact perforation tests were carried out by a gas gun system with the maximum allowable gas pressure of 15MPa as shown in Figure 7-6. The internal diameter of the barrel is 12.5mm, and a plastic holder was used to hold the projectiles, which had diameters of 7.5mm. The impact velocity can be controlled by adjusting gas pressure in
the charge chamber. A laser velocimeter was placed between the exit of the barrel and the fixed sample in order to measure the impact/incident velocity. A high-speed video camera was set up at the rear of the fixed sample to measure the rear/exit velocity. Calibrations were carried out before.

The square specimens were fully clamped at the peripheral regions, and the exposed circular area had a diameter of 100mm, which was the same as in the quasi-static case. The clamping device is illustrated in Figure 7-7.

Figure 7-6. Experimental set-up of the impact perforation

7.3 Experimental Observations

A high-speed video camera was also used to record the impact and perforation process and response of the sandwich structures for six samples. Figures 7-8(a) and (b) show typical photographic sequences of front face and back face, respectively, recorded at 20000 frames/s during perforation by a hemispherical-nosed projectile at 187.5m/s. The perforation process took approximately 0.45ms.
Depending on the impact energy level, the specimens after tests exhibit two damage modes: (1) full perforation and (2) partial perforation. The latter means that the front face is penetrated, but the projectile remains embedded in the foam core eventually. In each mode, the panels show similar failure patterns. Figure 7-9(a)–(c) shows the
typical failure pattern of a fully perforated panel for the front face, core cross-section and back face, respectively, impacted by a hemispherical-nosed projectile. The front face exhibits a circular crater without global deformation. A localised tunnel is evident in the foam core directly below the point of impact and throughout the thickness. A small amount of delamination can be observed between the core and two face-sheets. On the back face, a round hole is visible with a number of petals.

Figure 7-10 shows the typical failure pattern of a panel with partial perforation by a hemispherical-nosed projectile. The front face (Figure 7-10a) fails in the same way with the full perforation case. The projectile is stuck in the core as indicated in Figure 7-10(b), and the foam in front of the projectile nose is slightly crushed. No significant global deformation takes place in the panel, and the back face (Figure 7-10c) is almost undeformed. Based on the experimental observations it can be concluded that only a small amount of energy is absorbed due to global panel deflection or slippage at the clamped edges.
Figure 7-8. A typical high-speed photographic sequence of a specimen perforated by a hemispherical-nosed projectile (Specimen No.: AF06182515, skin thickness $h$: 0.6mm, core thickness $H$: 25mm, relative density of core $\bar{\rho}$: 18%)
Figure 7-9. A typical failure pattern of a fully perforated panel by a hemispherical-nosed projectile: (a) front face, (b) cross-section of core, and (c) back face (Specimen No.: AF06182504, skin thickness $h_f$: 0.6mm, core thickness $H_c$: 25mm, relative density of core $\rho$: 18%)
Figure 7-10. A typical failure pattern of a partially perforated panel by a hemispherical-nosed projectile: (a) front face, (b) cross-section of core, and (c) back face (Specimen No.: AF06105001, skin thickness $h_f$: 0.6mm, core thickness $H_c$: 50mm, relative density of core $\rho$: 10%). Note the projectile was cut through during sectioning.
7.4 Results and Analysis

Two key quantitative results that are critical for evaluating the penetration-resistant behaviour and energy-dissipating performance of the sandwich structures are analysed in detail in this section: (1) ballistic limit \( V_b \), which is defined as the velocity when the projectile is either stuck in the back face or else exits with the negligible velocity, and (2) perforation energy \( E_p \), which is essentially the energy absorbed by the structure during perforation.

If the mass of any small fragmentations is neglected and hence its kinetic energy, the change of the kinetic energy of the projectile is then equal to the energy dissipated by perforation, with the elastic energy negligible. Therefore,

\[
\frac{1}{2} m_p v_i^2 - \frac{1}{2} m_p v_r^2 = E_p
\]

(7-1)

where \( m_p \) is the mass of projectile. Assuming that the kinetic energy loss of the projectile is entirely dissipated by the sandwich structure, when \( v_r \) is ensured to be small, the corresponding perforation energy is given by

\[
E_p = \frac{1}{2} m_p v_b^2
\]

(7-2)

From this the value of \( V_b \) may be obtained. Strictly speaking, the ballistic limit would need to be obtained by conducting a series of experiments, gradually changing the velocity until the projectile has just perforated the target. This would be time consuming and costly. In the present study, the energy dissipated \( E_p \) is first calculated from Eq. 7-1, and then is assumed to be close to the value when \( v_r \) is zero. The ballistic limit is then estimated from Eq. 7-2. As will be seen later, \( E_p \) is indeed affected by the projectile velocity. Nevertheless, with the exception of two cases, the value of the calculated ballistic limit is close to the initial impact velocity and hence the error so introduced should be small.
If the structure is not fully perforated, that is, the projectile is embedded in the panel, then $V_r$ vanishes, and $V = V_i$. Then the energy absorption by the structure would be equal to the initial kinetic energy of the projectile: $(1/2) m_p V_i^2$.

In the subsequent sections, the effect of several parameters on the ballistic limit and perforation energy is examined. This is the impact velocity, skin thickness, core thickness and density and the projectile shape. In addition, a comparison is made between the monolithic and sandwich panels.

### 7.4.1 Effect of impact velocity

The effect of velocity on the composite laminates or sandwich panels with laminate skins has been studied extensively over the last 20 years [54, 55, 74, 75, 83, 84, 96]. It has been found that the energy dissipation by the structures during perforation can be significantly enhanced by increasing the impact velocity. However, few analytical models have been reported. Wen et al. [74] and Reid and Wen [75] performed perforation tests on FRP laminates and twin FRP-skinned sandwich panels. Dimensionless analysis was used to derive empirical equations to predict the ballistic limit and perforation energy of these structures. The approach was based on the assumption that the deformation is localised and the mean pressure provided by FRP laminate targets to resist the projectile can be decomposed into two parts. One part is the cohesive quasi-static resistive pressure due to the elasto-plastic deformations of the target materials, and the other is the dynamic resistive pressure arising from the effect of velocity.

In the impact tests by Zhao et al. [85] on the sandwich panels with aluminium foam cores and aluminium alloy faces, evident energy enhancement with impact velocity was observed, although the skin sheet and foam cores are nearly rate insensitive. They suggested that a possible reason is the difference in compressive strain levels of the foam core reached before the perforation of the top skin under static and impact loading because of different face-foam core interaction mechanisms. Such localised foam core
strength enhancement leads to the increase of the top skin peak loads. However, no
detailed quantitative analysis has been presented.

In this research, seven identical sandwich panels ($h_f$: 0.6mm; $H_c$: 25mm; $\bar{\rho}_c$: 18%) were
impacted by the hemispherical projectiles at various impact velocities. Based on the
impact and exit velocities recorded, the ballistic limit and energy dissipation were
obtained. The average ballistic limit is approximately equal to 105m/s. The panels’
specifications and results are list in Table 7-2.

A quasi-static perforation was performed on a panel with the configuration described
above (AF061825S-01HS), and the resultant perforation energy (22.37J) is used as a
benchmark for comparison with the dynamic impact cases. The ratio of dynamic and
quasi-static perforation energy ($E_d/E_s$) (defined as a dynamic enhancement factor $\phi$) is
plotted against the impact velocity ($V_i$) in Figure 7-11.

![Figure 7-11. A plot showing the linear relationship of impact velocity and the ratio of dynamic and quasi-static perforation energy](image)

Figure 7-11. A plot showing the linear relationship of impact velocity and the ratio of dynamic and quasi-static perforation energy
Table 7-2. Specifications and test results of a group of specimens, which have the identical configurations (\(h_f = 0.6\text{mm}; H_c = 25\text{mm}; \bar{\rho} = 0.18\)), and are loaded at various impact velocities, so that the effect of perforation velocity can be studied.

<table>
<thead>
<tr>
<th>Specimen name</th>
<th>Face thickness (h_f) (mm)</th>
<th>Core thickness (H_c) (mm)</th>
<th>Core relative density (\bar{\rho})</th>
<th>Impact velocity (V_i) (m/s)</th>
<th>Rear velocity (V_r) (m/s)</th>
<th>Ballistic limit (V_b) (m/s)</th>
<th>Perforation energy (E_p) (J)</th>
<th>Dynamic enhancement factor (\Phi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>AF06182507-3</td>
<td>0.6</td>
<td>25</td>
<td>0.18</td>
<td>186.34</td>
<td>143.89</td>
<td>109.63</td>
<td>49.28</td>
<td>2.20</td>
</tr>
<tr>
<td>AF06182508-1</td>
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<td>25</td>
<td>0.18</td>
<td>181.82</td>
<td>139.65</td>
<td>108.05</td>
<td>47.87</td>
<td>2.14</td>
</tr>
<tr>
<td>AF06182508-2</td>
<td>0.6</td>
<td>25</td>
<td>0.18</td>
<td>174.22</td>
<td>124.29</td>
<td>115.81</td>
<td>54.99</td>
<td>2.46</td>
</tr>
<tr>
<td>AF06182509-1</td>
<td>0.6</td>
<td>25</td>
<td>0.18</td>
<td>157.73</td>
<td>113.30</td>
<td>103.94</td>
<td>44.29</td>
<td>1.98</td>
</tr>
<tr>
<td>AF06182510-1</td>
<td>0.6</td>
<td>25</td>
<td>0.18</td>
<td>106.31</td>
<td>46.84</td>
<td>94.32</td>
<td>36.47</td>
<td>1.63</td>
</tr>
<tr>
<td>AF06182510-4</td>
<td>0.6</td>
<td>25</td>
<td>0.18</td>
<td>98.88</td>
<td>24.08</td>
<td>95.61</td>
<td>37.48</td>
<td>1.68</td>
</tr>
<tr>
<td>AF06182511-3</td>
<td>0.6</td>
<td>25</td>
<td>0.18</td>
<td>77.76</td>
<td>0</td>
<td>77.76</td>
<td>24.79</td>
<td>1.11</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(Stuck in the back face)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Fitting the data points reveals the linear relationship between $\phi$ and $V_i$ and gives

$$\Phi = \frac{\varepsilon_d}{\varepsilon_s} = 1 + 0.0065V_i$$

(7-3)

The enhancement factors calculated from the results by Zhao et al. [85] are also included in Figure 7-11. They display a similar trend to the current experimental data but with slightly higher values. This may be due to the fact that different materials and test protocols were used in their experiments.

