THEORETICAL ANALYSIS OF CFRP STRENGTHENED THIN-WALLED STEEL SQUARE HOLLOW SECTION (SHS) UNDER TORSION

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ABSTRACT
Carbon fibre reinforced polymer (CFRP) has been used in strengthening different types of steel members in bending and compression. However there is still a lack of understanding on the behaviour of CFRP reinforced steel beams subject to torsion. This paper presents a theoretical approach to investigate CFRP strengthening of thin-walled steel square hollow section (SHS) subjected to torsion. Various CFRP strengthening configurations including vertical, spiral and reverse-spiral wrapping are considered. Laminate composite mechanics is introduced to incorporate the contribution of CFRP strengthening to the torsional stiffness and strength of the SHS. By introducing reasonable failure criteria, the torsional behaviour of CFRP strengthened SHS can be predicted corresponding to different failure modes, i.e. CFRP shear, CFRP fracture, CFRP buckling, steel yielding. The relationship between torsion and twist angle have also been identified. The theoretical solutions are validated with lab testing results and reasonable agreement has been achieved.

KEYWORDS
CFRP, Square hollow section (SHS), torsion, theoretical analysis.

INTRODUCTION
In recent years, Carbon fibre reinforced polymer (CFRP) has become increasingly popular in strengthening different types of steel members (Hollaway and Teng 2008). The conventional method of repairing and strengthening aging steel structures involves replacing existing steel plates or attaching new ones, which are usually bulky, heavy, difficult to fix and prone to corrosion and fatigue. Therefore, CFRP has a great potential to strengthen steel structures due to its high tensile strength and lightweight characteristics.

There have been studies conducted for the flexural and axial behaviour of CFRP strengthened steel hollow sections (Zhao 2013). However, there is a considerable knowledge gap in the area of understanding the behaviour of CFRP strengthened steel hollow sections under the torsion effect. It is important to understand how CFRP strengthened steel hollow sections under torsion because such loading case may occur when used in bridges, buildings, offshore platforms, pipelines and crane structures. There has been some experimental work conducted in this area (Chahkand et al. 2013) but no theoretical analysis has been carried out. Therefore, this paper will fill the knowledge gap by investigating the torsion behaviour of CFRP strengthened thin-walled steel square hollow section (SHS) using theoretical approach.

THEORETICAL ANALYSIS
One complication when considering the contribution of CFRP in torsion is that CFRP is not an isotropic material, meaning that its material properties are different depending on the direction of the carbon fibres relative to the loading direction. As CFRP wrapping angles will change, it is firstly important to establish the coordinate system that is referred to in this article.
Referring to Figure 1, x and y are in the global direction while 1 and 2 are in the local direction depending on the CFRP wrapping angle ($\theta$).

When conducting theoretical analysis on the torsional behaviour, there are two main concepts that we are particularly interested in: the maximum torsion capacity and the torsion vs. rotational angle ($\alpha$) relationship. Maximum torsion capacity indicates how much torsion a structure can sustain before failure, whereas torsion vs. rotational angle relationship shows the torsion stiffness of the structure.

Existing torsion equations are applied here (Murray 1984):

$$ T = 2At \times \tau_{xy} \quad (1) $$

$$ \alpha = \frac{T \times S \times L}{4G_{xy} \times A^2 \times t} \quad (2a) $$

Rearranging to become torsion vs. rotational angle relationship,

$$ T = \frac{4G_{xy} \times A^2 \times t \times \alpha}{S \times L} \quad (2b) $$

Combining contribution from CFRP, the equation can be extended to:

$$ T = 2(A_t \times t_s \times \tau_s + A_c \times t_c \times \tau_{xy}) \quad (3) $$

$$ T = \left( \frac{4G_{xy} \times A^2 \times t_s}{S \times L} + \frac{4G_{xy} \times A^2 \times t_c}{S \times L} \right) \times \alpha \quad (4) $$

Again, it’s important to understand that CFRP is not an isotropic material like steel, which means the material properties of CFRP is not the consistent in different directions. For example, $E_x$ of CFRP is generally 11-15 times of that of $E_y$. This implies that with different wrapping angles $\theta$, properties of CFRP in the global (loading) direction such as $\tau_{xy}$ and $G_{xy}$ will be different.

Maximum torsional capacity depends on $\tau_{xy}$, the shear strength of CFRP in the direction that shear stress is acting at due to torsion (global direction). Depending on the wrapping angle of the CFRP, this value will change according to the failure criteria of CFRP in the local direction ($\sigma_1$, $\sigma_2$ and $\tau_{12}$). This will be discussed later when introducing appropriate failure criteria of CFRP.

