Variable mass structure and the Fundamental Plane

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Abstract. We examine the influence of broken structural homology upon the Fundamental Plane (FP). We fit the Sersic $R^{1/n}$ law, being the generalized $R^{1/4}$ law, where $n$ is a free parameter that accommodates structural differences between different galaxies. The galaxy light profiles show a trend of systematic departures from the de Vaucouleurs $R^{1/4}$ law, such that the larger galaxies have less curvature in their profiles than the $R^{1/4}$ profile and the smaller galaxies have greater curvature, as found by Caon, Capaccioli & D'Onofrio (1993) and Graham et al. (1996). This results in the effective half-light radii, $R_e$, and the mean surface brightness within these radii, $\Sigma_e$, having systematic biases if obtained from the $R^{1/4}$ law. The observed range in structural shapes implies a corresponding range in galaxy dynamics required to support the observed galaxy structure. Allowing for this, we find that broken structural and dynamical homology are partly responsible for the tilt of the FP, changing it from $R \propto \sigma^{1.33\pm0.10} \Sigma^{-0.79\pm0.11}$ to $R \propto \sigma^{1.48\pm0.13} \Sigma^{-1.00\pm0.09}$.

1. Introduction

The virial theorem predicts, under the assumptions of homologous galaxy structure and dynamics, that elliptical galaxies will be constrained to a 2D plane in the logarithmic space of radius, mass, and kinetic energy (Faber et al. 1987). In practice, one uses the observables: effective radius ($R_e$), mean surface brightness within this radius ($\Sigma_e$) and central velocity dispersion ($\sigma_0$) to describe this space. A decade has now past since it was firmly established that elliptical galaxies are indeed well represented by a two dimensional manifold in the larger space of their observable parameters (Brosche & Lentes 1983; Djorgovski 1987a; Lynden-Bell et al. 1988). This manifold/plane has come to be known as the Fundamental Plane (FP) (Djorgovski & Davis 1987; Dressler et al. 1987).

However, the observed FP departs from that predicted by theory. This departure is referred to as the tilt of the FP. The discrepancy between the FP and the virial plane is usually interpreted as being due to a varying mass-to-light ($M/L$) ratio of the ellipticals along the FP. This is typically expressed as

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$M/L \sim L^\alpha$, where $\alpha \sim 0.25 \pm 0.05$ (Faber & Jackson 1976; Faber et al. 1987; Djorgovski, de Carvalho & Han 1988). This dependence has been shown to vary with bandpass (Djorgovski & Santiago 1993; Pahre, Djorgovski & de Carvalho 1995) suggesting that the \textit{tilt} is in part due to stellar composition. Interestingly, a similar relation exists for the spiral galaxies with $\alpha \sim 0.2$ (Persic & Salucci 1990; Persic, Salucci & Stel 1996).

Stellar population differences along the FP (Djorgovski 1987b; Djorgovski & Santiago 1993; Renzini & Ciotti 1993) or a systematic variation in stellar age are unlikely to cause the \textit{tilt} because of the necessary fine-tuning in such explanations. In examining stellar population effects, Pahre et al. (1995) tested for the contribution from the mass-metallicity effect (Guzman, Lucey & Bower 1993) along the FP. By sampling the older distribution of stars, their near-infrared K-band photometry was less sensitive to line-blanketing effects which appear in the optical pass-bands (Faber et al. 1987). The metallicity effect was seen to be insufficient in explaining the entire \textit{tilt} of the FP, in concordance with previous studies by Dressler et al. (1987), Recillas-Cruz et al. (1990), and Djorgovski & Santiago (1993). Pahre et al. (1995) went on to suggest that systematic departures from structural and dynamical homology may be the cause of the \textit{tilt}.

Caon et al. (1993) and Graham et al. (1996), and references therein, have collectively demonstrated that a real range in galaxy profile shapes do exist, Again, in this study, it is shown that smaller galaxies have greater curvature in their radial light distribution than what the larger galaxies do. The idea that structural homology might explain the departure of the FP from the virial plane has been entertained before (Djorgovski et al. 1988; Djorgovski & Santiago 1993) and has started to be examined in more depth by other groups like Ciotti, Lanzoni & Renzini (1996) and Hjorth & Madsen (1995).

With quality CCD data, we explore the possibility that the \textit{tilt} of the FP may be caused by a range of galaxy structures (profile shapes) that have been largely ignored in the past. In this work we allow for observed differences in galaxy structure, as represented by the $R^{1/n}$ law, and we construct an \textit{improved} FP allowing one to better address the \textit{tilt} of the FP and hence the importance of dark matter in the formation and evolution of elliptical galaxies.