### 7.4.2 Effect of face and core thickness

A total of eight sandwich panels with four different face thicknesses and two different core thicknesses were tested. The specimens were designed into two groups, and in each group, the four panels had the identical core thickness. The specifications of the panels and results are listed in Table 7-3.

The ballistic limit versus face thickness is plotted graphically in Figure 7-12. The figure indicates that, as expected, the panels with a thicker core result in a higher ballistic limit. For the range of thickness tested, the ballistic limit is almost linearly proportional to the face thickness. Increasing the face thickness, the ballistic limits of the sandwich panels with 25mm and 50mm cores tend to converge. This suggested that for panels with thicker skins, the relative contribution from the core decreases. Using the ballistic limits of the panels with 0.6 mm skins as the benchmark, for $H_c=25$mm, the ballistic limit of the panels with 1.0 mm, 1.5 mm and 2.0 mm skins increases by 33.2%, 55.3% and 91.0%, respectively; while for $H_c=50$mm, the increases are 6.7%, 8.8% and 35.5%, respectively.

Thicker skins also lead to larger delamination zone of back faces, as shown in Figure 7-13. A large cavity can be observed between the back face and core in the case of $h_f=2.0$mm, which indicates a significant delamination of over 50% area of the back face. For the specimen with $h_f=0.6$ mm, in contrast, no evident delamination is shown.
Table 7-3. Specifications and test results of two groups of specimens, each of which has an identical core thickness ($H_c=25\text{mm}$ and $50\text{mm}$ respectively) and relative density ($\bar{\rho}=0.18$), but different face thicknesses ($h_f=0.6\text{mm}$, $1.0\text{mm}$, $1.5\text{mm}$ and $2.0\text{mm}$, respectively), so that the effect of core and face thickness can be studied.

<table>
<thead>
<tr>
<th>Specimen name</th>
<th>Face thickness $h_f$(mm)</th>
<th>Core thickness $H_c$(mm)</th>
<th>Core relative density $\bar{\rho}$</th>
<th>Impact velocity $V_i$(m/s)</th>
<th>Rear velocity $V_r$(m/s)</th>
<th>Ballistic limit $V_b$(m/s)</th>
<th>Perforation energy $E_p$(J)</th>
</tr>
</thead>
<tbody>
<tr>
<td>AF20182502</td>
<td>2.0</td>
<td>25</td>
<td>0.18</td>
<td>212.16</td>
<td>90.16</td>
<td>192.05</td>
<td>151.23</td>
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<tr>
<td>AF15182502</td>
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<td>25</td>
<td>0.18</td>
<td>192.80</td>
<td>113.14</td>
<td>156.11</td>
<td>99.92</td>
</tr>
<tr>
<td>AF10182501</td>
<td>1.0</td>
<td>25</td>
<td>0.18</td>
<td>147.78</td>
<td>62.49</td>
<td>133.92</td>
<td>73.53</td>
</tr>
<tr>
<td>AF06182514</td>
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<td>0.18</td>
<td>100.54</td>
<td>0</td>
<td>100.54</td>
<td>41.44</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(Stuck in the back face)</td>
<td></td>
</tr>
<tr>
<td>AF20185001</td>
<td>2.0</td>
<td>50</td>
<td>0.18</td>
<td>245.90</td>
<td>93.60</td>
<td>227.39</td>
<td>211.99</td>
</tr>
<tr>
<td>AF15185001</td>
<td>1.5</td>
<td>50</td>
<td>0.18</td>
<td>187.50</td>
<td>42.93</td>
<td>182.52</td>
<td>136.59</td>
</tr>
<tr>
<td>AF10185001</td>
<td>1.0</td>
<td>50</td>
<td>0.18</td>
<td>192.31</td>
<td>70.33</td>
<td>178.99</td>
<td>131.36</td>
</tr>
<tr>
<td>AF06185001</td>
<td>0.6</td>
<td>50</td>
<td>0.18</td>
<td>182.93</td>
<td>72.78</td>
<td>167.83</td>
<td>115.49</td>
</tr>
</tbody>
</table>
7.4.3 Effect of core density and thickness

In this section, 10 specimens were separated into two groups and in each of group the foam cores had an identical thickness (25mm and 50mm, respectively), but had different relative densities (20%, 18%, 15%, 10% and 5%), as listed in Table 7-4, together with the test results. A plot of ballistic limit against relative density is shown in Figure 7-14 together with the ballistic limit of a core-absent structure (two identical faces with an air core). The figure reveals the approximate linear relationship between ballistic limit and relative density. It can be seen that the ballistic limit of the specimens with a thicker core has a rapid increase pace than their counterparts with a thinner core. When the density of the core is low, the ballistic limits of the two groups are approaching. When $p \lessgtr 10\%$, the effect of core thickness can be neglected. However, the energy absorption is still higher than that of the core-absent structure. This may be due to the extra energy dissipation caused by the core/faces interaction during the perforation.
Figure 7-13. Photographs showing that thicker skins lead to larger delamination areas of back faces
7.4.4 Effect of projectile nose shape

The effect of the projectile shape is an important topic in ballistic mechanics, and a large amount of work has been conducted on the composite laminates [120–124] and sandwich structures with such skins and polymeric cores [74, 75, 77, 78]. In this research, a similar study is carried out on the sandwich specimens with an aluminium foam core and faces of aluminium alloy. Six identical specimens were perforated by quasi-static and impact loading, using three different projectiles: hemispherical-nosed, conical-nosed and flat-ended. The panels’ specifications and experimental results are given in Table 7-5. Analyses of the effect of the projectile shape are made in terms of (1) damage of back face, (2) piercing force-displacement history recorded in the quasi-static perforation tests, and (3) ballistic limit and perforation energy.

• Damage of back face

The back faces of the six panels after tests are separated into three groups and shown in Figure 7-15. In each group, the two panels were loaded by the identical projectiles. The figure shows the damage pattern of back face dependence on the projectile type and velocity. Blunter projectiles result in larger petal areas. Compared with the impact cases, quasi-static perforations produced larger tearing damage on the back face.

• Piercing force-displacement history

The piercing force-displacement histories of the three panels perforated quasi-statically were recorded using a PC connected to the MTS system, and are shown in Figure 7-16. It can be seen that the curves of the three panels have similar trends, and the perforation processes can be divided into three stages:
Table 7-4. Specifications and test results of two groups of specimens, each of which has identical core thickness ($H_c=25\text{mm}$ and $50\text{mm}$ respectively) and face thickness ($h_f=0.6\text{mm}$), but different core relative densities ($\bar{\rho}=0.05, 0.1, 0.15, 0.18$ and $0.20$ respectively), so that the effect of face thickness and core density can be studied.

<table>
<thead>
<tr>
<th>Specimen name</th>
<th>Face thickness $h_f$ (mm)</th>
<th>Core thickness $H_c$ (mm)</th>
<th>Core relative density $\bar{\rho}$</th>
<th>Impact velocity $V_i$ (m/s)</th>
<th>Rear velocity $V_r$ (m/s)</th>
<th>Ballistic limit $V_b$ (m/s)</th>
<th>Perforation energy $E_p$ (J)</th>
</tr>
</thead>
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<td>0.20</td>
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<td>45.24</td>
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<td>52.58</td>
</tr>
<tr>
<td>AF06202502</td>
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<td>25</td>
<td>0.20</td>
<td>109.09</td>
<td>20.01</td>
<td>107.24</td>
<td>47.15</td>
</tr>
<tr>
<td>AF06182514</td>
<td>0.6</td>
<td>25</td>
<td>0.18</td>
<td>100.54</td>
<td>0</td>
<td>100.54 (Stuck in the back face)</td>
<td>41.44</td>
</tr>
<tr>
<td>AF06152502</td>
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<td>25</td>
<td>0.15</td>
<td>113.04</td>
<td>55.03</td>
<td>98.74</td>
<td>39.97</td>
</tr>
<tr>
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<td>25</td>
<td>0.05</td>
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<td>92.75</td>
<td>70.43</td>
<td>20.34</td>
</tr>
<tr>
<td>AF06185001</td>
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<td>50</td>
<td>0.18</td>
<td>182.93</td>
<td>72.78</td>
<td>167.83</td>
<td>115.49</td>
</tr>
<tr>
<td>AF06155001</td>
<td>0.6</td>
<td>50</td>
<td>0.15</td>
<td>151.15</td>
<td>81.16</td>
<td>127.51</td>
<td>66.66</td>
</tr>
<tr>
<td>AF06105002</td>
<td>0.6</td>
<td>50</td>
<td>0.10</td>
<td>113.55</td>
<td>45.99</td>
<td>103.82</td>
<td>44.19</td>
</tr>
<tr>
<td>AF06055001</td>
<td>0.6</td>
<td>50</td>
<td>0.05</td>
<td>106.08</td>
<td>80.04</td>
<td>69.62</td>
<td>19.87</td>
</tr>
<tr>
<td>AF06055002</td>
<td>0.6</td>
<td>50</td>
<td>0.05</td>
<td>100.00</td>
<td>60.65</td>
<td>79.51</td>
<td>25.92</td>
</tr>
</tbody>
</table>
Figure 7-14. A plot showing the relationship of core density and ballistic limit

Figure 7-15. Damage patterns of back faces perforated quasi-statically and dynamically by the projectiles with different shapes
Table 7-5. Specifications and test results of two groups of specimens, which have the identical configurations \((h_f=0.6\text{mm}; \ H_c=25\text{mm}; \ \overline{\rho}=0.18\)), and loaded by different projectiles. The first group was perforated quasi-statically and the second group was perforated dynamically.