Torsion vs. rotational angle relationship depends on $G_{xy}$, the shear modulus of CFRP in the direction that shear stress is acting at due to torsion (global direction). Since we are considering a pure torsion case, the shear stress is acting along the global axis.

To determine $G_{xy}$, laminate composite mechanic is applied here (Kaw 2006):

$$ G_{xy} = 1/(2 \left( \frac{2}{E_1} + \frac{2}{E_2} - \frac{4G_{12}}{E_1} - \frac{1}{G_{12}} \right) \times s^2 \times c^2 + \frac{1}{G_{12}} (s^4 + c^4)) \quad (5) $$

$$ T = \left( \frac{4G_{xy} \times A^2 \times t_s}{S \times L} + \frac{4G_{xy} \times t_2 \times (s^4 + c^4)}{S \times L} \right) \times \alpha \quad (6) $$

This overall equation represents the relationship between torsion and rotational angle combining both contribution from both steel and CFRP.
EXPERIMENTAL VALIDATION

Some existing preliminary experimental work has been conducted using a single SHS section (50.3x50.3x2.7mm) with different wrapping configurations (Chahkand et al. 2012). Therefore this theoretical analysis aims to explain what happened in the experiment by simulating the experiment again using parameters and properties from the experiment.

To validate theory against existing experimental work, it is important to establish the property values used in the experiment so that the same values can be utilised in the theoretical analysis to maintain consistency. Since some of the CFRP properties are not given, they are estimated as follows:

E₁ from the experiment can be established by combining the contribution from pure carbon fibre and epoxy based on their percentage thickness used in the composite. E₁ is determined to be 32061 MPa.

E₂ and G₁₂ values used in the experiment are not given. Literature and manufacturers’ values have been researched. Generally E₁/E₂ values range from 11-15 with an average value of 13.782; E₁/G₁₂ values range from 22-30 with an average value of 26.834 (Bank 2006; Hollaway and Teng 2008; Keller 2003; Mays 1992; Sika 2007 & 2011; Sudarisman 2008; Wu 2006). Using the average ratios, experimental E₂ and G₁₂ values can be calculated to be 2326 MPa and 1195 MPa respectively.

σ₁, σ₂ and τ₁₂ are also determined in a similar fashion, with values of 516 MPa, 15.68 MPa and 32.88 MPa respectively.

<table>
<thead>
<tr>
<th>Specimen</th>
<th>No. of Layers</th>
<th>Experimental Torsional Capacity (Nm)</th>
<th>Theoretical Torsional Capacity (Nm)</th>
<th>Experimental/Theoretical ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Control</td>
<td></td>
<td>2782</td>
<td>2862</td>
<td>0.97</td>
</tr>
<tr>
<td>V5</td>
<td>5</td>
<td>3186</td>
<td>3679</td>
<td>0.87</td>
</tr>
<tr>
<td>RR</td>
<td>2</td>
<td>3689</td>
<td>3354</td>
<td>1.10</td>
</tr>
<tr>
<td>RRRS</td>
<td>4</td>
<td>4442</td>
<td>3954</td>
<td>1.12</td>
</tr>
<tr>
<td>SRSR</td>
<td>4</td>
<td>4462</td>
<td>3954</td>
<td>1.13</td>
</tr>
</tbody>
</table>

The above capacity numbers are based on a rotational angle of 60 degrees, as the experimental tests only test the samples up to 60 degree rotational angle. It can be seen that reasonable agreement has been achieved between the experimental results and theoretical results.

SENSITIVITY ANALYSIS

Gₓᵧ has a linear relationship with torsion capacity and it changes significantly as G₁₂, E₁ and E₂ changes according to the formula. Since E₂ and G₁₂ values are not directly measured from the experiment, it’s important to identify how sensitive Gₓᵧ changes when E₁/E₂ and E₁/G₁₂ changes within the general range between 11-15 and 22-30 respectively. This will be tested at a variety of different wrapping angles to monitor which ratios change Gₓᵧ the most and at what wrapping angle this happens at.

As it will be justified in the later section, generally wrapping angle at 45 degree is the most efficient way of strengthening against torsion and will be the wrapping angle used in application. At 45 degrees wrapping, Gₓᵧ is most sensitive to E₁/E₂, where Gₓᵧ values deviates up to a maximum value of 22% from the average as E₁/E₂ reaches the lower limit 11, with the majority of the Gₓᵧ values within 10% difference from the average. This deviation is acceptable and torsion capacity can be predicted with confidence.