### 2. Galaxy Parameters

The $R^{1/n}$ light profile, being the generalization of the de Vaucouleurs $R^{1/4}$ law (de Vaucouleurs 1948, 1953), where $n$ is a free parameter, was introduced by Sersic (1968) and re-vitalized by Capaccioli (1989). It allows for structural differences amongst galaxies, with it’s circular aperture luminosity form given as a function of radius by

$$L(R) = 2\pi n \frac{e^b}{b^{2n}} \gamma[2n, b(R/R_e)^{1/n}] I_e R_e^2,$$

where $I_e$ is the intensity at the radius $R_e$ and $\gamma$ is the incomplete gamma function. The parameter $b$ is chosen so that $R_e$ is the projected radius enclosing half of the total light from the galaxy and is well approximated by $b = 2.0n - 0.33$, for $n > 1$. While the $R^{1/4}$ law has two parameters, one which scales the radius.
Figure 1. The top diagram shows the shape parameter $n$, from the $R^{1/n}$ models, plotted against the spatial half-mass radius for the 26 E/S0 Virgo galaxies from Bower, Lucey, & Ellis (1992). The bottom diagram shows the total galaxy mass plotted against the shape parameter $n$. 
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Figure 2. The spatial ($\sigma(r)$, solid), line-of-sight ($\sigma_{ls}(R)$, dashed), and aperture ($\sigma_{ap}(R)$, dotted) velocity dispersion profiles are shown for a range of shape parameters $n$ for the $R^{1/n}$ luminosity profile. The abscissa is such that $s = r/R_e$ and $\eta = R/R_e$.

and the other the surface brightness, the $R^{1/n}$ profile has the additional parameter, $n$, which describes different shapes for the galaxy light profiles. The Sersic law can take the form of an exponential disk ($n=1$) and the de Vaucouleurs light profile ($n=4$), it can also approximate a power-law (large $n$) and provides intermediate forms between these common profile shapes by varying its shape parameter $n$.

The aperture magnitude profiles of 26 E/S0 Virgo cluster galaxies (Bower et al. 1992) were used in this study. Published dynamical data was taken from Dressler (1984,1987) and McElroy (1995).

The shape parameter is seen to have a physical association with the galaxies' other properties, such as size and mass, as shown in Figure 1. The bigger galaxies having less curvature in their light profiles and being described by a larger value of $n$. Not allowing for such variations in profile shape can have a substantial systematic effect on the derived half-light radius and associated surface brightness term.
In applying the virial theorem to a test particle in orbit at the spatial half-mass radius of the galaxy (equal to the spatial half-light radius for a constant M/L ratio with in each galaxy), it is the spatial quantities, not the projected quantities, that one should be using. It is possible to obtain these spatial (three dimensional) structural quantities by deprojecting the \( R^{1/n} \) light profiles (Ciotti 1991). This was done for \( R_e \) and \( \Sigma_e = -2.5 \log(I_e) \) giving the spatial half-light radius and the associated luminosity term within this radius. It was found that both of these spatial quantities are approximately equal to a constant value times their corresponding projected quantities, and thus this does not effect the tilt of the FP, being constructed in logarithmic space. The volumetric kinematical quantity, being the spatial half-light velocity dispersion term, can be derived from application of the Jeans hydrodynamical equation to the \( R^{1/n} \) model, calibrated by the projected central velocity dispersion \( \sigma_0 \) (Graham & Colless 1996). A range of velocity dispersion profiles are displayed in Figure 2 for different \( n \). showing the line-of-sight velocity dispersion as a function of the projected radius \( R \), the aperture velocity dispersion as a function of the projected radius \( R \), and the spatial velocity dispersion as a function of the spatial radius \( r \).

3. The Fundamental Plane

We used the computer code from Murtagh & Heck (1987) to perform a Principal Component Analysis (PCA) on both data sets \(( \log \sigma_0, \log R_{e,4}, \Sigma_{e,4} \) and \(( \log \sigma_{e,n}, \log R_{e,n}, \Sigma_{e,n} \). Both data sets were found to be well represented by a 2D plane in their respective 3-space of observables, with more than 98 per cent of the variance being in the plane, thus confirming the appropriate use of a plane to describe the location of the elliptical galaxies in this space.

Our objective was to construct a FP for comparison with what theory predicts. In doing this, all variables had to be treated equally; there were no dependent/independent variables. Our preferred method of construction is the bisector method of linear regression, giving a greater certainty for the slope of the fitted plane than other methods, and also having a symmetrical treatment of all the variables. (An enlightening and thorough treatment of the two dimensional case is presented in Isobe et al. (1990) and Feigelson & Babu (1992).)