<table>
<thead>
<tr>
<th>Loading condition</th>
<th>Specimen name</th>
<th>Projectile shape</th>
<th>Face thickness (h_f) (mm)</th>
<th>Core thickness (H_c) (mm)</th>
<th>Core relative density (\overline{\rho})</th>
<th>Impact velocity (V_i) (m/s)</th>
<th>Rear velocity (V_r) (m/s)</th>
<th>Ballistic limit (V_b) (m/s)</th>
<th>Perforation energy (E_p) (J)</th>
<th>Dynamic enhancement factor (\Phi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quasi-static perforation</td>
<td>AF061825S-02HS</td>
<td>Hemi-Spherical</td>
<td>0.6</td>
<td>25</td>
<td>0.18</td>
<td>0</td>
<td>0</td>
<td>--</td>
<td>23.26</td>
<td>--</td>
</tr>
<tr>
<td></td>
<td>AF061825S-03CH</td>
<td>Conical</td>
<td>0.6</td>
<td>25</td>
<td>0.18</td>
<td>0</td>
<td>0</td>
<td>--</td>
<td>21.33</td>
<td>--</td>
</tr>
<tr>
<td></td>
<td>AF061825S-04FH</td>
<td>Flat</td>
<td>0.6</td>
<td>25</td>
<td>0.18</td>
<td>0</td>
<td>0</td>
<td>--</td>
<td>25.03</td>
<td>--</td>
</tr>
<tr>
<td>Impact perforation</td>
<td>AF06182510-1</td>
<td>Hemi-Spherical</td>
<td>0.6</td>
<td>25</td>
<td>0.18</td>
<td>106.31</td>
<td>46.84</td>
<td>94.32</td>
<td>36.47</td>
<td>1.57</td>
</tr>
<tr>
<td></td>
<td>AF061825S-07CH</td>
<td>Conical</td>
<td>0.6</td>
<td>25</td>
<td>0.18</td>
<td>109.49</td>
<td>73.86</td>
<td>80.83</td>
<td>26.78</td>
<td>1.26</td>
</tr>
<tr>
<td></td>
<td>AF061825S-08FH</td>
<td>Flat</td>
<td>0.6</td>
<td>25</td>
<td>0.18</td>
<td>112.78</td>
<td>37.74</td>
<td>106.28</td>
<td>46.32</td>
<td>1.85</td>
</tr>
</tbody>
</table>
(1) Front face failure – the contact force between the indenter and specimen sharply increases from zero to the first peak, then drops quickly, implying the sudden failure of the front face;

(2) Core failure – the piercing force reaches a plateau and keeps constant for a period of time, indicating the core failure due to shear and a small amount of compression; and

(3) Back face failure – the force goes up again to the second peak, where the core becomes densified, and then the back face fails. When the indenter penetrates the back face, the force-displacement curve drops down to zero gradually, due to the friction effect.

The first peak loads of the flat-ended, hemispherical-nosed and conical-nosed projectiles are 882 N, 984 N and 1249 N respectively. The largest initial slope (the slope formed between the original point and first peak) was produced by the flat-ended indenter, followed by the hemispherical-nosed and conical-nosed indenters. In the case of the flat-ended indenter, penetration of the faces did not occur until the force almost reached the maximum level, and the initial slope would be close to the elastic stiffness of the panel. For the conical projectile, due to its sharp tip, penetration of the front face is much smoother. The hemispherical projectile is in between the two cases. For the flat and hemispherical projectiles, their second peaks are 98% and 50% higher than their first ones, respectively, which indicates the densification of the foam core in front of the projectile head, which leads to a sudden jump of the stress [2]. In the case of the conical indenter, as very little foam is densified, the two peaks are almost identical.

- **Ballistic limit and perforation energy**

Based on the experimental results presented in Table 7-5, the ballistic limits and perforation energies of the six panels were calculated. The results indicate that the blunter impactor can raise the ballistic limit and energy dissipation. Figure 7-17 shows the perforation energy by the six projectiles. The specimen impacted by the flat-ended projectile has the maximum perforation energy, in both quasi-static and impact loading
conditions. In the quasi-static case, compared with the flat impactor, the hemispherical-nosed projectile leads to 7.87% less energy dissipation. Minimum results are produced by the conical-nosed projectile, which is 14.78% reduction. In the impact perforation, again, using the flat-ended projectile as a benchmark, the energy absorptions by the hemispherical and conical impactors reduce by 21.27% and 42.19%, respectively. A possible explanation for this is that the flat-ended projectile has more foam material compressed inwards due to its blunter tip, which causes more energy consumption; while the conical tip trends to push the material sideways and less energy is dissipated during compression. The detailed mechanism in this process is complex and open for the further investigation.

Figure 7-16. Piercing force histories of the three types of projectiles in quasi-static perforation
In this section, a 0.6mm monolithic skin and an aluminium foam panel with 25mm thickness and 18% relative density were perforated by a hemispherical projectile in quasi-static and impact-loading conditions, and then compared with the results of the corresponding sandwich panels with the same skins and cores. Specifications of the specimens and test results are available in Table7-6. It is clear that both the face-sheets and foam cores do not exhibit the velocity effect, but the dynamic perforation energy of the sandwich structure is almost two times of its quasi-static dissipation, which agrees with the observations by Zhao et al. [85].
Table 7-6. Specifications and test results of two groups of specimens, in each of which a 0.6mm monolithic skin, an aluminium foam panel with 25mm thickness and 18% relative density, and a corresponding sandwich panel with such skins and core were perforated. The first group was loaded quasi-statically and the second group was loaded dynamically.

<table>
<thead>
<tr>
<th>Loading condition</th>
<th>Specimen name</th>
<th>Face thickness $h_f$ (mm)</th>
<th>Core thickness $H_c$ (mm)</th>
<th>Core relative density $\bar{\rho}$</th>
<th>Impact velocity $V_i$ (m/s)</th>
<th>Rear velocity $V_r$ (m/s)</th>
<th>Ballistic limit $V_b$ (m/s)</th>
<th>Perforation energy $E_p$ (J)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quasi-static perforation</td>
<td>Face0601</td>
<td>0.6</td>
<td>--</td>
<td>--</td>
<td>0</td>
<td>0</td>
<td>--</td>
<td>3.37</td>
</tr>
<tr>
<td></td>
<td>Core0201</td>
<td>--</td>
<td>25</td>
<td>0.18</td>
<td>0</td>
<td>0</td>
<td>--</td>
<td>22.14</td>
</tr>
<tr>
<td></td>
<td>AF061825S-01HS</td>
<td>0.6</td>
<td>25</td>
<td>0.18</td>
<td>0</td>
<td>0</td>
<td>--</td>
<td>22.37</td>
</tr>
<tr>
<td>Impact perforation</td>
<td>Face0602</td>
<td>0.6</td>
<td>--</td>
<td>--</td>
<td>105.93</td>
<td>102.23</td>
<td>27.76</td>
<td>3.20</td>
</tr>
<tr>
<td></td>
<td>Core0202</td>
<td>--</td>
<td>25</td>
<td>0.18</td>
<td>103.45</td>
<td>65.85</td>
<td>79.79</td>
<td>26.42</td>
</tr>
<tr>
<td></td>
<td>AF061825S-06HS</td>
<td>0.6</td>
<td>25</td>
<td>0.18</td>
<td>102.74</td>
<td>0</td>
<td>102.74 (Stuck in the back face)</td>
<td>43.28</td>
</tr>
</tbody>
</table>
7.5 Finite Element Simulation

Finite element simulations have been carried out using LS-DYNA software based on the experiments described in the previous sections. In this section a description of the models and simulation results is presented.

The structural responses of sandwich panels are studied in terms of two aspects: (1) deformation/failure patterns of the sandwich panels under ballistic impact loading, and (2) quantitative assessment, which mainly focuses on the energy absorption and dissipation of the panels.

7.5.1 Modelling geometry of the sandwich panel

The geometric model of the sandwich panel used in the simulation is based on the experimental dimensions and depicted in Figure 7-18a, while Figure 7-18b shows an enlarged view of the indenter. Since the circular sandwich panel is symmetrical for x-z and y-z planes, only a quarter of the panel was modelled.