With confidence in the CFRP properties, theoretical torsional capacity are calculated and compared with the existing experimental results.

FAILURE CRITERIA

Failure criteria need to be introduced because the increase in torsion capacity due to CFRP strengthening will not be infinite. Three types of failure criteria are considered: Shear, Fibre fracture and Fibre separation.

As mentioned earlier, σ₁, σ₂ and τ₁₂ are also determined with average values of 516 MPa, 15.68 MPa and 32.88 MPa respectively. σ₁ is the tensile strength of CFRP in the major fibre direction and this corresponds to fibre
fracture failure; $\sigma_2$ is the tensile strength of CFRP in the minor direction perpendicular to fibre and this corresponds to fibre separation failure; $\tau_{12}$ is the shear strength of CFRP and this corresponds to shear failure.

To take into consideration of different wrapping angles, the transformation matrix is used to transform local strength $\sigma_1, \sigma_2$ and $\tau_{12}$ into the global stress that need to be applied to achieve these strengths at different wrapping angles. These global stresses will be the stresses that cause failure at different wrapping angles.

Since we are considering a pure torsion, the only stress in the global direction will be $\tau_{xy}$ (Kaw 2006):

\[
\begin{bmatrix}
\sigma_1 \\
\sigma_2 \\
\tau_{12}
\end{bmatrix} = \begin{bmatrix}
2\sigma_c \\
-2\sigma_c \\
\sigma_2 - \sigma_1
\end{bmatrix} \times \begin{bmatrix}
\tau_{xy}
\end{bmatrix}
\]  \hspace{1cm} (7)

Now the shear stress $\tau_{xy}$ we need to apply in the global direction to achieve these different failure criteria at different wrapping angles can be established.

For shear failure mode, ultimate local shear strength $\tau_{12}$ is 32.88 MPa. For fracture failure mode, ultimate tensile strength along fibre direction $\sigma_1$ is 516 MPa. For fibre separation failure mode, ultimate tensile strength perpendicular to fibre direction $\sigma_2$ is 15.68 MPa.

Table 2. Failure Criteria Summary

<table>
<thead>
<tr>
<th>Wrapping Angle (Degrees)</th>
<th>0</th>
<th>15</th>
<th>30</th>
<th>45</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shear Failure</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ultimate global stress $\tau_{xy}$ (MPa)</td>
<td>32.88</td>
<td>37.97</td>
<td>65.76</td>
<td>$\infty$</td>
</tr>
<tr>
<td>Ultimate Torsional Capacity (Nm)</td>
<td>154</td>
<td>177</td>
<td>307</td>
<td>$\infty$</td>
</tr>
<tr>
<td>Fracture Failure</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ultimate global stress $\tau_{xy}$ (MPa)</td>
<td>$\infty$</td>
<td>1032</td>
<td>596</td>
<td>516</td>
</tr>
<tr>
<td>Ultimate Torsional Capacity (Nm)</td>
<td>$\infty$</td>
<td>4819</td>
<td>2782</td>
<td>2409</td>
</tr>
<tr>
<td>Fibre Separation Failure</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ultimate global stress $\tau_{xy}$ (MPa)</td>
<td>$\infty$</td>
<td>31.3</td>
<td>18.11</td>
<td>15.68</td>
</tr>
<tr>
<td>Ultimate Torsional Capacity (Nm)</td>
<td>$\infty$</td>
<td>146</td>
<td>84.53</td>
<td>73.20</td>
</tr>
</tbody>
</table>

It can be seen from the table that at 45 degrees, shear failure will never occur as an infinite amount of torsion is required for the fibre to undergo shear failure. Shear failure is therefore the dominant failure criteria at 0 and 90 degree wrapping angles. It can be seen from the graph that at 0 degrees, fracture and fibre separation failure will never occur as an infinite amount of torsion is required for the fibre to undergo failure. Fracture and fibre separation failure modes are therefore the dominant failure criteria at 45 degree wrapping angles.

At wrapping angles other than 0, 45 and 90 degrees, a combined failure mode will occur, with shear and fibre separation failure generally occurring first and ultimately followed by fracture failure. The CFRP is still expected to have capacity after fibre separation failure occurs because fibres can still work in tension. But the ultimate fracture failure torsion load will not be reached as fibres have been debonded and lost its integrity.

Limitations of failure criteria approach:
- No combined failure criteria can be derived. Thus failure modes can only be predicted independently from each other.
- Theoretical analysis assumes constant stiffness that grows up until failure occurs. In reality, CFRP stiffness flattens out after a certain rotational angle due to combined failure taking place.
- Fibre orientation is assumed not to be changed in the analysis, whereas in reality as sample is rotated under torsion, the fibre orientation (i.e. wrapping angle) is constantly increasing.