We constructed the standard FP, obtained by fitting \( R^{1/4} \) profiles, using the effective projected half-light radius \( R_{e,4} \), the mean surface brightness within this radius, \( \Sigma_{e,4} \), and the measured aperture velocity dispersion \( \sigma_0 \). We then constructed an improved FP, using the spatial (volumetric) parameters from the \( R^{1/n} \) model and the spatial half-light velocity dispersion term \( \sigma_{e,n} \), as detailed in Graham & Colless (1996). Describing the FP relation as

\[
\log R(kpc) = A \log \sigma - 0.4B\Sigma + C,
\]

the virial theorem predicts that \( A \) should be 2 and \( B \) should be -1. The bisector method, produced

\[
R \propto \sigma^{1.33\pm0.10} \Sigma^{-0.79\pm0.11}
\]

for the standard FP, and

\[
R \propto \sigma^{1.48\pm0.13} \Sigma^{-1.00\pm0.09}
\]

for our improved FP. Both planes are shown in Figure 3.
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4. Discussion/Implications

If galaxy structure is dependent on galaxy environment, then the FP may also be dependent on the galaxies environment. In fitting an $R^{1/n}$ profile, we take into account different galaxy structures, and thus possibly different galaxy environments. The new FP that we derive may therefore implicitly take into consideration possible environmental effects. It will be of interest to construct FP's for other galaxy clusters to explore this possibility.

The photometry used in this study, taken from Bower et al. (1992), consisted of optical data, leaving us (and most studies to date) somewhat contaminated by metallicity effects. This is not such a problem for infrared studies, but it will slightly effect our exponents, derived from V-band data (Pahre et al. 1995). Despite this, because we have constructed the FP using both $R^{1/4}$ and $R^{1/n}$ profiles, the change in the observed FP due to structural effects will still be apparent in our analysis.

The exponent for the surface brightness term in our expression for the improved FP is an exact match with the virial theorem prediction of -1. Thus, systematic departures from structural and dynamical homology and the use of spatial rather than projected kinematical quantities are able to explain past departures between observation and theory for this exponent. The exponent on the $\sigma$ term is also closer to the value expected from the virial theorem, changing from 1.33 to 1.48 as we allow for such changes.

Figure 3. a) The FP constructed using an $R^{1/4}$ law to derive the half-light radius (R) and mean surface brightness ($\mu$); and using the central projected aperture velocity dispersion measure for $\sigma$. b) Constructed with the parameters from an $R^{1/n}$ law and the spatial half-mass velocity dispersion term.
The deviation of the observed FP from the virial plane is often attributed to the mass-to-light ratio of the galaxies varying with galaxy mass or luminosity. Assuming the mass-to-light ratio is a power law function of the observables, it can then be easily computed from the slope of the FP. For the plane constructed using the $R^{1/4}$ model and aperture velocity dispersions, we have that $M/L \sim L^{0.25} \Sigma^{-0.06}$, whereas for the plane that allowed for broken structural homology and used the volumetric quantities, we find that $M/L \sim L^{0.17} \Sigma^{0.17}$, or $M/L \sim \sigma^{1/2}$.

It seems probable that the tilt of the FP may actually be due to a combination of influences. For example, Pahre et al. (1995) measured a change in $\alpha$, with $M/L \sim M^\alpha$, from 0.23 (Lucey et al. 1991; de Carvalho & Djorgovski 1992) in the optical (V-band) to $\alpha=0.16$ in their near-infrared K-band study. They found that the exponent on the velocity dispersion term, $\alpha$, increased by $0.19 \pm 0.06$ dex from the V-band to the K-band. Our result is expected to change by a similar amount when one uses near-infrared data; thus further reducing the departure between the observed and the virial plane and once more changing the assumed mass-to-light ratio dependence along the FP.

While we have better treated the mass structure of galaxies, showing they exhibit a range of shapes, as described by the Sersic model, and we have dealt with the associated range in dynamical structures, there are still other factors that influence the tilt of the FP. For instance, stellar populations may vary with galactic radius, resulting in $M/L$ varying with radius, or worse still, light may not trace mass in a simple fashion. Furthermore, until we have greater kinematical information on the galaxies (i.e., velocity dispersion profiles and rotation curves) we cannot fully address the problem of broken dynamical homology and contributions from rotational support of the elliptical galaxies. These effects may work in symmetry to account for the FP tilt, and they should all be addressed before one can use the FP as an accurate probe of galaxy formation and evolution.

Acknowledgments. We thank Richard Bower, John Lucey, and Richard Ellis for kindly making their galaxy profile data available to us in electronic form. We are grateful for discussions with Luca Ciotti, George Djorgovski, Reinaldo de Carvalho and Mike Pahre that helped to improve this work. We wish to thank Eric Feigelson and Michael Akritas at the Statistical Consulting Centre for Astronomy at Pennsylvania State University, for their advice on the construction of the Fundamental Plane. We are grateful for the use of Eric Feigelson's computer code SLOPES.

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