7.5.2 Materials modelling of the sandwich panel

- Modelling of the aluminium foam core

In the LS-DYNA model, the aluminium foam core was modelled using material model no.154, "Mat_DESHPANDE_FLECK_FOAM. The same material model was used in the numerical simulation of the aluminium foam under static shear loading in Chapter 3. The foam core was meshed into the eight-node brick (solid) elements as shown in Figure 7-19.
Modelling of aluminium skin sheets

The aluminium face-sheets are made from material Al-5005H34 and four different thicknesses were used in the experiments: 0.6mm, 1.0mm, 1.5mm and 2.0mm, respectively. The stress-strain relationships of face and foam cores were determined via standard tensile tests and they are presented in the previous section (Figure 7-2). In the Finite Element Simulation model, the face-sheets were meshed using 2D shell elements, which give a high computational efficiency. The mesh is as illustrated in Figure 7-20.
Figure 7-19. Mesh of the aluminium foam core
The material of the skin sheets was modelled using the simplified Johnson-Cook constitutive relationship (material model 99 in LS-DYNA 970). Compared with the full Johnson-Cook implementation [102], the simplified model neglects material softening caused by high temperature; the flow stress of a material is expressed in a multiplicative form of strain and strain-rate terms as:

$$\sigma_{Td} = (A + B\bar{\varepsilon}^P)(1 + C\varepsilon^*)$$  \hspace{1cm} (7-4)

where $A$, $B$ and $C$ are material constants, and $n$ is the work hardening exponent; $\bar{\varepsilon}^P$ is the effective plastic strain; $\varepsilon^* = \bar{\varepsilon}/\varepsilon_0$, being the effective strain rate for $\varepsilon_0 = 1s^{-1}$. This model is suitable for problems where strain rates vary over a large range but without temperature effects. Damage begins when a tensile fracture strain ($\tilde{\varepsilon}_f$) is reached and after fracture is detected at all nodes of an element, that element is deleted from further calculation.

The standard strain–stress curves obtained from dogbone tests were converted into true strain-stress curves, and the parameters $A$, $B$, $n$ were found by using the least-squares method fitting relating Eq. 7-4 to the data points on these curves. Parameter $C$, which describes the increase of flow stress with increasing plastic strain rate, is taken as 0.001 from the work of Borvik et al. [119]. The parameters of the aluminium sheets are listed in Table 7-7.
Table 7-7. Parameters of Al-5005-H34 used in LS-DYNA simulation

<table>
<thead>
<tr>
<th>Thickness of face sheets</th>
<th>$E$ (GPa)</th>
<th>$\nu$</th>
<th>$\rho$ (kg/m$^3$)</th>
<th>$A$ (MPa)</th>
<th>$B$ (MPa)</th>
<th>$n$</th>
<th>$\varepsilon_f$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.6mm</td>
<td>65.04</td>
<td>0.33</td>
<td>2700</td>
<td>134.43</td>
<td>200.07</td>
<td>0.56</td>
<td>2.7%</td>
</tr>
<tr>
<td>1.0mm</td>
<td>62.57</td>
<td>0.33</td>
<td>2700</td>
<td>135.57</td>
<td>151.35</td>
<td>0.63</td>
<td>8.9%</td>
</tr>
<tr>
<td>1.5mm</td>
<td>77.09</td>
<td>0.33</td>
<td>2700</td>
<td>144.47</td>
<td>152.47</td>
<td>0.65</td>
<td>7.6%</td>
</tr>
<tr>
<td>2.0mm</td>
<td>66.69</td>
<td>0.33</td>
<td>2700</td>
<td>136.34</td>
<td>160.24</td>
<td>0.68</td>
<td>7.8%</td>
</tr>
</tbody>
</table>

- **Modelling of the indenter**

  The hemispherical-ended indenter was simulated in this numerical model. The material type of the indenter was described as a rigid body in LS-DYNA. The mesh is generated automatically using the topology meshing method.

- **7.5.3 Modelling of boundary and loading conditions**

  The circumference of specimens were fully fixed with a 100mm diameter opening at the centre part. The boundary condition is modelled as shown in Figure 7-21. The impact loading is assigned to the indenter as an initial velocity condition.

Figure 7-21. Boundary conditions of the numerical simulation
7.6 FE Model Validation

Separate models of the skin sheet and foam core under impact loading were designed respectively in order to study validity of the material type, boundary conditions and initial loading of the FE sandwich model.

7.6.1 Validation of the FE model of the face skin

A separate FE model is designed for the aluminium skin sheet under impact loading using the material model of the simplified Johnson-Cook constitutive relationship (material model 99 in LS-DYNA 970). The model has with the same boundary condition and loading methods. The FE simulation result of ballistic limits for the three different skin thicknesses was compared with the experimental results as shown in Figure 7-22. It is observed that the FE result is in fairly good agreement with the experiment data for the 0.6mm thickness aluminium sheet. For the thicker specimens, 1.5mm and 2.0mm, FE-simulated ballistic limits are 55.78m/s and 85.11m/s, which slightly underestimated the experiment data of 59.92 and 89.85 respectively. Nevertheless, the difference is less than 7%, which is acceptable, as our focus is on the overall performance and its energy dissipation of the sandwich panels.

![Figure 7-22. A comparison of the FE simulation and experimental data of aluminium sheets under ballistic-impact loading](image)
7.6.2 Validation of the FE model of the aluminium foam core

The aluminium foam core without skin sheets is also modelled separately using material model no. 154, *Mat_DESHPANDE_FLECK_FOAM, as described in the previous section. The aluminium foam with relative density of 18% was simulated and compared with the impact experimental results.

An FE model of the Deshpande_Fleck foam was calculated with the parameters derived from previous quasi-static experiments (described in Chapter 4). The perforation velocity obtained is 45.64m/s when the foam core is under a simulated impact velocity of 80m/s, which is nearly 43.6% less than the experimental result of an average 81m/s. The same model was also tested with material model no. 66, *Mat_CRUSHABLE_FOAM. This time the perforation velocity is 41.46m/s, 49% less than the experiment data. Both material models have dramatically underestimated the experiment. This may be due to the fact that both material models have not considered the effect of impact loading, which in fact has caused a much more complex structural response of the aluminium foam under high-velocity dynamic loadings.

Reid et al. [116–118] developed the shock front theory applied to cellular materials. When a cellular material is crushed at a sufficiently high velocity, it might be expected that a structural shock is generated and propagates through the cellular material. Following this conjecture, to estimate the enhancement in the crushing stress as a function of impact speed, a simple one-dimensional ‘shock’ model is proposed.

The equations governing the propagation of the shock are made up of kinematical equations and equations of conservation of mass and momentum for the material crossing the shock front. The dynamic crushing stress $\sigma$ at the proximal end was derived as follows [118]:

$$
\sigma = \sigma_0 + \rho \frac{v^2}{\varepsilon_d}
$$  \hspace{1cm} \text{(7-5)}
where $\sigma_0$ is the quasi-static crushing strength, $\rho$ is the initial density of the cellular material and $V$ is the velocity of the striker. This equation provides an estimate of the crushing stress at a given impact velocity $V$.

Eq. 7-5 is used to calculate the dynamic enhancement of impact velocity on the crushing strength of aluminium foams. The calculated new dynamic collapse stress obtained from the above equation is 11.3MPa for aluminium foam of a relative density of 18% under an impact velocity of 80m/s, which is 2.91 times the quasi-static collapse strength.

A new set of parameters is obtained from the modified real strain-stress curve for aluminium foam of a relative density of 18% as shown in Table 7-8.

The ballistic limit of the aluminium foam with 18% nominal relative density and a thickness of 25mm obtained from the finite element simulation with the parameters enhanced with shock front theory is 82.5m/s, which is obtained by several different simulations with impact velocity slightly over 80m/s. The result slightly overestimated the impact experimental average of 81m/s but the difference is less than 1.8%. Hence we conclude that the material model with shock-front dynamic enhancement can well present the property of aluminium foam under high-velocity impact loading.

Table 7-8. Dynamic enhanced parameters of aluminium foam with relative density 18% under impact velocity of 80m/s.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Young’s modulus $E$ (MPa)</td>
<td>890</td>
</tr>
<tr>
<td>Poisson’s ratio $\nu_p$</td>
<td>0.3</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>2.12</td>
</tr>
<tr>
<td>$\gamma$ (MPa)</td>
<td>139</td>
</tr>
<tr>
<td>$\varepsilon_p$</td>
<td>0.35</td>
</tr>
<tr>
<td>$\sigma_2$ (MPa)</td>
<td>56</td>
</tr>
<tr>
<td>$\beta$</td>
<td>2.32</td>
</tr>
<tr>
<td>$\sigma_p$ (MPa)</td>
<td>11.3</td>
</tr>
<tr>
<td>$\varepsilon_{cr}$</td>
<td>0.1</td>
</tr>
</tbody>
</table>
7.7 Numerical Simulation of Sandwich Panel – Results and Analysis

The focus of numerical simulation is to analyse and discuss two aspects: (1) the deformation process of the sandwich panel under high-velocity impact loading, (2) energy dissipation during the impact process.

7.7.1 The deformation and failure of the sandwich panel under impact loading

A sandwich panel with 0.6mm skin thickness and a foam core with a relative density of 18% and thickness of 25mm is used as a basic configuration in the FE model. Figure 7-23 illustrates the simulated procedure of the deformation of the sandwich panel under an impact loading with initial velocity of 105m/s.

The deformation process is observed having been divided into three stages as shown in Figure 7-23:

Stage 1: The failure and penetration of the top skin; it is shown that the top skin is first damaged by the indenter. There is no obvious global bending of the top skin

Stage 2: The penetration of the foam core; the foam core was observed to be deformed and failed as the indenter penetrated further into the panel. This process is continued until the entire thickness of the foam core is perforated.

Stage 3: The tearing and stretching of the bottom skin sheet is shown at this stage until the entire sandwich panel is totally penetrated.
Figure 7-23. FE simulation of the deformation of the sandwich panel under ballistic impact loading.
Figure 7-24 shows the cross-section view of the final deformation of the FE model and a nominally identical specimen after the impact experiment. The FE simulation shows a similar damage pattern with the experimental sample. In the experiment one has a slightly larger indenter tunnel compared with the FE simulation. This may be due to the imperfection of the aluminium foam, and the densification of the foam core is not fully reflected in the finite element models.