Due to the limitations of the failure criteria approach in determining when failure occurs, other approaches have been considered. Firstly, it’s possible to consider when CFRP stiffness starts to flatten. One way to determine when the curve starts to flatten out is by looking at the number of reverse wrapping layers. The reverse wrapping layers withstand torsion in fibre compression instead of tension. The torsion capacity contribution from fibre compression is hard to quantify, but the presence of these reverse layers will enhance the torsion carrying capacity of spirally wrapped layers by delaying the failure to a larger rotational angle (Haedir et al. 2009). For example, from experimental results, SRSR’s curve starts to flatten out at about 48 degrees where RRSS’s curve remains stiff passing 48 degrees due to the presence of an extra R layer.
This can become useful in determining the actual failure angle, as not much torsional capacity increase will be expected after the curve starts to flatten. Therefore, predicting when the curve starts to flatten becomes important. This issue will be addressed with more experimental tests being conducted so that a reasonable consensus can be made.

Another way is using experimental results, it’s possible to back-calculate at what rotational angle fracture failure will occur based on theoretical results. From the experiment results, a torsional capacity increase of 1660Nm is observed for the RRRS specimen. This translates to a rotational angle of 430 degrees from theoretical analysis. A torsional capacity of 1680 Nm is observed for the RSRS specimen. This translates to a rotational angle of 210 degrees from theoretical analysis. These results confirm that with more reverse wrapping layers, the CFRP has greater ductility show more gradual failure as opposed to sudden failure. This is an advantage because a gradual failure mode is more desired as failure sign can be observed and rectified become real failure occurs. However, these results are still far beyond the normal rotational angles expected in a structure’s life and further experimental works are required to verify this.

PARAMETRIC STUDIES

There are 3 sets of parameters to be investigated: wrapping angle, number of CFRP wrapping layers and steel section width-to-thickness (D/t) ratio.

Wrapping Angle and Number of layers

To investigate the impact of wrapping angle, the other 2 parameters are set to default: 1 CFRP layer and original section size used in the experiment (50.21x50.21x2.7 mm). From the graph below, it can be seen that 45 degree wrapping angle is the most efficient wrapping angle because the CFRP contribution grows the fastest compare to other wrapping configuration. This reaffirms what was found earlier.

To investigate the impact of wrapping layers, the other 2 parameters are set to default: 45 degree wrapping angle and original section size used in the experiment (50.21x50.21x2.7 mm). Torsion capacity increases with the number of layers. It can be seen that the increase in torsion capacity is marginal with additional layer. This is because an extra layer has a marginal increase in enclose area compared to the previous layer.

Size of specimen

To investigate the impact of size of SHS, the other 2 parameters are set to default: 1 CFRP layer and 45 degree wrapping angle.

Firstly by fixing D at 200mm, we test D/t between 10 and 50. It’s evident that a higher D/t ratio increases the CFRP contribution effectiveness. This is mainly because since D is fixed, increasing D/t will decrease the steel section’s torsion capacity itself. CFRP contribution remains constant in terms of absolute value but appears more effective on a weaker steel section.

Secondly by fixing t = 5 mm, we test D/t between 10 and 50. It’s evident again that a higher D/t ratio increases CFRP contribution effectiveness. Therefore, it can be concluded that higher D/t ratio will generally make CFRP contribution more effective, due to the weaker steel section contribution.
CONCLUSION

Using the theoretical approach presented in this paper, it is evident that CFRP does increase the torsion capacity of steel hollow sections under pure torsion. Reasonable agreement has been achieved between the theoretical studies and experimental results shown in Table 1. The following conclusions can be made:

1. 45 degree wrapping angle is the most efficient wrapping angle in terms of increasing torsion stiffness. Ultimate torsion capacity based on fracture failure criteria is the smallest at 45 degrees wrapping. However the amount of rotation when that happens far exceeds the rotation structures generally experience.
2. At 45 degrees wrapping angle, $G_{xy}$ can be sensitive to different $E_1/E_2$ ratios, up to a maximum of 22%. $E_2$ properties should be estimated with reasonable accuracy to give confidence to predicted results.
3. A combination of failure criteria and other approaches mentioned in the paper can be used to predict the failure torsion load CFRP can sustain.
4. Increasing an extra layer of CFRP has marginal increase in capacity contribution due to bigger enclosed area. CFRP strengthening also proves to be more effective in steel sections with larger $D/t$ ratios.

REFERENCES