![FE simulated deformation](image)

(a) FE simulated deformation

![Experiment deformation](image)

(b) Experiment deformation

Figure 7-24. A comparison of the FE simulation and experiment deformation – cross-section view of the specimen

### 7.7.2 Energy absorption and dissipation of the sandwich structure

The FE-simulated perforation energy absorption by the sandwich structure under an impact velocity of 106m/s is 35.98J, which is slightly lower than the 36.47J obtained from the ballistic impact experiment with a measured impact loading of 106.31m/s. Several different impact velocities are simulated with the FE model. Figure 7-25 presents the enhancement factor against the impact velocity with FE results of energy absorption in comparison with corresponding experimental data.
The FE-simulated energy dissipation of the sandwich panel with a skin thickness of 0.6mm and foam core of a relative density of 20% and a thickness of 25mm is illustrated in Figure 7-26. The impact velocity is 106m/s. The energy absorption is dominated by the foam core, which is 29.83J or 87% of the overall perforation energy of 34.17J, while the top face skin and bottom face skin absorbed 2.91J and 1.43J or 8.5% and 4.2% of the overall perforation energy respectively. The top skin absorbed more energy than the bottom skin, which may due to the fact that the impact velocity is higher when the projectile hits the top skin and the foam core underneath the top skin may also be strengthened by the energy absorption capacity of it.

The energy dissipation of the same sandwich panel is shown in Figure 7-26. The energy absorbed by different parts of the sandwich structure is illustrated. The three different stages of perforation are observed on the curves. In the first stage of perforation, the top face skin absorbed energy through tearing the top skin; then the foam core gradually dominated the absorption of the perforation energy by continued shearing and
compression damage while the indenter penetrated the thickness of the foam core; in the third stage the energy is mainly used to bend and tear off the bottom face skin.

![Energy dissipation of aluminium foam sandwich structures under impact loading](image)

Different impact velocity on the same sandwich structure is also simulated. Figure 7-27 shows the energy dissipation of the sandwich structure with the increasing impact velocities. The overall energy increases as the impact velocity increases. From the chart, it is obvious that the energy absorption of the foam core increases dramatically while the impact velocity increases. In contrast, the energy absorption of top and bottom skins shows little variation. Hence we conclude the aluminium foam works as a core of the sandwich panel and has strengthened the overall energy absorption of the panel. This property of the sandwich panel makes them ideal energy protection structures in a wide range of applications.
Quasi-static and impact-perforation tests were carried out to test the ballistic performance and energy absorption of metallic sandwich panels. The sandwich specimens consisted of two aluminium alloy skins and a core made from aluminium foam, and were impacted by three projectiles with different shapes: flat-ended, hemispherical-nosed and conical-nosed. The perforated specimens showed similar damage patterns: the front face exhibits a circular crater without global deformation. A localised tunnel is evident in the foam core directly below the point of impact and through the thickness. A small amount of delamination can be observed between the core and two face-sheets. On the back face, a round hole is visible with a number of petals, but without significant global deflection.

Based on the quantitative results obtained, the effect of several key parameters – impact velocity, skin thickness, thickness and density of foam core and projectile nose shapes –
on the ballistic limit and energy absorption of the panels during perforation is discussed. It has been found that the dynamic perforation can significantly raise the perforation energy, which has a linear relationship with the impact velocity. A simplified perforation model was proposed to calculate the energy dissipation during quasi-static perforation, based on which an empirical equation has been suggested to approximately estimate the energy absorption of the panels during perforation. It has been found that the dynamic perforation can significantly raise the perforation energy, which has a linear relationship with the impact velocity. Thicker skins and cores with higher thickness and density are prone to producing higher ballistic limits, and thicker skins also result in larger delamination areas between the core and back face. Blunter projectiles result in larger petalling areas and tend to increase the ballistic limits and energy dissipation. Compared with the impact cases, quasi-static perforations produced larger tearing damage on the back face. Dynamic energy enhancement has been observed for the all three types of projectiles, although both the face and core materials do not exhibit evident dynamic effect.

LS-DYNA 970 was used to construct numerical models of sandwich structures to simulate the deformation and energy absorption of sandwich structures under ballistic-impact loading. Individual models are also designed to validate the material models and boundary conditions used in the simulation. Material model no. 154 was used for the aluminium foam core, which is modified with a dynamic enhancement factor calculated based on the shock front theory. Good agreement was achieved for both the skin and foam core simulations compared with the impact experiment results.

The FE-simulated damage and deformation of the aluminium sandwich panel was illustrated. The energy dissipation of the sandwich structure was analysed. It is observed that the aluminium foam core dominated 87% of energy absorption while the top face skin and bottom face skin shared 8.5% and 4.2% of the overall perforation energy when the sandwich panel was under an impact velocity of 106m/s.

The simulated perforation energy absorption by the sandwich structure will increase when the impact velocity increases, which agrees with the findings from previous impact experiments. The FE analysis also found that the major portion of perforation
energy was absorbed by the foam core of the structures; when the impact velocity reached 200m/s, the energy dissipated to the foam core was 91%. The aluminium foam working as a core of the sandwich panel dramatically enhanced the overall energy absorption of the panel.
CHAPTER EIGHT

CONCLUSIONS AND FUTURE WORK

8.1 Conclusions

A systematic study is presented in this thesis, which includes experimental and computational investigations on the resistant behaviour and energy-absorbing performance of circular metallic sandwich panels with aluminium foam cores under quasi-static, low-velocity and ballistic impact loadings. As the shearing properties of the foam core are important in analysis of the overall sandwich structures, an experimental study of the shearing properties of the aluminium foam is also carried out and reported.

CYMAT closed-cell aluminium foams were cut into rectangular beams and tested under quasi-static shear loading using an MTS test system. After analysis of the experimental results, it is found that the maximum shear load and total energy absorption of the foam beams shows a linear relationship with the beam width. Both the shear strength and essential energy under shear have been given, empirically, as a function of the relative density, using a power law relationship. Good agreement has been observed between the ultimate shear stress from the shearing tests and that derived from uniaxial compression incorporating the Deshpande-Fleck yield criterion. The present data agree with those obtained by other workers using different test methods. Low-velocity shearing tests using a drop hammer were also reported. There was no obvious dynamic enhancement found during the impact test range. ABAQUS/Explicit was used to investigate the static shearing behaviour of CYMAT aluminium foams.

Quasi-static tests were conducted on the sandwich panels made of aluminium skin with CYMAT foam core using an MTS universal testing machine and a hemispherical indenter. Experimental observation discovered four different types of failure modes. It was found that the skin thickness, the relative density and the thickness of the foam core have significant influence on the failure modes as well as the energy absorption of the
sandwich panels. The supporting condition is another dominant factor in the deformation and final failure of the panels.

The drop hammer tests on the sandwich panels with similar geometrical structures show four similar types of failure modes under quasi-static loading tests. The impact velocity has enhanced the overall strength and energy absorption significantly. Other factors such as skin thickness, the relative density and the thickness of the foam core as well as the supporting method also have important influence on the failure modes, strength and energy absorption of the sandwich panels.

An FE model has been proposed to describe the structural responses of the sandwich panels under quasi-static loading with large deflections. The numerical model is compared with an analytical model that considered the effect of membrane force on the load-bearing behaviour of circular sandwich plates (either simply supported or fully clamped). The analysis is based on the assumption that global deformation is the main deformation mechanism and no local indentation takes place. The large deflection response is estimated by assuming a velocity field, which is defined according to the initial deformation pattern of the flat panel and the boundary condition. A parametric study is then carried out to examine the effect of (1) boundary conditions, (2) face and core thickness ratios, and (3) the core strength on the structural response. Good agreement between the simulation and analytical models confirms that the overall deformation behaviour of circular sandwich plates is well captured by the analytical model. Finally, a comparative study with monolithic plates on the energy absorption using the analytical model is given for sandwich plates with the identical mass per unit area, but with various different face-to-core thickness ratios.

To test the ballistic performance and energy absorption of sandwich panels, further quasi-static and impact perforation tests were carried out on the sandwich specimens consisting of two aluminium alloy skins and a core made from aluminium foam, and impacted by projectiles with different shapes: flat-ended, hemispherical-nosed and conical-nosed. Similar damage patterns were observed with the perforated specimens: (1) the front face exhibits a circular crater without global deformation, (2) a localised
tunnel is evident in the foam core directly below the point of impact and throughout the thickness, (3) a small amount of delamination can be observed between the core and two face-sheets, and (4) on the back face, a round hole is visible with a number of petals, but no significant global deflection.

Based on the experimental results obtained, the effect of several key parameters – impact velocity, skin thickness, thickness and density of the foam core and projectile shapes – on the ballistic limit and energy absorption of the panels during perforation is discussed. It has been found that the perforation energy can be raised significantly by the dynamic perforation, and the perforation energy has a linear relationship with the impact velocity. Based on the experimental data, an empirical equation has been suggested to estimate approximately the energy enhancement of the dynamic perforation. Thicker skins and the cores with higher thickness and density are prone to produce a higher ballistic limit, and thicker skins also result in a larger delamination area between the core and back face. Blunter projectiles result in larger petal areas and a trend to increase the ballistic limit and energy dissipation. Quasi-static perforations produced larger tearing damage on the back face compared with the impact cases. Although both the face and core materials do not exhibit evident dynamic effects, dynamic energy enhancement has been observed for the all three types of projectiles in impact loading.

A numerical model of sandwich structures under ballistic loading using LS-DYNA 970 was constructed to simulate the deformation and energy absorption of the sandwich structure under ballistic impact loading. Separate models were also designed to validate the material model and boundary condition used in the simulation. The material model used for the foam core was modified by shock-front theory. Good agreement was achieved for both the skin and foam core simulations compared with the impact experiment results.

The FE model of the sandwich panel was used to simulate the progress of the damage and deformation as well as the energy dissipation of the sandwich structure. When the sandwich panel was simulated under the impact velocity of 106m/s, it was observed that
the aluminium foam core dominated 87% of energy absorption while the top face skin and bottom face skin shared 8.5% and 4.2% of the overall perforation energy.

The simulated energy absorption by the sandwich structure will increase when the impact velocity increases. The major increase of perforation energy is absorbed by the foam core of the structures; when the impact velocity reached 200m/s, the energy dissipated to the foam core was 91%.

8.2 Future Work

Some future work is recommended in this section.

• Experimental and computational studies

In the present study, no response time history was recorded for ballistic impacts, due to the limitation of measurement means. This issue could be addressed in future work. More physical tests and numerical simulations should be carried out on the sandwich panels with different configurations, for instance, rectangular panels with different side length ratios, and the two faces with unequal thicknesses. Also, the ballistic-resistant performances of several novel core topologies should be studied: square honeycombs, corrugated lattices, and octet, tetrahedral, pyramidal or Kagome trusses, which could offer lightweight structures high strength and stiffness.

• Analytical modelling

More analytical analysis and modelling regarding the deformation, structural strength and energy absorption need to be considered in future work. In the previous analyses, the total energy of the structure obtained from the ballistic load was assumed to be entirely dissipated during core crushing and subsequent overall plastic bending and stretching, and the energy loss due to front face–core delaminating, shear at the supports
and vibratory motion was neglected. This additional dissipation therefore needs to be considered in future work. Also, the analytical solution was based on the assumption that the complete deformation process is split into three phases, which can significantly simplify the problem. However, the rationale of this assumption might be still arguable, and therefore, the coupling effect between Phases I and II and Phases II and III should be investigated in detail. Detailed optimisation should also be considered in future work.
APPENDIX A

ANALYTICAL MODELLING OF CIRCULAR ALUMINIUM SANDWICH PLATES AT LARGE DEFLECTION

In this section, analytical solutions for the elastic stiffness, initial collapse strength, and post-yield response of simply supported and clamped circular sandwich panels are derived, based on a yield locus. This analytical model can be used to assess the load-bearing capacities of the sandwich constructions, and to optimise the geometric parameters of the structures in different loading conditions. Here, both face-sheets and core are considered as elastic-perfectly plastic solids that obey the Tresca yield criterion and the associated flow rule. The circular sandwich panel has a radius R, and two identical face-sheets with thickness, which are perfectly adhered to a metallic foam core with thickness, as shown in Figure A-1. The sandwich panel is loaded in the central area by a lateral pressure via a flat-ended punch with radius.

Figure A-1. Sketches of simply supported and clamped circular sandwich panels transversely loaded by a flat punch
Define the symbols $E_f$, $\sigma_f$, $\nu_f$, $\rho_f$ and $E_c$, $\sigma_c$, $\nu_c$, $\rho_c$ to Young’s modulus, yield strength, Poisson’s ratio, and density of the face-sheets and core, respectively, and introduce the following non-dimensional geometrical and material parameters

$$\bar{\sigma} = \frac{\sigma_c}{\sigma_f}, \quad \bar{h} = \frac{h_f}{H_c}, \quad \bar{w}_m = \frac{w_m}{H_c}, \quad \bar{\rho} = \frac{\rho_c}{\rho_f}$$

where $w_m$ is the central-point deflection of the sandwich panel.

The load-versus-deflection response may be roughly subdivided into three phases (shown in Figure A-2): (1) elastic-bending phase, where the structure deflects elastically until the applied load reaches the initial collapse load associated with the operative collapse mechanism, (2) transition phase, where once initial collapse has been attained, the bending-dominated deformation mode gradually turns into the stretching-dominated mechanism until the order of the deflection reaches that of its thickness, and (3) large deflection phase, where the membrane effect is the key deformation mechanism. This stretching phase can be characterised by a rapidly increasing the linear load-versus-deflection response and the structure deflects until there is a sudden loss of load-carrying capacity due to face-sheet tearing when the practical strain attains the material ductility. These phases are discussed in detail in the subsequent sections.

![Figure A-2. Sketch of the three stages of the load-versus-deflection response of circular sandwich panel transversely loaded by a flat punch](image-url)
1. Elastic-bending deformation

In this phase, the panels deflect elastically until the applied load is equal to the initial collapse load. When a concentrated load \( P \) acts at the plate centre, the transverse deflection \( w_m \) at the centre of the circular plate (with radius \( R \)) is given by [111]:

\[
 w_m = \frac{PR^2}{16\pi D_{eq}} \left( \frac{3 + \nu_f}{1 + \nu_f} \right) \tag{A-1}
\]

for the simply supported boundary condition, and

\[
 w_m = \frac{PR^2}{16\pi D_{eq}} \tag{A-2}
\]

for the fully clamped boundary condition. The equivalent flexural rigidity \( D_{eq} \) gives as

\[
 D_{eq} \approx \frac{E_f h_f (H_c + h_f)^2}{2(1 - \nu_f^2)} \tag{A-3}
\]

Here the shear deformation of the core is ignored.

2. Plastic collapse

Three main plastic collapse modes for the sandwich beams with metallic face-sheets and metallic foam core have been identified: face yield, core shear and indentation [36]. The initial limit load corresponding to each collapse mechanism has been obtained using the plastic limit analysis. Due to the axisymmetricical geometry, it is reasonable to assume that circular sandwich panels have similar plastic collapse modes and failure initiation mechanisms. In this study, we shall concentrate on thin sandwich panels, where only the face yield mode is considered. For simplicity, core shear and local indentation are neglected.

During the face yielding, the collapse behaviour of a circular sandwich panel is characterised by the rotation of the annular portion of the face-sheets about the circular
plastic hinge adjacent to the central punch, as sketched in Figure A-3. The plastic-bending moment of a sandwich structure is

\[ M_0 = \sigma_f h_f (H_c + h_f) + \sigma_c H_c^2 / 4 \]  

(A-4)

The plastic limit load for the face yield of a simply supported circular sandwich panel is given by

\[ P_S = 2\pi M_0 / (1 - 2a/3R) \]  

(A-5)

which can be rewritten in the non-dimensional form as

\[ \bar{P}_S = \frac{P_S}{M_0} = 2\pi \left[ (1 + 2\bar{h})^2 - (1 - \bar{\sigma}) \right] / (1 - 2a/3R) \]  

(A-6)

where \( M_0 = \sigma_f H_c^2 / 4 \). For the clamped circular sandwich panels, the corresponding normalised plastic limit load is written as

\[ \bar{P}_c = \frac{P_c}{\sigma_f H_c^2 / 4} = 2\pi \left[ (1 + 2\bar{h})^2 - (1 - \bar{\sigma}) \right] \chi_0 / \alpha_i \]  

(A-7)

where \( \alpha_0 = 2 + \ln \frac{R}{\rho}, \) \( \alpha_1 = 1 + \ln \frac{R}{\rho} - \frac{2a}{3\rho}, \) \( a/R \leq 0.606 \) and \( \rho \) is obtained from the following equation

\[ \frac{3\rho}{2a} - \left( 1 + \ln \frac{R}{\rho} \right) = 0 \]  

(A-8)

Note that the above formulae can also be obtained using the lower bound method

3. Yield locus of the sandwich panel

With the deflection increasing, the effect of the membrane force becomes critical. When the deflection is beyond the original thickness of the panel, the membrane effect actually dominates the structural response. Therefore, in this chapter, a yield criterion [97] is adopted for the cross-section of the sandwich panel, where both plastic bending and stretching have been taken into account, together with the strength of the cellular core. Figure A-4 shows the distribution of normal stresses on a sandwich cross-section,
subjected to both a bending moment $M$ and a membrane force $N$ simultaneously. Depending on the magnitude of $N$, there are two cases for the stresses distribution:

\[ 0 \leq \left| \frac{N}{N_0} \right| \leq \frac{\sigma}{\sigma + 2h} \]  
(Figure A-4a), and \[ \frac{\sigma}{\sigma + 2h} \leq \left| \frac{N}{N_0} \right| \leq 1 \]  
(Figure A-4b), where $N_0 = 2\sigma_f h_f + \sigma_s H_s$.

The profile of the stresses can be described by the combination of a symmetric component with respect to the central axis (stretching effect) and an antisymmetric component (bending effect).

Figure A-3. Initial collapse by face yielding of the circular sandwich panels for both simply supported and fully clamped boundary conditions

Based on the distribution of the stresses, the circumferential bending moment and membrane force in both cases can be calculated by

\[ \frac{N}{N_0} = \frac{\xi \bar{\sigma}}{\sigma + 2h}, \quad \frac{M}{M_0} = 1 - \frac{\xi^2 \sigma}{4h(1 + h) + \sigma} \]  
for \[ 0 \leq \left| \frac{N}{N_0} \right| \leq \frac{\sigma}{\sigma + 2h} \]  
(A-9a)

\[ \frac{N}{N_0} = 1 - \frac{2\xi \bar{\sigma}}{\sigma + 2h}, \quad \frac{M}{M_0} = \frac{2\xi \bar{\sigma}}{4h(1 + h) + \sigma} \]  
for \[ \frac{\sigma}{\sigma + 2h} \leq \left| \frac{N}{N_0} \right| \leq 1 \]  
(A-9b)
where $\xi$ is a constant and $0 \leq \xi \leq 1$. Eliminating $\xi$, the corresponding yield locus is expressed as

$$\frac{M}{M_0} + \frac{(\bar{\sigma} + 2\bar{h})^2}{4\bar{\sigma}h(1 + \bar{h}) + \bar{\sigma}^2} \left( \frac{N}{N_0} \right)^2 = 1$$

when $0 \leq \left| \frac{N}{N_0} \right| \leq \frac{\bar{\sigma}}{\bar{\sigma} + 2\bar{h}}$ \hfill (A-10a)

$$\frac{M}{M_0} + \frac{\left[ \frac{N}{N_0} (\bar{\sigma} + 2\bar{h}) + (1 - \bar{\sigma}) \right]^2 - (1 + 2\bar{h})^2}{4\bar{h}(1 + \bar{h}) + \bar{\sigma}} = 0$$

when $\frac{\bar{\sigma}}{\bar{\sigma} + 2\bar{h}} \leq \left| \frac{N}{N_0} \right| \leq 1$ \hfill (A-10b)

Figure A-4. Sketch of the normal stresses profile on a sandwich cross-section
Obviously, when $\sigma = \sigma_e / \sigma_f = 1$, the above locus reduces to that of a solid monolithic plate: $|M/M_0| + (N/N_0)^2 = 1$; while if $\sigma = \sigma_e / \sigma_f \leq 1$ and $\bar{h} = h_f / H_c \leq 1$, then Eq. A-10 reduces to the classical locus for the sandwich structures with thin, strong faces but a thick, weak core: $|M/M_0| + |N/N_0| = 1$. The related yield surfaces are illustrated in Figure A-5.

![Figure A-5. Yield loci for monolithic and sandwich structures together with their circumscribing and inscribing squares](image)

4. Finite deflection of the circular sandwich panel

In the present model, it is assumed that the sandwich panels deform in a global pattern without local failure. Based on the yield locus described above, the existing finite deflection theories [102–110] for monolithic solids are extended to sandwich structures. The analysis for their post-yield behaviour is discussed in the simply supported and fully clamped cases, respectively.

- Simply supported case
Now consider a circular sandwich panel with the geometry and loading conditions described above. The panel is simply supported at the outer circular edge. At this stage, both the core and faces are assumed as rigid plastic solids. It is reasonable to assume that, similar to a monolithic solid plate, the initial-velocity field is such that the sandwich plate would deform into a circular cone with the apex at the centre. Therefore, the deflection field can be expressed as

\[ w = w_m (1 - \rho_s) \tag{A-11} \]

with \( w_m \) being the central deflection of the panel and \( \rho_s = r/R \). Due to the nonlinearity in geometry, the strain and curvature components of a circular sandwich panel subjected to a bending moment and a membrane force are expressed as follows:

\[
\varepsilon_r = \frac{du}{dr} + \frac{1}{2} \left( \frac{dw}{dr} \right)^2, \quad \varepsilon_\theta = \frac{u}{r}, \quad \kappa_r = -\frac{\partial^2 w}{\partial r^2}, \quad \kappa_\theta = -\frac{1}{r} \frac{\partial w}{\partial r} \tag{A-12}
\]

where \( \varepsilon_r \) and \( \varepsilon_\theta \) are components of normal strain; \( \kappa_r, \kappa_\theta \) are principal curvatures of the middle plane. \( w \) and \( u \) denote the transverse deflection and the radial displacement of the middle plane, respectively. It is noted that the radial strain component should be small for the simply supported case; the radial membrane force at the edge would be zero. Thus it is reasonable to assume that the strain component of the middle plane is zero at the outer edge. For simplicity, the component is taken as zero throughout the plate, that is \( \varepsilon_r = 0 \). On the other hand, if \( u = 0 \) at \( \rho_s = 0 \), the following relation can be obtained from Eq. A-12

\[ u = -\frac{1}{2} \int \left( \frac{dw}{dr} \right)^2 dr = -\frac{w_m^2}{2R} \rho_s \tag{A-13} \]

Thus the generalised displacement field can be defined using only one parameter \( w \) and the corresponding strain and curvature rates:

\[
\dot{\varepsilon}_r = 0, \quad \dot{\varepsilon}_\theta = \frac{w_m}{R^2}, \quad \dot{\kappa}_r = 0, \quad \dot{\kappa}_\theta = \frac{1}{R^2 \rho_s} \tag{A-14}
\]
Therefore, bending moments and membrane forces corresponding to the strain rates defined above are dependent on the following parameters of the Prager-Onat plastic condition [106]

\[
\beta_1 = -\frac{1}{d} \dot{\varepsilon}_r + \frac{1}{\kappa_r} = 0,
\beta_2 = -\frac{1}{d} \dot{\varepsilon}_\theta + \frac{w_m}{h} \rho_s,
\beta_3 = -\frac{1}{d} \dot{\varepsilon}_r + \dot{\varepsilon}_\theta = \beta_2
\]

(A-15)

where \( d \) is the overall thickness of the sandwich panel. Then the value of the membrane force can be further obtained from the following equation

\[
\frac{N_0 \dot{\varepsilon}_\theta}{M_0 \kappa_\theta} = 2n_0 \frac{(\sigma + 2\bar{h})^2}{4\bar{h}(1 + \bar{h}) + \bar{\sigma}^2}
\]

(A-16)

Substituting the membrane force into the yield criterion, we have

\[
n_\theta = -\frac{2\bar{w}_m \rho_s}{\sigma + 2\bar{h}}, \quad m_\theta = 1 - \frac{(\sigma + 2\bar{h})^2}{4\bar{h}(1 + \bar{h}) + \bar{\sigma}^2} n_\theta^2
\]

for \( |n_\theta| \leq \frac{\sigma}{\sigma + 2\bar{h}} \) \hspace{1cm} (A-17a)

\[
n_\theta = -\frac{(2\bar{w}_m \rho_s - (1 - \bar{\sigma}))}{\sigma + 2\bar{h}}, \quad m_\theta = \frac{(1 + 2\bar{h})^2 - \left(\frac{\sigma_0}{\sigma + 2\bar{h}} + (1 - \bar{\sigma})\right)}{4\bar{h}(1 + \bar{h}) + \bar{\sigma}}
\]

for \( \frac{\sigma}{\sigma + 2\bar{h}} \leq |n_\theta| \leq 1 \) \hspace{1cm} (A-17b)

where \( n_\theta = N_\theta / N_0, \quad m_\theta = M_\theta / M_0, \quad \bar{w}_m = w_m / H_c, \quad N_0 = 2\sigma_0 \bar{h} + \sigma_c H_c \).

Then energy dissipation with respect to the generalised displacement \( w \) can be obtained as

\[
D = \int_0^\infty \dot{D} \ dw_w = \int_0^\infty \int_0^\infty 2\pi (N_\theta \dot{\varepsilon}_\theta + M_\theta \kappa_\theta) R^2 \rho_s d\rho_s \ dw_w
\]

(A-18)

The external work by the applied pressure \( p \) is

\[
E = 2\pi \int_0^\infty p \bar{w}_m (1 - \frac{r}{R}) r \ dr = P \left(1 - \frac{2a}{3R}\right) \bar{w}_m
\]

(A-19)

where \( P = \pi a^2 p \). The total energy of the circular sandwich plate \( \Pi = D - E \). Then from the condition \( d\Pi/dw_w = 0 \), we have
\[ \frac{P}{P_{Sl}} = \frac{4}{3} \bar{w}_m^2 \sigma + \left[ (1 + 2 \bar{h})^2 - (1 - \bar{\sigma}) \right] \quad \text{when } w_m \leq \frac{H_c}{2} \quad \text{(A-20a)} \]

\[ \frac{P}{P_{Sl}} = \frac{4}{3} \bar{w}_m^2 - 2 \bar{w}_m (1 - \bar{\sigma}) + \left( 1 + 2 \bar{h} \right)^2 - \frac{(1 - \bar{\sigma})}{6 \bar{w}_m} \quad \text{when } w_m \leq \frac{H_c}{2} (1 + 2 \bar{h}) \quad \text{(A-20b)} \]

and

\[ \frac{P}{P_{Sl}} = 2 \bar{w}_m \left( \sigma + 2 \bar{h} \right) + \frac{(2 \bar{h} + 1)^2}{6 \bar{w}_m} - \frac{(1 - \bar{\sigma})}{6 \bar{w}_m} \quad \text{when } w_m \geq \frac{H_c}{2} (1 + 2 \bar{h}) \quad \text{(A-20c)} \]

where \( P_{Sl} = 2\pi M_0 / (1 - 2a/3R) \).

It is evident that Eq. A-20 is a general load-deflection relationship, which is valid for circular sandwich plates with different dimensions and core strengths. If \( \bar{\sigma} = \sigma_c / \sigma_f = 1 \), Eq. A-20 reduces to the relationship for monolithic circular plates [110]:

\[ \frac{P}{P_0} = \begin{cases} 
1 + \frac{4}{3} \left( \frac{w_m}{h} \right)^2 & \text{when } \frac{w_m}{h} \leq 1/2 \\
2 \frac{w_m}{h} + \frac{h}{6 \bar{w}_m} & \text{when } \frac{w_m}{h} \geq 1/2 
\end{cases} \quad \text{(A-21)} \]

where \( P_0 = 2\pi M_0 / (1 - 2a/3R) \) being the limit load for the monolithic circular plate.

- Fully clamped case

In this case, the circular sandwich panel is fully clamped at its outer radius \( r = R \). For this boundary condition, it would not be reasonable to assume the strain rate to be zero at the mid-surface of the panel. The simplest assumption is to let the radial velocity be zero, so that all the points in the middle surface can only move vertically. Therefore, the initial velocity field consists of two parts [103]: in a central zone with the radius \( r = \rho \), the plate deforms to a conical shape, as in the simply supported case; outside the zone the deformation shape is logarithmic. Note that the finite rotation occurs just inside the clamped edge, implying the formation of a plastic hinge at that radius. The displacement field can be determined by
\[ w = \begin{cases} C \left(1 + \ln \frac{R}{\rho}\right) - C \frac{r}{\rho} & 0 \leq r \leq \rho \\ C \ln \frac{R}{r} & \rho \leq r \leq R \end{cases} \]  \hspace{1cm} (A-22)

where \( C = w_m/(1 + \ln R/\rho) \), is a function of the central deflection \( w_m \).

The deflection rate with respect to \( C \) is

\[ \dot{w} = \begin{cases} \left(1 + \ln \frac{R}{\rho}\right) - \frac{r}{\rho} & 0 \leq r \leq \rho \\ \ln \frac{R}{r} & \rho \leq r \leq R \end{cases} \]  \hspace{1cm} (A-23)

In this analysis, we assume that the radii of the punch and circular plate have the following relationship: \( a/R \leq e^{-0.5} \). Thus \( \rho \) can be obtained from Eq. A-8.

The strain-deflection relationship of a plate subjected to a bending moment and a membrane force is

\[ \kappa_r = -\frac{\partial^2 w}{\partial r^2} = 0, \quad \kappa_\theta = -\frac{1}{r} \frac{\partial w}{\partial \theta} = \frac{C}{r \rho}, \quad \epsilon_r = \frac{du}{dr} + \frac{1}{2} \left(\frac{dw}{dr}\right)^2 = \frac{C^2}{2 \rho^2}, \quad \epsilon_\theta = \frac{u}{r} = 0 \]  \hspace{1cm} (A-24)

for \( 0 \leq r \leq \rho \) and

\[ \kappa_r = -\frac{\partial^2 w}{\partial r^2} = -\frac{C}{r \rho^2}, \quad \kappa_\theta = -\frac{1}{r} \frac{\partial w}{\partial \theta} = -\frac{C}{r^2}, \quad \epsilon_r = \frac{du}{dr} + \frac{1}{2} \left(\frac{dw}{dr}\right)^2 = \frac{C^2}{2 r^2}, \quad \epsilon_\theta = \frac{u}{r} = 0 \]  \hspace{1cm} (A-25)

for \( \rho \leq r \leq R \). Therefore, the relative parameters in the Prager-Onat plastic condition [106] are as follows

\[ \beta_1 = -\frac{1}{d \dot{\kappa}_r}, \quad \beta_2 = -\frac{1}{d \dot{\kappa}_\theta}, \quad \beta_3 = -\frac{1}{d (\dot{\kappa}_r + \dot{\kappa}_\theta)} \]  \hspace{1cm} (A-26)

where \( \dot{\kappa}_r, \dot{\kappa}_\theta, \dot{\kappa}_r, \dot{\kappa}_\theta \) are rates of \( \epsilon_r, \epsilon_\theta, \kappa_r, \kappa_\theta \) with respect to \( C \), respectively.

Utilising these parameters, the membrane forces and bending moments can be obtained as
\[
\begin{align*}
n_r &= \frac{1}{2} + \frac{C_H \tilde{\sigma}}{\tilde{\sigma} + 2h} \frac{r}{\rho}, \quad M_\theta = 1 - 2C_H^2 \frac{\tilde{\sigma}}{4\hat{h}(1 + \frac{\tilde{\sigma}}{r} + \frac{2}{\rho})} r^2 \quad &\text{for } 0 \leq r \leq \rho \quad (A-27a) \\
n_r &= \frac{1}{2} + \frac{C_H \tilde{\sigma}}{\tilde{\sigma} + 2h} \frac{r}{\rho}, \quad M_\theta = \frac{1}{2}, \quad M_r = 2C_H^2 \frac{\tilde{\sigma}}{4\hat{h}(1 + \frac{\tilde{\sigma}}{r} + \frac{2}{\rho})} - \frac{1}{2} \quad &\text{for } \rho \leq r \leq R \quad (A-27b)
\end{align*}
\]

when \( \frac{r}{\rho} \leq \frac{1}{2C_H} \), and

\[
\begin{align*}
n_r &= \frac{1}{2} + \frac{\tilde{\sigma} - 1}{2(\tilde{\sigma} + 2h)} + \frac{C_H r}{(\tilde{\sigma} + 2h) \rho}, \quad M_\theta = \frac{1}{2}, \quad M_r = 2C_H^2 \frac{1}{4\hat{h}(1 + \frac{\tilde{\sigma}}{r} + \frac{2}{\rho})} - \frac{1}{2} \quad (A-28a)
\end{align*}
\]

for \( 0 \leq r \leq \rho \)

\[
\begin{align*}
n_r &= \frac{1}{2} + \frac{\tilde{\sigma} - 1}{2(\til\sigma + 2h)} + \frac{C_H r}{(\til\sigma + 2h) \rho}, \quad M_\theta = \frac{1}{2}, \quad M_r = \frac{2C_H^2}{4\hat{h}(1 + \frac{\til\sigma}{r} + \frac{2}{\rho})} - \frac{1}{2} \quad (A-28b)
\end{align*}
\]

for \( \rho \leq r \leq R \)

when \( \frac{r}{\rho} \leq \frac{1 + 2h}{2C_H} \), where \( C_H = \frac{C}{H_c} \).

The energy dissipation with respect to the generalised displacement \( C \) can be expressed as

\[
D = \int_0^C \frac{\partial D}{\partial C} \, dC = \int_0^C \left[ 2\pi M_\theta + 2\pi \int_0^R (N_r \dot{\varepsilon}_r + N_\theta \dot{\varepsilon}_\theta + M_r \dot{\kappa}_r + M_\theta \dot{\kappa}_\theta) \, rdr \right] \, dC \quad (A-29)
\]

The external work done by the pressure \( p \) is

\[
E = \int_0^R (2\pi Cr p) \, dr = P \left( 1 + \ln \frac{R}{\rho} - \frac{2a}{3\rho} \right) C = P \alpha C \quad (A-30)
\]

where \( P = \pi a^2 p \) is the total load. The total energy of the circular sandwich panel \( \Pi = D - E \), then when \( d\Pi/dC = 0 \), we have

\[
\frac{P}{P_{cl}} = \left( \frac{(1 + 2\til{h})^2}{(1 - \til{\sigma})} \right) \left( 2 + \ln \frac{R}{\rho} \right) + \left( 1 + 2\ln \frac{R}{\rho} \right) \til{\sigma} + \frac{2}{3} \frac{2 + \ln \frac{R}{\rho}}{\til{\sigma} C^2_H} \quad (A-31a)
\]

when \( C_H \leq \frac{1}{2} \).
\[
\frac{P}{P_{cl}} = \left(1 + 2\bar{h}\right)^2 - (1-\bar{\sigma}) \left(\frac{3}{2} + \ln \frac{R}{\rho}\right) + \frac{(1+2\bar{h})^2 - 1-\bar{\sigma}}{2} \ln \frac{R}{\rho} + \left(1 + 2\ln \frac{R}{\rho}\right) (\bar{\sigma} + 2\bar{h}) C_{\bar{\sigma}}
\]

\[
- \left(1 + 2\ln \frac{R}{\rho}\right) (1-\bar{\sigma}) C_{\bar{\sigma}} + \left(\frac{2}{3} + 2\ln \frac{R}{\rho}\right) C_{\bar{\sigma}}^2 - \frac{1-\bar{\sigma}}{12 C_{\bar{\sigma}}}
\]

(A-31b)

when \( C_{\bar{\sigma}} \leq \frac{1}{2} \left(1 + 2\bar{h}\right) \), and

\[
\frac{P}{P_{cl}} = \left(1 + 2\bar{h}\right)^2 - (1-\bar{\sigma}) \left(\frac{3}{2} + \ln \frac{R}{\rho}\right) + \left(2 + 4\ln \frac{R}{\rho}\right) (\bar{\sigma} + 2\bar{h}) C_{\bar{\sigma}} - \frac{1-\bar{\sigma}}{12 C_{\bar{\sigma}}} + \frac{(1+2\bar{h})^3}{12 C_{\bar{\sigma}}}
\]

(A-31c)

when \( C_{\bar{\sigma}} \geq \frac{1}{2} \left(1 + 2\bar{h}\right) \), where \( P_{cl} = 2\pi M_0 / \alpha_1 \).

Similarly, Eq. A-31 is a general load-deflection relationship for the fully clamped sandwich panels. If \( \bar{\sigma} = \sigma_c / \sigma_f = 1 \), then Eq. A-31 can also be reduced to the relationship for monolithic plates, which is written as

\[
\frac{P}{P_0} = \begin{cases} 
2 + \ln \frac{R}{\rho} + \frac{C}{h} \left(1 + 2\ln \frac{R}{\rho}\right) + \frac{C^2}{h^2} \left(\frac{2}{3} + 2\ln \frac{R}{\rho}\right) & \frac{C}{h} \leq 1/2 \\
\frac{3}{2} + \frac{1}{2} \ln \frac{R}{\rho} + \frac{C}{h} \left(2 + 4\ln \frac{R}{\rho}\right) + \frac{h}{12C} & \frac{C}{h} \geq 1/2
\end{cases}
\]

(A-32)

where \( P_0 = \pi \sigma_c h^2 / 2\alpha_1 \). The value of \( \rho/R \) is obtained from Eq. (A-8).
REFERENCES


